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EE24BTECH11035 - KOTHAPALLI AKHIL

1) Let $f(z) = 2z^2 - 1$. Then the maximum value of $|f(z)|$ on the unit disc $D = \{z \in \mathbb{C} : |z| \leq 1\}$ equals
   a) 1
   b) 2
   c) 3
   d) 4
2) Let
$$f(z) = \frac{1}{z^2 - 3z + 2}. \tag{1}$$
   Then the coefficient of $\frac{1}{z^2}$ in the Laurent series expansion of $f(z)$ for $|z| > 2$ is
   a) 0
   b) 1
   c) 3
   d) 5
3) Let $f : \mathbb{C} \to \mathbb{C}$ be an arbitrary analytic function satisfying $f(0) = 0$ and $f(1) = 2$. Then
   a) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| > n$
   b) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| \leq n$
   c) there exists a bounded sequence $\{z_n\}$ such that $|f(z_n)| > n$
   d) there exists a sequence $\{z_n\}$ such that $z_n \to 0$ and $f(z_n) \to 2$
4) Define $f : \mathbb{C} \to \mathbb{C}$ by
$$f(z) = \begin{cases} 0, & \text{if } \mathrm{Re}(z) = 0 \text{ or } \mathrm{Im}(z) = 0, \\ z, & \text{otherwise.} \end{cases} \tag{2}$$
   Then the set of points where f is analytic is
   a) $\{z : \mathrm{Re}(z) \neq 0 \text{ and } \mathrm{Im}(z) \neq 0\}$
   b) $\{z : \mathrm{Re}(z) \neq 0\}$
   c) $\{z : \mathrm{Re}(z) \neq 0 \text{ or } \mathrm{Im}(z) \neq 0\}$
   d) $\{z : \mathrm{Im}(z) \neq 0\}$
5) Let $U(n)$ be the set of all positive integers less than n and relatively prime to n. Then $U(n)$ is a group under multiplication modulo n. For $n = 248$, the number of elements in $U(n)$ is
   a) 60
   b) 120
   c) 180
   d) 240

6) Let $\mathbb{R}[x]$ be the polynomial ring in x with real coefficients and let $I = (x^2 + 1)$ be the ideal generated by the polynomial $x^2 + 1$ in $\mathbb{R}[x]$. Then
   a) I is a maximal ideal
   b) I is a prime ideal but NOT a maximal ideal
   c) I is NOT a prime ideal
   d) $\mathbb{R}[x]/I$ has zero divisors
7) Consider $\mathbb{Z}_5$ and $\mathbb{Z}_{20}$ as rings modulo 5 and 20, respectively. Then the number of homomorphisms $\phi : \mathbb{Z}_5 \to \mathbb{Z}_{20}$ is
   a) 1
   b) 2
   c) 4
   d) 5
8) Let $\mathbb{Q}$ be the field of rational numbers and consider $\mathbb{Z}_2$ as a field modulo 2. Let
$$f(x) = x^3 - 9x^2 + 9x + 3. \tag{3}$$
   Then $f(x)$ is
   a) irreducible over $\mathbb{Q}$ but reducible over $\mathbb{Z}_2$
   b) irreducible over both $\mathbb{Q}$ and $\mathbb{Z}_2$
   c) reducible over $\mathbb{Q}$ but irreducible over $\mathbb{Z}_2$
   d) reducible over both $\mathbb{Q}$ and $\mathbb{Z}_2$
9) Consider $\mathbb{Z}_5$ as a field modulo 5 and let
$$f(x) = x^5 + 4x^4 + 4x^3 + 4x^2 + x + 1 \tag{4}$$
   Then the zeros of $f(x)$ over $\mathbb{Z}_5$ are 1 and 3 with respective multiplicity.
   a) 1 and 4
   b) 2 and 3
   c) 2 and 2
   d) 1 and 2
10) Consider the Hilbert space $\ell^2 = \left\{x = \{x_n\}; x_n \in \mathbb{R}, \sum_{n=1}^{\infty} x_n^2 < \infty\right\}$. Let
$$E = \left\{x = \{x_n\}; |x_n| \leq \frac{1}{n} \text{ for all } n\right\} \tag{5}$$
   be a subset of ℓ^2. Then:
   a) $E^\circ = \left\{x : |x_n| < \frac{1}{n} \text{ for all } n\right\}$
   b) $E^\circ = E$
   c) $E^\circ = \left\{x : |x_n| < \frac{1}{n} \text{ for all but finitely many } n\right\}$
   d) $E^\circ = \emptyset$
11) Let X and Y be normed linear spaces and let $T : X \to Y$ be a linear map. Then T is continuous if
   a) Y is finite dimensional
   b) X is finite dimensional
   c) T is one to one

d) T is onto

12) Let X be a normed linear space and let $E_1, E_2 \subseteq X$. Define

$$E_1 + E_2 = \{x + y : x \in E_1, y \in E_2\}. \tag{6}$$

Then $E_1 + E_2$ is

a) open if E_1 or E_2 is open
b) NOT open unless both E_1 and E_2 are open
c) closed if E_1 or E_2 is closed
d) closed if both E_1 and E_2 are closed

13) For each $a \in \mathbb{R}$, consider the linear programming problem

$$\text{Max. } z = x_1 + 2x_2 + 3x_3 + 4x_4, \tag{7}$$

subject to

$$ax_1 + 2x_3 \leq 1, \quad x_1 + ax_3 + 3x_4 \leq 2, \quad x_1, x_2, x_3, x_4 \geq 0. \tag{8}$$

Let $S = \{a \in \mathbb{R} : \text{ the given LP problem has a basic feasible solution}\}$. Then:

a) $S = \emptyset$
b) $S = \mathbb{R}$
c) $S = (0, \infty)$
d) $S = (-\infty, 0)$

14) Consider the linear programming problem

$$\text{Max. } z = x_1 + 5x_2 + 3x_3, \tag{9}$$

subject to

$$2x_1 - 3x_2 + 5x_3 \leq 3, \quad 3x_1 + 2x_3 \leq 5, \quad x_1, x_2, x_3 \geq 0. \tag{10}$$

Then the dual of this LP problem:

a) has a feasible solution but does NOT have a basic feasible solution
b) has a basic feasible solution
c) has infinite number of feasible solutions
d) has no feasible solution

15) Consider a transportation problem with two warehouses and two markets. The warehouse capacities are $a_1 = 2$ and $a_2 = 4$, and the market demands are $b_1 = 3$ and $b_2 = 3$. Let x_{ij} be the quantity shipped from warehouse i to market j and c_{ij} be the corresponding unit cost. Suppose that $c_{11} = 1, c_{12} = 1$ and $c_{22} = 2$. Then $(x_{11}, x_{12}, x_{21}, x_{22}) = (2, 0, 1, 3)$ is optimal for every

a) $c_{12} \in [2, 3]$
b) $c_{12} = [0, 3]$
c) $c_{12} \in [1, 3]$
d) $c_{12} \in [2, 4]$

16) The smallest degree of the polynomial that interpolates the data

x	-2	-1	0	1	2	3
$f(x)$	-58	-21	-12	-13	-6	27

is:

(A) 3
(B) 4
(C) 5
(D) 6

17) Suppose that x_0 is sufficiently close to 3. Which of the following iterations $x_{n+1} = g(x_n)$ will converge to the fixed point $x = 3$?

a)
$$x_{n+1} = -16 + 6x_n + \frac{3}{x_n} \tag{11}$$

b)
$$x_{n+1} = \sqrt{3 + 2x_n} \tag{12}$$

c)
$$x_{n+1} = \frac{3}{x_n - 2} \tag{13}$$

d)
$$x_{n+1} = \frac{x_n^2 - 3}{2} \tag{14}$$