Eigen value calculation

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In this report, we will know about Eigenvalues, Various algorithms for finding eigenvalues of matrices, Comparison between different algorithms and their complexities, The code structure for finding eigenvalues.

1 Introduction

Eigen values plays a vital role in understanding the behaviour of Linear Transformations . These are fundamental in many scientific and Engineering fields.

2 Eigen values and Eigen vectors

2.1 Overview of Eigenvalue

The eigenvalues of a matrix $A \in \mathbf{C}^{n\times n}$ are the n roots of its characteristic polynomial $p(z) = det(zI - A)$. The set of these roots is called the spectrum and is denoted by $\lambda(A)$. If $\lambda(A) = \lambda_1, \dots, \lambda_n$, then it follows that

$$\det(A) = \lambda_1\lambda_2\dots\lambda_n$$

Moreover, Trace of A is sum of eigen values, i.e,

$$tr(A) = \lambda_1 + \lambda_2 + \lambda_3 \dots + \lambda_n$$

2.2 Overview of Eigenvector

If $\lambda \in \lambda(A)$, then the nonzero vectors $x \in \mathbf{C}^n$ that satisfy

$$Ax = \lambda x$$

are referred to as eigenvectors. More precisely, x is a right eigenvector for λ if $Ax = \lambda x$ and a left eigenvector if $x^H A = \lambda x^H$. Unless ostherwise stated, "eigenvector" means "right eigenvector".

3 Different Methods of finding eigen values

3.1 Methods

Method	Large Matrices	Non-Symmetric	Sparse	Complexity
Lanczos	✓	×	✓	$O(nk)$
Arnoldi	✓	✓	✓	$O(nk)$
Jacobi	×	×	✓	$O(n^3)$
QR Algorithm	✓	✓	✓	$O(n^3)$

3.2 Comparison

Method	Applicability	Accuracy	Convergence Rate
Jacobi Method	$Symmetricmatrices$	*High (small values)*	$Moderate$
QR Algorithm	$Generalmatrices$	*Very High*	$Fast$
Lanczos Method	$Largesparsesymmetricmatrices$	*Moderate*	$Fastforlargen.$
Arnoldi Method	$Generallargematrices$	*High*	$Fast$

4 Jacobi Method Overview

- Start with a symmetric matrix. check whether the matrix is diagonal or not. If it is diagonal matrix , the elements of principle diagonal are Eigen values of the matrix.
- Find the Largest off-diagonal element A_{pq} of the given Non-diagonal symmetric matrix.
- compute Rotation Angle θ i.e.,

$$\theta = \frac{1}{2} \arctan \frac{2A_{pq}}{A_{pp} - A_{qq}}$$

- construct a matrix J , which makes rotation in p-q plane.
- Update the given matrix A as $A' = J^T AJ$ till we get a diagonal matrix.

4.1 Reason for Choosing Jacobi's Method

- Accuracy, Simplicity.
- works well for small to medium sized elements.
- It can sometimes compute tiny eigenvalues and their eigenvectors with much higher accuracy than other methods.
- Easy to code in C language.

5 Implementation

5.1 Code Structure

- Contains the functions: matrix multiplications , matrix transposing, checking whether the matrix is diagonal or not,e.t.c.
- In main function, There is a while loop ,For iteration of matrices till the matrix becomes diagonal matrix.

6 Conclusion

This report mainly concentrates on Calculation of Eigen values for a Symmetric Matrix using Jacobi's method. Which is an iterative way of rotating a matrix and making the matrix diagonal. Since, jacobi's iterative method is simple and gives high precision for small values it is used more widely for symmetric matrices.

7 References

Matrix computations,Book by Gene H. Golub
Linear Algebra and its applications ,Gilbert strang