# Question-1-1.11-11

EE24BTECH11035 - AKHIL

# Question

The scalar product of the vector $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vector $\mathbf{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\mathbf{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of $\lambda$ and hence find the unit vector along $\mathbf{b} + \mathbf{c}$.

# Inputs

| Point | Coordinates |
|---|---|
| **A** | $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ |
| **B** | $\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$ |

# Solution

Given vector **c** is:

$$\mathbf{c} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \tag{1}$$

The sum of vectors **b** and **c** is:

$$\mathbf{b} + \mathbf{c} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} + \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} 2+\lambda \\ 4+2 \\ -5+3 \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} \tag{4}$$

# Solution

Now, we find the norm of $\mathbf{b}+\mathbf{c}$. The norm is given by:

$$\|\mathbf{b}+\mathbf{c}\| = \sqrt{(\mathbf{b}+\mathbf{c})^\top(\mathbf{b}+\mathbf{c})} \tag{5}$$
$$= \sqrt{(2+\lambda)^2+6^2+(-2)^2} \tag{6}$$
$$= \sqrt{(2+\lambda)^2+36+4} \tag{7}$$
$$= \sqrt{(2+\lambda)^2+40} \tag{8}$$

The unit vector along $\mathbf{b}+\mathbf{c}$ is:

$$\hat{\mathbf{u}} = \frac{\mathbf{b}+\mathbf{c}}{\|\mathbf{b}+\mathbf{c}\|} \tag{9}$$

# Solution

Given,The scalar product of **a** with this unit vector as 1:

$$\mathbf{a}^\top \hat{\mathbf{u}} = 1 \tag{10}$$

Substituting the expressions for **a** and $\hat{\mathbf{u}}$, we get:

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{(2+\lambda)^2 + 40}} \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} = 1 \tag{11}$$

$$\frac{1}{\sqrt{(2+\lambda)^2 + 40}} \left((2+\lambda) + 6 - 2\right) = 1 \tag{12}$$

$$\frac{1}{\sqrt{(2+\lambda)^2 + 40}} (\lambda + 6) = 1 \tag{13}$$

# Solution

Squaring both sides:

$$\frac{(\lambda+6)^2}{(2+\lambda)^2+40} = 1 \tag{14}$$

$$(\lambda+6)^2 = (2+\lambda)^2 + 40 \tag{15}$$

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 4 + 40 \tag{16}$$

# solution

Simplifying:

$$12\lambda + 36 = 4\lambda + 44 \tag{17}$$
$$8\lambda = 8 \tag{18}$$
$$\lambda = 1 \tag{19}$$

Thus, $\lambda = 1$. Substituting $\lambda = 1$ into $\mathbf{b} + \mathbf{c}$:

$$\mathbf{b} + \mathbf{c} = \begin{pmatrix} 2+1 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} \tag{20}$$

The norm of $\mathbf{b} + \mathbf{c}$ is:

$$\|\mathbf{b} + \mathbf{c}\| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \tag{21}$$

Thus, the unit vector along $\mathbf{b} + \mathbf{c}$ is:

$$\hat{\mathbf{u}} = \frac{1}{7}\begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} \frac{3}{7} \\ \frac{6}{7} \\ \frac{-2}{7} \end{pmatrix} \tag{22}$$

# Diagram

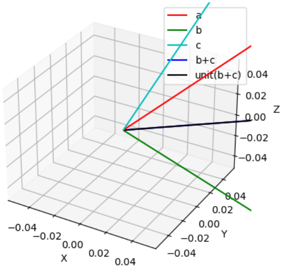


Figure: $\mathbf{a}, \mathbf{b}, \mathbf{c}$,unit vector in the direction of $\mathbf{b} + \mathbf{c}$

# Python code for graph

```
import numpy as np
import matplotlib.pyplot as plt

a = np.array([1, 1, 1])
b = np.array([2, 4, -5])
c = np.array([1, 2, 3]) # lambda = 1

b_plus_c = b + c

magnitude_b_plus_c = np.linalg.norm(b_plus_c)
unit_vector_b_plus_c = b_plus_c / magnitude_b_plus_c
```

# Python code for graph

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

ax.quiver(0, 0, 0, a[0], a[1], a[2], color='r', label='a')

ax.quiver(0, 0, 0, b[0], b[1], b[2], color='g', label='b')

ax.quiver(0, 0, 0, c[0], c[1], c[2], color='c', label='c')

ax.quiver(0, 0, 0, b_plus_c[0], b_plus_c[1], b_plus_c[2], color='
    b', label='b+c')
```

# Python code for graph

```
# Plot the unit vector along b + c (black)
ax.quiver(0, 0, 0, unit_vector_b_plus_c[0], unit_vector_b_plus_c
    [1], unit_vector_b_plus_c[2], color='k', label='unit(b+c)')

# Set labels
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
# Show legend
ax.legend()
# Show plot
plt.show()
```

# C code for solution

```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "libs/matfun.h"
void printMatToFile(double **p, int m, int n, FILE *fp) {
    for (int i = 0; i < m; i++) {
        for (int j = 0; j < n; j++) {
            fprintf(fp, "%lf ", p[i][j]);
        }
        fprintf(fp, "\n");
    }
}
```

# C code for solution

```c
int main() {
 double lambda;
 double **a, **b, **c, **sum, **unitVec;
 double scalarProduct;
 FILE *outputFile = fopen("output.dat", "w");
 if (outputFile == NULL) {
     printf("Error opening file!\n");
     return 1;
 }
 a = createMat(3, 1);
 a[0][0] = 1;
 a[1][0] = 1;
 a[2][0] = 1;
b = createMat(3, 1);
 b[0][0] = 2;
 b[1][0] = 4;
 b[2][0] = -5;
```

# C code for solution

```
c = createMat(3, 1);
   for (lambda = -100; lambda <= 100; lambda += 0.01) {
       c[0][0] = lambda;
       c[1][0] = 2;
       c[2][0] = 3;
       sum = Matadd(b, c, 3, 1);
       unitVec = Matscale(sum, 3, 1, 1 / Matnorm(sum, 3));
      scalarProduct = Matdot(a, unitVec, 3);
       if (fabs(scalarProduct - 1.0) < 1e-6) {
           // Write the results to the output file
           fprintf(outputFile, "Found lambda = %lf\n", lambda);
           fprintf(outputFile, "Unit vector along b + c:\n");
           printMatToFile(unitVec, 3, 1, outputFile);
           break;
       }
   }
```

# C code for solution

```
        free(sum);
        free(unitVec);
    }

    free(a);
    free(b);
    free(c);
    fclose(outputFile);

    return 0;
}
```