27-July-2021 Shift-1

EE24BTECH11035 - KOTHAPALLI AKHIL

1) If the mean and variance of the following data: $6, 10, 7, 13, a, 12, b, 12$ are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to:
   a) 24
   b) 12
   c) 32
   d) 16
2) The value of
$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{(2j-1)+8n}{(2j-1)+4n} \tag{2.1}$$
   is equal to:
   a) $5 + \log_e\left(\frac{3}{2}\right)$
   b) $2 - \log_e\left(\frac{2}{3}\right)$
   c) $3 + 2\log_e\left(\frac{2}{3}\right)$
   d) $1 + 2\log_e\left(\frac{3}{2}\right)$
3) Let $\mathbf{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\mathbf{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product
$$(\mathbf{a}+\mathbf{b}) \times ((\mathbf{a} \times (\mathbf{a}-\mathbf{b}) \times \mathbf{b}) \times \mathbf{b}) \tag{3.1}$$
   is equal to:
   a) $5(34\hat{i} - 5\hat{j} + 3\hat{k})$
   b) $7(34\hat{i} - 5\hat{j} + 3\hat{k})$
   c) $7(30\hat{i} - 5\hat{j} + 7\hat{k})$
   d) $5(30\hat{i} - 5\hat{j} + 7\hat{k})$
4) The value of the definite integral
$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{\cos^3 x})(\sin^4 x + \cos^4 x)} \tag{4.1}$$
   is equal to:
   a) $-\frac{\pi}{2}$
   b) $\frac{\pi}{2\sqrt{2}}$
   c) $-\frac{\pi}{4}$
   d) $\frac{\pi}{\sqrt{2}}$
5) Let C be the set of all complex numbers. Let $S_1 = \{z \in \mathbb{C} \mid |z-3-2i| = 8\}$, $S_2 = \{z \in \mathbb{C} \mid \mathrm{Re}(z) \geq 5\}$, $S_3 = \{z \in \mathbb{C} \mid |z-\bar{z}| \geq 8\}$ Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to:
   a) 1

b) 0
c) 2
d) Infinite

6) If the area of the bounded region $R = \{(x, y) : \max\{0, \log_2 x\} \le y \le 2^x, \frac{1}{2} \le x \le 2\}$ is $\alpha(\log_2 2)^{-1} + \beta(\log_2 2) + \gamma$, then the value of $(\alpha + \beta - 2\gamma)^2$ is equal to:
   a) 8
   b) 2
   c) 4
   d) 1

7) A ray of light through $(2, 1)$ is reflected at a point P on the y-axis and then passes through the point $(5, 3)$. If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be:
   a) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$
   b) $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$
   c) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$
   d) $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

8) If the coefficients of x^7 in $\left(x^2 + \frac{1}{bx^2}\right)^{11}$ and x^{-7} in $\left(x - \frac{1}{bx^2}\right)^{11}$, $\quad b \ne 0$ are equal, then the value of b is equal to:
   a) 2
   b) -1
   c) 1
   d) -2

9) The compound statement $(P \vee Q) \wedge (\sim P) \implies Q$ is equivalent to:
   a) $P \vee Q$
   b) $P \wedge \sim Q$
   c) $\sim (P \implies Q)$
   d) $\sim (P \implies Q) \Leftrightarrow P \wedge \sim Q$

10) If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin 2\theta + \cos 4\theta + \sin 6\theta)$ is equal to:
   a) 23
   b) -27
   c) -23
   d) 27

11) Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$ If $A^{-1} = \alpha I + \beta A, \quad \alpha, \beta \in \mathbb{R}$. If I is a 2×2 identity matrix, then $4(\alpha - \beta)$ is equal to:
   a) 5
   b) $\frac{8}{3}$
   c) 2
   d) 4

12) Let $f : \left(-\frac{\pi}{4}, \frac{4}{4}\right) \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} (1 + |\sin x|) \cdot \frac{3}{\pi} & , \quad -\frac{\pi}{4} < x < 0 \\ b & , \quad x = 0 \\ e^{\cot(4/x \cdot 2x)} & , \quad 0 < x < \frac{\pi}{4} \end{cases} \tag{12.1}$$

If f is continuous at $x = 0$, then the value of $6a + b^2$ is equal to:

a) $1 - e$
b) $2e - 1$
c) $1 + e$
d) $4e$

13) Let $y = y(x)$ be the solution of the differential equation

$$\log_e \left(\frac{dy}{dx}\right) = 3x + 4y, \quad y(0) = 0. \tag{13.1}$$

If $y\left(-\frac{2}{3}\log_2 2\right) = \alpha \log_2 2$, then the value of α is equal to:

a) $-\frac{1}{4}$
b) $\frac{1}{4}$
c) 2
d) $-\frac{1}{2}$

14) Let the plane passing through the point $(-1, 0, -2)$ and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of $a + b + c$ is equal to:

a) 3
b) 8
c) 5
d) 4

15) Two tangents are drawn from the point $P(-1, 1)$ to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that the lengths of the segments AB and AD are equal, then the area of the triangle ABD is equal to:

a) 2
b) $2\sqrt{3} + 2$
c) 4
d) $3\sqrt{2} - 1$