Question-9-9.3-25

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Question:
Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0, 2x + 3y - 21 = 0$and $x - 5y + 9 = 0$.
Solution:
The general form of line equation in matrix form is $h^T + m = 0$
Writing the Given lines in the form of matrices ,

Line Equation	**h value**	**m**
$3x - 2y + 1 = 0$	$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$	1
$2x + 3y - 21 = 0$	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	-21
$x - 5y + 9 = 0$	$\begin{pmatrix} 1 \\ -5 \end{pmatrix}$	9

TABLE 0: Lines and their parameters

Solving The Line equations to get points of intersections ,
By Solving First two equations, We get point of intersection as ,

$$P_1 = \begin{pmatrix} \frac{36}{13} \\ \frac{121}{26} \end{pmatrix} \tag{1}$$

Similarly,

$$P_2 = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, P_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2}$$

Equation for the Line Between P_1 and P_2:

$$m_1 = \frac{3 - \frac{121}{26}}{6 - \frac{36}{13}} = -\frac{43}{52} \tag{3}$$

$$y - 3 = -\frac{43}{52}(x - 6) \tag{4}$$

Equation for the Line Between P_2 and P_3:

$$m_2 = \frac{2 - 3}{1 - 6} = \frac{1}{5} \tag{5}$$

$$y - 3 = \frac{1}{5}(x - 6) \tag{6}$$

Equation for the Line Between P_3 and P_1:

$$m_3 = \frac{\frac{121}{26} - 2}{\frac{36}{13} - 1} = \frac{69}{52} \tag{7}$$

$$y - 2 = \frac{69}{52}(x - 1) \tag{8}$$

We now calculate the area using the definite integral:

$$A = \int_{x_1}^{x_2} (f(x) - g(x))\, dx \tag{9}$$

$$A = \int_1^6 \left(\left(-\frac{43}{52}(x-6) + 3\right) - \left(\frac{1}{5}(x-6) + 3\right)\right) dx \tag{10}$$

$$A = \int_1^6 \left(-\frac{43}{52}x + \frac{43 \cdot 6}{52} - \frac{1}{5}x + \frac{6}{5}\right) dx \tag{11}$$

$$A = \left[-\frac{43}{104}x^2 + \frac{43 \cdot 6}{52}x - \frac{1}{10}x^2 + \frac{6}{5}x\right]_1^6 \tag{12}$$

Substituting the limits,

$$A = 6.5\,\text{square units} \tag{13}$$

Thus, the area of the region bounded by the lines is 6.5 square units.

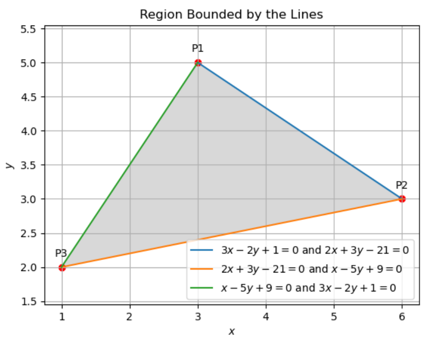

Fig. 0: Area enclosed between the 3 Lines