Eigen value calculation

Akhil kothapalli

November 18, 2024

In this report, we will know about Eigenvalues, Various algorithms for finding eigenvalues of matrices, Comparison between different algorithms and their complexities, The code structure for finding eigenvalues.

1 Introduction

Eigen values plays a vital role in understanding the behaviour of Linear Transformations . These are fundamental in many scientific and Engineering fields.

2 Eigen values and Eigen vectors

2.1 Overview of Eigenvalue

The eigenvalues of a matrix $A \in \mathbf{C}^{n \times n}$ are the n roots of its characteristic polynomial $p(z) = det(zI - A)$. The set of these roots is called the spectrum and is denoted by $\lambda(A)$. If $\lambda(A) = \lambda_1, \ldots, \lambda_n$, then it follows that

$$\det(A) = \lambda_1 \lambda_2 \ldots \lambda_n$$

Moreover, Trace of A is sum of eigen values, i.e,

$$tr(A) = \lambda_1 + \lambda_2 + \lambda_3 \cdots + \lambda_n$$

2.2 Overview of Eigenvector

If $\lambda \in \lambda(A)$, then the nonzero vectors $x \in \mathbf{C}^n$ that satisfy

$$Ax = \lambda x$$

are referred to as eigenvectors. More precisely, x is a right eigenvector for λ if $Ax = \lambda x$ and a left eigenvector if $x^H A = \lambda x^H$. Unless ostherwise stated, "eigenvector" means "right eigenvector".

3 Different Methods of finding eigen values

3.1 Methods

Method	Large Matrices	Non-Symmetric	Sparse	Complexity
Lanczos	✓	×	✓	$O(nk)$
Arnoldi	✓	✓	✓	$O(nk)$
Jacobi	×	×	✓	$O(n^3)$
QR Algorithm	✓	✓	✓	$O(n^3)$

3.2 Comparison

Method	Applicability	Accuracy	Convergence Rate
Jacobi Method	*Symmetric matrices*	*High (small values)*	*Moderate*
QR Algorithm	*General matrices*	*Very High*	*Fast*
Lanczos Method	*Large sparse symmetric matrices*	*Moderate*	*Fast for large n.*
Arnoldi Method	*General large matrices*	*High*	*Fast*

4 Jacobi Method Overview

- Start with a symmetric matrix. check whether the matrix is diagonal or not. If it is diagonal matrix , the elements of principle diagonal are Eigen values of the matrix.

- Find the Largest off-diagonal element A_{pq} of the given Non-diagonal symmetric matrix.

- compute Rotation Angle θ i.e.,

$$\theta = \frac{1}{2} tan^{-1} \frac{2A_{pq}}{A_{pp} - A_{qq}}$$

- construct a matrix J , which makes rotation in p-q plane.

- Update the given matrix A as $A' = J^T AJ$ till we get a diagonal matrix.

4.1 Reason for Choosing Jacobi's Method

- Accuracy, Simplicity.

- works well for small to medium sized elements.

- It can sometimes compute tiny eigenvalues and their eigenvectors with much higher accuracy than other methods.

- Easy to code in C language.

5 Implementation

5.1 Code Structure

- Contains the functions: matrix multiplications , matrix transposing, checking whether the matrix is diagonal or not,e.t.c.

- In main function, There is a while loop ,For iteration of matrices till the matrix becomes diagonal matrix.

6 Conclusion

This report mainly concentrates on Calculation of Eigen values for a Symmetric Matrix using Jacobi's method. Which is an iterative way of rotating a matrix and making the matrix diagonal. Since, jacobi's iterative method is simple and gives high precision for small values it is used more widely for symmetric matrices.

7 References

Matrix computations,Book by Gene H. Golub
Linear Algebra and its applications ,Gilbert strang