Question-9-9.3-25

EE24BTECH11035 - KOTHAPALLI AKHIL

Question:
Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$and $x - 5y + 9 = 0$.
Solution;
The lines Given are :
$3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$and $x - 5y + 9 = 0$
The general form of line equation in matrix form is $h^T + m = 0$
Writing the Given lines in the form of matrices ,

$$h_1^T + m_1 = 0 \quad where \quad h_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, x = \begin{pmatrix} x \\ y \end{pmatrix}, m_1 = 1 \tag{1}$$

$$h_2^T + m_2 = 0 \quad where \quad h_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, m_2 = -21 \tag{2}$$

$$h_3^T + m_3 = 0 \quad where \quad h_3 = \begin{pmatrix} 1 \\ -5 \end{pmatrix}, m_3 = 9 \tag{3}$$

Solving The Line equations to get points of intersections ,
By Solving First two equations, We get point of intersection as ,

$$P_1 = \begin{pmatrix} \frac{36}{13} \\ \frac{121}{26} \end{pmatrix} \tag{4}$$

Similarly,

$$P_2 = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, P_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{5}$$

We now calculate the area of the region using integration. The area is the integral of the top function minus the bottom function over the interval.
Equation for the Line Between P_1 and P_2: The slope of the line between P_1 and P_2 is:

$$m_1 = \frac{3 - \frac{121}{26}}{6 - \frac{36}{13}} = -\frac{43}{52} \tag{6}$$

The equation of the line is:

$$y - 3 = -\frac{43}{52}(x - 6) \tag{7}$$

Equation for the Line Between P_2 and P_3:
The slope of the line between P_2 and P_3 is:

$$m_2 = \frac{2 - 3}{1 - 6} = \frac{1}{5} \tag{8}$$

The equation of the line is:

$$y - 3 = \frac{1}{5}(x - 6) \tag{9}$$

Equation for the Line Between P_3 and P_1: The slope of the line between P_3 and P_1 is:

$$m_3 = \frac{\frac{121}{26} - 2}{\frac{36}{13} - 1} = \frac{69}{52} \tag{10}$$

The equation of the line is:

$$y - 2 = \frac{69}{52}(x - 1) \tag{11}$$

We now calculate the area using the definite integral:

$$A = \int_{x_1}^{x_2} (f(x) - g(x))\, dx \tag{12}$$

$$A = \int_1^6 \left(\left(-\frac{43}{52}(x - 6) + 3 \right) - \left(\frac{1}{5}(x - 6) + 3 \right) \right) dx \tag{13}$$

$$A = \int_1^6 \left(-\frac{43}{52}x + \frac{43 \cdot 6}{52} - \frac{1}{5}x + \frac{6}{5} \right) dx \tag{14}$$

$$A = \left[-\frac{43}{104}x^2 + \frac{43 \cdot 6}{52}x - \frac{1}{10}x^2 + \frac{6}{5}x \right]_1^6 \tag{15}$$

After evaluating the integral, we find the area is:

$$A = 6.5\,\text{square units} \tag{16}$$

Thus, the area of the region bounded by the lines is 6.5 square units.

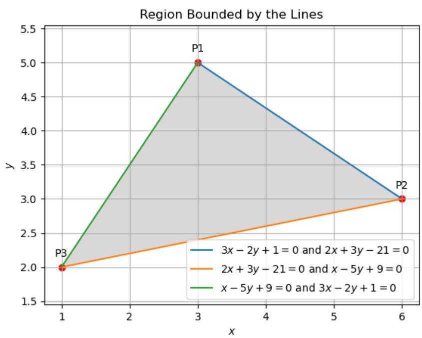

Fig. 0: Area enclosed between the 3 Lines