

1) Let $f(z) = 2z^2 - 1$. Then the maximum value of $|f(z)|$ on the unit disc $D = \{z \in \mathbb{C} : |z| \leq 1\}$ equals

- a) 1
- b) 2
- c) 3
- d) 4

2) Let

$$f(z) = \frac{1}{z^2 - 3z + 2}.$$

Then the coefficient of $\frac{1}{z^2}$ in the Laurent series expansion of $f(z)$ for $|z| > 2$ is

- a) 0
- b) 1
- c) 3
- d) 5

3) Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an arbitrary analytic function satisfying $f(0) = 0$ and $f(1) = 2$. Then

- a) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| > n$
- b) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| \leq n$
- c) there exists a bounded sequence $\{z_n\}$ such that $|f(z_n)| > n$
- d) there exists a sequence $\{z_n\}$ such that $z_n \rightarrow 0$ and $f(z_n) \rightarrow 2$

4) Define $f : \mathbb{C} \rightarrow \mathbb{C}$ by

$$f(z) = \begin{cases} 0, & \text{if } \operatorname{Re}(z) = 0 \text{ or } \operatorname{Im}(z) = 0, \\ \frac{1}{z}, & \text{otherwise.} \end{cases}$$

Then the set of points where f is analytic is

- a) $\{z : \operatorname{Re}(z) \neq 0 \text{ and } \operatorname{Im}(z) \neq 0\}$
- b) $\{z : \operatorname{Re}(z) \neq 0\}$
- c) $\{z : \operatorname{Re}(z) \neq 0 \text{ or } \operatorname{Im}(z) \neq 0\}$
- d) $\{z : \operatorname{Im}(z) \neq 0\}$

5) Let $U(n)$ be the set of all positive integers less than n and relatively prime to n . Then $U(n)$ is a group under multiplication modulo n . For $n = 248$, the number of elements in $U(n)$ is

- a) 60
- b) 120
- c) 180
- d) 240

- 6) Let $\mathbb{R}[x]$ be the polynomial ring in x with real coefficients and let $I = (x^2 + 1)$ be the ideal generated by the polynomial $x^2 + 1$ in $\mathbb{R}[x]$. Then
- I is a maximal ideal
 - I is a prime ideal but NOT a maximal ideal
 - I is NOT a prime ideal
 - $\mathbb{R}[x]/I$ has zero divisors

- 7) Consider \mathbb{Z}_5 and \mathbb{Z}_{20} as rings modulo 5 and 20, respectively. Then the number of homomorphisms $\phi : \mathbb{Z}_5 \rightarrow \mathbb{Z}_{20}$ is
- 1
 - 2
 - 4
 - 5

- 8) Let \mathbb{Q} be the field of rational numbers and consider \mathbb{Z}_2 as a field modulo 2. Let

$$f(x) = x^3 - 9x^2 + 9x + 3.$$

Then $f(x)$ is

- irreducible over \mathbb{Q} but reducible over \mathbb{Z}_2
 - irreducible over both \mathbb{Q} and \mathbb{Z}_2
 - reducible over \mathbb{Q} but irreducible over \mathbb{Z}_2
 - reducible over both \mathbb{Q} and \mathbb{Z}_2
- 9) Consider \mathbb{Z}_5 as a field modulo 5 and let

$$f(x) = x^5 + 4x^4 + 4x^3 + 4x^2 + x + 1$$

Then the zeros of $f(x)$ over \mathbb{Z}_5 are 1 and 3 with respective multiplicity.

- 1 and 4
 - 2 and 3
 - 2 and 2
 - 1 and 2
- 10) Consider the Hilbert space $\ell^2 = \{x = \{x_n\}; x_n \in \mathbb{R}, \sum_{n=1}^{\infty} x_n^2 < \infty\}$. Let

$$E = \left\{ x = \{x_n\}; |x_n| \leq \frac{1}{n} \text{ for all } n \right\}$$

be a subset of ℓ^2 . Then:

- $E^\circ = \{x : |x_n| < \frac{1}{n} \text{ for all } n\}$
 - $E^\circ = E$
 - $E^\circ = \{x : |x_n| < \frac{1}{n} \text{ for all but finitely many } n\}$
 - $E^\circ = \emptyset$
- 11) Let X and Y be normed linear spaces and let $T : X \rightarrow Y$ be a linear map. Then T is continuous if
- Y is finite dimensional
 - X is finite dimensional
 - T is one to one

d) T is onto

12) Let X be a normed linear space and let $E_1, E_2 \subseteq X$. Define

$$E_1 + E_2 = \{x + y : x \in E_1, y \in E_2\}.$$

Then $E_1 + E_2$ is

- a) open if E_1 or E_2 is open
- b) NOT open unless both E_1 and E_2 are open
- c) closed if E_1 or E_2 is closed
- d) closed if both E_1 and E_2 are closed

13) For each $a \in \mathbb{R}$, consider the linear programming problem

$$\text{Max. } z = x_1 + 2x_2 + 3x_3 + 4x_4,$$

subject to

$$ax_1 + 2x_3 \leq 1, \quad x_1 + ax_3 + 3x_4 \leq 2, \quad x_1, x_2, x_3, x_4 \geq 0.$$

Let $S = \{a \in \mathbb{R} : \text{the given LP problem has a basic feasible solution}\}$. Then:

- a) $S = \emptyset$
- b) $S = \mathbb{R}$
- c) $S = (0, \infty)$
- d) $S = (-\infty, 0)$

14) Consider the linear programming problem

$$\text{Max. } z = x_1 + 5x_2 + 3x_3,$$

subject to

$$2x_1 - 3x_2 + 5x_3 \leq 3, \quad 3x_1 + 2x_3 \leq 5, \quad x_1, x_2, x_3 \geq 0.$$

Then the dual of this LP problem:

- a) has a feasible solution but does NOT have a basic feasible solution
- b) has a basic feasible solution
- c) has infinite number of feasible solutions
- d) has no feasible solution

15) Consider a transportation problem with two warehouses and two markets. The warehouse capacities are $a_1 = 2$ and $a_2 = 4$, and the market demands are $b_1 = 3$ and $b_2 = 3$. Let x_{ij} be the quantity shipped from warehouse i to market j and c_{ij} be the corresponding unit cost. Suppose that $c_{11} = 1, c_{12} = 1$ and $c_{22} = 2$. Then $(x_{11}, x_{12}, x_{21}, x_{22}) = (2, 0, 1, 3)$ is optimal for every

- a) $c_{12} \in [2, 3]$
- b) $c_{12} \in [0, 3]$
- c) $c_{12} \in [1, 3]$
- d) $c_{12} \in [2, 4]$

16) The smallest degree of the polynomial that interpolates the data

x	-2	-1	0	1	2	3
$f(x)$	-58	-21	-12	-13	-6	27

is:

- (A) 3
- (B) 4
- (C) 5
- (D) 6

17) Suppose that x_0 is sufficiently close to 3. Which of the following iterations $x_{n+1} = g(x_n)$ will converge to the fixed point $x = 3$?

- a) $x_{n+1} = -16 + 6x_n + \frac{3}{x_n}$
- b) $x_{n+1} = \sqrt{3 + 2x_n}$
- c) $x_{n+1} = \frac{3}{x_n - 2}$
- d) $x_{n+1} = \frac{x_n - 3}{2}$