

# Question-1-1.11-11

EE24BTECH11035 - KOTHAPALLI AKHIL

## Question:

The scalar product of the vector  $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of the vector  $\mathbf{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\mathbf{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to 1. Find the value of  $\lambda$  and hence find the unit vector along  $\mathbf{b} + \mathbf{c}$ .

## Solution:

Point	Coordinates
<b>A</b>	(6, -4)
<b>B</b>	(-2, -7)

TABLE 0: given vectors

Given vectors  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  are:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad (1)$$

$$\mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}, \quad (2)$$

$$\mathbf{c} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \quad (3)$$

The sum of vectors  $\mathbf{b}$  and  $\mathbf{c}$  is:

$$\mathbf{b} + \mathbf{c} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} + \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \quad (4)$$

$$= \begin{pmatrix} 2 + \lambda \\ 4 + 2 \\ -5 + 3 \end{pmatrix} \quad (5)$$

$$= \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} \quad (6)$$

Now, we find the norm of  $\mathbf{b} + \mathbf{c}$ . The norm is given by:

$$\|\mathbf{b} + \mathbf{c}\| = \sqrt{(\mathbf{b} + \mathbf{c})^\top (\mathbf{b} + \mathbf{c})} \quad (7)$$

$$= \sqrt{(2 + \lambda)^2 + 6^2 + (-2)^2} \quad (8)$$

$$= \sqrt{(2 + \lambda)^2 + 36 + 4} \quad (9)$$

$$= \sqrt{(2 + \lambda)^2 + 40} \quad (10)$$

The unit vector along  $\mathbf{b} + \mathbf{c}$  is:

$$\hat{\mathbf{u}} = \frac{\mathbf{b} + \mathbf{c}}{\|\mathbf{b} + \mathbf{c}\|} \quad (11)$$

Given, The scalar product of  $\mathbf{a}$  with this unit vector as 1:

$$\mathbf{a}^\top \hat{\mathbf{u}} = 1 \quad (12)$$

Substituting the expressions for  $\mathbf{a}$  and  $\hat{\mathbf{u}}$ , we get:

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{(2 + \lambda)^2 + 40}} \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} = 1 \quad (13)$$

$$\frac{1}{\sqrt{(2 + \lambda)^2 + 40}} ((2 + \lambda) + 6 - 2) = 1 \quad (14)$$

$$\frac{1}{\sqrt{(2 + \lambda)^2 + 40}} (\lambda + 6) = 1 \quad (15)$$

Squaring both sides:

$$\frac{(\lambda + 6)^2}{(2 + \lambda)^2 + 40} = 1 \quad (16)$$

$$(\lambda + 6)^2 = (2 + \lambda)^2 + 40 \quad (17)$$

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 4 + 40 \quad (18)$$

Simplifying:

$$12\lambda + 36 = 4\lambda + 44 \quad (19)$$

$$8\lambda = 8 \quad (20)$$

$$\lambda = 1 \quad (21)$$

Thus,  $\lambda = 1$ . Substituting  $\lambda = 1$  into  $\mathbf{b} + \mathbf{c}$ :

$$\mathbf{b} + \mathbf{c} = \begin{pmatrix} 2+1 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} \quad (22)$$

The norm of  $\mathbf{b} + \mathbf{c}$  is:

$$\|\mathbf{b} + \mathbf{c}\| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \quad (23)$$

Thus, the unit vector along  $\mathbf{b} + \mathbf{c}$  is:

$$\hat{\mathbf{u}} = \frac{1}{7} \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} \frac{3}{7} \\ \frac{6}{7} \\ -\frac{2}{7} \end{pmatrix} \quad (24)$$

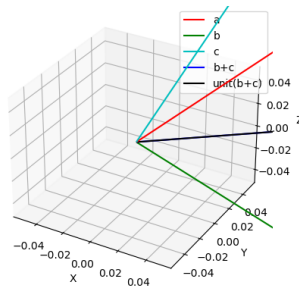


Fig. 0.1:  $\mathbf{a}, \mathbf{b}, \mathbf{c}$ , unit vector in the direction of  $\mathbf{b} + \mathbf{c}$