09-April-2024 Shift-2

EE24BTECH11035 - KOTHAPALLI AKHIL

16) Consider the line L passing through the points $(1, 2, 3)$ and $(2, 3, 5)$. The distance of the point

$$\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$$

from the line L along the line

$$\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$$

is equal to:

a) 5
b) 4
c) 3
d) 6

17) Let

$$B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$$

and A be a 2×2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and

$$C^4 + \alpha C^2 + \beta I = 0$$

then $2\beta - \alpha$ is equal to:

a) 8
b) 2
c) 16
d) 10

18) The area (in square units) of the region enclosed by the ellipse $x^2 + 3y^2 = 18$ in the first quadrant below the line $y = x$ is:

a) $\sqrt{3}\pi + 1$
b) $\sqrt{3}\pi + \frac{3}{4}$
c) $\sqrt{3}\pi$
d) $\sqrt{3}\pi + \frac{3}{4}$

19) Two vertices of a triangle ABC are $A(3, -1)$ and $B(2, -3)$, and its orthocenter is $P(1, 1)$. If the coordinates of the point C are (α, β) and the center of the circle circumscribing the triangle PAB is (h, k), then the value of $(\alpha + \beta) + 2(h + k)$ equals:

a) 81
b) 15
c) 5
d) 51

20) The integral

$$\int_{\frac{1}{4}}^{\frac{3}{4}} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right) dx$$

is equal to:

a) $1/2$
b) $1/4$
c) $-1/4$
d) $-1/2$

21) For a differentiable function $f : \mathbb{R} \to \mathbb{R}$, suppose $f'(x) = 3f(x) + a$, where $a \in \mathbb{R}$, $f(0) = 1$ and $\lim_{x\to\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to ....

22) Consider the circle $C : x^2 + y^2 = 4$ and the parabola $P : y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to ....

23) If

$$\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+1012}\right) - \left(\frac{1}{2\cdot 1} + \frac{1}{4\cdot 3} + \frac{1}{6\cdot 5} + \ldots + \frac{1}{2024\cdot 2023}\right) = \frac{1}{2024}$$

then α is equal to ....

24) The number of integers, between 100 and 1000 having the sum of their digits equal to 14, is ....

25) Consider the matrices

$$A = \begin{pmatrix} 2 & -5 \\ 3 & 20 \end{pmatrix}, B = \begin{pmatrix} 20 \\ m \end{pmatrix} \quad \textit{and} \quad X = \begin{pmatrix} x \\ y \end{pmatrix}.$$

Let the set of all n, for which the system of equations $AX = B$ has a negative solution (i.e., $x < 0$ and $y < 0$), be the interval (a, b). Then

$$8\int_a^b |A|\, dm$$

is equal to ....

26) Let $A = \{(x, y) : 2x + 3y = 23, x, y \in \mathbb{N}\}$ and $B = \{x, y \in \mathbb{A}\}$. Then the number of one-one functions from A to B is equal to ....

27) Let the inverse trigonometric functions take principal values. The number of real solutions of the equation

$$2\sin^{-1} x + 3\cos^{-1} x = \frac{2\pi}{5}$$

is ....

28) Let the set of all values of p, for which

$$f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$$

does not have any critical point, be the interval (a, b). Then $16ab$ is equal to ....

29) Let A, B, C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet

the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L. Then

$$\left(\frac{AM - BN}{CD}\right)^2$$

is equal to ....

30) The square of the distance of the image of the point $(6, 1, 5)$ in the line

$$\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$$

from the origin is ....