NCERT-9.4.14

EE24BTECH11023 - Murra Rajesh Kumar Reddy

Question: Find the solution of the following differential equation, Given that y=0 when x=1

$$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0 \tag{0.1}$$

Theoretical Solution:
Let $\frac{y}{x}$ be t

$$\implies \frac{dy}{dx} = t + x\frac{dt}{dx} \tag{0.2}$$

From (0.1) and (0.2) we get

$$t + \frac{dt}{dx} - t + \operatorname{cosec} t = 0 \tag{0.3}$$

$$\frac{dt}{dx} + \operatorname{cosec} t = 0 \tag{0.4}$$

$$\sin t dt = -dx \tag{0.5}$$

$$\tag{0.6}$$

By integrating on both sides,

$$\int \sin t dt = -\int dx \tag{0.7}$$

$$-\cos t = -(x + c) \tag{0.8}$$

$$\cos\frac{y}{x} = x + c \qquad \left(\because t = \frac{y}{x}\right) \tag{0.9}$$

Finding 'c' as given in question $y = 0$ when $x = 1$,

$$\cos 0 = 1 + c \tag{0.10}$$

$$c = 0 \tag{0.11}$$

Then Theoretical solution will be,

$$\cos\left(\frac{y}{x}\right) = x \tag{0.12}$$

Computational Solution:
The solution below is solved by method of **finitie differences:**
The finite difference method is a numerical technique for solving differential equations by approximating derivatives with differences. The first forward difference approximation

of the derivative of f (x) at x is given by:

$$\frac{dy}{dx} = \frac{y(x+h) - y(x)}{h} \tag{0.13}$$

From the equation (0.13) we get

$$y(x+h) = y(x) + h\frac{dy}{dx} \tag{0.14}$$

From the equations (0.1) and (0.14)

$$y(x+h) = y(x) + h\left(\frac{y}{x} - \text{cosec}\left(\frac{y}{x}\right)\right) \tag{0.15}$$

On taking the given point on the curve as the initial conditions (x0, y0),we can get

$$x_1 = x_0 + h \tag{0.16}$$

On assuming a value for h which is close to zero and by substituting the values of x_0 and y_0 in the above equations we get the point (x_1, y_1). what we have essentially done above is, obtaining a point which is very close to the initial point along the direction of derivative at that point. similarly we get,

$$x_n = x_{n-1} + h \tag{0.17}$$

$$y_n = y_{n-1} + h\left(\frac{y_{n-1}}{x_{n-1}} - \text{cosec}\,\frac{y_{n-1}}{x_{n-1}}\right) \tag{0.18}$$

we can obtain points on the curve by using the above expressions for y_n and x_n
and plot the curve by the points obtained. But there might be error in computation due to singualrities in equation, particularly when $\sin \frac{y}{x} = 0$. This results in division by division by zero.To resolve this we ensured that the numerical solver doesn't attempt to evaluate invalid points

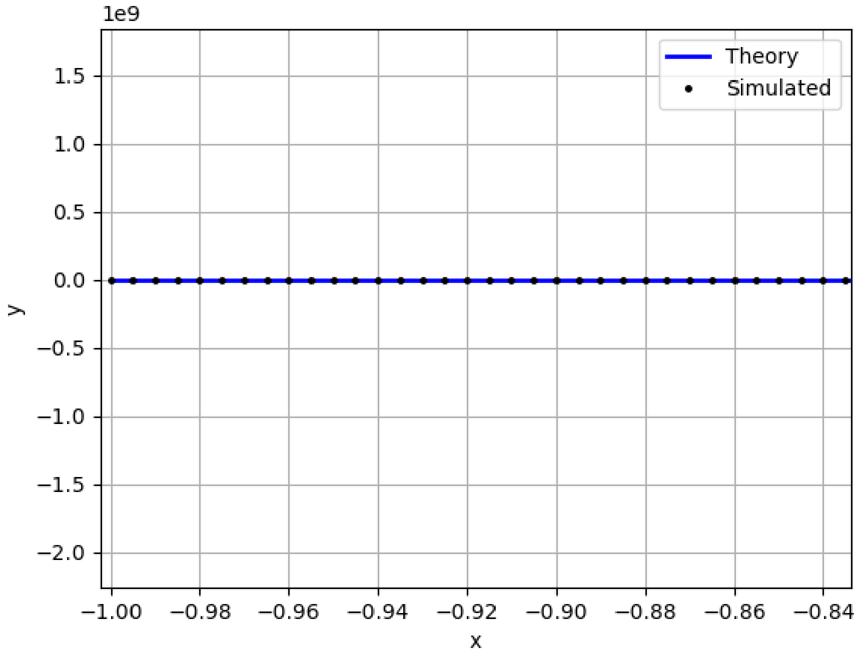

Fig. 0.1: Solution of given DE