

Homa Harshitha Vuddanti
(EE24BTECH11062)

Question: Find the particular solution of the differential equation $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x; y = 1$ when $x = 0$

Theoretical Solution:

Given,

$$\frac{dy}{dx} = \frac{2x^2 + x}{x^3 + x^2 + x + 1} \quad (1)$$

$$\frac{dy}{dx} = \frac{2x^2 + x}{(x + 1)(x^2 + 1)} \quad (2)$$

$$(3)$$

Splitting into partial fractions,

$$\frac{dy}{dx} = \frac{2x^2 + x}{(x + 1)(x^2 + 1)} = \frac{1}{2(x + 1)} + \frac{3x - 1}{2(x^2 + 1)} \quad (4)$$

$$\int dy = \int \frac{1}{2(x + 1)} dx + \int \frac{3x - 1}{2(x^2 + 1)} dx \quad (5)$$

$$y = \frac{1}{2} \ln|x + 1| + \frac{3}{4} \ln|x^2 + 1| - \frac{1}{2} \tan^{-1} x + c \quad (6)$$

Substituting $(0, 1)$ in the equation, we get the value of $c = 1$

$$y = \frac{1}{2} \ln|x + 1| + \frac{3}{4} \ln|x^2 + 1| - \frac{1}{2} \tan^{-1} x + 1 \quad (7)$$

Solving by Bilinear transformation : Applying Laplace transformation

$$\mathcal{L}(y') = \mathcal{L}\left(\frac{2x^2 + x}{x^3 + x^2 + x + 1}\right) \quad (8)$$

$$(9)$$

Let

$$\mathcal{L}\left(\frac{2x^2 + x}{x^3 + x^2 + x + 1}\right) = X(s) \quad (10)$$

$$sY(s) = X(s) \quad (11)$$

Then transfer function $H(s) = \frac{Y(s)}{X(s)}$

$$H(s) = \frac{1}{s} \quad (12)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s} \quad (13)$$

From bilinear transformation,

$$s = \frac{2}{h} \left(\frac{1 + z^{-1}}{1 - z^{-1}} \right) \quad (14)$$

Substituting in (13),

$$Y(s) - z^{-1}Y(s) = \frac{h}{2} (X(s) - z^{-1}X(s)) \quad (15)$$

Applying inverse \mathcal{Z} -transformation,

$$y_{n+1} - y_n = \frac{h}{2} \left(\frac{2x_{n+1}^2 + x_{n+1}}{x_{n+1}^3 + x_{n+1}^2 + x_{n+1} + 1} + \frac{2x_n^2 + x_n}{x_n^3 + x_n^2 + x_n + 1} \right) \quad (16)$$

$$x_{n+1} = x_n + h \quad (17)$$

Trapezoidal Rule:

Area is given by

$$\int_a^b f(x) dx \approx h \left(\frac{1}{2} f(a) + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + \frac{1}{2} f(b) \right) \quad (18)$$

$$h = \frac{b - a}{n} \quad (19)$$

$$A_{n+1} = A_n + \frac{h}{2} (f(x_{n+1}) + f(x_n)) \quad (20)$$

Substituting, we finally get

$$y_{n+1} = y_n + \frac{h}{2} \left(\frac{2x_{n+1}^2 + x_{n+1}}{x_{n+1}^3 + x_{n+1}^2 + x_{n+1} + 1} + \frac{2x_n^2 + x_n}{x_n^3 + x_n^2 + x_n + 1} \right) \quad (21)$$

$$x_{n+1} = x_n + h \quad (22)$$

Plotting: Taking the initial point (x_0, y_0) as $(0, 1)$ and the value of $h = 0.01$

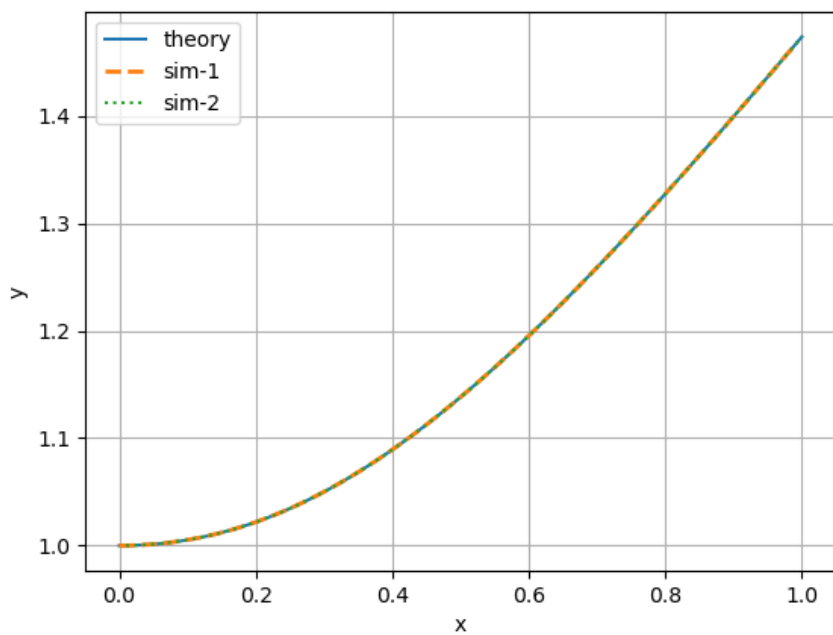


Fig. 0: Plot