Question-9.7.9

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

Question: Find the particular solution of the differential equation $\left(1+e^{2x}\right)$dy+$\left(1+y^2\right)e^x$dx=0,given that y=1 and when x=0

Solution:

The equation can de written as

$$\frac{dy}{dx} = -\frac{\left(1+y^2\right)e^x}{1+e^{2x}} \tag{0.1}$$

This is a first-order, separable differential equation.

$$\frac{1}{1+y^2}\ dy = -\frac{e^x}{1+e^{2x}}\ dx \tag{0.2}$$

The integral of $\frac{1}{1+y^2}$ is $\tan^{-1} y$
The integral of $\int -\frac{e^x}{1+e^{2x}}\ dx$ can be computed as shown below

$$\text{Let}\ \ e^x = t \tag{0.3}$$

$$e^x\ dx = dt \tag{0.4}$$

$$\int -\frac{e^x}{1+e^{2x}}\ dx = \int -\frac{1}{1+t^2}\ dt \tag{0.5}$$

$$\int -\frac{1}{1+t^2}\ dt = -\tan^{-1} t \tag{0.6}$$

$$\int -\frac{e^x}{1+e^{2x}}\ dx = -\tan^{-1} e^x \tag{0.7}$$

The final solution is

$$\tan^{-1} y = \tan^{-1} e^x + c \tag{0.8}$$

At x=0 the value of y is 1 on substituting

$$\tan^{-1} 1 = -\tan^{-1} e^0 + c \tag{0.9}$$

$$\frac{\pi}{4} = -\frac{\pi}{4} + c \tag{0.10}$$

$$c = \frac{\pi}{2} \tag{0.11}$$

The final solution is

$$\tan^{-1} y = -\tan^{-1} e^x + \frac{\pi}{2} \tag{0.12}$$

Numerical Approach:
1. I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x + h$. where h is the step size, representing the rate of change.
2. Assigned the values of y for different x-values using a for loop.

Using the Method of Finite Differences
The Method of Finite Differences is a numerical technique used to approximate solutions to differential equations.

We know that:

$$\lim_{h \to 0} \frac{y(x+h) - y(x)}{h} = \frac{dy}{dx} \tag{0.13}$$

For the given differential equation,

$$\frac{dy}{dx} = -\frac{\left(1 + y^2\right) e^x}{1 + e^{2x}} \tag{0.14}$$

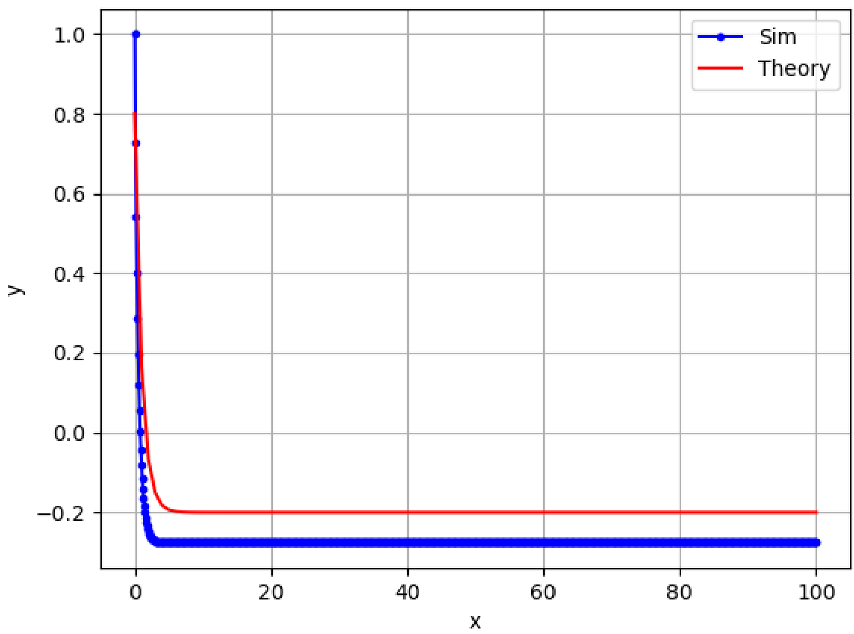
$$\frac{y_{n+1} - y_n}{h} \approx -\frac{\left(1 + y_n^2\right) e_n^x}{1 + e^{2x_n}} \tag{0.15}$$

$$y_{n+1} = y_n - h \cdot \frac{\left(1 + y_n^2\right) e_n^x}{1 + e^{2x_n}} \tag{0.16}$$

The iterative formula for updating x-values is:

$$x_n = x_{n-1} + h \tag{0.17}$$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match



Sim
Theory
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
y
0
20
40
60
80
100
x