

# 9.5.3

EE24BTECH11055 - Sai Akhila Reddy Turpu

QUESTION:

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \quad (0.1)$$

USING FINITE DIFFERENCES TO SOLVE THE GIVEN DIFFERENTIAL EQUATION:

The finite difference method is a numerical technique for solving differential equations by approximating derivatives with differences. The first forward difference approximation of the derivative of  $f(x)$  at  $x$  is given by:

$$\frac{dy}{dx} = \frac{f(x+h) - f(x)}{h} \quad (0.2)$$

Let  $f(x+h) = y_{n+1}$  and  $f(x) = y_n$

$$\Rightarrow y_{n+1} = h \times \frac{dy}{dx} + y_n \quad (0.3)$$

$$\Rightarrow y_{n+1} = h \left( x^2 - \frac{y}{x} \right) + y_n \quad (0.4)$$

We already know that our derivative is given by:

$$\frac{dy}{dx} = x^2 - \frac{y}{x} \quad (0.5)$$

Let's assume the initial conditions to be:  $x_0 = 1$  and  $y_0 = 0.25$  The derivative at the initial conditions can be calculated by substituting  $x_0$  and  $y_0$  in equation (1).

$$\frac{dy}{dx} \Big|_{x=x_0} = 1 - 0.25 = 0.75 \quad (0.6)$$

Let  $h = 10^{-3}$  and  $f_1(x) = f(x+h)$

We get  $f_1(x) = f(x) + h \times \frac{dy}{dx} \Big|_{x=x_0}$

$$f_1(x) = 0.25 + 10^{-3} \times 0.75 = 0.25075$$

Thus,  $x_1 = x_0 + h = 1 + 10^{-3} = 1.001$ ,

$$y_1 = y_0 + h \times \frac{dy}{dx} \Big|_{x=x_0} = 0.25075$$

What we've essentially done is, obtaining a point which is very close to the initial point along the direction of the derivative at that point.

To obtain the entire curve we repeat the process consecutively.

For finding  $f_2(x)$ , find  $\frac{dy}{dx} \Big|_{x=x_1}$  using equation (1) and substitute it in equation (2) Find  $f_2(x), f_3(x)$  and so on and plot the points to get the curve.

## JEE METHOD:

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \quad (0.7)$$

Finding the Integrating factor:

$$e^{\int \frac{1}{x} dx} = e^{\ln x} = x \quad (0.8)$$

$$\text{Thus } IF = x \quad (0.9)$$

$$y(x) = \int x^2 \times x dx \quad (0.10)$$

$$y.x = \frac{x^4}{4} + C \quad (0.11)$$

Let's assume  $C = 0$ , we get:

$$y = \frac{x^3}{4} \quad (0.12)$$

We can now verify that the given graph is  $y = \frac{x^3}{4}$

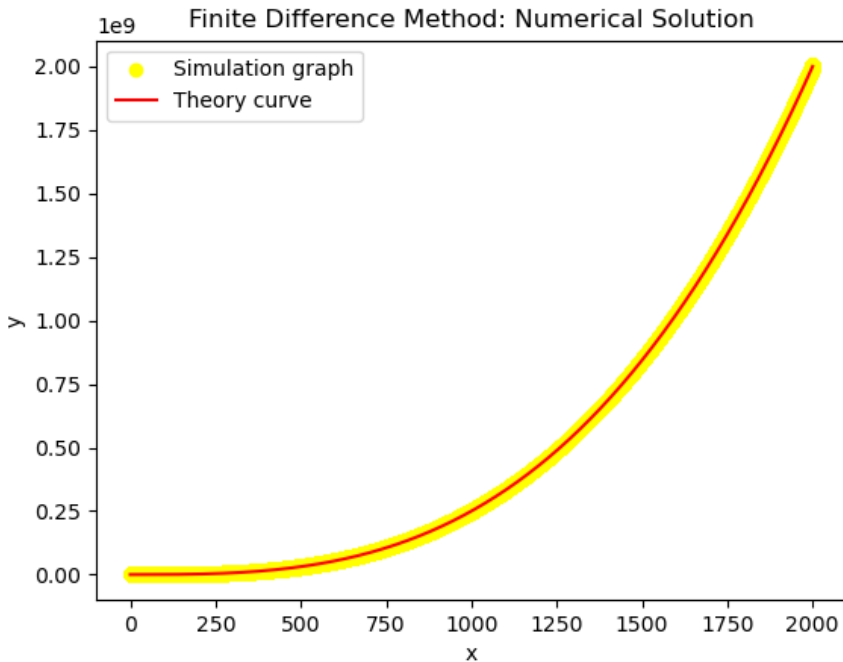


Fig. 0.1:  $y = \frac{x^3}{4}$