Robust Q-Learning under Corrupted Rewards

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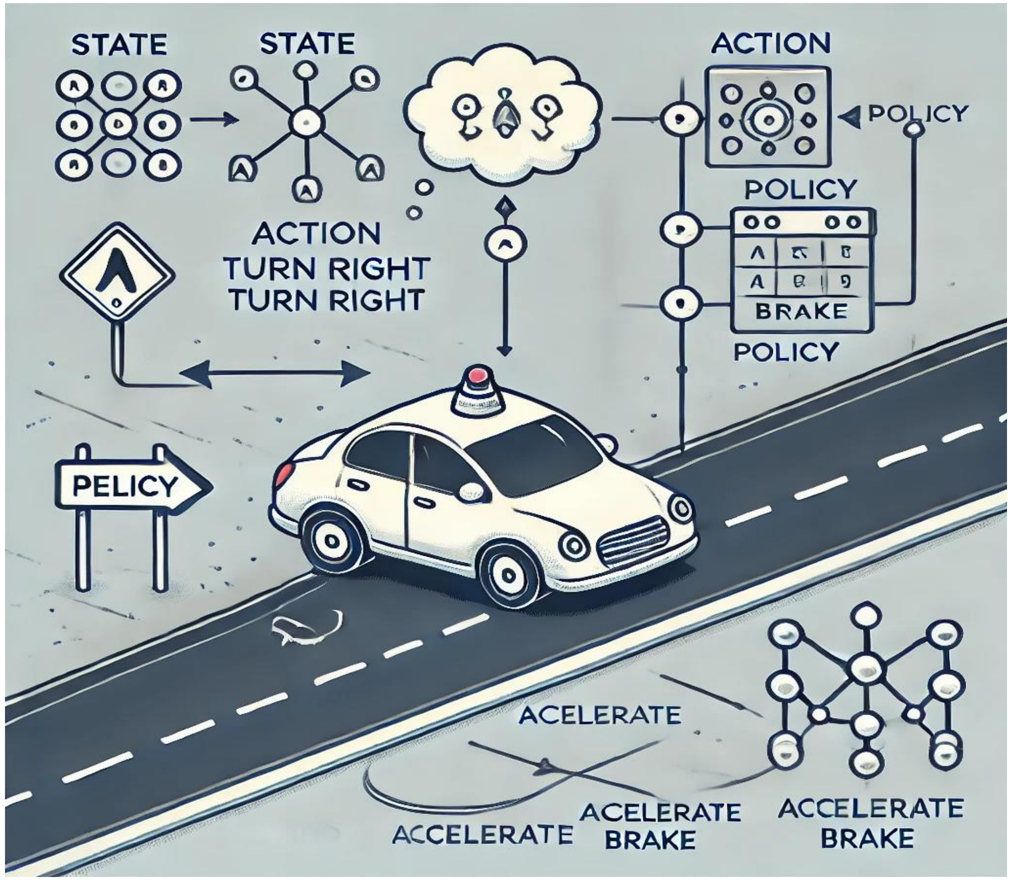
Are RL Algorithms always trustworthy ?

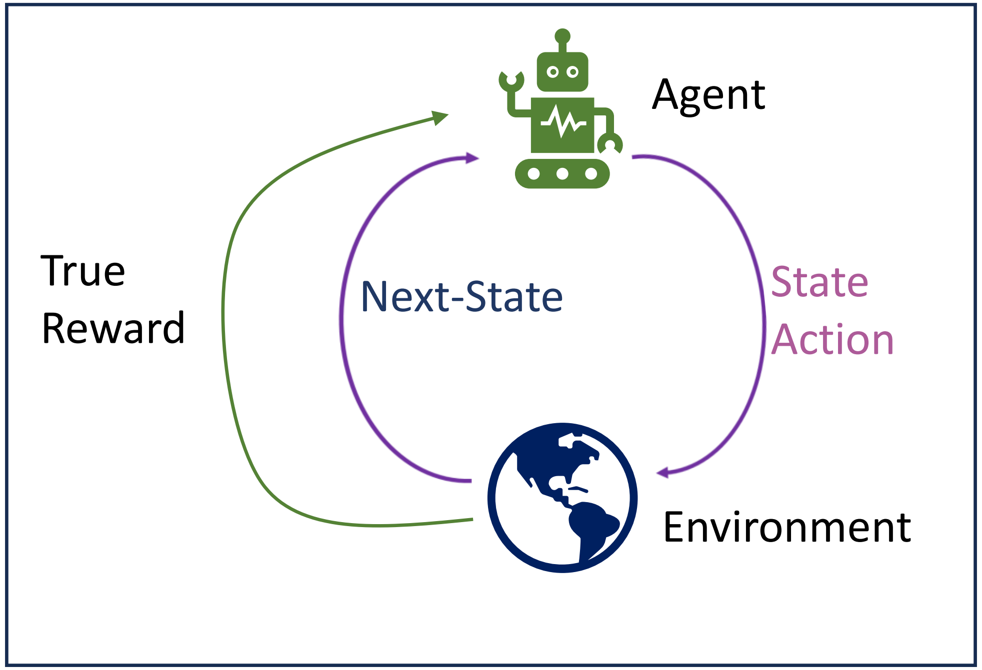
- How 'reliable' are RL Algorithms in real-world environments ?

- How critical is the assumption of "perfect" feedback in practical scenarios?

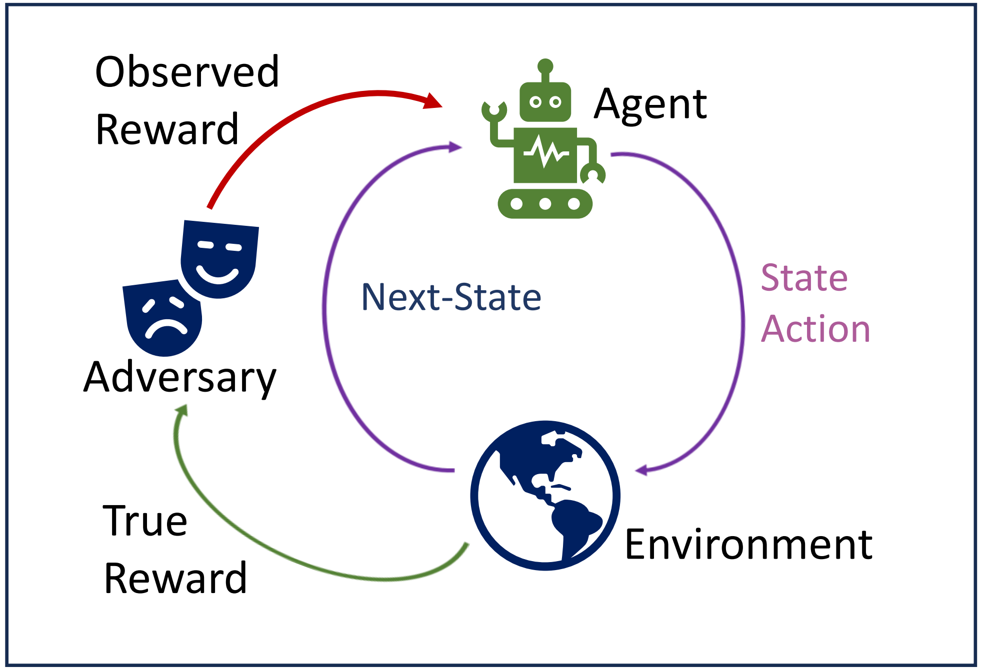
- Consequences of having blind faith in the correctness of feedback in safety-critical applications ?



Reinforcement Learning with Corrupted Rewards



Standard RL Pipeline



RL Pipeline with Corrupted Rewards

Brief Outline of the Talk

- **Contribution 1:** Vulnerability of the classical Q-Learning Algorithm (*Watkins et al.*, Machine Learning, Vol 8, 1992) with adversarial corruptions in rewards.

- **Contribution 2:** Design of a novel, Robust Q-Learning Algorithm to safeguard against corrupted rewards (adversary corrupts a fraction of rewards).

- **Contribution 3:** Finite-time analysis ***achieving near-optimal bounds*** with a small additive error proportional to corruption fraction.

Basic RL Setup

- We consider an MDP $\mathcal{M} = (S, A, P, R, \gamma)$ with finite state and action spaces.
- $R(s, a)$ is the immediate expected reward at state-action pair (s, a).
- $P(s'|s, a)$ is the probability of transitioning from s to s' under action a.
- A deterministic policy $\pi: S \mapsto A$.

- The "goodness" of a policy π is captured by the value function $V_\pi: S \mapsto \mathbb{R}$:

$$V_\pi(s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t(s_t, a_t) | s_0 = s\right]$$

Goal: Find optimal policy (π^*) that maximizes value function (without the knowledge of MDP). *One of most popular algorithm for this is Q-Learning.*

State-Action Value Function (Q-Function)

- The state-action value function $Q_\pi : S \times A \mapsto \mathbb{R}$:

$$Q_\pi(s,a) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t(s_t, a_t) | s_0 = s\,, a_0 = a\right]$$

Let $Q_{\pi^*} = Q^*$ be the optimal state-value function.

- Then Q^* is the unique fixed point of the Bellman optimality operator $\mathcal{T}^*$

$$\boxed{(\mathcal{T}^* Q)(s,a) = R(s,a) + \gamma \sum_{s' \in S} P(s'|s,a)\, max_{a' \in A} Q(s', a')}$$

Also, $||\mathcal{T}^* Q_1 - \mathcal{T}^* Q_2||_\infty \leq \gamma ||Q_1 - Q_2||_\infty$ for $Q_1, Q_2 \in \mathbb{R}^{|S||A|}$, $0 < \gamma < 1$.

Synchronous Q-Learning

At each iteration $t \in [T]$, for all (s, a), we observe:

- A new state $s_t(s, a) \sim P(.|s, a)$ (drawn independently).
- A stochastic reward $r_t(s, a) \sim \mathcal{R}(s, a)$ (drawn independently).

Reward Model

- Unbiased and Light Tailed.

Update Rule:

$$\boxed{Q_{t+1}(s, a) = (1-\alpha)Q_t(s, a) + \alpha\,[r_t(s, a) + \gamma \max_{a' \in A} Q_t(s_t(s, a), a')]}$$

Related Works

Asymptotic Analysis:

1) *Vivek S Borkar,* Springer, 2009.
2) *John N Tsitsiklis,* Machine Learning, 1994.
3) *Csaba Szepesvári,* NeuRIPS, 1997.

Finite-time Analysis (Our setting) :

1) *J Wainwright,* Arxiv, 2019.
2) *Wiermann and Qu,* PMLR, 2020.

Note: All the works assume *rewards* are sampled from the true distribution.

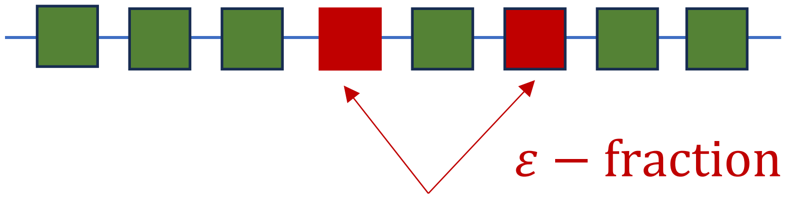
Heavy-Tailed Distribution $\mathcal{R}(s, a)$

In our setting, we further relaxed the standard assumptions on the true reward distribution. Assuming only the finiteness of second-moment, the infinite support reward distribution $\mathcal{R}(s, a)$ can potentially be

- Heavy-Tailed!

Strong Adversarial Corruption in Rewards

- The adversary observes the entire reward set $\{r_t(s,a)\}_{(s,a)\in S\times A}$ in each iteration t.
- Perturb $\varepsilon \in \left[0, \frac{1}{2}\right)$ fraction of these observed rewards up to t.



Strong Adversary:

- Has complete access to the MDP.
- Has access to all the observations.

Vulnerability of Q-Learning

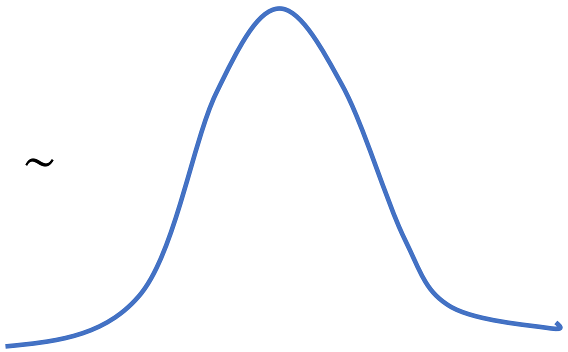
Assume an adversary *corrupts the reward at each iteration with probability ε* (Huber Contamination).

Observed Rewards

$$y_t(s,a) \sim (1-\varepsilon)\mathcal{R}(s,a) + \varepsilon\, \mathcal{C}(s,a)$$

$$\mathbb{E}\,[y_t(s,a)] = R_c(s,a)$$

$\sim$



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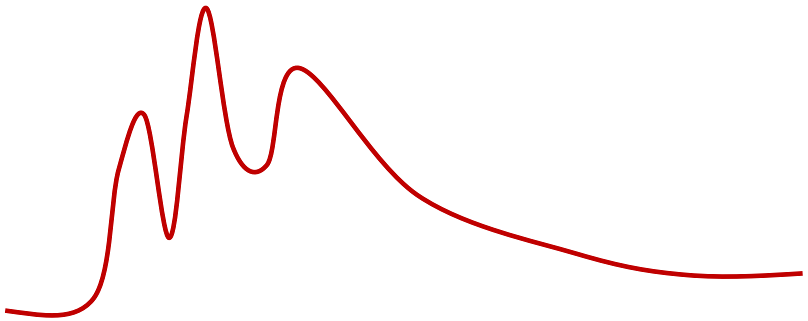


True Reward with prob. (1- ϵ)
$\mathcal{R}(s,a)$

Corrupted Reward with prob. (ϵ)
$\mathcal{C}(s,a)$

Vulnerability of Q-Learning

Under a weaker corruption model, assume an adversary *corrupts the reward at each iteration with probability ε* (Huber Contamination).

Theorem 1: Under Huber contamination, and a suitable step size α, with probability 1, $Q_t \to \widetilde{Q_c^*}$, where $\widetilde{Q_c^*}$ is the unique fixed point of the perturbed Bellman operator $\mathcal{T}_c^*$, satisfying:

$$(\mathcal{T}_c^* Q)(s,a) = R_c(s,a) + \gamma \sum_{s' \in S} P(s'|s,a)\, max_{a' \in A} Q(s',a')$$

$$R_c(s,a) = (1-\varepsilon) R(s,a) + \varepsilon\, C(s,a)$$

This is explicitly controlled by the adversary!

Vulnerability of Q-Learning (contd.)

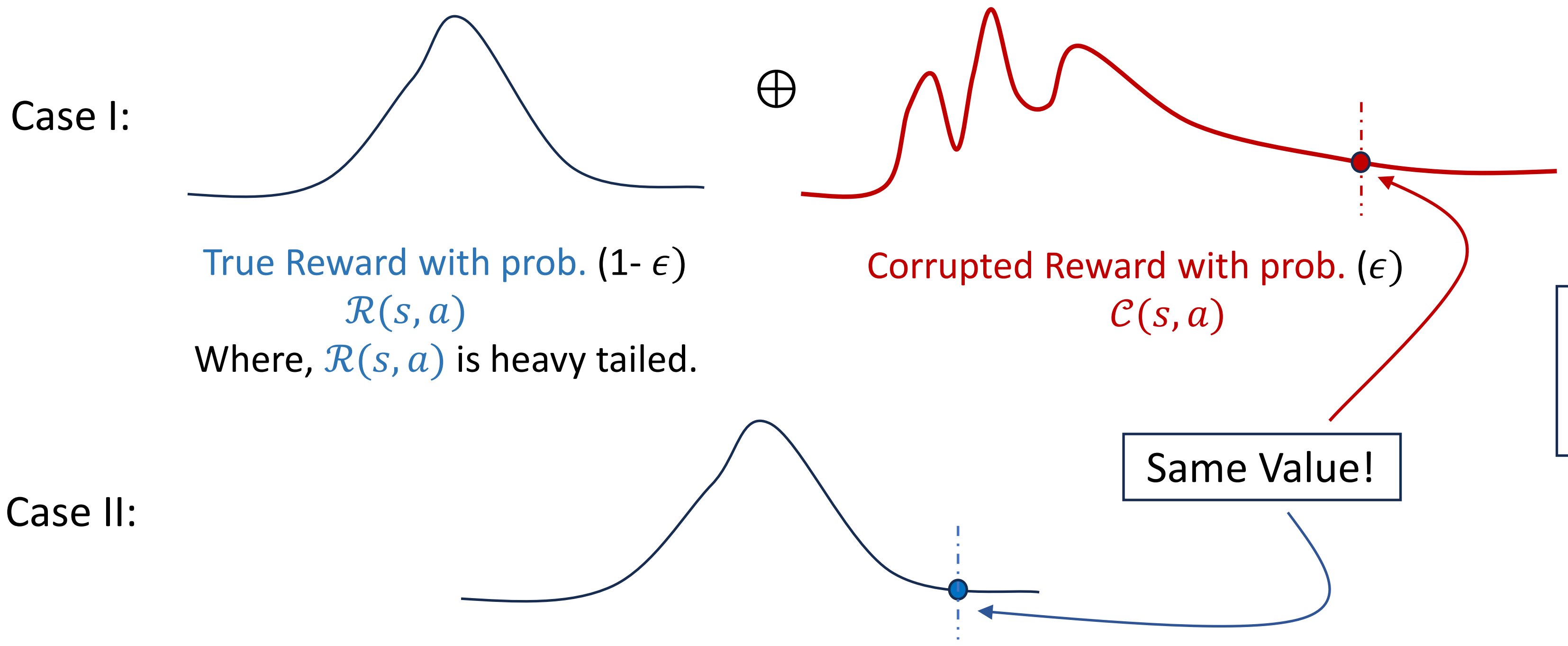
Can the gap between the true optimal Q^* and $\widetilde{Q_c^*}$ be arbitrarily large?

Yes!

Theorem 2: There exists an MDP with finite state-action spaces for which the gap $\left\| Q^* - \widetilde{Q_c^*} \right\|_\infty$ can be arbitrarily large under the Huber Corruption Model.

Proof: Details of MDP construction in our paper.

What makes Robust Q-Learning hard ?



Can we always differentiate between $\bullet$, and $\bullet$?

Proposed Robust Q-Learning Update Rule

- Vanilla Q-Learning is provably susceptible against adversarial corruptions (*Theorem* 1,2).

- Proposed Robust Q-Learning Update Rule:

$$Q_{t+1}(s,a) = (1-\alpha)Q_t(s,a) + \alpha[\tilde{r}_t(s,a) + \gamma \max_{a' \epsilon A} Q_t(s_t(s,a),a')]$$

Proposed Reward Proxy

Robust Q-Learning Algorithm

- Brief outline of our Algorithm is as follows (details in paper):

Iteratively, for all $(s, a) \in S \times A$, and t ∈ [T]

Inputs

- Q_0.
- Step Size α.
- Conf. Par δ.
- Corr. Fraction ε
- Iterations T.

- Sample $s_t(s,a) \sim P(.|s,a)$.
- Observe reward $y_t(s,a)$.
- $\widetilde{r}_t(s,a)$ = TRIM$[\{y_k(s,a)\}_{k\in[t]}, \delta_1, \varepsilon]$

Where, $\delta_1 = \delta/(2|S||A|T)$.

$\widetilde{r}_t(s,a)$

Threshold? (Yes/No)

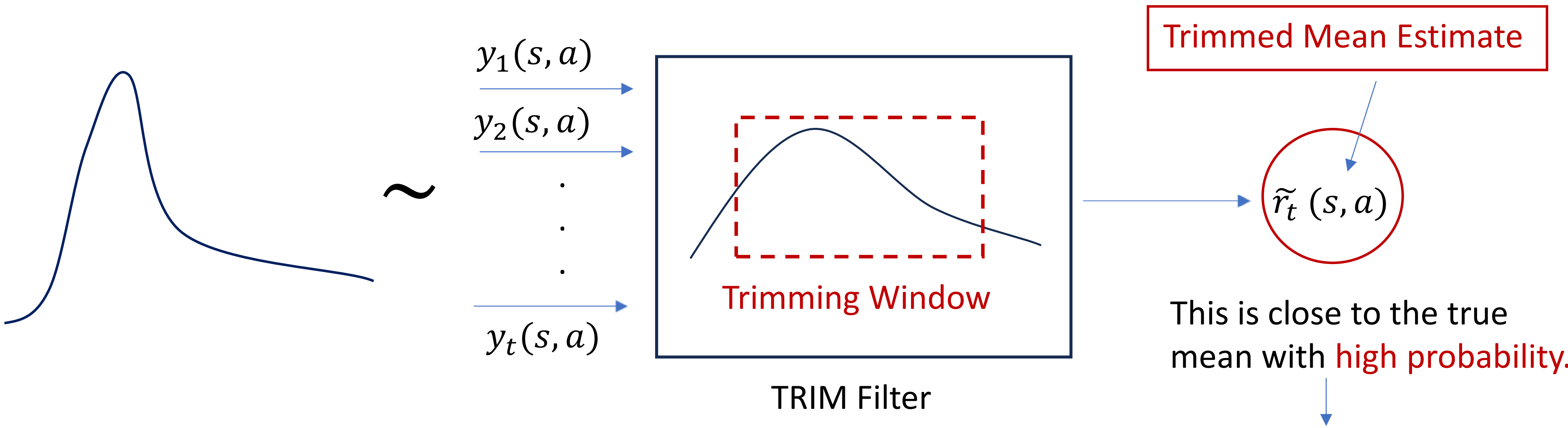
$\widetilde{r}_t(s,a)$

Update Eq. (1)

$$Q_{t+1}(s,a) = (1-\alpha)Q_t(s,a) + \alpha[\widetilde{r}_t(s,a) + \gamma \, max_{a' \epsilon A} Q_t(s_t(s,a), a')] \quad (1)$$

Key Algorithmic Components : TRIM

- $\widetilde{\mathrm{r}}_{\mathrm{t}}(\mathrm{s}, \mathrm{a})$ is the output of the `TRIM` Filter.
- For each set of reward obs. $[y_k\ (s, a)]_{k\in[t]}$, compute the Trimmed Mean Estimate from *Lugosi et al*, The Annals of Statistics, 2021 .



Is Robust Mean enough to ensure the convergence of Robust Q-Learning ? No.

Key Algorithmic Components: Thresholding

Is Trimmed Mean enough to ensure the convergence of Robust Q-Learning ? No.

- The TRIM output is close to the *true mean* with high probability.
- But what happens on extreme events ? Output of TRIM can be unbounded!
- For finite-time analysis, the iterates $\{Q_t\}_{t\in[T]}$ need to be uniformly bounded.
- Hence, we need to ensure the reward proxy to be bounded.

How to ensure that?

- First, we will design a novel conditional threshold for the TRIM output .
- In the "good" events (TRIM Output < Threshold), we won't threshold the TRIM output .
- Rather, we only need to threshold the TRIM output in "extreme" events.

Key Algorithmic Components: Thresholding

- The robust mean estimator works for $t \geq T_{lim} = \left\lceil 2\, log\left(\frac{4}{\delta_1}\right)\right\rceil$.
- Also, the guarantee holds with high probability.

How to design the threshold for the extreme events ?

Upper Bound on Reward Mean

Where, $|R(s,a)| \leq \mu$
$\forall\ (s,a) \in S \times A$, and $\mu \geq 1$

We design a curated deterministic threshold for all such outliers. Setting, $\mathcal{R}$ = max{ μ , σ }.

The threshold is motivated by the guarantees of *Trimmed Mean* described before.

$$G_t = \begin{cases} 2\,\mathcal{R}\,, & 0 \leq t \leq T_{lim} \\ C\mathcal{R}\left(\sqrt{\frac{log\left(\frac{4}{\delta_1}\right)}{t}}\right) + \mathcal{R}, & t \geq T_{lim} + 1 \end{cases}$$

Upper Bound on Reward SD

$\mathbb{E}_{r(s,a)\sim\mathcal{R}(s,a)}\left[\left(r(s,a) - R(s,a)\right)^2\right] \leq \sigma^2$
$\forall(s,a) \in S \times A$

Main Result

Theorem 3: With corruption fraction $\varepsilon \in [0, 1/16)$, failure probability $\delta \in (0, 1)$, and a suitable step-size α, the output of our Algorithm satisfies with probability at least 1- δ:

Corruption Fraction

$$||Q_T - Q^*||_\infty \le \underbrace{\frac{||Q_0 - Q^*||_\infty}{T} + O\left(\frac{\mathcal{R}}{(1-\gamma)^{\frac{5}{2}}} \frac{\log T}{\sqrt{T}} \sqrt{\log\left(\frac{|S||A|T}{\delta}\right)}\right.}_{\text{Matches the bound in Wainwright, Qu}} \left. + \underbrace{\frac{\mathcal{R}\sqrt{\varepsilon}}{1-\gamma}}_{\text{Additive Error Term}} \right)$$

Key Takeaways:

- Despite strong corruption, our proposed Algorithm achieves near-optimal, high probability l_∞ – error rate of $\tilde{O}\left(\frac{1}{\sqrt{T}}\right) + O(\sqrt{\varepsilon})$.
- In absence of corruption, it matches the prior established results.

Summary

- The standard Q-Learning is vulnerable to adversarial reward corruption (*Theorem 1, Theorem 2*).
- We propose a provably robust Q-Learning algorithm (Algorithm 2) with theoretical guarantees (*Theorem 3*).
- Our algorithm only requires the existence of second moments, allowing for rewards with infinite support and heavy tails!
- We conjecture that the additive error, dependent on corruption fraction, is unavoidable.

Future Directions:

- Extension to Asynchronous Q-Learning Algorithm.
- Lower Bounds to show the dependence on ε is unavoidable.
- Can we extend the algorithm when there is limited knowledge about the underlying reward distribution.
- What are some other possible adversaries ? State Adversaries.