BitVM3: Efficient Computation on Bitcoin

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Abstract

BitVM3 is a protocol for verifying SNARK proofs on Bitcoin that dramatically reduces the on-chain footprint of its predecessor, BitVM2. By leveraging optimistic computation with a garbled circuit, BitVM3 shifts the burden of verification off-chain. This design enables an evaluator to generate a compact fraud proof in the event of a dispute. The resulting on-chain transactions are highly efficient: the assertion transaction is approximately 56 kB, while the disproval transaction is just 200 bytes, reducing the on-chain cost of a dispute by over 1,000 times compared to the previous design.

1 Introduction

BitVM3 significantly enhances the on-chain efficiency of SNARK proof verification on Bitcoin. It addresses the primary drawback of BitVM2, where the 'assertTx' and 'disproveTx' were large (2-4 MB). In contrast, BitVM3 reduces the 'assertTx' to about 56 kB and the 'disproveTx' to a mere 200 bytes.
The core principle remains optimistic computation and the overall transaction graph remains unchanged. However, instead of using Bitcoin Script for on-chain computation, BitVM3 employs a garbled circuit to shift the computation off-chain. This circuit is designed to conditionally reveal a secret, which acts as a fraud proof, only if the garbler provides an invalid SNARK proof. This approach builds upon ideas from Jeremy Rubin and Liam Eagen.

2 Computing Gate Labels

The garbling scheme is founded on an RSA-based system.

- **Public Parameters:** The garbler selects and publishes an RSA modulus $N = P \cdot Q = (2p+1)(2q+1)$ (a product of two safe primes) and five public exponents: e, e_1, e_2, e_3, e_4. These exponents must be invertible modulo $\frac{\phi(N)}{4} = p \cdot q$, and for performance, small primes (e.g., 3, 5, 7, 11, 13) are suitable. Let their inverses be d, d_1, d_2, d_3, d_4. Additionally, the derived exponent $h \equiv (e_1 e_4 d_2 - e_3) \pmod{pq}$ must also be invertible.

- **Label Generation:** Using the secret trapdoor $\phi(N)$, the garbler computes the secret input wire labels $a_0, a_1, b_0, b_1 \in C_{pq} \subset (\mathbb{Z}/N\mathbb{Z})^*$ by solving the following system for output labels $c_0, c_1 \in C_{pq} \subset (\mathbb{Z}/N\mathbb{Z})^*$:

$$\begin{aligned}
a_0^e \cdot b_0^{e_1} &\equiv c_0 \pmod{N} \\
a_0^e \cdot b_1^{e_2} &\equiv c_0 \pmod{N} \\
a_1^e \cdot b_0^{e_3} &\equiv c_0 \pmod{N} \\
a_1^e \cdot b_1^{e_4} &\equiv c_1 \pmod{N}
\end{aligned}$$

The knowledge of pq allows the garbler to efficiently find a unique solution. The explicit solutions for the secret input labels are:

$$b_0 \equiv (c_1 c_0^{-1})^{h^{-1}} \pmod N$$
$$b_1 \equiv b_0^{e_1 d_2} \pmod N$$
$$a_0 \equiv c_0^d \cdot b_0^{-e_1 d} \pmod N$$
$$a_1 \equiv c_0^d \cdot b_0^{-e_3 d} \pmod N$$

2.1 Setup for Tree Circuits (Backward Pass)

For a circuit with a tree structure (fan-out of 1), the garbler generates labels by working backward from the final output gate.

1. For the final gate G', choose output labels c'_0, c'_1 and solve for its input labels (a'_0, a'_1, b'_0, b'_1).

2. For a preceding gate G whose output feeds into the first input wire of G', its output labels are determined by G''s requirements: $c_0 = a'_0$ and $c_1 = a'_1$.

3. Solve for gate G's input labels (a_0, a_1, b_0, b_1) using these newly defined c_0, c_1.

4. This process is repeated backward through the circuit. Since each gate feeds into exactly one subsequent gate, the output labels for every gate are uniquely determined.

2.2 Limitation of the Base Scheme: Fan-out > 1

The backward-pass setup fails for general circuits where a wire's fan-out is greater than one. Consider a gate G's output wire feeding into both gate G' (requiring input label a'_k) and gate G'' (requiring input label b''_k). The labels a'_k and b''_k are determined independently by the structures of G' and G'' respectively, meaning in general $a'_k \neq b''_k$. Gate G, however, can only produce a single output label c_k. This creates an impossible constraint where c_k must equal both a'_k and b''_k.

3 Static Fan-out Handling with Adaptor Elements

To handle fan-out in general circuits, we introduce static multiplicative "Adaptor Elements." If an output wire W_y (with labels $\ell_{y,0}, \ell_{y,1}$) feeds an input wire W_{xi} that requires different labels, the garbler pre-computes and publishes a static factor $T_{i,k}$:

$$\ell_{xi,k} \equiv \ell_{y,k} \cdot T_{i,k} \pmod N$$

The garbler, knowing all base labels during setup, computes this factor as $T_{i,k} \equiv \ell_{xi,k} \cdot (\ell_{y,k})^{-1} \pmod N$. These adaptors become part of the public circuit parameters.

4 Reblinding

To reblind the circuit, one can raise the input labels to a secret exponent. The adaptor elements must also be reblinded. For k rounds of reblinding (i.e. reusing the circuit k times), the garbler publishes pairwise coprime public exponents $u_1, \ldots, u_k$ and a secret-derived value $s = \prod_i u_i^{-1} \pmod{\phi(N)}$. The garbler then publishes the reblinded adaptor elements $T_{i,k}^s$.

This allows the evaluator to non-interactively compute any singly reblinded adaptor elements:

$$T_{i,k}^{\frac{1}{u_i}} = (T_{i,k}^{s})^{\prod_{j \neq i} u_j}$$

The evaluator can also recover the plaintext $T_{i,k}$ by raising $T_{i,k}^{s}$ to the power of $\prod_i u_i$.

5 Verifiability and Circuit Correctness

The evaluator can verify the correctness of the garbled circuit's structure by checking each gate in plaintext. Consequently, the garbler only needs to prove in zero-knowledge that the circuit's inputs and outputs (which are committed to) were reblinded correctly. Thus, the proving complexity amounts to proving in zero-knowledge about 2400 exponentiations with small exponents.

6 Communication Complexity and Onchain Footprint

The primary communication cost is the off-chain transfer of the garbled circuit. A SNARK verifier circuit (e.g., Groth16) may have ∼5 billion gates. With an average fan-out of 2-4 and a 256-byte RSA modulus, the adaptor elements dominate the circuit size. For each fan-out connection, two adaptors are needed (for logic 0 and 1).

- **Off-chain size:**

  $$5 \cdot 10^9 \text{ gates} \cdot 2\frac{\text{fan-out}}{\text{gate}} \cdot 2\frac{\text{elements}}{\text{fan-out}} \cdot 256\frac{\text{bytes}}{\text{element}} \approx 5 \text{ TB}$$

  Although sharing the circuit takes about 1.8 days with a 250 Mbps upload speed, this is a one-time setup cost.

- **On-chain 'assertTx' size:** For a proof of 128 bytes and a 20-byte public input, the garbler must commit to the circuit's input labels. This is optimized by publishing encrypted labels during setup and revealing 16-byte decryption keys on-chain.

  $$148 \text{ bytes} \cdot 8\frac{\text{wires}}{\text{byte}} \cdot \left(2\frac{\text{labels}}{\text{wire}} \cdot 16\frac{\text{bytes}}{\text{label}} + 1\frac{\text{dec\_key}}{\text{wire}} \cdot 16\frac{\text{bytes}}{\text{dec\_key}}\right) \approx 56 \text{ kB}$$

- **On-chain 'disproveTx' size:** This transaction is minimal. It simply reveals the hash of the output label for '0', signifying that the SNARK proof was invalid.

7 Reusable Sub-Circuits

A Groth16 SNARK verifier is dominated by ≈ 30,000 field-multiplication gates in the underlying scalar field (256-bit modulus for typical BN/BLS curves). Rather than garbling a monolithic 5×10^9-gate circuit, we factor the verifier into a small library of *sub-circuits*—one for field multiplication, plus analogous blocks for addition, subtraction and inversion—and reuse each block many times. This section explains how to reblind and splice these sub-circuits together while preserving zero-knowledge.

Reblind-and-reuse strategy: Because Section 4 already gives a non-interactive way to reblind *entire* circuits, we can instantiate a field-multiplication sub-circuit once, reblind it, and reuse the resulting encoding k times (here $k \approx 30{,}000$). All sub-circuits share the same public RSA modulus N and exponent set $\{e, e_1, \ldots, e_4\}$; only their wire labels differ.

Connector elements: To stitch sub-circuits together we introduce *connector elements*, which are to reusable blocks what adaptor elements were to single-use gates. Assume the final wire of a previously executed sub-circuit carries the (reblinded) label $y_1^{r_1}$, while the first wire of the next sub-circuit expects $x_2^{r_2}$. The evaluator locally derives a connector

$$C \;=\; \frac{x_2^{r_2}}{y_1^{r_1}}$$

publishes C_k once, and multiplies it into the hand-off label to produce the correct input for the subsequent block. In effect, each connector simultaneously *reblinds* the outgoing label and *adapts* it to the receiving block.

Circuit Size: A 256-bit field-multiplication block has 256 output wires, each with two labels, hence 2×256 connector elements are required per reuse. With $k = 30{,}000$ invocations and 256-byte RSA elements, the total off-chain payload becomes

$$k \;\times\; (2 \times 256) \;\times\; 256 \text{ bytes} \;\approx\; 4 \text{ GB},$$

a $\approx$1000× reduction relative to the naive 5 TB scheme of Section 6.

Limitations for the full verifier: While each *sub-circuit* is safely reusable, the *entire* SNARK-verifier composition is *not*: reblinding a connector for round i inherently reveals information about round $i{+}1$. Consequently the verifier as a whole must still be re-garbled for every reuse, but its size is dominated by the 4 GB connector set rather than by gate labels.

Proving complexity. Unlike adaptor elements, connector elements cannot be checked in plaintext and must be validated in zero knowledge. We mitigate the overhead by picking the connector reblinding factors from a set of small consecutive primes – their modular inverses serve as the blinding exponents. As a result, proving the correctness of each connector requires only one small-exponent exponentiation, and the quotient check $x_2^{r2}/x_1^{r1} = C$ is expressed as a single multiplication $x_2^{r2} = C \cdot x_1^{r1}$. The cumulative proving effort therefore grows linearly with the number of connectors, totaling at a few million lightweight exponentiations.

8 Conclusion

BitVM3 significantly advances Bitcoin's contracting capabilities by using an RSA-based garbled circuit to move SNARK verification off-chain. This approach reduces the on-chain footprint to a 56 kB assertTx and a 200-byte disproveTx, but requires a multi-terabyte off-chain data setup; introducing reusable sub-circuits and their connector elements cuts this requirement down to roughly 4 GB. While this trade-off enables much more capital efficient trust-minimized bridges for second layers like rollups and sidechains, future work must focus on reducing the off-chain data burden even further and on exploring techniques for safely reusing the entire verifier. Ultimately, BitVM3 demonstrates a viable path toward using Bitcoin as a secure settlement layer for arbitrarily complex computations.