Robotic Grasping and Contact: A Review

Antonio Bicchi

Centro “E. Piaggio
University of Pisa
56126 Pisa Italy
bicchi@ing.unipi.it

Vijay Kumar

GRASP Laboratory
University of Pennsylvania
Philadelphia, PA 19104
kumar@cis.upenn.edu

Abstract

In this paper, we survey the work in robotic grasping related areas that has been done over the last two decades, with a bias toward the development of the theoretical framework and analytical results in this area. In addition we assess the state of the art in this area and outline some of the important open problems.

1 Introduction

The human hand is used in a variety of ways. In particular, the three most important functions are to explore, to restrain objects, and to manipulate objects (relative to the wrist and to the palm). The first function falls within the realm of *haptics*, an active research area in its own merits [35]. We will not attempt an exhaustive coverage of this area. The work in robot grasping has tried to understand and to emulate the other two functions. We will distinguish between the task of restraining objects, sometimes called *fixturing*, and the task of manipulating objects with fingers (in contrast to manipulation with the robot arm), sometimes called *dexterous manipulation*.

The analysis of mechanical fixtures and jigs goes back to the work of Reuleaux [73] in 1875. He analyzed the conditions under which an object or part can be completely restrained and defined *form–closure*, the condition under which a positive combination of the contact wrenches acting on an object, without assuming any friction, can resist any disturbance force. He showed that planar bodies require at least four frictionless contacts for form closure. Somoff [78] in 1897, showed that at least seven frictional contacts are required for form closure in the spatial case [41].

While grippers and fixtures have been used extensively in industry, one can argue that the field of robot grasping started with the work of Asada and Hanafusa [1] and Salisbury's first attempts to develop a three-fingered robotic hand [49]. The most sophisticated multifingered hand built to date is the Utah-MIT hand [31]. While it was a beautifully designed and versatile hand with 32 actuators, it also illustrated some of the difficulties in robot control and motivated a lot of the interesting work that is reviewed later. In contrast to this work, there are a number of efforts to get away from general purpose, dexterous hands and that instead focus on reduced complexity multifingered hands. One of the first such attempts was a three fingered hand powered by four actuators [94] that was designed to grasp by *enveloping*.

Enveloping grasps [91], in contrast to fingertip grasps, are formed by wrapping the fingers (and the palm) around the fingers. Enveloping grasps are superior in terms of restraining objects. In fact, this is easily seen in human grasping where fingertips and distal phalanges are used for fine manipulation, while the inner parts of the hand (palm and proximal phalanges) enhance the stability of the grip [15, 30]). Variations of this basic theme are also seen in grippers designed for the so called *whole arm grasps* [80, 93] and *power grasps* [52, 74]. It is also interesting to note that inspite of the wide range of grippers and robotic hands used in industry, they are almost exclusively used for for restraint and for fixturing, and not for dexterous manipulation. On the other hand, research hands, for example [31, 49],

2 Closure properties of grasps

Consider an object grasped at N contacts. It is generally assumed in the literature that all contacts are point contacts and idealizations such as a line or surface contact can be approximated by two or more point contacts. Each point contact can be modeled as either a frictionless point contact, a frictional point contact, or a soft contact [79]. A frictionless contact is defined as a contact in which the finger can only exert a force along the common normal at the point of contact. A frictional contact (sometimes referred to as a point contact with friction) is defined as a contact that can transmit both a normal force and a tangential force, while a soft contact also allows the finger to exert a pure torsional moment about the common normal at the point of contact. Finally, note that the term "fingers" is used as a generic term to represent any link, finger, effector, or fixture in point contact with the object being grasped.

The choice of the contact model depends upon the surface properties at the contact and the coefficient of friction. For example, if the finger has a rubber-like "skin," the contacts are typically modeled as soft. Two relatively hard, rough surfaces in contacts are typically modeled by a frictional contact, while contact between objects where there is a low coefficient of friction may be (conservatively) modeled as frictionless.

At each contact, the grasped or fixture object is subjected to a (possibly zero) normal force, tangential force, and torsional moment about the normal. Let the applied wrenches are denoted by ${}^i w_N$, ${}^i w_T$, and ${}^i w_\theta$, respectively, where each wrench is a 6×1 unit vector composed of forces and moments. Let the corresponding wrench intensities are given by ${}^i c_N$, ${}^i c_T$, and ${}^i c_\theta$. We collect the N wrenches and intensities into submatrices w_N, w_T, w_θ, c_N, c_T, and c_θ, respectively. Finally, let us denote W as the combined wrench matrix, as the non zero vector of wrench intensities, and g as the (possibly zero) known external wrench. Now we are in a position to establish some basic definitions and properties of grasps that are useful for analysis.

A grasped object is in equilibrium if and only if [79]:

1. For all i, the contact forces and moments satisfy the contact constraints; and

2. The object is in static equilibrium:

$$Wc + g = 0$$

where the contact constraints include the inequality:

$${}^i c_N \geq 0,$$

and the inequalities that characterize the frictional forces and moments. Typically, Coulomb's law with an appropriately chosen coefficient of friction is used to limit the tangential forces:

$$\left|{}^i c_T\right| \leq {}^i \mu_T \; {}^i c_N.$$

Similarly, a Coulomb like frictional law can be applied to a frictional moment [49]:

$$\left|{}^i c_\theta\right| \leq \; {}^i \mu_\theta \; {}^i c_N.$$

Alternatively a coupled linear model of the friction cones [6] for tangential force and the torsional moment, which has been shown in some cases to show better experimental accuracy, can be used.

Many equilibrium grasps also meet the conditions of force closure. Note that in the grasping literature, there is a wide disparity in the terminology used (see [90] for a discussion of terminology). Adopting the terminology of Nguyen [60] and others, we define a grasp to be force closed if and only if it is in equilibrium for any arbitrary wrench $\hat{w}$.

In other words, a grasp is defined as force closed if and only if, for any arbitrary wrench $\hat{w}$, there exists an intensity vector λ satisfying the contact constraint inequalities, such that

$$W\lambda = \hat{w}.$$

Note that the intensity λ can be different from the actual intensity c, just as the hypothesized $\hat{w}$ can be different from g.

According to Lakshminarayana [41] and Nguyen [60], form closure is defined as a condition of complete restraint in which the grasped body can resist any external disturbance wrench, irrespective of the magnitude of the contact forces. Form closure is a stronger condition than force closure. More formally, a grasp is defined as form closed if and only if it is force closed with frictionless contacts.

Equivalently, a frictionless grasp with N unilateral wrenches is defined as form closed if and only if [90], there exist $\lambda > 0$, such that

$$W\lambda = 0,$$

and W is full rank.

In 1875, Reuleaux defined the term form closure in cases where a positive combination of the wrenches used to grasp an object could counter-react any possible disturbance force. He also gave a simple graphical technique to test for planar form closure [73]. This idea was refined much later by Nguyen [60] who gave a precise graphical test to determine whether or not a planar four contact frictionless grasp is form closed. Salisbury [79] gave a simple analytical procedure to test for either form or force closure. Trinkle [90] provided a quantitative test for form closure based on linear programming that provides a measure of how far the grasp is from losing its form closure.

There are several important papers dealing with the number of frictionless contacts required to obtain form closure. Reuleaux proved in 1875 that planar bodies require at least four frictionless contacts to achieve form closure. Reuleaux also showed that there exist various geometrical shapes in which it is impossible to completely constrain by any number of frictionless surface contacts. Selig and Rooney [83] classified Reuleaux's surfaces based on group theory. They derived a simple classification of surfaces which cannot be grasped, and showed that these surfaces are equivalent to those proposed by Reuleaux. In 1897, Somoff [78] extended Reuleaux's planar results to six dimensional space when he showed that at least seven frictionless contacts are necessary to achieve form closure of a spatial object, a result which Lakshminarayana independently re-proved in 1978 [41]. Mishra, Schwartz, and Sharir [53] were the first to set an upper bound of twelve frictionless contacts needed to achieve form closure on any spatial object with a piecewise smooth boundary (except Reuleaux's surfaces), and six frictionless contacts for any piecewise smooth planar object. Markenscoff, Ni, and Papadimitriou [47] showed that any planar object with a piecewise smooth boundary (except a circle) can be always form closed with no more than four frictionless contacts. For spatial objects, they showed that in most cases (including all polyhedra) a spatial object can be form closed with only seven frictionless contacts. Constructive procedures for placing contacts on given objects to achieve form–closure have attracted much attention in the literature, due also to the relevance to the fixturing problem (see e.g. the early work of [47], and more recently

[25, 86, 7, 45, 44, 95]). Other literature on the synthesis of force–closure grasps include [60, 63, 69, 70].

Based on the discussion thus far, the closure properties of a grasp depend on the locations of the contact points and the contact normals, but not on the shape of the object and the contacting effectors. This is because these closure properties depend on a first order kinematic analysis. If second order effects are examined, it is necessary to incorporate a model of the curvature of the contacting surfaces. Now even if a grasp is not form (force) closed, second order effects may guarantee the closure of the grasp. The formulation of such second order effects is discussed in [27, 76, 77, 92]). Higher order kinematic effects require the derivatives of the curvature and Christoffel symbols characterizing the contacting surfaces [81]. While third and higher order closure properties have not been formally defined, we now have a roadmap in this direction. We can now claim that such closure related properties are well understood and well-known techniques for analysing grasps exist.

3 Force Analysis

A crucial problem in robot grasping is the choice of grasp forces so as to avoid, or minimize the risk of, slippage. The *internal forces* [79], also sometimes called the *interaction forces* or the *squeeze forces*, are the contact forces lying in the nullspace of the grasp matrix W. It turns out, there is a unique decomposition of the grasp forces into the *equilibrating forces*, or the forces that lie in the range space of W, and the internal forces [39, 40], also called the *manipulation forces* [58]. The problem of choosing contact forces, or actuator forces if the kinematics and/or dynamics of the fingers are considered, so as to realize the required manipulating forces required by the task, while imposing constraints to prevent slip, is often referred to as the *force distribution* problem. This problem also occurs in other robot systems with closed kinematic chain, including legged locomotion systems and cooperating mainipulators (see, e.g. [61, 36, 43, 59, 97, 58, 62, 9]).

The problem of determining the appropriate internal forces can be posed as an optimization problem. Different approaches including linear programming [36], pseudo inverse [39], and mathematical programming [36] have been proposed. An important property of the nonlinear constrained optimization problem is convexity. This property, used first in [3], enables efficient solutions to an otherwise very complex problem: [3] proposed integration of an ODE as an iterative solution to the problem; [11] noticed that nonlinear friction constraints can be rewritten as positive–definiteness constraints on suitable matrices, and used projected gradient flow methods to optimize; [42] further exploited the matrix formulation of [11] to transform the problem in the format of a standard linear matrix inequality (LMI) problem, for which off–the–shelf, effective software exists.

It is important to note that force closure does *not* guarantee stability. Any definition of stability must regard the grasp as a dynamic system and describe the properties of the dynamic system when it is perturbed from an equilibrium configuration. Clearly, it is possible to use a classical Lyapunov definition to the stability of a grasp. Here, we pursue a definition that allows quasi-static analysis. Consider a grasp with elastic fingers [1, 15]. We can derive all contact forces from a potential function $\phi(q)$, where q describes the configuration of the dynamic system. If for every small perturbation δq from an equilibrium grasp, $\delta\phi > 0$, by Lagrange's theorem the equilibrium configuration is said to be stable. If on the other hand, a small perturbation δq such that $\delta\phi < 0$ exists, the equilibrium grasp is unstable [28]. Thus the stability of the grasp is effected by the local properties of the geometry of the grasp and the force distribution, in addition to the locations of the conctact points and the contact normals.

Salisbury [79] established a basic framework for testing the stability of a grasp. He showed that a grasp is stable if the stiffness matrix (which characterizes the grasp) is positive definite. Cutkosky and Wright [17] looked at the specific case of a two-fingered grasp, and established relationships between local geometry and stability with a simple decoupled model for the stiffness of the servo control loops. They examined a two point grasp of a rectangular object, where the grasping fingers were free to rotate, and found that the stability depended upon the forces applied and the local curvature. Their results were different for each of the various types of fingertips employed (pointed, curved, soft, etc). Cutkosky [15], in his doctoral dissertation, generalized these results to include any grasp (not just two-fingered grasps). He analyzed the stability of the grasp in much greater detail than Salisbury, showing that the stability is also a function of the fingertip models and the small changes in the grasp geometry. Cutkosky and Kao [16] extended the work of Cutkosky by including the compliance of the fingers grasping the object. They gave an example to show that a force closed grasp is not always stable, when the effects of the finger compliance are included. Utilizing a virtual spring at each contact, Nguyen [60] proved that force closed grasps can always be made stable, by adjusting the applied forces at each finger. He also showed that the curvature of the object can increase or decrease the relative stability of a grasp (however, he neglected the curvature of the contacting finger). Mishra and Silver [55] studied the relationship between stable human and robotic grasps. Montana [57] examined what he referred to as contact grasp stability, or the tendency of the grasped object to return to the same point of contact. He considered grasps in which the fingers were permitted to rotate and the grasped object was restricted to pure rolling on the surface of the fingers. With these assumptions, Montana showed that curvature plays a strong role in determining the contact grasp stability of non-force closed grasps. He showed (through examples) that two grasps could have differing contact stability, even though they were identical except for the curvature of the contacting surfaces.

Trinkle, Farahat, and Stiller [90, 92] were the first to look at a general formulation for the stability of non-force closed grasps. Their work, which is limited to polygons with vertex contacts, explores a linear programming approach to determining what they term "first and second order stability cells." They defined a first order stability cells as the neighborhood about a form closed grasp which is also form closed, and second order stability cells as the stable neighborhood about a stable grasp which is not form closed. Rimon and Burdick [76, 77] developed the concept of first and second order mobility of a grasp. They defined a grasp to be immobilized if the mobility index (which shows the first instantaneous degrees of freedom) is zero, and showed that a first order mobility index of zero is equivalent to form closure for frictionless grasps. They also proved that when the first order mobility index is greater than zero, by considering the curvature of the contacting surfaces a new lower mobility index was often achieved. This lower mobility index is what they defined as the second order mobility index. Rimon and Burdick [76] showed that deterministic, second order immobile grasps without any external force are stable. Howard and Kumar [28] incorporated the second order effect of contact curvature, the compliance at each contact, the magnitude of contact forces and friction, in their analysis of stability.

A more exhaustive analysis of stability must include the control laws used for actuating the hand joints, the mechanical impedance of the system, and the contact models that describe the interactions between the fingers and the objects. To our knowledge this has never been carried out. However, the modeling of finger object contacts has generated a lot of attention and we discuss this next.

4 Contact models

Kinematics of contact Contact kinematics is a study of the relationship between the location of the point of contact as a function of the relative motion of two contacting bodies. We refer to several fundamental papers in this area. Cai and Roth [12] studied rigid planar bodies in point contact. Using time derivatives of the planar curves, they derived a relationship for the velocity of the location of the point of contact as a function of the relative angular and linear velocity between the contacting bodies, as well as the curvature of the bodies. They also derived the relationship between the acceleration of the point of contact as a function of the relative angular and linear velocity and acceleration between the contacting bodies, as well as the curvature and change in curvature of the bodies. Montana [56] extended this work to rigid spatial bodies in point contact. He derived the relationship to describe the velocity of the point of contact on the surface of two spatial contacting bodies, using a differential geometric description of the surfaces. On the basis of on his equations, Montana explored such applications as rolling a sphere between two arbitrarily shaped fingers, determining the surface properties of an unknown object, and fine grip adjustment. Sarkar, Kumar, and Yun [81], extended the work of Cai and Roth, and Montana. Sarkar re-derived Montana's equations using standard terminology employed for intrinsic surface properties. He then derived the relationship between the acceleration of the point of contact as a function of the relative angular and linear velocity and acceleration between the contacting bodies, as well a differential geometric description of the surface of the bodies. These results were used to investigate the control of a rolling sphere between two planar fingers.

Contact compliance The importance of modeling the finger-object contact and the role of compliance in grasping has been investigated by many authors, beginning with [24]. However, it is particularly difficult to model the relationship between small object/finger displacements and changes in contact forces arising from these displacements.

Such contact problems have been studied extensively in the solid mechanics community in the context of rail-wheel interaction [34] and analysis of ball and roller bearings [33]. In particular, the uniqueness and existence of solutions of elastic bodies in static contact have been addressed by Duvaut and Lions [20]. In robotics, soft finger contacts have been explored by Cutkosky [15]. Sinha and Abel [85] proposed an elastic contact stress model for finger-object contacts in multifingered grasping and a variational approach for quasi-static analysis. Wang, Kumar, and Abel [98] proposed a similar approach for dynamic analysis. They developed a mathematical programming approach for frictional, elastic contacts as well as viscoelastic contacts in which the inertial forces due to the deformations at the contacts are neglected. Uniqueness and existence of solutions is proved in the elastic frictionless case. Paul and Hashemi [34] looked at non-conformal contacts. They developed a numerical method of modeling the pressure distribution on closely conforming elastic bodies. While the distributed parameter models yield accurate results, the solutions require computation-intensive numerical methods. A possible simplification for conformal contacts is provided by the Winkler elastic foundation model [33]. Analytical models for elastic contacts, including the solution by Hertz, are discussed by Johnson [33]. Hertz's model can be used to predict the elasticity at each contact as well as the pressure distribution across each contact patch. Hertzian contact theory is probably the most widely used analytical contact model, and variations of this are used in [28, 76, 77].

Cutkosky [15], in his analysis of the stability of the grasp, showed that the stability is a function of the fingertip models and the small changes in the grasp geometry. Cutkosky and Kao [16] extended the work of Cutkosky by including the compliance of the fingers grasping the object. They gave an example to show that a force closed grasp is not always stable, when the effects of the finger compliance are included. This line of thinking has been further refined over the last several decade. Relations of compliant (and rolling) contacts with the stability of the grasp have been considered by

[57, 90, 28, 89].

One of the very hard problems is getting an accurate and tractable model of contact compliance, particularly in the tangential direction. This has been recognized to be a difficult problem in the mechanics literature as well [33]. In addition to this, a tractable model of friction is necessary for grasp analysis, one that accurately predicts slip and one that lends itself to stability analysis.

5 Measures of grasp performance

More recent work in the literature has tried to develop quality measures for grasps. One such measure can be derived from the conditioning of the grasp or wrench matrix W and is directly connected with the closure properties of the grasp [43]. In a similar fashion, other structural properties can be derived from the characteristic matrices, for example, controllability and observability [71, 8, 72].

When an object is restrained or grasped with mutliple effectors, there are two, often conflicting, measures of grasp performance. First, if the fixtures can be accurately positioned, the system's ability to reject wrench disturbances is a measure of grasp stability. The grasp stiffness matrix, or a frame invariant measure of the minimum grasp stiffness [10], provides one choice for a performance metric. This assumption of being able to accurately position the end-effector is extensively used in the fixturing and grasping literature. However, when there are errors in positioning and orienting the end-effectors, it is important to choose a grasp so that the system performance is insensitive to these positioning errors. Thus, it also makes sense to minimize the dependence of grasp forces on such positioning errors.

Howard and Kumar [28] develop the theory needed to combine the stiffness matrices at each contact to calculate a grasp stiffness matrix. While the signs of the eigenvalues allow a test of grasp stablity, the eigenvalues themselves are not invariant with respect to changes in refererence frames [27]. Bruyninckx *et al.* [10] develops a frame invariant measure of stability that is based on the grasp stiffness matrix and a metric on the Euclidean group. Lin develops a frame-invariant quality measure that essentially minimizes the "object deflection" when the grasped object is subject to force disturbances [44]. The basic idea here to scale the eigenvalues measuring the rotational stiffness by a characterisic distance to an edge of the object. Thus it is possible to develop a scaled stiffness matrix and the smallest eigenvalue of the scaled matrix characterizes the system.

The focus in the above work is to quantify the ability of a fixture to reject disturbances due to external forces on the workpiece [18]. This is clearly a measure of performance that is relevant. However, the robustness of a grasp to errors in positioning the effectors has not been addressed in this literature. Sugar and Kumar develop a second measure of performance that characterizes this robustness and discuss an approach to optimizing fixtures based on both measures [88]. In this connection, the control of grasping [84, 75] and the effects of of uncertainties ([14, 19]) are particularly important.

Unfortunately most of these measures are based on the assumptions of small perturbations: displacements, forces and errors. There is no question that more global measures would be more useful. For example, in stability analysis, a figure relating to the size of the basin of attraction of the equilibrium, indicating how large a perturbation can be without causing instability would be desireable. However, the nonsmooth nature of grasp dynamics because of the unilateral constraints precludes such a thorough analysis.

6 Grasping and the kinematics of the hand

It is interesting that much of the literature in grasping actually ignores the kinematics of the fingers or the articulations that are involved in contacting the object. While Reuleaux's problem of form closure justifiably focuses on the geometry of the object and the arrangement of contacts, it is difficult to analyze a grasp without modeling the fingers and the interaction of the fingers with the object. Thus, for example, it is difficult to analyze force-closure without considering the dynamics or at least the kinematics of the contacting fingers.

Trinkle *et. al.* explore the kinematics of enveloping grasps [91] using the restrictive but conservative assumption of frictionless contacts. The kinematics of fingers with two or three point contacts with fingertips and palms have been studied by [66, 21]. While the analysis of form–closure is intrinsically geometric, force–closure is tightly linked to the kinematics and characteristics of the end–effector. In fact, it is possible that a geometric analysis of a grasp may predict force–closure, but a careful analysis of the kinematics may reveal that this is not the case [27]. Definitions of force–closure that take into account the kinematics of the gripping device were proposed in [3], along with an exact algorithm for testing such property. Yoshikawa proposes a new set of definitions for closure properties, including what he calls active and passive closures, to explicitly model the properties of the grasping mechanism [101]. Unfortunately, much of this, and other related work [**?**, 67, 29] is based on instantaneous kinematics .

An end–effector that has fewer degrees–of–freedom than necessary to arbitrarily impart motions/control forces at all contacts is sometimes referred to as *kinematically defective*). In fact such grasps cannot be considered be pathological - such statically indeterminate grasps are common in simple industrial grippers. In defective systems, where the hand Jacobian matrix is not full rank, it is not possible to command an arbitrary set of grasping forces [2]. This occurs when the nullspace of the grasp matrix and the nullspace of the transpose of the hand Jacobian have a nontrivial intersection. This is usually the case in power grasps. The modeling of the kinematics and manipulability of whole–hand manipulation in kinematically defective cooperating limbs is discussed in [5, 100, 71]. The more a grasp is defective, the more robust it is in restraining an object with respect to external disturbances, but the lower the "manipulability", and also lower the sensitivity to positioning errors. Grasp force optimization techniques for power grasping are discussed by [3, 102].

Many open problems remain to be solved in order to be able to design robot hands to effectively exploit defectivity to increase grasp robustness and reduce hardware complexity. Among these, perhaps the most important is the need for a reliable estimate of contact compliance, arising with statically indeterminate grasps. This allows the calculation of contact forces, and the development of models that relate joint displacements and torques to contact forces. While it is difficult to expect that such models can be precisely known, it is conceivable that the relevant parameters can be identified on–line, in a fashion similar to that used to estimate inertial parameters in adaptive controllers for robots.

7 Dynamics

The ability to predict the dynamic behavior of a grasp with a given model including the control algorithms, is critical to the design of the grasp. In multifingered grippers, as in legged locomotion systems, multi-arm systems, and other constrained robot systems, several limbs are used to constrain and manipulate an object [39, 43, 52]. The dynamic analysis and the simulation (the prediction of motion given the external forces and moments on the system) of such systems is central to the design of such systems and the development of control algorithms [99, 82].

When there are contacts between nominally rigid bodies, the constraints that are arise in such situations are called unilateral constraints because the contact forces (and relative displacements) can be defined so that they are non-negative. Featherstone [22], Lotstedt [46] and Mason and Wang [50] pointed out some of the inconsistencies which arise when rigid body models are used with Coulomb's empirical law of friction in unilateral systems. For example, if we consider the simulation of a rod sliding along a rough ground in a plane with a single contact, there are configurations in which no solutions (that are consistent with the constraints) exist, and others in which the solution is not unique. Wang, Kumar, and Abel [98, 99] performed a dynamic analysis of the peg-in-the-hole insertion problem and showed that there was a range of parameters during two-point-contact for which there were either no solutions or two solutions for the accelerations. Quasistatic analysis is also known to exhibit such inconsistencies [28].

The inconsistencies and ambiguities in the dynamic analysis of frictional contacts have been attributed to the approximate nature of Coulomb's model and to the incorrect assumption of rigidity. Recently, there has been some attention in the robotics community on overcoming these shortcomings by using rigid body models to predict the gross motion while using compliant contact models to predict the contact forces and the local deformations [38].

One of the main difficulties that is present in multifingered grasps, and a feature that is particularly true of such grasps as power grasps and enveloping grasps, is that the number of independent contact forces is much larger than the number of actuators. Thus, from a controllability standpoint, not all the contact forces are controllable.

The analysis of statically indeterminate grasps or grasps in which there is no unique solution to the inital value problem is simply not possible unless one explicitly models the compliance at the contacts [15, 28, 60, 38]. Of course such contact models tend to be more complex and the parameters are more difficult to identify. Further, it is harder to simulate systems in which the time scale for the dynamics of contact interactions is significantly different from the time scale of rigid body dynamics [64, 87]. Thus, although efficient, approximate algorithms for "impulsive dynamic simulation" that incorporate approximate impact models for collisions are available [54], it is very difficult to write accurate simulators for dexterous and fine manipulation where the contact forces may be finite and the results may be sensitive to the parameters in the contact model.

8 Concluding Remarks

This paper presented a survey of work in robotic grasping over the last twenty years. It is impossible to do justice to the work in this area, particularly because of the breadth of the field and its close connection to dexterous manipulation, fixturing, and haptics. We chose to focus on issues that are central to the mechanics of grasping and the finger–object contact interactions. In addition, the review mainly addressed research that has established the theoretical framework for grasp analysis, simulation and synthesis. Because of the limitations on space, we have not given the algorithmic aspects, and the applications the attention that they deserve. Finally, we have described some of the key problems that will most likely engage researchers in the years to come.

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