NETWORK FLOW EXAMPLES SOLVED

Book: Algorithm-Design-by-Jon-Kleinberg-Eva-Tardos.pdf

Chapter 7: Network Flow Exercise Question 2 [PDF pg. 441]

2. Figure 7.26 shows a flow network on which an *s*-*t* flow has been computed. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. (Edges without boxed numbers—specifically, the four edges of capacity 3—have no flow being sent on them.)

(a) What is the value of this flow? Is this a maximum (s,t) flow in this graph?

(b) Find a minimum *s*-*t* cut in the flow network pictured in Figure 7.26, and also say what its capacity is.

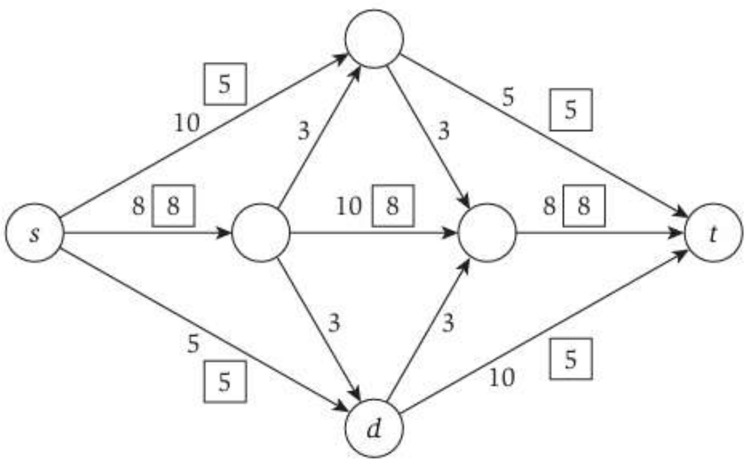


Figure 7.26 What is the value of the depicted flow? Is it a maximum flow? What is the minimum cut?

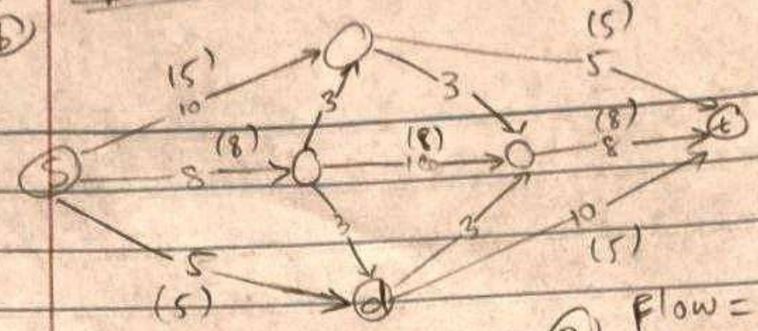
There’s a misprint, these nodes are *{a, b, c, d}*

Solution:

2 Fig 7.26

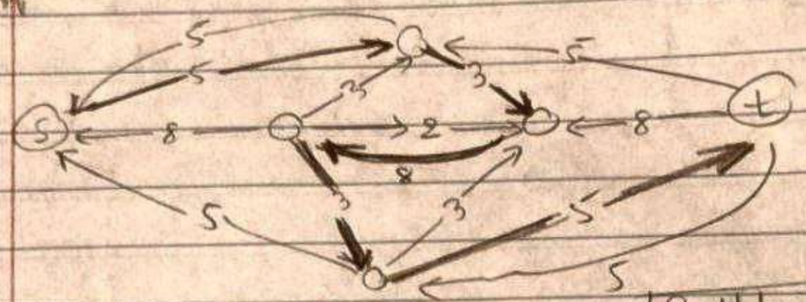
a & b

G_f:



a Flow = 5+8+5 = $\boxed{18}$

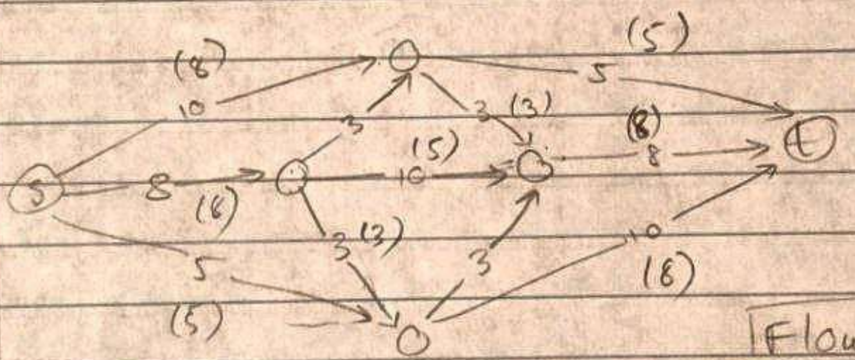
Residual Graph G_f':



$\boxed{\text{Bottleneck} = 3}$

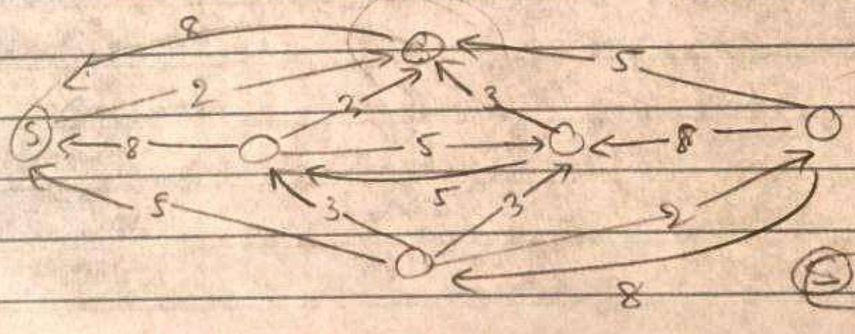
a As we still can go from s to t in the residual graph. The given flow is $\boxed{\text{NOT}}$ max (s-t) flow

G_f



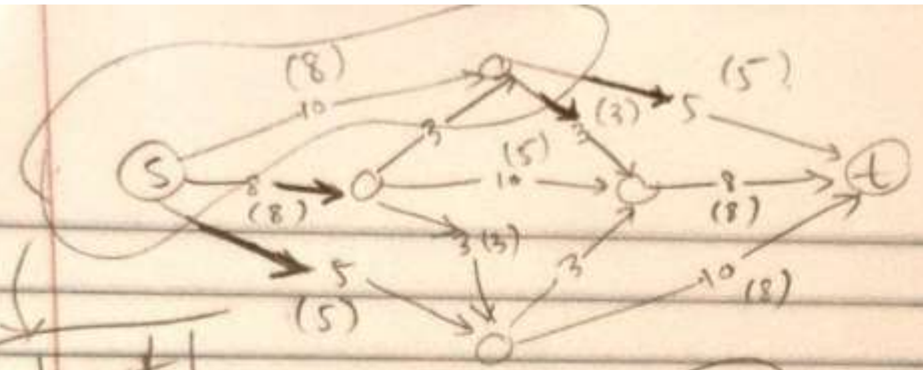
$\boxed{\text{Flow} = 21}$

G_f'



CANNOT GO FROM s to t anymore

s → 0 → min s-t cut

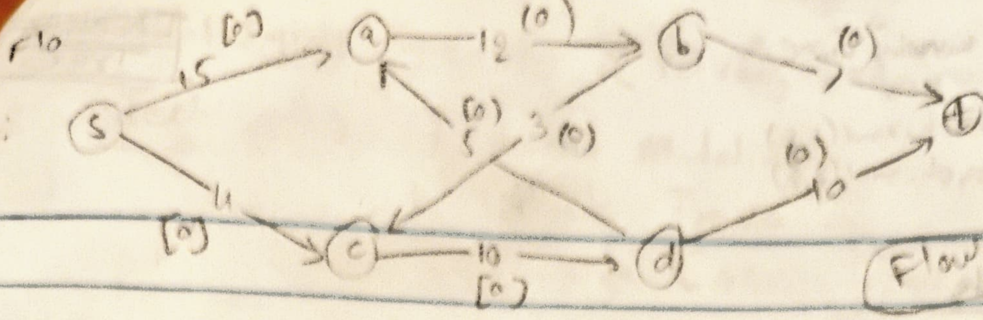


$\boxed{\text{s-t cut}}$ → Flow out of min s-t cut = 5+3+8+5

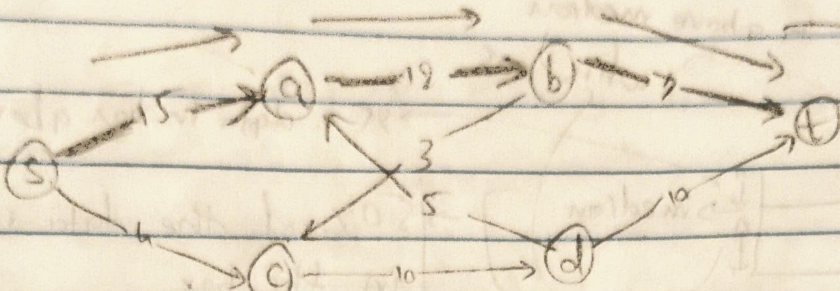
$\boxed{= 21}$

2) a) **18**, No it is **not** max s-t flow

b) **21** ← MAX s-t flow capacity

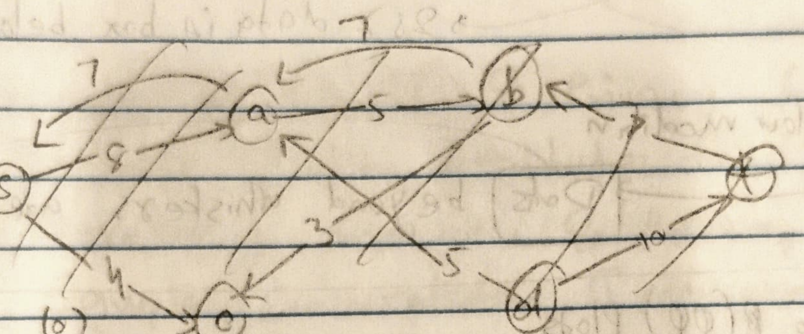


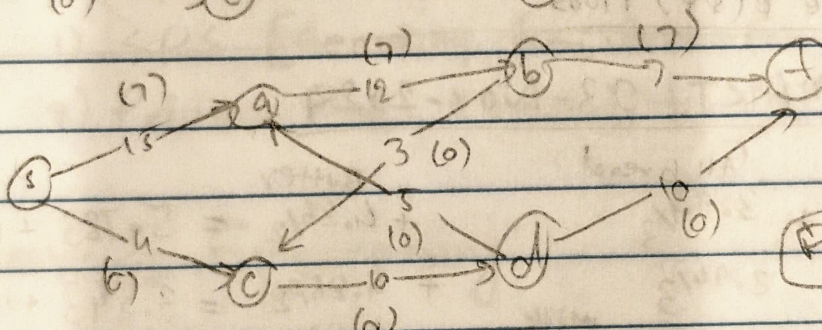
Flow = 0



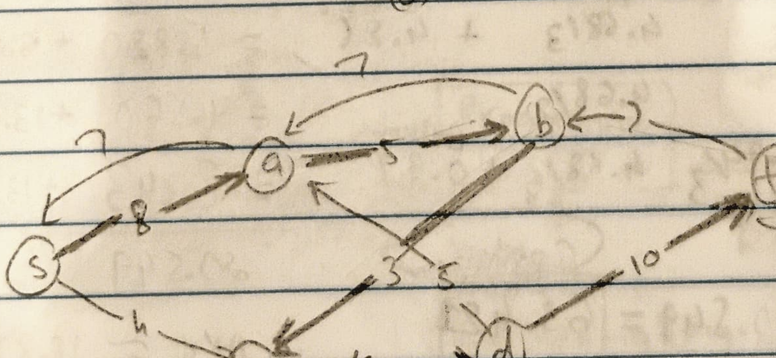
This path

Bottleneck = 7



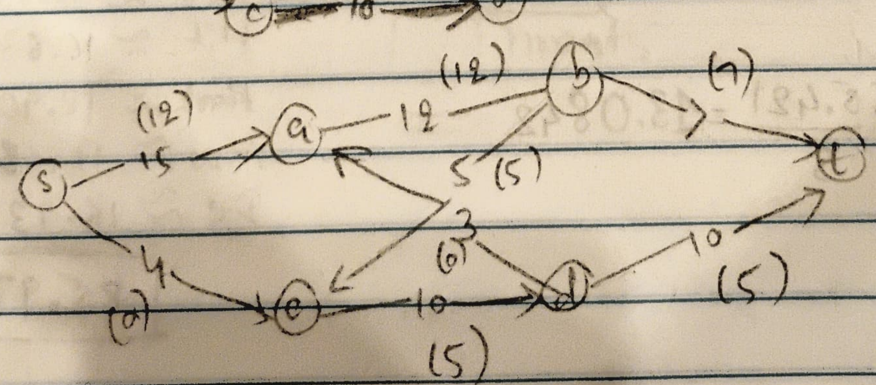


Flow = 7



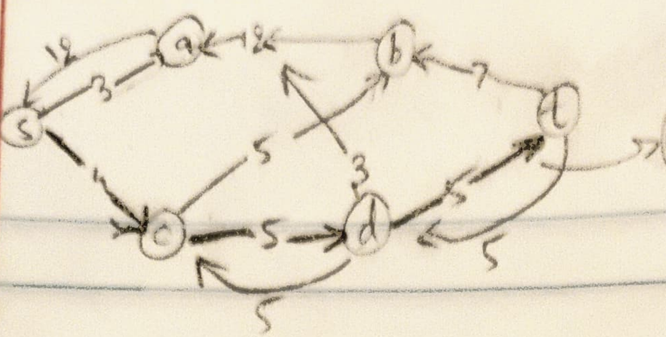
This path

Bottleneck = 5



Flow = 12

G_f:

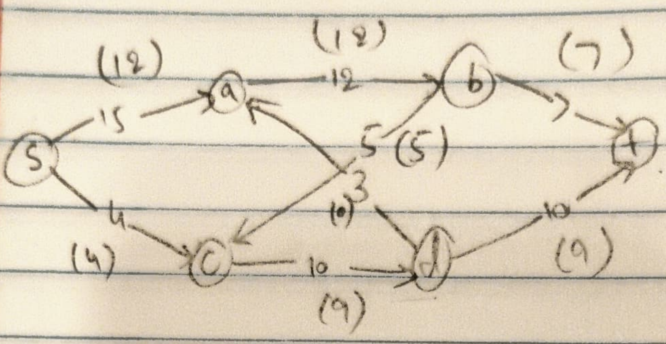


Bottleneck = $\boxed{4}$

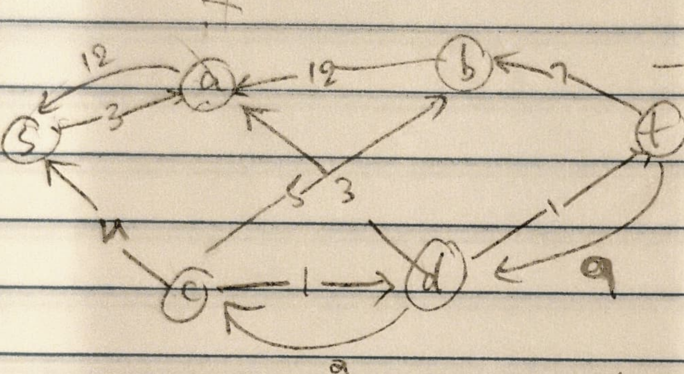
7+5=12+4

$\boxed{\text{flow} = 16}$

G:

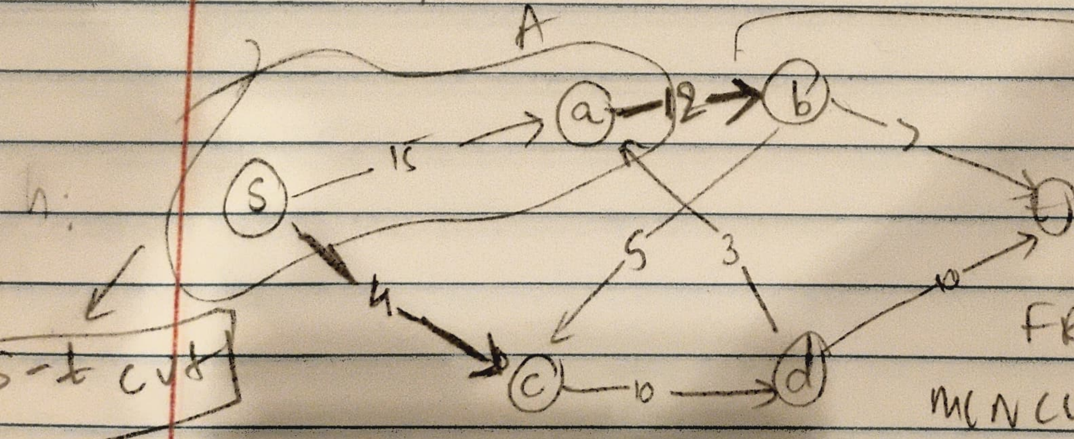


G_f:



→ At this point we are stuck that means we cannot go from source (s) to sink (t) anymore, hence

we found the optimal PATH!. Also, the minimum s-t cut will include all nodes ~~from~~ reachable from source (s) ~~...~~, in G_f i.e. residual graph G_f. It is clear that we can only ~~go~~ reach (a) from (s), hence the s-t cut will look like



This s-t cut is the min cut, AND FLOW OUT FROM THIS MINCUT IS MAX FLOW.

Which is $\xrightarrow{4} + \xrightarrow{12} = \boxed{16}$ ← This was the MAX FLOW!!!

amount equal to (cap...)
and remove flow part

Residual Graph

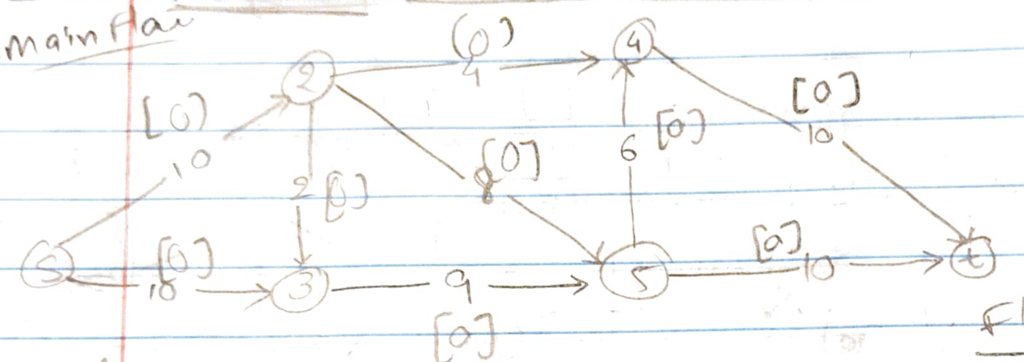
⇒ All such edges with positive residual capacity will make the Residual Graph

→ Now we can run an algorithm on this residual graph, i.e. to find MAX FLOW.

→ FORD-FULKERSON ALGORITHM

→ main flow

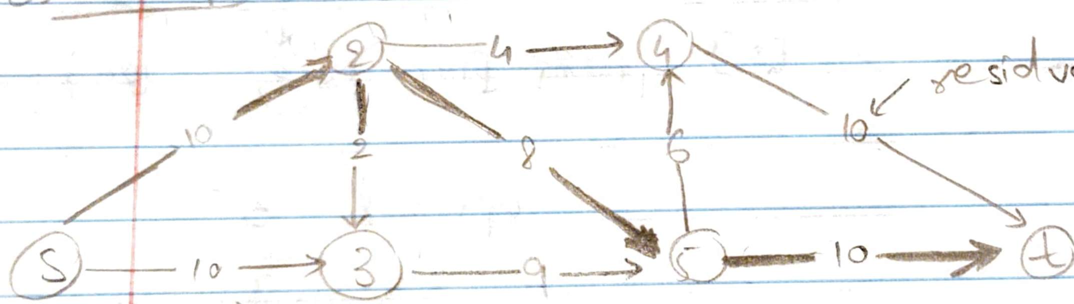
G =



Flow = 0

→ Residual Graph

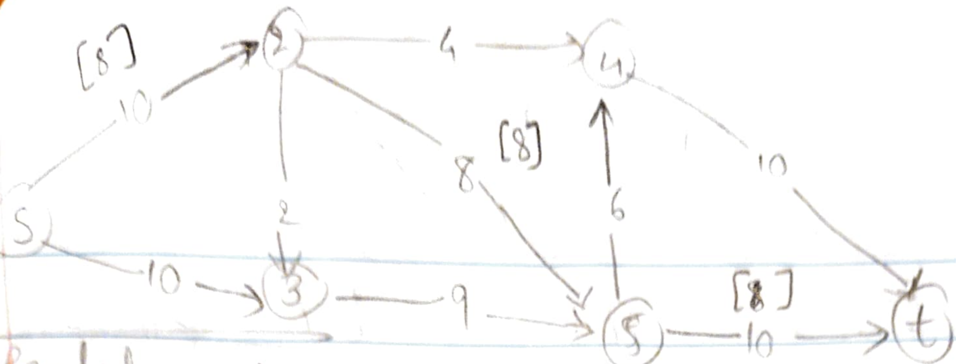
G_f



Lets choose this path, (you can choose any!)

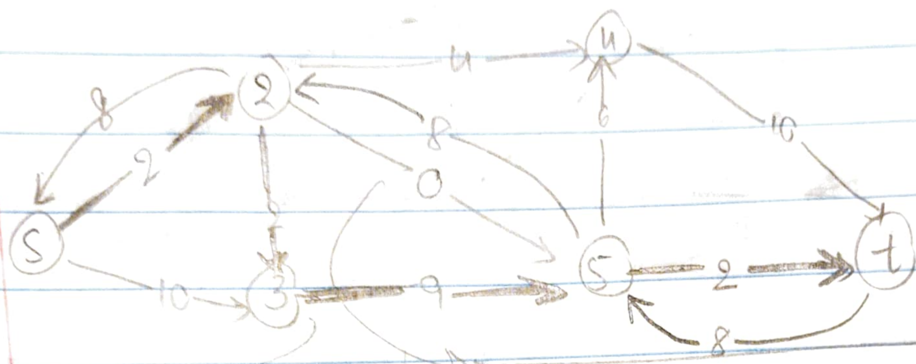
The bottleneck = edge with lowest capacity as that is the MAX Flow we can send in this p...

= 8 in this path, So take this and pass flow of 8 through this path in the main flow.



Residual graph

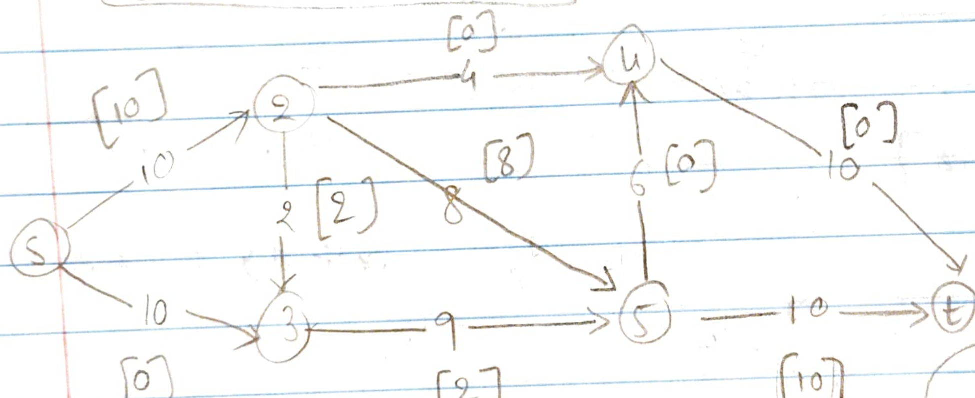
$\boxed{\text{Flow} = 8}$



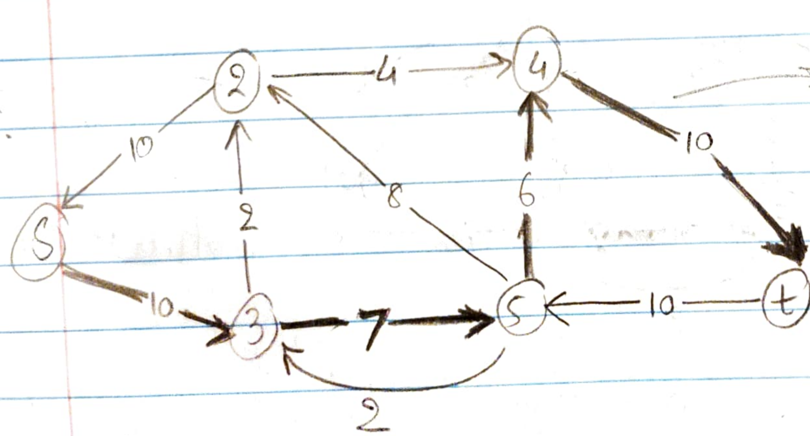
No need to draw edge if cap = 0, that means it DOES NOT EXIST

Afterwards, lets choose this path

$\boxed{\text{Bottleneck} = 2}$ → So take '2' flow on this path



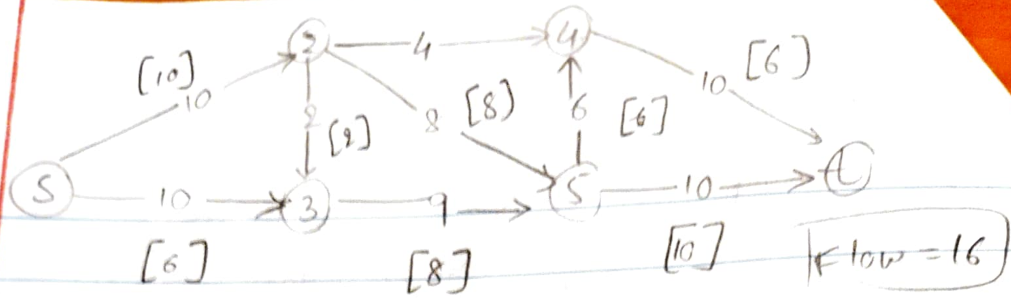
$\boxed{\text{Flow} = 10}$



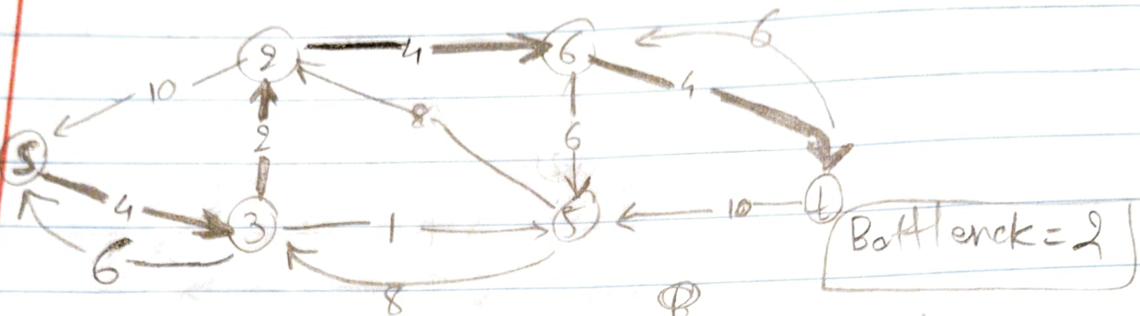
→ Next path

$\boxed{\text{Bottleneck} = 6}$

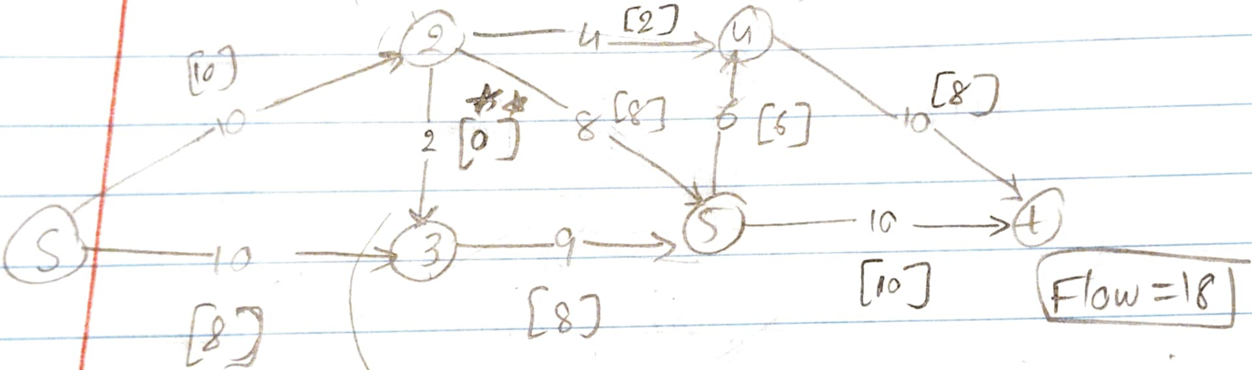
Add flow of '6' on this path in original "G"



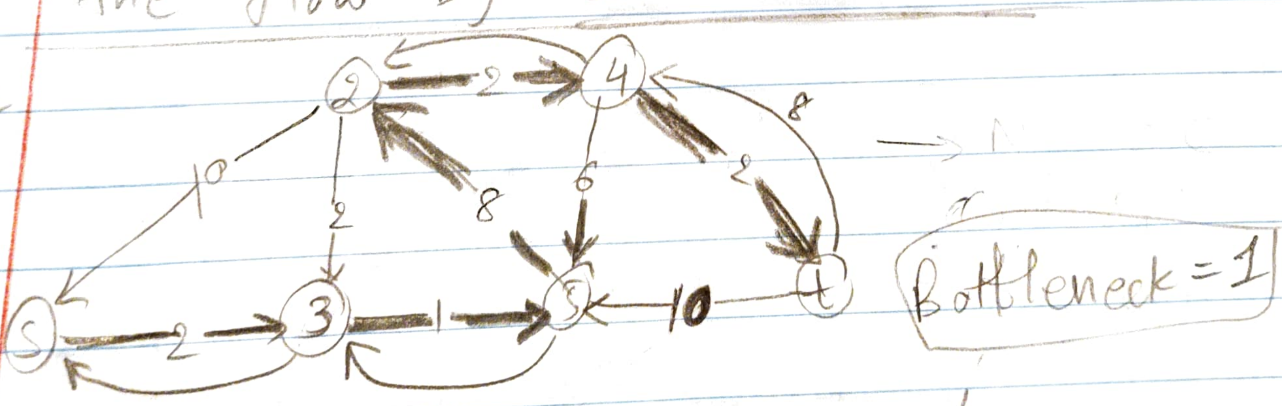
Update G_f:



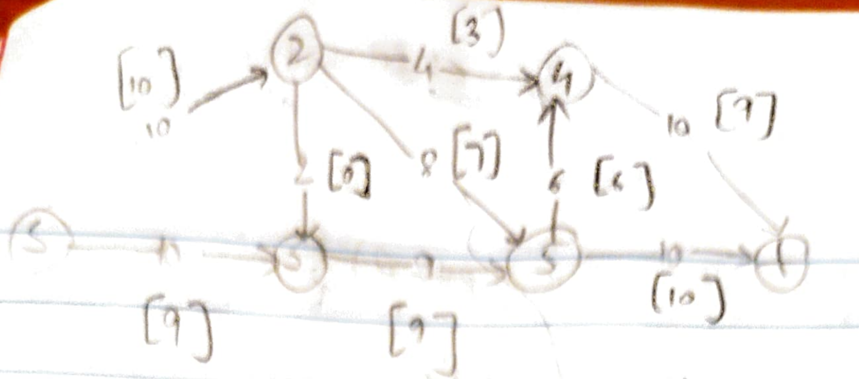
Add flow of '2' on this path in the original 'G'



This edge is actually reverse in the 'G_f'', hence it is telling us to decrease the flow by 'bottleneck amount'



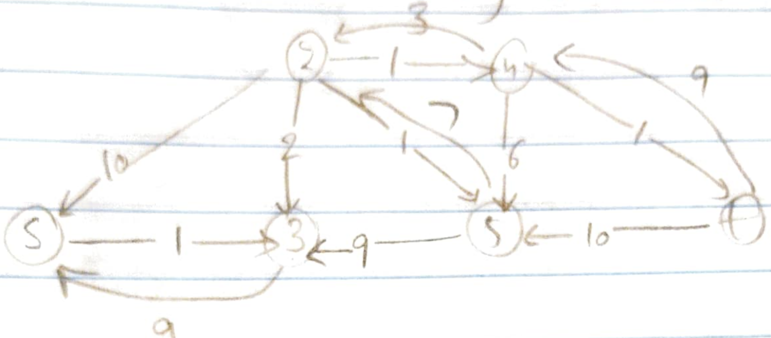
Increase flow by '1' in the main flow Graph 'G'



Flow = 19

→ This is reverse cycle in G_f, hence decrement flow by bottleneck = 1, Flow = 8 − 1 = 7

Update G_f



→ NOW WE CANNOT GO FROM SOURCE S to SINK t

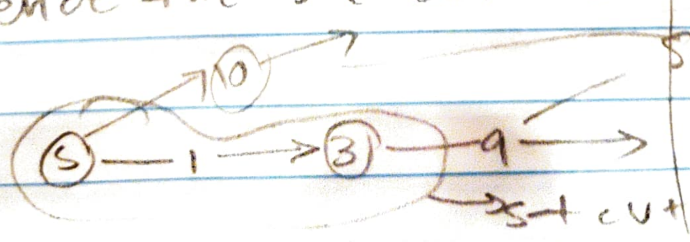
→ When this occurs, it means we found our optimal flow OR MAX FLOW OF THE SYSTEM. Hence max flow of this example = 19

→ Cut: All the nodes at this point of time that can be reached by sink S, are the "part" of S-t cut, which is NOTHING BUT THE MINIMUM CUT OR MAXIMUM FLOW → (minimum cut required to restrain maximum flow of the system)

S-t cut

In our case, only 3 is reacheable from S

Hence the s-t cut is



See the flow from s-t cut = 10 + 9 = 19 i.e. the MAX FLOW!