Assignment 2

CS1.201: Data Structures and Algorithms

Binary Trees

Due Date: 12 February 2024
Total Marks: 100

General Instructions

- The assignment must be implemented in C.
- The assignment needs to be submitted on OJ. This assignment comprises one question with two subtasks. Note that there will be two portals on OJ, one for each subtask. Even if you have submitted the second subtask, you must submit both subtasks separately to obtain full marks.
- Deadline extension requests will not be entertained.
- Resorting to any sort of plagiarism (as mentioned in the policy shared) will lead to strict penalisation at the least.

Astroworld Tragedy

We are dispatching a tech team to the remote land of Astroworld. It is inhabited by a single family, completely unaware of modern technology. The team's objective is to provide a minimal number of phones, ensuring that everyone has a means of communication and can stay informed about events in the broader world.

Subtask 1 (50 points)

You will be given the level order traversal of the family tree as an array *level_order* of size n. You have to construct this family tree from scratch, assuming it is a binary tree. Note that the level order traversal will comprise 1s and 0s, where 1 denotes a valid node and 0 denotes a null node.

Having created the tree of family members, you have to find the minimum number of phones m that needs to be distributed among them, given that each member who is given a phone can share their phone with their parent or their children. However, if someone is not given a phone, they can only be shared with i.e they can't share the phone further with others.

Subtask 2 (50 points)

Now that you have informed our team of the m number of phones that they need to distribute, they seem to have faced yet another hurdle. So, each of these m phones has a unique prime value, where the prime value of the i-th phone is the i-th smallest prime number. However, Astroworld only supports a range of l, r cacti (a unit of measurement) for two phones to communicate with each other. The cacti measure for a pair of phones can be calculated as the XOR of their prime values. You have to find out the number of such pairs whose cacti lie outside the $[l, r]$ range so that our tech team can come up with a solution before their next visit to Astroworld.

I/O Format

Subtask 1

Input Format

```
n
level_order[0] level_order[1] ...  level_order[n-1]
```

The first line contains n, the length of array *level_order*.
The second line contains *level_order*, which is the level-order traversal of the family tree.

Output Format

```
m
```

The output will just be m, the minimum number of phones required.

Subtask 2

Input Format

```
n
level_order[0] level_order[1] ...  level_order[n-1]
l r
```

The first line contains n, the length of array $level\_order$.
The second line contains $level\_order$, which is the level-order traversal of the family tree.
The third line contains the two integers l and r.

Output Format

```
m
x
```

The first line should contain m, the minimum number of phones required.
The second line should contain x, the number of pairs of phones that don't fall under their network range.

Constraints

1. $1 <= n <= 3e5$
2. $level\_order[i] \in \{0, 1\}$
3. $0 <= l <= r <= 1e5$

Sample Testcases

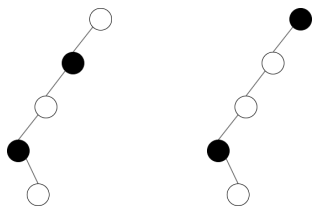
Subtask 1

Input 1

```
9
1 1 0 1 0 1 0 0 1
```

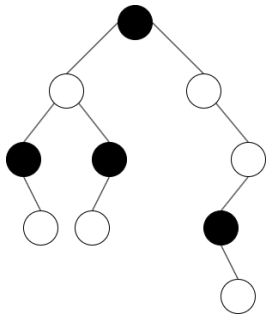
Output 1

```
2
```



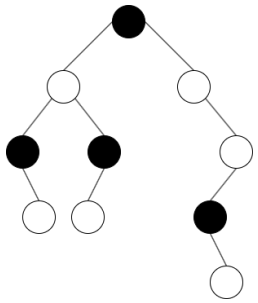
Explanation
The minimum number of phones required for every family member to have a mode of communication is 2. The images shown above are two of the valid distribution configurations.

Input 2
```
19
1 1 1 1 1 0 1 0 1 1 0 1 0 0 0 0 0 0 1
```
Output 2
```
4
```
Explanation
At least 4 phones are required for every family member to have a way to communicate.



Subtask 2

Input 1
```
19
1 1 1 1 1 0 1 0 1 1 0 1 0 0 0 0 0 0 1
3 5
```
Output 1
```
4
4
```



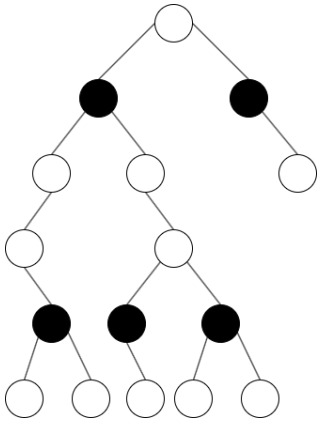
Explanation
At least 4 phones are needed to ensure everyone has a way to communicate. The above image shows one of the valid distributions of phones.
Now the prime values of these 4 phones will be $[2, 3, 5, 7]$, the first four prime numbers.
The cacti measure of all the pairs of phones will be:
$(2, 3) = 2 \oplus 3 = 1$
$(2, 5) = 2 \oplus 5 = 7$
$(2, 7) = 2 \oplus 7 = 5$
$(3, 5) = 3 \oplus 5 = 6$
$(3, 7) = 3 \oplus 7 = 4$
$(5, 7) = 5 \oplus 7 = 2$
So, the cacti of 4 pairs of phones lies outside the range of $[l, r]$ i.e $[3, 5]$ in this case.

Input 2

```
23
1 1 1 1 1 0 1 1 0 0 1 0 0 0 1 1 1 1 1 0 1 1
1
7 13
```

Output 2

```
5
6
```



Explanation

At least 5 phones are needed to ensure everyone has a way to communicate. The above image shows one of the valid distribution of phones.
Now the prime values of these 5 phones will be $[2, 3, 5, 7, 11]$, the first five prime numbers.
The cacti measure of all the pairs of phones will be:
$(2, 3) = 2 \oplus 3 = 1$
$(2, 5) = 2 \oplus 5 = 7$
$(2, 7) = 2 \oplus 7 = 5$
$(2, 11) = 2 \oplus 11 = 9$
$(3, 5) = 3 \oplus 5 = 6$
$(3, 7) = 3 \oplus 7 = 4$
$(3, 11) = 3 \oplus 11 = 8$
$(5, 7) = 5 \oplus 7 = 2$
$(5, 11) = 5 \oplus 11 = 14$
$(7, 11) = 7 \oplus 11 = 12$
The cacti of 6 pairs of phones lies outside the range of $[l, r]$ i.e $[3, 5]$ in this case.