RC CIRCUITS

Time Domain Response

Objectives

At the end of this module the student would be able to

- Explain Charging of RC Circuit with DC Source
- Explain Discharging of RC Circuit with DC Source
- Explain Square wave response of RC Circuit
- Explain RC Circuit as Integrator
- Explain RC Circuit as Differentiator

From our Earlier Lessons

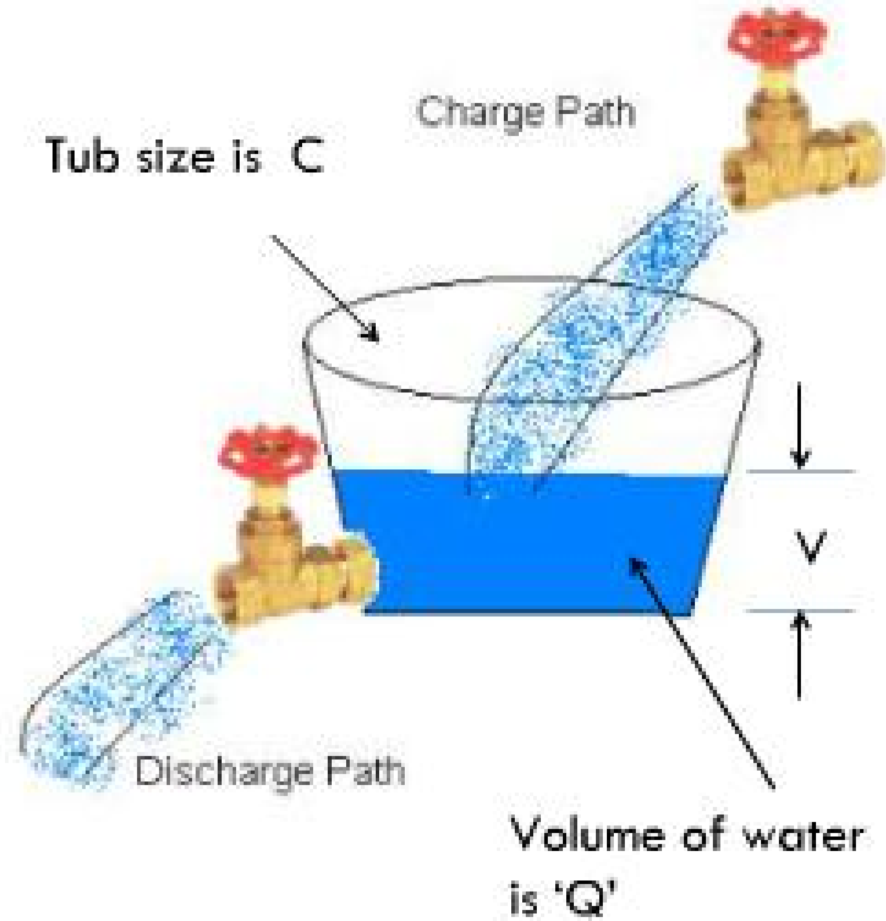
Capacitor helps us make circuits that ‘remember’ their recent history. This ability allows us to make ‘timing’ circuits – circuits that let ‘this’ happen a predetermined time after ‘that’ occurs.

Mathematical Notion

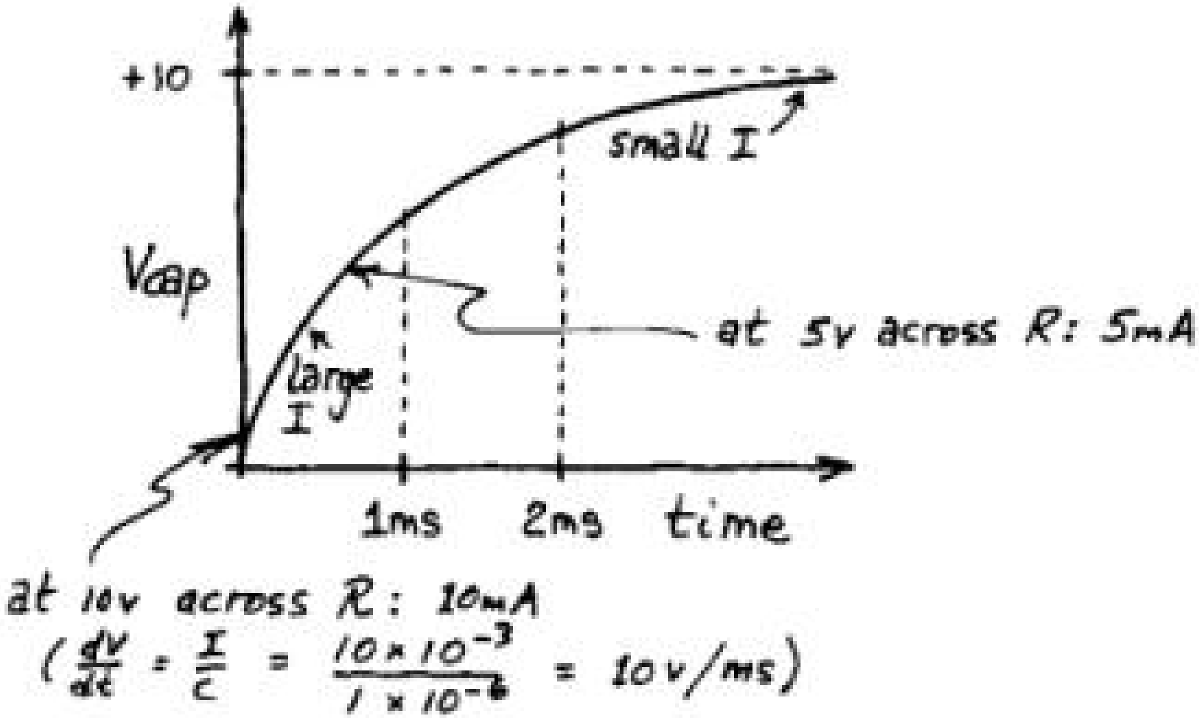
- A static description of the way a capacitor behaves would be to say $Q = CV$, where Q is the total charge, C is a measure of how big the capacitor is and V is the voltage across it.
- A dynamic description, ie one that changes with time would be to say $I = C\,dV/dt$. This is just the time derivative of the static description. C is constant wrt time, I is the rate at which charge flows. This essentially says - the bigger the current, the faster the capacitor's voltage changes.

Analogy

Think of Capacitor as a tub that can hold charge. A tub of large diameter (C), holds a lot of water (Q) for a given height (V). If you fill the tub with a thin straw (small I) then water level – V – will rise slowly. If you use a large pipe (large V) then water level will rise faster. Similar for draining (discharging) tub. Of course a tub of larger diameter takes longer to fill than a tub of smaller diameter.



Charging with DC Source



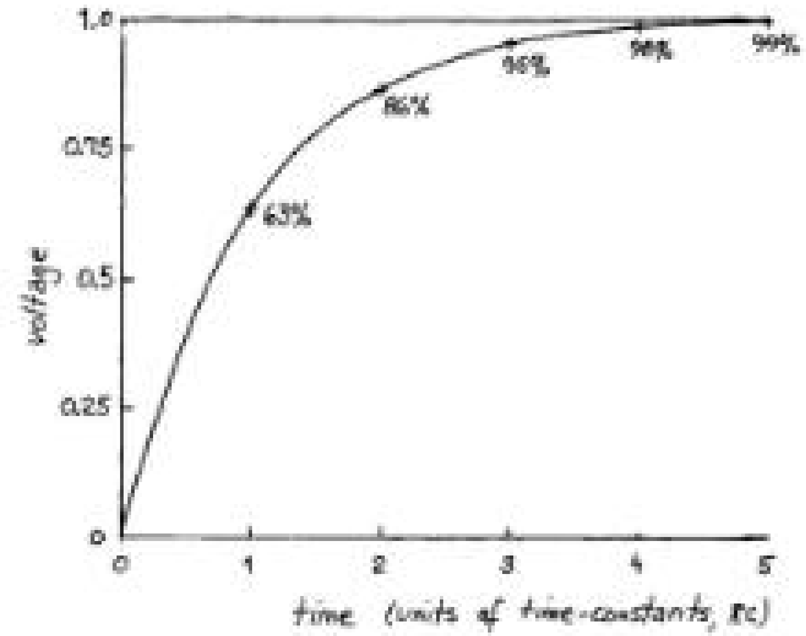
The Voltage across the capacitor approaches the applied voltage - but at a rate that diminishes towards zero as Vcap approaches the applied voltage. It starts out strongly charging at 10mA but as it is 1V away it has slowed to 1/10 its rate.

Question ?

Does Vcap ever reach the applied Voltage of 10 V ?

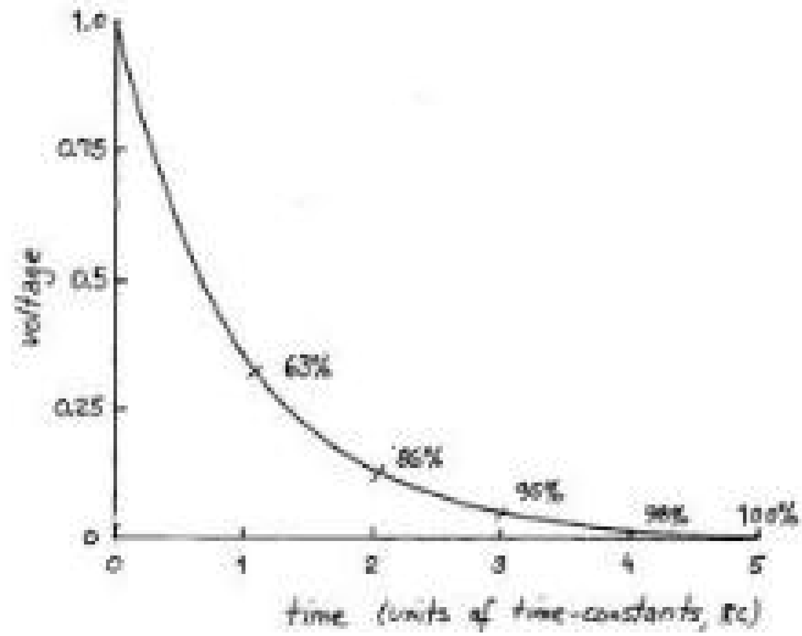
Charging with DC Source

- In 1 time constant = RC, the capacitor charges 63% of the way
- In 5 time constants = 5*RC, the capacitor charges 100% of the way
- Vcap never reaches Vapplies



Discharging with DC Source

- Discharging follows similar principles to charging
- In 1 time constant = RC, the capacitor discharges 63% of the way
- In 5 time constants = 5*RC, the capacitor charges 100% of the way

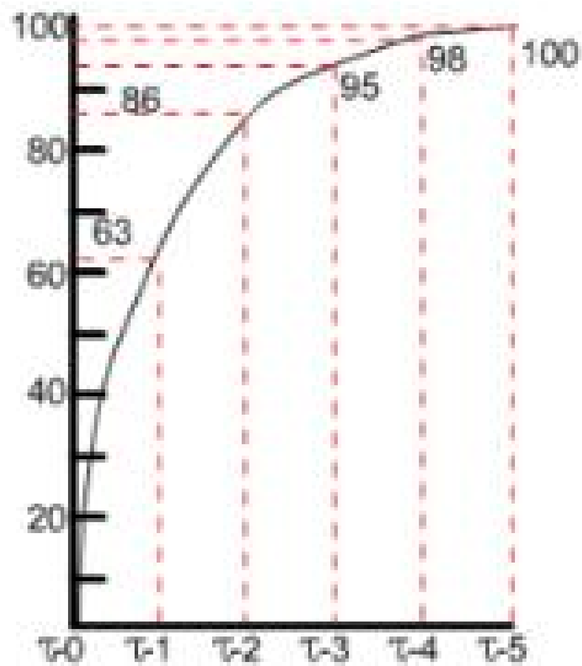


Exploratory – Charging with DC Source

How would the graph change if keeping C constant R is decreased ?

Will the rate of change of voltage increase or decrease ?

How will the time constant (RC) change ?

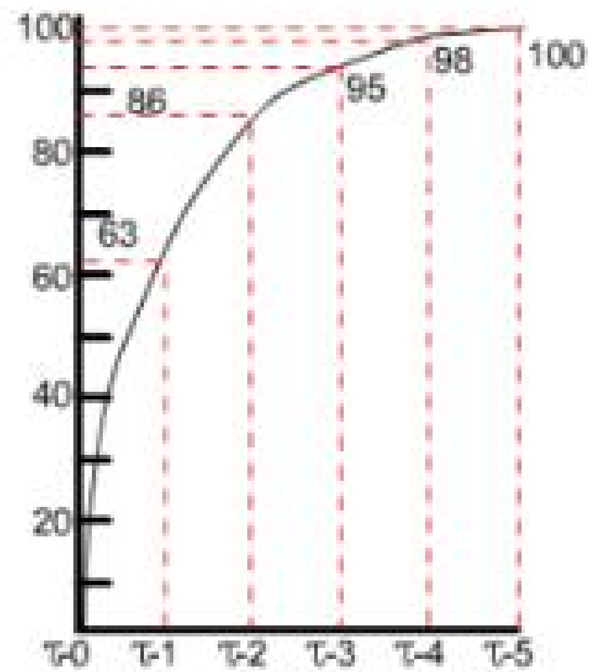


Exploratory – Charging with DC Source

Decreasing R would increase current, thus would increase $dV/dt = I/C$.

Rate of change of Voltage would increase.

Time constant (RC) would decrease, so it will charge faster.

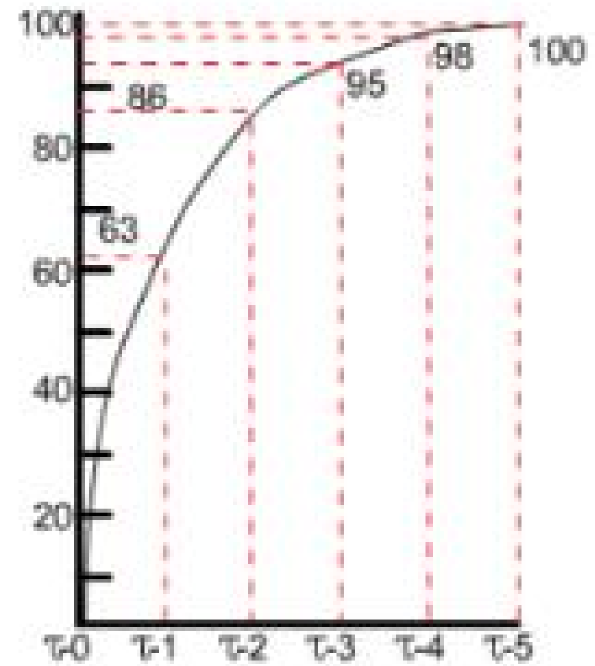


Exploratory – Charging with DC Source

How would the graph change if keeping R constant C is increased ?

Will the rate of change of voltage increase or decrease ?

How will the time constant (RC) change ?



Square Wave Response of RC Circuit

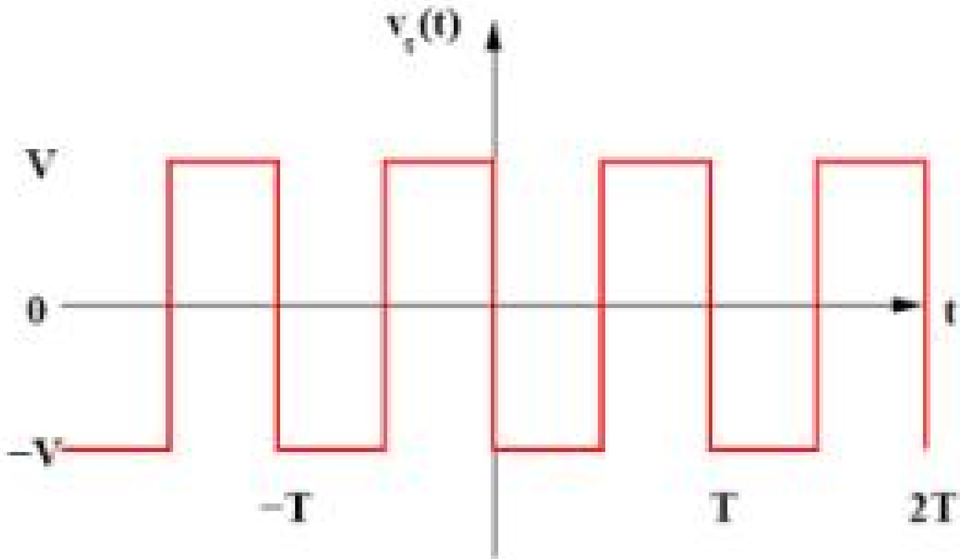
After studying how a constant source affects the charging-discharging of a RC circuit, let us change the constant source to a source which changes very fast and observe the response of an RC Circuit.

Square Wave Response of an RC Circuit

We want to study the transient behavior of the RC circuit when we suddenly change the voltage across the circuit. In order to do this we generate a square wave and observe the response of the voltage across the capacitor using an oscilloscope.

Characteristics of a Square Wave

In the laboratory, you will not study the response of an RC circuit to a single voltage step or voltage pulse, rather, you will study the response to a periodic square wave with the waveform illustrated below -



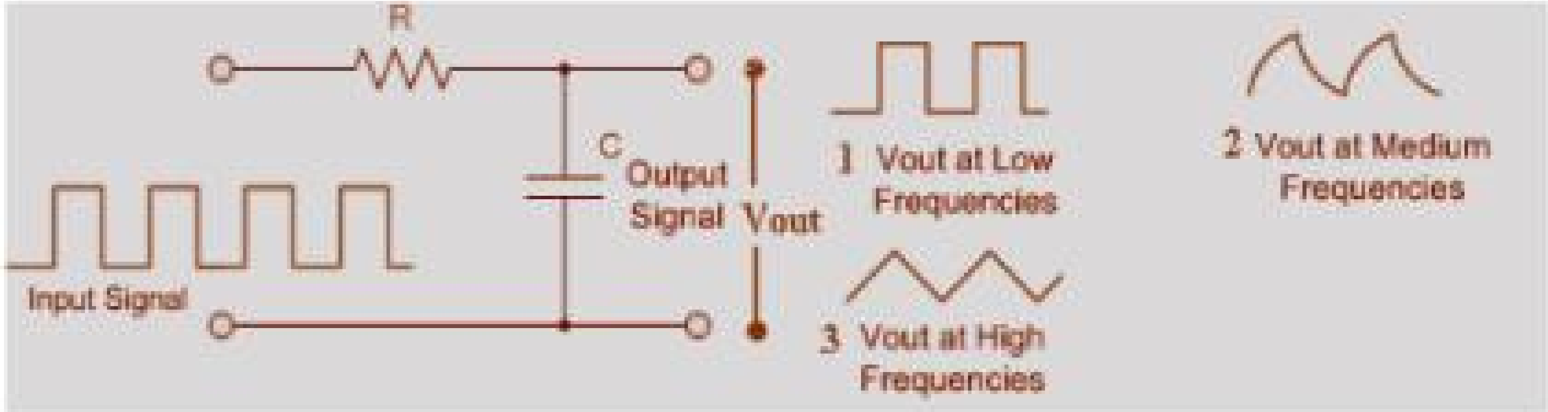
The pulse goes from −V to V and has a period T. What is the response of the RC circuit to this periodic square wave?

Square Wave Response – slow change

Since we have seen that a time of 5T where T = R*C is needed for the capacitor to charge fully, let us first take a time period of the square wave large enough to charge the capacitor. So we take Time period of Square wave > > RC. So in effect, this square wave is changing slowly.

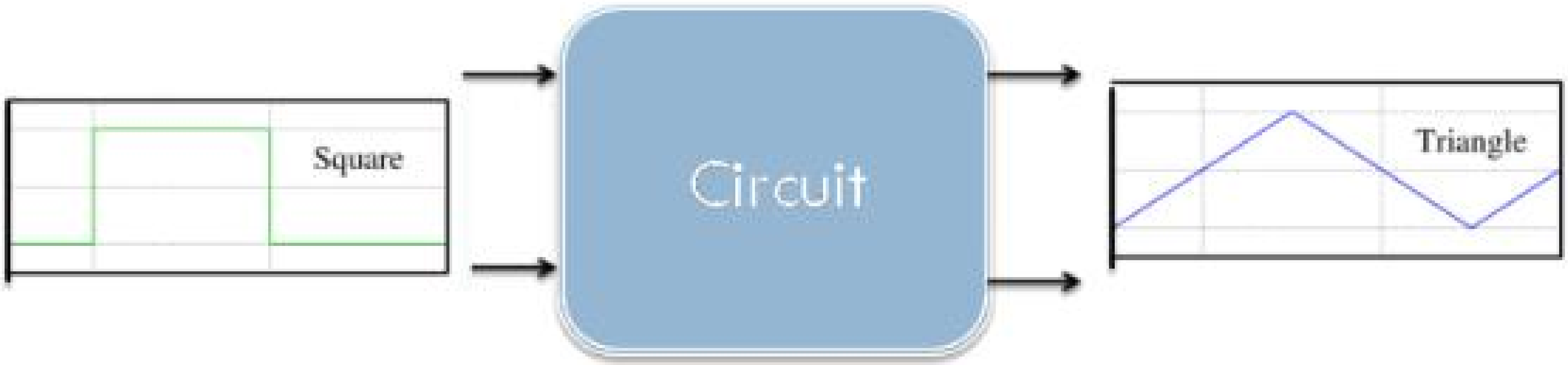
Important Observations

These experiments were done in the time domain. However in the frequency domain a signal which changes fast would mean it's a high frequency signal (or it has a high frequency components) , a signal which changes slowly would mean it has low frequency components and a signal which does not change very fast has medium frequency components. Clearly we saw the following response of RC circuit for the different frequencies as -



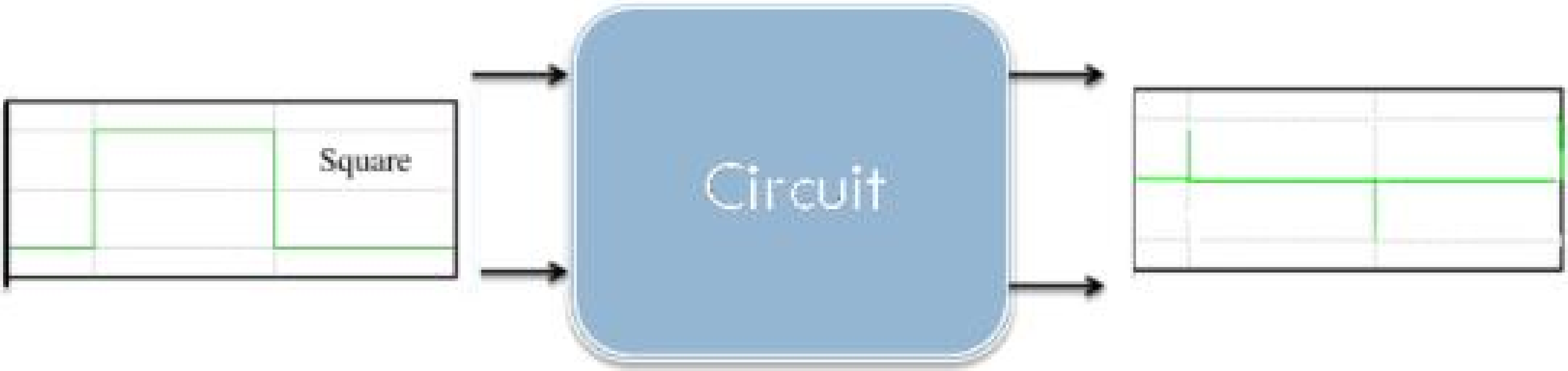
Integrators and Differentiators

What is an integrator ?



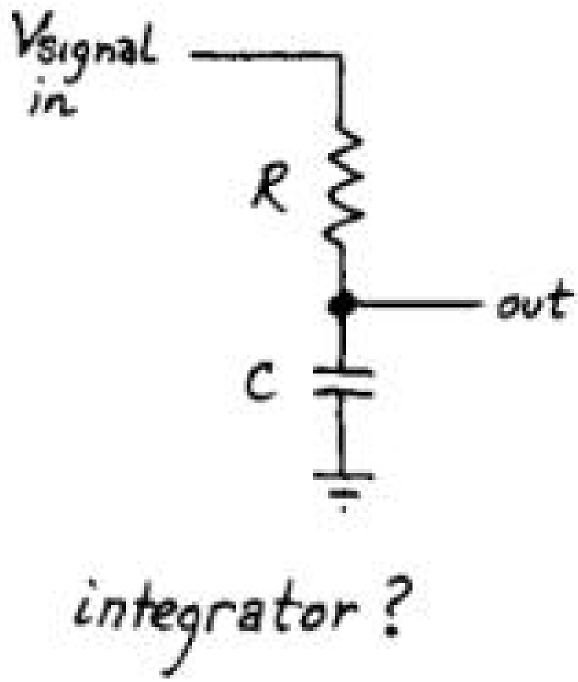
The **Integrator** is a circuit that converts or 'integrates' a square wave input signal into a triangular waveform output.

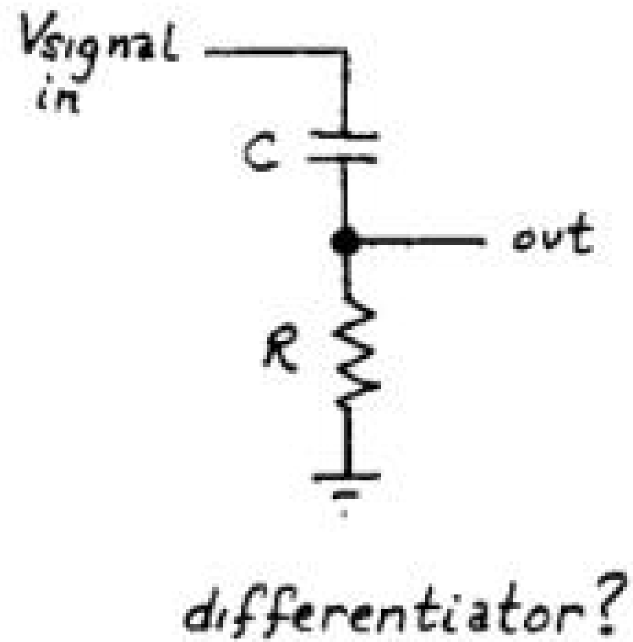
What is an differentiator ?



The **Differentiator** circuit converts or 'differentiates' a square wave input signal into high frequency spikes at its output.

RC as Integrators and Differentiators ?

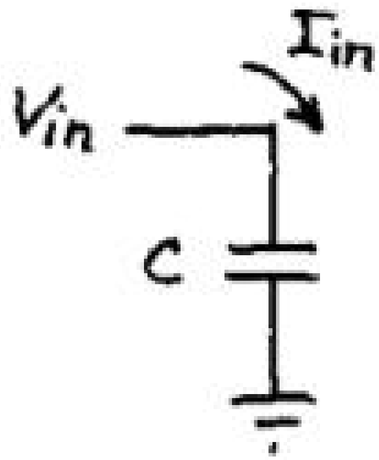




Can we exploit capacitors $I = CdV/dt$ to make differentiator and integrator ?

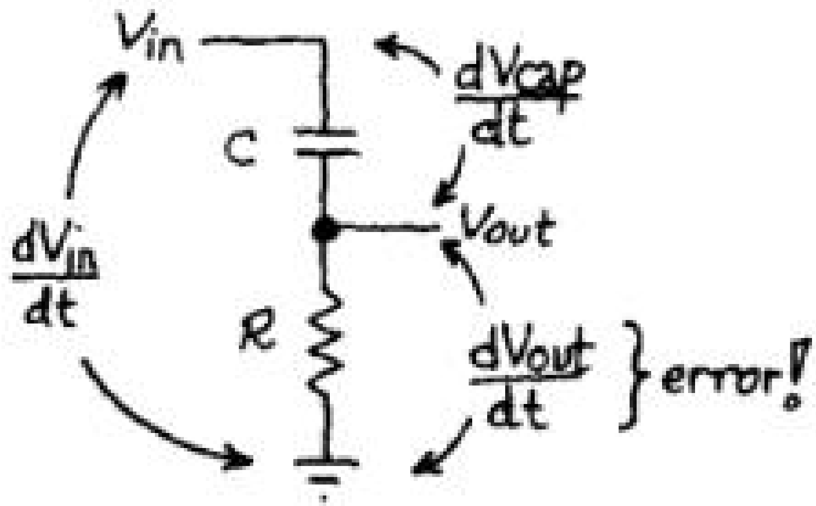
RC Circuit as Differentiator

Basic differentiation



Consider this circuit – the current that flows in the capacitor is proportional to dV/dt – ie, the circuit differentiates the input signal. But – we really cant measure the current here. Let us try to do that.

Basic differentiation

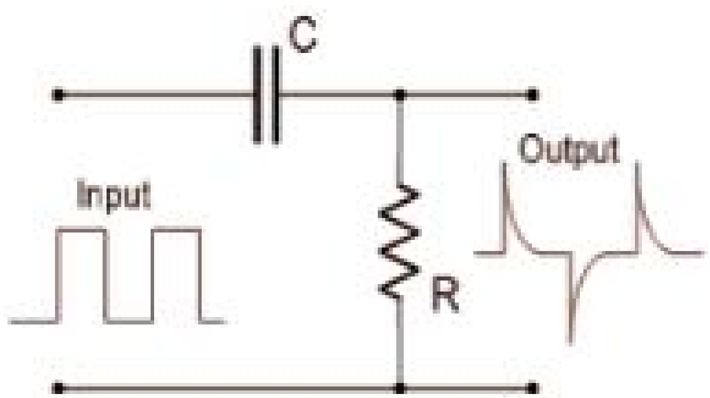


differentiator?

So, we put a resistor to measure the current but choose a very small resistor so that there will be very small voltage drop such that dVcap/dt ≈ dVin/dt.

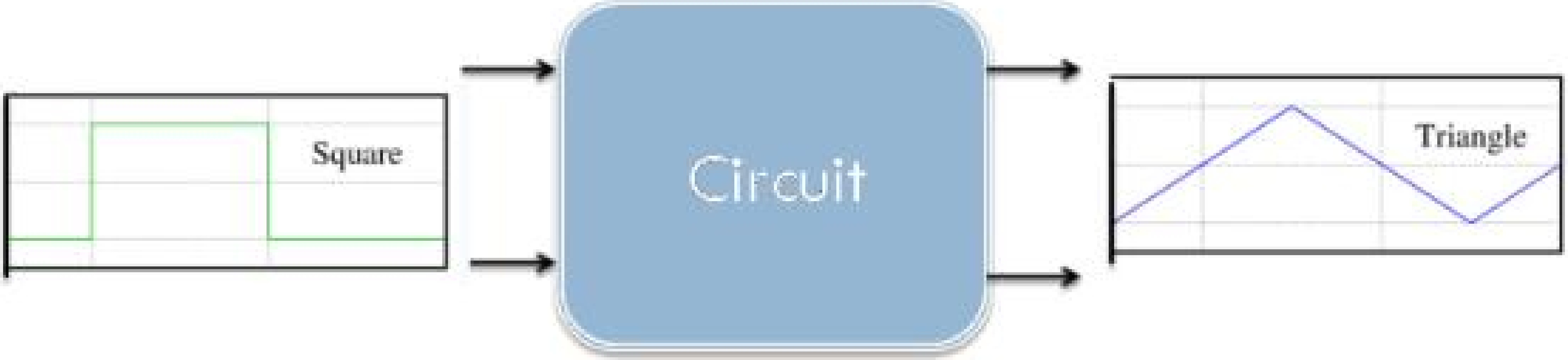
RC as Differentiator

In an RC circuit if we take the voltage drop across R, and if we keep RC time constant is very short compared to the time period of the input waveform we will be differentiating the square wave.



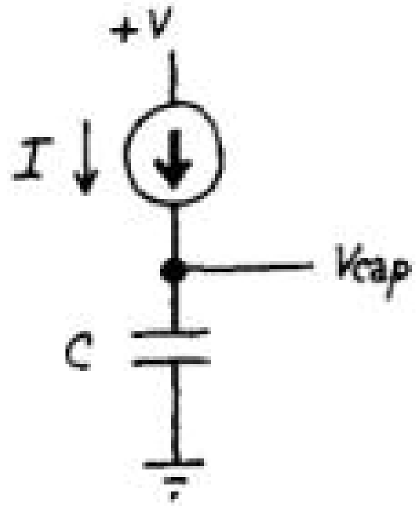
RC Circuit as Integrator

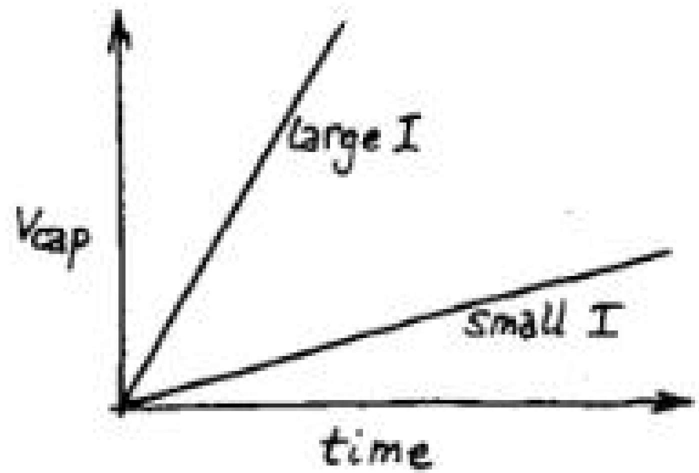
We understand – Integrator



The **Integrator** is a circuit that converts or 'integrates' a square wave input signal into a triangular waveform output.

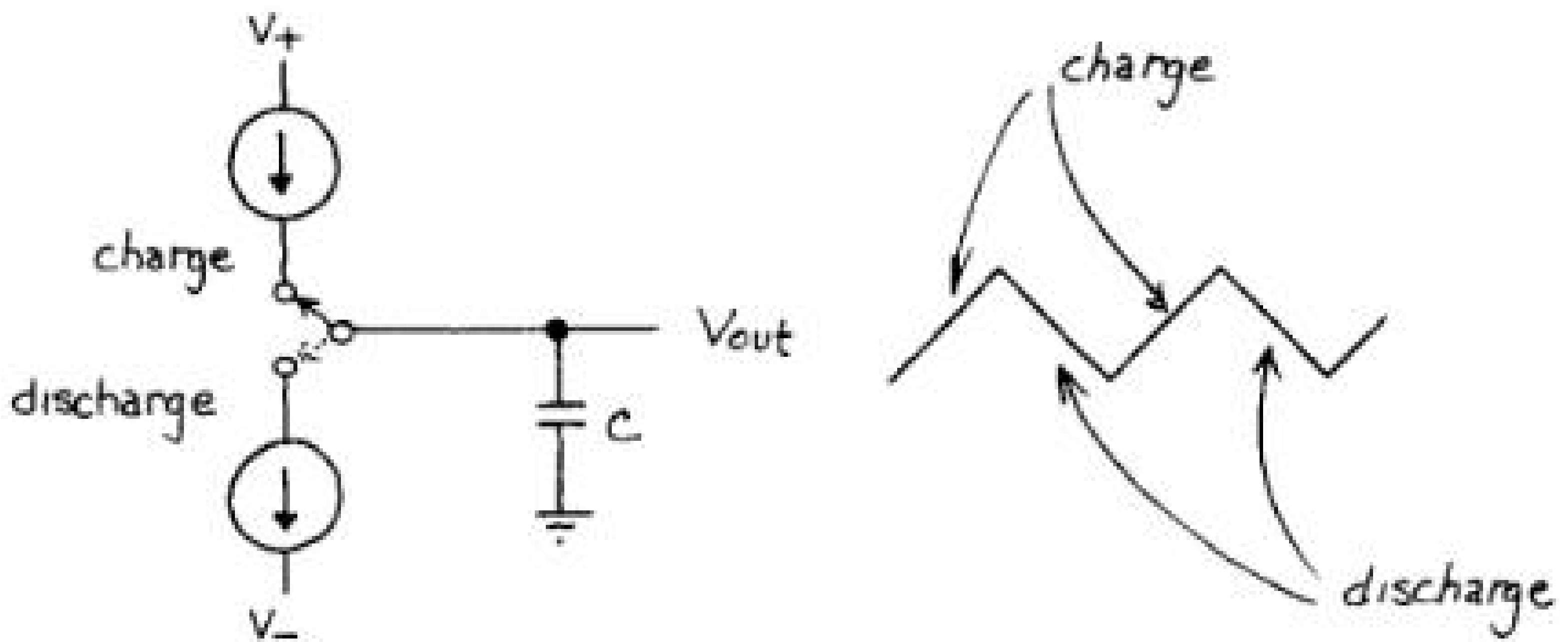
Simple Integrator





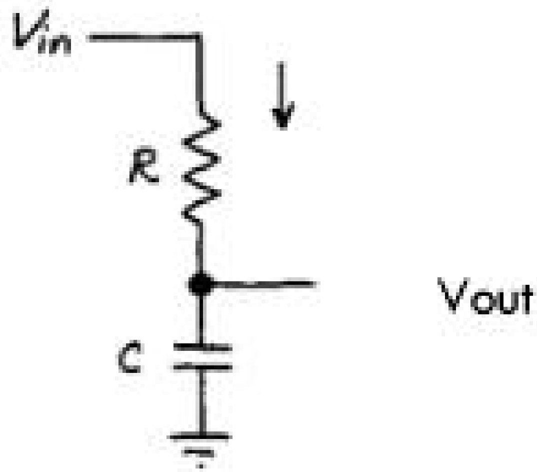
Consider this simple circuit – if we had a constant current source that flows in the capacitor, dVcap/dt – would be constant and we would have a ramp.

Simple Integrator



To generate a triangle wave – we can simply have the following setup. But of course – constant current sources are rare. So, how can we have this with a square wave generator ?

RC as Integrator



So, we put a Resistor such that we can control the current and ideally try to have dVout / dt to be a constant. Since the current does not change much during the initial part of the charging and discharging of the capacitor, the value of RC must be chosen such that it is large compared to the time period of the square wave.