Flipping a coin

1- head

2- tail

S_f I toss a fair coin what is the probability

$P(T) = \frac{1}{2} = \%50 = 0.5$

$P(H) = \frac{1}{2} = \%50 = 0.5$

sets

A set is an undenordered collection of things (elements)

$B = \{1,2,3,3\}$ $B = \{1,2,3\}$

Subset (alt küme)

set A is a subset of B if every element of A is also an element of B

" $\subset$ " subset

$A = \{x^3 : x = 2,3,4\} = \underbrace{\{2^3, 3^3, 4^3\}}_{\text{list}}$ → braces

↳ such that

N (natural numbers) = $\{1,2,3 \dots\}$

Z (integers) = $\{\dots -1, 0, 1, 2, \dots\}$

Rational numbers (ratio) → $\frac{a}{b}$

(Rasyonel) → Q   kesir

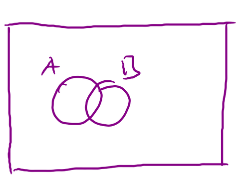
$Q = \{\frac{a}{b} : a \in Z \text{ and } b \in Z - \{0\}\}$

Real numbers (R)

infinity numbers:)

Superset if a is a subset of set B, then B is superset of A.

Venn diagrams



Universal set
(S) Ω omega

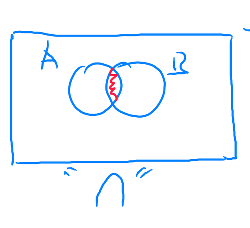
Set operators

Union (Birleşim) = the union of two sets A and B consists of all elements in A or B

$x \in (A \cup B) \Longleftrightarrow (x \in A) \text{ or } (x \in B)$

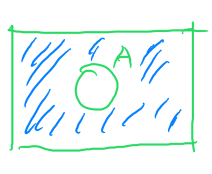
$\{1,2\}$ $\{2,4\}$ → $A \cup B = \{1,2,4\}$

Intersection (kesişim) → The intersection of two sets A and B consists of all elements in both a and b



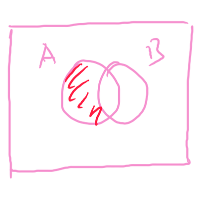
" $\cap$ "

(değil)
Complement → The complement of a set A is set of all elements in S that are not in A.



S
$A^c = A' = \bar{A}$

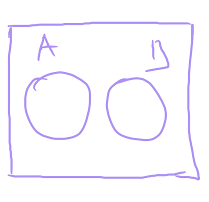
Diffrence (subtraction) → The diffrence of set B from A is all elements in A that are not in B.



$A - B \equiv x \in A$ and $x \notin B$

$A / B \equiv A \cap B^c$

Disjoint sets (Ayrık küme) = Mutually exclusive set → Two sets A and B are disjoint if $A \cap B = \emptyset$ → empty set

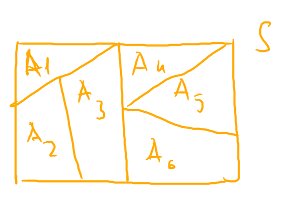


Partition → A collection of sets:

$A_1, A_2 \dots A_n$ is a Partition set if and only if

a- They are disjoint

b- $A_1 \cup A_2 \dots A_n = S$



De morgan's law

- $(A \cup B)^c \equiv \overline{(A \cup B)} = A' \cap B'$

Example

$S = \{1,2,3,4,5,6\}$ $A = \{1,2\}$ $B = \{2,4,6\}$

$A \cup B = \{1,2,4,6\}$ $A \cap B = \{2\}$ $A' = \{3,4,5,6\}$

$B' = \{1,3,5\}$ $(A \cup B)' = A' \cap B' = \{3,5\}$ proof (kanıt)

Theorem: Distributive law

$A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$

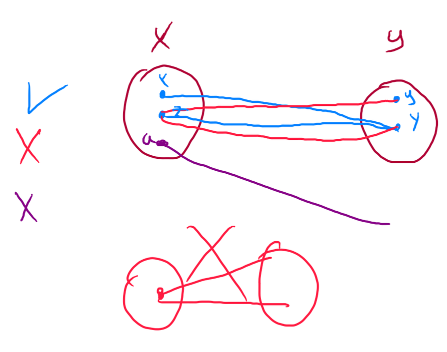
Functions: $f: X \to Y$
domain   co-domain

$\forall x \in X, f(x) \in Y$

Range: the set of all possible values of f(x) Range $\subset$ Y

$f: R \to R$ defined as $f(x) = x^2$

$X = Y = R$ Range$(f) = R^+ = \{x \in R \mid x \geq 0\}$



ya $f: R - \{a\} \to R$ olacak
ya da olmaz

Countable and uncountable sets

A set is finite of $|A| < \infty$

$|A|$ → cardinality

$A = \{3, 4, 5\}$

$|A| = 3$

A set countable if is finite or its elements can be enumarated or listed in a sequence.

$A = \bigcup_{k=1}^{\infty} \{a_k\} \to A = \{a_1, a_2 \dots\}$

- Finite set
$A = \{1, 2, 4, 8, 9\}$
$x \in [4, 8)$ and $x \in Z$
4, 5, 6, 7

- infinite set
$Z^+ = \{1, 2 \dots$ infinite

$N = \{1,2,3 \dots\}$ → infinite and countable (sayılabilir)

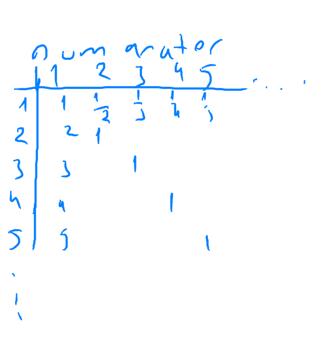
Uncountable → the elements cannot be enumarated

R → reel numbers → uncountable

- A set is countable infinite if is is in one to one carrespondance

Z → countable? $Z = \{\dots 0, 1, 2 \dots\}$ → infinite and countable
a_0 a_1 a_2

$Q^+ = \{\frac{a}{b} : a \text{ or } b \geq 0\}$ infinite, countable $Q^+ = \bigcup_{i,j=1}^{\infty} \{\frac{i}{j}\}$



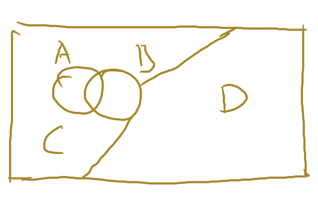
R → infinite, uncountable

$[a,b]$ $b \geq a$

Power set: Set of all subsets of a set A

$A \{1,2\} \to P(A) = \{\{1\}, \{2\}, \{1,2\}, \emptyset\}$

Exhaustive set



S

Sets A, B, C, D are exhaustive ie their union is Ω → $A \cup B \cup C \cup D \equiv \Omega$

Intersections are allowed

Probability Theory

random experiment: A phenomenon whose outcome cannot be predicted with certainty.

- Rolling a die
- flipping a coin
- Rolling a die 3 times

Outcome (Result of random experiment)

→ Rolling a die → 1, 3, 5, 6, 2, 4

→ flipping a coin → Head

→ Rolling a die 2 times → 6, 2

Event (is a collection of possible outcomes)

→ Rolling a die Event$_1$ $\{1, 5, 3\}$
Event$_2$ $\{4, 2\}$ Event $\subset$ SAmple Space

Sample space:

Set of all possible outcomes.

Roll a die $\Omega = \{1,2,3,4,5,6\}$

- Roll a die 3 times → $\Omega = \{(1,1,1), (1,1,2), (1,1,3), \dots (6,6,6)\}$ → 6x6x6 possible outcomes

Partition → is collectively exclusive and mutually exclusive set of events.

$A_1, A_2 \dots A_n$ is a partition

$A_1 \cup A_2 \dots A_n = \Omega$

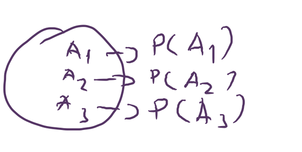
$A_i \cap A_j = \emptyset$, $i \neq j$

- We say that an even A is occured if the outcome of the experiment is an element of A.

Probability

- An event $A \to P(A)$: probability of A

We assign a probability $P(A)$ the every event A
(atanmak)



Axioms of probability.

P() is a function that maps events in the sample space S to real numbers.

1- For any event A, $P(A) \geq 0$

2- Probability of sample space Ω $P(\Omega) = 1$

3- For any countable collection $A_1, A_2 \dots$ disjoint events.

$P(A_1 \cup A_2) = P(A_1) + P(A_2) \dots P(A_n)$