MAT1320 LINEAR ALGEBRA EXERCISES IX-X

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1. (A points) Let $\vec{u}$ and $\vec{v}$ be two unit vectors. If $\vec{u}+2\vec{v}$ is orthogonal to $5\vec{u}-4\vec{v}$, then which of the followings is the angle between the vectors $\vec{u}$ and $\vec{v}$?

   a) $60°$ b) $90°$ c) $30°$ d) $\arccos\left(\frac{1}{3}\right)$ e) $\arccos\left(\frac{2}{7}\right)$

$(\vec{u}+2\vec{v})\cdot(5\vec{u}-4\vec{v})=0$

$\Rightarrow 5\vec{u}\cdot\vec{u} - 4\vec{u}\cdot\vec{v} + 10\underbrace{\vec{v}\cdot\vec{u}}_{\vec{u}\cdot\vec{v}} - 8\vec{v}\cdot\vec{v} = 0$

Since $\vec{u}\cdot\vec{u}=|\vec{u}|^2$ and $\vec{v}\cdot\vec{v}=|\vec{v}|^2$, and $|\vec{u}|=|\vec{v}|=1$,

$5+6\vec{u}\cdot\vec{v}-8=0 \Rightarrow \vec{u}\cdot\vec{v}=\frac{1}{2}$

$\Rightarrow \underbrace{\vec{u}\cdot\vec{v}}_{\frac{1}{2}} = \underbrace{|\vec{u}|}_{1}\underbrace{|\vec{v}|}_{1}\cos\alpha \Rightarrow \cos\alpha=\frac{1}{2} \Rightarrow \alpha=60°.$

$\begin{vmatrix} i & j & k \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}$

$(-3-7)i-(4-7)j+(7)k$

Let $\vec{u}=(u_1,u_2,u_3)$.

$\vec{u}\cdot\vec{a}=0 \Rightarrow 2u_1+0\cdot u_2+1\cdot u_3=0 \Rightarrow 2u_1+u_3=0$.

$\vec{c}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}=(-10,-3,7)$ $\quad -10i+3j+7k$

$\vec{u}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ 1 & 1 & 1 \end{vmatrix}=(u_2-u_3,\ u_1-u_3,\ u_1-u_2)$

$\begin{vmatrix} i & j & k \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

$(b-c)i-(a-c)j+(a-b)k$

$b-c=-10$

$a-c=-3$

$a-b=7$

$u_1-u_3=-3$ and $2u_1=-u_3 \Rightarrow u_1+2u_1=-3 \Rightarrow \boxed{u_1=-1}$

$\Rightarrow \boxed{u_3=2}$

$u_1-u_2=7 \Rightarrow \boxed{u_2=-8}$

$2a+c=0$

$a-c=-3$

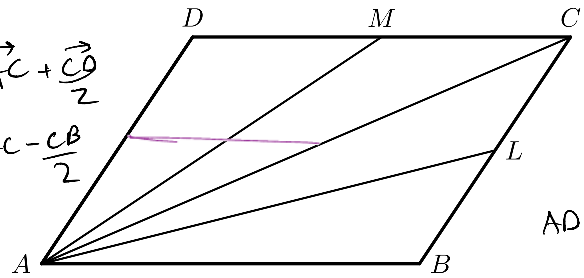
$3a=-3$, $a=-1$, $b=-8$, $c=2$

2. (D points) Let $\vec{a}=2\mathbf{i}+\mathbf{k}$, $\vec{b}=\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\vec{c}=4\mathbf{i}-3\mathbf{j}+7\mathbf{k}$. If $\vec{u}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{u}\cdot\vec{a}=0$, then which of the followings is the vector $\vec{u}$?

   a) $\mathbf{i}+8\mathbf{j}+\mathbf{k}$ b) $\mathbf{i}+8\mathbf{j}+2\mathbf{k}$ c) $2\mathbf{i}+\mathbf{j}-8\mathbf{k}$

   d) $-\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ e) $\mathbf{i}-8\mathbf{j}+\mathbf{k}$

3. (C points) For the following parallelogram $ABCD$, the points L and M are the middle points of the sides BC and CD, respectively. Then, which of the followings is the vector $AL+AM$?



$\vec{AC}+\frac{\vec{CD}}{2}$

$AC-\frac{CB}{2}$

$DC=AC-AD$

$AD+DC=AC$

   a) $\frac{1}{2}AC$ b) AC c) $\frac{3}{2}AC$ d) $2AC$

   e) None of them

$AC-\frac{(AC-AD)}{2}$

$\frac{3}{2}AC+\frac{AD}{2}$

$\vec{AL}=\vec{AB}+\vec{BL}$

$\vec{AM}=\vec{AD}+\vec{DM}$

$\Rightarrow \vec{AL}+\vec{AM}=\underbrace{\vec{AB}+\vec{AD}}_{\vec{AC}}+\underbrace{\vec{BL}+\vec{DM}}_{\frac{1}{2}\vec{AC}}=\frac{3}{2}\vec{AC}$

T: $3t^2\in T$, $3t\in T$ but $3t^2+3t\notin T$.

$\Rightarrow T$ is not subspace of P_2.

A: Let $a_1t^2+b_1t+c_1$, $a_2t^2+b_2t+c_2\in A$. Then,

$b_1=3c_1$, $b_2=3c_2$.

$\Rightarrow (a_1t^2+b_1t+c_1)+(a_2t^2+b_2t+c_2)=(a_1+a_2)t^2+(b_1+b_2)t+c_1+c_2\in A$ ✓

See that $(b_1+b_2)=3(c_1+c_2)$.

$\forall k\in\mathbb{R}$,

$\Rightarrow k(a_1t^2+b_1t+c_1)=ka_1t^2+kb_1t+kc_1\in A$ since $kb_1=3kc_1$ ✓

4. (A points) Let P_2 be the set of all polynomials over real numbers whose degrees are at most 2. Recall that P_2 is a vector space with usual addition and multiplication by a scalar on polynomials. Then, which of the following subsets is a subspace of P_2?

$$\mathcal{M}=\{at^2+bt+c \mid c=0\}$$
$$\checkmark\ \mathcal{A}=\{at^2+bt+c \mid b=3c\}$$
$$\times\ \mathcal{T}=\{at^2+bt+c \mid a+b+c=3\}$$

   a) $\mathcal{M}$ and $\mathcal{A}$ ~~b)~~ $\mathcal{M}$ and $\mathcal{T}$ ~~c)~~ $\mathcal{A}$ and $\mathcal{T}$

   ~~d)~~ Only $\mathcal{M}$ ~~e)~~ All of them

5. Which of the following subsets are subspaces of the given vector spaces ?

$$\mathcal{Y} = \left\{ \begin{bmatrix} x \\ x^2 \end{bmatrix} \mid x \in \mathbb{R} \right\} \subset \mathbb{R}^2$$

$$\mathcal{T} = \left\{ \begin{bmatrix} x \\ x+1 \\ 0 \end{bmatrix} \mid x \in \mathbb{R} \right\} \subset \mathbb{R}^3$$

a b

a^2 b^2

$r.a$

$r.a^2$

$a+b$

a^2+b^2

$$\mathcal{U} = \left\{ \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} \mid x \in \mathbb{R} \right\} \subset \mathbb{R}^3$$

a) ~~Only $\mathcal{Y}$~~ b) ~~Only $\mathcal{T}$~~ c) Only $\mathcal{U}$

d) ~~$\mathcal{Y}$ and $\mathcal{T}$~~ e) ~~$\mathcal{T}$ and $\mathcal{U}$~~

$\mathcal{Y}$: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix} \in \mathcal{Y}$ but $\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \notin \mathcal{Y}$ because $3^2 \neq 5$.

$\mathcal{T}$: $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \in \mathcal{T}$ but $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} \notin \mathcal{T}$ since $5 \neq 3+1$.

$\Rightarrow$ Then, see that $\mathcal{U}$ is closed under addition and multiplication by scalars.

$\Rightarrow \mathcal{U}$ is a subspace