Regular Operations

A ∪ B ⇒

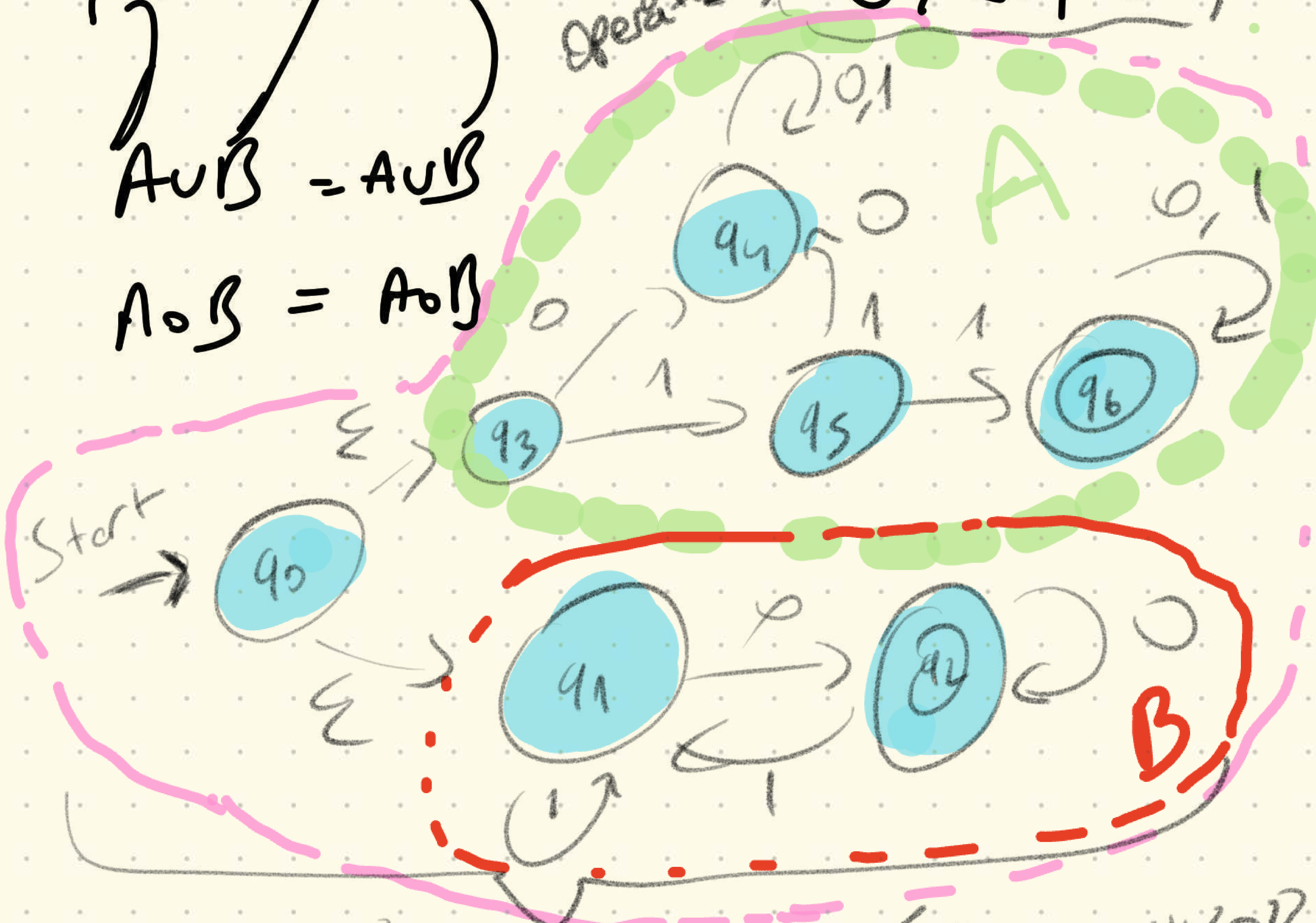
Regular Regular

$A \cup B = A \cup B$

$A \circ B = A \circ B$

The class of regular languages is closed under union or

Regular Operations → ∪, *, ∘



{Starts with {1}} ← A ∪ B → {Ends with {0}}

With all ∪, *, ∘ = Σ (Alphabet)

You can create all Regular Languages

Regular languages are closed under regular operations

From previous lessons

CONCATENATION (∘)

A = {mango, orange} B = {green, orange}

orangeorange

A ∘ B = {mangogreen, mangoorange, orangegreen, orangeorange}

KLEENE STAR (*)

Concatenate together strings in the language ZERO or more times

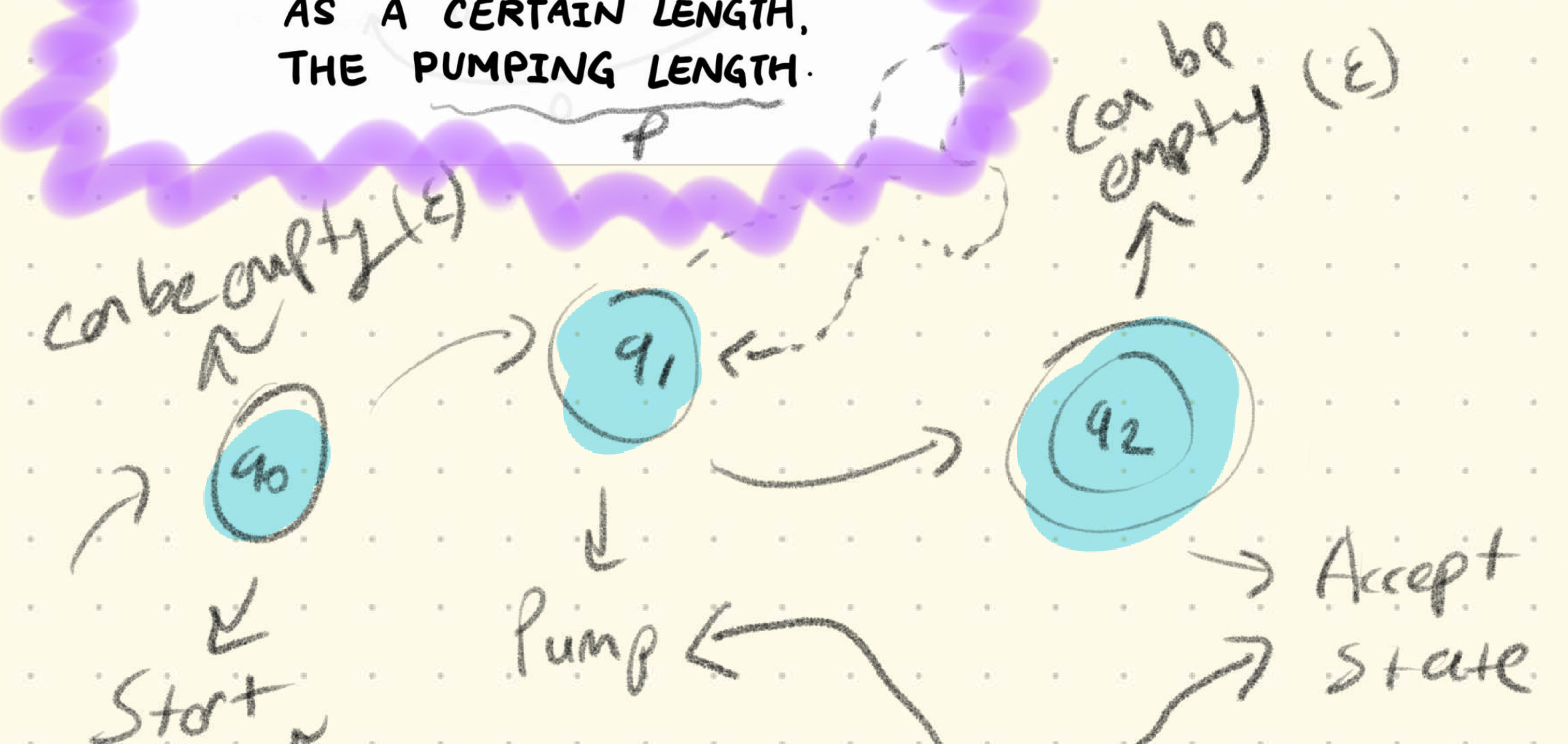
A = {mango, orange}

A^* = { orange, mango, orangeorange, orangemango, mangoorange, mangomango, orangeorangeorange, orangeorangemango, orangemangoorange

Pumping Lemma

PUMPING LEMMA:

ALL STRINGS IN THE LANGUAGE CAN BE REPEATED — "PUMPED" — IF THEY ARE AT LEAST AS LONG AS A CERTAIN LENGTH, $\underbrace{\text{THE PUMPING LENGTH}}_{p}$.



1. for each $i \geq 0$, $xy^iz \in A$
2. $|y| > 0$ → CANNOT BE EMPTY (ε)
3. $|xy| \leq p$ → pumping length

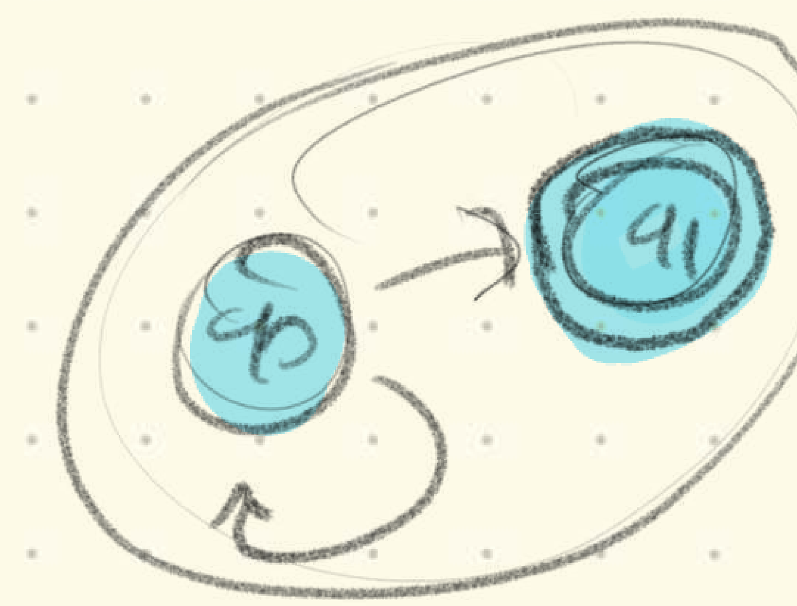
if $p=3$ → 0 1 1 1 1

Must be within first $|xy| \leq 3$ ✓

This "111" is not in the area between x and y

Start ← x and y → Pump

We know that Regular Languages can be expressed by



Finite Automaton OR {"011"}{"1011"} Regular expression

We also know closure properties

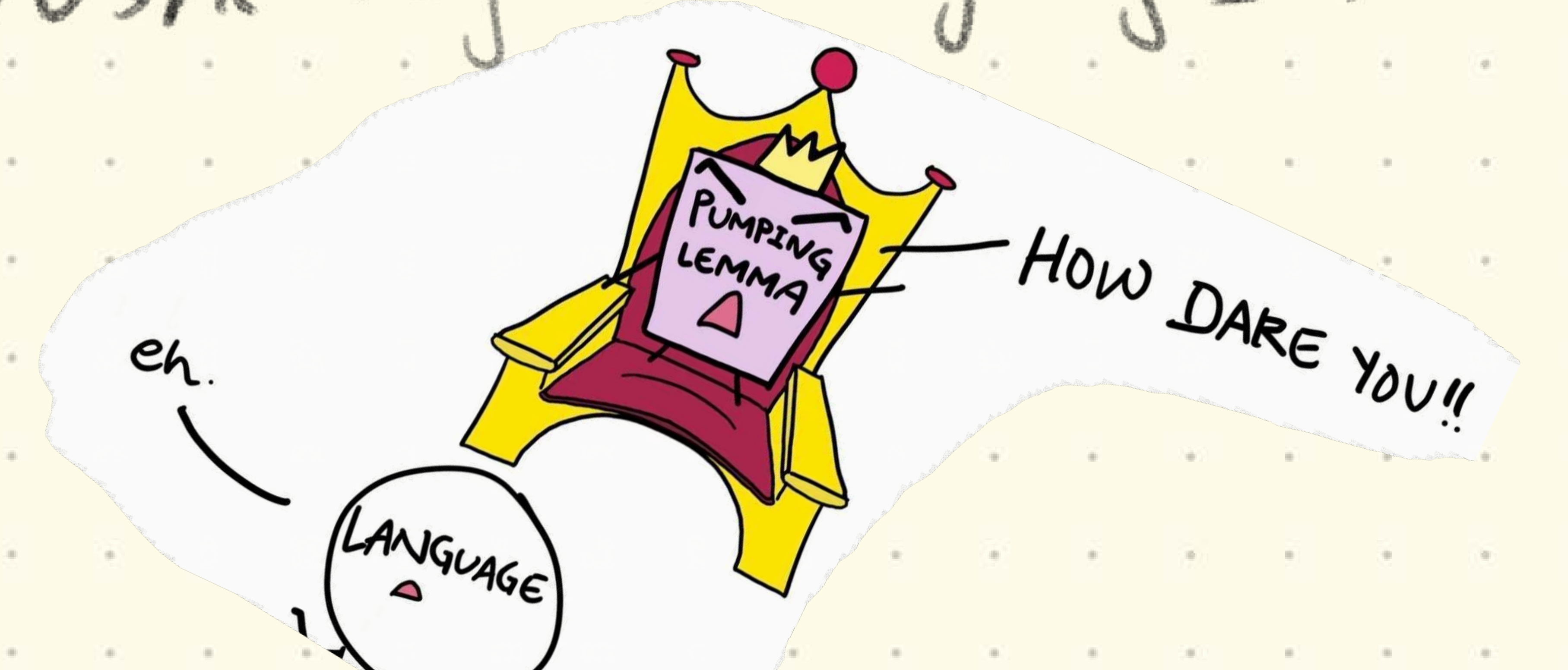
$A \cup B$, $A \cap B$, $A \circ B$, A^*

Proof By Contradiction

Is for satisfying the pumping lemma

- Regular Languages satisfies p.l.

but Non-regular languages not.



Exp $B = \{0^n 1^n \mid n \geq 0\}$

Is this language regular or non-regular?

Try yourself

Answer) Let's see if this language satisfies the pumping lemma

so we have xy^iz $(i \geq 0)$ rule in p.l.

$|y| > 0$ and $|xy| \leq p$

1. $\underbrace{0}_{y}{}^{p}1^p \Rightarrow 00000.0^p1^p \notin B$ (num of 0 > num of 1)

2. $0^p\underbrace{1}_{y}{}^{p} \Rightarrow 0^p1^p11111 \rightarrow \notin B$ (num of 1 > num of 0)

3. $\underbrace{0^p1^p}_{y} \Rightarrow 0\,001\,0101\,01...1 \notin B$ (language is not regular)

so this language doesn't SATISFY

Also if we apply the $|xy| \leq p$ rule, we can eliminate 2 and 3 because The pump wasn't at the start of the regular expression. Which has to be at the start.