Homework 4 (Due October 31, 2016, 9:00 AM)

BLM 1541: Statistics and Probability — Fall 2016

Print family (or last) name: ———————————————————————————

Print given (or first) name: ———————————————————————————

Print given student number: ———————————————————————————

I see that this homework has 3 questions in total.

I agree that I have to submit my homework solution before the deadline (October 31, 2016, 9 AM) otherwise my homework solution will not be graded.

I accept that ***I will add the signed version of this instruction page as a first page into my homework solution***; otherwise my homework solution will not be graded.

I know that ***I have to give my solutions written on white A4-sized pages that are stapled on the left-up corner***; otherwise my homework solution will not be graded.

I will take care of the readability of my solutions, from which I may lose 10 points.

For any proofs, I am sure to provide a step-by-step argument, with justifications for every step.

I understand that, during solving this homework, it is prohibited to exchange information about solutions with any other person in any way, including by talking or ex-changing solutions / papers.

I know that the course book is "Probability and Statistics for Computer Scientists, by Michael Baron (2nd Edition) ".

I have read, understand and accept all of the instructions above. On my honor, I pledge that I have not violated the provisions of the Academic Integrity Code of Yıldız Technical University.

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Signature and Date

Problem 1 [30 Points]: Show that the following equalities are correct for any discrete random variable X,

a) $P(a \leq X \leq b) = P(X = a) + F_X(b) - F_X(a)$,
b) $P(a < X < b) = F_X(b) - F_X(a) - P(X = b)$,
c) $P(a \leq X < b) = P(X = a) + F_X(b) - F_X(a) - P(X = b)$,

where $F_X(x)$ denotes the cumulative distribution function of the random variable X.

Problem 2 [35 Points]: The way TÜBİTAK hires graduated students from Yıldız Technical University is exemplified in the following table. Each applicant is rated to a discrete "graduation point average (mezuniyet not ortalaması)" X (horizontal axis) and a discrete "personal rating index" Y (vertical axis). The top number in each cell (in bold) is the number of applicants is a given year with the associated combination. The bottom number in each cell (in italic) is the probability of being accepted. *(Note that, although this is assumed the way TÜBİTAK handles applications, all numbers are fictitious)*.

← Graduation Point Average X →

← Personal Rating Y →	$90-100$	$80-90$	$70-80$	$60-70$	$50-60$	≤ 50
10	**27** 1.0	**32** 0.9	**43** 0.7	**30** 0.5	**10** 0.4	**6** 0.3
9	**60** 0.9	**95** 0.7	**110** 0.5	**150** 0.4	**45** 0.3	**20** 0.2
8	**86** 0.7	**175** 0.5	**305** 0.4	**350** 0.3	**78** 0.2	**62** 0.1
7	**39** 0.5	**173** 0.4	**250** 0.3	**165** 0.2	**102** 0.1	**50** 0.0
6	**17** 0.4	**54** 0.3	**118** 0.2	**151** 0.1	**97** 0.0	**68** 0.0
≤ 5	**1** 0.3	**12** 0.2	**28** 0.1	**45** 0.0	**17** 0.0	**21** 0.0

a) Plot the marginal PMF of the two indices.
b) Plot the conditional PMFs $(X|Y = 8)$ and $(X|Y = 6)$.
c) Plot the conditional PMF of $(Y|X \leq 50)$.
d) What is the probability that an applicant with $Y = 7$ is accepted.
e) Are X and Y independent? Why?

Problem 3 [35 Points]: Let X and Y be two discrete random variables with the joint probability mass function (PMF) given by

$$P_{XY}(x,y) = \begin{cases} k(x+1)^y, & \text{if } x \in \{-1,0,1\} \text{ and } y \in \{1,2\} \\ 0, & \text{Otherwise} \end{cases}$$

a) Find the value of k in order to make $P_{XY}(x,y)$ a joint PMF .
b) Find the PMFs of X and Y. Are X and Y are independent? Why?
c) Find the PMF of $Z = X^2$.
d) Find the PMF of $Z = X^Y$.