Fourier Örnek 1

$A\cos(7t) = ?$

$$\cos(wt) = \frac{e^{jwt} + e^{-jwt}}{2} \Rightarrow \frac{A}{2}e^{7jt} + \frac{A}{2}e^{-7jt}$$

★ $\boxed{\frac{-1}{j} = j = e^{j0.5\pi}}$

★ $$\boxed{\cos(wt+\phi) = \frac{e^{(jwt+j\phi)} + e^{(-jwt+-j\phi)}}{2}}$$

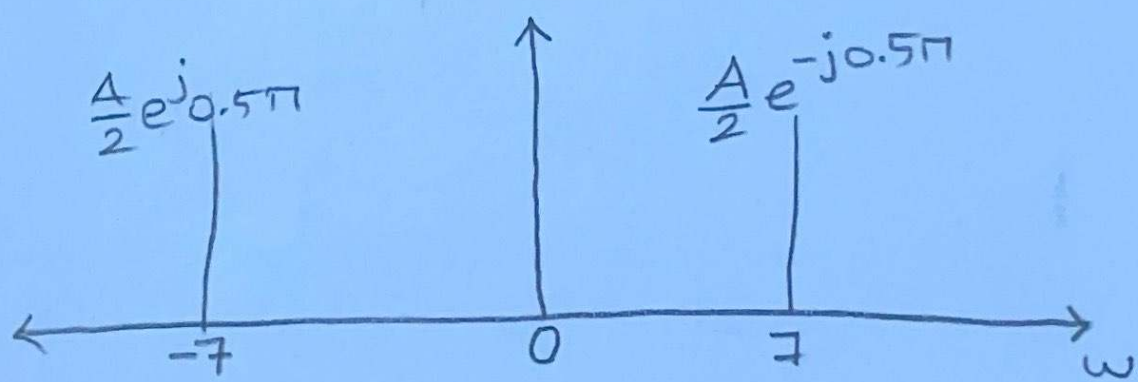
$A\sin(7t) = ?$

$$\frac{A}{2j}e^{7jt} - \frac{A}{2j}e^{-j7t}$$

$$\frac{A}{2}e^{-j0.5\pi}e^{j7t} + \frac{A}{2}e^{j0.5\pi}e^{-j7t}$$

pozitif frekans fazı -0.5π

negatif " " 0.5π



Fourier Transform örnek 2

$y'(t)+2y(t)=x(t)+x'(t)$ freq. response?

$$\boxed{h(w)=\frac{y(w)}{x(w)}}$$

$$\boxed{\begin{aligned} x(t) &\xleftrightarrow{F} x(w) \\ x^{n}(t) &\longrightarrow x(w)(jw)^{n} \end{aligned}}$$

$(jw)y(w)+2y(w)=x(w)+(jw)x(w)$

$$\frac{y(w)}{x(w)}=\frac{jw+1}{jw+2}=h(w)=1-\frac{1}{2+jw}$$

$$\underbrace{F^{-1}\{1\}}-\underbrace{F^{-1}\left\{\frac{1}{2+jw}\right\}}=h(t)=F^{-1}\{h(w)\}=F^{-1}\left\{1-\frac{1}{2+jw}\right\}$$

$\delta(t)-e^{-2t}u(t)=h(t)$

$$\boxed{y(t)=h(t)*x(t)}$$

Fourier örnek 3

$x(t) = \cos(t) + 0.5\cos(4t + \pi/3) + 0.25\cos(8t + \pi/2)$

Amp. formda ve $w_0 = 1$ iken exp. ve sine forma dönüştürünüz.

$A_1 = 1$ $\quad A_4 = 0.5$ $\quad A_8 = 0.25$ $\quad$ diğer A_k'ler 0

$\Phi_1 = 0$ $\quad \Phi_4 = \pi/3$ $\quad \Phi_8 = \pi/2$

$c_0 = A_0$

$c_k = \frac{1}{2} A_k e^{j\Phi_k}$

$c_{-k} = \frac{1}{2} A_k e^{-j\Phi_k}$

$c_1 = \frac{1}{2} \cdot 1 \cdot e^0 = 0.5$ $\qquad c_4 = \frac{1}{2} \cdot \frac{1}{2} \cdot e^{j\pi/3}$

$c_{-1} = 0.5$ $\qquad c_{-4} = 0.25 e^{-j\pi/3}$

$c_8 = \frac{1}{4} \cdot \frac{1}{2} \cdot e^{j\pi/2} = 0.125 e^{j\pi/2}$ $\qquad c_{-8} = 0.125 e^{-j\pi/2}$

$x(t) = [0.5e^{jt} + 0.5e^{-jt}] + [0.25e^{j\pi/3} e^{j4t} + 0.25e^{-j\pi/3} e^{-j4t}] +$

$[0.125 e^{j\pi/2} e^{j8t} + 0.125 e^{-j\pi/2} e^{-j8t}]$

↓

exp. form

$a_0 = c_0$

$a_k = A_k \cos(\Phi_k)$

$b_k = -A_k \sin(\Phi_k)$

$a_1 = 1 \cdot \cos(0) = 1$ $\qquad a_4 = 0.5\cos(\pi/3) = 0.25$

$b_1 = -1 \cdot \sin(0) = 0$ $\qquad -b_4 = 0.5\sin(\pi/3) = 0.43$

$a_8 = 0.25\cos(\pi/2) = 0$ $\qquad b_8 = 0.25 \cdot -\sin(\pi/2) = -0.25$

$x(t) = \cos(t) + [0.25\cos(4t) - 0.43\sin(4t)] + [-0.25\sin(8t)]$

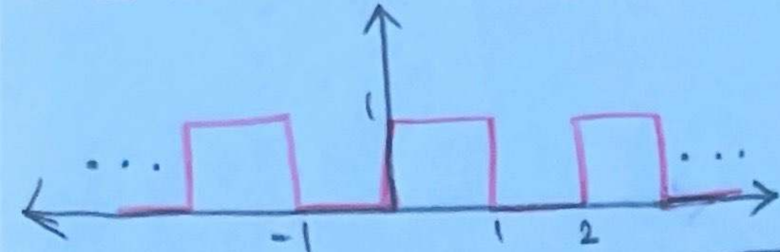
diğer a_k, b_k'ler 0

↓

sine-cosine form

Fourier örnek ④

FS coefficient bul.



sin-cosine form

$$\boxed{x(t)=\frac{1}{2}+\frac{2}{1\pi}\sin(1\omega_0 t)+\frac{2}{3\pi}\sin(3\omega_0 t)\ldots}$$

$$x(t)=\frac{1}{2}+\cdots+\frac{-j}{-3\pi}e^{-j3\omega_0 t}+\frac{-j}{-1\pi}e^{-j1\omega_0 t}+\frac{1}{2}+\frac{-j}{1\pi}e^{j1\omega_0 t}+\frac{-j}{3\pi}e^{j3\omega_0 t}+\ldots$$

exp. form

① $T=2$

$\omega_0=2\pi/2=\pi$

② $$c_k=\frac{1}{T}\int_{t_0}^{t_0+T}x(t)e^{-jk\omega_0 t}\,dt$$

$t_0 \leftarrow$ $t_0=0$ al.

③ $$c_k=\frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}\,dt=\frac{1}{2}\left[\int_0^1 1e^{-jk\pi t}\,dt+\int_1^2 0e^{-jk\pi t}\,dt\right]$$

$$=\frac{1}{2}\int_0^1 e^{-jk\pi t}\,dt=\frac{1}{2}\left[\frac{1}{-jk\pi}e^{-jk\pi t}\right]_0^1 \qquad \boxed{-\frac{1}{j}=j}$$

$$=\frac{j}{2k\pi}\left[e^{-jk\pi}-1\right] \qquad e^{-jk\pi}=\cos k\pi-j\sin k\pi$$

k'nın tek değerleri	k'nın çift değerleri
$e^{-jk\pi}=-1$	$e^{-jk\pi}=1$

$$c_k=\begin{cases}0, & k \text{ even}, \neq 0\\ \frac{-j}{k\pi}, & k \text{ odd}\\ \frac{1}{2}, & k=0\end{cases}$$

$$C_0=\frac{1}{2}\int_0^1 1.e^{-j0\pi t}=\frac{1}{2}$$

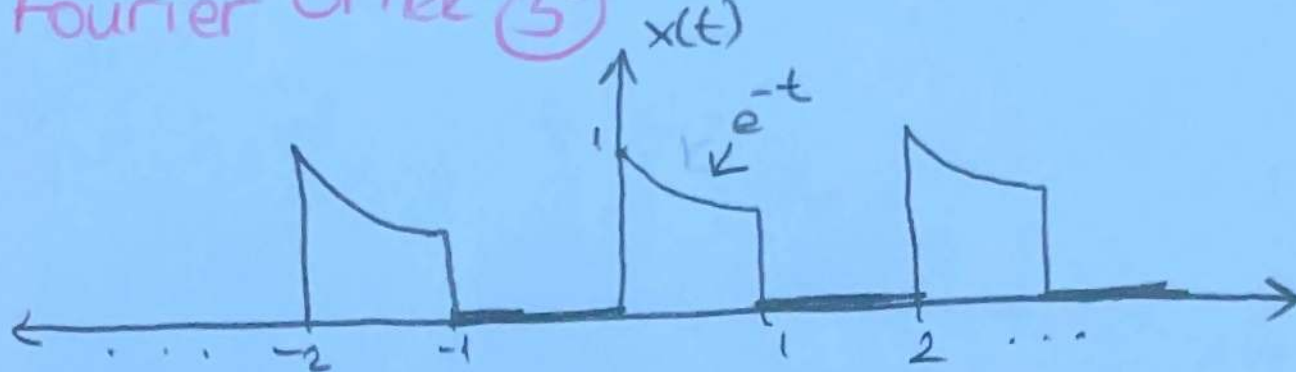
$a_0=C_0$

$\rightarrow a_k=2Re\{c_k\},\ k=1,2,\ldots$

$b_k=-2\,Im$ " " "

$$a_k=\begin{cases}\frac{1}{2}, & k=0\\ 0, & k\neq 0\end{cases} \qquad b_k=\begin{cases}0, & k \text{ even}\\ \frac{2}{k\pi}, & k \text{ odd}\end{cases}$$

Fourier örnek 5



① $T=2 \qquad \omega_0 = \frac{2\pi}{T} = \pi$

② $c_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}\,dt \qquad t_0=0$

$$e^{-jk\pi} = \underbrace{\begin{Bmatrix} 1, & \text{even } k \\ -1, & \text{odd } k \end{Bmatrix}}_{(-1)^k}$$

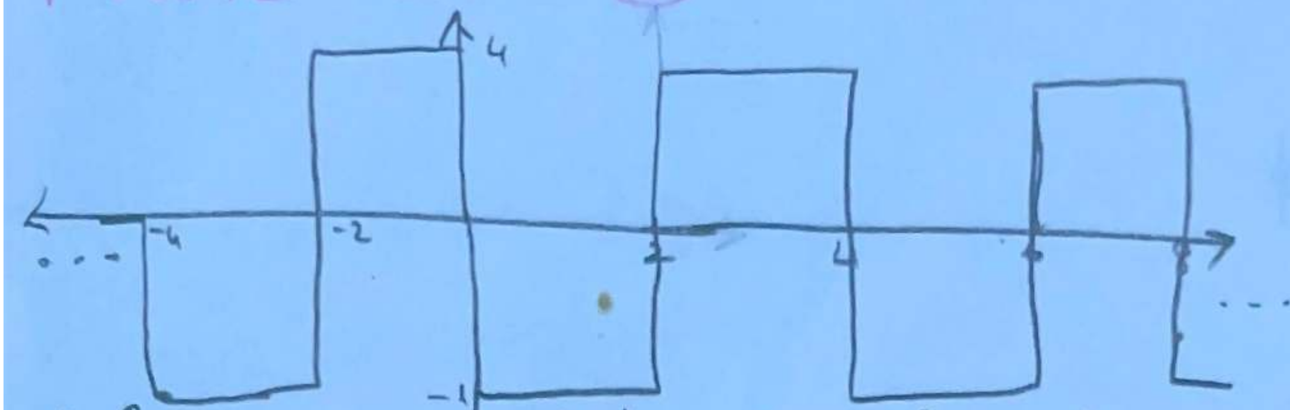
$$= \frac{1}{2}\int_0^1 e^{-t}e^{-jk\pi t}\,dt + \int_1^2 0\,dt$$

$$= \frac{1}{2}\int_0^1 e^{-t(1+jk\pi)}\,dt = \frac{1}{2}\left[\frac{e^{-t(1+jk\pi)}}{1+jk\pi}\cdot -1\right]_0^1 = \frac{-1}{2(1+jk\pi)}\left(e^{-(1+jk\pi)}-1\right)$$

$$= \frac{1-e^{-1}e^{jk\pi}}{2(1+jk\pi)} = \boxed{\frac{1-e^{-1}(-1)^k}{2(1+jk\pi)} = c_k}$$

$$c_1 = \frac{1+e^{-1}}{2(1+j\pi)} \qquad c_2 = \frac{1-e^{-1}}{2(1+2j\pi)} \qquad \ldots$$

Fourier Örnek 6



a) $f_0=?$

$T_0 = 4sn$ $f_0 = 1/4 = 0.25 Hz$

b) DC bileşen = ?

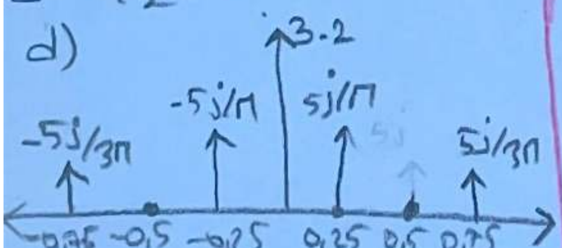
$a_0 = c_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$

$= \frac{1}{4}\left[\int_0^2 -1\,dt + \int_2^4 4\,dt\right]$

$= \frac{1}{4}\left[-t\right]_0^2 + \frac{1}{4}\left[4t\right]_2^4$

$= -1/2 + 4 - 2 = 1.5$

d)



c) $c_k=?$, $a_k=?$, $b_k=?$

$c_k = \frac{1}{4}\left[\int_0^2 -1e^{-jk\frac{\pi}{2}t}dt + \int_2^4 4e^{-jk\frac{\pi}{2}t}dt\right]$

$\frac{1}{4}\left[\frac{+1\cdot e^{-jk\pi/2\, t}}{+jk\pi/2}\right]_0^2 + \frac{1}{4}\left[\frac{4e^{-jk\pi/2\, t}}{-jk\pi/2}\right]_2^4$

$= \frac{-j}{2k\pi}\left[5e^{-jk\pi} - 4e^{-jk2\pi} - 1\right]$

$= \frac{j(1-5(-1)^k+4(-1)^{2k})}{2k\pi} = \underbrace{\frac{5j[1-(-1)^k]}{2k\pi}}_{c_k}$

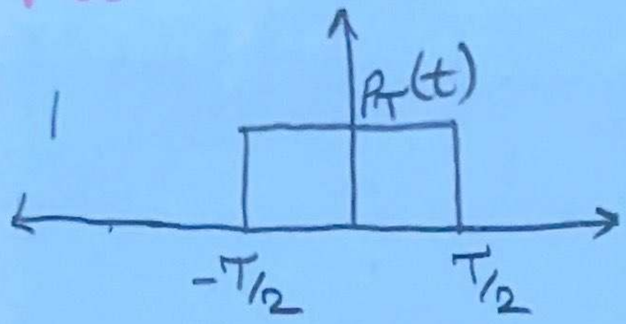
$a_k = 2Re\{c_k\} = 0$

$b_k = -2Im\{c_k\} = \frac{-5}{\pi k}\left(1-(-1)^k\right)$

$c_1 = \frac{5j}{2\pi}\cdot 2 = \frac{5j}{\pi}$ $c_2 = 0$ $c_{-2} = 0$

$c_{-1} = \frac{-5j}{\pi}$ $c_3 = \frac{5j}{3\pi}$ $c_{-3} = \frac{-5j}{3\pi}$

Fourier Transform Örnek 1



find $P_T(\omega)$

1

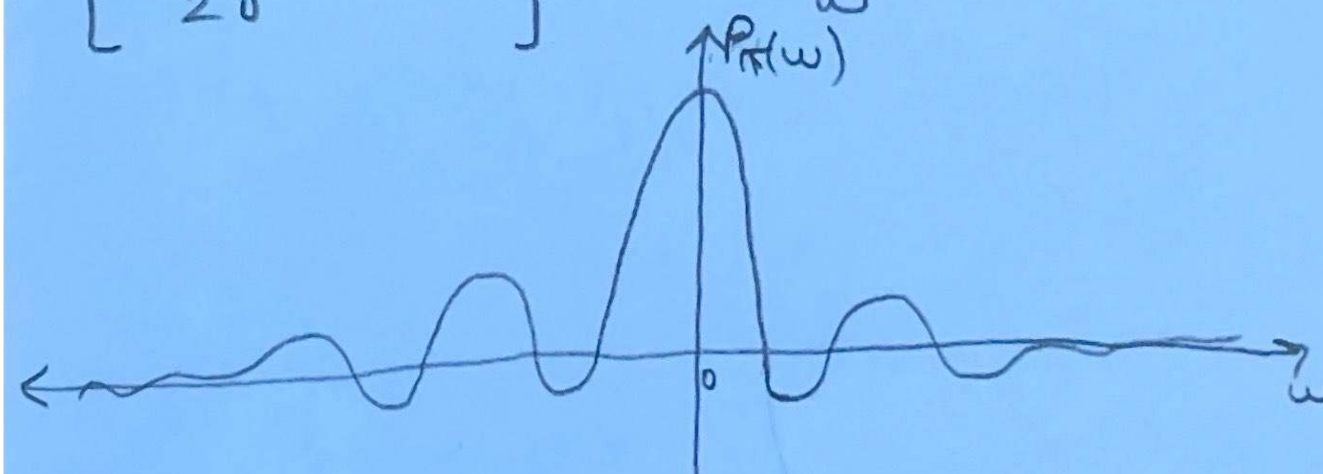
$$P_T(t) = \begin{cases} 1, & -T/2 \le t \le T/2 \\ 0, & \text{otherwise} \end{cases}$$

2

$$P_T(\omega) = \int_{-\infty}^{\infty} P_T(t)\, e^{-j\omega t}\, dt = \int_{-T/2}^{T/2} 1 e^{-j\omega t}\, dt$$

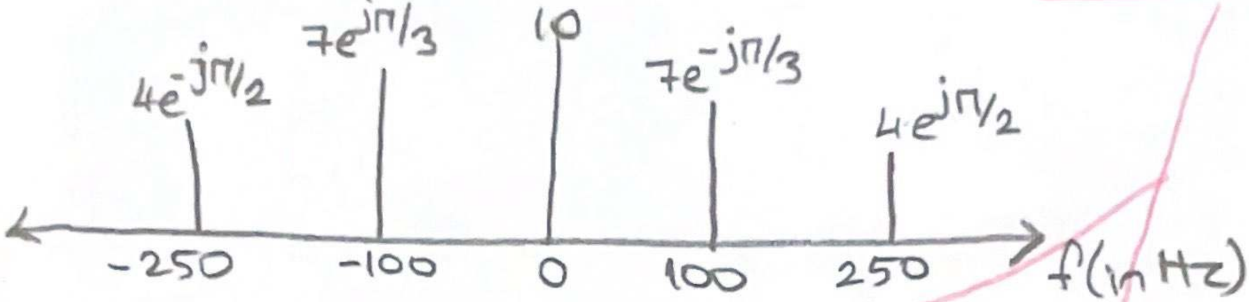
$$= \left[\frac{-1}{j\omega} e^{-j\omega t}\right]_{-T/2}^{T/2} = \frac{-1}{j\omega}\left[e^{-j\omega T/2} - e^{j\omega T/2}\right]$$

$$= \left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j}\right]\frac{2}{\omega} = \frac{2\sin\left(\omega \frac{T}{2}\right)}{\omega} = P_T(\omega)$$



Fourier Örnek (2)

$\boxed{w = 2\pi f t}$



$$x(t) = 10 + 7 \cdot e^{-j\pi/3} \cdot e^{j2\pi(100)t} + 7 \cdot e^{j\pi/3} \cdot e^{-j2\pi(100)t} +$$

$$\underbrace{4 \cdot e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} \cdot e^{-j2\pi(250)t}}$$

tersine euler dönüşümü yaparsak

$$x(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$

frekans	faz	genlik
-250	$-\pi/2$	8
-100	$\pi/3$	14
0	0	10
100	$-\pi/3$	14
250	$\pi/2$	8

önce tablo çizilir.

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \Phi_k)$$

$A_0 = 10$
$A_1 = 14$ $\quad \Phi_1 = -\pi/3$
$A_2 = 8$ $\quad \Phi_2 = \pi/2$
$f_0 = 50$
$f_1 = 100$
$f_2 = 250$

Pratik ters euler
1) $7 \rightarrow 14$ $\quad 4 \rightarrow 8$
2) frekansı poz. olanın fazı alınır. cos'un içine yazılır.
3) frek. poz. olan alınır.

Fourier Transform örnek 3

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \qquad y(t) = ?$$

a) $x(t) = e^{-t}u(t)$ b) $x(t) = u(t)$

$$(jw)y(w) + 2y(w) = x(w)$$

$$\frac{y(w)}{x(w)} = \frac{1}{2+jw} = h(w) \Rightarrow h(t) = e^{-2t}u(t)$$

a)

$$x(w) = \frac{1}{1+jw} \qquad y(w) = h(w)\,x(w) = \frac{1}{1+jw} \cdot \frac{1}{2+jw}$$

$$y(w) = \frac{1}{(1+jw)(2+jw)} = \left.\frac{1}{1+jw} - \frac{1}{2+jw}\right\} \text{ters fourier}$$

$$y(t) = e^{-t}u(t) - e^{-2t}u(t) = u(t)\cdot(e^{-t} - e^{-2t})$$

b) $x(w) = \frac{1}{jw}$

$$\frac{1}{jw} \cdot \frac{1}{2+jw} \Rightarrow \frac{1}{jw} - \frac{1}{2+jw}$$

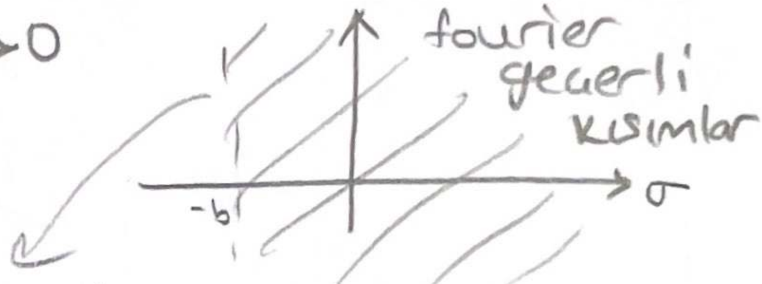
Laplace örnek 1

$x(t) = e^{-bt}u(t)$

$s+b>0$ $\quad$ $b>0$

$$X(s) = \int_0^\infty e^{-bt}e^{-st}\,dt = \int_0^\infty e^{-(s+b)t}\,dt = -\frac{e^{-(s+b)t}}{(s+b)}\Big|_0^\infty$$

$$= \frac{1}{s+b} \quad \left.\begin{matrix} Re\{s\}+b>0 \\ \sigma+b>0 \\ \sigma>-b \end{matrix}\right.$$



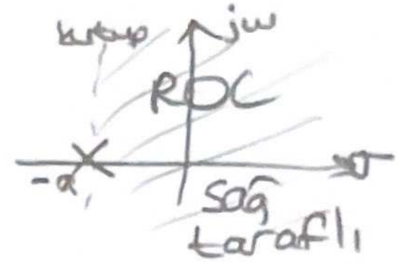
* $b>0$ iken $j\omega$ de bu alanda yani hem LT hem FT var.
* $b<0$ " " bu alanda değil, yani sadece LT var.

Laplace örnek (2)

$x(t)=e^{-at}.u(t)$ $\qquad$ $x_2(t)=-e^{-at}u(-t)$

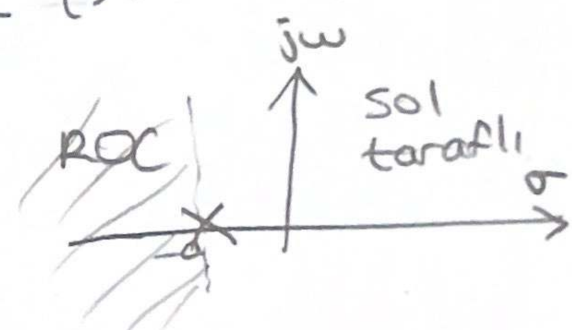
$$X(s)=\int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}dt = \int_{0}^{\infty} e^{-(a+s)t}dt = \left.\frac{e^{-(s+a)t}}{-(s+a)}\right|_{0}^{\infty}$$

$$X(s)=\frac{1}{s+a} \quad (s+a>0 \text{ ise}), (\sigma > -a)$$



$$X(s)=\int_{-\infty}^{\infty} -e^{-at}u(t)e^{-st}d(t) = \int_{-\infty}^{0} -e^{-(a+s)t}dt$$

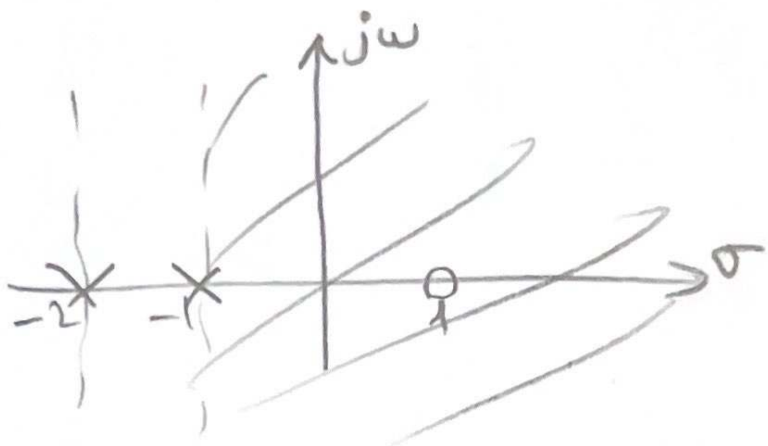
$$= +\left(\frac{e^{-(s+a)t}}{+(s+a)}\right)\Bigg|_{-\infty}^{0} = \frac{1}{s+a} \quad (s+a<0 \text{ ise}), (\sigma<-a)$$



Laplace örnek 3

$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$

$X(s) = \frac{3}{s+2} - \frac{2}{s+1}$ $\begin{cases} Re\{s\} > -2, \sigma > -2 \\ Re\{s\} > -1, \sigma > -1 \end{cases}$



$X(s) = \frac{s-1}{(s+2)(s+1)}$, $s-1=0$

$s=1 \rightarrow$ sistemin zero'su