

# **CENG 222**

## **Statistical Methods for Computer Engineering**

### **Week 14**

Chapter 6 Stochastic Processes  
Counting Processes  
Simulation of Stochastic Processes

# Counting Processes

- $X$  is a counting process if  $X(t)$  shows the number of items counted by time  $t \in T$ .
- Counting processes are non-decreasing
- Since they show count, they are discrete-state processes
- Can be *discrete-time* (Binomial Process) or *continuous-time* (Poisson Process)
- Examples:
  - Counting emails received by time  $t$
  - Counting total number of goals scored in a game by time  $t$

# Binomial process

- Discrete-time (i.e., each time step contains a Bernoulli trial)
- A binomial process  $X(t)$  is the number of successes by the time  $t$  in a sequence of independent Bernoulli trials.
- $X(t)$  number of successes by the time  $t$ 
  - $\text{Binomial}(tp)$
- $Y$  number of trials between two successes
  - $\text{Geometric}(p)$

# Binomial Process

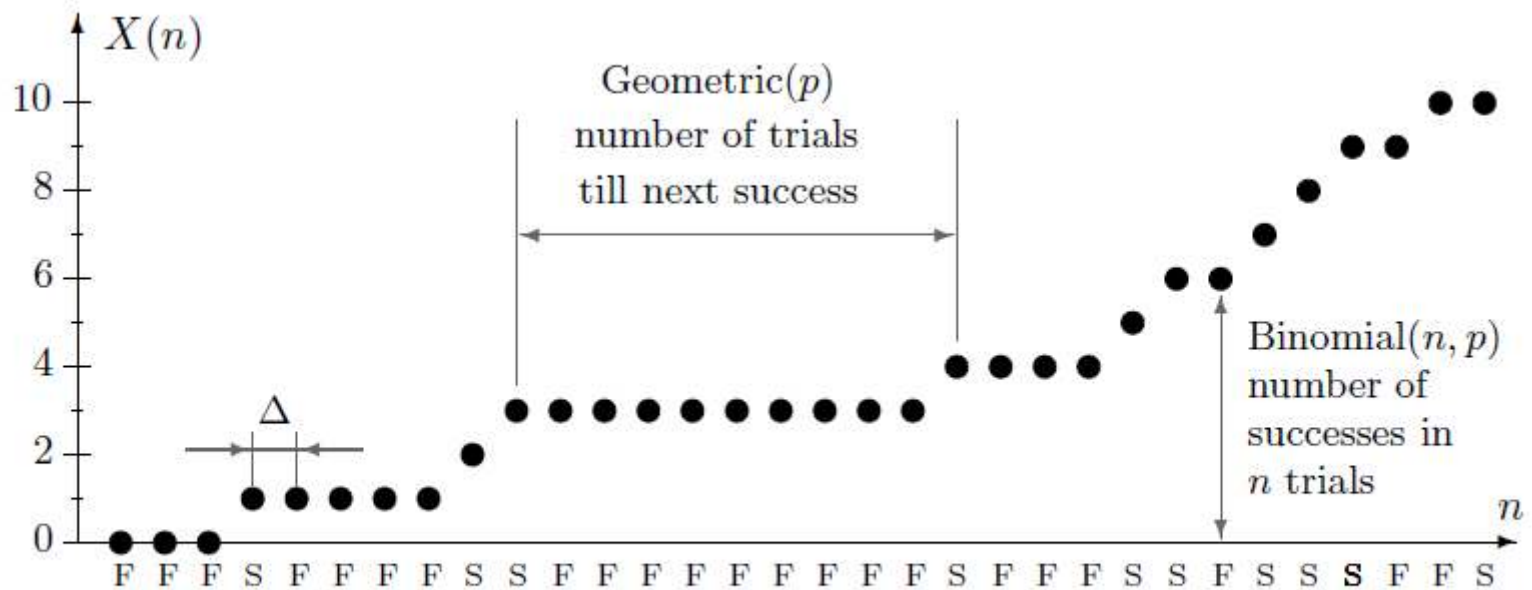


FIGURE 6.7: Binomial process (sample path). Legend: S = success, F=failure.

## # of trials versus time

- Although discrete, if the time unit for each trial is not 1 (second, minute, etc.), we may need to be careful in using the value of  $t$  in our computations.
- For example: if a Bernoulli trial occurs every 3 seconds,  $X(6)$  is **not** Binomial(6, $p$ ) but it is Binomial(2, $p$ ) (2 trials in 6 seconds).
- The time interval  $\Delta$  of each Bernoulli is called a frame.
  - Number of trials equals to  $t/\Delta$

## Arrival/Success rate $\lambda$

- If  $p$  is the success rate at  $\Delta$  units of time, then  $\lambda$  is the success rate per 1 unit of time
  - $\lambda = p/\Delta$
- $T$  is the time between two success. ( $Y$  was the number of trials ( $\Delta$ s) between two success.
  - $T = Y\Delta$

# Transition probabilities of a Binomial Process

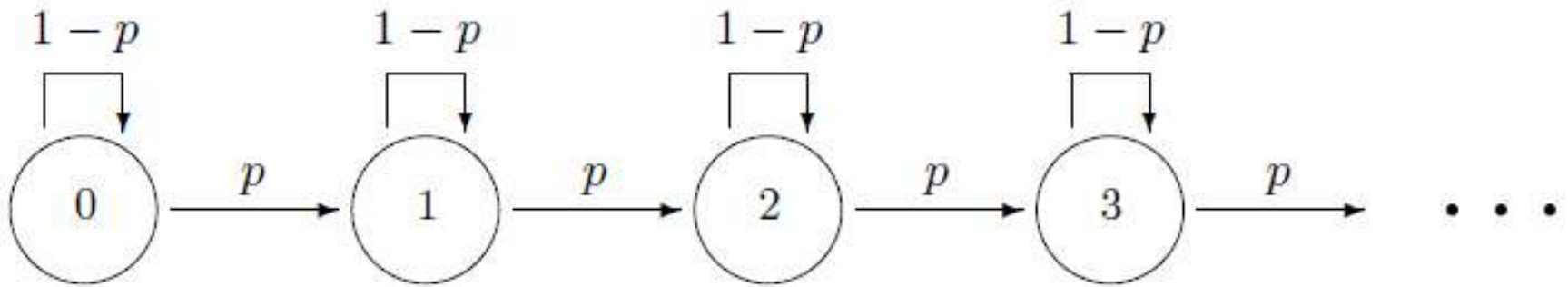


FIGURE 6.8: *Transition diagram for a Binomial counting process.*

- Is it regular?

# Poisson counting process

- When the frame  $\Delta$  approaches 0, we approach a continuous-time counting process.
  - Note that as  $\Delta$  approaches 0,  $p$  also approaches 0.
- Taking the success/arrival rate per unit of time,  $\lambda$ , as constant, we may model such continuous counting processes.
- $X(t)$  becomes a  $\text{Poisson}(\lambda t)$  variable.  $T$  becomes an  $\text{Exponential}(\lambda)$  variable.



# Using the Gamma-Poisson formula

- Recall that Poisson problems could be solved using the Gamma distribution (Chapter 4)
  - Gamma-Poisson formula (Eq. 4.14)
- Time needed for the  $k$ th success,  $T_k$ , is a  $\text{Gamma}(k, \lambda)$  variable
- $P(T_k \leq t) = P(k \text{ successes before time } t) = P(X(t) \geq k)$ 
  - $T_k$  is  $\text{Gamma}(k, \lambda)$  and  $X(t)$  is  $\text{Poisson}(\lambda t)$

# Poisson Process

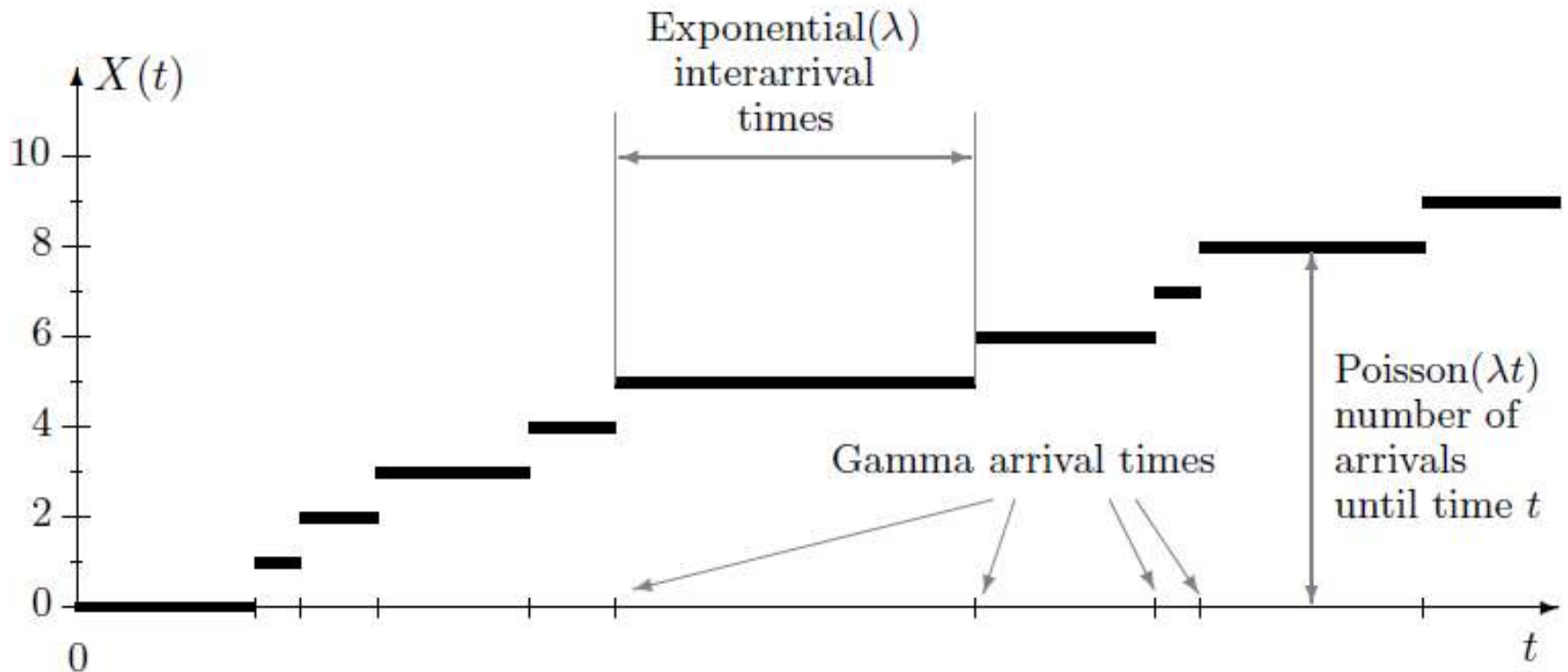


FIGURE 6.10: *Poisson process (sample path).*

# Simulation of Stochastic Processes

- We can use random sampling techniques we learned in Chapter 5 to simulate stochastic processes.
  - For example: state transitions are discrete with specific *pmfs*, which could be simulated by using Algorithm 5.1 (or the Alias method for efficiency)
- Steady-state distributions of a regular Markov chain can also be found using an iterative simulation and checking whether two successive state-distributions are equal (or very close).