

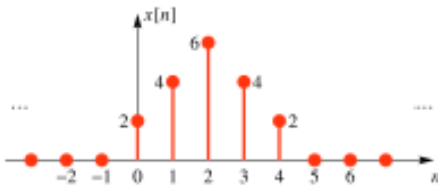
Q13

Determine the output of a *centralized averager*

$$y[n] = (1/3)(x[n+1] + x[n] + x[n-1])$$

for the following input. Is this filter causal or noncausal?

What is the support of the output for this input?



A13

$$y[0] = \frac{1}{3}(x[1] + x[0] + x[-1]) = \frac{1}{3}(4 + 2 + 0) = 2$$

$$y[-1] = \frac{1}{3}(x[0] + x[-1] + x[-2]) = \frac{1}{3}(2 + 0 + 0) = 2/3$$

$$y[-2] = 0$$

$$y[1] = \frac{1}{3}(x[2] + x[1] + x[0]) = \frac{1}{3}(6 + 4 + 2) = 4$$

Make a table:

n	≤ -2	-1	0	1	2	3	4	5	6	≥ 7
x[n]	0	0	2	4	6	4	2	0	0	0
y[n]	0	2/3	2	4	14/3	4	2	2/3	0	0

Since y[n] starts before x[n] ⇒ NOT causal

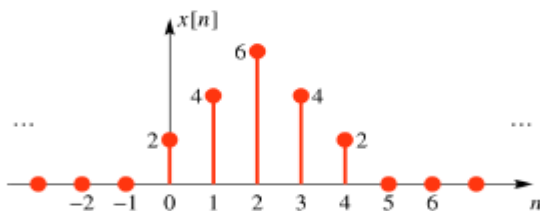
SUPPORT is:  
-1 ≤ n ≤ 5

Q14

Compute the output y[n] for the length-4 filter whose coefficients are {b<sub>k</sub>} = {3, -1, 2, 1}. Use the the following signal as input.

Verify that the answers tabulated here are correct, then fill in the missing values.

n	n < 0	0	1	2	3	4	5	6	7	8	n > 8
x[n]	0	2	4	6	4	2	0	0	0	0	0
y[n]	0	6	10	18	?	?	?	8	2	0	0



A14

$$y[n] = \underset{b_0}{3}x[n] - \underset{b_1}{x}[n-1] + \underset{b_2}{2}x[n-2] + \underset{b_3}{x}[n-3]$$

$$y[2] = 3x[2] - x[1] + 2x[0] + x[-1]$$

$$= 3(6) - 4 + 2(2) + 0 = 18 \checkmark$$

$$y[3] = 3x[3] - x[2] + 2x[1] + x[0]$$

$$= 3(4) - 6 + 2(4) + 2 = \boxed{16}$$

$$y[4] = 3(2) - 4 + 2(6) + 4 = \boxed{18}$$

$$y[5] = 3(0) - 2 + 2(4) + 6 = \boxed{12}$$

$$y[6] = 3(0) - 0 + 2(2) + 4 = 8 \checkmark$$

Q15

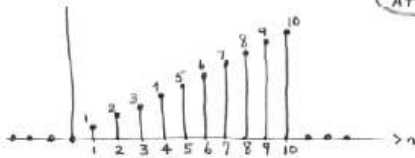
Determine and plot the impulse response of the  
FIR system

$$y[n] = \sum_{k=0}^{10} kx[n-k]$$

A15

$$y[n] = \sum_{k=0}^{10} k \delta[n-k] = 0 \cdot \delta[n] + \delta[n-1] + 2 \delta[n-2] + \dots + 10 \delta[n-10]$$

IMPULSE  
AT  $n=2$

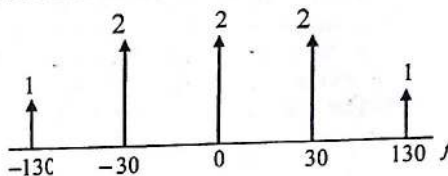


- b. The same ADC is used to record a speech signal for 2 minutes. If the sampling frequency of the ADC is 20 kHz (k samples/second), calculate the required memory space (in terms of the byte) to store the speech signal. (05)

$$\text{required memory space} = 2 \times 60 \times 20000 = 2400000 \text{ bytes}$$

- Q4. Frequency spectrum of a signal is given as the following:

- a. Write an equation for the signal  $x(t)$  defined by this frequency spectrum. (05)



$$\begin{aligned} x(t) &= 2 + 4 \cos(2\pi 30t) + 2 \cos(2\pi 130t) \\ &= 2 + 4 \cos(60\pi t) + 2 \cos(260\pi t) \end{aligned}$$

- b. Write  $x[n]$  after the signal digitized by an ADC with a sampling frequency of 100 Hz. (05)

$$x[n] = x[nT_s] = 2 + 4 \cos(60\pi nT_s) + 2 \cos(260\pi nT_s), \quad T_s = \frac{1}{f_s} = \frac{1}{100}$$

$$x[n] = 2 + 4 \cos\left(\frac{60\pi n}{100}\right) + 2 \cos\left(\frac{260\pi n}{100}\right)$$

$$x[n] = 2 + 4 \cos(0.6\pi n) + 2 \cos(2.6\pi n)$$

$$x[n] = 2 + 4 \cos(0.3 \times 2\pi n) + 2 \cos(1.3 \times 2\pi n)$$

- c. Is there any aliasing in (b)? If there is prove it. (05)

$$\begin{aligned} x[n] &= 2 + 4 \cos(0.6\pi n) + 2 \cos((2+0.6)\pi n) \\ &= 2 + 4 \cos(0.6\pi n) + 2 \cos(2\pi n + 0.6\pi n) \\ &= 2 + 4 \cos(0.6\pi n) + 2 \underbrace{\cos(2\pi n)}_1 \cos(0.6\pi n) - 2 \underbrace{\sin(2\pi n)}_0 \sin(0.6\pi n) \end{aligned}$$

$$= 2 + 4 \cos(0.6\pi n) + 2 \cos(0.6\pi n) = \underline{\underline{2 + 6 \cos(0.6\pi n)}}$$

So, there is an aliasing

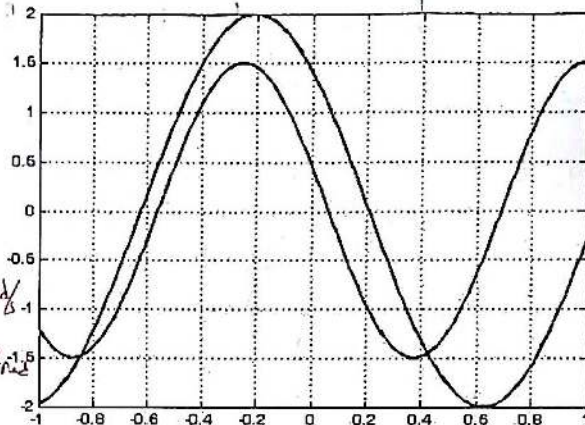
d. What happens to the signal  $x(t)$  when the sampled signal in (b) is reconstructed? (05)

$$X(t) = X\left[\frac{t}{T_s}\right] = 2 + 6 \cos\left(\frac{0.6\pi t}{0.01}\right) = 2 + 6 \cos(60\pi t)$$

$$X(t) = 2 + \cos(2\pi 30t)$$

Because of aliasing, signal with 130 Hz is lost. It appears as a 30 Hz signal. Therefore, magnitude of 30 Hz signal became 6.

Q5. Find the three important parameters amplitude  $A$ , phase  $\phi$  and fundamental angular frequency  $\omega_0$  which define a particular sinusoid for the two signals on the following graph. Take time delays approximately. (15)



Signal 1:

$$A = 1.5, T \approx 1.25 \text{ s}, \omega_0 = \frac{2\pi}{T} = 1.6\pi \text{ rad/s}$$

$$\phi \approx 0.4\pi = -\omega_0 t_m = -1.6\pi \times (-0.25) = 0.4\pi \text{ rad}$$

$$t_m = -0.25 \text{ s}$$

Signal 2:

$$A = 2, T \approx 1.6 \text{ s}, \omega_0 = \frac{2\pi}{T} = 1.25\pi \text{ rad/s}, \phi = 0.25\pi \text{ rad}$$

$$t_m = -0.2 \text{ s}, \phi = -\omega_0 t_m = -1.25\pi \times (-0.2) \approx 0.25\pi \text{ rad} = \frac{\pi}{4} \text{ rad}$$

Q6. A Periodic signal is defined by the equation

$$x(t) = 2 + 4 \cos\left(40\pi t - \frac{1}{5}\pi\right) + 3 \sin(60\pi t) + 4 \cos\left(120\pi t - \frac{1}{3}\pi\right)$$

a. Determine the fundamental frequency  $\omega_0$ , the fundamental period  $T_0$ , and coefficients  $a_k$  in the Fourier representation for that signal. (10)

$$X(t) = 2 + \frac{4}{2} \left( e^{j(40\pi t - \frac{\pi}{5})} + e^{-j(40\pi t - \frac{\pi}{5})} \right) + \frac{3}{2j} \left( e^{j60\pi t} - e^{-j60\pi t} \right) + \frac{4}{2} \left( e^{j(120\pi t - \frac{\pi}{3})} + e^{-j(120\pi t - \frac{\pi}{3})} \right)$$

$$X(t) = 2 + 2e^{j\frac{\pi}{5}} e^{j40\pi t} + 2e^{-j\frac{\pi}{5}} e^{-j40\pi t} + 1.5e^{-j\frac{\pi}{2}} e^{j60\pi t} + 1.5e^{j\frac{\pi}{2}} e^{-j60\pi t} + 2e^{-j\frac{\pi}{3}} e^{j120\pi t} + 2e^{j\frac{\pi}{3}} e^{-j120\pi t}$$

$$\text{not: } \frac{1}{j} = -j = e^{-j\frac{\pi}{2}}, -\frac{1}{j} = j = e^{j\frac{\pi}{2}}$$

$$f_1 = 20 \text{ Hz}, f_2 = 30 \text{ Hz}, f_3 = 60 \text{ Hz}, \text{ so } f_0 = 10 \text{ Hz} \Rightarrow T_0 = \frac{1}{10} = 0.1 \text{ s}$$

$$\omega_0 = 20\pi \text{ rad/s}$$

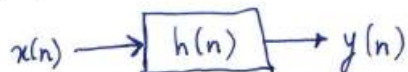


1. (35 pts.) (a) Compute the following convolution:

$$\{a, b, c, d, e\} * \{e, d, c, b, a\}$$

where  $a, b, c, d, e$  are some numbers. Both sequences above start with the  $n=0$  index in time. (15 pts.)

(b) The impulse response of a LTI system is given as  $\mathbf{h} = \{1, 2, 1\}$ . The output sequence is known to be  $\mathbf{y} = \{1, 4, 8, 12, \dots\}$ . Both sequences start with the  $n=0$  index in time. Find the input sequence  $x(n)$ . (20 pts.)



Answer:

1. (a)  $x = \{a, b, c, d, e\}$ ,  $y = \{e, d, c, b, a\}$   
 $\uparrow$   $\uparrow$   
 $n=0$   $n=0$

$$y(-k) = \{a, b, c, d, e\}$$

$$z(n) = \sum_{k=-\infty}^{\infty} x(k) y(n-k) = \sum_k x(k) y(-(k-n))$$

$$z(0) = ae, \quad z(1) = ad + be, \quad z(2) = ac + bd + ce$$

$$z(3) = ab + bc + cd + de, \quad z(4) = a^2 + b^2 + c^2 + d^2 + e^2,$$

$$z(5) = ba + cb + dc + ed, \quad z(6) = ca + db + ec,$$

$$z(7) = da + eb, \quad z(8) = ea, \quad z(n) = 0 \text{ for } n < 0 \text{ and } n > 8.$$

(b) The freq. response:  $H(e^{j\omega}) = \sum_n h(n) e^{-j\omega n} = 1 + 2e^{-j\omega} + e^{-j2\omega}$

Output Fourier transform:  $Y(e^{j\omega}) = 1 + 4e^{-j\omega} + 8e^{-j2\omega} + 12e^{-j3\omega} + \dots$   
 $= H(e^{j\omega}) X(e^{j\omega})$

The input Fourier transform:

$$X(e^{j\omega}) = \text{bTFT}\{x(n)\} = \frac{Y(e^{j\omega})}{H(e^{j\omega})}$$

$$\begin{array}{r} 1 + 4e^{-j\omega} + 8e^{-j2\omega} + 12e^{-j3\omega} + \dots \\ \underline{1 + 2e^{-j\omega} + e^{-j2\omega}} \\ 2e^{-j\omega} + 7e^{-j2\omega} + 12e^{-j3\omega} + \dots \\ \underline{2e^{-j\omega} + 4e^{-j2\omega} + 2e^{-j3\omega}} \\ 3e^{-j2\omega} + 10e^{-j3\omega} + 16e^{-j4\omega} + \dots \end{array} \quad \begin{array}{r} 1 + 2e^{-j\omega} + e^{-j2\omega} \\ \underline{1 + 2e^{-j\omega} + 3e^{-j2\omega}} \\ 4e^{-j3\omega} + \dots \end{array}$$

(2)

$$\begin{aligned}
 X(e^{j\omega}) &= 1 + 2e^{-j\omega} + 3e^{-j2\omega} + 4e^{-j3\omega} + \dots \\
 &= \text{DTFT}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}
 \end{aligned}$$

$$\therefore x(n) = \{1, 2, 3, 4, 5, \dots\} = \begin{cases} n+1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

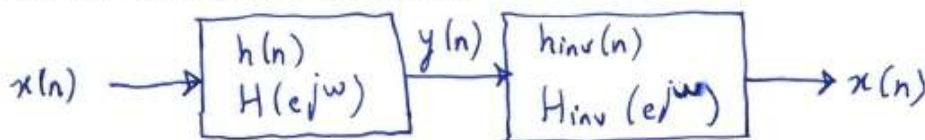
$\uparrow$   
 $n=0$

2. (35 pts.) Consider a LTI system described by the following difference equation:

$$y(n) - (3/4)y(n-1) + (1/8)y(n-2) = 2x(n-1)$$

with  $y(-1) = y(-2) = 0$ .  $x(n)$  and  $y(n)$  denote input and output sequences, respectively.

- (a) Find the frequency response  $H(e^{j\omega})$  of this system. (10 pts.)  
 (b) Find the impulse response  $h(n)$  of this system. (Hint: Apply partial fraction expansion before taking the inverse DTFT.) Is this system causal? Is it stable? (10 pts.)  
 (c) Consider the inverse system of this system:



Find the frequency response  $H_{inv}(e^{j\omega})$  of the inverse system. (7.5 pts.)

(d) Find the impulse response  $h_{inv}(n)$  of the inverse system. Is the inverse system causal? Is it stable? (7.5 pts.)

Answer:

$$2. (a) \quad y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 2x(n-1)$$

Let's take Fourier tr. of both sides:  $Y(e^{j\omega}) = \text{DTFT}\{y(n)\}$ ,

$$X(e^{j\omega}) = \text{DTFT}\{x(n)\}$$

and use  $\text{DTFT}\{y(n-k)\} = e^{-j\omega k} Y(e^{j\omega})$ :

$$\left[1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}\right] Y(e^{j\omega}) = 2e^{-j\omega} X(e^{j\omega})$$

$$\boxed{H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2e^{-j\omega}}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}}$$

$$(b) \quad h(n) = \text{IDTFT}\{H(e^{j\omega})\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

$$\text{where } H(e^{j\omega}) = \frac{2e^{-j\omega}}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}} = \frac{2e^{-j\omega}}{(1 - \frac{1}{4}e^{-j\omega})(1 - \frac{1}{2}e^{-j\omega})}$$

$$\frac{1}{(1 - \frac{1}{4}e^{-j\omega})(1 - \frac{1}{2}e^{-j\omega})} = \frac{2}{1 - \frac{1}{2}e^{-j\omega}} - \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$$

(3)

$$H(e^{j\omega}) = \frac{4e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}} - \frac{2e^{-j\omega}}{1 - \frac{1}{4}e^{-j\omega}}$$

$$\boxed{h(n) = 4 \cdot (1/2)^{n-1} u(n-1) - 2 \cdot (1/4)^{n-1} u(n-1),}$$

with  $u(n) = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$   
 $h(n) = 0$  for  $n < 0 \Rightarrow$  the system is causal.

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=1}^{\infty} |4 \cdot (1/2)^{n-1} - 2 \cdot (1/4)^{n-1}|$$

$$= \sum_{n=0}^{\infty} [4 \cdot (1/2)^n - 2 \cdot (1/4)^n] = 4 \cdot \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n -$$

$$2 \cdot \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= 4 \cdot \frac{1}{1 - 1/2} - 2 \cdot \frac{1}{1 - 1/4} = 8 - 8/3 = 16/3 < \infty$$

$\therefore$  The system is stable.

$$(c) H_{inv}(e^{j\omega}) = \frac{X(e^{j\omega})}{Y(e^{j\omega})} = \frac{1}{H(e^{j\omega})} = \frac{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}{2e^{-j\omega}}$$

$$= \boxed{\frac{1}{2}e^{j\omega} - \frac{3}{8} + \frac{1}{16}e^{-j\omega}}$$

$$(d) H_{inv}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_{inv}(n)e^{-j\omega n} \Rightarrow h_{inv}(n) = \frac{1}{2}\delta(n+1) - \frac{3}{8}\delta(n) + \frac{1}{16}\delta(n-1)$$

$$h_{inv}(n) = \begin{cases} 1/2, & n = -1 \\ -3/8, & n = 0 \\ 1/16, & n = 1 \\ 0, & \text{otherwise} \end{cases}$$

$h_{inv}(-1) = 1/2 \neq 0$   
 $\Downarrow$   
the system is noncausal.

(4)

$$\sum_n |h(n)| = \frac{1}{2} + \frac{3}{8} + \frac{1}{16} = \frac{15}{16} < \infty \Rightarrow \text{the system is } \underline{\underline{\text{stable}}}.$$

3. (30 pts.) A system with input signal  $x(n]$  and output signal  $y(n]$ , is described as

$$y(k) = x(-\infty) = \lim_{i \rightarrow -\infty} x(i)$$

for all  $k$  values.

(a) Is this system linear or not? Justify your answer.

(b) Is this system time-invariant or not? Justify your answer.

(c) Let the input be  $x(n) = \delta(n)$ , the unit sample sequence (discrete-time impulse signal). Find the output, i.e., the impulse response  $h(n]$  of this system.

Can the output signal  $y(n]$  be computed by convolving the impulse response  $h(n]$  with an input signal  $x(n]$ , if  $x(-\infty) \neq 0$ ?

Answer:

3. (a)  $y(k) = \lim_{i \rightarrow -\infty} x(i) = x(-\infty) = T\{x(n)\}$

Let  $y_1(n) = T\{x_1(n)\}$  and  $y_2(n) = T\{x_2(n)\}$  be two input/output pairs.

$$y_1(k) = x_1(-\infty) = \lim_{i \rightarrow -\infty} x_1(i)$$

for all  $k$ .

$$y_2(k) = x_2(-\infty) = \lim_{i \rightarrow -\infty} x_2(i)$$

If  $x(n) = a x_1(n) + b x_2(n)$  is a new input, the corresponding



output will be:  $y(k) = x(-\infty) = \lim_{i \rightarrow -\infty} x(i) = \lim_{i \rightarrow -\infty} [ax_1(i) + bx_2(i)]$

$$= a \cdot \lim_{i \rightarrow -\infty} x_1(i) + b \cdot \lim_{i \rightarrow -\infty} x_2(i)$$

$$= ax_1(-\infty) + bx_2(-\infty) = ay_1(k) + by_2(k)$$

for any  $a, b$ . for all  $k$ ,

$\therefore$  The system is linear.

(b)  $y(k) = \lim_{i \rightarrow -\infty} x(i) = x(-\infty)$ , for all  $k$ , for an input/output pair.

Let  $x_1(n) = x(n-m)$  be a new input signal to this system, for a fixed integer  $m$ .

The corresponding output:  $y_1(k) = \lim_{i \rightarrow -\infty} x_1(i) = \lim_{i \rightarrow -\infty} x(i-m)$

$$= x(-\infty), \text{ for all } k.$$

$$y(n-m) = x(-\infty) = \text{constant, for all } n.$$

(5)

$\therefore y_1(n) = y(n-m) = x(-\infty)$  for any fixed  $m$ .

$\therefore$  The system is also time-invariant.

(c)  $h(n) = \delta(-\infty) = \lim_{i \rightarrow -\infty} \delta(i) = 0$ , for all  $n$ .

If  $x(-\infty) \neq 0$  for an input signal  $x(n)$ , the output;

$$y(n) = x(-\infty) \neq 0, \text{ for all } n.$$

$$x(n) * h(n) = \sum_k \underbrace{h(k)}_0 x(n-k) = 0, \text{ for all } n.$$

$\therefore y(n) \neq x(n) * h(n)$ , if  $x(-\infty) \neq 0$ ,

although the system is a LTI system.

Some LTI systems are so-called "nonconvolutional" systems, i.e., they can't be modeled by convolution operation, although they are LTI systems.

This system is an example of such a nonconvolutional system.

1. (35 pts.) Consider a LTI system described by

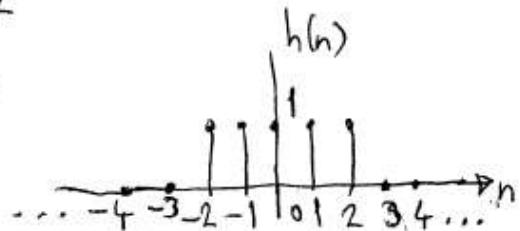
$$y(n] = x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2),$$

where  $x(n]$  and  $y(n]$  denote input and output sequences, respectively.

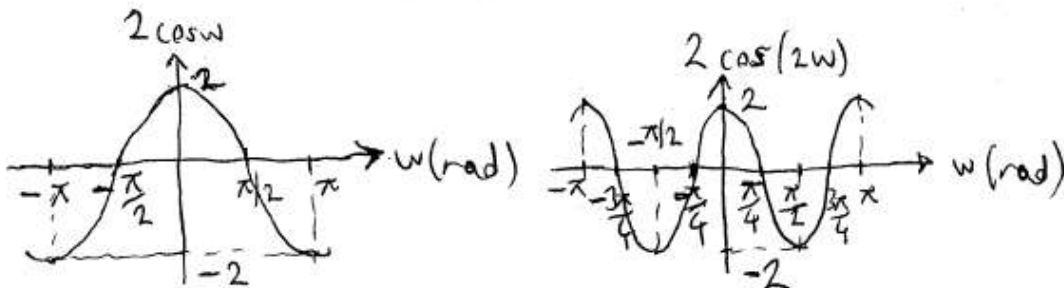
- Find and plot impulse response  $h(n]$  of this system.
- Find the frequency response  $H(e^{j\omega})$  of the system, and plot it for  $-\pi \leq \omega \leq \pi$  rad.
- Is this system stable or not? Is it causal or not?
- Find a recursive difference equation expressing this system.
- Find a difference equation for the "inverse system" of this system.
- Find the impulse response of the inverse system.
- Is the inverse system stable? Is it causal?

Answer: (a)  $h(n] = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)$

$$= \begin{cases} 1, & -2 \leq n \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$



(b)  $H(e^{j\omega}) = e^{-j2\omega} + e^{-j\omega} + 1 + e^{j\omega} + e^{j2\omega} = 1 + 2\cos\omega + 2\cos(2\omega),$

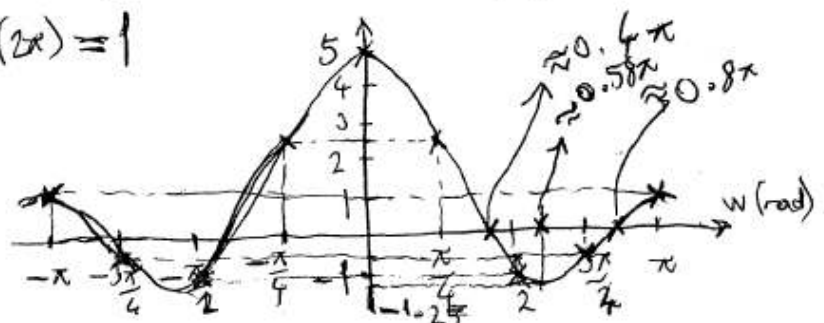


$$H(e^{j0}) = 5, \quad H(e^{j\pi/4}) = 1 + 2\underbrace{\cos(\pi/4)}_{1/\sqrt{2}} + 2\underbrace{\cos(\pi/2)}_0 = 1 + \sqrt{2} \approx 2.4142$$

$$H(e^{j\pi/2}) = 1 + 2\cos(\pi/2) + 2\cos(\pi) = -1$$

$$H(e^{j3\pi/4}) = 1 + 2\cos(3\pi/4) + 2\cos(3\pi/2) = 1 - \sqrt{2} \approx -0.4142$$

$$H(e^{j\pi}) = 1 + 2\cos(\pi) + 2\cos(2\pi) = 1$$



$$(c) \sum_n |h(n)| = 5 < \infty \Rightarrow \text{stable system.}$$

$$h(-1) \neq 0, h(-2) \neq 0 \Rightarrow \text{noncausal system.}$$

$$(d) y(n) = x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2)$$

$$y(n+1) = x(n+3) + x(n+2) + x(n+1) + x(n) + x(n-1)$$

$$\therefore y(n+1) - y(n) = x(n+3) - x(n-2) \Rightarrow \boxed{y(n) - y(n-1) = x(n+2) - x(n-3)}$$

(e)  $y(n) \rightarrow \boxed{h_{inv}(n)} \rightarrow x(n)$   $x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2) = y(n)$

$$\boxed{x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4) = y(n-2)}$$

$$\text{or } x(n+2) - x(n-3) = y(n) - y(n-1) \Rightarrow \boxed{x(n) - x(n-5) = y(n-2) - y(n-3)}$$

$x(n)$ : output,  $y(n)$ : input of the inverse system.  
Both of them are correct.

$$(f) H_{inv}(e^{j\omega}) = \frac{X(e^{j\omega})}{Y(e^{j\omega})} = \frac{e^{-j2\omega}}{1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega}}$$

$$\frac{1}{e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} + e^{-j5\omega} + \dots} \cdot \frac{e^{-j2\omega}}{1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega}}$$

$$\frac{e^{-j\omega} - e^{-j2\omega} + e^{-j3\omega} - e^{-j4\omega} + e^{-j5\omega} - e^{-j6\omega} + e^{-j7\omega} - e^{-j8\omega} + e^{-j9\omega} - e^{-j10\omega} + \dots}{e^{-j5\omega} + e^{-j6\omega} + e^{-j7\omega} + e^{-j8\omega} + e^{-j9\omega} + \dots}$$

$$\frac{e^{-j10\omega}}{e^{-j5\omega} + e^{-j6\omega} + e^{-j7\omega} + e^{-j8\omega} + e^{-j9\omega} + \dots}$$

$$\vdots$$

$$\therefore H_{inv}(e^{j\omega}) = e^{-j2\omega} - e^{-j3\omega} + e^{-j7\omega} - e^{-j8\omega} + e^{-j12\omega} - e^{-j13\omega} + e^{-j17\omega} - e^{-j18\omega} + \dots$$

$$= \sum_{n=-\infty}^{\infty} h_{inv}(n) e^{-j\omega n}$$

{ continues on back of card }

2.(30 pts.) The sequence of daily sunspot (güneş lekesi) numbers is smoothed by taking five-day running totals. For each day we add the sunspot numbers for the preceding and following two days:

$$y(n) = x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2),$$

where  $x(n)$  and  $y(n)$  denote the sunspot number observed on day  $n$  and the running total value on day  $n$ , respectively. Here is a sequence of five-day running totals:

$y = \{45, 35, 25, 15, 5, 0, 0, 0, 0, 15, 50, 80, 100, 125, 125, 100, 80, 70, 45, 30, 30, 30, 35, 60, 80, 90, 95, 100, 90, 85, 75\}$ . Find the actual sequence of daily sunspot numbers,  $x$ .

Hint: Daily sunspot numbers should be nonnegative integers.

Answer: Let the marked index be  $n = 0$ :

$$y(0) = 0 = x(-2) + x(-1) + x(0) + x(1) + x(2)$$

$$\therefore x(-2) = x(-1) = x(0) = x(1) = x(2) = 0,$$

since  $x(n) \geq 0$  and integer always ( $x(n)$ : # of sunspots on  $n^{\text{th}}$  day) for all  $n$ .

For  $n > 2$ :  $x(n) = y(n-2) - x(n-1) - x(n-2) - x(n-3) - x(n-4)$

can be used to recover  $x(n)$  from the above initial values

For  $n < -2$ :  $x(n-4) = y(n-2) - x(n) - x(n-1) - x(n-2) - x(n-3)$

can be used to recover  $x(n)$ , recursively.

$y$		45	35	25	15	5	0	0	0	0	15	50	80	100
$x$	10	10	10	10	5	0	0	0	0	0	0	0	15	35

$y$	125	125	100	80	70	45	30	30	30	35	60	80
$x$	30	20	25	15	10	10	10	0	0	10	10	15

$\Rightarrow \{ \text{Back page} \}$

$y$	90	95	100	90	85	75					
$x$	25	20	20	15	20	15	15	10	...		