Homework 4 (Due October 31, 2016, 9:00 AM)

BLM 1541: Statistics and Probability — Fall 2016

Print family (or last) name: ———————————————————————————

Print given (or first) name: ———————————————————————————

Print given student number: ———————————————————————————

I see that this homework has 3 questions in total.

I agree that I have to submit my homework solution before the deadline (October 31, 2016, 9 AM) otherwise my homework solution will not be graded.

I accept that *I will add the signed version of this instruction page as a first page into my homework solution*; otherwise my homework solution will not be graded.

I know that *I have to give my solutions written on white A4-sized pages that are stapled on the left-up corner*; otherwise my homework solution will not be graded.

I will take care of the readability of my solutions, from which I may lose 10 points.

For any proofs, I am sure to provide a step-by-step argument, with justifications for every step.

I understand that, during solving this homework, it is prohibited to exchange information about solutions with any other person in any way, including by talking or ex-changing solutions / papers.

I know that the course book is "Probability and Statistics for Computer Scientists, by Michael Baron (2nd Edition) ".

I have read, understand and accept all of the instructions above. On my honor, I pledge that I have not violated the provisions of the Academic Integrity Code of Yıldız Technical University.

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Signature and Date

Problem 1 [30 Points]: Show that the following equalities are correct for any discrete random variable X,

    a) $P(a \leq X \leq b) = P(X = a) + F_X(b) - F_X(a)$,

    b) $P(a < X < b) = F_X(b) - F_X(a) - P(X = b)$,

    c) $P(a \leq X < b) = P(X = a) + F_X(b) - F_X(a) - P(X = b)$,

where $F_X(x)$ denotes the cumulative distribution function of the random variable X.

Problem 2 [35 Points]: The way TÜBİTAK hires graduated students from Yıldız Technical University is exemplified in the following table. Each applicant is rated to a discrete "graduation point average (mezuniyet not ortalaması)" X (horizontal axis) and a discrete "personal rating index" Y (vertical axis). The top number in each cell (in bold) is the number of applicants is a given year with the associated combination. The bottom number in each cell (in italic) is the probability of being accepted. *(Note that, although this is assumed the way TÜBİTAK handles applications, all numbers are fictitious).*

$\leftarrow$ Graduation Point Average $X \rightarrow$

		$90-100$	$80-90$	$70-80$	$60-70$	$50-60$	≤ 50
	10	**27**	**32**	**43**	**30**	**10**	**6**
		1.0	0.9	0.7	0.5	0.4	0.3
	9	**60**	**95**	**110**	**150**	**45**	**20**
		0.9	0.7	0.5	0.4	0.3	0.2
$\leftarrow$ Personal Rating $Y \rightarrow$	**8**	**86**	**175**	**305**	**350**	**78**	**62**
		0.7	0.5	0.4	0.3	0.2	0.1
	7	**39**	**173**	**250**	**165**	**102**	**50**
		0.5	0.4	0.3	0.2	0.1	0.0
	6	**17**	**54**	**118**	**151**	**97**	**68**
		0.4	0.3	0.2	0.1	0.0	0.0
	≤ 5	**1**	**12**	**28**	**45**	**17**	**21**
		0.3	0.2	0.1	0.0	0.0	0.0

    a) Plot the marginal PMF of the two indices.

    b) Plot the conditional PMFs $(X|Y = 8)$ and $(X|Y = 6)$.

    c) Plot the conditional PMF of $(Y|X \leq 50)$.

    d) What is the probability that an applicant with $Y = 7$ is accepted.

    e) Are X and Y independent? Why?

Problem 3 [35 Points]: Let X and Y be two discrete random variables with the joint probability mass function (PMF) given by

$$P_{XY}(x,y) = \begin{cases} k(x+1)^y, & \text{if } x \in \{-1,0,1\} \text{ and } y \in \{1,2\} \\ \qquad 0, & \text{Otherwise} \end{cases}$$

    a) Find the value of k in order to make $P_{XY}(x,y)$ a joint PMF .

    b) Find the PMFs of X and Y. Are X and Y are independent? Why?

    c) Find the PMF of $Z = X^2$.

    d) Find the PMF of $Z = X^Y$.