MAT1071 MATEMATİK 1

ALIŞTIRMALAR 4 - TÜREV

1. $f(x) = \arccos\frac{x-5}{2} + \log(6-x) + \sin\sqrt[3]{x-2}$ fonksiyonunun tanım kümesini bulunuz.

- $\arccos\frac{x-5}{2}$ $\left\{ \begin{array}{l} -1 \leq \frac{x-5}{2} \leq 1 \\ -2 \leq x-5 \leq 2 \\ 3 \leq x \leq 7 \end{array} \right.$

- $\log(6-x)$ $\left\{ \begin{array}{l} 6-x > 0 \\ 6 > x \end{array} \right.$

- $\sin\sqrt[3]{x-2}$ $\left\{ \mathbb{R} \right.$

$D(f) = [3, 6)$

$-1 \leq \frac{x-5}{2} \leq 1$

$-2 \leq x-5 \leq 2$

$\boxed{3 \leq x \leq 7}$

$6-x > 0$

$6 > x$

$[3, 6)$

2. $f(x) = \arcsin(1-x) + \ln(\ln x)$ fonksiyonunun tanım kümesini bulunuz.

- $\arcsin(1-x)$ $\left\{ \begin{array}{l} -1 \leq 1-x \leq 1 \\ -2 \leq -x \leq 0 \\ 0 \leq x \leq 2 \end{array} \right.$

- $\ln(\ln x)$ $\left\{ \begin{array}{l} \ln x > 0 \quad , \quad x > 0 \\ x > 1 \end{array} \right.$

$D(f) = (1, 2]$

$-1 \leq 1-x \leq 1$

$-2 \leq -x \leq 0$

$0 \leq x \leq 2$

$\ln x > 0$

$x > 0$

$x > 1$

$(1, 2]$

3. $\lim_{x\to e} \frac{\ln x - 1}{x-e} = ?$

$\frac{\frac{1}{x}}{1} = \frac{1}{x} \Rightarrow \frac{1}{e}$

$\lim_{x\to e} \frac{\ln x - 1}{x-e} = \lim_{x\to e} \frac{\ln x - \ln e}{x-e} = \lim_{x\to e} \frac{\ln(\frac{x}{e})}{x-e}$

$= \lim_{x\to e} \frac{\ln(\frac{x}{e})}{e(\frac{x}{e}-1)}$

$\left[\lim_{n\to 0} (1+n)^{1/n} = e \quad \lim_{n\to\infty} \left(1+\frac{k}{n}\right)^n = e^k \quad \lim_{n\to 0} \frac{\ln(1+n)}{n} = 1 \right]$

$\frac{x}{e} - 1 = t \Rightarrow x \to e \Rightarrow t \to 0$

$= \lim_{t\to 0} \frac{1}{e} \cdot \frac{\ln(t+1)}{t} = \frac{1}{e} \lim_{t\to 0} \ln(t+1)^{1/t}$

$= \frac{1}{e} \ln\left(\underbrace{\lim_{t\to 0} (t+1)^{1/t}}_{e}\right) = \frac{1}{e} \cdot \ln e = \frac{1}{e}$ //

4. $\lim_{x\to \frac{\pi}{4}} \frac{\ln(\tan x)}{1-\cot x} = ?$

$\frac{\frac{(\tan x)'}{\tan x}}{(1-\cot x)'}$

$\frac{\frac{1}{\cancel{(1+x^2)}} \cdot (\tan x)}{\frac{1}{\cancel{1+x^2}}}$

$\frac{\pi}{4}$

$\cot x$

$\cot 45 \quad \frac{1}{\tan}$

1

$\tan x = u$

$x \to \frac{\pi}{4} \Rightarrow u \to 1$

$\lim_{u\to 1} \frac{\ln u}{1-\frac{1}{u}} = \lim_{u\to 1} u \cdot \frac{\ln u}{u-1}$

$u - 1 = t$

$u \to 1 \Rightarrow t \to 0$

$= \lim_{t\to 0} (t+1) \cdot \frac{\ln(t+1)}{t}$

$= \underbrace{\lim_{t\to 0} (t+1)}_{1} \cdot \underbrace{\lim_{t\to 0} \frac{\ln(t+1)}{t}}_{1}$

$= \frac{1}{2}$

5. $\lim\limits_{x\to -1} \dfrac{\sqrt{\pi}-\left(\sqrt{\arccos x}\right)'}{\left(\sqrt{x+1}\right)'} = ?$

$\dfrac{\dfrac{\dfrac{+1}{\sqrt{1-x^2}}}{2\sqrt{\arccos x}}}{\dfrac{1}{2\sqrt{x+1}}}$ $\quad \dfrac{\cancel{1}.\cancel{2}\sqrt{x+1}}{\cancel{2}\sqrt{1-x^2}.\sqrt{\arccos x}}$ $\quad \sqrt{\dfrac{1}{2\pi}}$ $\quad \sqrt{\dfrac{+1}{+2.\pi}}$

$\sqrt{\dfrac{x+1}{1-x^2}.\arccos x}$

$\sqrt{\dfrac{\cancel{x+1}}{-\cancel{(x+1)}(x-1)\arccos x}}$

$\sqrt{\dfrac{-1}{(x-1)(\arccos x)}}$

$\arccos x = t \qquad x\to -1 \Rightarrow t\to\pi$

$x=\cos t$

$\left(\cos t = 2\cos^2\frac{t}{2}-1\right)$

$= \lim\limits_{t\to\pi} \dfrac{\sqrt{\pi}-\sqrt{t}}{\sqrt{\cos t+1}} . \dfrac{(\sqrt{\pi}+\sqrt{t})}{(\sqrt{\pi}+\sqrt{t})} = \lim\limits_{t\to\pi} \dfrac{\pi-t}{\sqrt{2\cos^2\frac{t}{2}}} . \dfrac{1}{(\sqrt{\pi}+\sqrt{t})}$

$\pi - t = u$

$t\to\pi \Rightarrow u\to 0$

$= \lim\limits_{u\to 0} \dfrac{u}{\sqrt{2}.\cos\left(\frac{\pi}{2}-\frac{u}{2}\right)} . \dfrac{1}{(\sqrt{\pi}+\sqrt{\pi+u})}$

$= \lim\limits_{u\to 0} \dfrac{\frac{u}{2}.2}{\sqrt{2}.\underbrace{\sin\frac{u}{2}}_{1}} . \dfrac{1}{(\sqrt{\pi}+\sqrt{\pi+u})}$

$= \dfrac{1}{\sqrt{2\pi}}$

6. $f(x) = \begin{cases} \dfrac{\ln(\sin x)}{\cos^2 x} &, x\neq \frac{\pi}{2} \\ -\dfrac{1}{2} &, x=\frac{\pi}{2} \end{cases}$ $\quad x=\frac{\pi}{2}$ de sürekli midir?

$\dfrac{\frac{\cos x}{\sin x}}{-2\cos x.\sin x}$ $\quad \dfrac{\cancel{\cos x}}{\sin x}.\dfrac{1}{-2.\cancel{\cos x}.\sin x}$ $\quad \dfrac{-1}{2\sin^2 x}$ $\quad \dfrac{-1}{2}$

$\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\ln(\sin x)}{\cos^2 x} = \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\ln(1-\cos^2 x)^{1/2}}{\cos^2 x} = \lim\limits_{x\to\frac{\pi}{2}} \dfrac{1}{2}\dfrac{\ln(1-\cos^2 x)}{\cos^2 x}$

$\cos^2 x = -t$

$x\to\frac{\pi}{2} \Rightarrow t\to 0$

$= \lim\limits_{t\to 0} -\dfrac{1}{2}\dfrac{\ln(1+t)}{t}$

$= -\dfrac{1}{2}\underbrace{\lim\limits_{t\to 0}\dfrac{\ln(1+t)}{t}}_{1} = -\dfrac{1}{2} = f\left(\frac{\pi}{2}\right)$ oldugundan

$f(x)$ süreklidir.

7. $y=\ln(\sin x)$, $x=\sqrt{\arccos 2^{-3t}}$ ise $\dfrac{dy}{dt}=?$ $\dfrac{1}{2^{3t}}$

$$\frac{dy}{dt}=\frac{dy}{dx}\cdot\frac{dx}{dt}$$

$\dfrac{\cos x}{\sin x}$ $\dfrac{dy}{dx}\cdot\dfrac{dx}{dt}$ 2^{-3x} 2^{-3t} $2^{-3x}\cdot -3\ln 2$

$$=\frac{\cos x}{\sin x}\cdot\frac{3.\ln 2.2^{-3t}}{\sqrt{1-2^{-6t}}}\cdot\frac{1}{2\sqrt{\arccos 2^{-3t}}}$$

$\dfrac{-3.2^{-3t}.\ln 2}{\sqrt{1-2^{-6t}}}\cdot\dfrac{1}{2\sqrt{arc}}$

8. $f(x)=e^{\pi x}-\pi x$ eğrisine teğet olan yatay doğruları bulunuz.

$f'(x)=e^{\pi x}.\pi-\pi$ $\pi(e^{\pi x}-1)=0$

$f'(x)=\pi.e^{\pi x}-\pi$ $(0,1)$

yatay doğruların eğimi "0 (sıfır)" olduğundan, $e^{\pi x}=1$

$$\pi(e^{\pi x}-1)=0 \Rightarrow e^{\pi x}=1 \Rightarrow \pi x=0 \Rightarrow \underbrace{x=0}$$

$x=0$

$$x=0 \Rightarrow f(x)=y \Rightarrow y=e^{0}=1 \Rightarrow \underbrace{y=1} \text{ doğrusudur.}$$

9. $\cos(3x+y)+\sin(x+3y)=-1$ eğrisinin $A\left(0,\frac{\pi}{2}\right)$ noktasındaki teğet ve normal doğru denklemlerini bulunuz.

$-\sin(3x+y).(3+y')+\cos(x+3y).(1+3y')=0$

$-\sin\frac{\pi}{2}.(3+y')+\cancel{\cos\frac{3\pi}{2}.(1+3y')}=0$

$$-(3+y')\cdot\sin(3x+y)+(1+3y')\cdot\cos(x+3y)=0$$

$-3-y'=0$ $y'=-3$

$\boxed{y=-3x+\frac{\pi}{2}}$

$$A\left(0,\tfrac{\pi}{2}\right) \Rightarrow -(3+y')\cdot\sin\frac{\pi}{2}+(1+3y')\cdot\cos\left(3\frac{\pi}{2}\right)=0$$

$\boxed{y=\frac{x}{3}+\frac{\pi}{2}}$

$$-(3+y')=0 \Rightarrow y'=-3=m_T \quad , m_N=\frac{1}{3}$$

T.D.D. : $y-\frac{\pi}{2}=-3(x-0) \Rightarrow \underbrace{y=-3x+\frac{\pi}{2}}$ //

N.D.D. : $y-\frac{\pi}{2}=\frac{1}{3}(x-0) \Rightarrow \underbrace{y=\frac{1}{3}x+\frac{\pi}{2}}$ //

10. $(1{,}002)^3 - 2\sqrt{1{,}002} + 3$ yaklasık degerini lineerleştirme ve diferansiyel yardımıyla bulunuz.

$f(x) = x^3 - 2\sqrt{x} + 3 \qquad a=1 \quad , \ f(1) = 2$

$f'(x) = 3x^2 - \frac{1}{\sqrt{x}} \qquad , \ f'(1) = 2$

a) $f(x) \simeq L(x) = f(1) + f'(1)(x-1) = 2 + 2.(x-1)$

$f(1{,}002) \simeq L(1{,}002) = 2 + 2 .\ 0{,}002 = 2{,}004$

b) $dy \approx \Delta y \qquad \Delta x = 0{,}002 = dx$

$dy = f'(x).dx = f'(x).\ \Delta x = f'(1).\ 0{,}002 = 0{,}004$

$\Delta y = f(1{,}002) - f(1) = f(1{,}002) - 2$

$dy \approx \Delta y \Rightarrow f(1{,}002) \approx 2 + 0{,}004 = 2{,}004$

11. $f(x) = \left(\tan\frac{x}{2}\right)^{x \arcsin 2x}$ ise $f'(x) = ?$

$\ln f(x) = (x.\arcsin 2x).\ \ln\left(\tan\frac{x}{2}\right)$

$$\frac{f'(x)}{f(x)} = \left(\arcsin 2x + \frac{2x}{\sqrt{1-4x^2}}\right).\ln\left(\tan\frac{x}{2}\right) + x.\arcsin 2x.\ \frac{\frac{1}{2}.\sec^2\frac{x}{2}}{\tan\frac{x}{2}}$$

$$f'(x) = \left(\tan\frac{x}{2}\right)^{x\arcsin 2x} . \left[\left(\arcsin 2x + \frac{2x}{\sqrt{1-4x^2}}\right).\ln\left(\tan\frac{x}{2}\right) + x.\arcsin 2x.\frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}}\right]$$

$\ln f(x) = x.\arcsin 2x.\left(\ln \tan\frac{x}{2}\right)$

$\frac{f'(x)}{f(x)} =$

12. g ve h fonksiyonları $g(1)=h'(1)=1$, $g'(1)=h(1)=2$ şartlarını sağlayan pozitif değerli ve türevlenebilen birer fonksiyon olmak üzere f fonksiyonu da $f(x)=\left[g(x^2)\right]^{h(x)}$ ile tanımlı olsun. $f'(1)$ değerini bulunuz.

$$f(x)=\left[g(x^2)\right]^{h(x)} \quad , f(1)=(g(1))^{h(1)}=1^2=1$$

$$\ln f(x)=h(x).\ln(g(x^2))$$

$$\frac{f'(x)}{f(x)}=h'(x).\ln(g(x^2))+h(x).\frac{2x.g'(x^2)}{g(x^2)}$$

$$f'(1)=\underbrace{f(1)}_{1}.\left[\underbrace{h'(1)}_{1}.\underbrace{\ln(\underbrace{g(1)}_{1})}_{0}+\underbrace{h(1)}_{2}.\frac{2.1.g'(1)}{\underbrace{g(1)}_{1}}\right]$$

$$=2.2.2=8$$

13. $f(x)=2^{x^2+\cos x}+3^{x\ln(x+1)}$ ise $f'(x)=?$

$$f'(x)=(2x-\sin x).\ln 2.2^{x^2+\cos x}+\ln 3.3^{x\ln(x+1)}.\left(\ln(x+1)+\frac{x}{x+1}\right)$$

$$2^{x^2+\cos x}.(2x-\sin x).\ln 2+3^{x\ln(x+1)}.\ln 3.\left(\ln(x+1)+\frac{x}{x+1}\right)$$

14. $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x\sqrt{8+x^2}$ fonksiyonunun tersinin mevcut olduğunu gösterip $(f^{-1})'(3)$ değerini bulunuz.

$\sqrt{8x^2+x^4}$ $\quad \frac{16x+4x^3}{2\sqrt{8x^2+x^4}}$ $\quad \frac{20}{6}=$

$$f'(x)=\sqrt{8+x^2}+x.\frac{2x}{2\sqrt{8+x^2}}=\frac{2x^2+8}{\sqrt{8+x^2}}>0$$

$\forall x\in\mathbb{R}$ için $f'(x)>0$ olduğundan f artan $\Rightarrow f$, 1-1 $\Rightarrow f$'in tersi mevcuttur.

$$x\sqrt{8+x^2}=3 \Rightarrow \underline{x=1}$$

$$\frac{1}{f'(f^{-1}(3))} \quad \frac{1}{f'(1)} \quad \frac{6}{20}$$

$$(f^{-1})'(3)=\frac{1}{f'(1)}=\frac{1}{\frac{10}{3}}=\frac{3}{10}$$

$\frac{3}{10}$

(Fonksiyonun tanım kümesi üzerinde çalışıldığı için örtenlik sağlanır.)

15. $y=(s+3)^2$, $s=\sqrt{t-3}$, $t=x^2$ ise $\left.\dfrac{dy}{dx}\right|_{x=2}=?$

$$\frac{dy}{dx}=\frac{dy}{ds}\cdot\frac{ds}{dt}\cdot\frac{dt}{dx}$$

$x=2 \Rightarrow t=4 \Rightarrow s=1$

$$=2\cdot(s+3)\cdot\frac{1}{2\sqrt{t-3}}\cdot 2x$$

$$2(s+3)\cdot\frac{1}{2\sqrt{t-3}}\cdot 2x$$

$$\left.\frac{dy}{dx}\right|_{x=2}=2\cdot(1+3)\cdot\frac{1}{2\cdot 1}\cdot 2\cdot 2=16$$

$$\cancel{2}\cdot 4\cdot\frac{1}{\cancel{2}}\cdot 4=16$$

16. $\tan(xy)=xy \Rightarrow y'=?$

$$\sec^2(xy)\cdot(y+x\cdot y')=y+x\cdot y'$$

$$(y+xy')\cdot\sec^2(xy)=y+xy'$$

$$xy'\cdot\sec^2(xy)-xy'=y-y\cdot\sec^2(xy)$$

$$xy'\cdot\sec^2(xy)+y\sec^2(xy)=y+x\cdot y'$$

$$y'\,(x\cdot\sec^2(xy)-x)=y\,(1-\sec^2(xy))$$

$$xy'(\sec^2(xy)-1)=-y\sec^2(xy)$$

$$y'=\frac{y\cdot(1-\cancel{\sec^2(xy)})}{-x\cdot(1-\cancel{\sec^2(xy)})}$$

$(\sec^2(xy)\neq 1)$

$$y'=\frac{-y\cdot\sec^2(xy)}{x\cdot(\sec^2(xy)-1)}$$

$$y'=-\frac{y}{x}$$

17. $f(x)=\dfrac{1-x}{1+x}$ ise $f^{(n)}=?$

$$f'(x)=\frac{-(1+x)-(1-x)}{(1+x)^2}=\frac{-1-x-1+x}{(1+x)^2}=-2\cdot(1+x)^{-2}$$

$-1(1+x)-(1-x)$

$-1-\cancel{x}-1+\cancel{x}$

$-2\,(1+x)^{-2}$

$$f''(x)=(-2)\cdot(-2)\cdot(1+x)^{-3}$$

$$f'''(x)=(-2)\cdot(-2)\cdot(-3)\cdot(1+x)^{-4}$$

$$f^{(4)}(x)=(-2)\cdot(-2)\cdot(-3)\cdot(-4)\cdot(1+x)^{-5}$$

$\vdots$

$$f^{(n)}(x)=2\cdot n!\cdot(-1)^n\cdot(1+x)^{-(n+1)}$$

$f(x)=\dfrac{1-x}{1+x}$

$f'(x)=\dfrac{-2}{(1+x)^2}$

$f''(x)=\dfrac{-2\cdot-2}{(1+x)^3}$

$\dfrac{-2\cdot-2}{(1+x)^3}$

$f^{(3)}(x)=\dfrac{-2\cdot-2\cdot-3}{(1+x)^4}$

$$\boxed{\frac{2\cdot n!\cdot(-1)^n}{(1+x)^{n+1}}}$$

18. $f(x)=\left[\cos(x^4)\right]^{\arctan x^2}$ fonksiyonu türevlenebilen bir fonksiyon olmak üzere $f'(0)=?$

$\ln f(x) = \arctan x^2 . \ln \cos x^4$

$\frac{f'(x)}{f(x)} = \frac{2x}{1+x^4} . \ln \cos x^4 + \arctan x^2 . \frac{-\sin x^4 . 4x^3}{\cos x^4}$

0,,

$\ln f(x) = \arctan x^2 . \ln(\cos(x^4))$

$$\frac{f'(x)}{f(x)} = \frac{2x}{1+x^4} . \ln(\cos(x^4)) - \arctan x^2 . \frac{4x^3 . \sin(x^4)}{\cos(x^4)}$$

$f'(0) = f(0) . \left[0 - 0 \right] = 0$

$12x^3-12x^2-24x$ $\quad 12x(x^2-x-2)$ $\quad 12x(x-2)(x+1)$

$x=0$ $\quad x=2$ $\quad x=-1$

	-1		0		2	
$-$	ϕ	$+$	ϕ	$-$	ϕ	$+$
↘		↗		↘		↗

19. $f(x)=3x^4-4x^3-12x^2+5$ fonksiyonunun artan ve azalan olduğu aralıkları belirleyiniz.

$f'(x) = 12x^3-12x^2-24x = 0$

$x^3 - x^2 - 2x = 0$

$x(x^2-x-2)=0$

$x(x-2)(x+1)=0$

x	$-\infty$		-1		0		2		∞
$f'(x)$		$-$	ϕ	$+$	ϕ	$-$	ϕ	$+$	
$f(x)$		↘		↗		↘		↗	

artan : $(-1,0) \cup (2,\infty)$

azalan : $(-\infty,-1) \cup (0,2)$

20. $y=x^3-2x^2+x-2$ eğrisinin hangi noktadaki teğeti x-ekseni-ne paraleldir.

$3x^2-4x+1$

$3x \quad -1$

$x \quad -1$

$(3x-1)(x-1)$

$y' = 3x^2-4x+1$, teğet x-eksenine paralel ise $\frac{dy}{dx}=0$ olmalıdır.

$x=\frac{1}{3}$

$x=1$

$3x^2-4x+1=0$

$(x-1)(3x-1)=0$

$\Rightarrow x=1 \Rightarrow y=-2$

$\Rightarrow 3x-1=0 \Rightarrow x=\frac{1}{3} \Rightarrow y=\frac{-50}{27}$

$A(1,-2)$, $B\left(\frac{1}{3}, \frac{-50}{27}\right)$