

BLM1612 - Circuit Theory

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Energy Storage Devices

Capacitors and Inductors

Objective of Lecture

- Describe
 - the construction of a capacitor
 - how charge is stored.
 - Introduce several types of capacitors
- The electrical properties of a capacitor
 - Relationship between charge, voltage, and capacitance; power; and energy
 - Equivalent capacitance when a set of capacitors are in series and in parallel
- Describe
 - The construction of an inductor
 - How energy is stored in an inductor
 - The electrical properties of an inductor
 - Relationship between voltage, current, and inductance; power; and energy
 - Equivalent inductance when a set of inductors are in series and in parallel

Capacitors

Energy Storage Devices

Capacitors

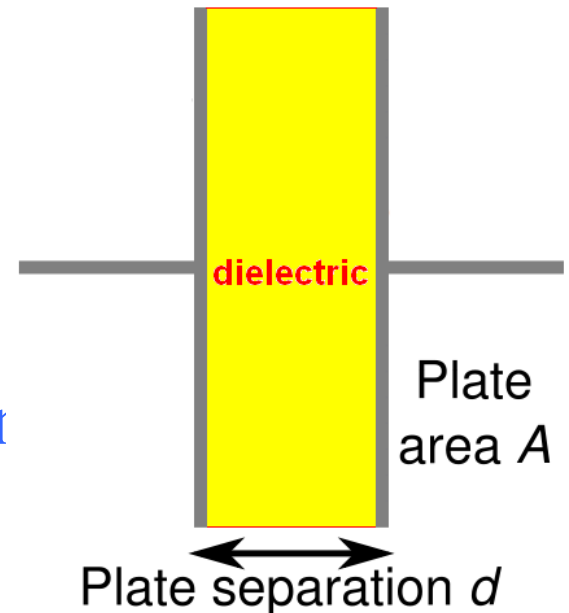
- Composed of two conductive plates separated by an insulator (or dielectric).
 - Commonly illustrated as two parallel metal plates separated by a distance, d .

$$C = \epsilon A/d$$

where $\epsilon = \epsilon_r \epsilon_0$

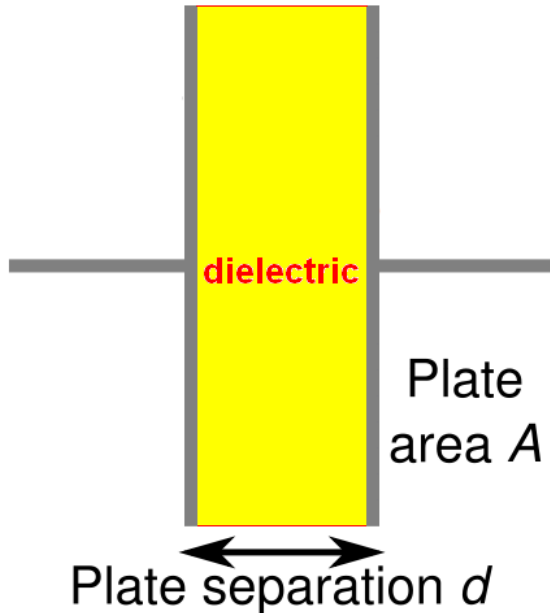
ϵ_r is the relative dielectric constant

ϵ_0 is the vacuum permittivity



Effect of Dimensions

- Capacitance increases with



- increasing surface area of the plates,
- decreasing spacing between plates, and
- increasing the relative dielectric constant of the insulator between the two plates.

Types of Capacitors

- Fixed Capacitors

- Nonpolarized

- May be connected into circuit with either terminal of capacitor connected to the high voltage side of the circuit.

- Insulator: Paper, Mica, Ceramic, Polymer

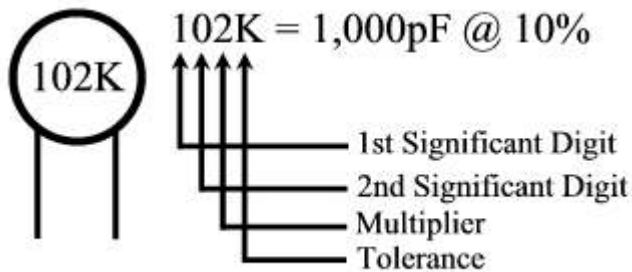
- Electrolytic

- The negative terminal must always be at a lower voltage than the positive terminal

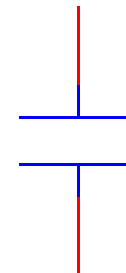
- Plates or Electrodes: Aluminum, Tantalum

Nonpolarized

- Difficult to make nonpolarized capacitors that store a large amount of charge or operate at high voltages.
 - Tolerance on capacitance values is very large
 - +50%/-25% is not unusual



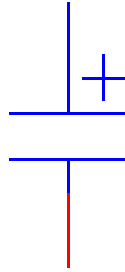
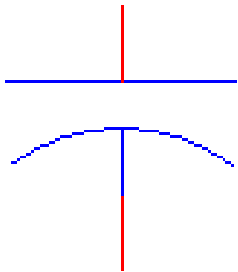
PSpice Symbol



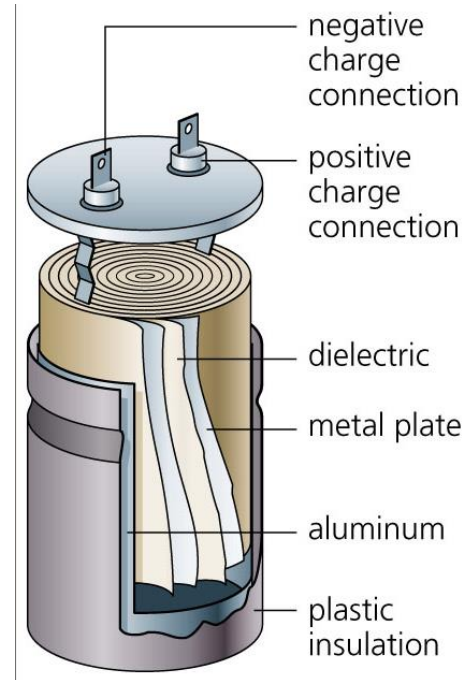
http://www.marvac.com/fun/ceramic_capacitor_codes.aspx

Electrolytic

Pspice Symbols



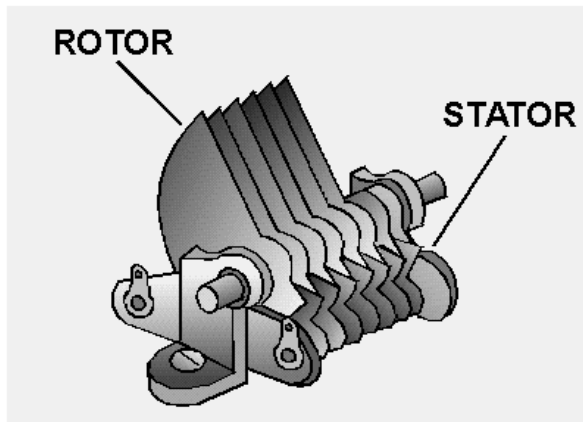
Fabrication



<http://www.digitivity.com/articles/2008/11/choosing-the-right-capacitor.html>

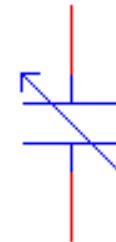
Variable Capacitors

- Cross-sectional area is changed as one set of plates are rotated with respect to the other.



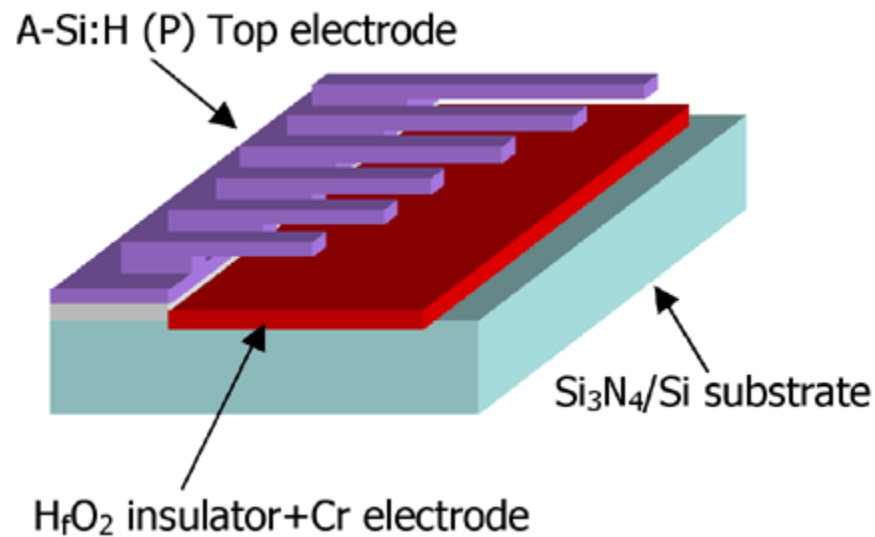
<http://www.tpub.com/neets/book2/3f.htm>

PSpice Symbol



MEMS Capacitor

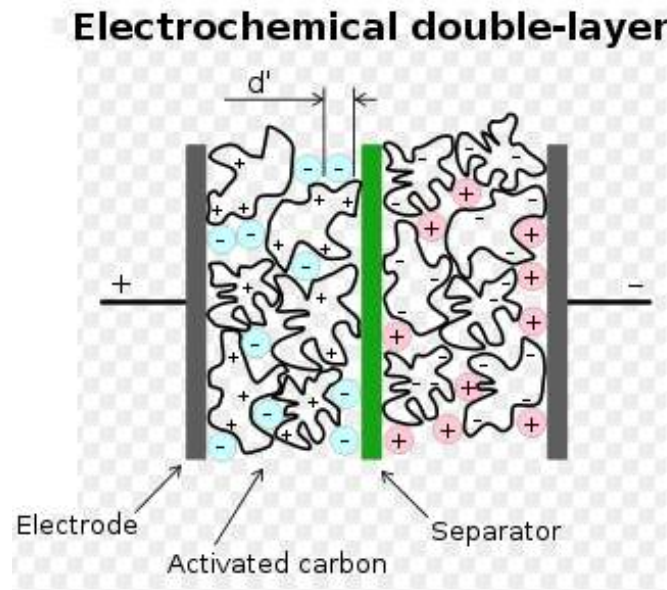
- MEMS (Microelectromechanical system)
 - Can be a variable capacitor by changing the distance between electrodes.
 - Use in sensing applications as well as in RF electronics.



http://www.silvaco.com/tech_lib_TCAD/simulationstandard/2005/aug/a3/a3.html

Electric Double Layer Capacitor

- Also known as a supercapacitor or ultracapacitor
 - Used in high voltage/high current applications.
 - Energy storage for alternate energy systems.



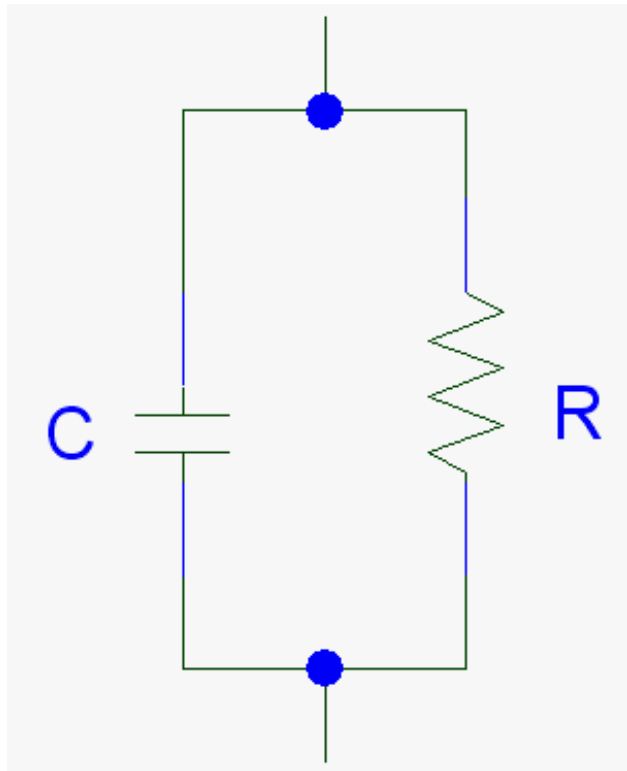
http://en.wikipedia.org/wiki/File:Supercapacitor_diagram.svg

Electrical Properties of a Capacitor

- Acts like an open circuit at steady state when connected to a d.c. voltage or current source.
- Voltage on a capacitor must be continuous
 - There are no abrupt changes to the voltage
- An ideal capacitor does not dissipate energy, it takes power when storing energy and returns it when discharging.

Properties of a Real Capacitor

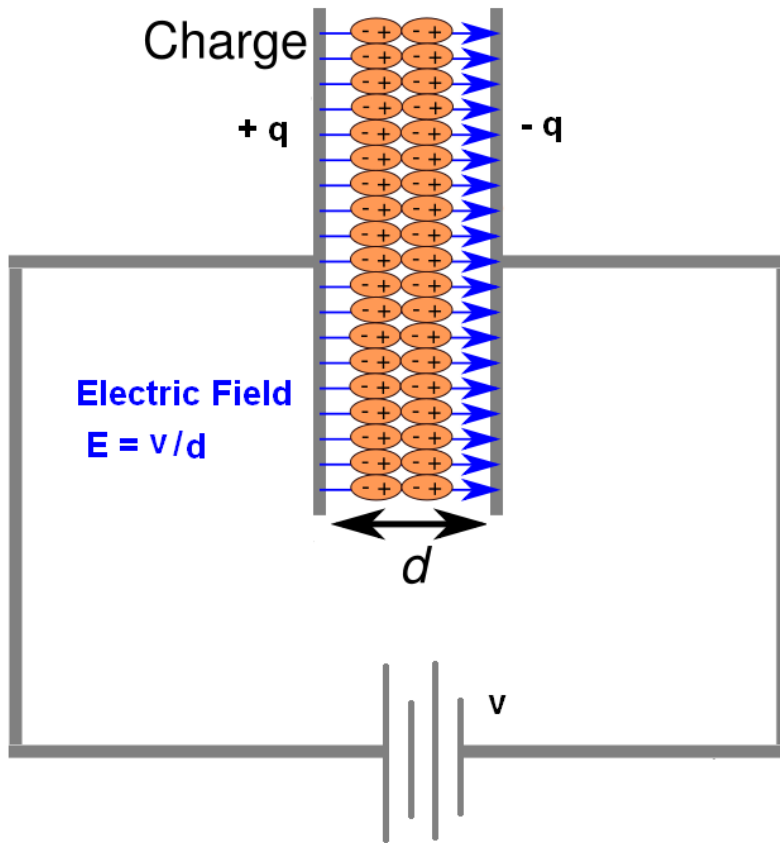
- A real capacitor does dissipate energy due to leakage of charge through its insulator.



- This is modeled by putting a resistor in parallel with an ideal capacitor.

Energy Storage

- Charge is stored on the plates of the capacitor.



Equation:

$$Q = CV$$

Units:

Coulomb = Farad·Voltage

$$C = F/V$$

Adding Charge to Capacitor

- The ability to add charge to a capacitor depends on:
 - the amount of charge already on the plates of the capacitorand
 - the force (voltage) driving the charge towards the plates (i.e., current)

Charging a Capacitor

- At first, it is easy to store charge in the capacitor.
- As more charge is stored on the plates of the capacitor, it becomes increasingly difficult to place additional charge on the plates.
 - Coulombic repulsion from the charge already on the plates creates an opposing force to limit the addition of more charge on the plates.
 - Voltage across a capacitor increases rapidly as charge is moved onto the plates when the initial amount of charge on the capacitor is small.
 - Voltage across the capacitor increases more slowly as it becomes difficult to add extra charge to the plates.

Discharging a Capacitor

- At first, it is easy to remove charge in the capacitor.
 - Coulombic repulsion from the charge already on the plates creates a force that pushes some of the charge out of the capacitor once the force (voltage) that placed the charge in the capacitor is removed (or decreased).
- As more charge is removed from the plates of the capacitor, it becomes increasingly difficult to get rid of the small amount of charge remaining on the plates.
 - Coulombic repulsion decreases as the charge spreads out on the plates. As the amount of charge decreases, the force needed to drive the charge off of the plates decreases.
 - Voltage across a capacitor decreases rapidly as charge is removed from the plates when the initial amount of charge on the capacitor is small.
 - Voltage across the capacitor decreases more slowly as it becomes difficult to force the remaining charge out of the capacitor.

Current-Voltage Relationships

$$i_C = \frac{dq}{dt}$$

$$q = Cv_C$$

$$i_C = C \frac{dv_C}{dt}$$

$$v_C = \frac{1}{C} \int_{t_o}^{t_1} i_C dt$$

Power and Energy

$$p_C = i_C v_C$$

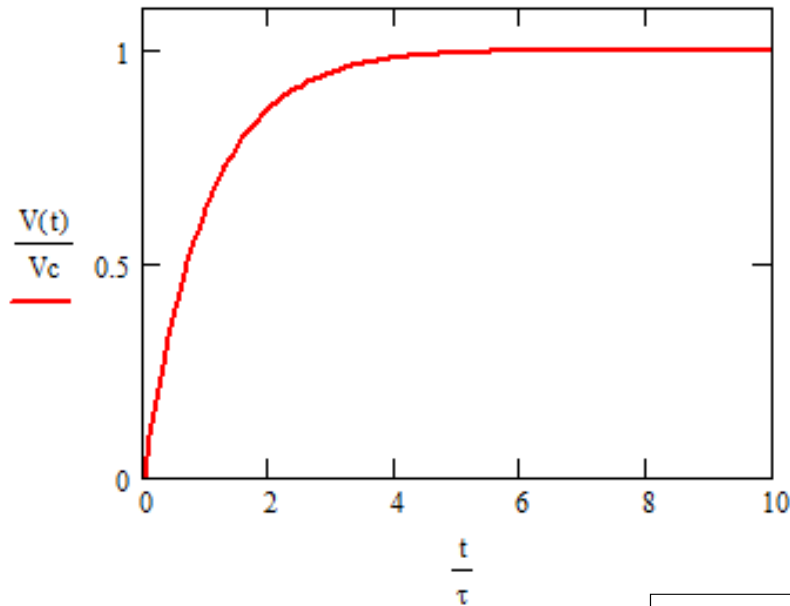
$$p_C = C v_C \frac{dv_C}{dt}$$

$$w_C = \frac{1}{2} C v_C^2$$

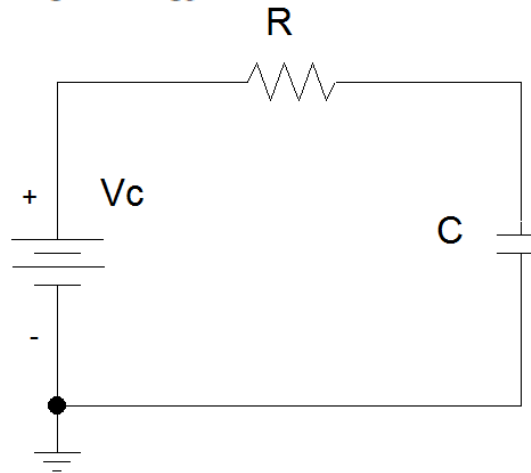
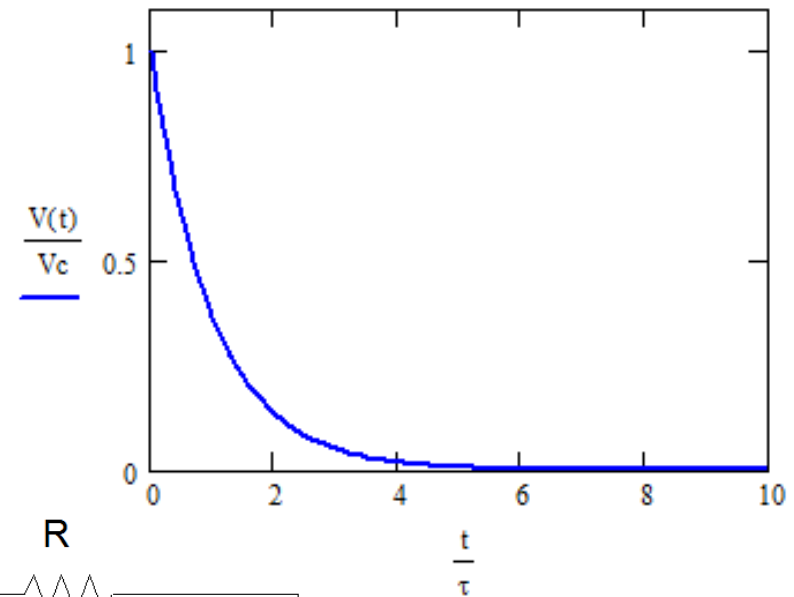
$$w_C = \frac{q^2}{2C}$$

Capacitor Voltage vs. Time

d.c. voltage, V_c , is applied at $t = 0s$



d.c. voltage, V_c , is removed at $t = 0s$



Time constant, τ

- The rate at which charge can be added to or removed from the plates of a capacitor as a function of time can be fit to an exponential function.

Charging

$$V(t) = V_c \left(1 - e^{-t/\tau} \right)$$

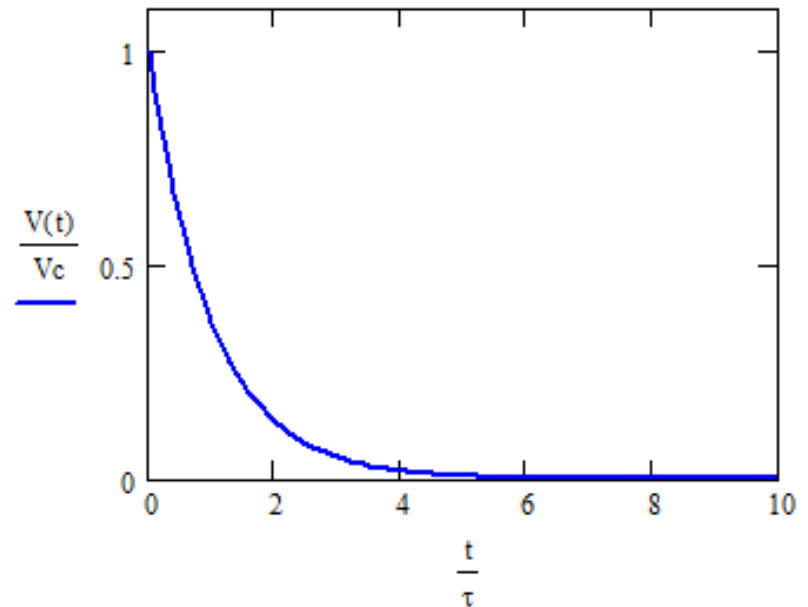
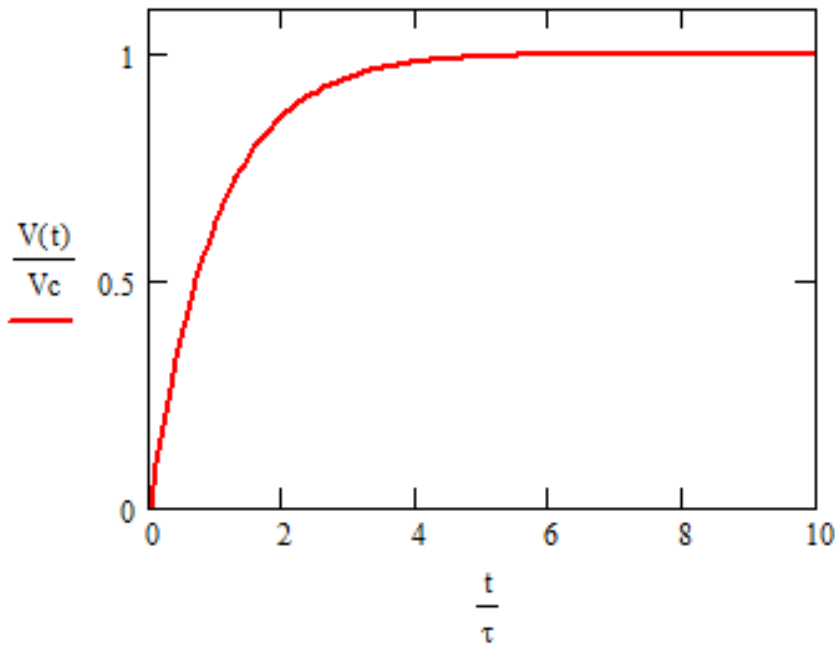
Discharging

$$V(t) = V_c e^{-t/\tau}$$

$$\tau = RC$$

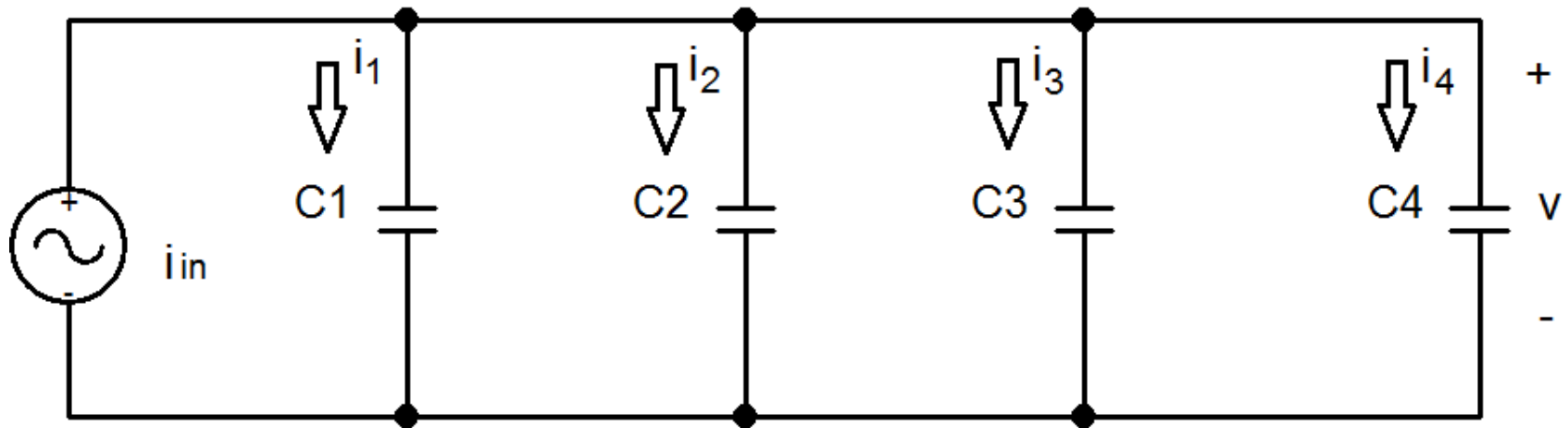
Transition to steady state

- We approximate that the exponential function reaches its final value when the charging or discharging time is equal to 5τ .



Equivalent Capacitance

- Capacitors in parallel



C_{eq} for Capacitors in Parallel

$$i_{in} = i_1 + i_2 + i_3 + i_4$$

$$i_1 = C_1 \frac{dv}{dt} \qquad i_2 = C_2 \frac{dv}{dt}$$

$$i_3 = C_3 \frac{dv}{dt} \qquad i_4 = C_4 \frac{dv}{dt}$$

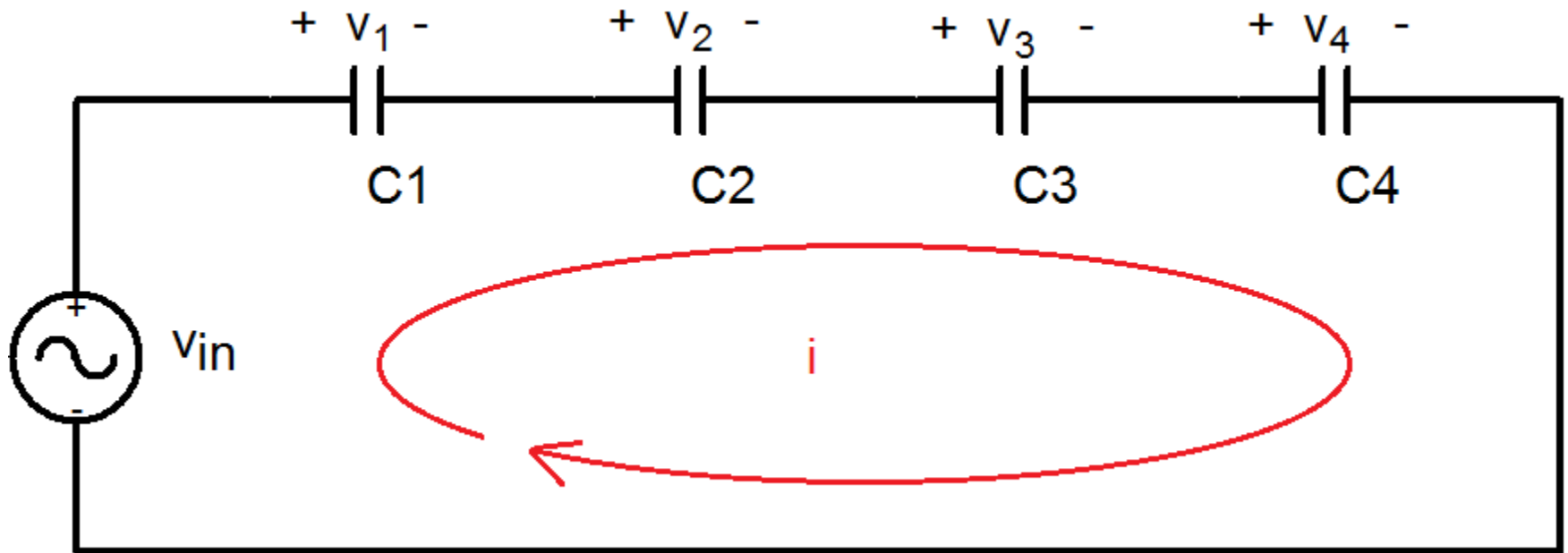
$$i_{in} = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + C_4 \frac{dv}{dt}$$

$$i_{in} = C_{eq} \frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3 + C_4$$

Equivalent Capacitance

- Capacitors in series



C_{eq} for Capacitors in Series

$$v_{in} = v_1 + v_2 + v_3 + v_4$$

$$v_1 = \frac{1}{C_1} \int_{t_o}^{t_1} i dt$$

$$v_2 = \frac{1}{C_2} \int_{t_o}^{t_1} i dt$$

$$v_3 = \frac{1}{C_3} \int_{t_o}^{t_1} i dt$$

$$v_4 = \frac{1}{C_4} \int_{t_o}^{t_1} i dt$$

$$v_{in} = \frac{1}{C_1} \int_{t_o}^{t_1} i dt + \frac{1}{C_2} \int_{t_o}^{t_1} i dt + \frac{1}{C_3} \int_{t_o}^{t_1} i dt + \frac{1}{C_4} \int_{t_o}^{t_1} i dt$$

$$v_{in} = \frac{1}{C_{eq}} \int_{t_o}^{t_1} i dt$$

$$C_{eq} = [(1/C_1) + (1/C_2) + (1/C_3) + (1/C_4)]^{-1}$$

General Equations for C_{eq}

Parallel Combination

- If P capacitors are in parallel, then

$$C_{eq} = \sum_{p=1}^P C_P$$

Series Combination

- If S capacitors are in series, then:

$$C_{eq} = \left[\sum_{s=1}^S \frac{1}{C_s} \right]^{-1}$$

Summary

- Capacitors are energy storage devices.
- An ideal capacitor act like an open circuits when a DC voltage or current has been applied for at least 5τ .
- The voltage across a capacitor must be a continuous function; the current flowing across a capacitor can be discontinuous.
- The equation for equivalent capacitance for

capacitors in parallel

$$C_{eq} = \sum_{p=1}^P C_P$$

capacitors in series

$$C_{eq} = \left[\sum_{s=1}^S \frac{1}{C_s} \right]^{-1}$$

Inductors

Energy Storage Devices

Inductors

- Generally - coil of conducting wire



- Usually wrapped around a solid core.
- If no core is used, then the inductor is said to have an ‘air core’.

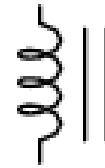
<http://bzupages.com/f231/energy-stored-inductor-uzma-noreen-group6-part2-1464/>

Symbols

Inductor symbols



generic, or air-core



iron core



iron core
(alternative)



PSpice

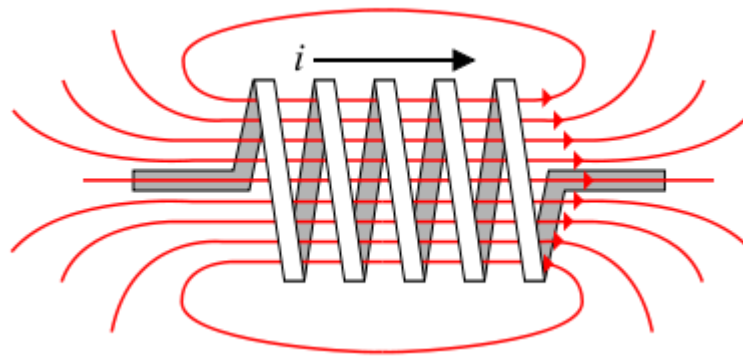
http://www.allaboutcircuits.com/vol_1/chpt_15/1.html

Alternative Names for Inductors

- Reactor
 - inductor in a power grid
- Choke
 - designed to block a particular frequency while allowing currents at lower frequencies or d.c. currents through
 - Commonly used in RF (radio frequency) circuitry
- Coil
 - often coated with varnish and/or wrapped with insulating tape to provide additional insulation and secure them in place
 - A winding is a coil with taps (terminals).
- Solenoid
 - a three dimensional coil.
 - Also used to denote an electromagnet where the magnetic field is generated by current flowing through a toroidal inductor.

Energy Storage

- The flow of current through an inductor creates a magnetic field (right hand rule).



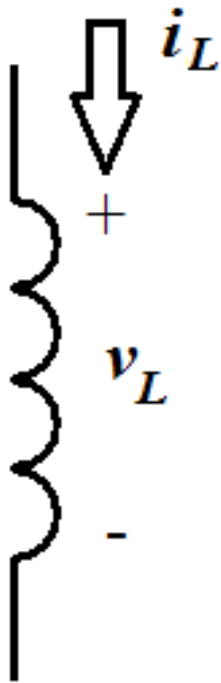
B field

http://en.wikibooks.org/wiki/Circuit_Theory/Mutual_Inductance

- If the current flowing through the inductor drops, the magnetic field will also decrease and energy is released through the generation of a current.

Sign Convention

- The sign convention used with an inductor is the same as for a power dissipating device.



- When current flows into the positive side of the voltage across the inductor, it is positive and the inductor is dissipating power.
- When the inductor releases energy back into the circuit, the sign of the current will be negative.

Current and Voltage Relationships

- L , inductance, has the units of Henries (H)

$$1 \text{ H} = 1 \text{ V-s/A}$$

$$v_L = L \frac{di}{dt}$$

$$i_L = \frac{1}{L} \int_{t_o}^{t_1} v_L dt$$

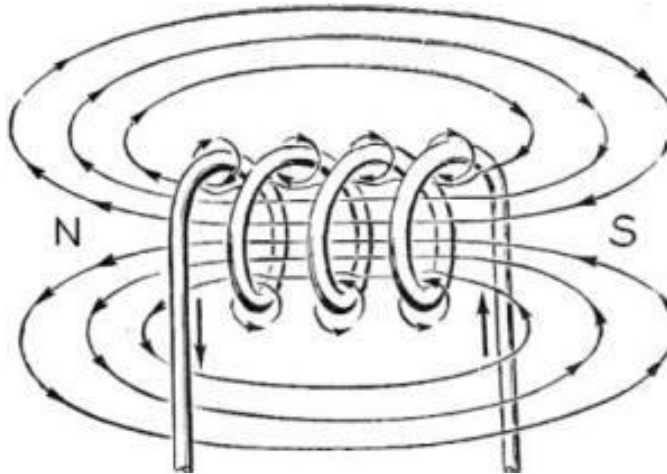
Power and Energy

$$p_L = v_L i_L = L i_L \int_{t_o}^{t_1} i_L dt$$

$$w = \int_{t_o}^{t_1} L \frac{di_L}{dt} i_L dt = L \int_{t_o}^{t_1} i_L di_L$$

Inductors

- Stores energy in an magnetic field created by the electric current flowing through it.
 - Inductor opposes change in current flowing through it.
 - Current through an inductor is continuous; voltage can be discontinuous.



<http://www.rfcafe.com/references/electrical/Electricity%20-%20Basic%20Navy%20Training%20Courses/electricity%20-%20basic%20navy%20training%20courses%20-%20chapter%2012.htm>

Calculations of L

- For a solenoid (toroidal inductor)

$$L = \frac{N^2 \mu A}{\ell} = \frac{N^2 \mu_r \mu_o A}{\ell}$$

N is the number of turns of wire

A is the cross-sectional area of the toroid in m².

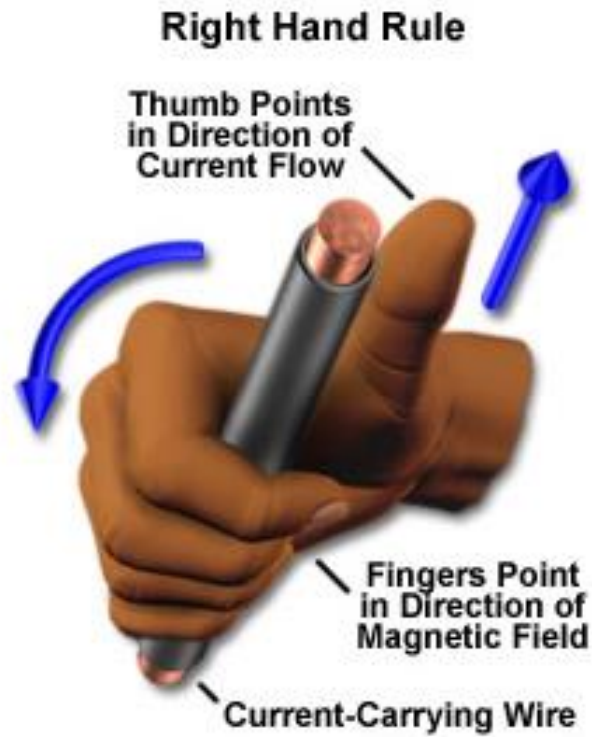
μ_r is the relative permeability of the core material

μ_o is the vacuum permeability ($4\pi \times 10^{-7}$ H/m)

ℓ is the length of the wire used to wrap the toroid in meters

Wire

- Unfortunately, even bare wire has inductance.



$$L = \ell \left[\ln \left(4 \frac{\ell}{d} \right) - 1 \right] (2 \times 10^{-7}) H$$

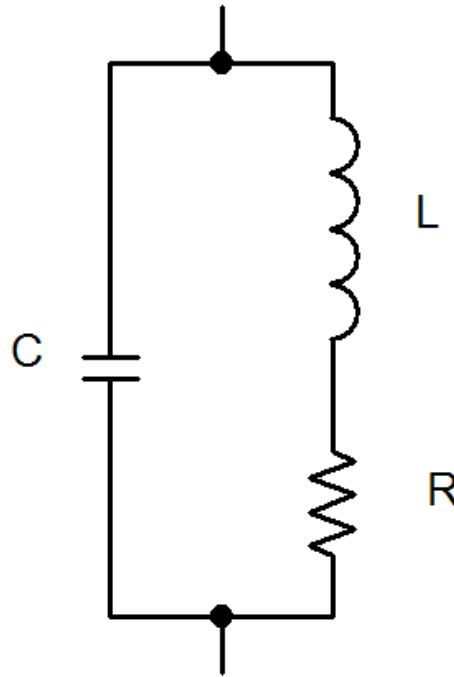
– d is the diameter of the wire in meters.

Properties of an Inductor

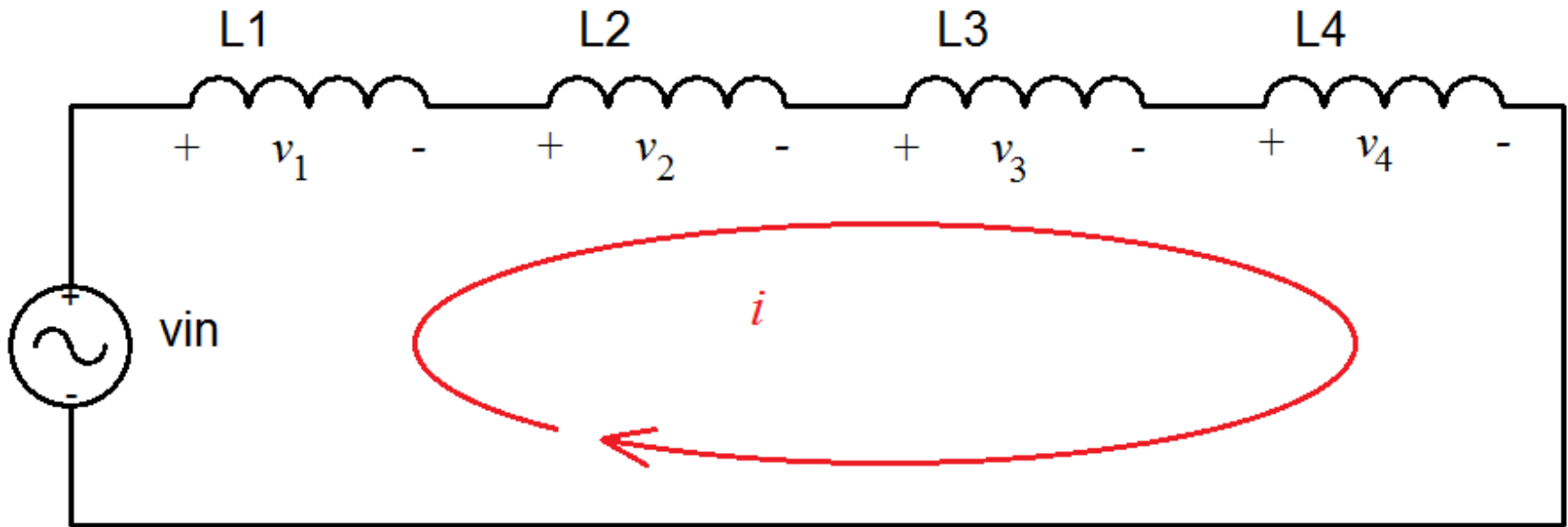
- Acts like an short circuit at steady state when connected to a d.c. voltage or current source.
- Current through an inductor must be continuous
 - There are no abrupt changes to the current, but there can be abrupt changes in the voltage across an inductor.
- An ideal inductor does not dissipate energy, it takes power from the circuit when storing energy and returns it when discharging.

Properties of a Real Inductor

- Real inductors do dissipate energy due to resistive losses in the length of wire and capacitive coupling between turns of the wire.



Inductors in Series



L_{eq} for Inductors in Series

$$v_{in} = v_1 + v_2 + v_3 + v_4$$

$$v_1 = L_1 \frac{di}{dt}$$

$$v_2 = L_2 \frac{di}{dt}$$

$$v_3 = L_3 \frac{di}{dt}$$

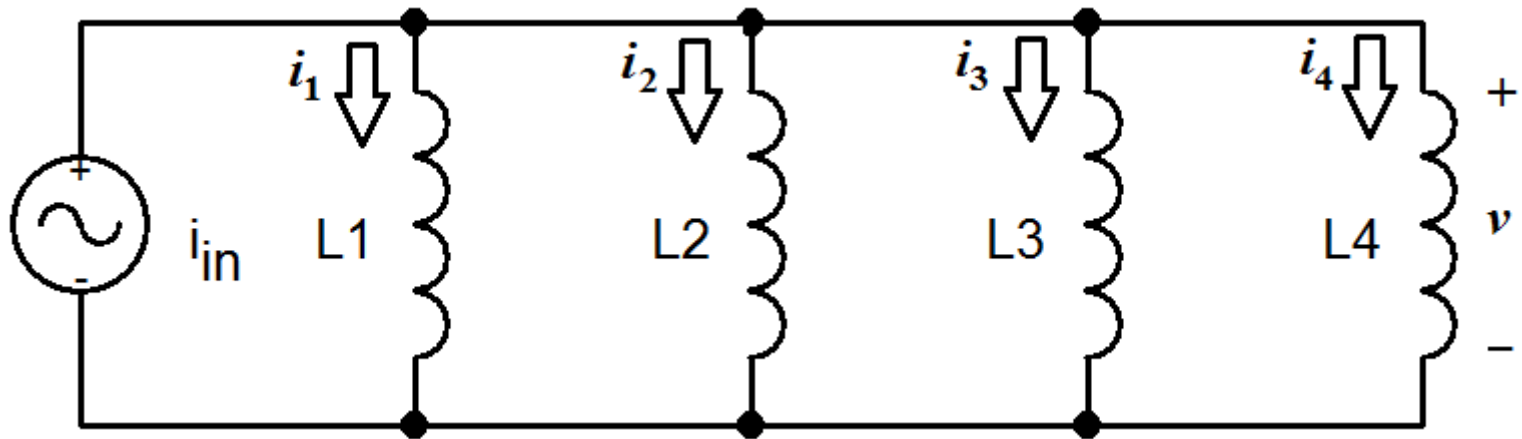
$$v_4 = L_4 \frac{di}{dt}$$

$$v_{in} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + L_4 \frac{di}{dt}$$

$$v_{in} = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + L_4$$

Inductors in Parallel



L_{eq} for Inductors in Parallel

$$i_{in} = i_1 + i_2 + i_3 + i_4$$

$$i_1 = \frac{1}{L_1} \int_{t_o}^{t_1} v dt$$

$$i_2 = \frac{1}{L_2} \int_{t_o}^{t_1} v dt$$

$$i_3 = \frac{1}{L_3} \int_{t_o}^{t_1} v dt$$

$$i_4 = \frac{1}{L_4} \int_{t_o}^{t_1} v dt$$

$$i_{in} = \frac{1}{L_1} \int_{t_o}^{t_1} v dt + \frac{1}{L_2} \int_{t_o}^{t_1} v dt + \frac{1}{L_3} \int_{t_o}^{t_1} v dt + \frac{1}{L_4} \int_{t_o}^{t_1} v dt$$

$$i_{in} = \frac{1}{L_{eq}} \int_{t_o}^{t_1} v dt$$

$$L_{eq} = \left[\left(\frac{1}{L_1} \right) + \left(\frac{1}{L_2} \right) + \left(\frac{1}{L_3} \right) + \left(\frac{1}{L_4} \right) \right]^{-1}$$

General Equations for L_{eq}

Series Combination

- If S inductors are in series, then

$$L_{eq} = \sum_{s=1}^S L_s$$

Parallel Combination

- If P inductors are in parallel, then:

$$L_{eq} = \left[\sum_{p=1}^P \frac{1}{L_p} \right]^{-1}$$

Summary

- Inductors are energy storage devices.
- An ideal inductor act like a short circuit at steady state when a DC voltage or current has been applied.
- The current through an inductor must be a continuous function; the voltage across an inductor can be discontinuous.
- The equation for equivalent inductance for

inductors in series

$$L_{eq} = \sum_{s=1}^S L_s$$

inductors in parallel

$$L_{eq} = \left[\sum_{p=1}^P \frac{1}{L_p} \right]^{-1}$$