

Flipping a coin

1- head

2- tail

If I toss a fair coin what is the probability

$$P(T) = \frac{1}{2} = 0.50 = 0.5$$

$$P(H) = \frac{1}{2} = 0.50 = 0.5$$

Sets

A set is an unordered collection of things (elements)

$$B = \{1, 2, 3, 3\} \quad B = \{1, 2, 3\}$$

Subset (alt küme)

Set A is a subset of B if every element of A is also an element of B

" \subset " subset

$$A = \{x \mid x = 2, 3, 4\} = \{2^1, 3^1, 4^1\}$$

such that braces
 list

$$N (\text{natural numbers}) = \{1, 2, 3, \dots\}$$

$$Z (\text{integers}) = \{\dots, -1, 0, 1, 2, \dots\}$$

$$R (\text{rational numbers}) \rightarrow \frac{a}{b}$$

(Rasyonel) kesir

$$Q = \left\{ \frac{a}{b} : a \in Z \text{ and } b \in Z - \{0\} \right\}$$

Real numbers (R)

infinity numbers

Superset if A is a subset of set B, then B is superset of A.

Venn diagrams



Set operations

Union (Birleşim) = the union of two sets A and B consists of

all elements in A or B

$$x \in (A \cup B) \iff (x \in A) \text{ or } (x \in B)$$

$$\{1, 2\} \cup \{2, 3\} \rightarrow A \cup B = \{1, 2, 3\}$$

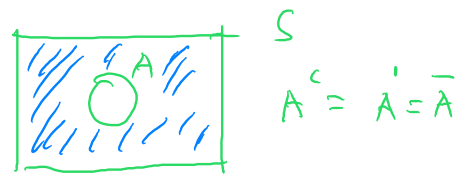
Intersection (kesişim) → The intersection of two sets A and B consists

of all elements in both A and B

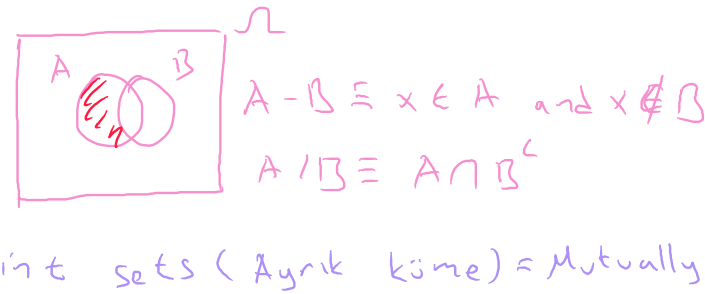


(doğul)

Complement → The complement of a set A is set of all elements in S that are not in A.

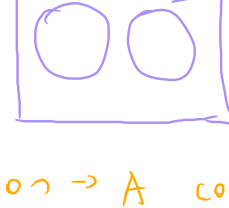


Difference (subtraction) → The difference of set B from A is all elements in A that are not in B.



Disjoint sets (Ayrık küme) = Mutually exclusive set → Two sets A and B

are disjoint if $A \cap B = \emptyset \rightarrow$ empty set



Partition → A collection of sets

A_1, A_2, \dots, A_n is a Partition set if and only if

a- They are disjoint

b- $A_1 \cup A_2 \cup \dots \cup A_n = S$



De Morgan's law

$$(A \cup B)^c = \overline{(A \cup B)} = A' \cap B'$$

Example

$$S = \{1, 2, 3, 4, 5, 6\} \quad A = \{1, 2\} \quad B = \{2, 4, 6\}$$

$$A \cup B = \{1, 2, 4, 6\} \quad A \cap B = \{2\} \quad A' = \{3, 4, 5, 6\}$$

$$B' = \{1, 3, 5\} \quad (A \cup B)' = A' \cap B' = \{3, 5\} \quad \text{proof (kariye)}$$

Theorem: Distributive law

$$A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$$

Functions: $f: X \rightarrow Y$

domain co-domain

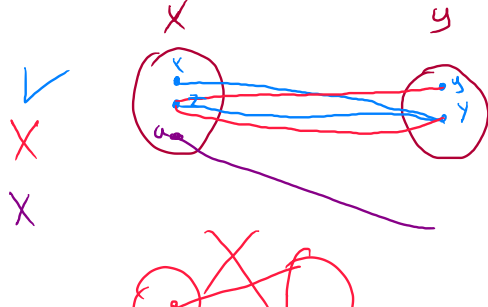


$$\forall x \in X, f(x) \in Y$$

Range: the set of all possible values of $f(x)$ $\text{Range} \subset Y$

$f: R \rightarrow R$ defined as $f(x) = x^2$

$$x = y = R \quad \text{Range}(f) = R^+ = \{x \in R \mid x \geq 0\}$$



ya $f: R - \{a\} \rightarrow R$ olacak

ya da olmaz

Countable and uncountable sets

A set is finite if $|A| < \infty$

A set countable if $|A| \rightarrow$ cardinality

is finite or its elements can be

enumerated or listed

in a sequence.

$$A = \bigcup_{k=1}^{\infty} \{a_k\} \rightarrow A = \{a_1, a_2, \dots\}$$

$N = \{1, 2, 3, \dots\} \rightarrow$ infinite and countable (sayılabilir)

Uncountable → the elements cannot be enumerated

$R \rightarrow$ real numbers → uncountable

- A set is countable infinite if it is in one to one correspondence

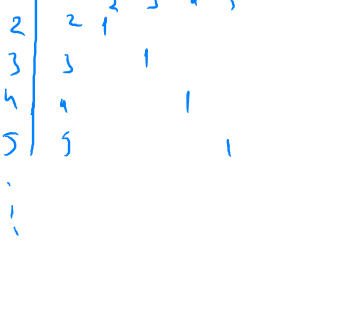
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$Z \rightarrow$ countable?

$Z = \{\dots, -2, -1, 0, 1, 2, \dots\} \rightarrow$ infinite and countable

a_0, a_1, a_2

$$\Phi^+ = \left\{ \frac{a}{b} : a \text{ or } b \neq 0 \right\} \quad \text{infinite, countable} \quad \Phi^+ = \bigcup_{i,j=1}^{\infty} \left\{ \frac{i}{j} \right\}$$



$R \rightarrow$ infinite, uncountable

$C, b, 3, b, a$

Power set: Set of all subsets of a set A

$$A = \{1, 2\} \rightarrow P(A) = \{\{1\}, \{2\}, \{1, 2\}, \emptyset\}$$

Exhaustive set



sets A, B, C, D are exhaustive if their union is S

Intersections are allowed

Probability Theory

random experiment: A phenomenon whose outcome cannot be predicted with certainty.

- Rolling a die

- Flipping a coin

- Rolling a die 3 times

Outcome (Result of random experiment)

→ Rolling a die → $\{1, 2, 3, 4, 5, 6\}$

→ Flipping a coin → Head

→ Rolling a die 2 times → $6, 2$

Event (is a collection of possible outcomes)

→ Rolling a die $\text{Event}_1 = \{1, 5, 3\}$

$\text{Event}_2 = \{4, 2\}$ $\text{Event} \subset \text{Sample Space}$

Sample space:

Set of all possible outcomes.

Roll a die $S = \{1, 2, 3, 4, 5, 6\}$

Roll a die 3 times → $S = \{(1, 1, 1), (1, 1, 2), (1, 1, 3), \dots, (6, 6, 6)\} \rightarrow 3, 6, 6$ possible outcomes

Partition → is collectively exclusive and mutually exclusive set of events.

A_1, A_2, \dots, A_n is a partition

$$A_1 \cup A_2 \cup \dots \cup A_n = S$$

$$A_1 \cap A_2 = \emptyset, \text{ if } j$$

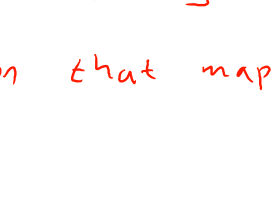
- We say that an event A is occurred if the outcome of the experiment is an element of A.

Probability

• An event $A \rightarrow P(A)$: probability of A

We assign a probability $P(A)$ to every event A

(atanımlı)



Axioms of probability.

P() is a function that maps events in the sample space S to real numbers

①- For any event A, $P(A) \geq 0$

②- Probability of sample space S $P(S) = 1$

③- For any countable collection A_1, A_2, \dots disjoint events.

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) + \dots + P(A_n)$$