2017' Final Sorusu:

$a>0$ olmak üzere $6+\int_a^x \frac{f(t)}{t^2}dt = 2\sqrt{x}$ eşitliğini sağlayan f fonksiyonu ve a sayısını bulunuz.

$$6+\int_a^x \frac{f(t)}{t^2}dt = 2\sqrt{x} \xrightarrow{\text{Türev}} \frac{f(x)}{x^2} = \frac{1}{\sqrt{x}} \Rightarrow \boxed{f(x) = x^{3/2}}$$

$\downarrow x=a$ yazarsak

$$6+\underbrace{\int_a^a \frac{f(t)}{t^2}dt}_{0} = 2\sqrt{a} \Rightarrow 3 = \sqrt{a} \Rightarrow \boxed{a=9}$$

2017 Bütünleme Sorusu

$$\int \sec^2\sqrt{x}\,dx = ?$$

$$\sqrt{x}=t \qquad \frac{1}{2\sqrt{x}}dx = dt \Rightarrow dx = 2\sqrt{x}\,dt = 2t\,dt$$

$$\int \sec^2\sqrt{x}\,dx = 2\int t\sec^2 t\,dt \qquad \begin{cases} t=u & dt=du \\ \sec^2 t\,dt = dv & v=\tan t \end{cases}$$

$$= 2\left[t\tan t - \int \tan t\,dt\right] = 2t\tan t + 2\ln|\cos t| + c$$

$$= 2\sqrt{x}\tan\sqrt{x} + 2\ln|\cos\sqrt{x}| + c$$

2017 2. Vize

$$I = \int_0^{\pi/4} e^{\tan x - 2\ln(\cos x)}dx = ?$$

$$e^{\tan x}\cdot \frac{e^{\ln(\cos x)^{-2}}}{\frac{1}{\cos^2 x}} = \frac{e^{\tan x}}{\cos^2 x} \Rightarrow I = \int_0^{\pi/4}\frac{e^{\tan x}}{\cos^2 x}dx \qquad \tan x = u \quad \frac{dx}{\cos^2 x} = du$$

$x=\frac{\pi}{4} \to u=1$

$x=0 \to u=0$

$$= \int_0^1 e^u du = e^u\Big|_0^1$$

$$= e-1$$

2017 Mazeret Sınavı

$x \geq 1$, $f(x) = \int_{2}^{x+1} (t-1)^t \, dt \Rightarrow f''(1) = ?$

↓ Türev

$f'(x) = (x+1-1)^{x+1} = x^{x+1}$ $\quad \rightarrow f'(1) = 1^2 = 1$

↓ logaritmik türev

$\ln f'(x) = (x+1)\ln x$

↓ Türev

$\dfrac{f''(x)}{f'(x)} = \ln x + \dfrac{x+1}{x} \Rightarrow \dfrac{f''(1)}{1} = \ln 1 + \dfrac{2}{1} \Rightarrow \boxed{f''(1) = 2}$

2017 - Mazeret Sınavı

$$\int_1^e \frac{\ln x^2}{x(1+\ln^2 x)} = \int_1^e \frac{2\ln x}{x(1+\ln^2 x)} dx = \int_0^1 \frac{2u}{1+u^2} du = \ln|1+u^2| \Big|_0^1 = \underline{\underline{\ln 2}}$$

$\ln x = u \quad \dfrac{dx}{x} = du$

$x = e \rightarrow u = 1$

$x = 0 \rightarrow u = 0$

2017 - final

(*) $f(x) = x\sqrt{16 - x^2}$ fonksiyonunun yerel ve mutlak ekstremumlarını bulunuz.

$\boxed{D(f) = [-4, 4]}$

$f'(x) = \sqrt{16-x^2} + \dfrac{x \cdot -2x}{2\sqrt{16-x^2}} = \dfrac{16 - 2x^2}{\sqrt{16-x^2}}$ $\nearrow f' = 0 \rightarrow x^2 = 8 \rightarrow \boxed{x = \mp 2\sqrt{2}}$ K.N.

$\searrow f'$ tanımsız $\rightarrow \boxed{x = \mp 4}$ K.N.

$\boxed{x = \mp 4 \text{ Uç Nokta}}$

x	-4		$-2\sqrt{2}$		$2\sqrt{2}$		4
f'		$-$	0	$+$	0	$-$	
f		↘		↗		↘	
	y.max		y.min		y.max		y.min

$f(-4) = f(4) = 0$

$f(2\sqrt{2}) = 8$

$f(-2\sqrt{2}) = -8$

Yerel Max

$(-4, 0), (2\sqrt{2}, 8)$

Yerel Min

$(-2\sqrt{2}, -8), (4, 0)$

Mutlak Max

$(2\sqrt{2}, 8)$

Mutlak Min

$(-2\sqrt{2}, -8)$

(*) $f(x)=\dfrac{x^5-3}{x^4}$ eğrisinin asimptotlarını, artan / azalan olduğu aralıkları, konkavlığını, eksenleri kesim noktalarını araştırıp eğriyi çizin.

① $x=0$ Düşey asimptot

② $f(x)=\dfrac{x^5-3}{x^4}=\boxed{x}-\dfrac{3}{x^4}$ $\Rightarrow y=x$ eğik asimptot (veya $\begin{array}{r|l} x^5-3 & x^4 \\ \hline -x^5 & \boxed{x} \rightarrow y=x \\ -3 & \text{E.A.} \end{array}$)

③ $f'(x)=\dfrac{5x^4\cdot x^4-4x^3(x^5-3)}{x^8}=\dfrac{x^3(x^5+12)}{x^8}=\dfrac{x^5+12}{x^5}$

$f'=0 \swarrow$ $\searrow$ f' tanımsız

$x^5+12=0$ $\boxed{x=0}$ K.N değil (f tanımsız)

$\downarrow$

$\boxed{x=-\sqrt[5]{12}}$ K.N.

④ $f''(x)=\dfrac{5x^4\cdot x^5-5x^4(x^5+12)}{x^{10}}=-\dfrac{60}{x^6}$

$\swarrow$ $\searrow$

$f''=0$ XX f'' tanımsız

$\boxed{x=0}$ G.K.K.

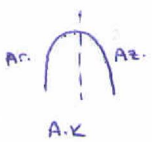
Büküm Nok. olamaz (f tanımsız)

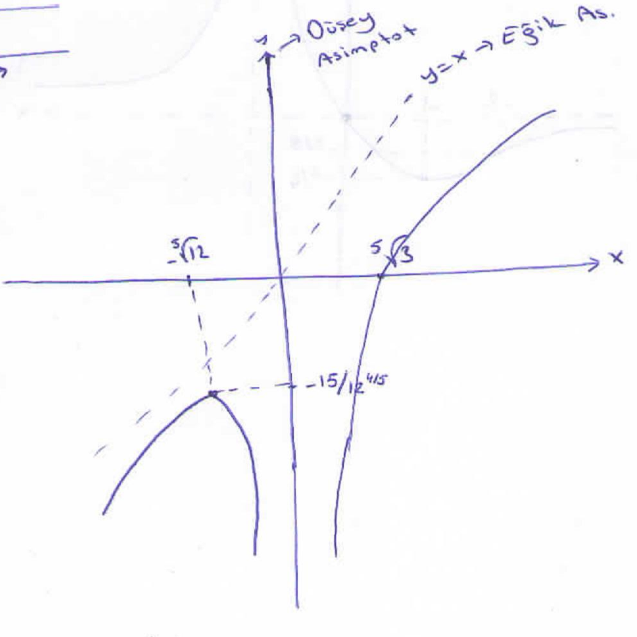
⑤ $y=0 \Rightarrow x^5-3=0 \Rightarrow \boxed{x=\sqrt[5]{3}}$

⑥

x	$-\infty$	$-\sqrt[5]{12}$	0	∞
f'	$+$	0 $-$	T.sız	$+$
f''	$-$	$-$	T.sız	$-$
f	$\nearrow$	$\searrow$	T.sız	$\nearrow$

$x=-\sqrt[5]{12} \Rightarrow y=\dfrac{-12-3}{(-\sqrt[5]{12})^4}=-\dfrac{15}{12^{4/5}}$





⊛ $y=\dfrac{x}{(x-1)^2}$ eğrisini çiziniz.

① $x=1$ Düşey As. ② $\lim\limits_{x\to\mp\infty}\dfrac{x}{(x-1)^2}=0$ olduğundan $\boxed{y=0}$ Yatay As.

$y=0 \Rightarrow x=0$ (0,0) da yatay asimptot ve eğri kesişir

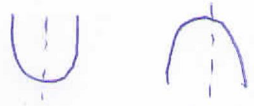
③ $y'=\dfrac{(x-1)^{\cancel{2}}-2\cancel{(x-1)}.x}{(x-1)^{4}} = -\dfrac{x+1}{(x-1)^3}$ $\xrightarrow{y'=0}$ $\boxed{x=-1}$ K.N.

$\xrightarrow{y' \text{ tanımsız}}$ $\boxed{x=1}$ K.N. olamaz (fonk. tanımsız)

④ $y''=-\dfrac{(x-1)^{\cancel{3}}-3\cancel{(x-1)^2}.(x+1)}{(x-1)^{\cancel{6}\,4}} = \dfrac{2x+4}{(x-1)^4}$ $\xrightarrow{y''=0}$ $\boxed{x=-2}$

$\xrightarrow{y'' \text{ t.sız}}$ $\boxed{x=1}$ Ç.K.K. (B.N. olamaz)

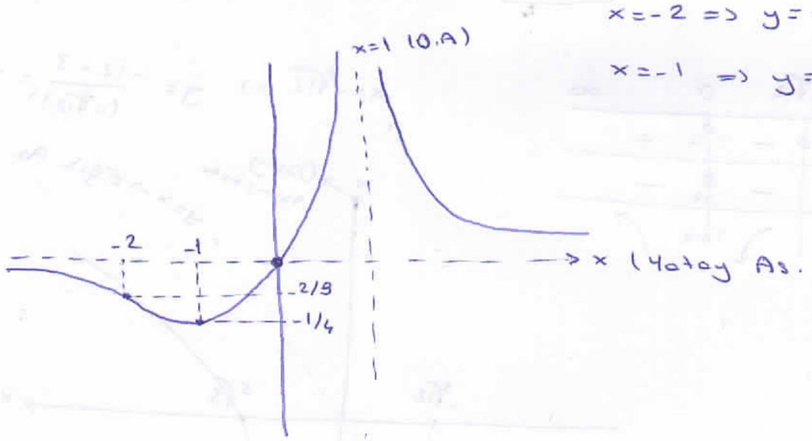
x	$-\infty$		-2		-1		1		∞
y'		$-$		$-$	0	$+$	T.sız	$-$	
y''		$-$	0	$+$		$+$	T.sız	$+$	
y	0	↘		↘		↗	T.sız	↘	0



Ç.K. A.K.

$x=-2 \Rightarrow y=-\frac{2}{9}$

$x=-1 \Rightarrow y=-\frac{1}{4}$



2017 Mazeret

ODT kullanarak her $x>0$ için $\frac{x}{1+x} < \ln(1+x) < x$ olduğunu gösterin.

$f(x)=\ln(1+x)$, olsun. $[0,x]$ aralığı için:

① $f(x)$, $[0,x]$ de süreklidir ② $f'(x)=\frac{1}{1+x}$, $(0,x)$ de tanımlıdır.

O.D.T. şartları sağlanır ✓

O halde:

$f'(c)=\frac{f(x)-f(0)}{x-0}$ olacak şekilde bir $c\in(0,x)$ vardır.

$$\downarrow$$

$$\frac{1}{1+c}=\frac{\ln(1+x)}{x}$$

$c>0 \Rightarrow \frac{1}{1+c}=\frac{\ln(1+x)}{x}<1$

$$\downarrow$$

$$\boxed{\ln(1+x)<x}$$

$c<x \Rightarrow \frac{1}{1+c}=\frac{\ln(1+x)}{x}>\frac{1}{1+x}$

$$\downarrow$$

$$\boxed{\ln(1+x)>\frac{x}{1+x}}$$

(*) $\int x^3e^{x^2}dx = \int x.\underbrace{x^2}_{u}.\underbrace{e^{x^2}}_{e^u}dx=\frac{1}{2}\int u e^u du = \frac{1}{2}\left(ue^u-\int e^u du\right)$

$x^2=u \quad 2xdx=du \quad \frac{du}{2}$

$u=t \quad du=dt$

$e^u du=dv \quad v=e^u$

$= \frac{1}{2}\left(ue^u-e^u\right)+c$

$= \frac{1}{2}x^2e^{x^2}-\frac{e^{x^2}}{2}+c$

2016 - Mazeret

$f(x)=(1+\ln x)^x$, $g(x)=\int_x^{f(x)}\cos^2 x\,dx \Rightarrow f'(1)=?\quad g'(1)=?\quad (x>0$ için$)$

$f(x)=(1+\ln x)^x \xrightarrow{L.\text{Türev}} \ln f(x)=x\ln(1+\ln x) \xrightarrow{\text{Türev}} \frac{f'(x)}{f(x)}=\ln(1+\ln x)+\frac{x.\frac{1}{x}}{1+\ln x}$

$$\downarrow x=1$$

$\frac{f'(1)}{\underbrace{f(1)}_{1}}=\overbrace{\ln(1+\ln 1)}^{0}+\frac{1}{1+\underbrace{\ln 1}_{0}}$

$$\downarrow$$

$$\boxed{f'(1)=1}$$

$g(x)=\int_x^{f(x)}\cos^2 x\,dx$

$\downarrow$ Türev

$g'(x)=f'(x).\cos^2 f(x)-\cos^2 x \xrightarrow[x=1]{} g'(1)=\underbrace{f'(1)}_{1}.\cos^2 f(1)-\cos^2 f(1)$

$$\boxed{g'(0)=0}$$

(*) $\int \text{Cot}^n x\,dx$ integrali için indirgeme formülü bulup $\int \text{Cot}^3 x\,dx$ integralini bu formülle hesaplayın.

$\text{Cosec}^2 x = 1 + \text{Cot}^2 x$

$$I_n = \int \text{Cot}^n x\,dx = \int (\text{Cot}^n x)^{n-2} \cdot \underbrace{\text{Cot}^2 x}_{(\text{Cosec}^2 x - 1)}\,dx$$

$$= \int \underbrace{(\text{Cot}^{n-2} x)}_{u^{n-2}} \cdot \underbrace{\text{Cosec}^2 x\,dx}_{-du} - \underbrace{\int (\text{Cot}^{n-2} x)\,dx}_{I_{n-2}}$$

$$I_n = -\frac{u^{n-1}}{n-1} - I_{n-2}$$

$$\boxed{I_n = -\frac{(\text{Cot}\,x)^{n-1}}{n-1} - I_{n-2}} \Rightarrow I_3 = -\frac{\text{Cot}^2 x}{2} - \overbrace{I_1}^{\int \text{Cot}\,x\,dx}$$

$$= -\frac{\text{Cot}^2 x}{2} - \ln|\text{Sin}\,x| + C$$

2017 Final Sorusu:

$$I = \int \text{Cos}(\ln x)\,dx = ?$$

↓ Kısmi int.

$\text{Cos}(\ln x) = u \quad dx = dv$

$-\frac{1}{x}\text{Sin}(\ln x)dx = du \quad v = x$

$$I = x\,\text{Cos}\ln x + \int \text{Sin}(\ln x)\,dx$$

$\text{Sin}(\ln x) = u \quad dx = dv$

$\frac{1}{x}\text{Cos}(\ln x)dx = du \quad v = x$

$$= x\,\text{Cos}(\ln x) + x\,\text{Sin}(\ln x) - \underbrace{\int \text{Cos}(\ln x)\,dx}_{I}$$

$\Downarrow$

$$2I = x\,\text{Cos}(\ln x) + x\,\text{Sin}(\ln x)$$

$$I = \frac{1}{2}(x\,\text{Cos}(\ln x) + x\,\text{Sin}(\ln x)) + C$$

⊛ $$\int_0^{\pi/2} \frac{\sin 2x}{e^{\sin^2 x}+e^{-\sin^2 x}}\,dx = \int_0^1 \frac{du}{e^u+\underbrace{e^{-u}}_{\frac{1}{e^u}}} = \int_0^1 \frac{e^u\,du}{e^{2u}+1} = \int_1^e \frac{dt}{t^2+1} = \operatorname{Arctan} t \Big|_1^e$$

$\sin^2 x = u$

$\underbrace{2\sin x\cos x}_{\sin 2x}\,dx = du$

$x=\frac{\pi}{2} \Rightarrow u=1$

$x=0 \Rightarrow u=0$

$e^u = t$

$e^u\,du = dt$

$u=1 \Rightarrow t=e$

$= \operatorname{Arctan} e - \operatorname{Arctan} 1$

$= \operatorname{Arctan} e - \frac{\pi}{4}$

⊛ $$\int \frac{e^{1/1-x}}{1-3x+3x^2-x^3}\,dx = ?$$

$1-3x+3x^2-x^3 = (1-x)^3$

• $\frac{1}{1-x} = u \Rightarrow \frac{dx}{(1-x)^2} = du$

$$I = \int \frac{\overbrace{e^{1/x}}^{e^u}\,\overbrace{dx}^{du}}{\underbrace{(1-x)}_{\frac{1}{u}}.(1-x)^2} = \underbrace{\int u e^u\,du}_{\text{Kısmi int.}}$$

$u=t \qquad du=dt$

$e^u du = dv \qquad v=e^u$

$$\Rightarrow \quad = ue^u - \int e^u\,du = ue^u - e^u + c$$

$$= \frac{1}{1-x}.e^{1/1-x} - e^{1/1-x} + c$$

⊛ $$\int \frac{dx}{\tan^5 x} = \int \frac{\cos^5 x}{\sin^5 x}\,dx = \int \frac{(\cos^2 x)^2 . \cos x}{\sin^5 x}\,dx$$

$\sin x = u$

$\cos x\,dx = du$

$$= \int \frac{(1-u^2)^2}{u^5}\,du = \int (u^{-5} - 2u^{-3} - u^{-1})\,du$$

$$= \frac{u^{-4}}{-4} - \frac{2u^{-2}}{-2} - \ln|u| + c =$$

$$= -\frac{1}{4\sin^4 x} + \frac{1}{\sin^2 x} - \ln|\sin x| + c$$

(*) $\displaystyle\int \frac{x+3}{x^2-2x+10}\,dx = ?$

$$\frac{1}{2}\int \frac{2x+6}{x^2-2x+10}\,dx = \frac{1}{2}\int \frac{2x-2+8}{x^2-2x+10}\,dx = \frac{1}{2}\int \frac{\overbrace{2x-2}^{du}}{\underbrace{x^2-2x+10}_{u}}\,dx + 4\int \frac{\overbrace{dx}^{du}}{(\underbrace{x-1}_{u})^2+9}$$

$$= \frac{1}{2}\ln|x^2-2x+10| + \frac{4}{3}\,\text{Arctan}\,\frac{x-1}{3} + c$$

(*) $\displaystyle I = \int \text{Arc}\cos x\,dx = ?$

$$\left.\begin{array}{ll} u = \text{Arc}\cos x & dv = dx \\ du = -\dfrac{dx}{\sqrt{1-x^2}} & v = x \end{array}\right\} I = x\,\text{Arc}\cos x + \int \frac{x}{\sqrt{1-x^2}}\,dx \Rightarrow \begin{array}{l} 1-x^2 = u \\ -2x\,dx = du \end{array}$$

$$= x\,\text{Arc}\cos x - \frac{1}{2}\int \frac{du}{\sqrt{u}}$$

$$= x\,\text{Arc}\cos x - \sqrt{u} + c = x\,\text{Arc}\cos x - \sqrt{1-x^2} + c$$

(*) $\displaystyle I = \int e^{2x}.\cos 3x\,dx = ?$

$$\begin{array}{ll} u = \cos 3x & e^{2x}dx = dv \\ du = -3\sin 3x\,dx & v = \dfrac{e^{2x}}{2} \end{array} \qquad \begin{array}{ll} u = \sin 3x & e^{2x}dx = dv \\ du = 3\cos 3x\,dx & v = \dfrac{e^{2x}}{2} \end{array}$$

$$I = \frac{e^{2x}}{2}.\cos 3x + \frac{3}{2}\int e^{2x}.\sin 3x\,dx$$

$$= \frac{e^{2x}}{2}.\cos 3x + \frac{3}{2}\left[\frac{e^{2x}}{2}.\sin 3x - \frac{3}{2}\underbrace{\int e^{2x}\cos 3x\,dx}_{I}\right]$$

$$\left(\overbrace{1+\frac{9}{4}}^{\frac{13}{4}}\right) I = \frac{e^{2x}}{2}.\cos 3x + \frac{3}{4}e^{2x}\sin 3x$$

$$I = \frac{2}{13}e^{2x}\cos 3x + \frac{3}{13}e^{2x}\sin 3x + c$$

(*) $\int_0^1 e^{\text{ArcSin}x}\,dx = ?$

$\text{ArcSin}x = u \Rightarrow x = \text{Sin}u$

$dx = \text{Cos}u\,du$

Ë.S Y.S

$x = 1 \Rightarrow u = \pi/2$

$x = 0 \Rightarrow u = 0$

$I = \int_0^{\pi/2} e^u \text{Cos}u\,du \xrightarrow[\text{int.}]{\text{Kısmi}}$ $\text{Cos}u = t$, $-\text{Sin}u\,du = dt$, $e^u du = dv$, $v = e^u$

$I = e^u.\text{Cos}u\Big|_0^{\pi/2} + \int_0^{\pi/2} e^u \text{Sin}u$ Kısmi inş. $\text{Sin}u = t$, $e^u du = dv$, $\text{Cos}u\,du = dt$, $v = e^u$

$I = \underbrace{e^{\pi/2}.\text{Cos}\frac{\pi}{2}}_{0} - \underbrace{e^0.\text{Cos}0}_{1} + e^u\text{Sin}u\Big|_0^{\pi/2} - \underbrace{\int_0^{\pi/2} e^u \text{Cos}u\,du}_{I}$

$2I = -1 + e^{\pi/2}\underbrace{\text{Sin}\frac{\pi}{2}}_{1} - \underbrace{e^0.\text{Sin}0}_{0} \Rightarrow 2I = e^{\pi/2} - 1 \Rightarrow \boxed{I = \frac{e^{\pi/2}-1}{2}}$

(*) $\int \text{Arctan}\frac{1}{x}\,dx = x.\text{Arctan}\frac{1}{x} + \int \frac{\overbrace{x}^{du/2}}{\underbrace{1+x^2}_{u}}\,dx$

$\text{Arctan}\frac{1}{x} = u$ $dx = dv$

$\frac{-\frac{1}{x^2}dx}{1+\frac{1}{x^2}} = du$ $v = x$

$-\frac{dx}{1+x^2} = du$

$= x\,\text{Arctan}\frac{1}{x} + \frac{1}{2}\ln|1+x^2| + C$

Hepinize 2. vizelerinizde başarılar dilerim...

Sevgiler,

Pınar Albayrak