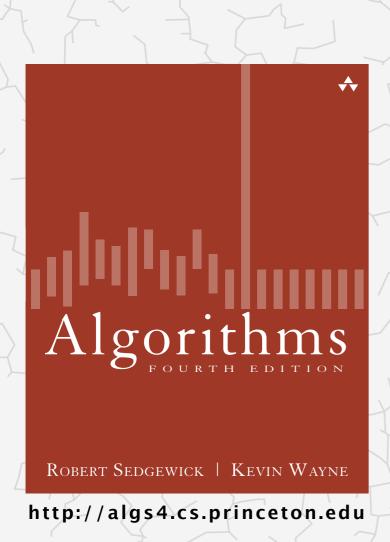
# Algorithms



# PRIM'S ALGORITHM DEMO

- Prim's algorithm
- lazy implementation
- eager implementation

# PRIM'S ALGORITHM DEMO

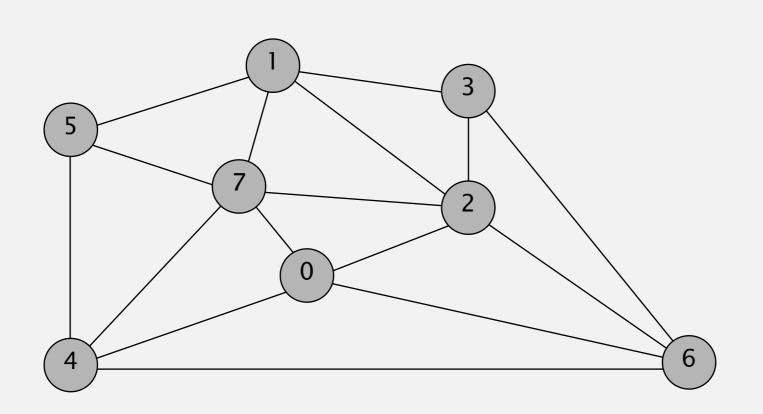
- Prim's algorithm
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Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

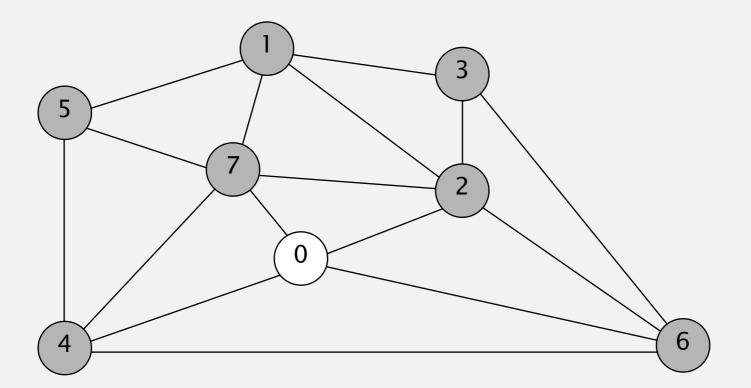
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



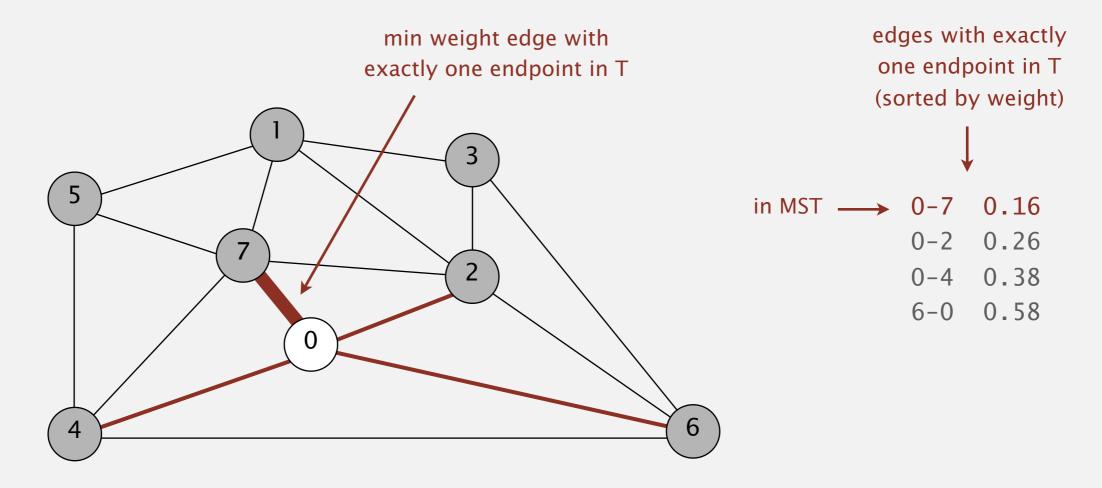
an edge-weighted graph

0-7	0.16
2-3	0.17
1-7	0.19
0-2	0.26
5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
4-5	0.35
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
6-4	0.93

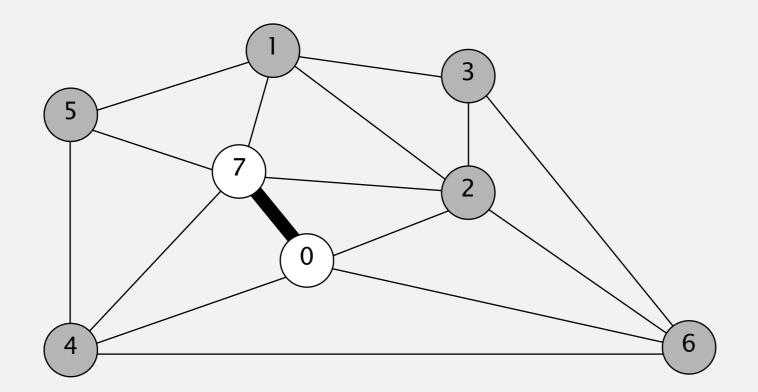
- Start with vertex 0 and greedily grow tree *T*.
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- Start with vertex 0 and greedily grow tree *T*.
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- Repeat until V-1 edges.

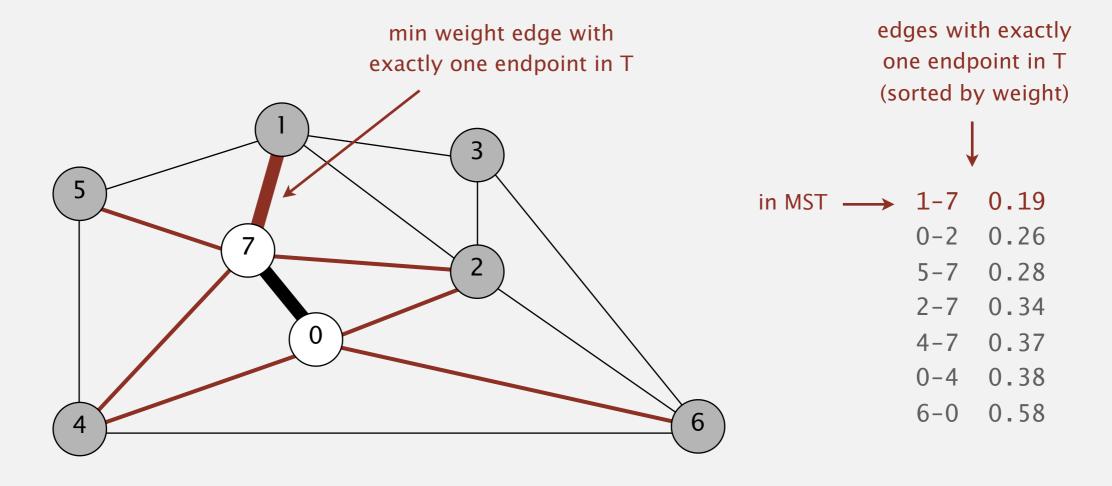


- Start with vertex 0 and greedily grow tree *T*.
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- Repeat until V-1 edges.



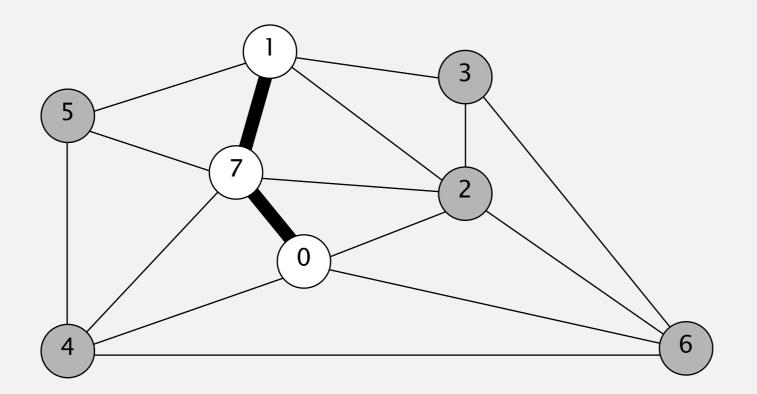
**MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



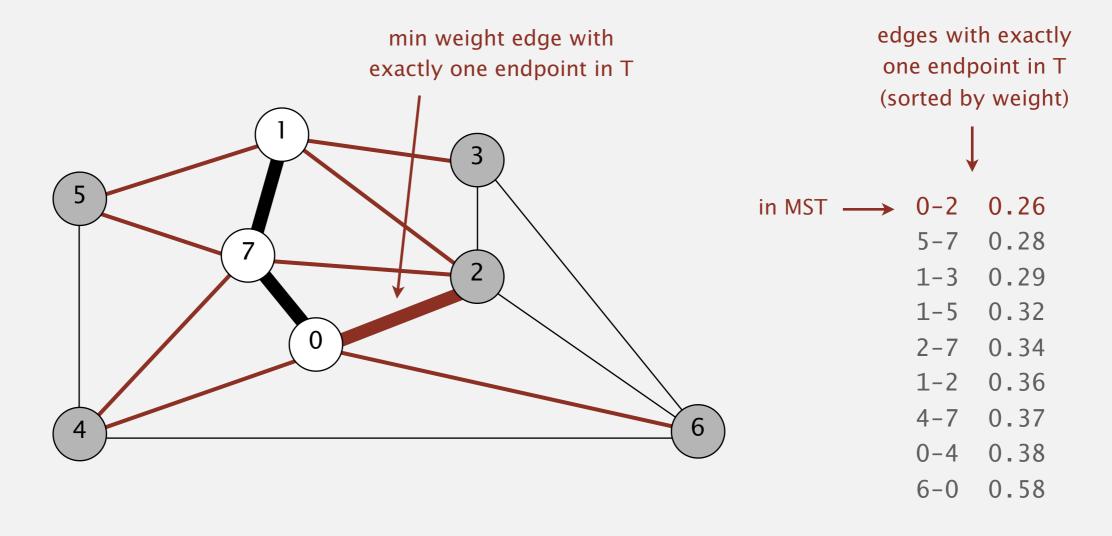
**MST** edges

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
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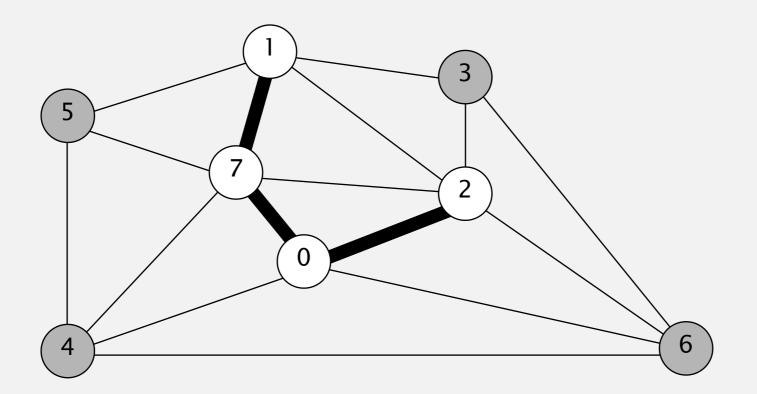
**MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST** edges

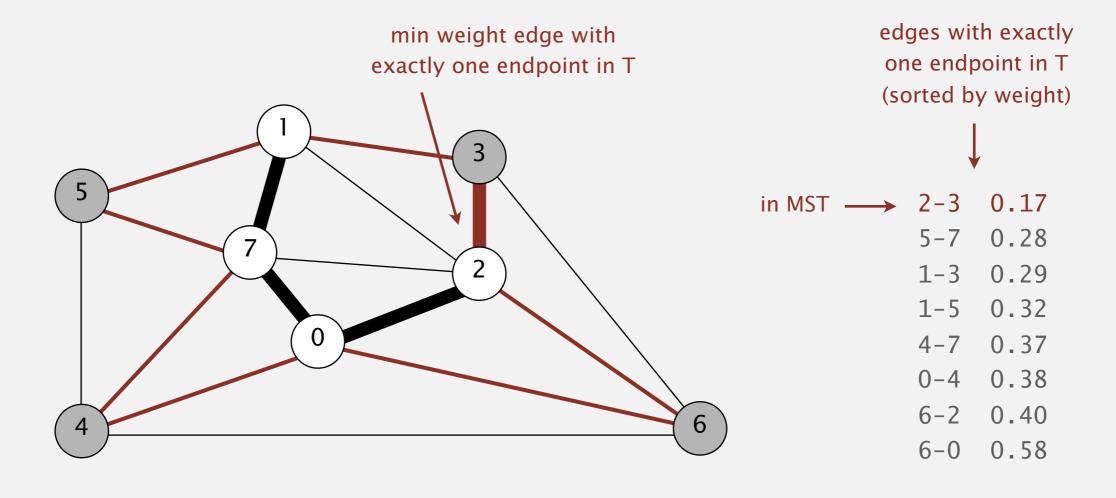
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST edges** 

0-7 1-7 0-2

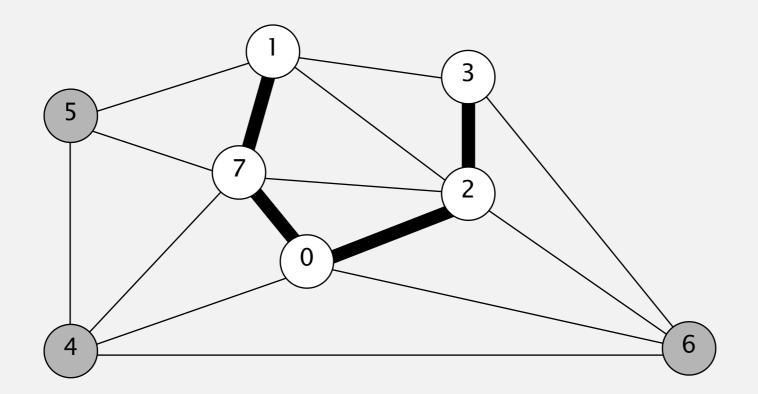
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST** edges

0-7 1-7 0-2

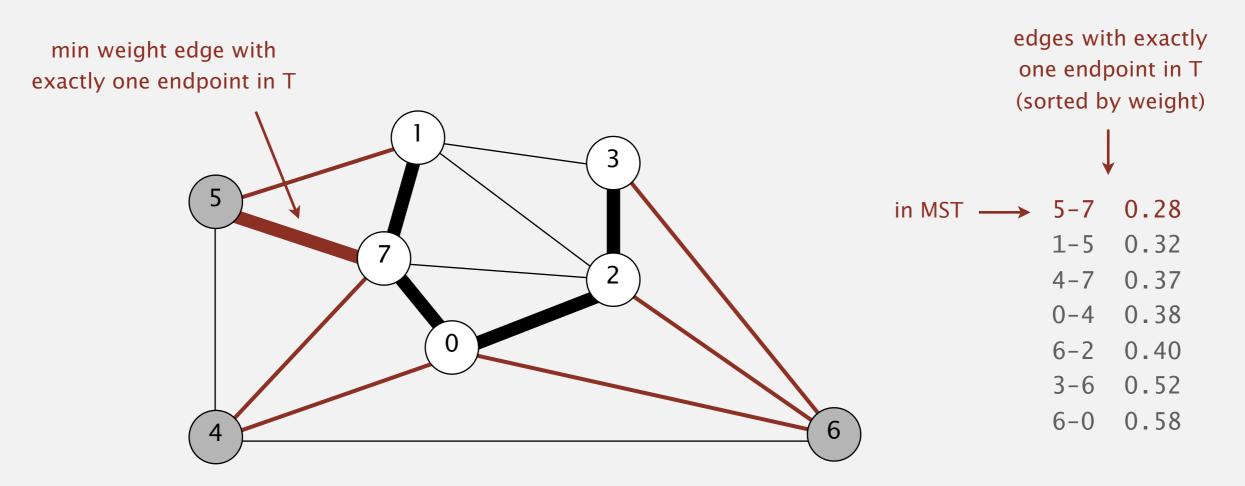
- Start with vertex 0 and greedily grow tree *T*.
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- Repeat until V-1 edges.



#### **MST edges**

0-7 1-7 0-2 2-3

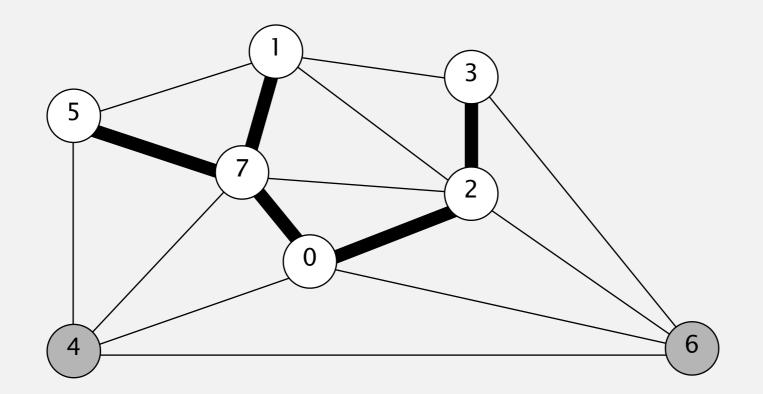
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST edges** 

0-7 1-7 0-2 2-3

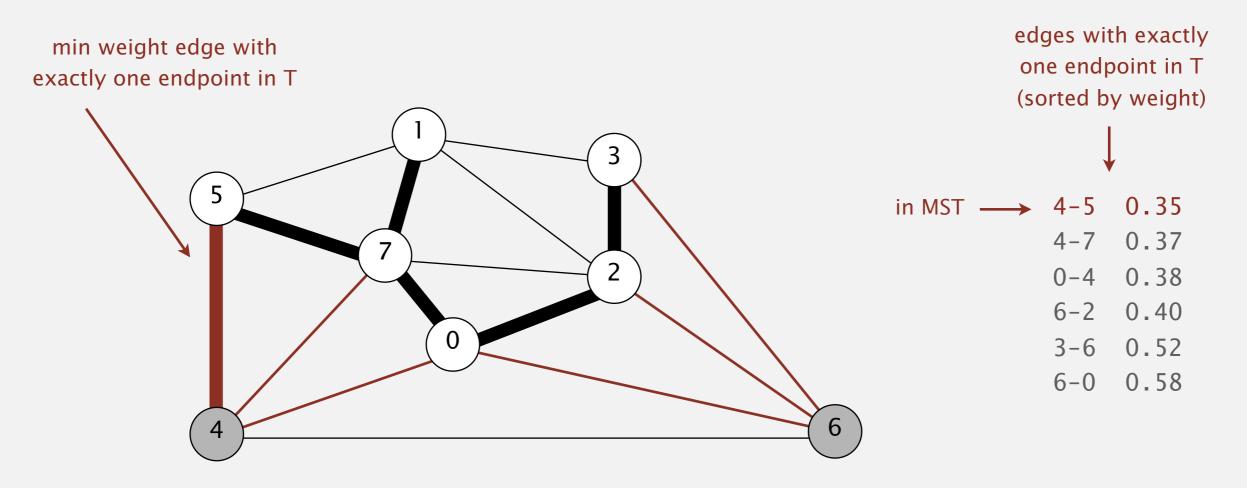
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



#### **MST edges**

0-7 1-7 0-2 2-3 5-7

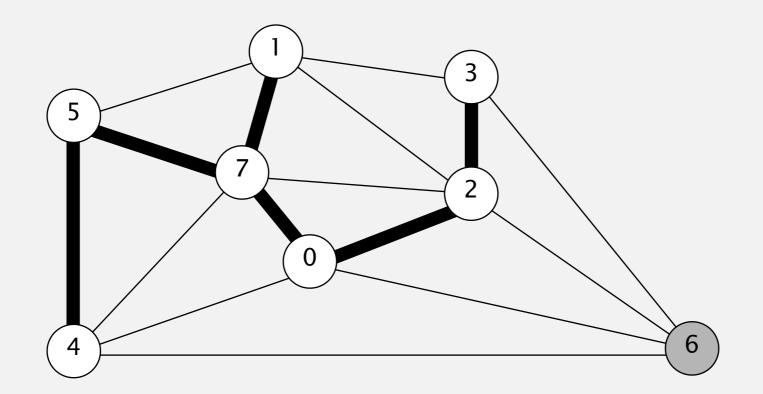
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



**MST edges** 

0-7 1-7 0-2 2-3 5-7

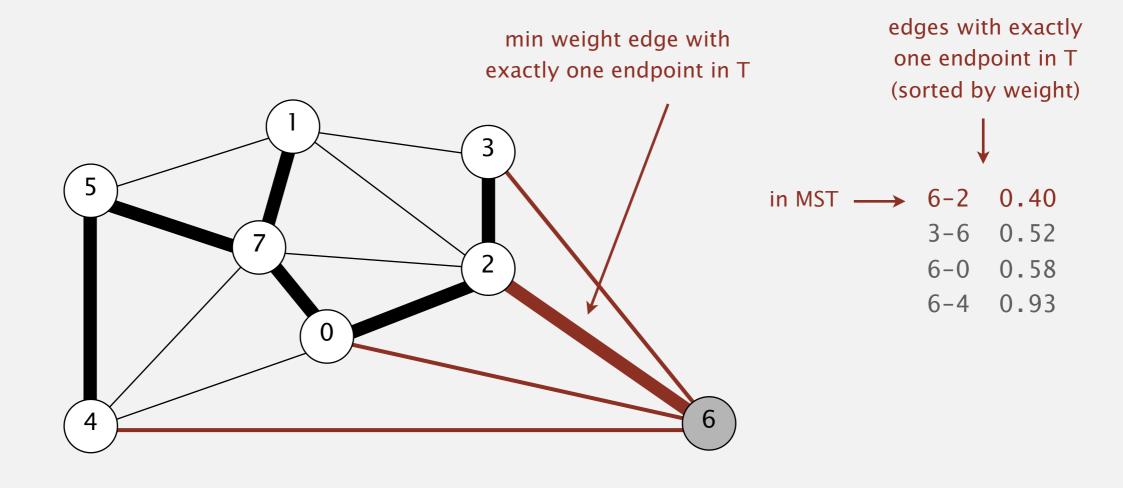
- Start with vertex 0 and greedily grow tree *T*.
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- Repeat until V-1 edges.



#### **MST edges**

0-7 1-7 0-2 2-3 5-7 4-5

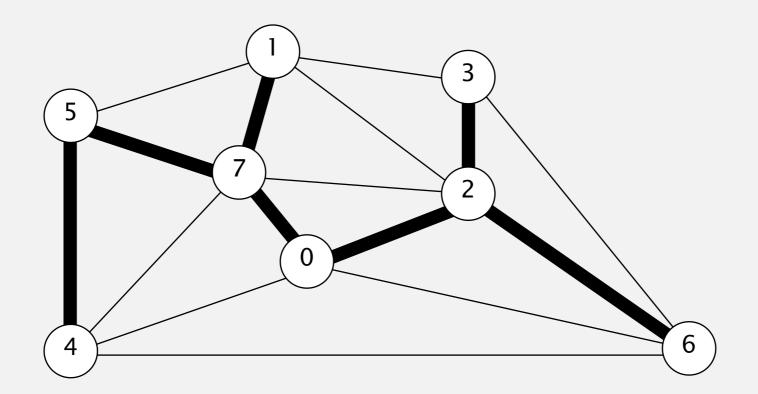
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST** edges

0-7 1-7 0-2 2-3 5-7 4-5

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



#### **MST edges**

0-7 1-7 0-2 2-3 5-7 4-5 6-2

# PRIM'S ALGORITHM DEMO

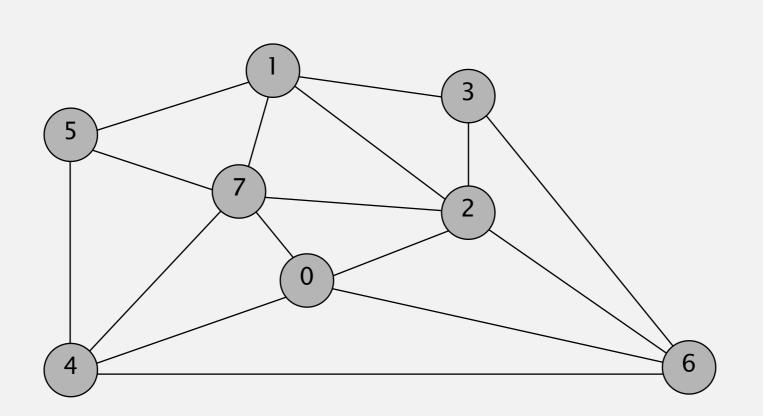
- Prim's algorithm
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Algorithms

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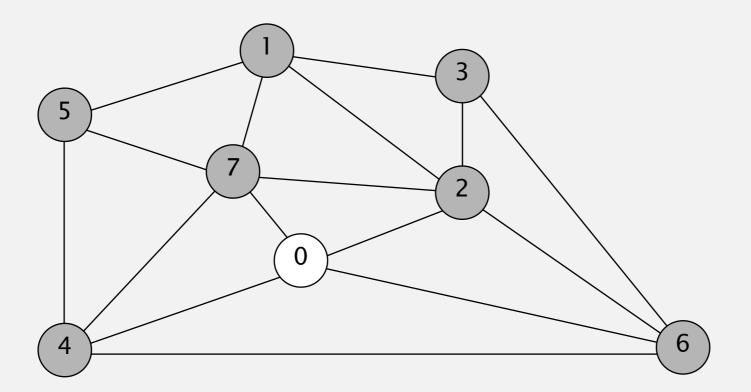
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



an edge-weighted graph

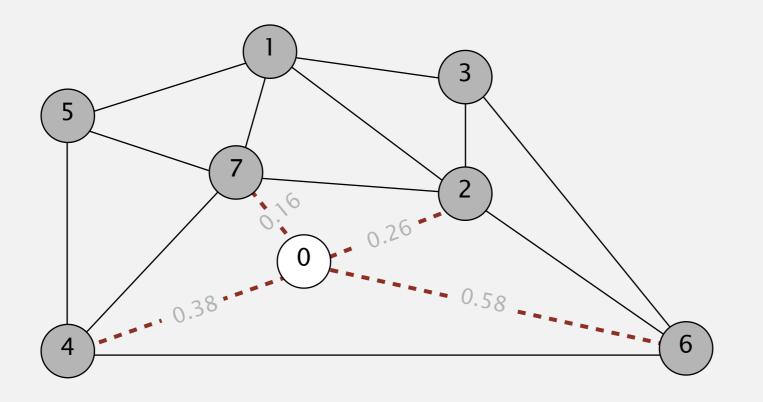
0-7 0.16 0.17 1-7 0.19 0-2 0.26 5-7 0.28 0.29 1-3 1-5 0.32 2-7 0.34 4-5 0.35 1-2 0.36 4-7 0.37 0.38 0.40 6-2 3-6 0.52 6-0 0.58  $6-4 \quad 0.93$ 

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### add to PQ all edges incident to 0



edges on PQ (sorted by weight)

**\*** 0-7 0.16

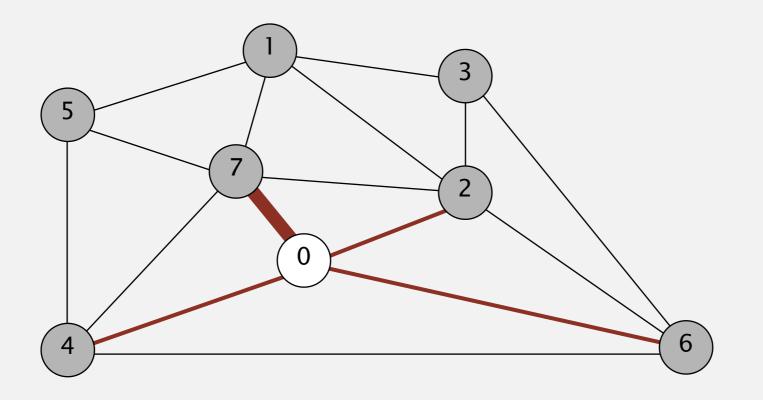
\* 0-2 0.26

**\*** 0-4 0.38

**\*** 6-0 0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 0-7 and add to MST



edges on PQ (sorted by weight)

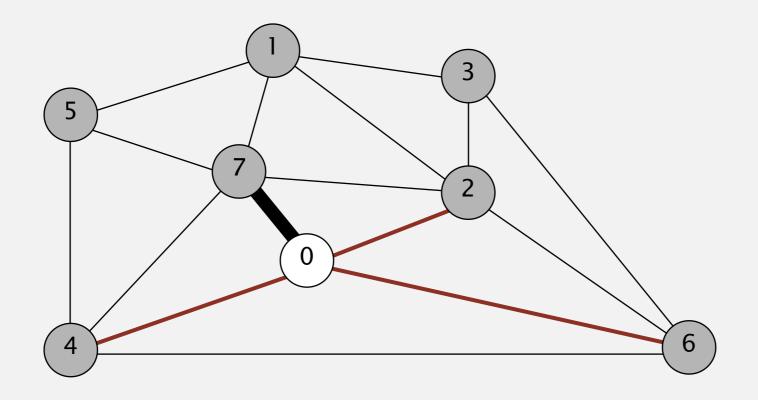
0-7 0.16

0-2 0.26

0-4 0.38

6-0 0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

0-2 0.26

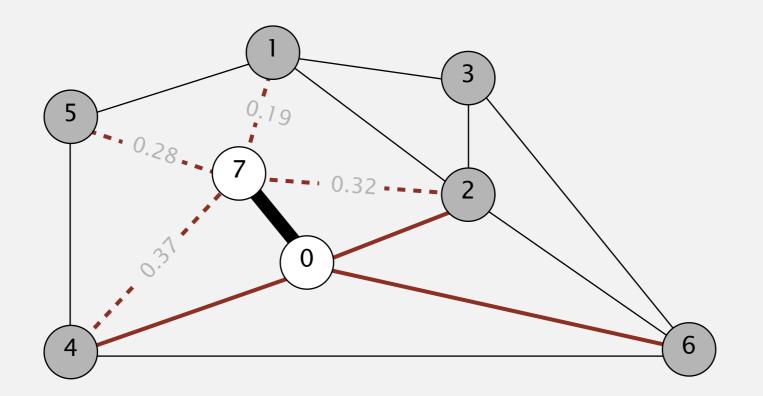
0-4 0.38

6-0 0.58

**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### add to PQ all edges incident to 7



edges on PQ (sorted by weight)

**\*** 1-7 0.19

0-2 0.26

**\*** 5-7 0.28

**\*** 2-7 0.34

**\*** 4-7 0.37

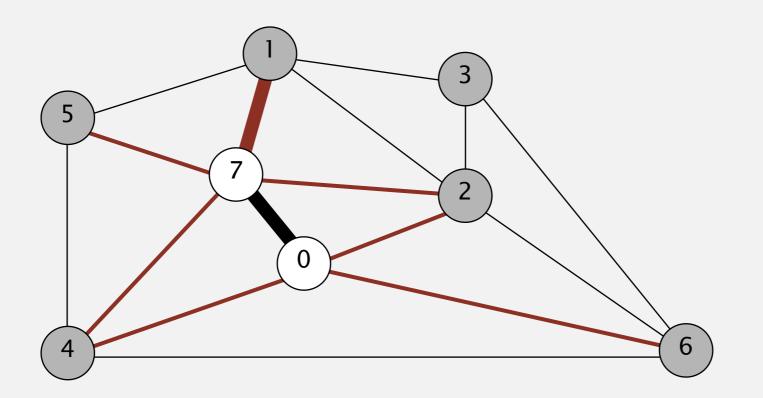
0-4 0.38

6-0 0.58

**MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 1-7 and add to MST



edges on PQ (sorted by weight)

1-7 0.19

0-2 0.26

5-7 0.28

2-7 0.34

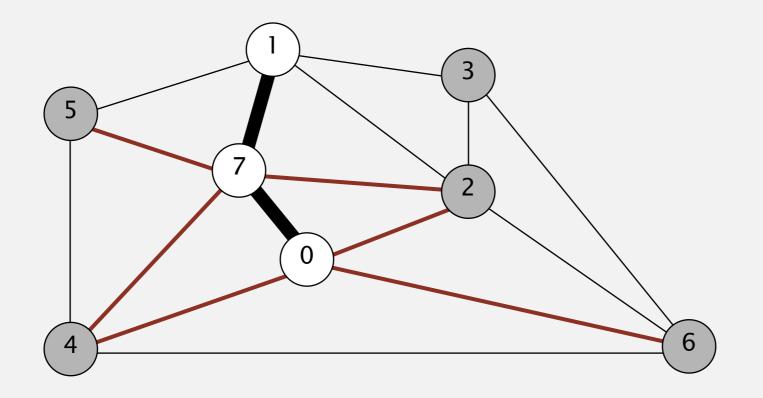
4-7 0.37

0-4 0.38

6-0 0.58

**MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

0-2 0.26

5-7 0.28

2-7 0.34

4-7 0.37

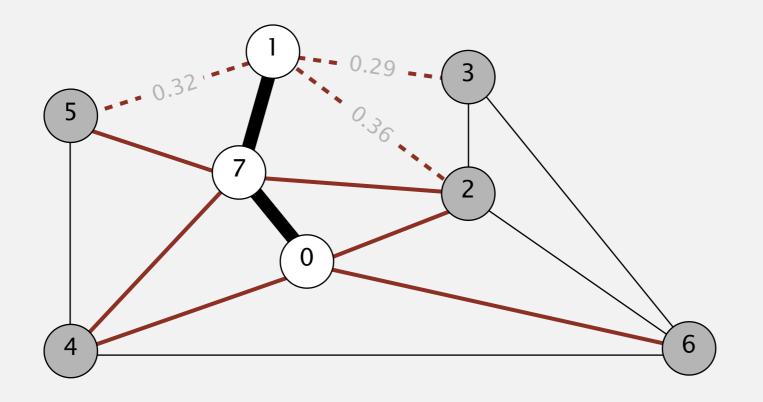
0-4 0.38

6-0 0.58

**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### add to PQ all edges incident to 1



edges on PQ (sorted by weight)

0-2 0.26

5-7 0.28

**\*** 1-3 0.29

**\*** 1-5 0.32

2-7 0.34

**\*** 1-2 0.36

4-7 0.37

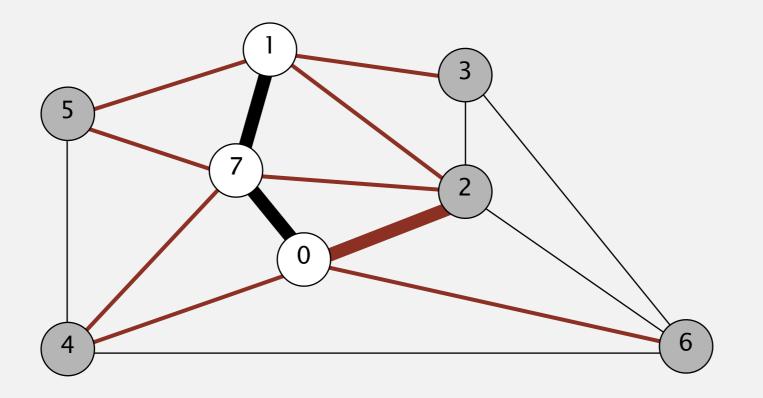
0-4 0.38

6-0 0.58

**MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete edge 0-2 and add to MST



edges on PQ (sorted by weight)

0-2 0.26

5-7 0.28

1-3 0.29

1-5 0.32

2-7 0.34

1-2 0.36

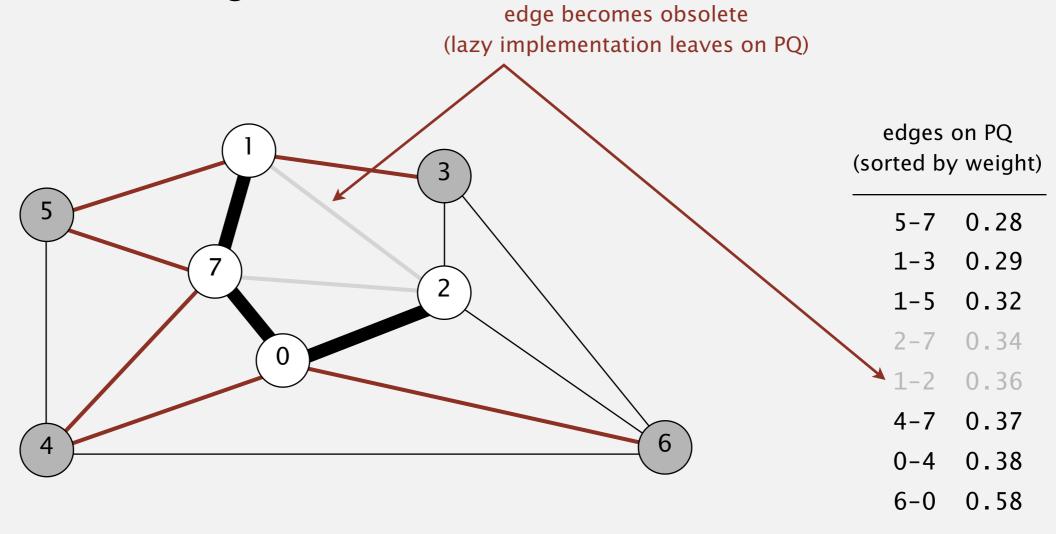
4-7 0.37

0-4 0.38

6-0 0.58

#### **MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST edges** 

0-7 1-7 0-2

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.

no need to add edge 1-2 or 2-7

• Repeat until V-1 edges.

add to PQ all edges incident to 2

because it's already obsolete

7

0.17

7

0.17

6

**MST edges** 

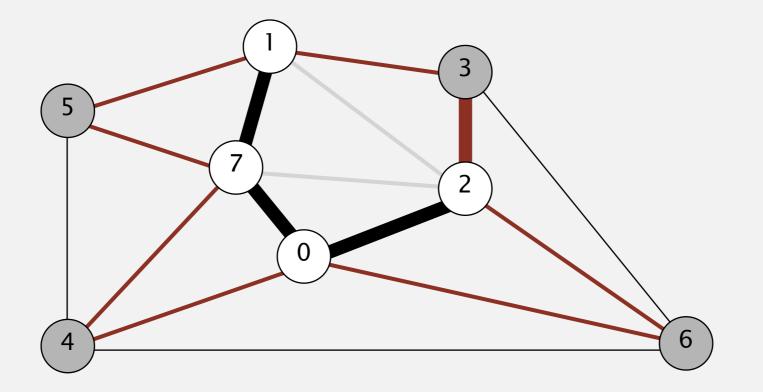
0-7 1-7 0-2

# edges on PQ (sorted by weight)

_		
	* 2-3	0.17
	5-7	0.28
	1-3	0.29
	1-5	0.32
	2-7	0.34
	1-2	0.36
	4-7	0.37
	0-4	0.38
	<b>*</b> 6-2	0.40
	6-0	0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 2-3 and add to MST



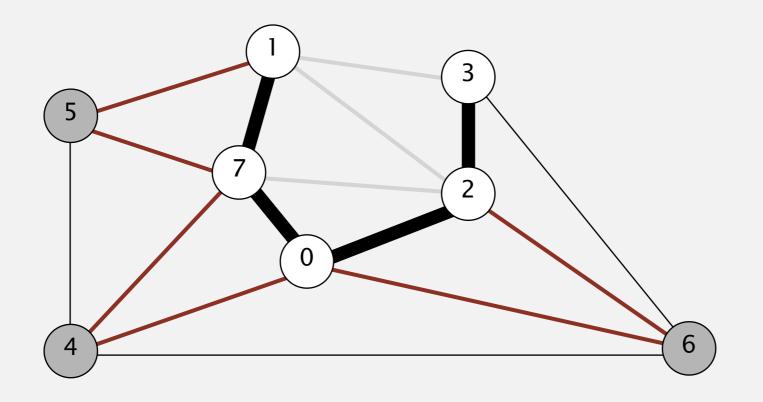
MST edges

0-7 1-7 0-2

# edges on PQ (sorted by weight)

* 2-3	0.17
5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
1-2	0.36
4-7	0.37
0-4	0.38
<b>*</b> 6-2	0.40
6-0	0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

5-7 0.28

 $1-3 \quad 0.29$ 

1-5 0.32

2-7 0.34

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

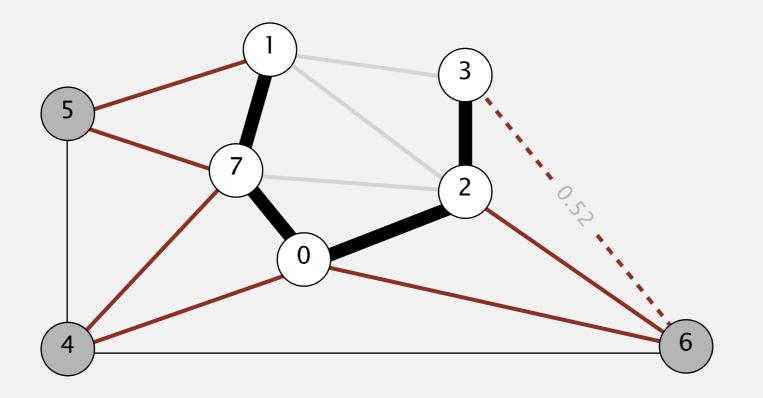
6-0 0.58

#### **MST** edges

0-7 1-7 0-2 2-3

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### add to PQ all edges incident to 3



#### MST edges

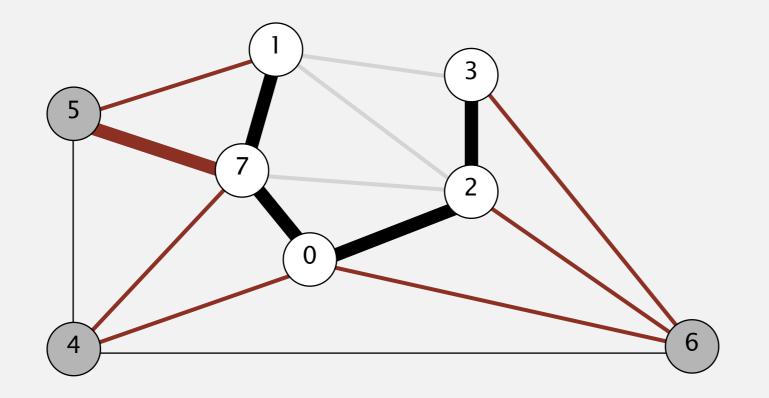
0-7 1-7 0-2 2-3

# edges on PQ (sorted by weight)

5-7	0.28

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 5-7 and add to MST



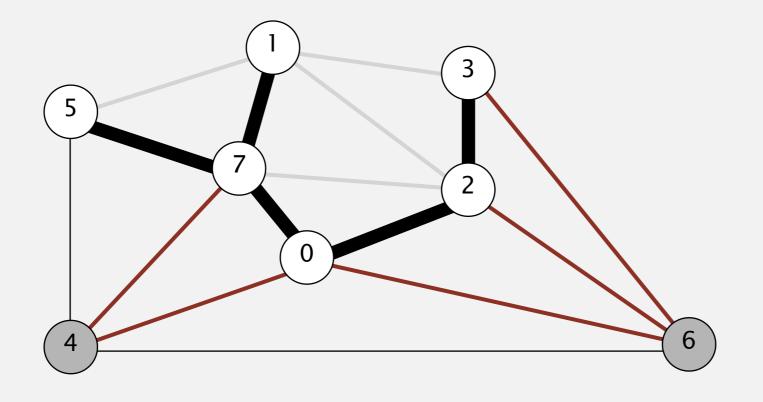
#### **MST** edges

0-7 1-7 0-2 2-3

# edges on PQ (sorted by weight)

	· J ·/
5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



# edges on PQ (sorted by weight)

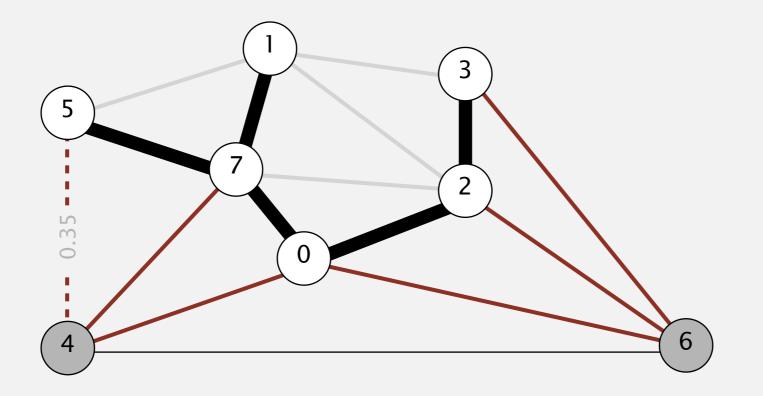
1-3	0.29
1-5	0.32
2-7	0.34
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

**MST edges** 

0-7 1-7 0-2 2-3 5-7

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### add to PQ all edges incident to 5



#### **MST** edges

0-7 1-7 0-2 2-3 5-7

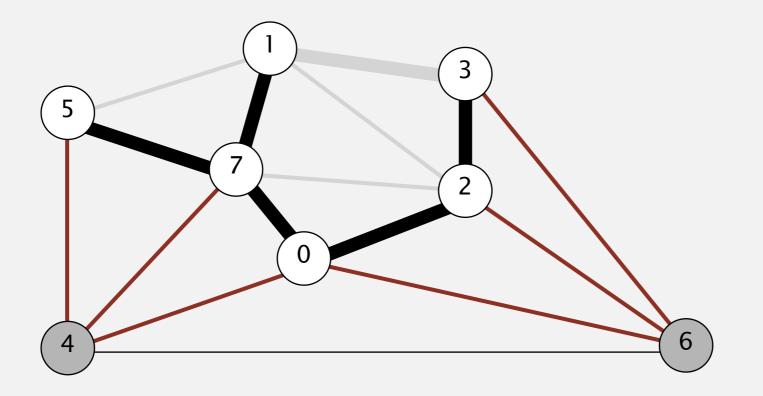
# edges on PQ (sorted by weight)

1-	3	0	2	9

$$6-0$$
 0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 1-3 and discard obsolete edge



#### **MST** edges

0-7 1-7 0-2 2-3 5-7

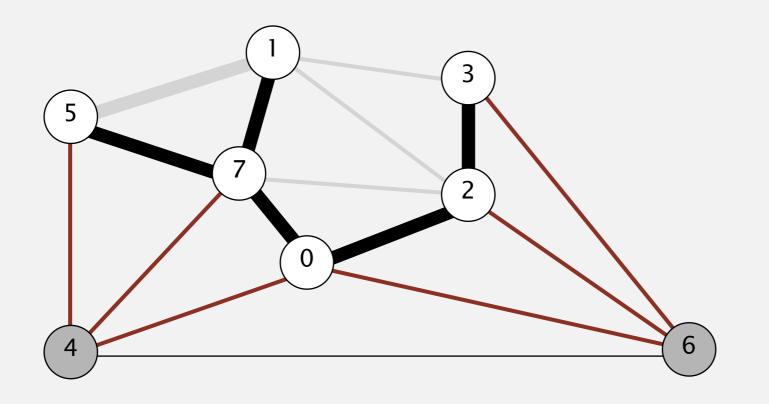
# edges on PQ (sorted by weight)

1	3	$\cap$	70
$\perp$	$\supset$	U.	4

$$6-0$$
 0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 1-5 and discard obsolete edge



edges on PQ (sorted by weight)

1-5 0.32

2-7 0.34

4-5 0.35

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

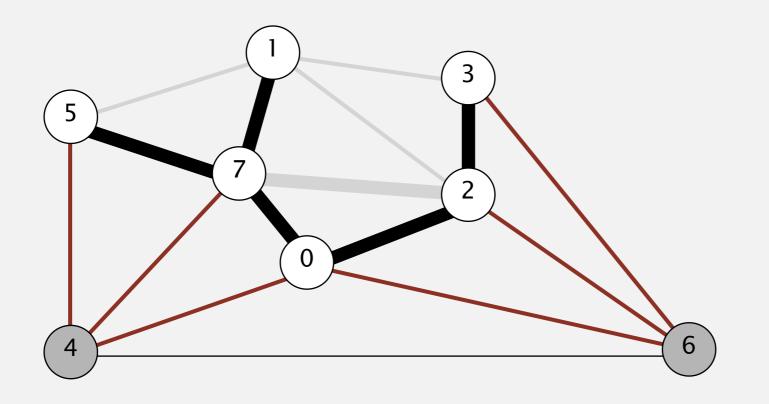
3-6 0.52

6-0 0.58

#### **MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 2-7 and discard obsolete edge



edges on PQ (sorted by weight)

2-7 0.34

4-5 0.35

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

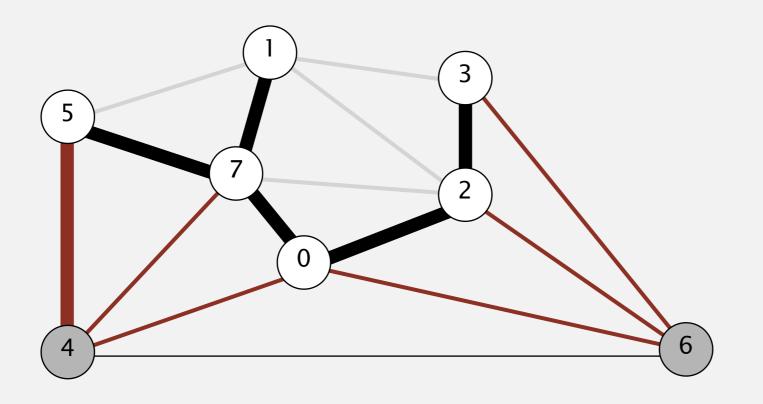
3-6 0.52

6-0 0.58

#### **MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 4-5 and add to MST



edges on PQ (sorted by weight)

4-5 0.35

1-2 0.36

4-7 0.37

0-4 0.38

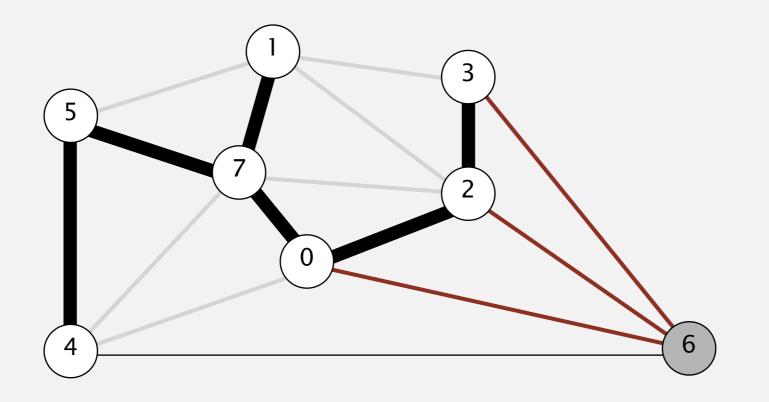
6-2 0.40

3-6 0.52

6-0 0.58

#### **MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

1-2 0.36 4-7 0.37

0-4 0.38

6-2 0.40

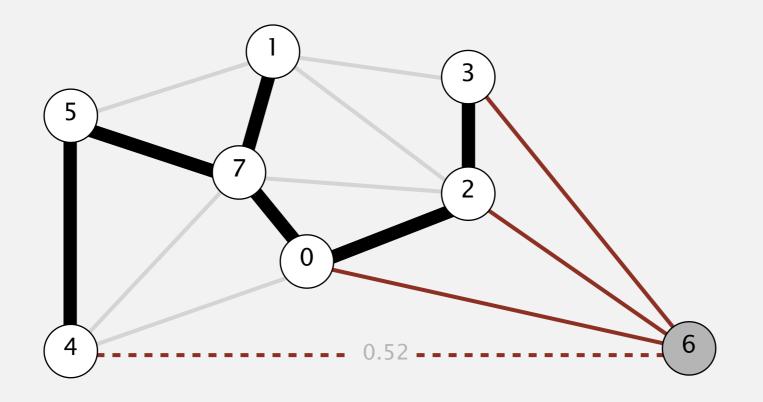
3-6 0.52

6-0 0.58

**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### add to PQ all edges incident to 4



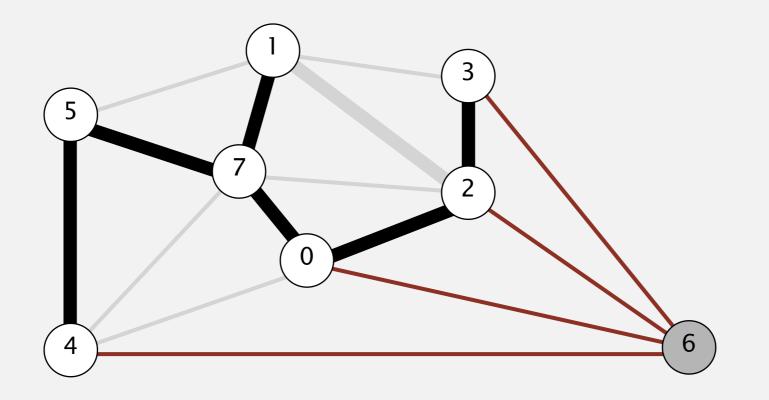
# edges on PQ (sorted by weight)

1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
<b>*</b> 6-4	0.93

**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 1-2 and discard obsolete edge



edges on PQ (sorted by weight)

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

3-6 0.52

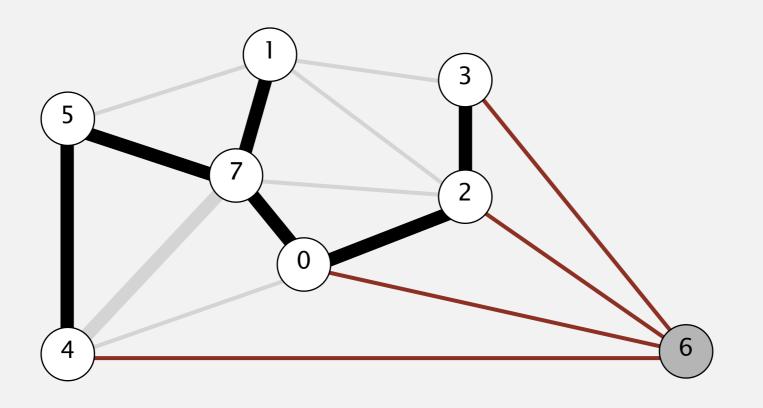
6-0 0.58

6-4 0.93

#### **MST** edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 4-7 and discard obsolete edge



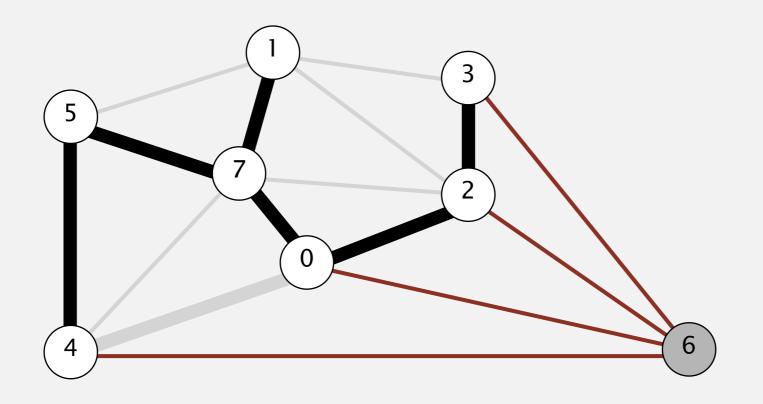
edges on PQ (sorted by weight)

4-7 0.37 0-4 0.38 6-2 0.40 3-6 0.52 6-0 0.58 6-4 0.93

**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 0-4 and discard obsolete edge



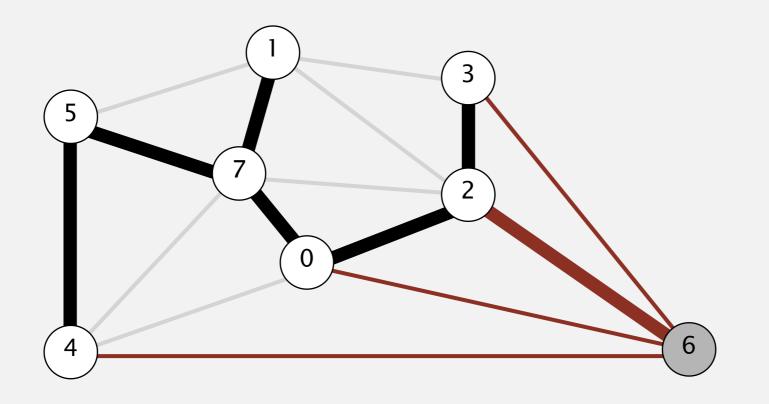
edges on PQ (sorted by weight)

0-4 0.38 6-2 0.40 3-6 0.52 6-0 0.58 6-4 0.93

**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 6-2 and add to MST



edges on PQ (sorted by weight)

6-2 0.40

3-6 0.52

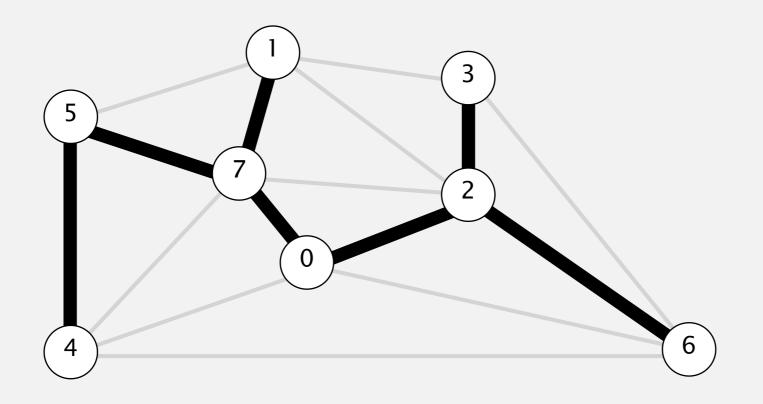
6-0 0.58

6-4 0.93

**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### delete 6-2 and add to MST



edges on PQ (sorted by weight)

3-6 0.52

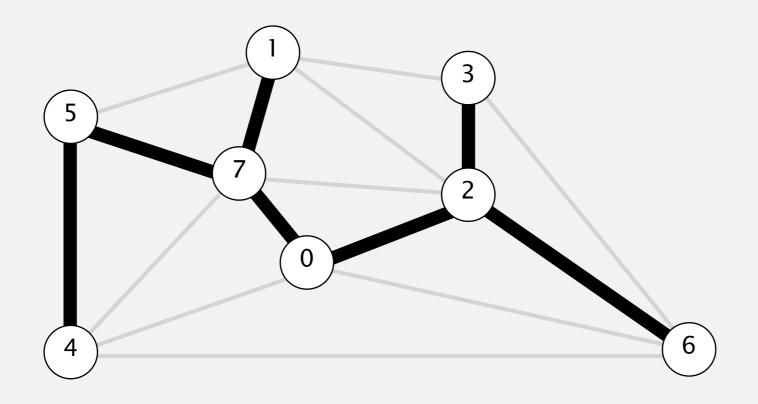
6-0 0.58

 $6-4 \quad 0.93$ 

**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

#### stop since V-1 edges



edges on PQ (sorted by weight)

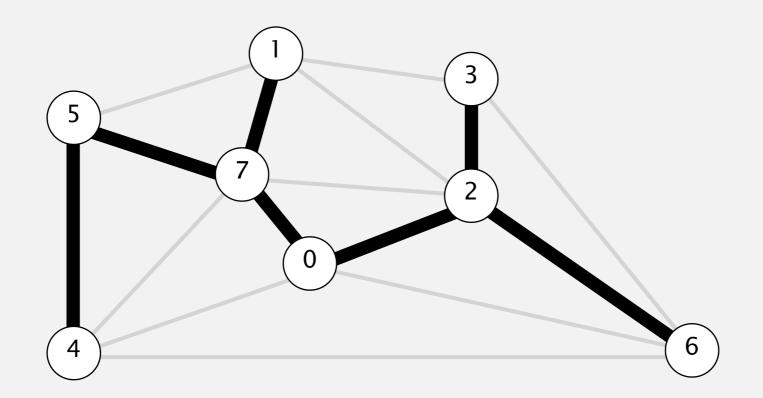
3-6 0.52

6-0 0.58

 $6-4 \quad 0.93$ 

#### **MST edges**

- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



**MST edges** 

# Algorithms

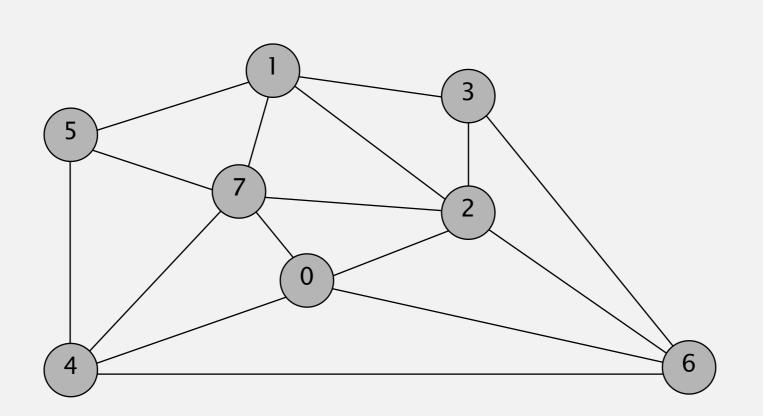
ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

# PRIM'S ALGORITHM DEMO

- Prim's algorithm
- lazy implementation
- eager implementation

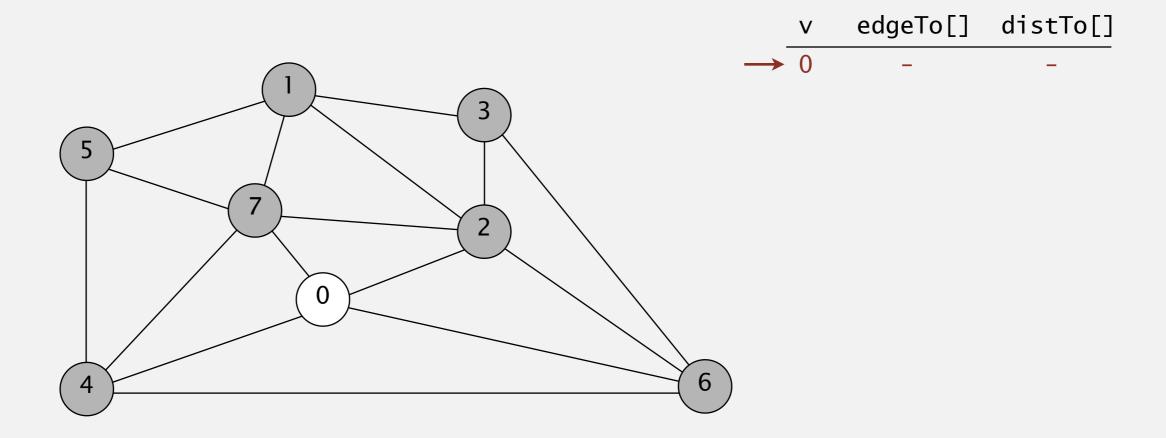
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



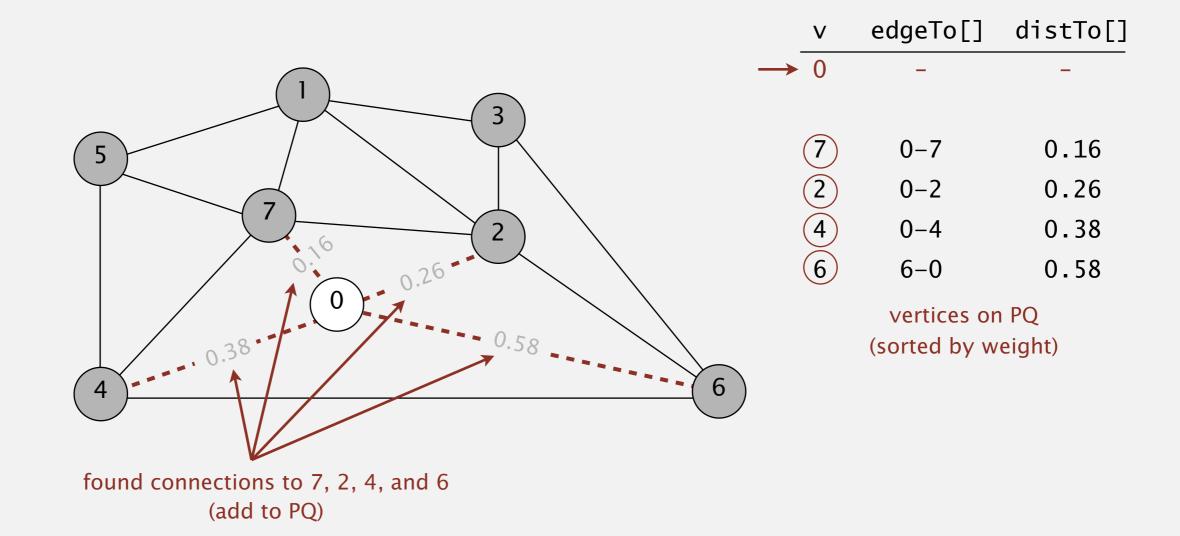
an edge-weighted graph

0-7 0.16 0.17 1-7 0.19 0-2 0.26 5-7 0.28 0.29 1-3 1-5 0.32 2-7 0.34 4-5 0.35 1-2 0.36 4-7 0.37 0.38 6-2 0.40 3-6 0.52 6-0 0.58  $6-4 \quad 0.93$ 

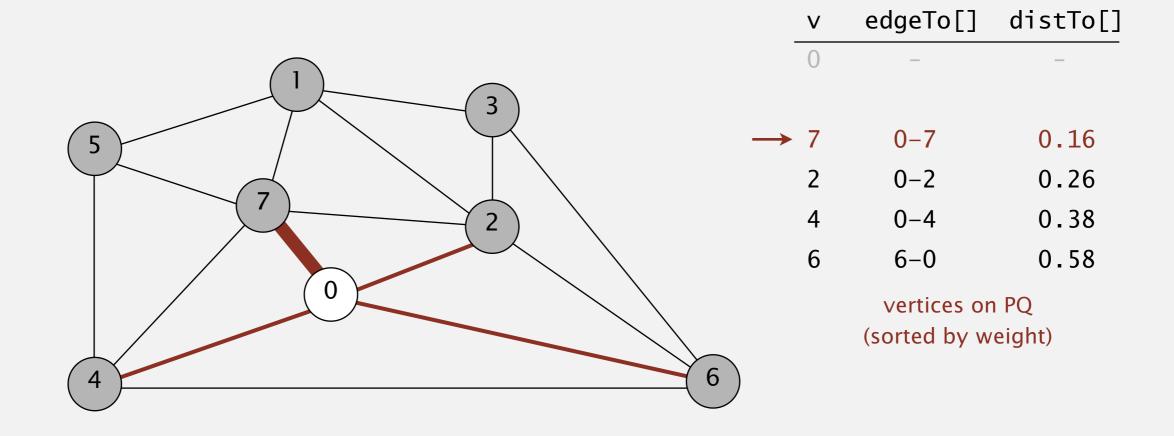
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



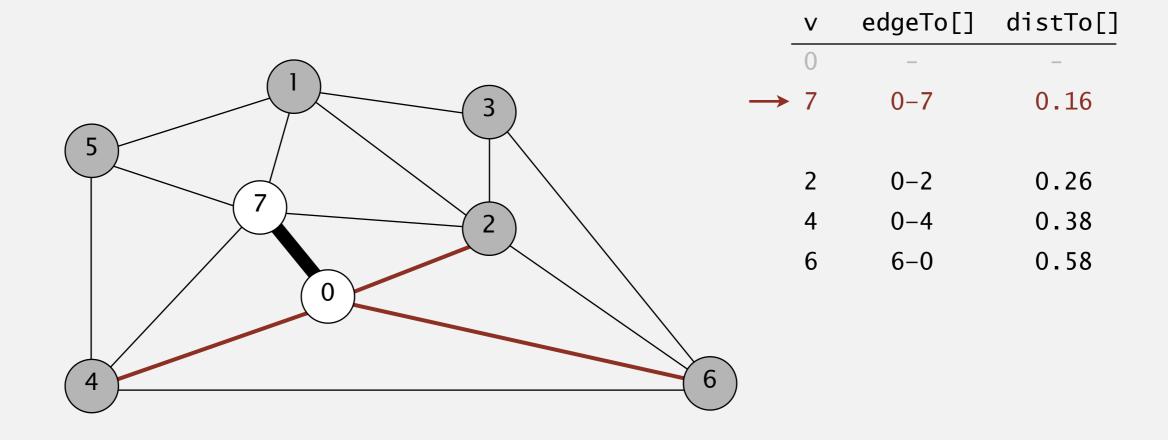
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
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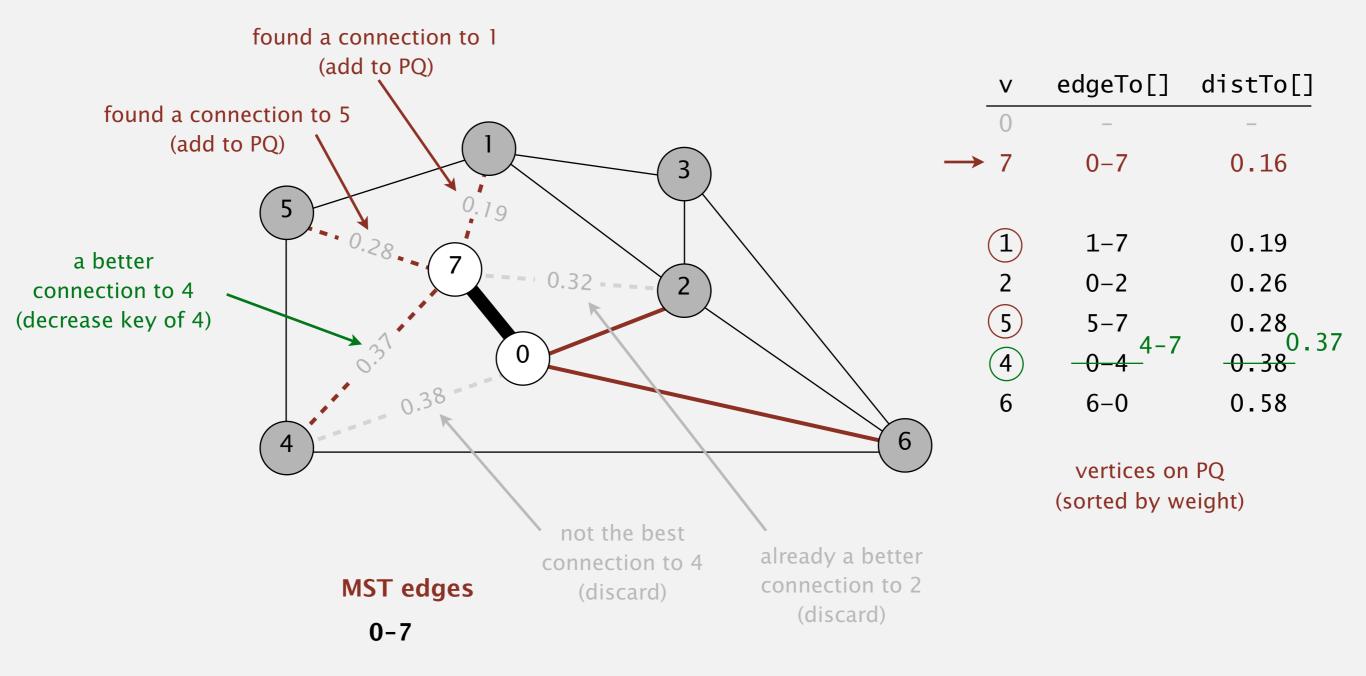
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



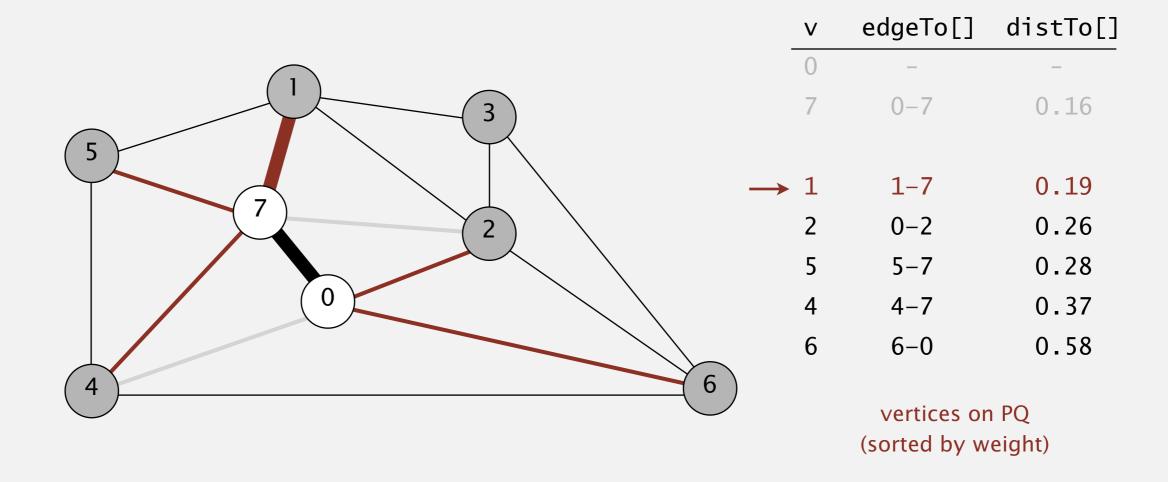
**MST edges** 

0-7

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



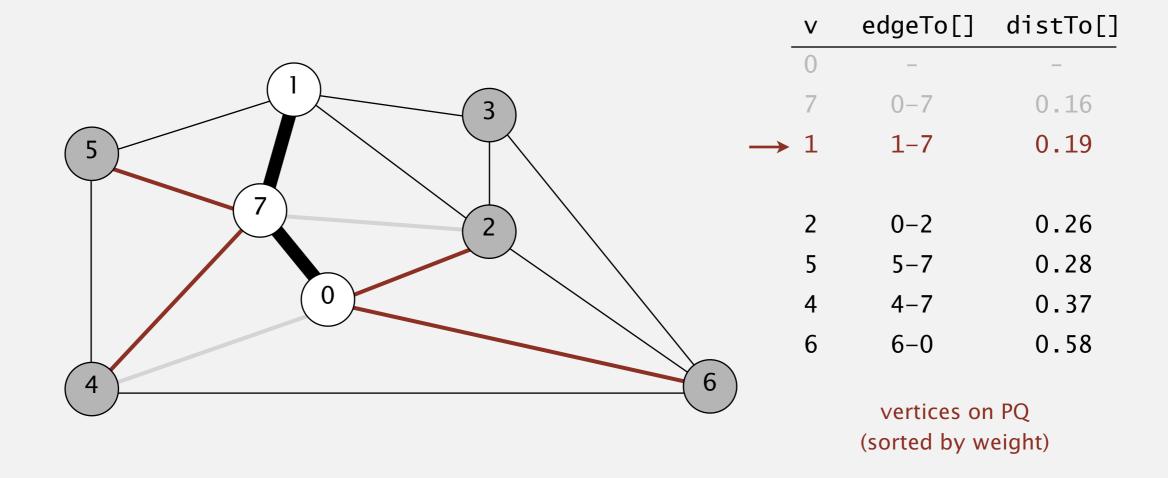
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST** edges

0-7 1-7

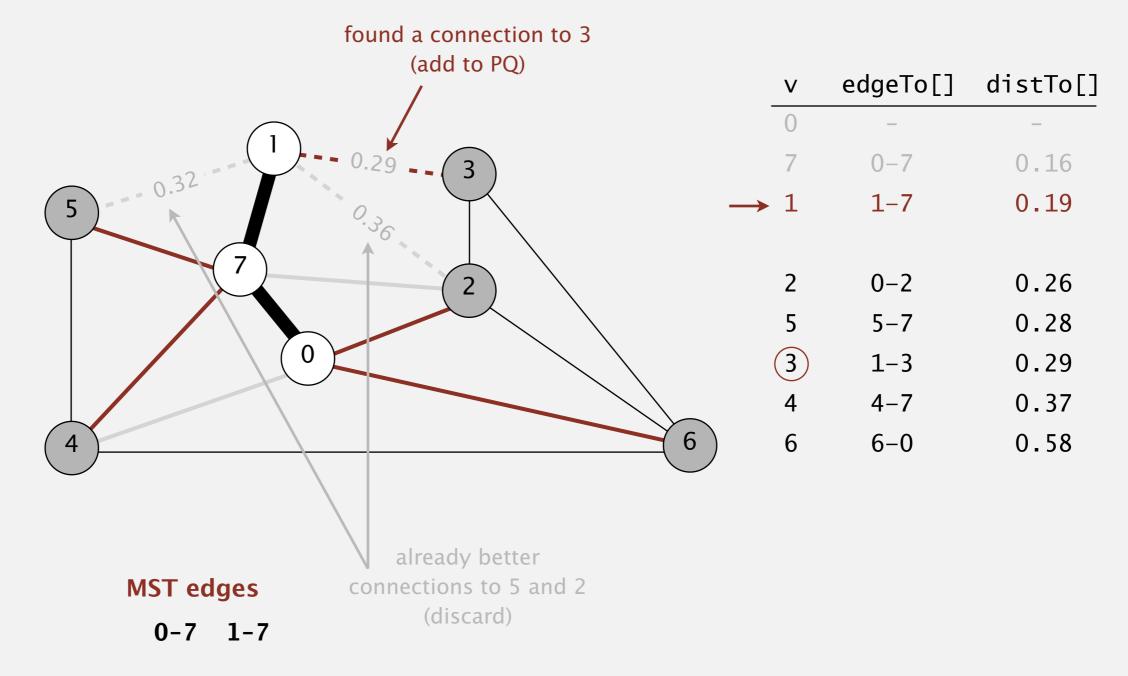
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



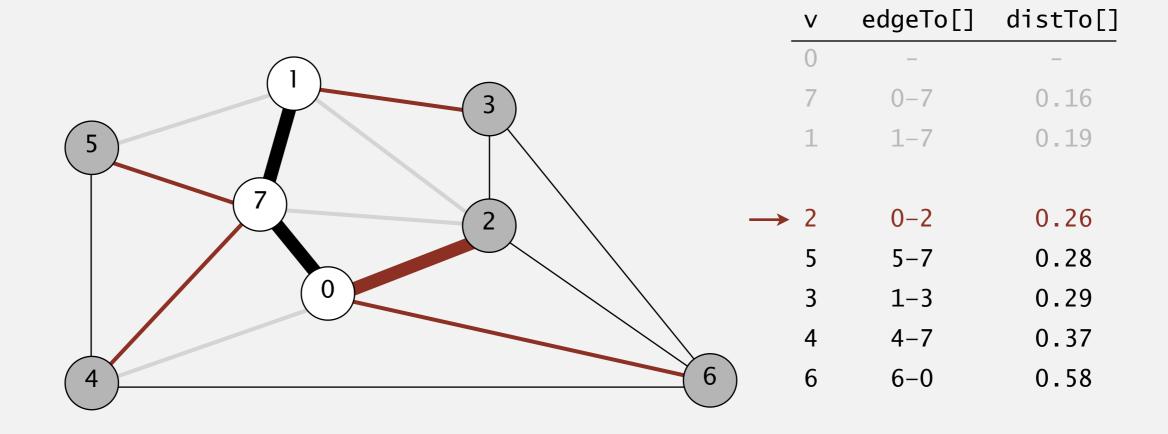
**MST** edges

0-7 1-7

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



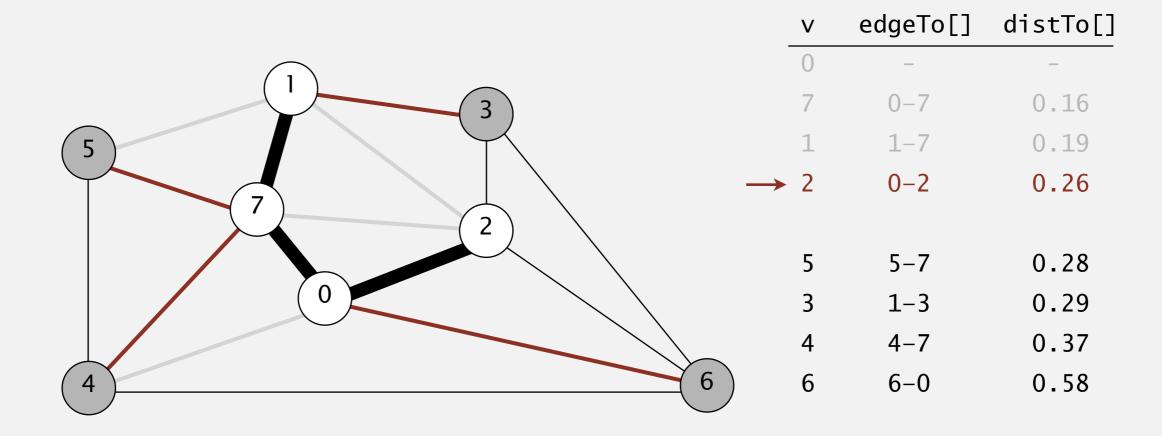
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST edges** 

0-7 1-7

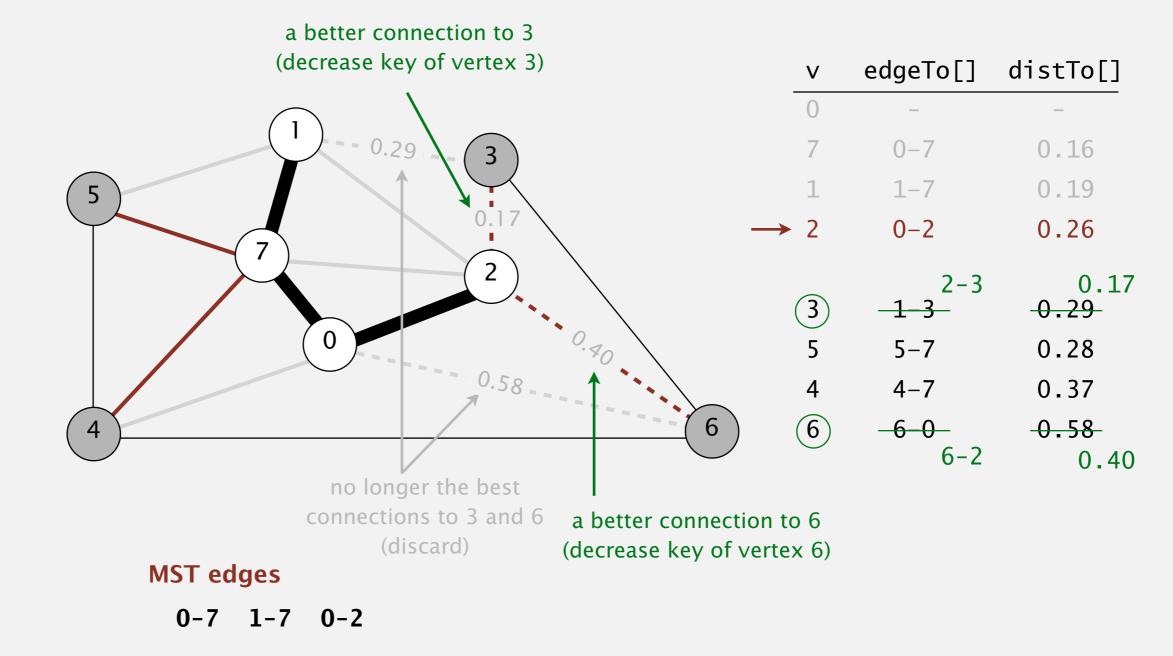
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



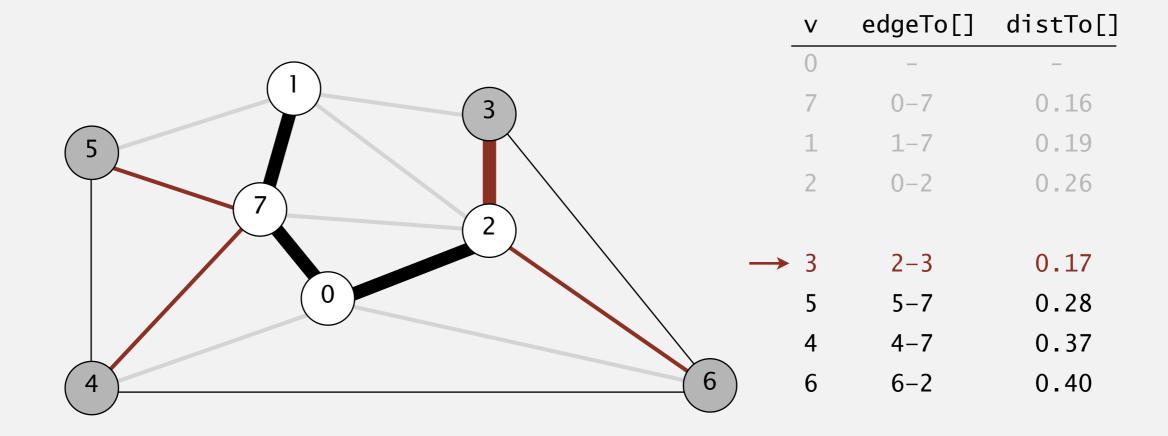
**MST edges** 

0-7 1-7 0-2

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



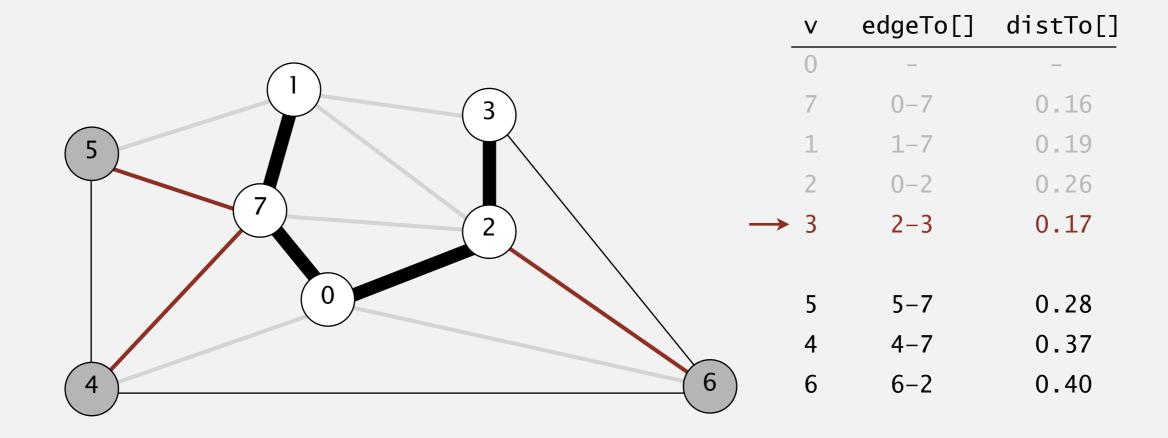
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



**MST edges** 

0-7 1-7 0-2 2-3

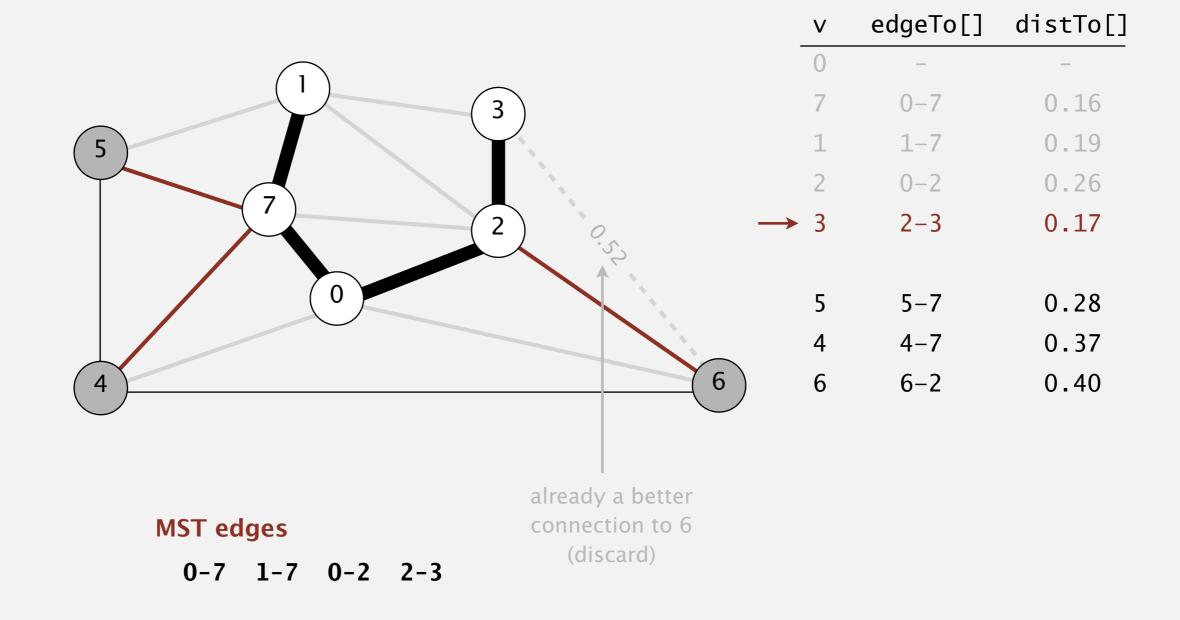
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



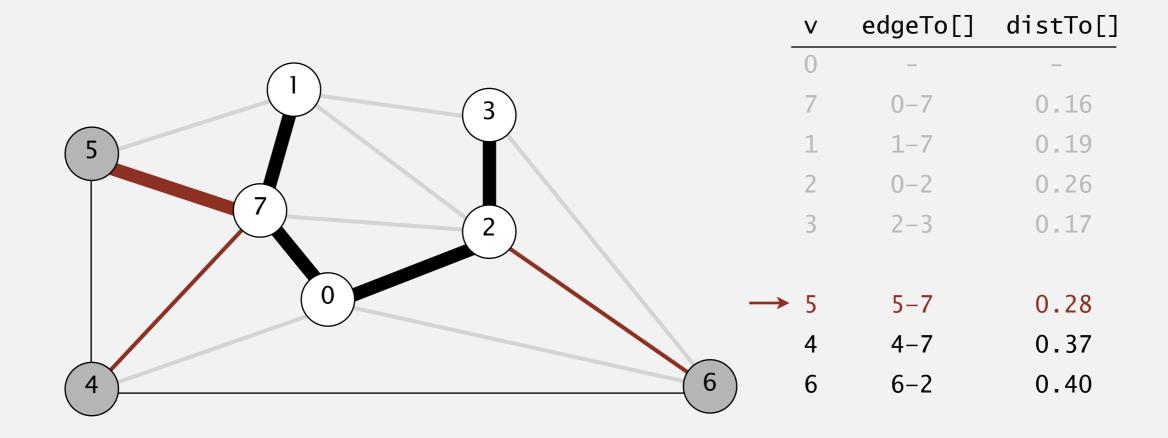
**MST edges** 

0-7 1-7 0-2 2-3

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
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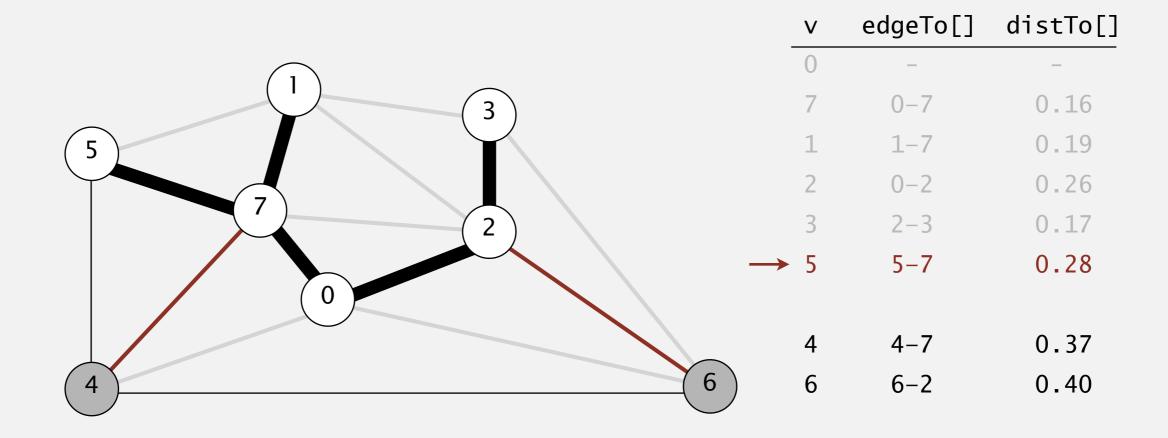
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



**MST edges** 

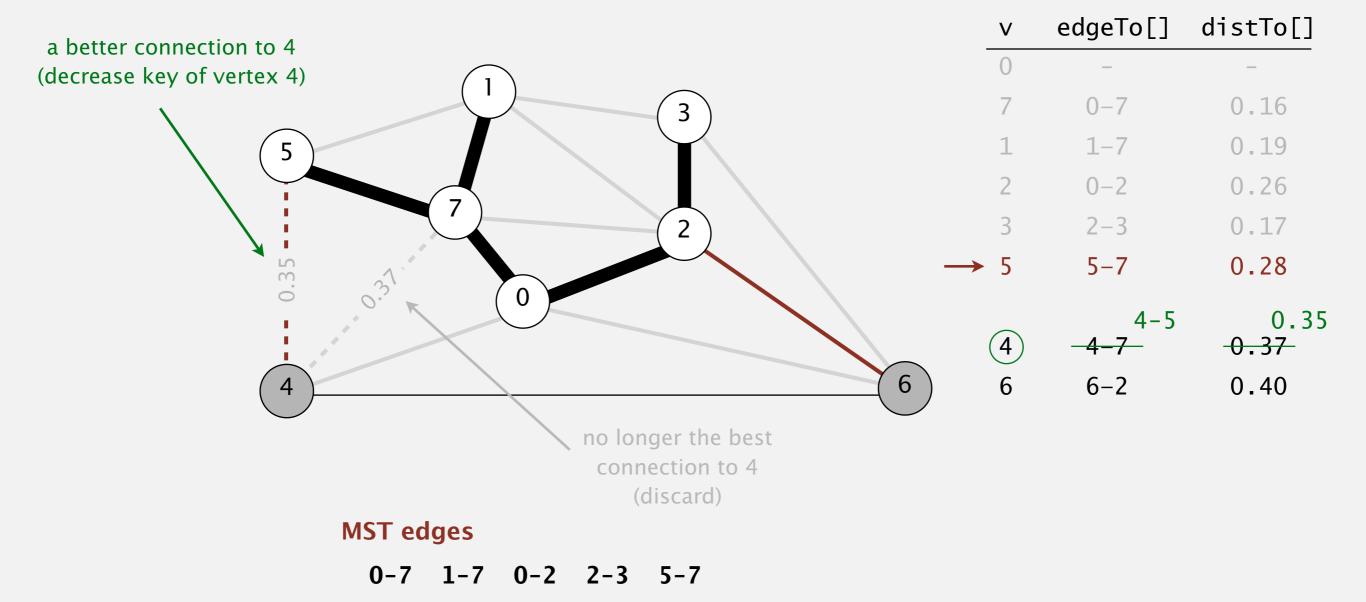
0-7 1-7 0-2 2-3

- Start with vertex 0 and greedily grow tree T.
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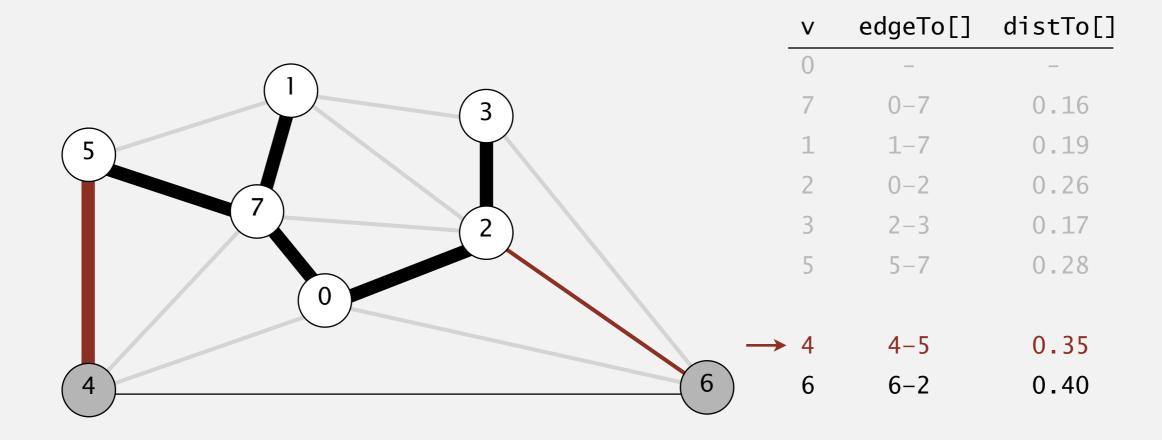


#### **MST edges**

- Start with vertex 0 and greedily grow tree T.
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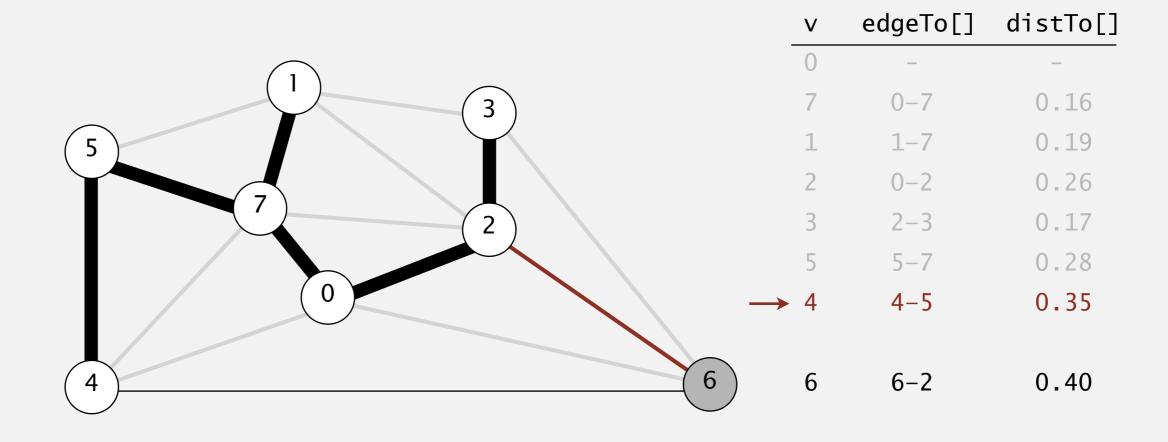


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#### **MST edges**

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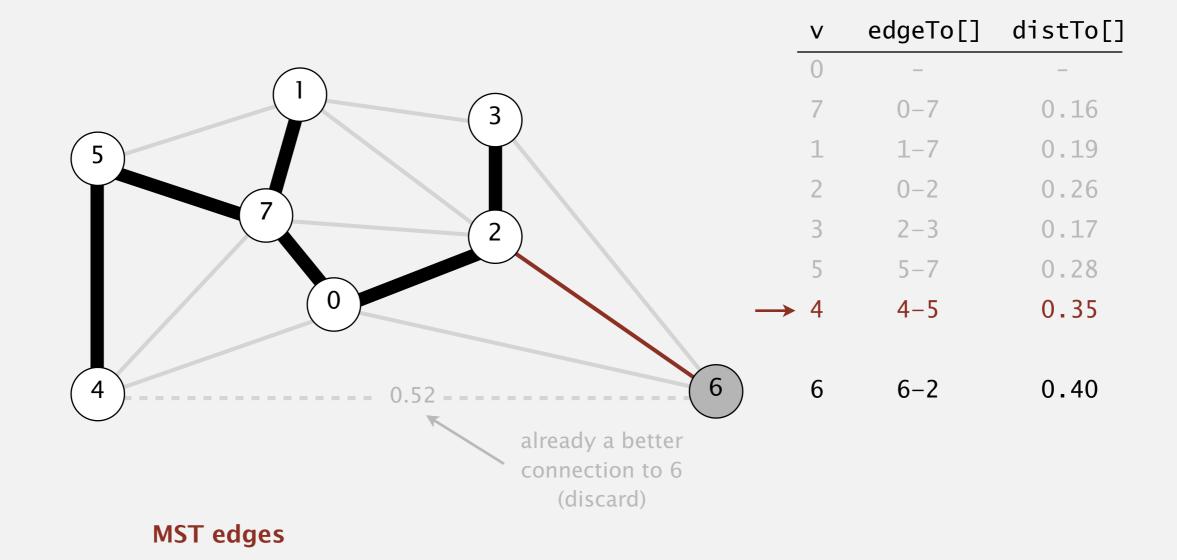


**MST edges** 

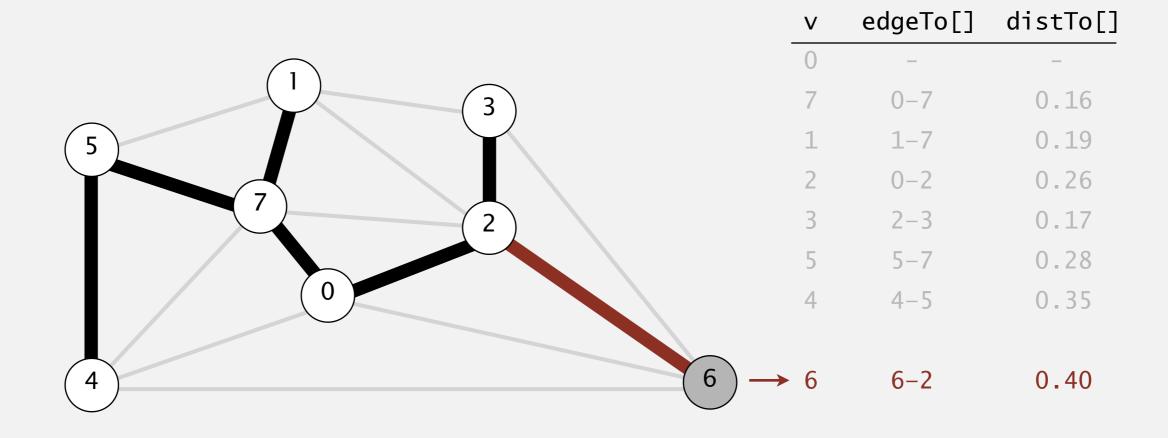
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.

0-7 1-7 0-2 2-3 5-7 4-5

• Repeat until V-1 edges.

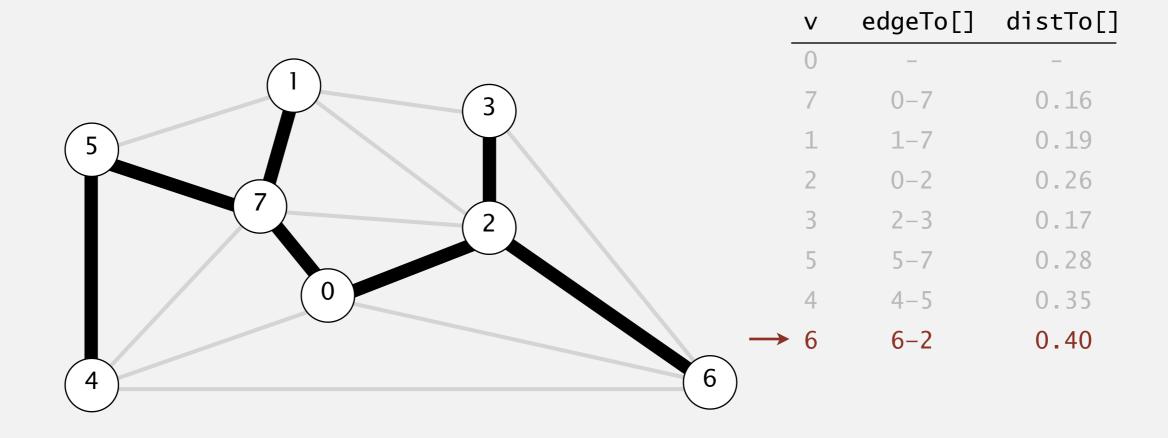


- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



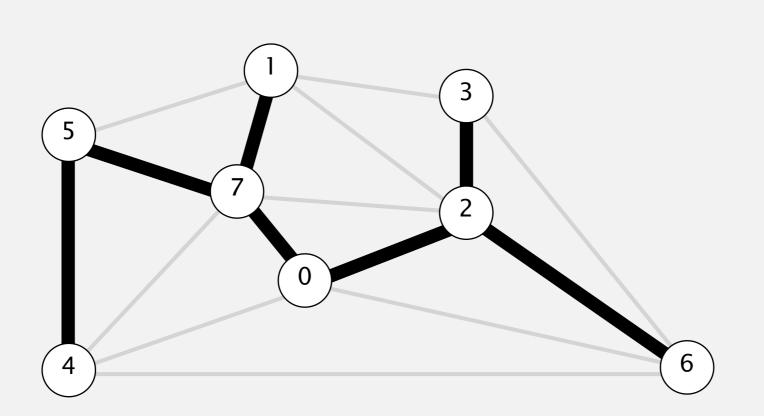
**MST edges** 

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- Repeat until V-1 edges.



**MST edges** 

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



V	edgeTo[]	distTo[]	
0	_	_	
7	0-7	0.16	
1	1-7	0.19	
2	0-2	0.26	
3	2–3	0.17	
5	5-7	0.28	
4	4-5	0.35	
6	6–2	0.40	

**MST edges**