Fourier Örnek ①

$A\cos(7t) = ?$

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \Rightarrow \frac{A}{2}e^{7jt} + \frac{A}{2}e^{-7jt}$$
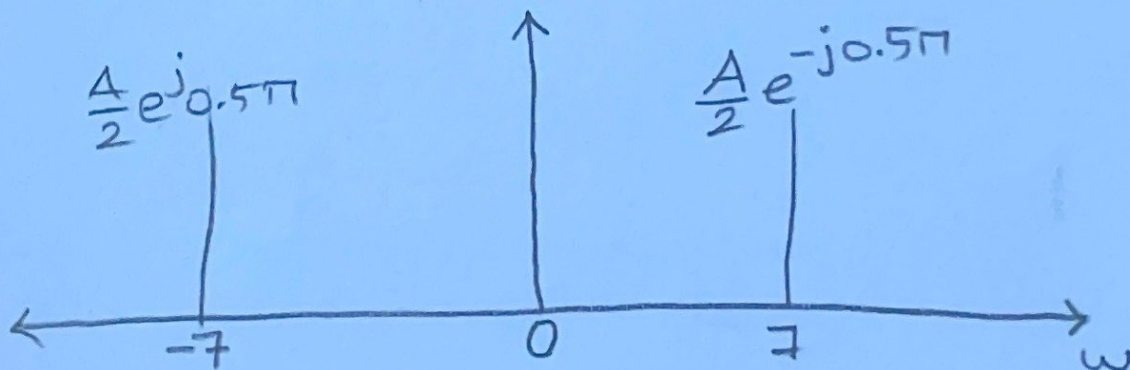
$A\sin(7t) = ?$

$$\frac{A}{2j}e^{7jt} - \frac{A}{2j}e^{-j7t}$$

$$\star \quad \cos(\omega t + \Phi) = \frac{e^{(j\omega t + j\Phi)} + e^{(-j\omega t + j\theta)}}{2}$$

$$\frac{A}{2}e^{-j0.5\pi}e^{j7t} + \frac{A}{2}e^{j0.5\pi}e^{-j7t}$$

pozitif frekans fazı -0.5π
negatif " " 0.5π

$\frac{A}{2}e^{j0.5\pi}$

$\frac{A}{2}e^{-j0.5\pi}$

-7   0   7 ω

Fourier Transform örnek ②

$$y'(t) + 2y(t) = x(t) + x'(t) \quad \text{freq. response}?$$

$$h(\omega) = \frac{y(\omega)}{x(\omega)}$$

$$x(t) \overset{F}{\longleftrightarrow} x(\omega)$$
$$x^n(t) \longrightarrow x(\omega)(j\omega)^n$$

$$(j\omega)y(\omega) + 2y(\omega) = x(\omega) + (j\omega)x(\omega)$$

$$\frac{y(\omega)}{x(\omega)} = \frac{j\omega + 1}{j\omega + 2} = H(\omega) = 1 - \frac{1}{2 + j\omega}$$

$$F^{-1}\{1\} - F^{-1}\left\{\frac{1}{2+j\omega}\right\} = h(t) = F^{-1}\{H(\omega)\} = F^{-1}\left\{1 - \frac{1}{2+j\omega}\right\}$$

$$\delta(t) - e^{-2t}u(t) = h(t)$$

$$y(t) = h(t) * x(t)$$

Fourier Örnek ③

$x(t) = \cos(t) + 0.5\cos(4t + \pi/3) + 0.25\cos(8t + \pi/2)$

Amp. formda ve $\omega_0 = 1$ iken exp. ve sine forma dönüş-türünüz.

$A_1 = 1$	$A_4 = 0.5$	$A_8 = 0.25$	diğer A_k'ler 0
$\Phi_1 = 0$	$\Phi_4 = \pi/3$	$\Phi_8 = \pi/2$	

$C_0 = A_0$

$C_k = \frac{1}{2} A_k e^{j\Phi_k}$

$\underline{C_{-k} = \frac{1}{2} A_k e^{-j\Phi_k}}$

$C_1 = \frac{1}{2} \cdot 1 \cdot e^0 = 0.5 \qquad C_4 = \frac{1}{2} \cdot \frac{1}{2} \cdot e^{j\pi/3}$

$C_{-1} = 0.5 \qquad\qquad\qquad C_{-4} = 0.25 e^{-j\pi/3}$

$C_8 = \frac{1}{4} \cdot \frac{1}{2} \cdot e^{j\pi/2} = 0.125 e^{j\pi/2} \quad C_{-8} = 0.125 e^{-j\pi/2}$

$x(t) = \left[0.5 e^{jt} + 0.5 e^{-jt}\right] + \left[0.25 e^{j\pi/3} e^{j4t} + 0.25 e^{-j\pi/3} e^{-j4t}\right] +$

↓ exp. form

$\left[0.125 e^{j\pi/2} e^{j8t} + 0.125 e^{-j\pi/2} e^{-j8t}\right]$

$a_0 = C_0$

$a_k = A_k \cos(\Phi_k)$

$\underline{b_k = -A_k \sin(\Phi_k)}$

$a_1 = 1 \cdot \cos(0) = 1 \qquad a_4 = 0.5 \cos(\pi/3) = 0.25$

$b_1 = -1 \cdot \sin(0) = 0 \qquad -b_4 = 0.5 \sin(\pi/4) = 0.43$

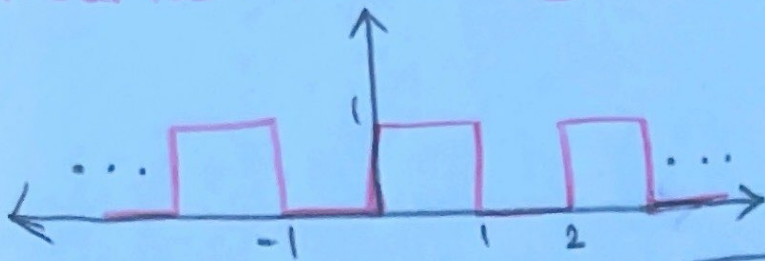$a_8 = 0.25 \cos(\pi/2) = 0 \quad b_8 = 0.25 \cdot -\sin(\pi/2)$

$\qquad\qquad\qquad\qquad\qquad = -0.25$

$x(t) = \cos(t) + \left[0.25\cos(4t) - 0.43\sin(4t)\right] + \left[-0.25\sin(8t)\right]$

diğer a_k, b_k'ler 0

↓

sine-cosine form

Fourier örnek ④

FS coefficient bul.

sin-cosine form

① $T = 2$

$\omega_0 = 2\pi/2 = \pi$

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\sin(\omega_0 t) + \frac{2}{3\pi}\sin(3\omega_0 t)\ldots$$

② $c_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}\,dt$

t_0 ⟵ ($t_0 = 0$ al.)

$$x(t) = \frac{1}{2} + \cdots + \frac{-j}{-3\pi}e^{-j2\omega_0 t} + \frac{-j}{-\pi}e^{-j1\omega_0 t} + \frac{1}{2}t$$

exp. form

$$\frac{-j}{\pi}e^{j1\omega_0 t} + \frac{-j}{3\pi}e^{j3\omega_0 t} + \cdots$$

③ $c_k = \frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}\,dt = \frac{1}{2}\left[\int_0^1 1e^{-jk\pi t}\,dt + \int_1^2 0e^{-jk\pi t}\,dt\right]$

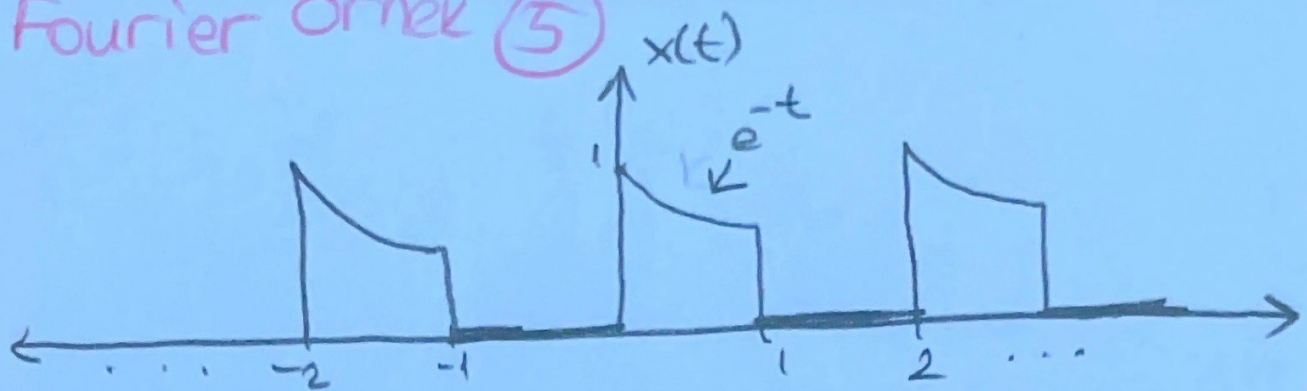$= \frac{1}{2}\int_0^1 e^{-jk\pi t}\,dt = \frac{1}{2}\left[\boxed{\frac{1}{-jk\pi}}\,e^{-jk\pi t}\right]_0^1$ $\boxed{-\frac{1}{j} = j}$

$= \frac{j}{2k\pi}\left[e^{-jk\pi} - 1\right]$ } $e^{-jk\pi} = \cos k\pi - j\sin k\pi$

k'nın tek değerleri	k'nın çift değerleri
$e^{-jk\pi} = -1$	$e^{-jk\pi} = 1$

$c_k = \begin{cases} 0, & k \text{ even}, \neq 0 \\ \frac{-j}{k\pi}, & k \text{ odd} \\ \frac{1}{2}, & k = 0 \end{cases}$

$c_0 = \frac{1}{2}\int_0^1 1\cdot e^{-j0\pi t} = \frac{1}{2}$

$a_0 = c_0$

$\rightarrow a_k = 2\text{Re}\{c_k\}; \quad k = 1,2,\ldots$

$b_k = -2\,\text{Im}\,"$ " "

$a_k = \begin{cases} \frac{1}{2}, & k = 0 \\ 0, & k \neq 0 \end{cases}$ $b_k = \begin{cases} 0, & k \text{ even} \\ \frac{2}{k\pi}, & k \text{ odd} \end{cases}$

Fourier Örnek ⑤

$x(t)$, with e^{-t} labeled

① $T = 2$ $\qquad$ $\omega_0 = \frac{2\pi}{T} = \pi$

② $c_k = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-jk\omega_0 t}\, dt$ $\qquad$ ($t_0 = 0$)

$$e^{-jk\pi} = \left\{ \begin{array}{l} 1,\ \text{even } k \\ -1,\ \text{odd } k \end{array} \right\}$$

$$\underbrace{\phantom{e^{-jk\pi}}}_{(-1)^k}$$

$$= \frac{1}{2} \int_0^1 e^{-t} e^{-jk\pi t}\, dt + \int_1^2 0\, dt$$

$$= \frac{1}{2} \int_0^1 e^{-t(1+jk\pi)}\, dt = \frac{1}{2}\left[\frac{e^{-t(1+jk\pi)}}{1+jk\pi} \cdot -1 \right]_0^1 = \frac{-1}{2(1+jk\pi)}\left(e^{-(1+jk\pi)} - 1 \right)$$
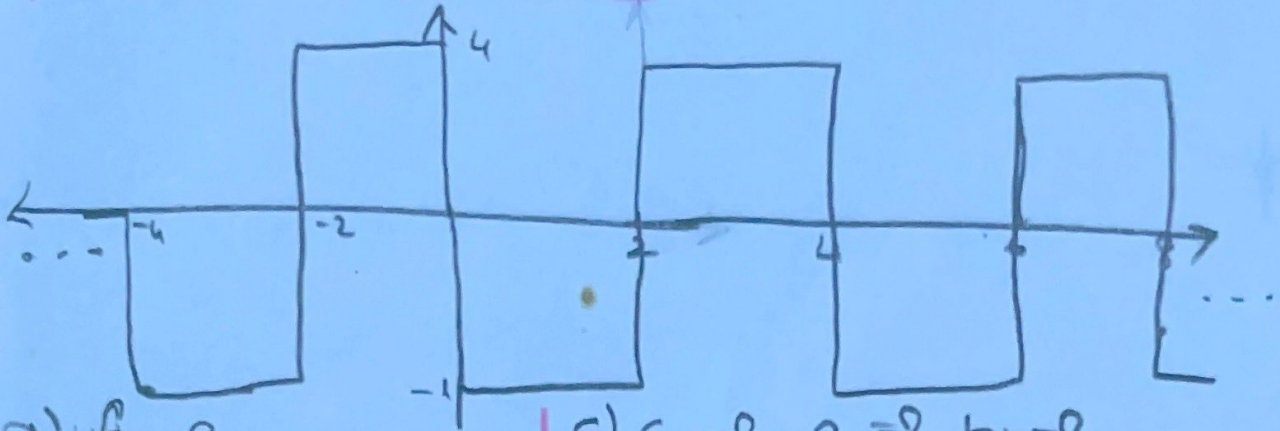
$$= \frac{1 - e^{-1} e^{-jk\pi}}{2(1+jk\pi)} = \boxed{\frac{1 - e^{-1}(-1)^k}{2(1+jk\pi)} = c_k}$$

$$c_1 = \frac{1 + e^{-1}}{2(1+j\pi)} \qquad c_2 = \frac{1 - e^{-1}}{2(1+2j\pi)} \qquad \ldots$$

Fourier Örnek ⑥

a) $f_0 = ?$

$T_0 = 4sn \quad f_0 = 1/4 = 0.25 Hz$

b) DC bileşen = ?

$a_0 = C_0 = \dfrac{1}{T_0} \displaystyle\int_0^{T_0} x(t)\, dt$

$= \dfrac{1}{4}\left[\displaystyle\int_0^2 -1\,dt + \int_2^4 4\,dt\right]$

$= \dfrac{1}{4}\left[-t\right]_0^2 + \dfrac{1}{4}\left[4t\right]_2^4$

$= -1/2 + 4 - 2 = 1.5$

d)
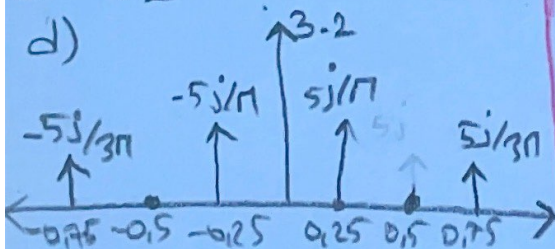
c) $C_k = ? \ , \ a_k = ? \ , \ b_k = ?$

$C_k = \dfrac{1}{4}\left[\displaystyle\int_0^2 -1 e^{-jk\frac{\pi}{2}t}\,dt + \int_2^4 4 e^{-jk\frac{\pi}{2}t}\,dt\right]$

$\dfrac{1}{4}\left[\dfrac{+1 \cdot e^{-jk\pi/2\,t}}{+jk\pi/2}\right]_0^2 + \dfrac{1}{4}\left[\dfrac{4 e^{-jk\pi/2\,t}}{-jk\pi/2}\right]_2^4$

$= \dfrac{-j}{2k\pi}\left[5e^{-jk\pi} - 4e^{-jk2\pi} - 1\right]$

$= \dfrac{j(1 - 5(-1)^k + 4(-1)^{2k})}{2k\pi} = \underbrace{\dfrac{5j\left[1 - (-1)^k\right]}{2k\pi}}_{C_k}$

$a_k = 2Re\{C_k\} = 0$
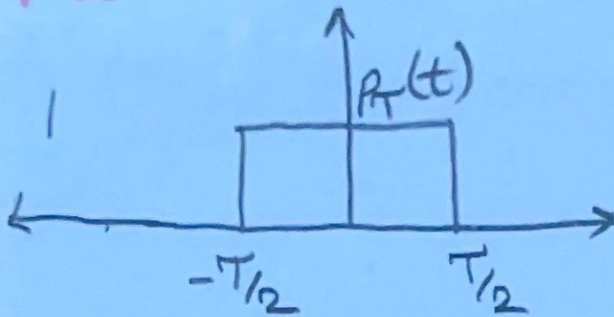
$b_k = -2 Im\{C_k\} = \dfrac{-5}{\pi k}\left(1 - (-1)^k\right)$

$C_1 = \dfrac{5j}{2\pi} \cdot 2 = \dfrac{5j}{\pi} \qquad C_2 = 0 \quad C_{-2} = 0$

$C_{-1} = \dfrac{-5j}{\pi} \qquad C_3 = \dfrac{5j}{3\pi} \qquad C_{-3} = \dfrac{-5j}{3\pi}$
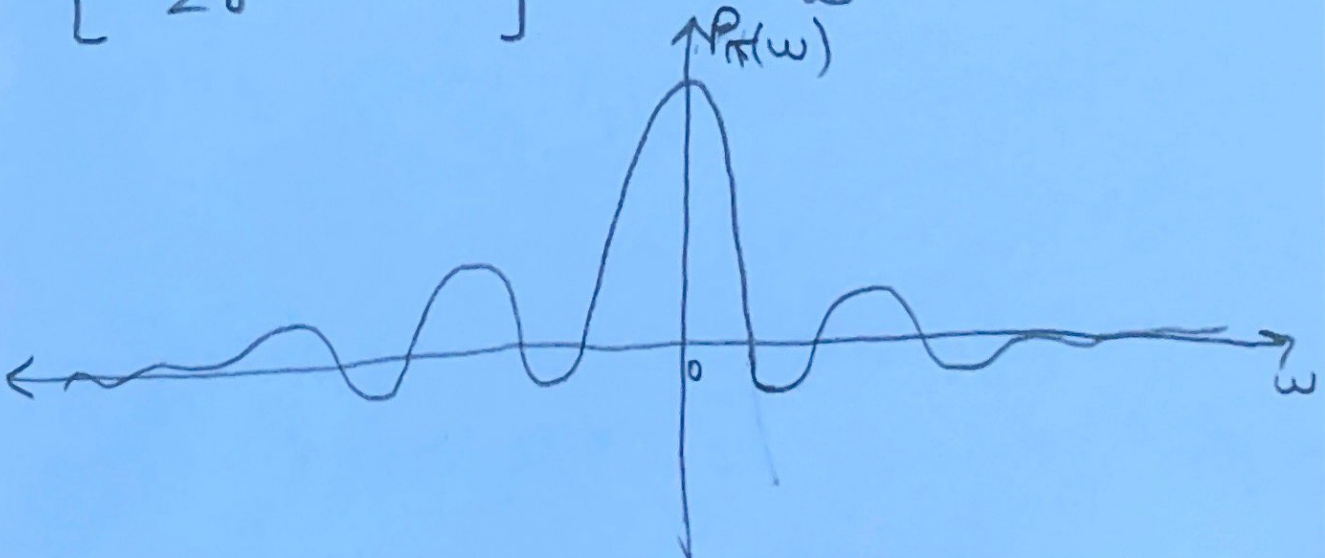
Fourier Transform örnek ①

find $P_T(\omega)$

①
$$P_T(t) = \begin{cases} 1 & -T/2 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

②
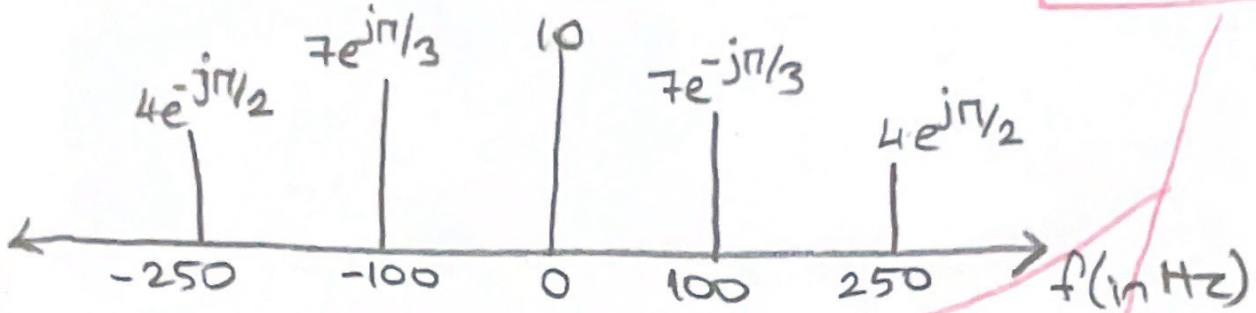$$P_T(\omega) = \int_{-\infty}^{\infty} P_t(t) e^{-j\omega t} dt = \int_{-T/2}^{T/2} 1 e^{-j\omega t} dt$$

$$= \left[\frac{-1}{j\omega} e^{-j\omega t} \right]_{-T/2}^{T/2} = \frac{-1}{j\omega} \left[e^{-j\omega T/2} - e^{j\omega T/2} \right]$$

$$= \left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j} \right] \frac{2}{\omega} = \frac{2\sin(\omega T/2)}{\omega} = P_T(\omega)$$

Fourier Örnek ②

$\omega = 2\pi f t$

$4e^{-j\pi/2}$ $7e^{j\pi/3}$ 10 $7e^{-j\pi/3}$ $4e^{j\pi/2}$

-250 -100 0 100 250 f (in Hz)

$$X(t) = 10 + 7 \cdot e^{-j\pi/3} \cdot e^{j2\pi(100)t} + 7 \cdot e^{j\pi/3} \cdot e^{-j2\pi(100)t} +$$

$$4 \cdot e^{j\pi/2} \cdot e^{j2\pi(250)t} + 4e^{-j\pi/2} \cdot e^{-j2\pi(250)t}$$

aşağıda ───── tersine euler dönüşümü yaparsak

$$X(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$

frekans	faz	genlik
-250	$-\pi/2$	8
-100	$\pi/3$	14
0	0	10
100	$-\pi/3$	14
250	$\pi/2$	8

önce tablo çizilir.

Pratik ters euler
① $7 \to 14$ $4 \to 8$
② frekansı poz. olanın fazı alınır. cos'un içine yazılır.
③ frek. poz. olan alınır.

$$X(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \Phi_k)$$

$A_0 = 10$
$A_1 = 14$ $\Phi_1 = -\pi/3$
$A_2 = 8$ $\Phi_2 = \pi/2$
$f_0 = 50$
$f_1 = 100$
$f_2 = 250$

Fourier Transform örnek ③

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \qquad y(t) = ?$$

a) $x(t) = e^{-t}u(t)$ \qquad b) $x(t) = u(t)$

$$(jw)y(w) + 2y(w) = x(w)$$

$$\frac{y(w)}{x(w)} = \frac{1}{2+jw} = h(w) \Rightarrow h(t) = e^{-2t}u(t)$$

a) $x(w) = \frac{1}{1+jw}$ \qquad $y(w) = h(w)x(w) = \frac{1}{1+jw} \cdot \frac{1}{2+jw}$

$$y(w) = \frac{1}{(1+jw)(2+jw)} = \frac{1}{1+jw} - \frac{1}{2+jw} \Big\} \begin{array}{l}\text{ters} \\ \text{fourier}\end{array}$$

$$y(t) = e^{-t}u(t) - e^{-2t}u(t) = u(t) \cdot (e^{-t} - e^{-2t})$$

b) $x(w) = \frac{u(t)}{jw}$

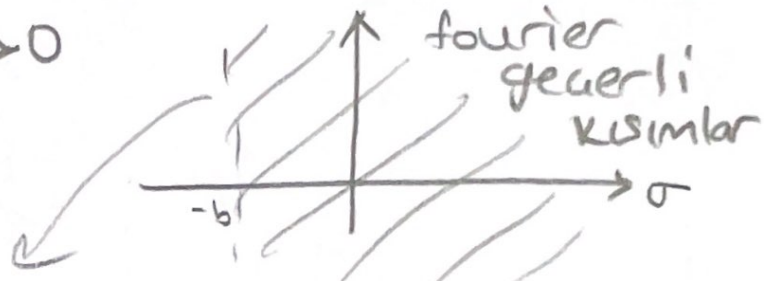$$\frac{1}{jw} \cdot \frac{1}{2+jw} \Rightarrow \frac{1}{jw} \quad \sim \quad \frac{1}{2+jw}$$

Laplace örnek ①

$x(t) = e^{-bt} u(t)$.

$\boxed{s+b>0}$ $\boxed{b>0}$

$$X(s) = \int_0^\infty e^{-bt} e^{-st} dt = \int_0^\infty e^{-(s+b)t} dt = -e^{\dfrac{-(s+b)t}{(s+b)}}\bigg|_0^\infty$$

$$= \frac{1}{s+b} \left.\begin{array}{l} Re\{s\}+b>0 \\ \sigma + b > 0 \\ \sigma > -b \end{array}\right.$$

fourier geçerli kısımlar

* $b>0$ iken jw de bu alanda ıyani hem LT hem FT var.

* $b<0$ "   " bu alanda değil, yani sadece LT var.

Laplace örnek ②

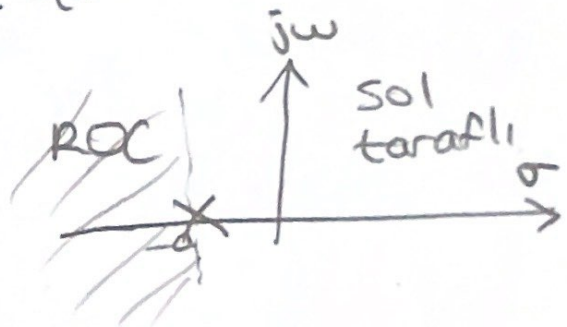$x(t) = e^{-at} \cdot u(t)$ $\qquad$ $x_2(t) = -e^{-at} u(-t)$

$$X(s) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-st} dt = \int_{0}^{\infty} e^{-(a+s)t} dt = \frac{e^{-(s+a)t}}{-(s+a)} \Big|_{0}^{\infty}$$

$$X(s) = \frac{1}{s+a} \quad (s+a > 0 \text{ ise}), (\sigma > -a)$$

$$X(s) = \int_{-\infty}^{\infty} -e^{-at} u(t) e^{-st} d(t) = \int_{-\infty}^{0} -e^{-(a+s)t} dt$$
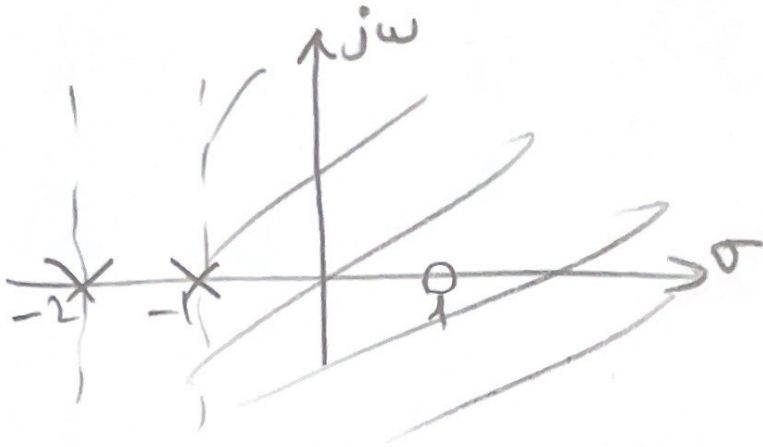
$$= +\left(\frac{e^{-(s+a)t}}{+(s+a)} \right) \Big|_{-\infty}^{0} = \frac{1}{s+a} \quad (s+a < 0 \text{ ise}), (\sigma < -a)$$

Laplace örnek ③

$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$

$X(s) = \dfrac{3}{s+2} - \dfrac{2}{s+1}$ $\qquad \begin{cases} Re\{s\} > -2, \sigma > -2 \\ Re\{s\} > -1, \sigma > -1 \end{cases}$

$X(s) = \dfrac{s-1}{(s+2)(s+1)}$ $\qquad \begin{array}{l} s-1=0 \\ s=1 \rightarrow \text{sistemin zero'su} \end{array}$