① $\int \frac{dx}{x\,lnx(ln^4x-1)}$ integralinin basit kesirlere ayrılmış hali aşağıdakilerden hangisi olabilir?

A) $\int \left(\frac{A}{u} + \frac{B}{(u-1)} + \frac{C}{(u+1)} + \frac{Du+E}{(u^2+1)}\right) du$

B) $\int \left(\frac{A}{u} + \frac{Bu+C}{(u^2-1)} + \frac{D}{(u+1)} + \frac{Eu+F}{(u^2+1)}\right) du$

C) $\int \left(\frac{Au+B}{(u^2-1)} + \frac{Cu+D}{(u^2+1)}\right) du$

D) $\int \left(\frac{A}{(u+1)^2} + \frac{B}{(u+1)} + \frac{C}{(u^2+1)}\right) du$

E) $\int \left(\frac{A}{u} + \frac{B}{(u-1)} + \frac{C}{(u+1)} + \frac{D}{(u^2+1)}\right) du$

Önce $u = lnx$ $du = \frac{dx}{x}$ dönüşümü yapalım.

$$\int \frac{dx}{x\,lnx\cdot(ln^4x-1)} = \int \frac{du}{u\cdot(u^4-1)}$$

$$\frac{1}{u\cdot(u^4-1)} = \frac{A}{u} + \frac{B}{u-1} + \frac{C}{u+1} + \frac{Du+E}{u^2+1}$$

$u\cdot(u-1)(u+1)(u^2+1)$

4 çarpan ⟹ 4 Basit Kesir olmalı

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② $\int \frac{cosx\,dx}{(sin^6x-sin^2x)}$ integralinin basit kesirlere ayrılmış hali aşağıdakilerden hangisi olabilir?

A) $\int \left(\frac{A}{t} + \frac{B}{t^2} + \frac{C}{(t-1)} + \frac{D}{(t+1)} + \frac{Et+F}{(t^2+1)}\right) dt$

B) $\int \left(\frac{A}{t^2} + \frac{B}{(t-1)} + \frac{C}{(t+1)} + \frac{Dt+E}{(t^2+1)}\right) dt$

C) $\int \left(\frac{A}{t} + \frac{B}{t^2} + \frac{Ct}{(t^2-1)} + \frac{E}{(t^2+1)}\right) dt$

D) $\int \left(\frac{A}{t} + \frac{B}{t^2} + \frac{Ct}{(t^2-1)} + \frac{Et}{(t^2+1)}\right) dt$

E) $\int \left(\frac{A}{t} + \frac{B}{(t-1)} + \frac{C}{(t+1)} + \frac{Dt+E}{(t^2+1)}\right) dt$

$Sinx = u$ $Cosx\,dx = du$

$$\int \frac{Cosx\,dx}{(Sin^6x - Sin^2x)} = \int \frac{du}{u^6 - u^2}$$

$$\frac{1}{u^6-u^2} = \frac{1}{u^2(u^4-1)} = \frac{1}{u\cdot u\cdot(u-1)(u+1)(u^2+1)}$$

5 çarpan
5 kesir olmalı

$$= \frac{A}{u} + \frac{B}{u^2} + \frac{C}{u-1} + \frac{D}{u+1} + \frac{Eu+F}{u^2+1}$$

③ $\int \dfrac{e^x\,dx}{e^{5x}-4e^{3x}}$ integralini basit kesirlere ayırarak çözmek için kullanılacak kesirler aşağıdakilerden hangisi olabilir?

A) $\dfrac{A}{t^3}+\dfrac{Ct+D}{t^2-4}$

B) $\dfrac{A}{t^5}+\dfrac{B}{t^4}+\dfrac{C}{t^3}+\dfrac{D}{t^2}+\dfrac{E}{t}+\dfrac{F}{2t}+\dfrac{G}{4t^3}$

C) $\dfrac{A}{t}+\dfrac{B}{t^2}+\dfrac{C}{t^3}+\dfrac{D}{t-2}+\dfrac{E}{t+2}$

D) $\dfrac{At+B}{t^2-4t}+\dfrac{C}{t-2}+\dfrac{D}{t^3}+\dfrac{Et+F}{t^2+2t}+\dfrac{G}{t+2}$

E) $\dfrac{A}{t+2}+\dfrac{B}{t+4}+\dfrac{C}{t-4}+\dfrac{Dt+E}{t^2+4}$

$e^x = t \qquad e^x dx = dt$

$\int \dfrac{e^x dx}{e^{5x}-4e^{3x}} = \int \dfrac{du}{t^5-4t^3}$

$\dfrac{1}{t^5-4t^3} = \dfrac{1}{t^3(t^2-4)} = \dfrac{1}{t^3\cdot(t-2)(t+2)} \rightarrow 5 \text{ çarpan} \rightarrow 5 \text{ kesir}$

$= \dfrac{A}{t}+\dfrac{B}{t^2}+\dfrac{C}{t^3}+\dfrac{D}{t-2}+\dfrac{E}{t+2}$

---

④ $\int \dfrac{e^x\,dx}{e^{5x}-9e^{3x}}$ integralini basit kesirlere ayırarak çözmek için kullanılacak kesirler aşağıdakilerden hangisi olabilir? $e^x = t \qquad e^x dx = dt$

A) $\dfrac{A}{t^3}+\dfrac{Ct+D}{t^2-9}$

B) $\dfrac{A}{t^5}+\dfrac{B}{t^4}+\dfrac{C}{t^3}+\dfrac{D}{t^2}+\dfrac{E}{t}+\dfrac{F}{3t}+\dfrac{G}{9t^3}$

C) $\dfrac{A}{t}+\dfrac{B}{t^2}+\dfrac{C}{t^3}+\dfrac{D}{t-3}+\dfrac{E}{t+3}$

D) $\dfrac{At+B}{t^2-9t}+\dfrac{C}{t-3}+\dfrac{D}{t^3}+\dfrac{Et+F}{t^2+3t}+\dfrac{G}{t+3}$

E) $\dfrac{A}{t+3}+\dfrac{B}{t+9}+\dfrac{C}{t-9}+\dfrac{Dt+E}{t^2+3}$

$\int \dfrac{dt}{t^5-9t^3}$

$\dfrac{1}{t^3(t^2-9)} = \dfrac{A}{t}+\dfrac{B}{t^2}+\dfrac{C}{t^3}+\dfrac{D}{t-3}+\dfrac{E}{t+3}$

$\underbrace{}_{(t-3)(t+3)}$

⑤ Aşağıdakilerden hangisi $\int \dfrac{dx}{\left(x^2+1\right)\left(x-1\right)^2}$ integralindeki

fonksiyonun basit kesirlere ayrılmış halidir ?

A) $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$

B) $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{Cx+D}{x^2+1}$

C) $\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{C}{x^2+1}$

D) $\dfrac{Ax+B}{x^2-1}+\dfrac{Cx+D}{x^2+1}$

E) $\dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{Cx+D}{x^2+1}$

$\dfrac{1}{(x^2+1)(x-1)(x-1)} \to 3$ çarpan

$\to 3$ kesir

$=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{Cx+D}{x^2+1}$

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⑥ $y=2x^{3/2}$ eğrisinin $[0,1]$ kapalı aralığındaki yay uzunluğu aşağıdakilerden hangisidir?

A) $\dfrac{2}{27}\left(10\sqrt{10}-1\right)$ B) $\dfrac{2}{3}\left(10\sqrt{10}-1\right)$ C) $\dfrac{2}{9}\left(10\sqrt{10}-1\right)$

D) $\dfrac{1}{9}\left(10\sqrt{10}-1\right)$ E) $\dfrac{1}{6}\left(10\sqrt{10}-1\right)$

$S=\displaystyle\int_0^1 \sqrt{1+(y')^2}\, dx$

$y'=2\cdot\dfrac{3}{2}\cdot x^{1/2}=3\sqrt{x}$

$\sqrt{1+(y')^2}=\sqrt{1+9x}$

$S=\displaystyle\int_0^1 \sqrt{1+9x}\, dx$

$1+9x=u$
$9dx=du$

E.S   Y.S
$x=1 \to u=10$
$x=0 \to u=1$

$=\displaystyle\int_1^{10}\sqrt{u}\cdot\dfrac{du}{9}=\dfrac{1}{9}\cdot\dfrac{u^{3/2}}{\frac{3}{2}}\Big|_1^{10}=\dfrac{2}{27}\left[10\sqrt{10}-1\right]$

7. $y = \dfrac{1}{2}x^2 - \dfrac{1}{4}\ln x$, $1 \le x \le 2$ eğrisinin yay uzunluğu kaçtır ?

$$S = \int_1^2 \sqrt{1 + (y')^2}\, dx$$

A) $\dfrac{3}{2} + \dfrac{1}{4}\ln 2$

B) $\dfrac{1}{2} + \dfrac{1}{3}\ln 2$

C) $2 + \dfrac{1}{8}\ln 2$

D) $4 + 2\ln 2$

E) $1 + \ln 2$

$$y' = x - \frac{1}{4x} \qquad \rightarrow \quad y'^2 = x^2 - \frac{1}{2} + \frac{1}{16x^2}$$

$$1 + (y')^2 = 1 + x^2 - \frac{1}{2} + \frac{1}{16x^2} = x^2 + \frac{1}{2} + \frac{1}{16x^2}$$

$$= \left(x + \frac{1}{4x}\right)^2$$

$$\sqrt{1+(y')^2} = \sqrt{\left(x + \frac{1}{4x}\right)^2} = \left| x + \frac{1}{4x} \right|$$

$$S = \int_1^2 \left| x + \frac{1}{4x} \right| dx = \int_1^2 \left(x + \frac{1}{4x} \right) dx = \left. \frac{x^2}{2} + \frac{1}{4}\ln x \right|_1^2$$

$$= 2 + \frac{1}{4}\ln 2 - \frac{1}{2} = \frac{3}{2} + \frac{1}{4}\ln 2$$

8) $x = \frac{y^3}{3} + \frac{1}{4y}$ eğrisinin $1 \leq y \leq 3$ aralığındaki yay uzunluğu aşağıdakilerden hangisidir?

A) $\frac{23}{12}$ B) $\frac{53}{6}$ C) $\frac{15}{11}$ D) $\frac{33}{6}$ E) $\frac{32}{11}$

$$S = \int_1^3 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$\frac{dx}{dy} = y^2 - \frac{1}{4y^2} \rightarrow \left(\frac{dx}{dy}\right)^2 = y^4 - \frac{1}{2} + \frac{1}{16y^4}$$

$$1 + \left(\frac{dx}{dy}\right)^2 = y^4 + \frac{1}{2} + \frac{1}{16y^4} = \left(y^2 + \frac{1}{4y^2}\right)^2$$

$$S = \int_1^3 \sqrt{\left(y^2 + \frac{1}{4y^2}\right)^2}\, dy = \int_1^3 \left(y^2 + \frac{1}{4y^2}\right) dy = \left. \frac{y^3}{3} - \frac{1}{4y} \right|_1^3$$

$$= \frac{27}{3} - \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = \frac{53}{6}$$

---

9)

Türev fonksiyonu $f'(x) = \sqrt{\sec^4 x - 1}$ olarak tanımlı olan $y = f(x)$ eğrisinin $\left[0, \frac{\pi}{3}\right]$ aralığındaki yayının uzunluğu kaç birimdir?

A) 1   B) $\frac{\sqrt{3}}{3}$ C) 2   D) $\sqrt{3}$ E) $\sqrt{3} - 1$

$$S = \int_0^{\pi/3} \sqrt{1 + (f'(x))^2}\, dx$$

$$(f'(x))^2 = \sec^4 x - 1 \rightarrow \sqrt{1 + (f'(x))^2} = \sqrt{\sec^4 x} = \sec^2 x$$

$$S = \int_0^{\pi/3} \sec^2 x\, dx = \left. \tan x \right|_0^{\pi/3} = \sqrt{3}$$

(10) $y = \frac{1}{2}\sqrt{x-1}$ eğri yayının $x = \frac{3}{2}$ ile $x = 4$ doğruları arasında kalan kısmının x —ekseni etrafında döndürülmesi ile meydana gelen yüzeyin alanını veren integral aşağıdakilerden hangisidir?

A) $2\pi \int_{\frac{3}{2}}^{4} x\sqrt{x-1}\,dx$

B) $2\pi \int_{\frac{3}{2}}^{4} \sqrt{x-1}\,dx$

C) $\pi \int_{\frac{3}{2}}^{4} x\sqrt{x-1}\,dx$

D) $\frac{\pi}{4} \int_{\frac{3}{2}}^{4} \sqrt{16x-15}\,dx$ ⭕

E) $2\pi \int_{\frac{3}{2}}^{4} \sqrt{16x-15}\,dx$

$$S = 2\pi \int_{3/2}^{4} f(x) \cdot \sqrt{1 + (f'(x))^2}\, dx$$

$$= 2\pi \int_{3/2}^{4} \frac{1}{2} \cdot \sqrt{x-1} \cdot \frac{\sqrt{16x-15}}{4\sqrt{x-1}}\, dx$$

$$S = \frac{\pi}{4} \int_{3/2}^{4} \sqrt{16x-15}\, dx$$

$f(x) = \frac{1}{2}\sqrt{x-1}$

$f'(x) = \frac{1}{4\sqrt{x-1}}$

$1 + (f'(x))^2 = 1 + \frac{1}{16(x-1)}$

$= \frac{16x-15}{16(x-1)}$

$\sqrt{1 + (f')^2} = \frac{\sqrt{16x-15}}{4 \cdot \sqrt{x-1}}$

---

(11) $y = 2\sqrt{3-x}$ eğri yayının $x = 0$ ile $x = 3$ doğruları arasında kalan kısmının x —ekseni etrafında döndürülmesi ile meydana gelen yüzeyin alanını veren integral aşağıdakilerden hangisidir?

A) $2\pi \int_{0}^{3} x\sqrt{3-x}\,dx$

B) $2\pi \int_{0}^{3} \sqrt{3-x}\,dx$

C) $4\pi \int_{0}^{3} x\sqrt{3-x}\,dx$

D) $2\pi \int_{0}^{3} x\sqrt{4-x}\,dx$

E) $4\pi \int_{0}^{3} \sqrt{4-x}\,dx$ ⭕

$$S = 2\pi \int_{0}^{3} f(x) \cdot \sqrt{1 + (f'(x))^2}\, dx$$

$$= 2\pi \int_{0}^{3} 2 \cdot \sqrt{3-x} \cdot \frac{\sqrt{4-x}}{\sqrt{3-x}}\, dx$$

$$= 4\pi \int_{0}^{3} \sqrt{4-x}\, dx$$

$f(x) = 2\sqrt{3-x}$

$f'(x) = 2 \cdot \frac{-1}{2\sqrt{3-x}} = -\frac{1}{\sqrt{3-x}}$

$1 + (f'(x))^2 = 1 + \frac{1}{3-x} = \frac{4-x}{3-x}$

$\sqrt{1 + (f'(x))^2} = \frac{\sqrt{4-x}}{\sqrt{3-x}}$

12) $y = x^3$ eğrisinin $[0,1]$ aralığı üstünde kalan kısmı $x-ekseni$ etrafında döndürülürse meydana gelen dönel yüzeyin alanı kaç birim karedir?

A) $\dfrac{\pi}{54}\left(10^{3/2} - 1\right)$ B) $\dfrac{\pi}{45}\left(10^{3/2} - 1\right)$ C) $\dfrac{\pi}{36}\left(10^{3/2} - 1\right)$

D) $\dfrac{\pi}{27}\left(10^{3/2} - 1\right)$ E) $\dfrac{\pi}{18}\left(10^{3/2} - 1\right)$

$$y' = 3x^2$$
$$\sqrt{1+(y')^2} = \sqrt{1 + 9x^4}$$

$$S = 2\pi \int_0^1 x^3 \cdot \sqrt{1+(y')^2}\, dx$$

$$= 2\pi \int_0^1 x^3 \cdot \sqrt{1 + 9x^4}\, dx$$

$1 + 9x^4 = u \qquad 36x^3 dx = du$

$\dfrac{E.S}{x=1} \rightarrow \dfrac{Y.S}{u=10}$

$x=0 \rightarrow u=1$

$$= 2\pi \int_1^{10} \sqrt{u} \cdot \dfrac{du}{36} = \dfrac{\pi}{18} \cdot \dfrac{u^{3/2}}{\frac{3}{2}}\Big|_1^{10} = \dfrac{\pi}{27}\left[10^{3/2} - 1\right]$$

12) Aşağıdaki integrallerden hangileri yakınsaktır?

I. $\displaystyle\int_0^3 \dfrac{dx}{(x-2)^2}$ II. $\displaystyle\int_0^3 \dfrac{dx}{\sqrt{3-x}}$ III. $\displaystyle\int_3^\infty \dfrac{dx}{\sqrt{x}}$

a) I   b) II   c) III   d) I ve II   e) Hiçbiri

I. $\displaystyle\int_0^3 \dfrac{dx}{(x-2)^2} = \int_0^2 \dfrac{dx}{(x-2)^2} + \int_2^3 \dfrac{dx}{(x-2)^2} = -\dfrac{1}{x-2}\Big|_0^{2^-} - \dfrac{1}{x-2}\Big|_{2^+}^3$

$$= -\underbrace{\dfrac{1}{2^- - 2}}_{\frac{-\infty}{\infty}} + \dfrac{1}{2} - \dfrac{1}{3-2} + \underbrace{\dfrac{1}{2^+ - 2}}_{\infty} = \infty \rightarrow \text{Iraksak}$$

$$\text{II.} \quad \int_0^3 \frac{dx}{\sqrt{3-x}} = \int_0^3 (3-x)^{-1/2}\, dx = -\frac{(3-x)^{1/2}}{\frac{1}{2}} \Big|_0^{3^-} = 0 + 2\sqrt{3} = 2\sqrt{3}$$

$$\text{III.} \quad \int_3^\infty \frac{dx}{\sqrt{x}} = 2\sqrt{x}\,\Big|_3^\infty = \infty \;\rightarrow\; \text{Iraksak}$$

---

⑬ ✱ Aşağdaki int hangileri yakınsaktır?

$$\text{I.} \int_0^1 \frac{dx}{\sqrt{x}} \qquad \text{II.} \int_0^2 \frac{dx}{x-1} \qquad \text{III.} \int_{10}^\infty \frac{dx}{x^2+1}$$

a) I   b) II   c) III   ⓓ) **I ve III**   e) II ve III

$$\text{I.} \quad \int_0^1 \frac{dx}{\sqrt{x}} = 2\sqrt{x}\,\Big|_{0^+}^1 = \boxed{2} \;\rightarrow\; \text{Yakınsak}$$

$$\text{II.} \quad \int_0^2 \frac{dx}{x-1} = \int_0^1 \frac{dx}{x-1} + \int_1^2 \frac{dx}{x-1} = \ln|x-1|\,\Big|_0^{1^-} + \ln|x-1|\,\Big|_{1^+}^2$$

$$= \underbrace{\ln|\overset{0^+}{1^--1}| - \ln|1|}_{-\infty} + \underbrace{\ln|1| - \ln|\overset{0^+}{1^+-1}|}_{+\infty}$$

$$= -\infty + \infty \;\rightarrow\; \text{Sonuç belirsiz}$$
$$\downarrow$$
$$\text{Iraksak}$$

$$\text{III.} \quad \int_{10}^\infty \frac{dx}{x^2+1} = \text{Arctan}x \,\Big|_{10}^\infty = \text{Arctan}\,\infty - \text{Arctan}\,10$$
$$\underbrace{\phantom{\text{Arctan}\infty}}_{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} - \text{Arctan}\,10 \;\rightarrow\; \text{Yakınsak}$$

14 $\int_0^\infty 2xe^{-x^2}\,dx$ integralinin sonucu aşağıdakilerden hangisidir?

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

A) ∞

B) -1

C) $-\infty$

(D) 1

E) 0

$u = -x^2 \qquad du = -2x\,dx$

$$\frac{A.S}{x\to\infty} \qquad \frac{Ü.S}{u\to-\infty}$$

$$x=0 \qquad u=0$$

(—) li sınırları değiştirmek için kullandım

$$\int_0^\infty 2x\,e^{-x^2}\,dx = \int_0^{-\infty} -e^u\,du = \int_{-\infty}^0 e^u\,du$$

$$= e^u\Big|_{-\infty}^0 = \underbrace{e^0}_{1} - \underbrace{e^{-\infty}}_{\frac{1}{e^\infty}=0} = 1$$

---

15 $\displaystyle\int_1^e \frac{dx}{\ln x^x}$ integrali için hangisi doğrudur?

a) integralin sonucu 0 dır, Yakınsaktır

b) " " $+\infty$, Iraksaktır

c) " " $-\infty$, "

d) " " e , Yakınsaktır

$$\int_1^e \frac{dx}{\ln x^x} = \int_1^e \frac{dx}{x\ln x} = \int_0^1 \frac{du}{u} = \ln|u|\Big|_{0^+}^1 = \underbrace{\ln 1 - \ln|0^+|}_{-\infty} = \infty$$

Iraksak

$\ln x = u \qquad \frac{dx}{x} = du$

$$\frac{E.S}{x=e} \rightarrow \frac{Ü.S}{u=1}$$

$$x=1 \rightarrow u=0$$

16 $\int_{-\infty}^{\infty} e^{-|x|}\, dx$ int.   için   hangisi   doğrudur!

a) Sonucu ∞, Iraksak

b) Sonucu 0, Yakınsak

c) Sonucu 2, ''

d) '' ∞, Iraksak

$$\int_{-\infty}^{\infty} e^{-|x|}\, dx = \int_{-\infty}^{0} e^{+x}\, dx + \int_{0}^{\infty} e^{-x}\, dx = e^{x}\Big|_{-\infty}^{0} - e^{-x}\Big|_{0}^{\infty}$$

$$= \underbrace{e^{0}}_{1} - \underbrace{e^{-\infty}}_{0} - \underbrace{e^{-\infty}}_{0} + \underbrace{e^{0}}_{1} = \underset{\downarrow}{2}$$
Yak.

---

17 $\int \dfrac{dx}{1-Cosx+Sinx}$ integralinin uygun dönüşüm altında alacağı ifadesi aşağıdakilerden hangisi olabilir?
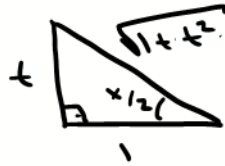
$t = Tan\dfrac{x}{2}$ yapmalıyız.

A) $\int \left(\dfrac{1}{t+1} - \dfrac{1}{t}\right)\, dt$

B) $\int \left(\dfrac{1}{t+1} - \dfrac{1}{2t}\right)\, dt$

C) $\int \left(\dfrac{1}{2t} - \dfrac{1}{t+1}\right)\, dt$

D) $\int \left(\dfrac{1}{t} - \dfrac{1}{2(t+1)}\right)\, dt$

E) $\int \left(\dfrac{1}{t} - \dfrac{1}{t+1}\right)\, dt$

$x = 2\,Arctant \rightarrow dx = \dfrac{2dt}{1+t^2}$

$Cosx = 2Cos^2\dfrac{x}{2} - 1 = \dfrac{1-t^2}{1+t^2}$

$Sinx = 2Cos\dfrac{x}{2}\cdot Sin\dfrac{x}{2}$

$= \dfrac{2t}{1+t^2}$

Yerine yazalım

$$\int \dfrac{dx}{1-Cosx+Sinx} = \int \dfrac{\frac{2dt}{1+t^2}}{1-\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}} = \int \dfrac{2dt}{1+t^2-1+t^2+2t}$$

$$= \int \dfrac{2dt}{2(t^2+t)} = \int \dfrac{dt}{t(t+1)} =$$

$$\overbrace{\dfrac{1}{t(t+1)} = \dfrac{A}{t} + \dfrac{B}{t+1}}^{=} = \dfrac{At + A + Bt}{t(t+1)} \qquad \begin{array}{l} A+B=0 \uparrow \\ A=1 \quad B=-1 \end{array}$$

$$\underset{(t+1)}{} \qquad \underset{t}{}$$

$$\int \dfrac{dt}{t\cdot(t+1)} = \int\left(\dfrac{1}{t} - \dfrac{1}{t+1}\right) dt$$

---

⑱ $\displaystyle\int \dfrac{dx}{4+3\cos x}$ uygun dönüşüm ile oluşacak yeni integral?

Ⓐ) $\displaystyle\int \dfrac{2\,du}{u^2+7}$ b) $\displaystyle\int \dfrac{du}{u^2+3}$ c) $\displaystyle\int \dfrac{2\,du}{u^2+5}$ d) $\displaystyle\int \dfrac{du}{u^2+1}$

$$u = \tan\dfrac{x}{2} \quad\rightarrow\quad dx = \dfrac{2\,du}{1+u^2} \qquad\qquad \cos x = \dfrac{1-u^2}{1+u^2}$$

$$\int \dfrac{dx}{4+3\cos x} = \int \dfrac{\dfrac{2\,du}{1+u^2}}{4 + \dfrac{3(1-u^2)}{1+u^2}} = \int \dfrac{2\,du}{4+4u^2+3-3u^2} = \int \dfrac{2\,du}{u^2+7}$$

---

⑲ $\displaystyle\int \dfrac{dx}{1+\sin x} = ?$ $\qquad u = \tan\dfrac{x}{2} \qquad dx = \dfrac{2\,du}{1+u^2} \qquad \sin x = \dfrac{2u}{1+u^2}$

$$\int \dfrac{dx}{1+\sin x} = \int \dfrac{\dfrac{2\,du}{1+u^2}}{1 + \dfrac{2u}{1+u^2}} = \int \dfrac{2\,du}{u^2+2u+1} = \int \dfrac{2\,du}{(1+u)^2}$$

$$= 2\cdot\left(-\dfrac{1}{1+u}\right) + c$$

$$= -\dfrac{2}{\tan\dfrac{x}{2}+1}$$