

Introduction

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BLM5106- Advanced Algorithm Analysis and Design

Algorithm Analysis

- Resources :

- Introduction to Design & Analysis of Algorithms, Anany Levitin, 2011
- Cormen, Leiserson, Rivest, Stein, "Introduction to Algorithms, 3E", MIT Press, 2009
- Algorithms, Fourth Edition, R. Sedgewick and K. Wayne (<http://algs4.cs.princeton.edu>), 2013
- The Algorithm Design Manual, Steven Skiena, 2010

- Grading :

- Project: 1
- Midterm Exam(s) 1
- Final Exam

Contents

1. **Fundamentals for Algorithms**
2. **Fundamentals of the Analysis of Algorithms Efficiency, Asymptotic Analysis**
Algorithm Analysis, Complexities, Big OH, Big Theta, Big Omega, Orders of Growth
3. **Analysis of Non-Recursive and Recursive Algorithms**
Running Time, Recurrence Relation, Backward Substitution
4. **Analysis of Divide and Conquer Algorithms**
Brute Force, Exhaustive Search, Decrease and Conquer
5. **Hashing Algorithms**
6. **Dynamic Programming**
Rod cutting, Matrix-chain multiplication, Elements of dynamic programming, Optimal binary search trees

Contents

7. Greedy Algorithms

Activity-selection problem , Elements of the greedy strategy

8. Midterm

9. Amortized Analysis

Aggregate analysis, The accounting method, The potential method, Dynamic tables

10. Advanced Data Structures

B-trees, Fibonacci Heaps, AVL trees

11. Elementary Graph Algorithms

Representations of graphs, Breadth-first search, Depth-first search, Topological sort, strongly connected components

12. Presentations

13. String Matching

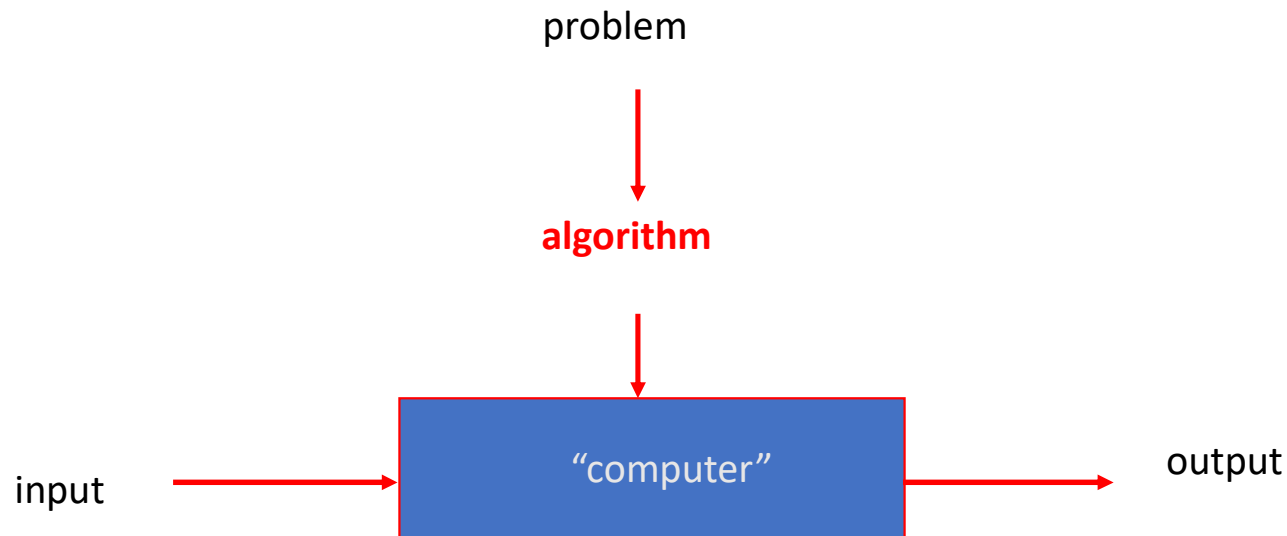
The naive string-matching algorithm, The Rabin-Karp algorithm, String matching with finite automata , The Knuth-Morris-Pratt algorithm

14. Approximation Algorithms

The vertex-cover problem, The traveling-salesman problem, The set-covering problem, Randomization and linear programming, The subset-sum problem

Notion of Algorithm

- An algorithm is a **sequence of unambiguous instructions** for solving a problem, for obtaining a required output for any legitime input **in a finite amount of time**.



Analyzing an Algorithm

- How good is the algorithm?

Correctness

Efficiency (Time, Space) *An algorithm is efficient if it has a polynomial running time.*

Simplicity

Generality

- Does there exist a better algorithm?

Lower bounds

Optimality

Fundamental Data Structures

- **Linear Data Structures**

- Arrays, Linked lists, Stack, Queue

- **Graphs**

- Nodes, Edges, Adjacency matrix and Adjacency lists

- **Trees**

- Rooted Trees, Ordered Trees

- **Sets and Dictionaries**

- *Universal set, Bit vector*, Using the list structure

Fundamentals of the Analysis of Algorithm Efficiency

- System independent effects :
 - Algorithm
- System dependent effects
 - Hardware : CPU, memory, cache
 - Software : compiler, interpreter, etc.
 - System : OS, network, etc.

Fundamentals of the Analysis of Algorithm Efficiency

- Measuring an Input's Size
 - Product of two matrices: total number of elements N in the matrices being multiplied
 - Evaluating a polynomial: polynomial's degree
- Units for Measuring Running Time

$$T(n) \approx c_{op} C(n)$$

Assuming that $C(n) = 1/2 n(n - 1)$, how much longer will the algorithm run if we double its input size?

$$C(n) = \frac{1}{2}n(n - 1) = \frac{1}{2}n^2 - \frac{1}{2}n \approx \frac{1}{2}n^2$$

$$\frac{T(2n)}{T(n)} \approx \frac{c_{op}C(2n)}{c_{op}C(n)} \approx \frac{\frac{1}{2}(2n)^2}{\frac{1}{2}n^2} = 4.$$

Orders of growth

order of growth	name	typical code framework	description	example	$T(2N) / T(N)$
1	constant	<code>a = b + c;</code>	statement	add two numbers	1
$\log N$	logarithmic	<pre>while (N > 1) { N = N / 2; ... }</pre>	divide in half	binary search	~ 1
N	linear	<pre>for (int i = 0; i < N; i++) { ... }</pre>	loop	find the maximum	2
$N \log N$	linearithmic	[see mergesort lecture]	divide and conquer	mergesort	~ 2
N^2	quadratic	<pre>for (int i = 0; i < N; i++) for (int j = 0; j < N; j++) { ... }</pre>	double loop	check all pairs	4
N^3	cubic	<pre>for (int i = 0; i < N; i++) for (int j = 0; j < N; j++) for (int k = 0; k < N; k++) { ... }</pre>	triple loop	check all triples	8
2^N	exponential	[see combinatorial search lecture]	exhaustive search	check all subsets	

Worst-Case, Best-Case, and Average-Case Efficiencies

ALGORITHM *SequentialSearch*($A[0..n - 1]$, K)

//Searches for a given value in a given array by sequential search

//Input: An array $A[0..n - 1]$ and a search key K

//Output: The index of the first element in A that matches K

// or -1 if there are no matching elements

$i \leftarrow 0$

while $i < n$ **and** $A[i] \neq K$ **do**

$i \leftarrow i + 1$

if $i < n$ **return** i

else return -1

- **Worst-case** $C_{\text{worst}}(n) = n$
- **Best-case** $C_{\text{best}}(n) = 1$
- **Average-case** $C_{\text{avg}}(n) = [1 * p/n + 2 * p/n + \dots + i * p/n + \dots + n * p/n] + n(1-p)$

(a) probability of a successful search is equal to p ($0 \leq p \leq 1$)

(b) probability of the first match occurring in the i th position of the list is the same for every i