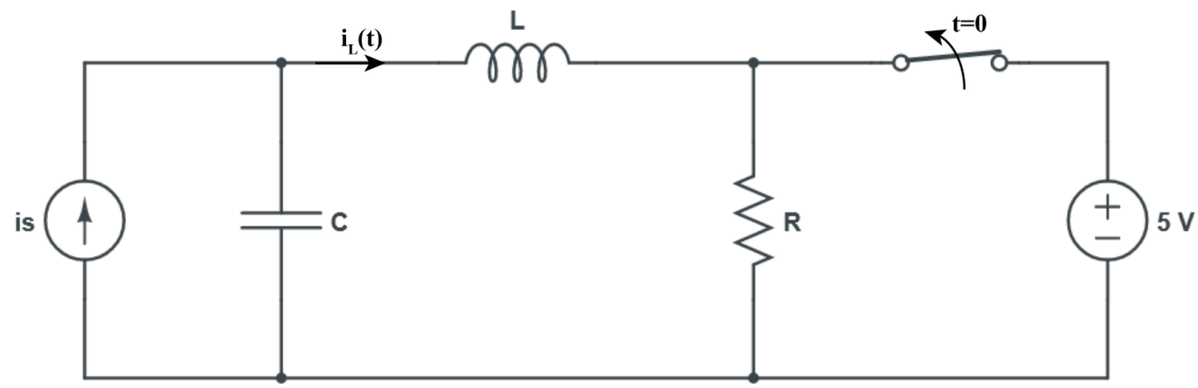
BLM1032 – Circuit Theory
Short Exam 2 – 03/June/2021

Upload your answer as a single PDF file at max. 5MB of size.

For the given circuit switch is opened at t=0 after being closed for a long time.
R=40 Ohm, C=25mF, L=10H

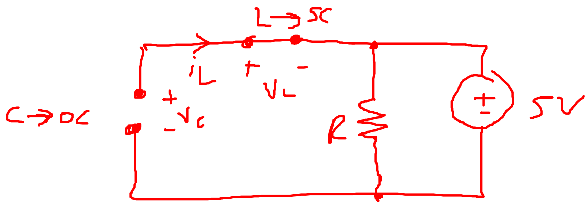


1) **If $i_s(t) = 10\,u(t)A$**
   a. $i_L(0^-) = ?$
   b. $i_L(t) = ?$

2) **If $i_s(t) = e^{-3t}\,u(t)A$**
   a. $i_L(\infty) = ?$
   b. $i_L(t) = ?$

1 a) for $t=0^-$ circuit becomes :

SC : short circuit
OC : open circuit



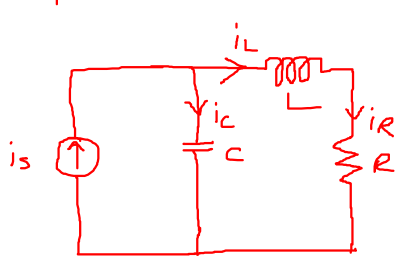
$\boxed{i_L(0^-) = 0A}$

$V_c(0^-) = 5V$

$V_L(0^-) = 0V$

$i_c(0^-) = 0A$

1b) for $t>0$ circuit becomes



$i_s = i_L + i_c$

$i_R = i_L$

$i_c = C \cdot \frac{dV_c}{dt}$

$V_L + V_R = V_c$

$L \cdot \frac{di_L}{dt} + i_L \cdot R = V_c$

$$i_c = C \cdot \frac{d}{dt}\left(L\, i_L' + R i_L\right) = C L\, i_L'' + R C\, i_L'$$

$$C L\, i_L'' + R C\, i_L' + i_L = i_s \qquad i_s(t) = 10$$

$$i_L'' + \frac{R}{L}\, i_L' + \frac{1}{CL}\, i_L = \frac{i_s}{CL}$$

$i_L \rightarrow$ natural solution $\Longrightarrow$ $i_L'' + 4 i_L' + 4 i_L = 0 \Rightarrow i_L = A \cdot e^{\alpha t}$

$i_L \rightarrow$ forced solution $\Longrightarrow$ $i_L = B$

$$(\alpha^2 + 4\alpha + 4)\, A \cdot e^{\alpha t} = 0 \Longrightarrow \alpha_{1,2} = -2$$

$$i_{L1}(t) = A_1 \cdot e^{-2t}$$

$$i_{L2}(t) = A_2 t \cdot e^{-2t}$$

$$4 \cdot B = 4 \cdot 10 \Longrightarrow B = 10$$

$$\Longrightarrow \quad i_L(t) = A_1 e^{-2t} + A_2 t e^{-2t} + 10$$

- $i_L(0^+) = i_L(0^-) = 0A$

$$A_1 + 10 = 0 \Longrightarrow A_1 = -10$$

- $V_c(0^-) = 5V$

$$L \cdot i_L' + R i_L = V_c$$

$$i_L'(0) = \frac{V_c - R\, i_L(0)}{L} = \frac{5 - 40 \cdot 0}{10} = 0,5$$

$$i_L'(t) = -2A_1 e^{-2t} + A_2 e^{-2t} - 2A_2 t e^{-2t}$$

$$i_L'(0) = -2A_1 + A_2 = 0,5$$

$$A_2 = -19,5$$

$$\Longrightarrow \quad i_L(t) = -10 \cdot e^{-2t} - 19,5 \cdot t e^{-2t} + 10$$

2a) $t \to \infty \qquad i_s(t) = e^{-3t} u(t) \to 0$

$i_L(\infty) = 0$

2b) for $t = 0^-$ $\qquad i_L(0) = 0 \qquad V_L(0) = 0 \qquad i_c(0) = 0 \qquad V_c(0) = 5$

for $t > 0$

$CL\, i_L'' + RC\, i_L' + i_L = i_s \qquad i_s(t) = e^{-3t}$

$$i_L'' + \frac{R}{L} i_L' + \frac{1}{CL} i_L = \frac{i_s}{CL}$$

$i_L \to$ natural solution $\Longrightarrow$ $i_L'' + 4 i_L' + 4 i_L = 0 \Rightarrow i_{L_N} = A.e^{\alpha t}$

$i_L \to$ forced solution $\Longrightarrow$ $i_{L_F} = B.e^{-3t}$

$$i_{L_N}(t) = A_1.e^{-2t} + A_2.t.e^{-2t}$$

$$9Be^{-3t} - 12B.e^{-3t} + 4Be^{-3t} = 4.e^{-3t}$$

$$B = 4$$

$$i_{LF}(t) = 4e^{-3t}$$

$$i_L(t) = A_1 e^{-2t} + A_2.t e^{-2t} + 4.e^{-3t}$$

- $i_L(0) = 0 \Longrightarrow A_1 + 4 = 0 \Longrightarrow A_1 = -4$

- $i_L'(0) = 0{,}5$

$$i_L'(t) = -2A_1.e^{-2t} + A_2 e^{-2t} - 2A_2 t e^{-2t} - 12e^{-3t}$$

$$i_L'(0) = -2A_1 + A_2 - 12 = 0{,}5$$

$$-2A_1 + A_2 = 12{,}5$$

$$A_2 = 4{,}5$$

$$\Longrightarrow \quad i_L(t) = -4e^{-2t} + 4{,}5.t.e^{-2t} + 4e^{-3t}$$