

Ters Trigonometrik Fonksiyonlar.

$$y = \arcsin x = \sin^{-1} x \quad \text{TK: } [-1, 1] \quad \text{GK: } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

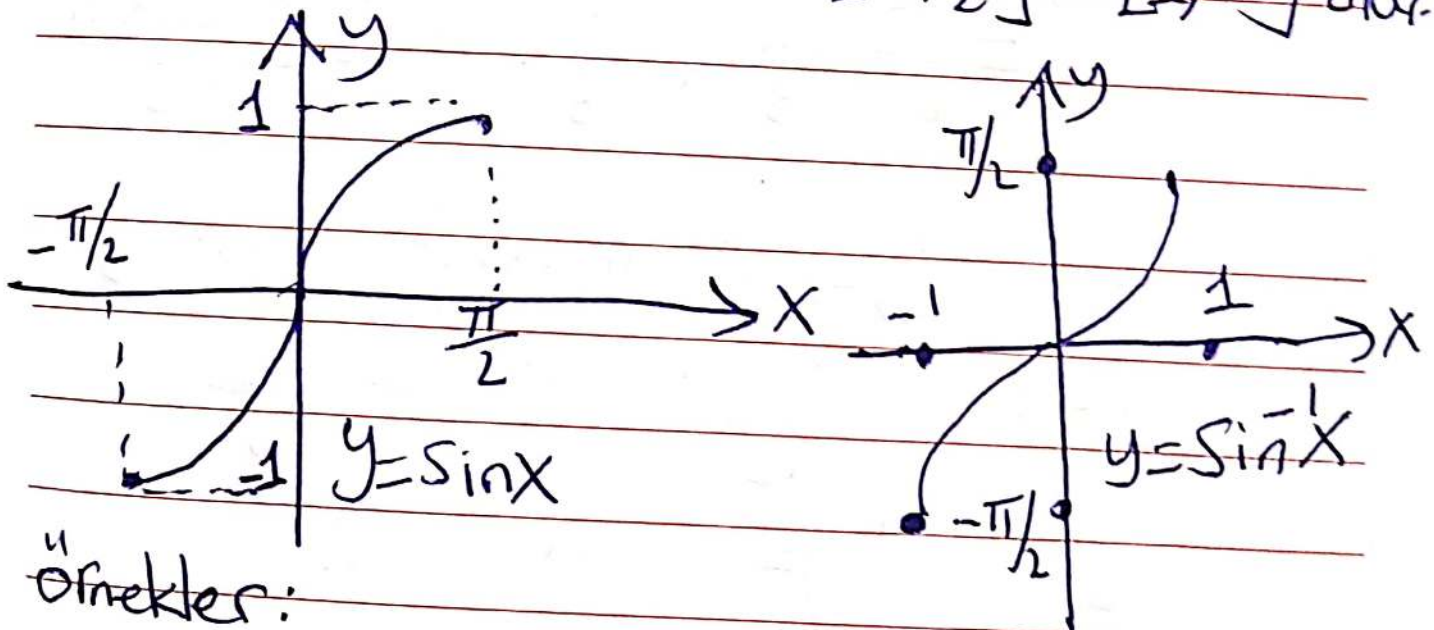
her iki tarafın Sinüsü alınırsa.

$$\sin y = \sin(\arcsin x) \quad f \circ f^{-1}(x) = x \text{ den.}$$

$$\sin y = x \quad f^{-1}(x) = \sin x \quad \text{yani } \sin^{-1} x \text{ in}$$

ters fonksiyonu $f^{-1}(x) = \sin x$ olur.

$y = \sin x$ fonksiyonunda $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow [-1, +1]$ olur.



örnekler:

$$1-) \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = ? \quad 2) \sin^{-1}\left(\frac{1}{2}\right) = ? \quad 3) \sin^{-1}(1) = ?$$

$$C: -\frac{\pi}{4}$$

$$C: \frac{\pi}{6}$$

$$C: \frac{\pi}{2}$$

$$2) \cos\left(\arcsin \frac{1}{2} - \arcsin 0\right) = ? \quad \arcsin 0 = 0$$

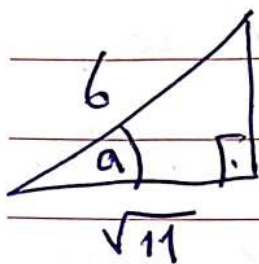
$$\cos\left(\frac{\pi}{6} - 0\right) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\arcsin \frac{1}{2} = \frac{\pi}{6}$$

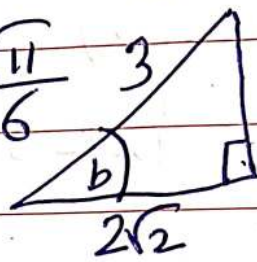
$$3) \sin\left(\arcsin \frac{5}{6} + \arcsin \frac{1}{3}\right) = ? \quad b = \arcsin \frac{1}{3}$$

$$\sin b = \frac{1}{3}$$

$$a = \arcsin \frac{5}{6} \quad \sin a = \frac{5}{6}$$



$$\cos a = \frac{\sqrt{11}}{6}$$



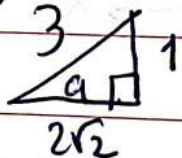
$$\cos b = \frac{2\sqrt{2}}{3}$$

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$= \frac{5}{6} \cdot \frac{2\sqrt{2}}{3} + \frac{1}{3} \cdot \frac{\sqrt{11}}{6} = \frac{10\sqrt{2} + \sqrt{11}}{18} //$$

$$4) \cos\left(\arcsin \frac{1}{3}\right) = ? \quad a = \arcsin \frac{1}{3} \quad \sin a = \frac{1}{3}$$

$$\cos a = ?$$

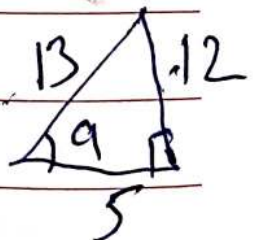


$$\cos a = \frac{2\sqrt{2}}{3} //$$

$$5) \tan\left(\frac{3\pi}{2} - \cos^{-1} \frac{5}{13}\right) = ?$$

$$a = \cos^{-1} \frac{5}{13}$$

$$\cos a = \frac{5}{13}$$



$$\tan\left(\frac{3\pi}{2} - a\right) = \cot a = \frac{12}{5} //$$

$$6) \sin(2 \arccot 2) = ? \quad a = \arccot 2 \quad \sin 2a = ?$$

$$2 \sin a \cos a$$



$$\cot a = 2$$

$$\sin a = \frac{1}{\sqrt{5}}$$

$$2 \cdot \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} = \frac{4}{5} //$$

$$\cos a = \frac{2}{\sqrt{5}}$$

$$7) \tan(\sin^{-1} \frac{3}{4} - \cos^{-1} \frac{1}{2\sqrt{2}}) = ?$$

$$a = \sin^{-1} \frac{3}{4} \quad \sin a = \frac{3}{4} \quad \begin{array}{c} 4 \\ \swarrow \searrow \\ 3 \quad \sqrt{7} \end{array} \quad \cos a = \frac{\sqrt{7}}{4}$$

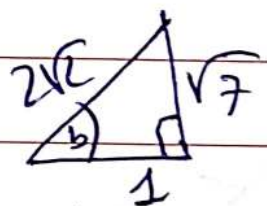
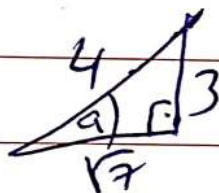
$$b = \cos^{-1} \frac{1}{2\sqrt{2}} \quad \cos b = \frac{1}{2\sqrt{2}} \quad \begin{array}{c} 2\sqrt{2} \\ \swarrow \searrow \\ 1 \quad \sqrt{7} \end{array} \quad \sin b = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b} \quad \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$$

$$\tan a = ? \quad \tan b = ?$$

$$\tan(a-b) = ?$$



$$\tan a = \frac{3}{\sqrt{7}} \quad \tan b = \frac{\sqrt{7}}{1}$$

$$\tan(a-b) = \frac{\frac{3}{\sqrt{7}} - \sqrt{7}}{1 + \frac{3}{\sqrt{7}} \cdot \sqrt{7}} = \frac{\frac{-4}{\sqrt{7}}}{4} = -\frac{\sqrt{7}}{7} //$$

$$8) \arcsin(\sin \frac{4\pi}{3}) = ? \quad \arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$$

$$9) \tan(\arcsin \frac{24}{25}) = ? \quad a = \arcsin \frac{24}{25} \quad \begin{array}{c} 25 \\ \swarrow \searrow \\ 24 \quad 7 \end{array} \quad \sin a = \frac{24}{25} \quad \cos a = \frac{7}{25} \quad \tan a = \frac{24}{7}$$

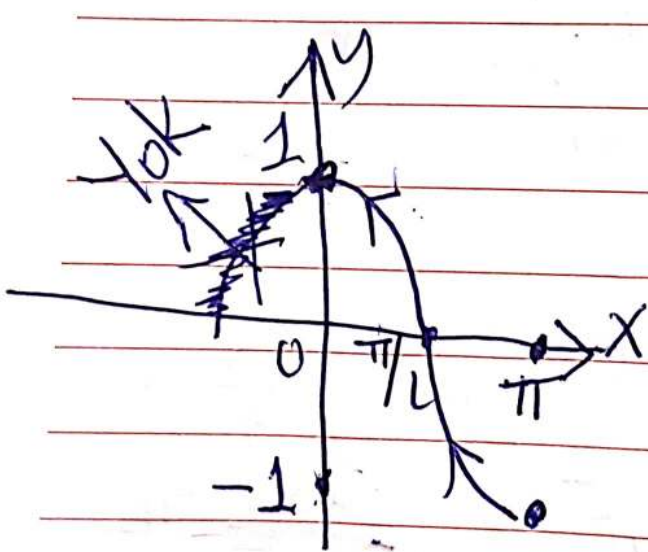
$y = \arccos x = \cos^{-1} x$ fonksiyonu

TK: $[-1, 1]$ G.K: $[0, \pi]$ GK: Görüntü
TK: Tanım Kome,

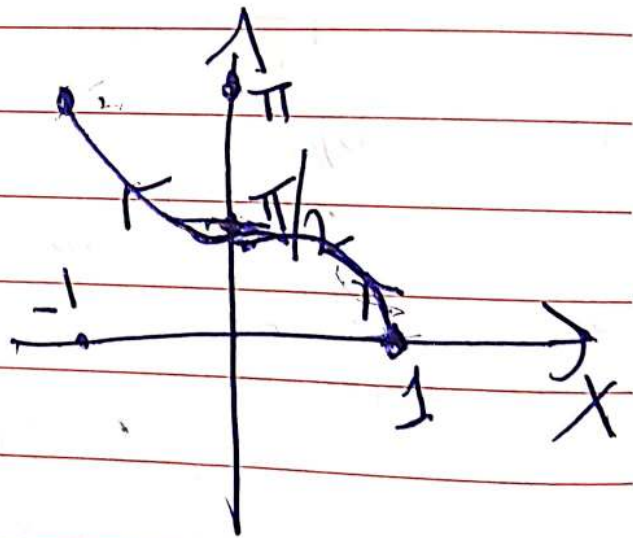
her iki tarafın Cosinüsü alınırsa -

$\cos y = x$ $f^{-1}(x) = \cos x$ (\cos^{-1} 'in
Ters fonksiyonu
($\cos x$ 'in ters fonksiyonu $\cos^{-1} x = \arccos x$ dir.

$y = \cos x$ $[0, \pi] \rightarrow [-1, 1]$



$y = \cos x$
fonksiyonunun
Grafiği



$y = \cos^{-1} x$
fonksiyonunun
Grafiği

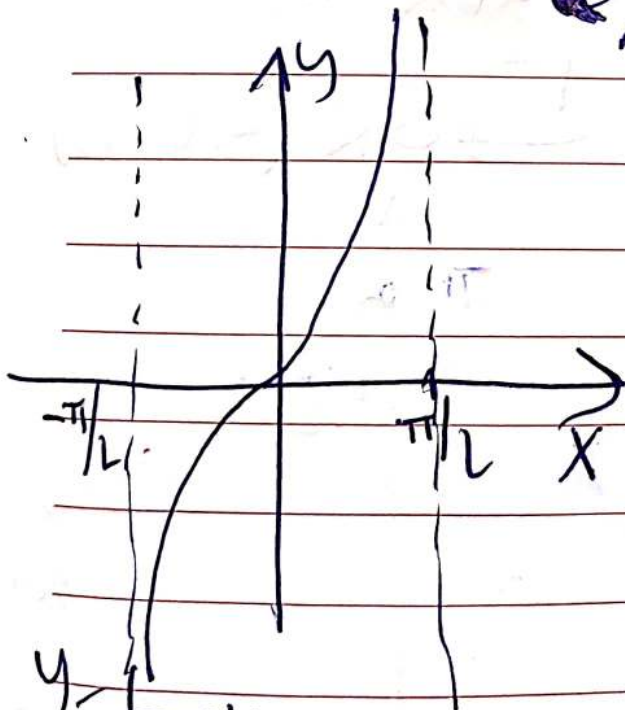
$y = \arctan x = \tan^{-1} x$ fonksiyonu

T.K: $(-\infty, \infty)$ \xrightarrow{f} G.K: $(-\pi/2, \pi/2)$

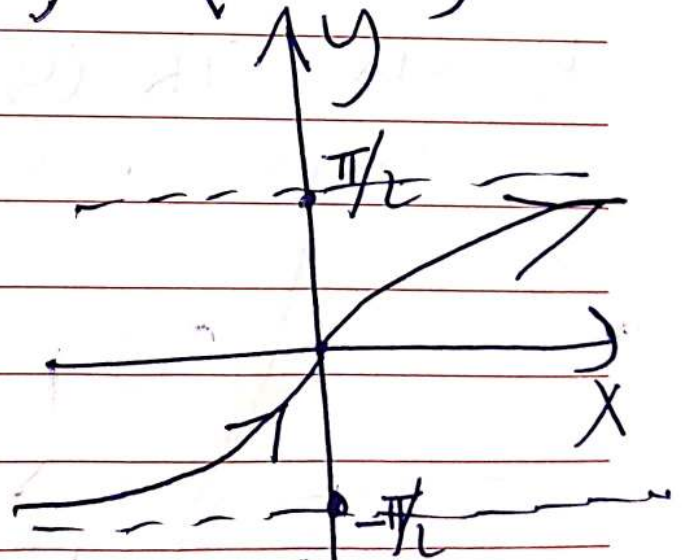
her iki tarafın tanjantı alınırsa.

$\tan y = x$ $f^{-1}(x) = \tan x$ ($\tan^{-1} x$ 'in ters fonksiyonu)
 $\tan x$ 'in ters fonksiyonu ise $\tan^{-1} x$ dir.

$y = \tan x$ $(-\pi/2, \pi/2) \xrightarrow{f} (-\infty, \infty)$



$y = \tan x$
fonksiyonunun
grafigi



$y = \tan^{-1} x$ fonksiyonunun
grafigi

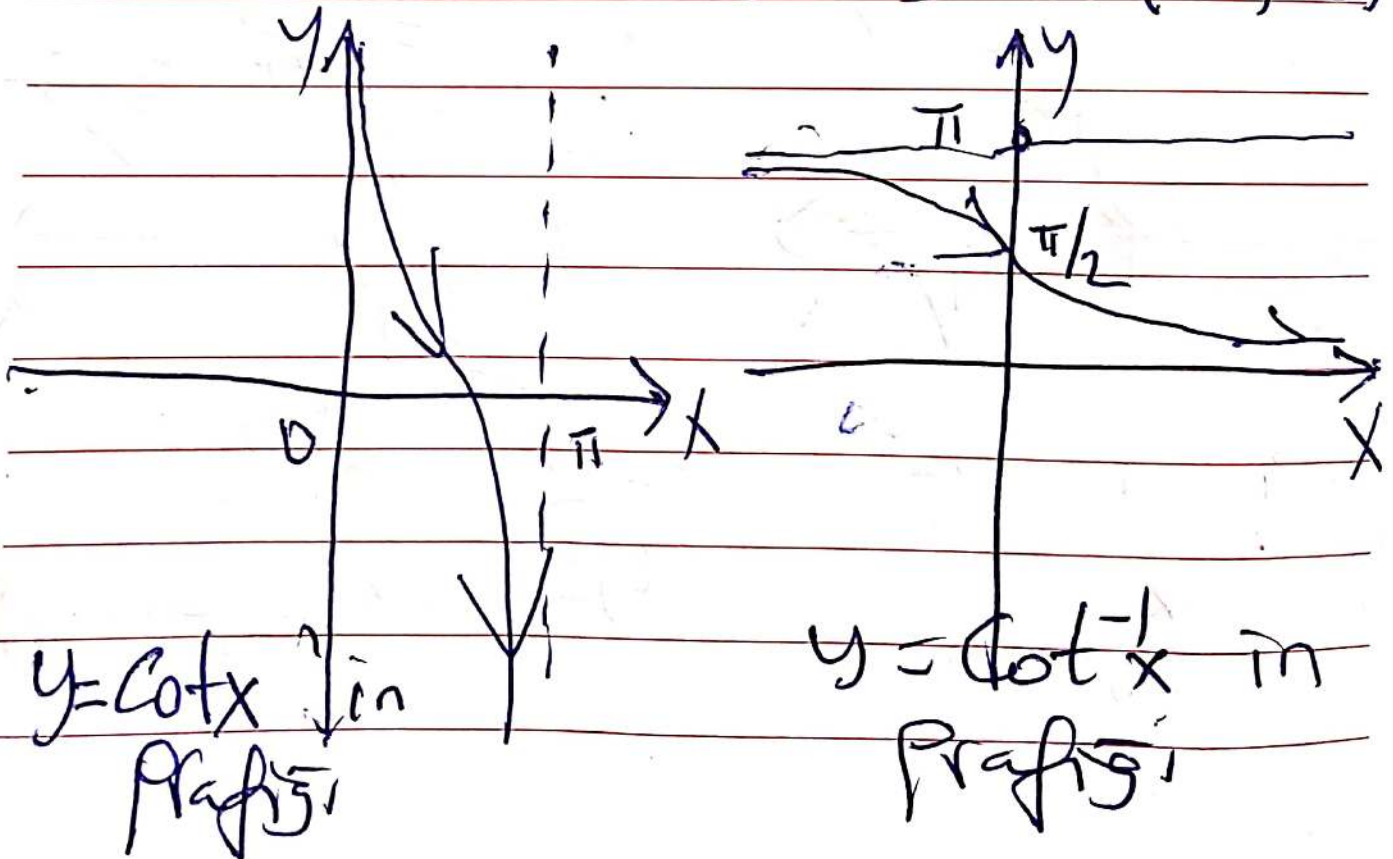
$y = \cot^{-1} x = \arccot x$ fonksiyonu

T.K. $(-\infty, \infty) \xrightarrow{f} G.K. (0, \pi)$

her iki tarafın Kotanjanti alınır.

$\cot y = x$, $f^{-1}(x) = \cot x$ elde edilir. Yani $y = \cot^{-1} x$ in ters fonksiyonu $y = \cot x$, $y = \cot x$ ' in ters fonksiyonu $\cot^{-1} x$ dir.

$y = \cot x$ T.K. $(0, \pi) \xrightarrow{f} G.K. (-\infty, \infty)$



$y = \text{Sec}^{-1}x = \text{arcsec}x$ fonksiyonu

TK: $x \leq -1 \cup x \geq 1$ \xrightarrow{f} GK: $[0, \pi] \setminus \{\frac{\pi}{2}\}$

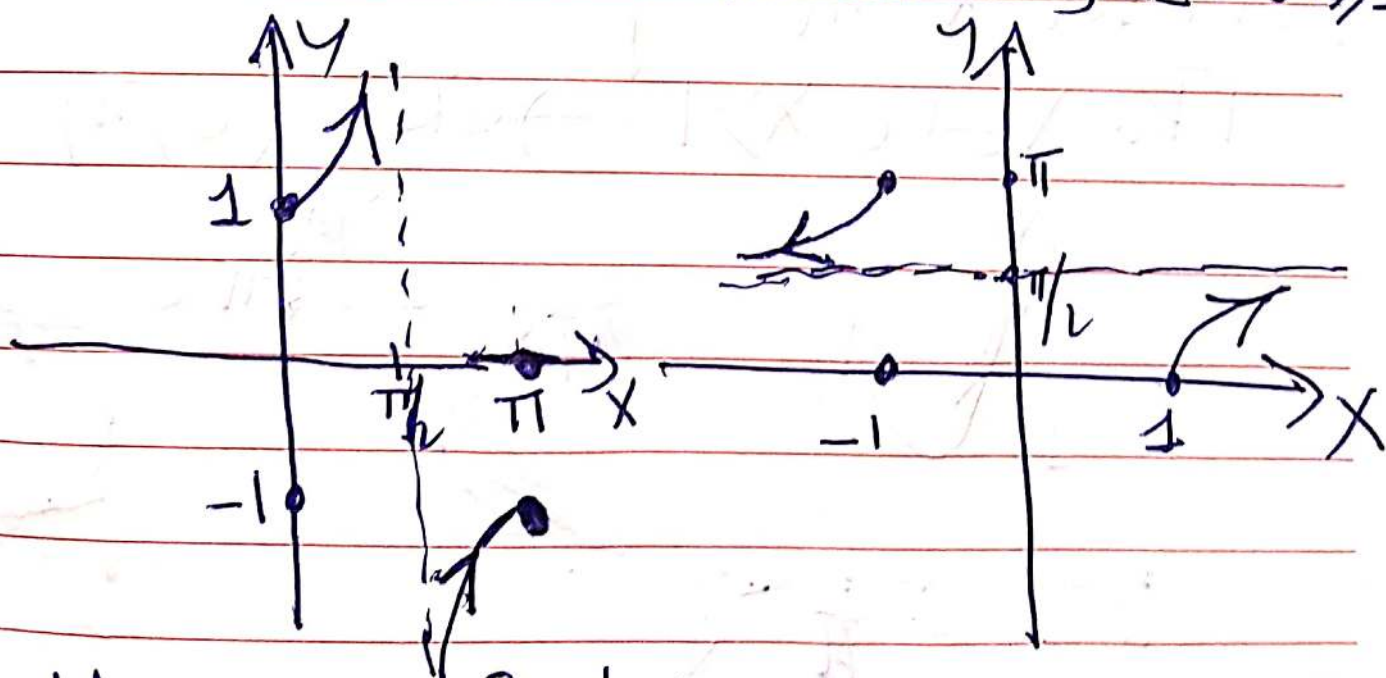
$y = \text{arcsec}x$ 'in her iki tarafının sekanti alınırsa.

$\text{Sec}y = x$ $f^{-1}(x) = \text{Sec}x$ elde edilir.

$\text{Sec}^{-1}x$ 'in ters fonksiyonu $\text{sec}x$,

$\text{sec}x$ 'in ters fonksiyonu $\text{Sec}^{-1}x$ 'dir.

$y = \text{Sec}x$ TK $[0, \pi] \setminus \{\frac{\pi}{2}\}$ \xrightarrow{f} $x \leq -1 \cup x \geq 1$



$y = \text{sec}x$ fonksiyonunun
Grafiği



$y = \operatorname{Cosec}^{-1} x = \arccsc x$ fonksiyonu.

her iki tarafın kosecantı alınır.

$$\operatorname{Cosec} y = x, \quad f^{-1}(x) = \operatorname{Cosec} x$$

$\operatorname{Cosec} x$ 'in ters fonksiyonu $\operatorname{Cosec}^{-1} x$

$\operatorname{Cosec}^{-1} x$ in " " " " $\operatorname{Cosec} x$ dir.

$y = \operatorname{Cosec}^{-1} x$ in f

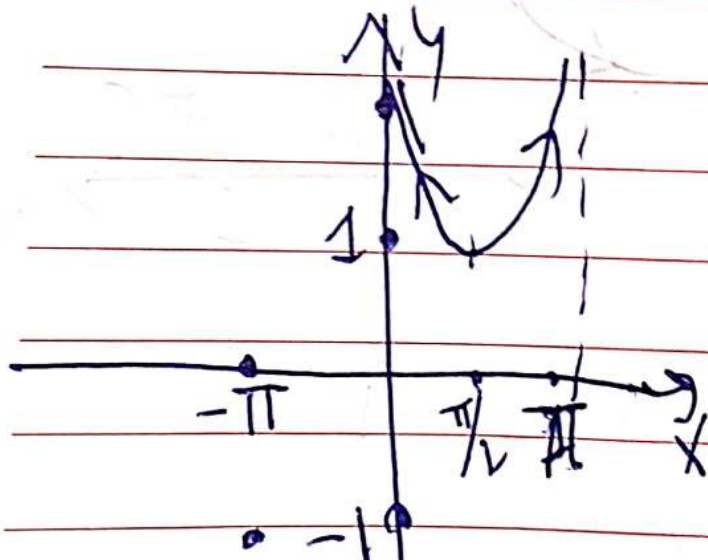
TK: $(0, \pi)$

GK: $x \leq -1 \cup x \geq 1$

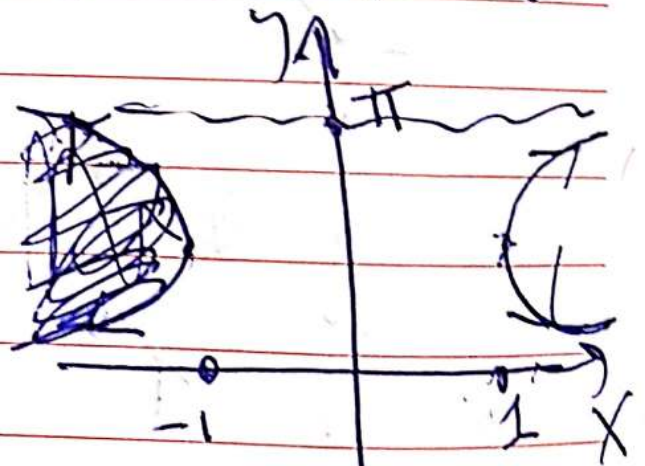
$y = \operatorname{Cosec}^{-1} x$ in f

TK: $x \leq -1 \cup x \geq 1$

GK: $(0, \pi)$



$y = \operatorname{Cosec} x$ fonksiyonunun grafiği



$y = \operatorname{Cosec}^{-1} x$ fonksiyonunun grafiği

$$y = \arcsin x \quad y' = \frac{1}{\sqrt{1-x^2}}$$

$$y = \arccos x \quad y' = -\frac{1}{\sqrt{1-x^2}}$$

$$y = \arctan x \quad y' = \frac{1}{1+x^2}$$

$$y = \operatorname{arccot} x \quad y' = -\frac{1}{1+x^2}$$

$$y = \operatorname{arcsec} x \quad y' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$y = \operatorname{arccsc} x \quad y' = \frac{-1}{|x|\sqrt{x^2-1}}$$

