# BLM3620 Digital Signal Processing

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# Course Materials

***Important Materials:***

- James H. McClellan, R. W. Schafer, M. A. Yoder, *DSP First Second Edition*, Pearson, 2015.

- Lizhe Tan, Jean Jiang, *Digital Signal Processing: Fundamentals and Applications*, Third Edition, Academic Press, 2019.

***Auxilary Materials:***

- Prof. Sarp Ertürk, *Sayısal İşaret İşleme*, Birsen Yayınevi.

- Prof. Nizamettin Aydin, DSP Lecture Notes.

- J. G. Proakis, D. K. Manolakis, *Digital Signal Processing Fourth Edition*, Peason, 2014.

- J. K. Perin, *Digital Signal Processing*, *Lecture Notes,* Standford University, 2018.

# Syllabus

| Week | Lectures |
|---|---|
| 1 | Introduction to DSP and MATLAB |
| 2 | Sinuzoids and Complex Exponentials |
| 3 | Spectrum Representation |
| 4 | Sampling and Aliasing |
| 5 | Discrete Time Signal Properties and Convolution |
| 6 | Convolution and FIR Filters |
| 7 | Frequency Response of FIR Filters |
| 8 | Midterm Exam |
| 9 | Discrete Time Fourier Transform and Properties |
| 10 | Discrete Fourier Transform and Properties |
| 11 | Fast Fourier Transform and Windowing |
| 12 | z- Transforms |
| 13 | FIR Filter Design and Applications |
| 14 | IIR Filter Design and Applications |
| 15 | Final Exam |

For more details -> Bologna page: http://www.bologna.yildiz.edu.tr/index.php?r=course/view&id=5730&aid=3

# Lecture #8 – Frequency Response of FIR Filters

- Frequency Response
- Digital Filtering
- Frequency Scaling
- Exercises
- FIR Filter Application

# Remember: Classification of Impulse Response h[n]

## FIR – Finite Impulse Response:

- Number of impulses are limited.
- Always stable.

$For\ example: h[n] = \delta[n-1] + 5\delta[n-5]$

## IIR – Infinite Impulse Response:

- Number of impulses are infinite.
- Sometimes these systems are not stable.

$For\ example: h[n] = u[n-1] + 5u[n-5]$

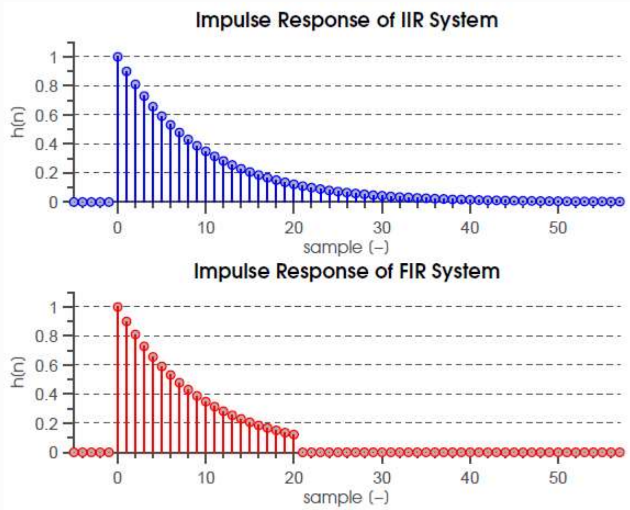
Finite impulse response (FIR):

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

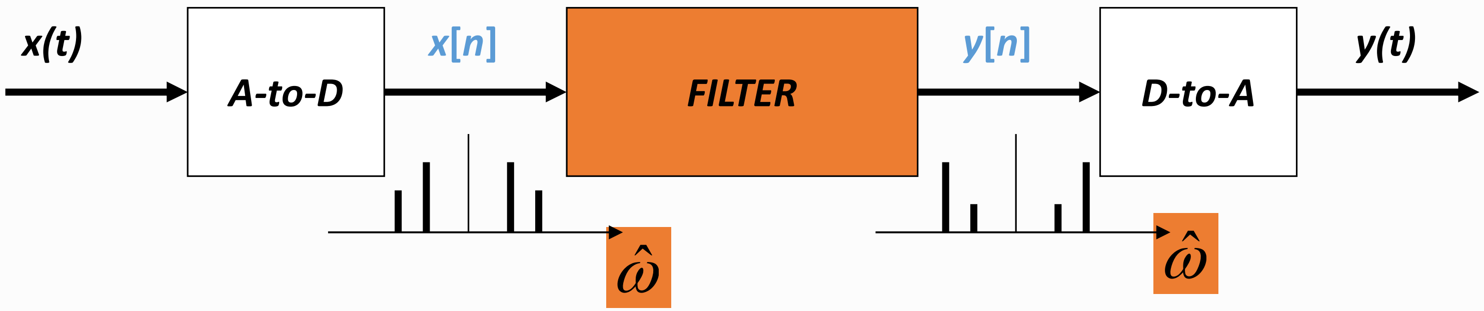
Infinite impulse response (IIR):

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

Another example:



# DOMAINS: Time & Frequency




- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

- **Time-Domain: "n" = time**
  - x[n] discrete-time signal
  - x(t) continuous-time signal

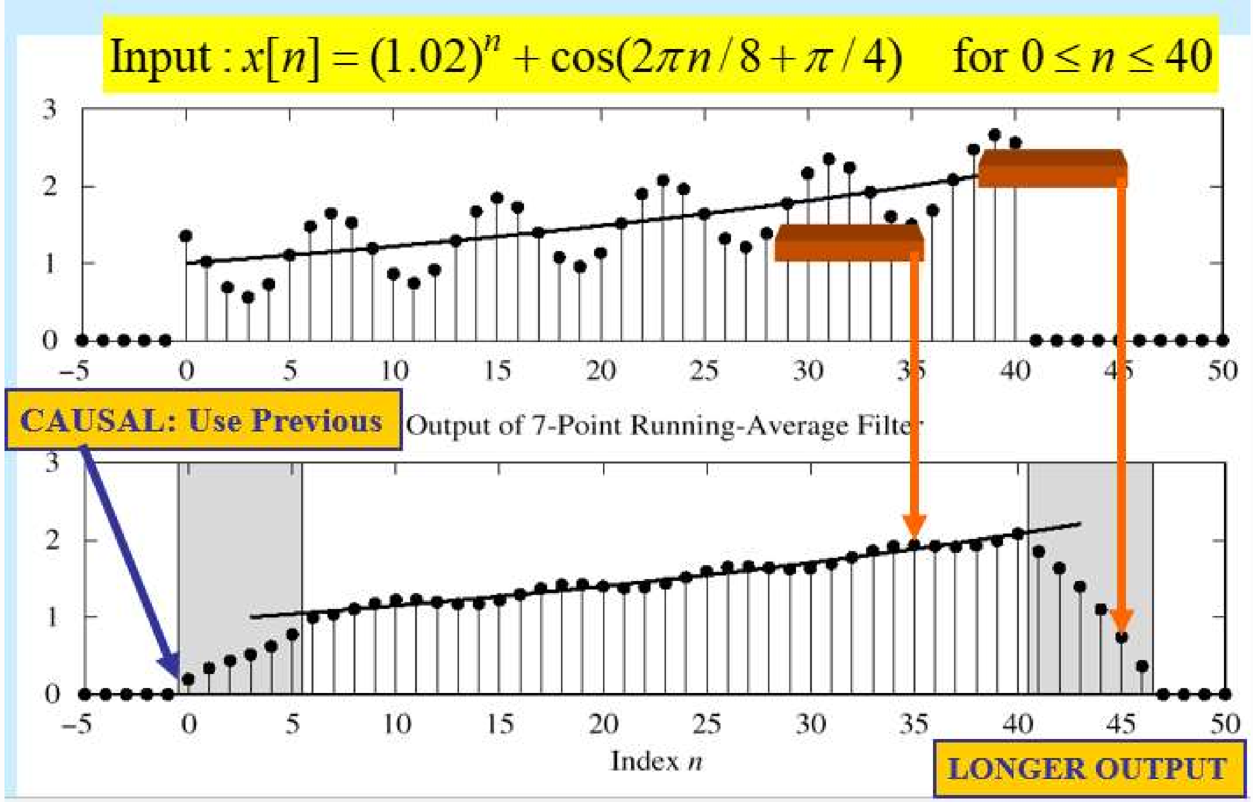
# Example FIR Filters

- 3-point AVERAGER

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

- 7-point AVERAGER

$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$



## But...

How can I calculate the effects of this filter on digital frequency?

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

**x[n] is the input signal—a complex exponential**

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

**FIR DIFFERENCE EQUATION**

$H(e^{j\hat{\omega}}) = H(\hat{\omega})$

$= \sum h[k] \cdot e^{-j\hat{\omega}k}$

- Use the FIR "Difference Equation"

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} b_k Ae^{j\varphi}e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)}\right) Ae^{j\varphi}e^{j\hat{\omega}n}$$

$$= H(\hat{\omega}) Ae^{j\varphi}e^{j\hat{\omega}n}$$

# New Term: Frequency Response $H(e^{j\hat{\omega}})$

- At each frequency, we can DEFINE

$$\boxed{H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}} \leftarrow \boxed{\text{FREQUENCY RESPONSE}}$$

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency
- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$

$|H(e^{j\hat{\omega}})| \rightarrow$ even symmetric

$\angle H(e^{j\hat{\omega}}) \rightarrow$ odd ''

$$A e^{j\hat{\omega}n} e^{j\varphi} \longrightarrow \boxed{\text{System}} \longrightarrow A e^{j\hat{\omega}n} e^{j\varphi} . H(e^{j\hat{\omega}})$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \cdots$$
$$= \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})}$$

Complex Number:

1- A Phase Component

2- A Magnitude Component

$H(e^{j\omega}) = \sum_{k=0}^{\infty} h[k] \cdot e^{-j\hat\omega k}$

$H(e^{j\hat\omega}) = \sum_{k=0}^{2} h[k] \cdot e^{-j\hat\omega k}$ $\quad 1 + 2 \cdot e^{-j\hat\omega} + 1 \cdot e^{-j\hat\omega 2}$

$\to e^{-j\hat\omega}(e^{j\hat\omega} + 2 + e^{-j\hat\omega})$ $\quad (2\cos\hat\omega + 2) \cdot e^{-j\hat\omega}$

# Example

$$H(e^{j\hat\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat\omega k}$$

$$H(e^{j\hat\omega}) = (2 + 2\cos\hat\omega)e^{-j\hat\omega}$$

not → $\hat\omega$ olduğu için $-\pi$ – $\pi$ arasında değer alır çünkü dijital frekans

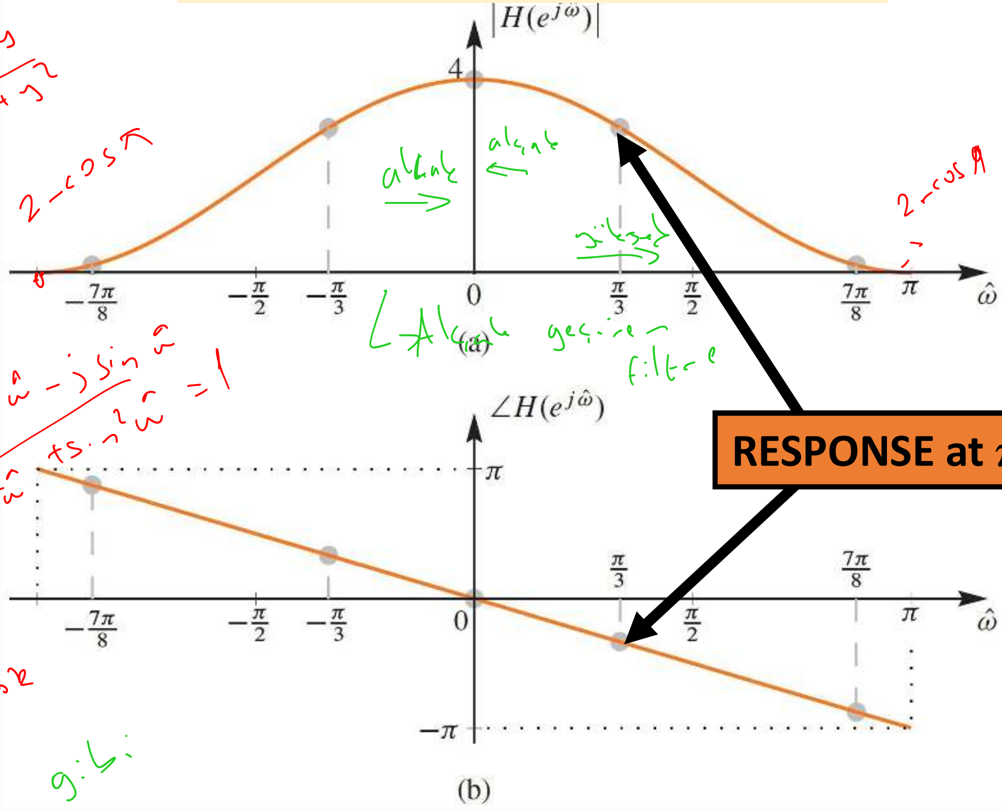
$$\{b_k\} = \{1,\ 2,\ 1\}$$

$z = x + jy$ $\quad |z| = \sqrt{x^2 + y^2}$

$$H(e^{j\hat\omega}) = 1 + 2e^{-j\hat\omega} + e^{-j2\hat\omega}$$
$$= e^{-j\hat\omega}(e^{j\hat\omega} + 2 + e^{-j\hat\omega})$$
$$= e^{-j\hat\omega}(2 + 2\cos\hat\omega)$$

**EXPLOIT SYMMETRY**

$2 - \cos\pi$




alçak → ← alçak

yüksek

$2 - \cos\pi$

↳ alçak geçiren filtre

$e^{-j\hat\omega} = \cos\hat\omega - j\sin\hat\omega$

$|e^{-j\hat\omega}| = \sqrt{\cos^2\hat\omega + \sin^2\hat\omega} = 1$

↳ genliğe etkisi yok

**RESPONSE at**

Since $(2 + 2\cos\hat\omega) \geq 0$

Magnitude is $|H(e^{j\hat\omega})| = (2 + 2\cos\hat\omega)$ → genlik yanıtı

and Phase is $\angle H(e^{j\hat\omega}) = -\hat\omega$ → faz yanıtı

$y = -x$ gibi

What does this filter do in frequency domain?

# Example – 2 : For the previous system...

Find $y[n]$ when $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\, \hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4} e^{j(\pi/3)n} = 6\, e^{-j\pi/12} e^{j(\pi/3)n}$$

# Example - 3 : For the previous system...

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$e^{-j\omega} \cdot (2 \cdot \cos \omega n)$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3+\pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

Use Linearity

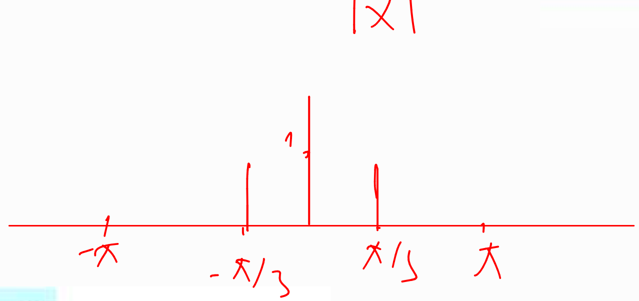
$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3+\pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3+\pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

# Example - 3 : For the previous system…

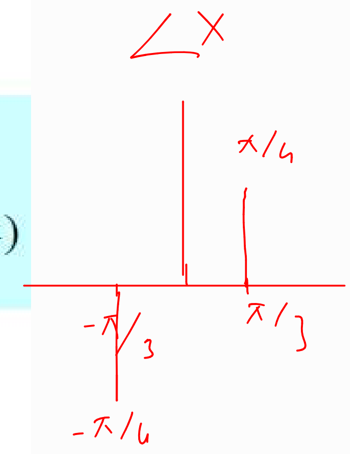
Find $y[n]$ when $x[n]=2\cos(\frac{\pi}{3}n+\frac{\pi}{4})$




$$H(e^{j\hat{\omega}})=(2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

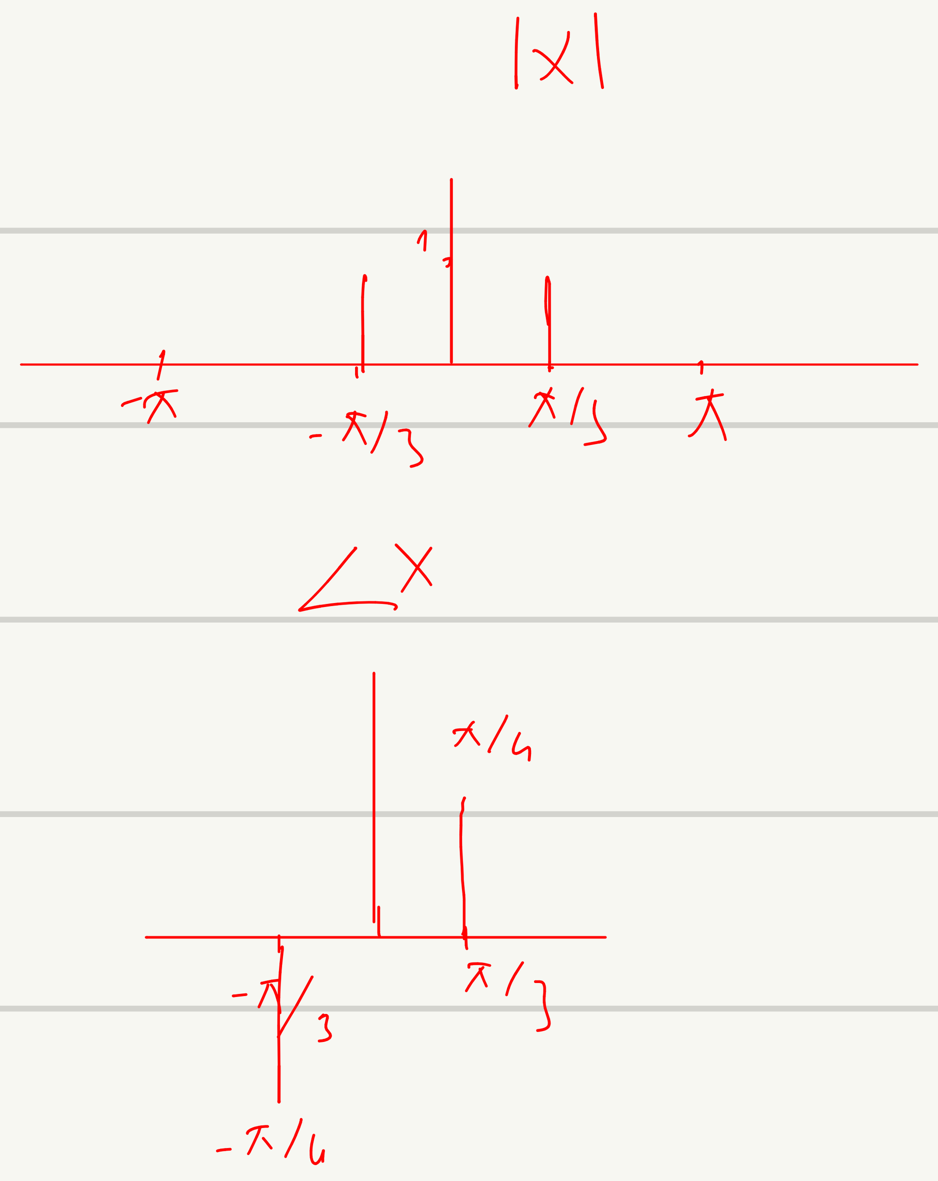
$$y_1[n]=H(e^{j\pi/3})e^{j(\pi n/3+\pi/4)}=3e^{-j(\pi/3)}e^{j(\pi n/3+\pi/4)}$$

$$y_2[n]=H(e^{-j\pi/3})e^{-j(\pi n/3+\pi/4)}=3e^{j(\pi/3)}e^{-j(\pi n/3+\pi/4)}$$




$$y[n]=3e^{j(\pi n/3-\pi/12)}+3e^{-j(\pi n/3-\pi/12)}$$

$$\Rightarrow y[n]=6\cos(\frac{\pi}{3}n-\frac{\pi}{12})$$

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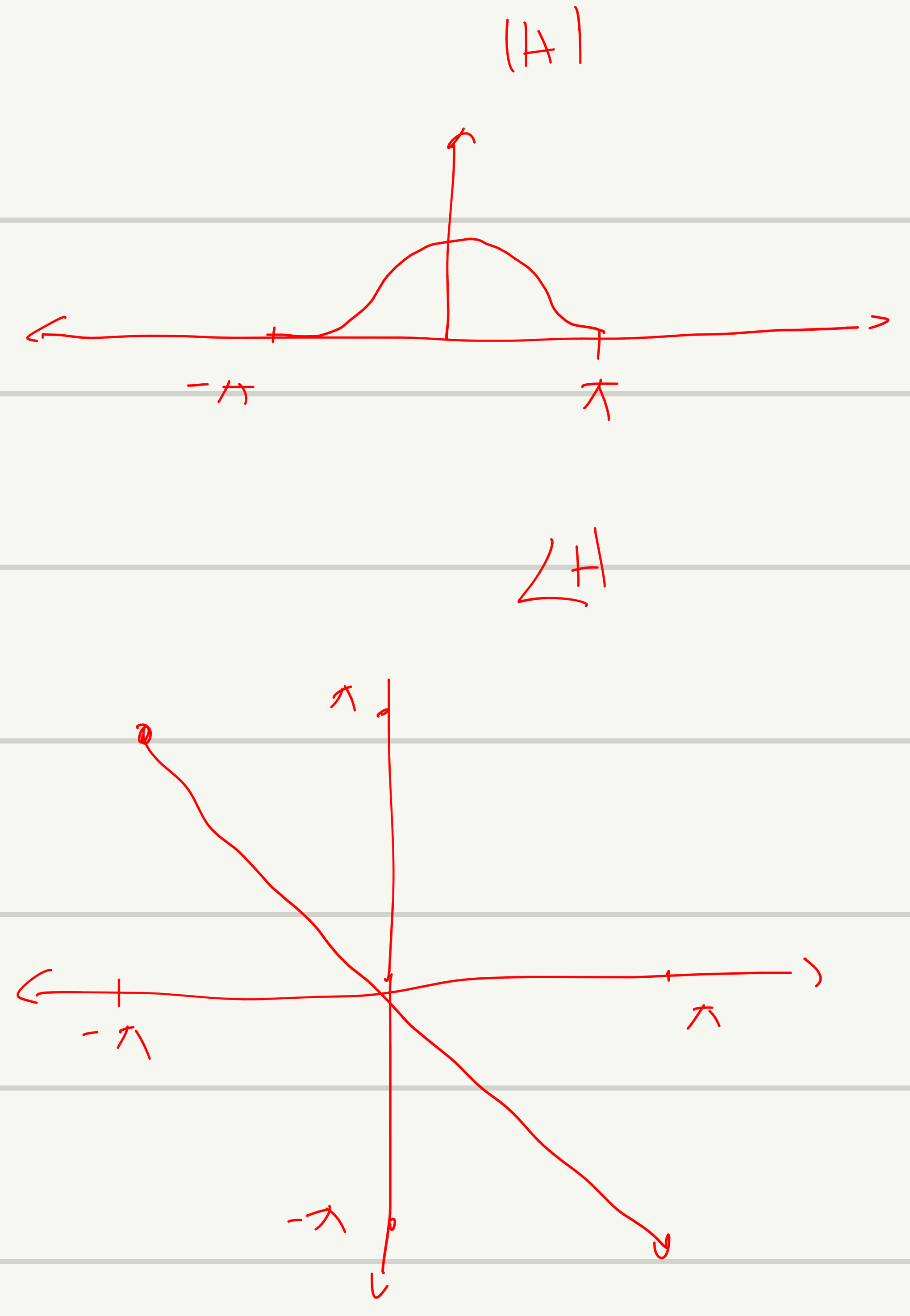
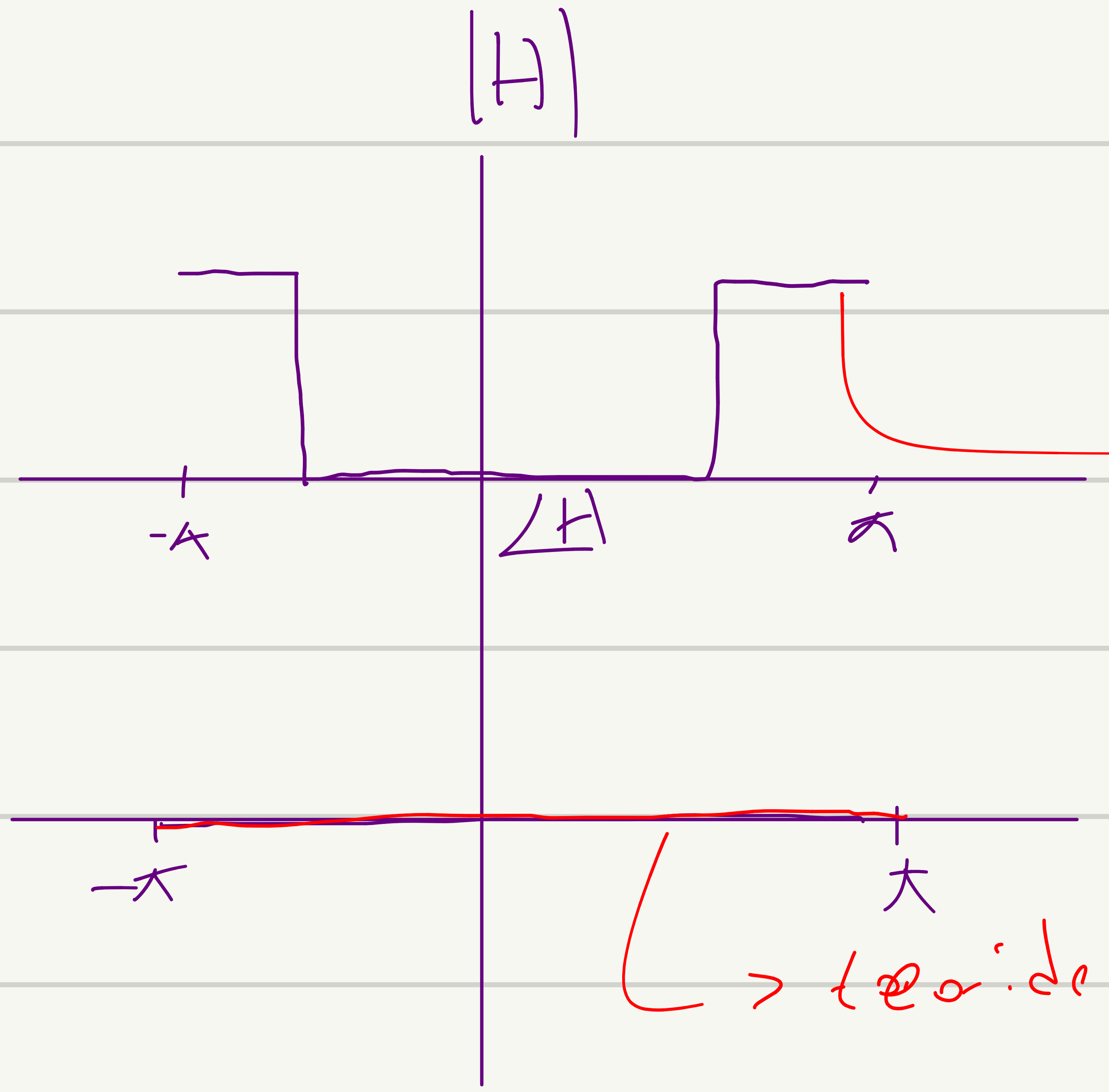



→ 3 3

$\rightarrow \left(-\frac{\pi}{4} + \frac{\pi}{3}\right) \quad \left(+\frac{\pi}{4} - \frac{\pi}{3}\right)$

$3e^{+\frac{\pi}{12}} + 3e^{-\frac{\pi}{12}}$




$x[n] = 5\cdot\cos(\pi n) + 3\cdot\cos(0.1\pi n)$ → 0

$y[n] = 5\cdot\cos(\pi n)$

→ köşeli yapılar oluşturmak için çok uzun H değerleri gerekir

→ teoride böyle olur ama pratikte → böyle olur.

$h[n] = \delta[n] - \delta[n-1] \rightarrow \sum_{k=0}^{1} h[k]\, e^{-j\omega k} \rightarrow 1 - e^{-j\omega} = H(e^{j\omega})$

$\hookrightarrow 1 - \cos\hat{\omega} + j\sin\hat{\omega}$

$A = \sqrt{(1-\cos\hat{\omega})^2 + (-\sin\hat{\omega})^2}$

$\sum_{k=0}^{1} h[k] \cdot e^{-j\omega k}$

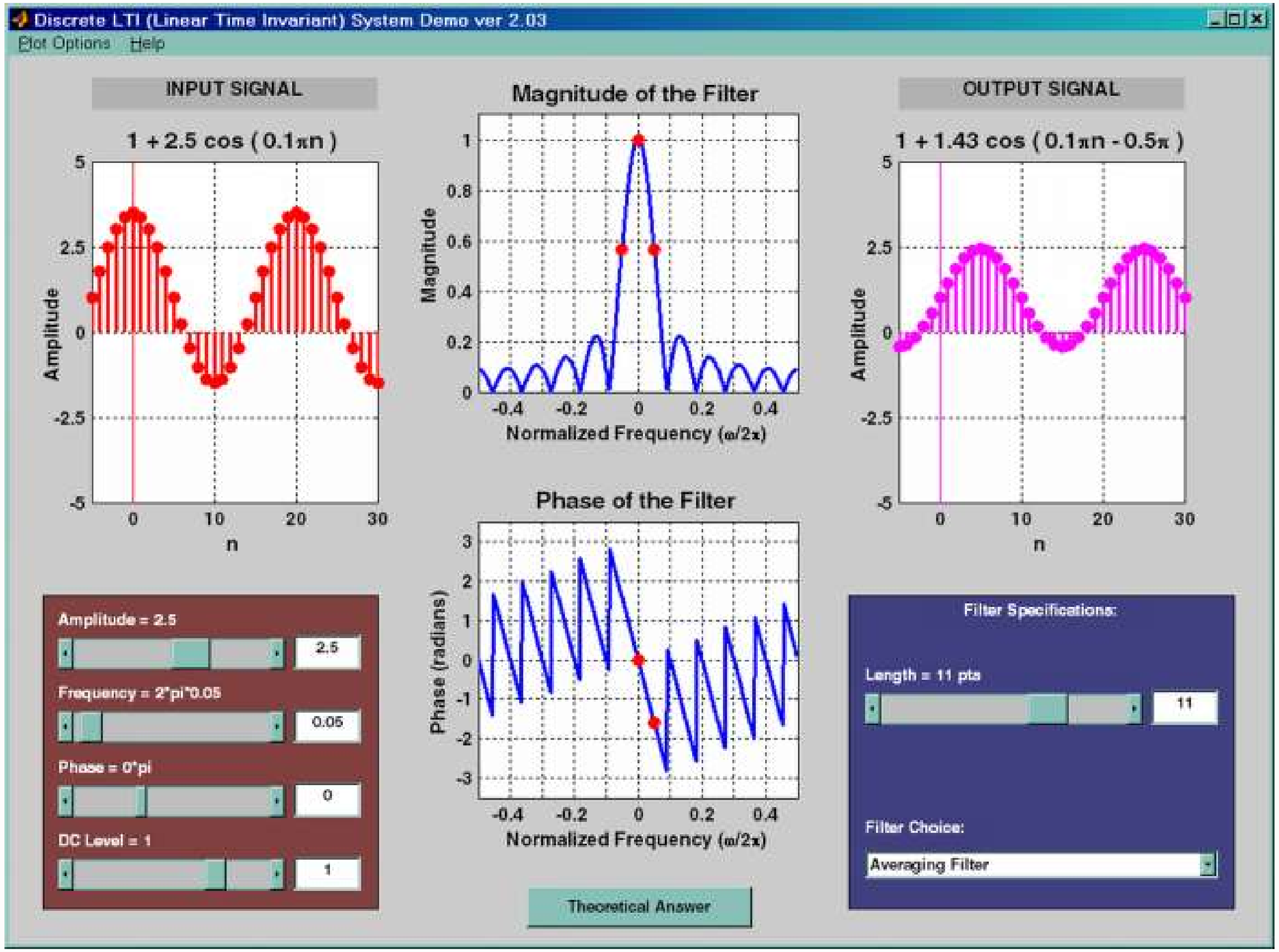
$1 - e^{-j\omega k}$

$\hat{\omega} = 0$ için $\rightarrow \sqrt{(1-1)^2 + 0^2} = 0$

$\hat{\omega} = \pi$ için $\rightarrow \sqrt{2^2 + 0^2} \rightarrow 2$

$|H|$

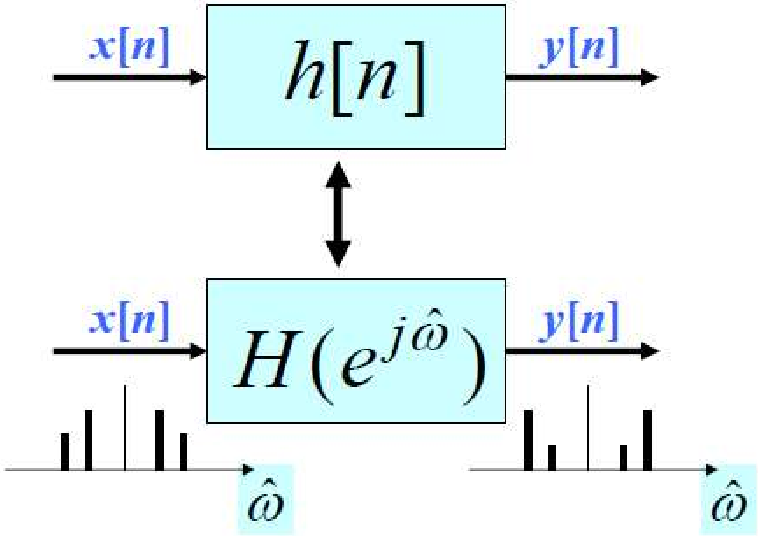
# DLTI Demo with Sinuzoids



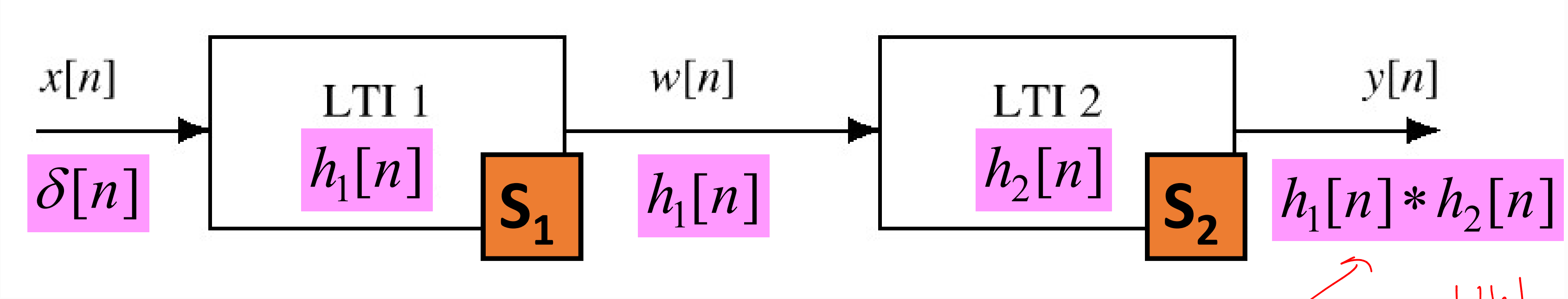

https://dspfirst.gatech.edu/matlab/#dltidemo

# Summary over Block Diagrams

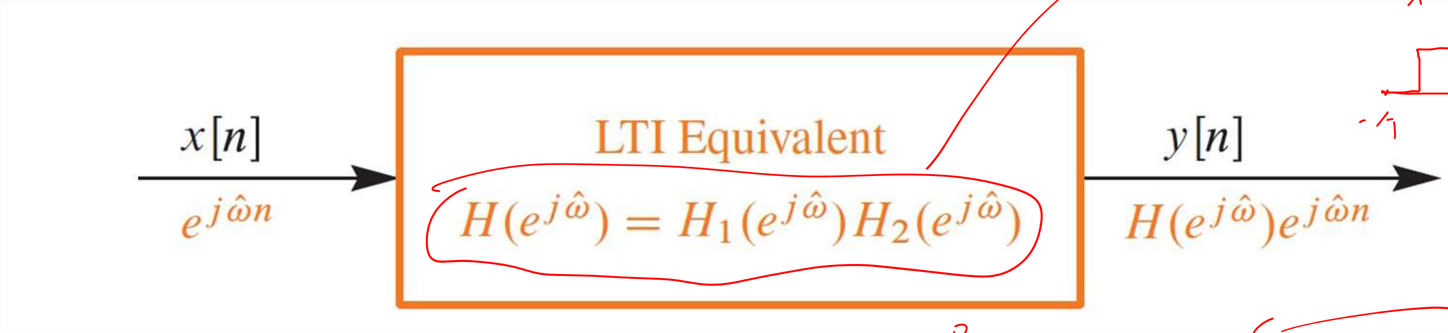
- Equivalent Representations

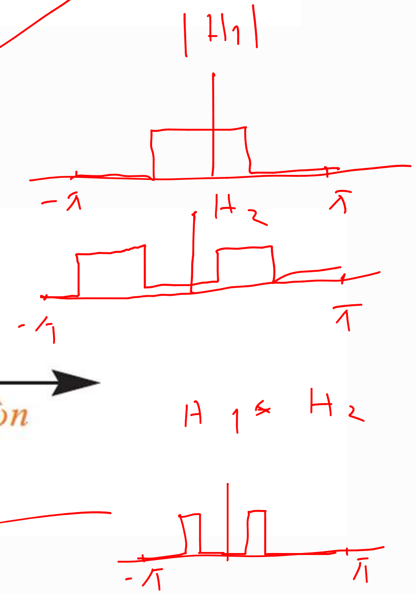


# Cascaded LTI Systems




**WHAT is the overall FREQUENCY RESPONSE ?**




Band geçiren filtre

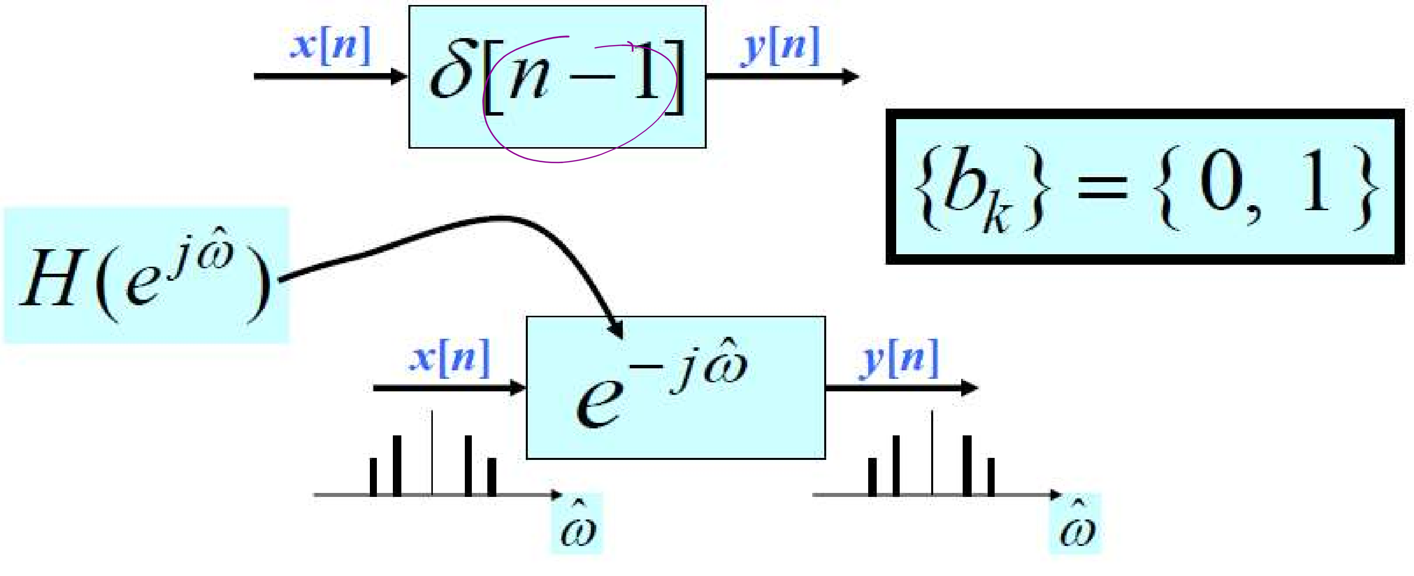
$$\boxed{H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})}$$

https://dspfirst.gatech.edu/chapters/06firfreq/demos/blockd/index.html

# Example – 4: Unit Delay System

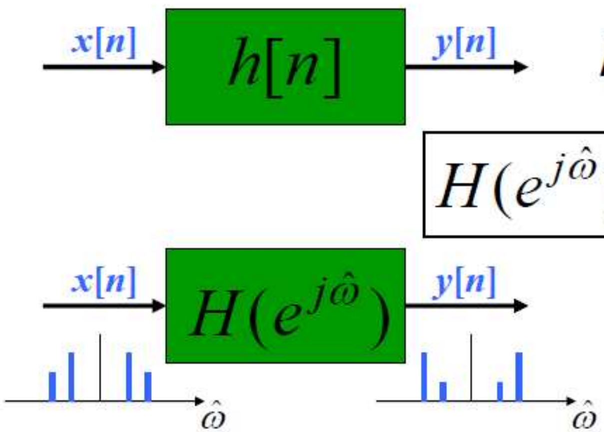
Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-1]$

$\sum_{k=0}^{\infty} h[k]\, e^{-j\hat{\omega}k}$



# Example – 5: Freq. Domain to Time

$H(e^{j\hat{\omega}})$ and find $h[n]$ or $b_k$




$h[n] = ?$

$$\boxed{H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})}$$

$$\boxed{H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}}$$

$$3.5\,\frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\left(e^{-j\omega t} + e^{-3j\omega t}\right) 3.5$$

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$

**EULER's Formula**

$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$b_1 = 3.5$

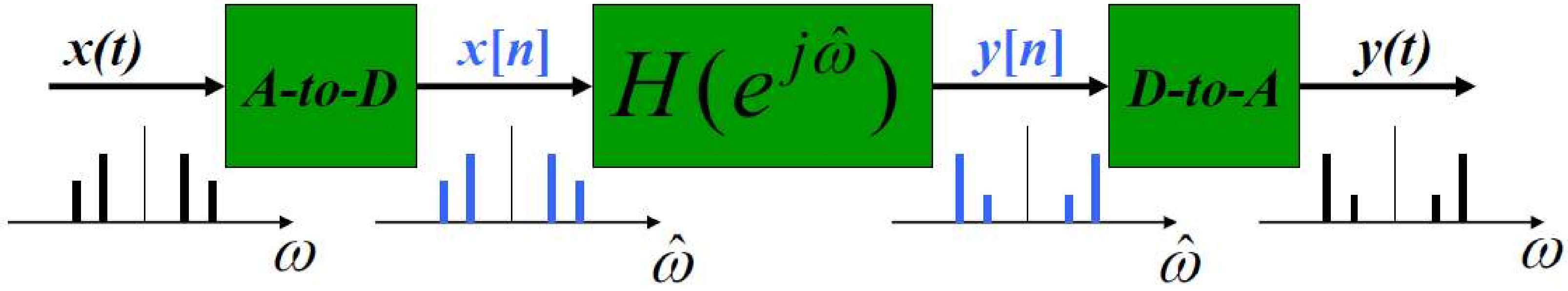
$b_3 = 3.5$

$h[n] = \{0, 3.5, 0, 3.5\}$

$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0, 3.5, 0, 3.5\}$$

# Digital Filtering




$\boxed{\omega}$ – SPECTRUM of x(t) (SUM of SINUSOIDS)

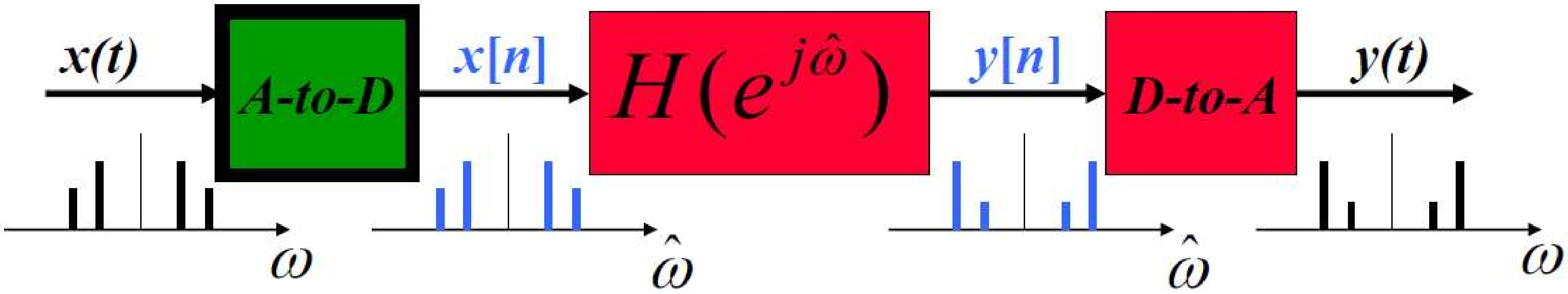
$\boxed{\hat{\omega}}$

– SPECTRUM of x[n]
  - Is ALIASING a PROBLEM ?

– SPECTRUM y[n] (FIR Gain or Nulls)

$\boxed{\omega}$ – Then, OUTPUT y(t) = SUM of SINUSOIDS

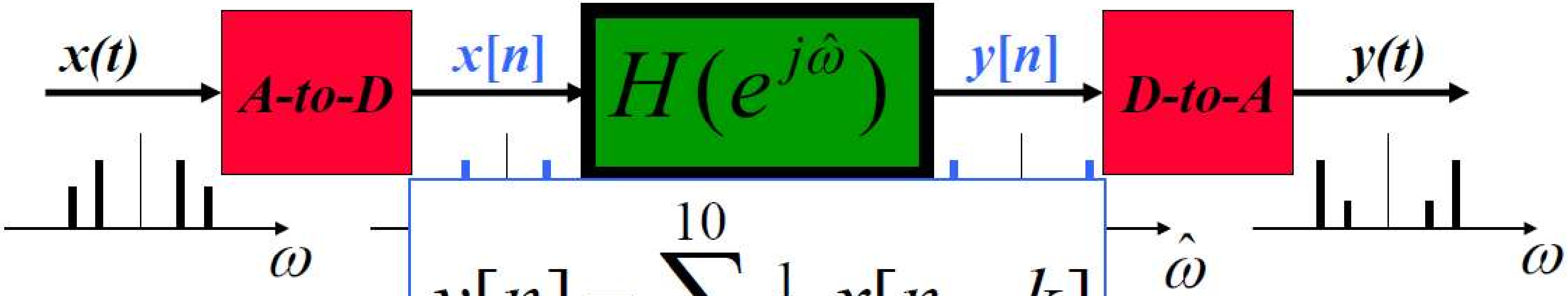
# Frequency Scaling




- TIME SAMPLING: $t = nT_s$
  - IF NO ALIASING:
  - FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

# 11-pt Averager




$$y[n] = \sum_{k=0}^{10} \frac{1}{11} x[n-k]$$

250 Hz

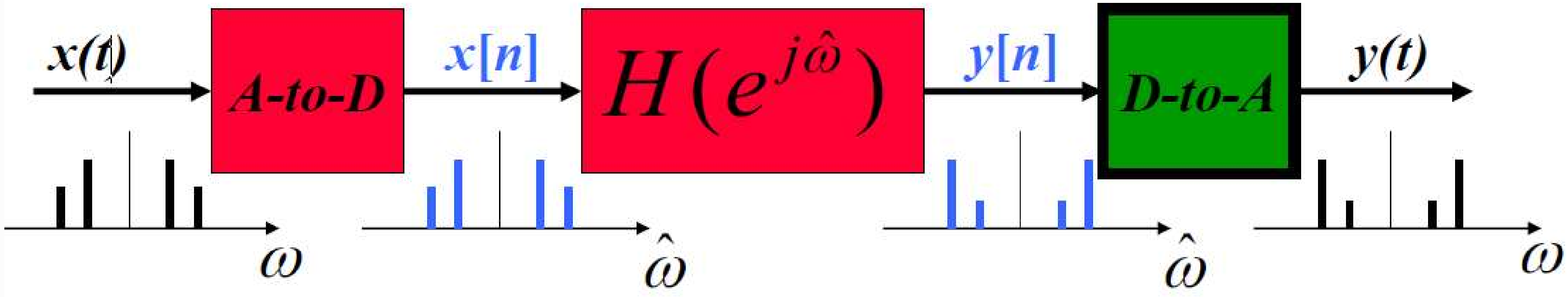
25 Hz

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

?

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

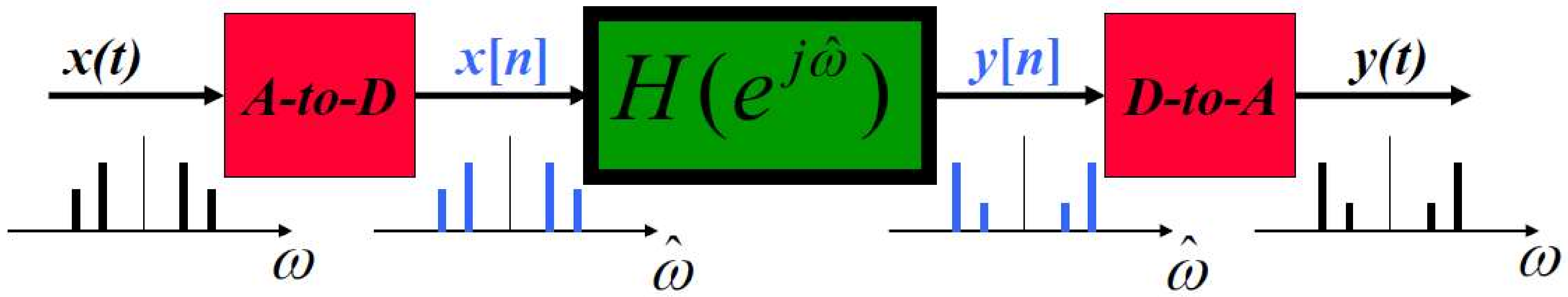
# D-A Frequency Scaling




- TIME SAMPLING: $\boxed{t = nT_s \Rightarrow n \leftarrow t f_s}$
- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

$$\boxed{\omega = \hat{\omega} f_s}$$

# Summary



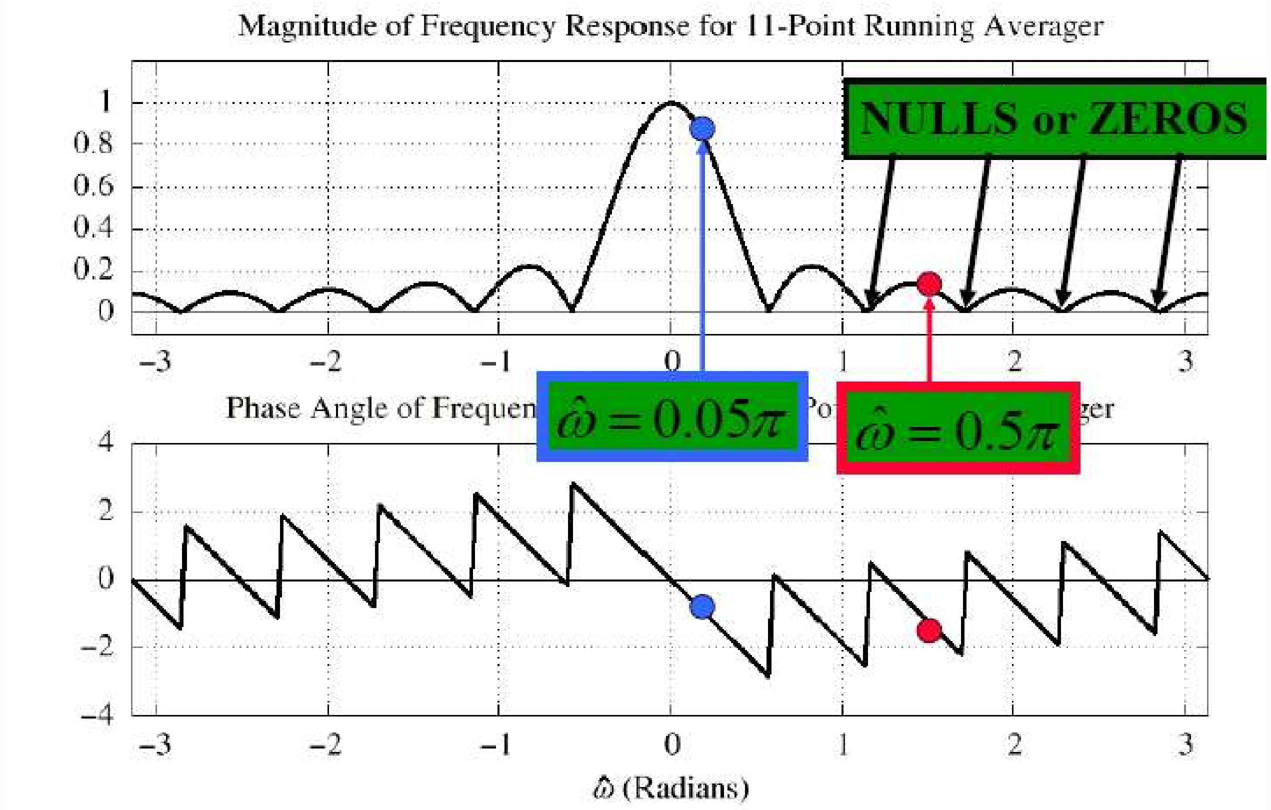

| | | | | |
|---|---|---|---|---|
| • 250 Hz | • $0.5\pi$ | $H(e^{j0.5\pi})$ | • $0.5\pi$ | • 250 Hz |
| • 25 Hz | • $.05\pi$ | $H(e^{j0.05\pi})$ | • $.05\pi$ | • 25 Hz |

**Fs = 1000 Hz**

**NO new freqs**

$$t = nTs = n/1000 \qquad cos\,(2pi250t) \rightarrow cos\left(2.pi.250.\frac{n}{100}\right) = cos(0.5pi)$$

# Magnitude of Frequency Response




$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

At $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}(0.5\pi))}{11\sin(\frac{1}{2}(0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

# Magnitude of Frequency Response

$H(e^{j2\pi(25)/1000})$

$f_s = 1000$

$$H(e^{j2\pi(25)/1000}) = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

$$= 0.8811 e^{-j\pi/4}$$

$H(e^{j2\pi(250)/1000})$

$$H(e^{j2\pi(250)/1000}) = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

$$= 0.0909 e^{-j\pi/2}$$

MAG SCALE

PHASE CHANGE

$$y(t) = \underline{0.8811}\cos(2\pi(25)t \underline{- \pi/4}) + \underline{0.0909}\sin(2\pi(250)t \underline{- \pi/2})$$

# Remember: 17-pt Centralized Average filter to Noisy Audio

```
clc; clear all;

%% Load Sound
load ('piano2.mat');
x = x(1:16000);
soundsc(x,Fs);

%% Add noise
K = awgn(x,40);
soundsc(K,Fs);

%%  Filter
N = 17;
h = 1/N*ones(1,N);

%% Apply Convolution
y = conv(K,h,'same');
soundsc(y,Fs);
%%
plot(x,'r'); hold on; plot(y,'b');
```

LOAD THE SIGNAL

ADD A NOISE TO SIGNAL

FILTER THE SIGNAL

But How it works? What is the frequency response?

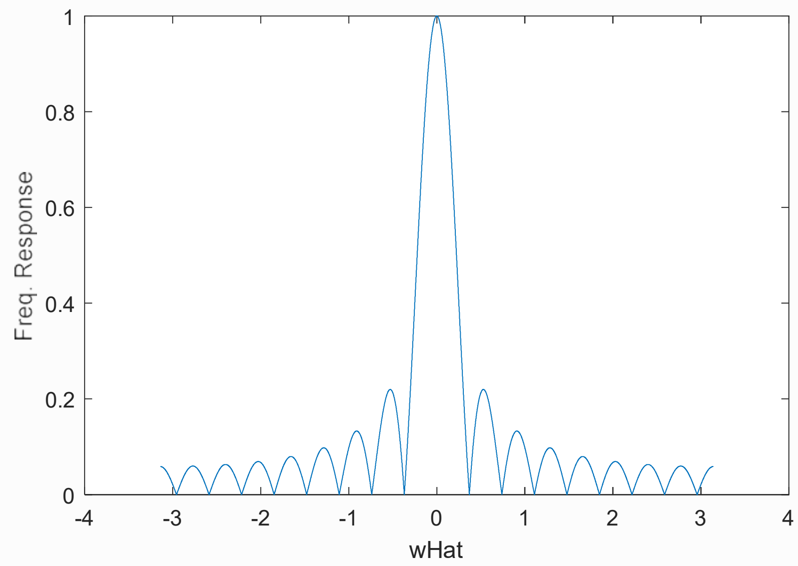
# 17-pt Averager

$$y(n) = \frac{1}{17}\sum_{k=0}^{16} x(n-k)$$

$$h(n) = \frac{1}{17}\sum_{k=0}^{16} \delta(n-k) = \frac{1}{17}\delta(n) + \cdots + \frac{1}{17}\delta(n-16)$$

$$\boxed{H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}}$$

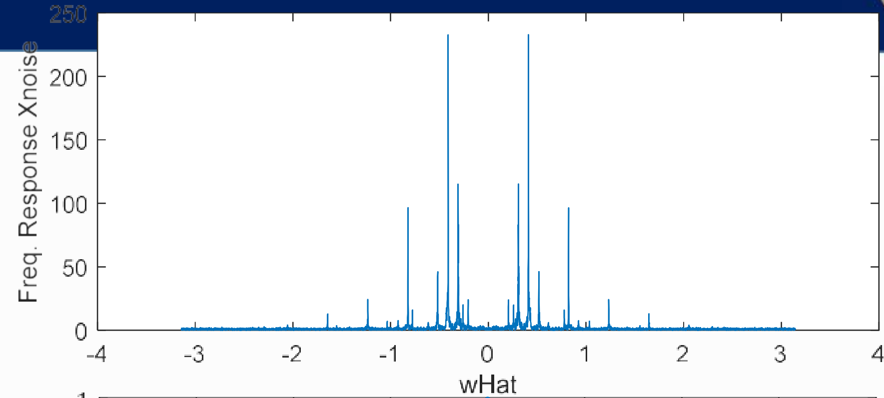
```
wHat = linspace(-pi,pi,Fs);
b = (1/17)*ones(1,17);
%%
H = zeros(1,Fs);
for k = 1:17
    H = H + b(k)*exp(-1j*wHat*k);
end

plot (wHat, abs(H));
xlabel('wHat');
ylabel('Freq. Response');
```



# Let’s make a deep analysis

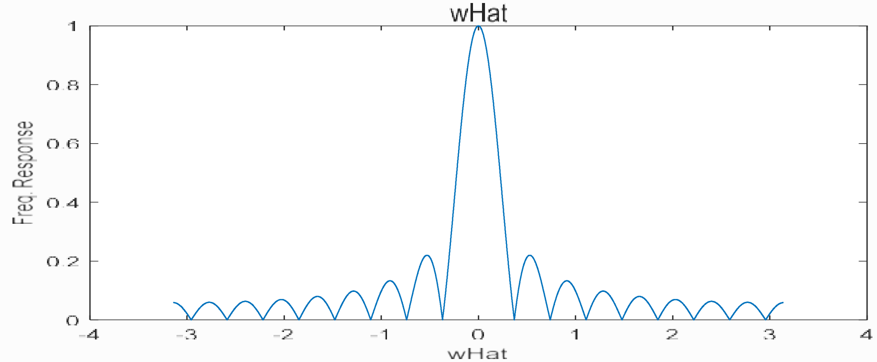
```
clc; clear all;
%% Load Sound
load ('piano2.mat');
x = x(1:16000);
X = fftshift(fft(x,Fs));
wHat = linspace(-pi,pi,Fs);
plot(wHat,abs(X));
xlabel('wHat');
ylabel('Freq. Response X');

%% Add Noise
Xnoise = awgn(x,40);
Xnoisef = fftshift(fft(Xnoise,Fs));
figure(2);
plot(wHat,abs(Xnoisef));
xlabel('wHat');
ylabel('Freq. Response Xnoise');

%%  Filter
N = 17; h = 1/N*ones(1,N);
y = conv(Xnoise,h,'same');
yf = fftshift(fft(y,Fs));
figure(3);
plot(wHat,abs(yf));
xlabel('wHat');
ylabel('Freq. Response Y');
```

Input




x

Filter

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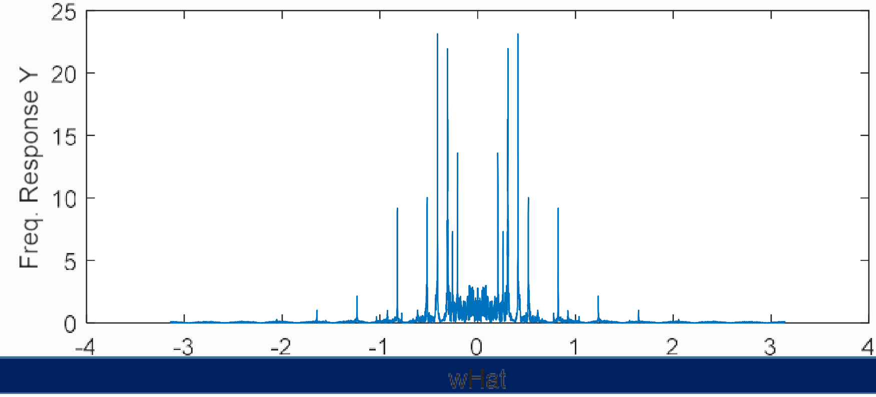
Output



Çok önemli değil

# Hearing Test – Audiometry Test (Try it at home)

Conduct a test of your hearing, and present the results as a frequency response plot.

https://dspfirst.gatech.edu/chapters/06firfreq/labs/HearingTestFreqResponse/HearingTestFreqResponse.pdf

http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/maxsens.html

Define a sampling frequency (Fs)

From 20 Hz to 22000 Hz with 100 Hz step do:

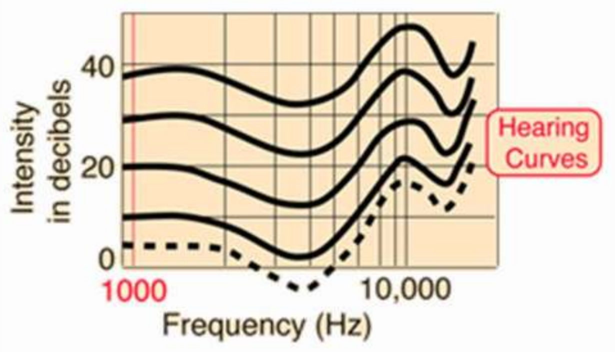
Play a tone with the selected frequency

Did you hear it: Give a score from 0-100.

Save this value for the last plot

Continue loop.

Use the hearing test to determine the frequency where your hearing sensitivity starts to drop significantly.




- Plot analog frequency vs. |H|. (Freq in logspace)
- Plot digital frequency vs. |H|. (Freq in logspace)

# Exercise - 1

A linear time-invariant system is described by the FIR difference equation

$$y[n] = x[n] - 3x[n-1] + 9x[n-2] - 3x[n-3] + x[n-4]$$

(a) Write a simple formula for the magnitude of the frequency response $|H(e^{j\hat{\omega}})|$. Express your answer in terms of real-valued functions only.

$1 - 3e^{-jw} + 9.e^{-2jw} - 3.e^{-3jw} + 1.e^{-4jw}$

(b) Derive a simple formula for the phase of the frequency response $\angle H(e^{j\hat{\omega}})$.

$$H(e^{j\hat{\omega}}) = 1 - 3e^{-j\hat{\omega}} + 9e^{-j2\hat{\omega}} - 3e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$= e^{-j2\hat{\omega}}\left(e^{j2\hat{\omega}} - 3e^{j\hat{\omega}} + 9 - 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right)$$

$$= e^{-j2\hat{\omega}}\left(2\cos 2\hat{\omega} - 6\cos\hat{\omega} + 9\right)$$

$\Rightarrow$

$$|H(e^{j\hat{\omega}})| = 2\cos 2\hat{\omega} - 6\cos\hat{\omega} + 9$$

$e^{-2jw}(e^{2jw} - 3.e^{jw} + 9 - 3.e^{-jw} + e^{-2jw})$

$2.\cos(2w) - 6.\cos(w) + 9$

$$\angle H(e^{j\hat{\omega}}) = -2\hat{\omega}$$

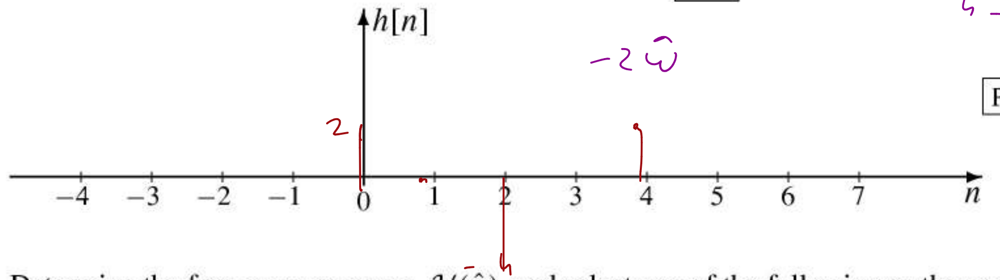
# Exercise - 2

**PROBLEM:**

The following FIR filter is specified by the filter coefficients $\{b_k\} = \{2, 0, -4, 0, 2\}$


$e^{-2j\omega}\,(2\cdot e^{+2j\omega} + 2\cdot e^{-2j\omega} - 4\cdot 1)$ $\quad 2 - 4\cdot e^{-2j\omega} + 2\cdot e^{-4j\omega}$ $\quad 4\cos(2\omega) - 4$ $\quad 4 - 4\cos(2\omega)$

(a) Determine the impulse response: give your answer as a $\boxed{\text{plot}}$ of $h[n]$ vs. $n$.




$-2\hat{\omega}$

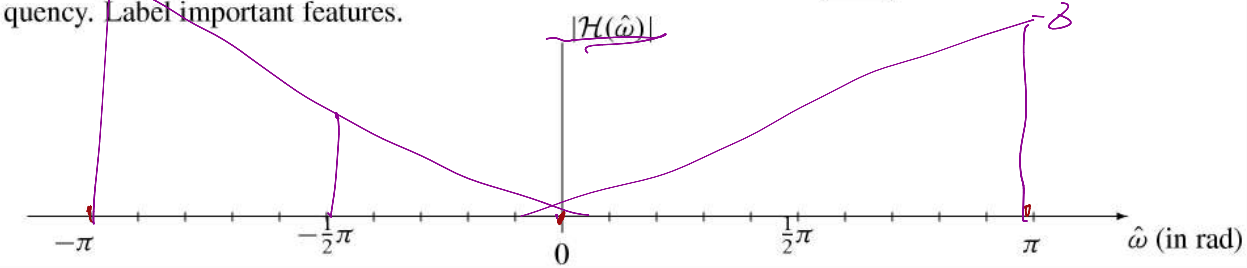
Plot zero values also

(b) Determine the frequency response, $\mathcal{H}(\hat{\omega})$, and select one of the following as the correct answer:

**(A)** $(4 - 4\cos(2\hat{\omega}))e^{-j(2\hat{\omega}-\pi)}$ **(B)** $2\cos\hat{\omega} + 4e^{-j(2\hat{\omega}+\pi)}$ **(C)** $(4\cos(2\hat{\omega}) - 4)e^{-j\hat{\omega}}$

**(D)** $2\cos(2\hat{\omega}) - 4$

(c) Determine the magnitude of $\mathcal{H}(\hat{\omega})$ and present your answer as a a $\boxed{\text{plot}}$ of the magnitude vs. frequency. Label important features.



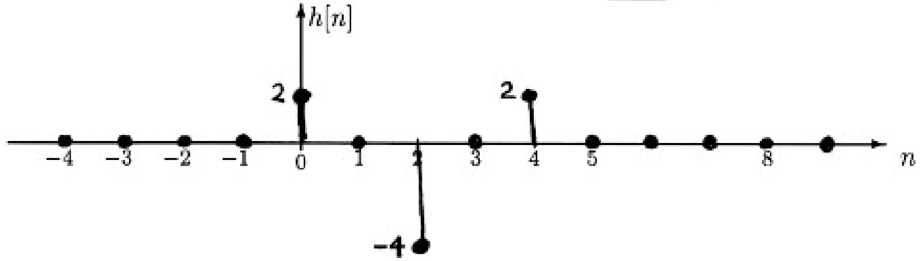
# Solution

The following FIR filter is specified by the filter coefficients $\{b_k\} = \{2, 0, -4, 0, 2\}$


$y[n] = 2x[n] - 4x[n-2] + 2x[n-4]$

$h[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-4]$.

(a) Determine the impulse response: give your answer as a $\boxed{\text{plot}}$ of $h[n]$ vs. $n$.




(b) Determine the frequency response, $\mathcal{H}(\hat{\omega})$, and select one of the following as the correct answer:

**(A)** $(4 - 4\cos(2\hat{\omega}))e^{-j(2\hat{\omega}-\pi)}$ **(B)** $2\cos\hat{\omega} + 4e^{-j(2\hat{\omega}+\pi)}$ **(C)** $(4\cos(2\hat{\omega}) - 4)e^{-j\hat{\omega}}$

**(D)** $2\cos(2\hat{\omega}) - 4$

$$\mathcal{H}(\hat{\omega}) = 2 - 4e^{-j2\hat{\omega}} + 2e^{-j4\hat{\omega}}$$

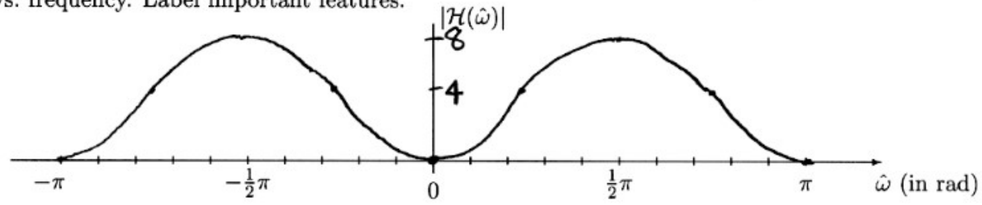
$$= e^{-j2\hat{\omega}}(2e^{+j2\hat{\omega}} - 4 + 2e^{-j2\hat{\omega}})$$

$$= e^{-j2\hat{\omega}}(4\cos 2\hat{\omega} - 4)$$

$$= e^{-j2\hat{\omega}}e^{j\pi}(4 - 4\cos 2\hat{\omega})$$

A

(c) Determine the magnitude of $\mathcal{H}(\hat{\omega})$ and present your answer as a a $\boxed{\text{plot}}$ of the magnitude vs. frequency. Label important features.




$$|\mathcal{H}(\hat{\omega})| = |\underbrace{4 - 4\cos 2\hat{\omega}}_{\text{This is non-negative}}| = 4 - 4\cos 2\hat{\omega}$$

$\hat{\omega} = 0 \Rightarrow 4 - 4 = 0$

$\hat{\omega} = \pi \Rightarrow 4 - 4 = 0$

$\hat{\omega} = \pi/2 \Rightarrow 4 - 4(-1) = 8$

$\hat{\omega} = \pi/4 \Rightarrow 4 - 4(0) = 4$

# Exercise - 3

**PROBLEM:**

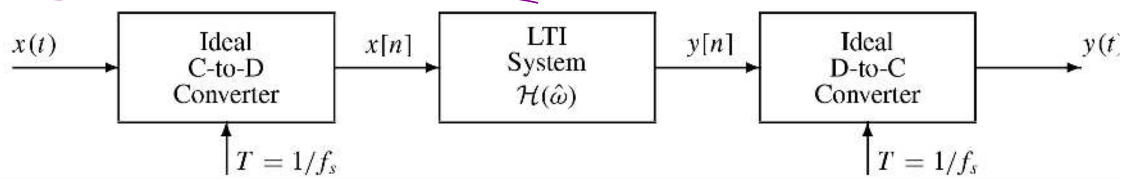
The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 2\cos(6000\pi t - \pi/4) + 11\cos(12000\pi t - \pi/3)$$

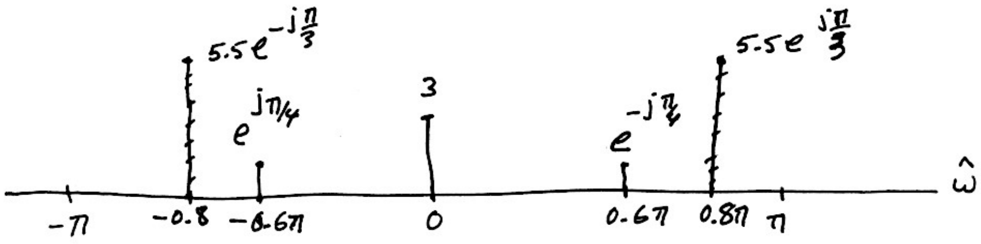
The frequency response for the digital filter (LTI system) is

$$\mathcal{H}(\hat{\omega}) = \frac{\sin(5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

If $f_s = 10000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.




$$x[n] = 3 + 2\cos(0.6\pi n - \pi/4) + 11\cos(1.2\pi n - \pi/3)$$




$$H(\hat{\omega}) = \frac{\sin(5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

$$H(0) = 10$$

$$H(0.6\pi) = \frac{\sin 3\pi}{\sin 0.3\pi} e^{-j3\pi} = 0$$

$$H(0.8\pi) = \frac{\sin 4\pi}{\sin 0.4\pi} e^{-j4\pi} = 0$$

$$y[n] = 10 \times 3 = 30, \quad y(t) = 30$$