BLM1612 Circuit Theory
Voltage and Current Laws

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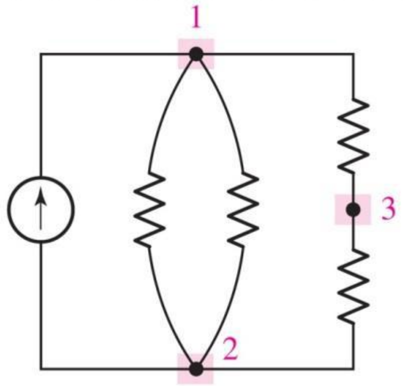
Circuit Terminology

- **node**: point at which 2+ elements have a common connection

Node 1
Node 2
Node 3



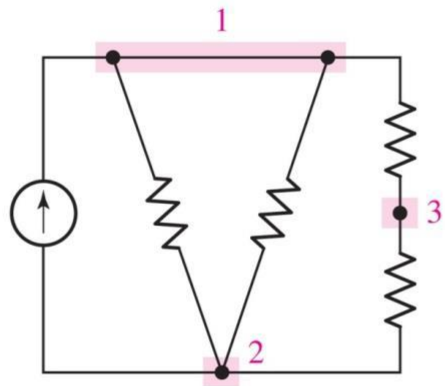
(*a*)

- **path**: a route through a network, through nodes that never repeat (except for possibly the last node)

1→3 →2
1 →2 → 3

- **loop**: a path that starts & ends on the same node

3 →1 → 2 → 3



(*b*)

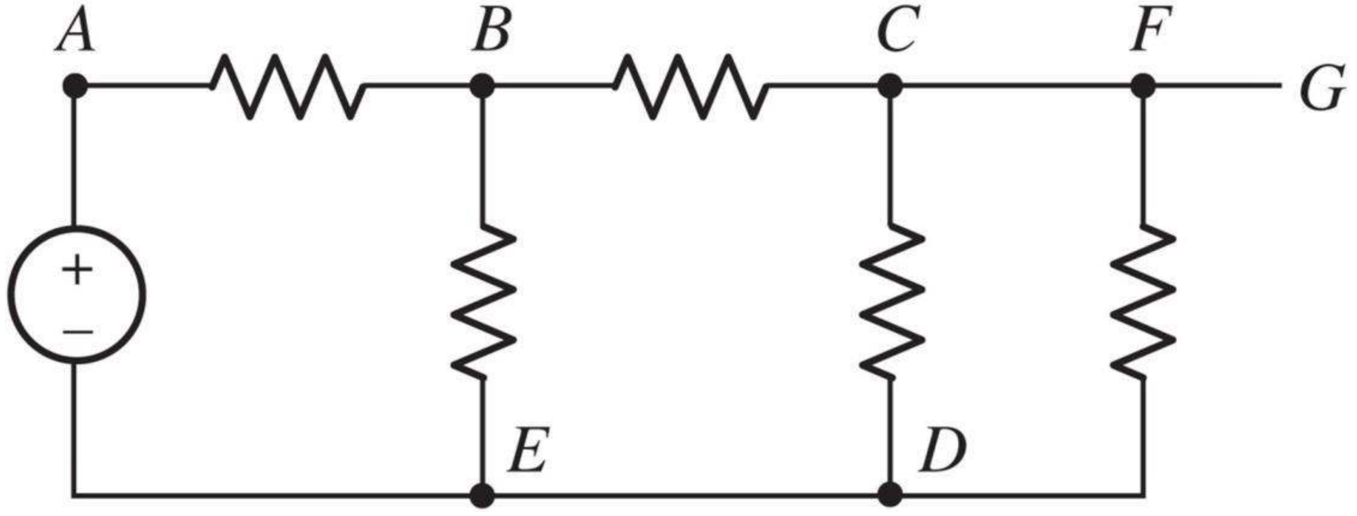
- **branch**: a single path in a network; contains one element and the nodes at the 2 ends

1 →2 (x 3)
1 → 3 (x 1)
3 → 2 (x 1)

(a,b) 3 nodes, 5 branches

Exercise 3.4

- For the circuit below:
  - (a) Count the number of circuit elements.
  - (b) If we move from *B* to *C* to *D*, have we formed a path and/or a loop?
  - (c) If we move from *E* to *D* to *C* to *B* to *E*, have we formed a path and/or a loop?



Kirchhoff's Current Law

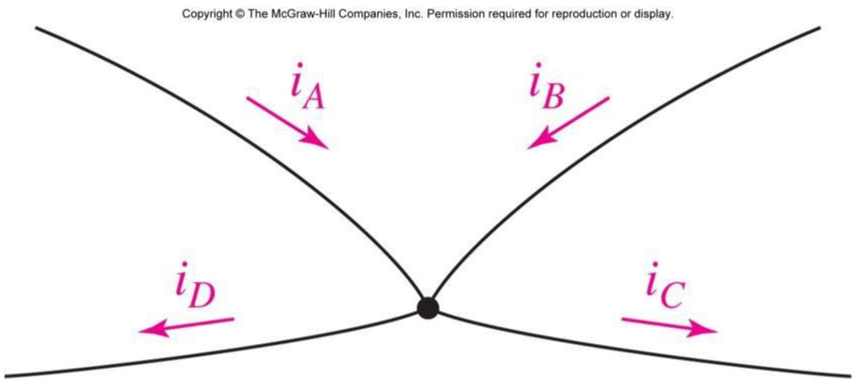
- Gustav Robert Kirchhoff: German university professor, born while Ohm was experimenting
- The algebraic sum of the currents entering any node is zero.

$$\sum_{n=1}^{N} i_n = 0$$

- i.e. Charge cannot accumulate at a *node* (which is *not* a circuit element).

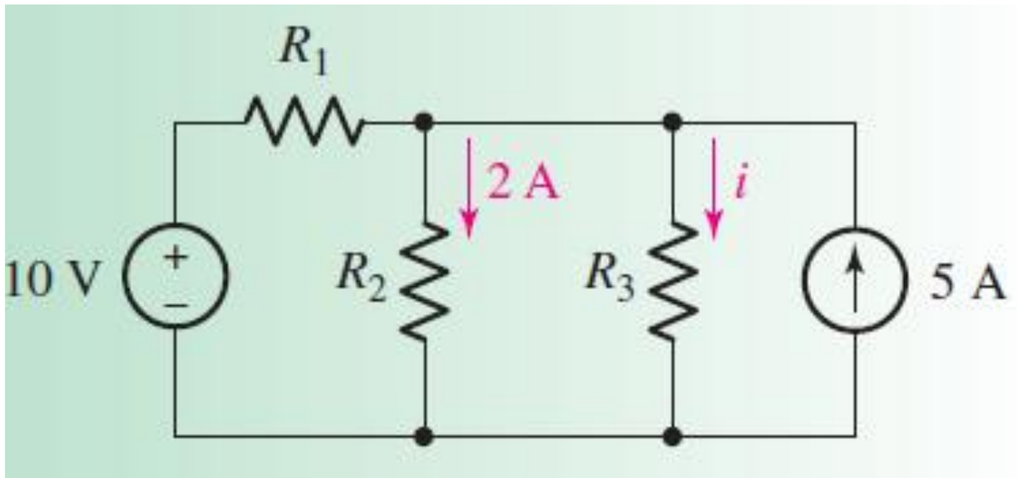
$$i_A + i_B - i_C - i_D = 0$$

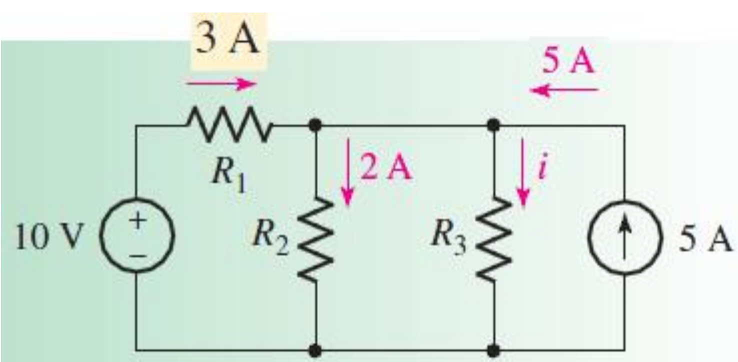
$$-i_A - i_B + i_C + i_D = 0$$



Example 3.1

- For the circuit, compute the current through R_3 if it is known that the voltage source supplies a current of 3 A.



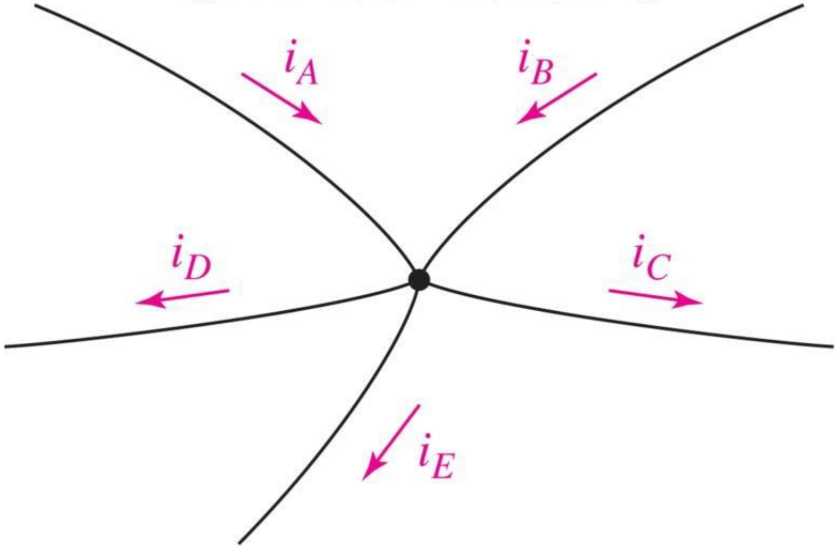


$$3 - 2 - i + 5 = 0$$

$$i = 3 - 2 + 5 = 6\text{ A}$$

Exercises 3.7

- Referring to the single node below, compute:
  - (a) i_B, given $i_A = 1$ A, $i_D = –2$ A, $i_C = 3$ A, and $i_E = 4$ A
  - (b) i_E, given $i_A = –1$ A, $i_B = –1$ A, $i_C = –1$ A, and $i_D = –1$ A



$$i_A + i_B - i_C - i_D - i_E = 0$$

(a) $i_B = -i_A + i_C + i_D + i_E$

$$i_B = -1 + 3 - 2 + 4 = 4 \text{ A}$$

(b) $i_E = i_A + i_B - i_C - i_D$

$$i_E = -1 - 1 + 1 + 1 = 0 \text{ A}$$

Kirchhoff's Voltage Law

- The algebraic sum of the voltages around any closed path is zero.

$$\sum_{n=1}^{N} v_n = 0$$

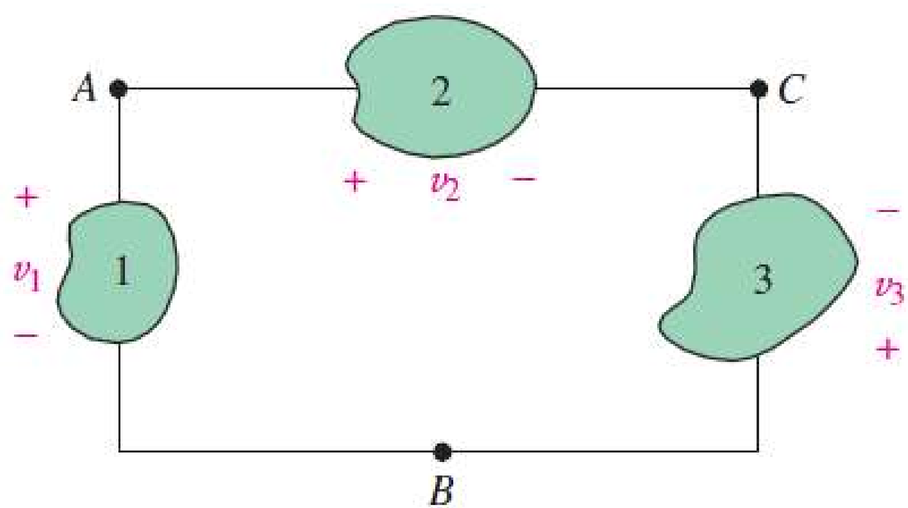
- i.e. The energy required to move a charge from point *A* to point *B* must have a value independent of the path chosen.

Clockwise

$$-v_1 + v_2 - v_3 = 0$$

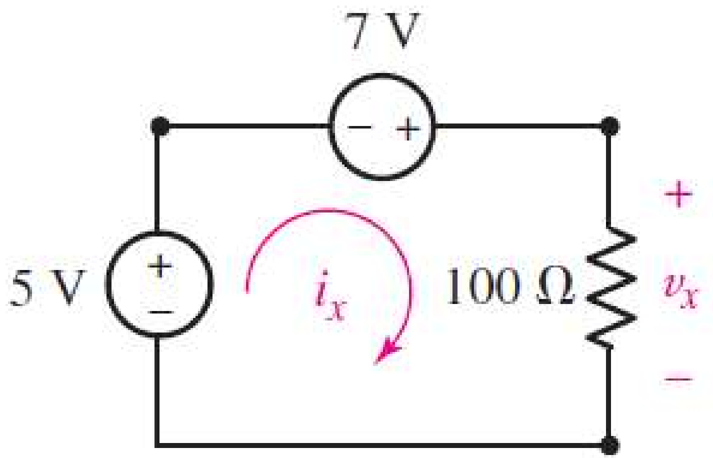
$$v_1 - v_2 + v_3 = 0$$

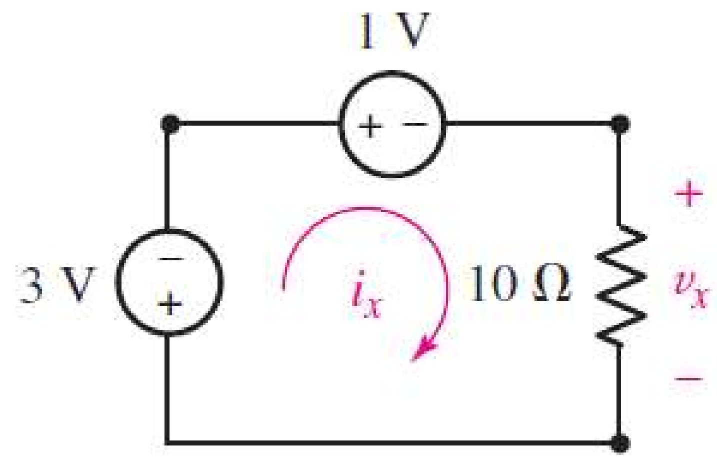
AntiClockwise



Example 3.2 and Practice 3.2

- For each of the circuits in the figure below, determine the voltage v_x and the current i_x.





If we apply KVL clockwise around the loop:

$$-5 - 7 + v_x = 0$$

$$v_x = 12 \text{ V}$$

Ohm's law,

$$i_x = \frac{v_x}{100} = \frac{12}{100} \text{ A} = 120 \text{ mA}$$

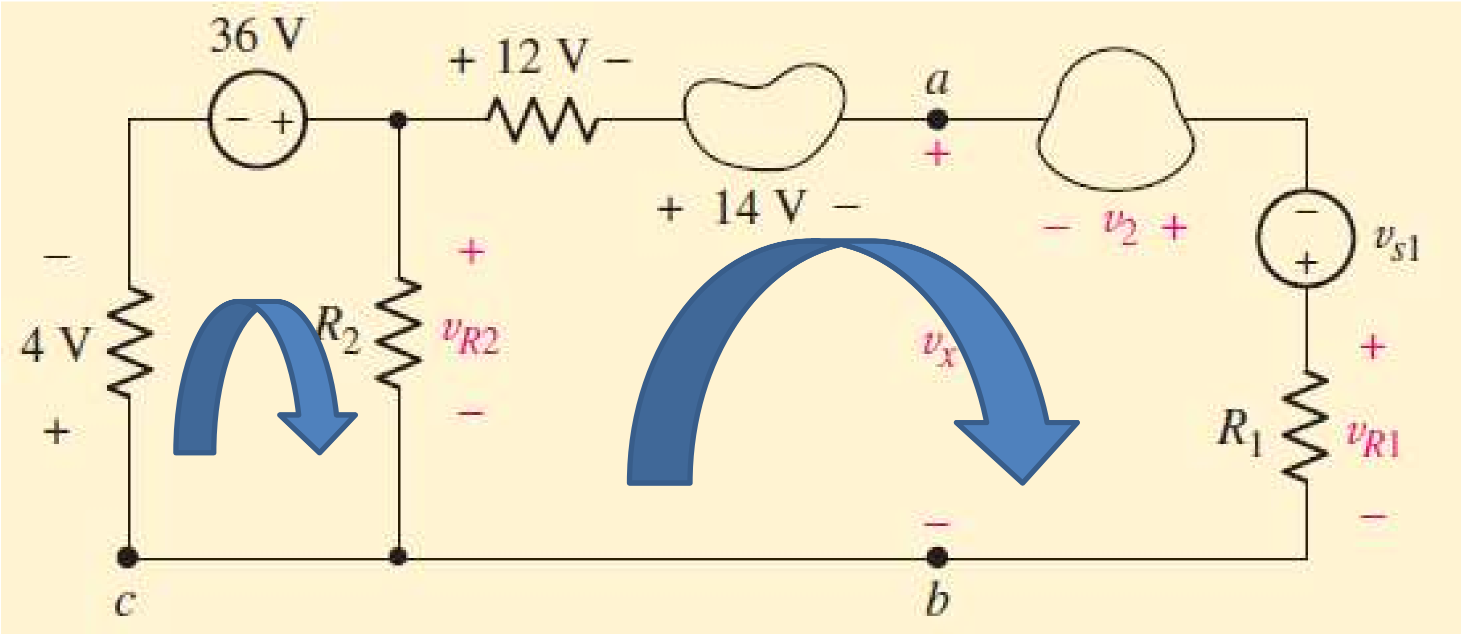
By KVL, $+3 + 1 + v_x = 0$

so $v_x = \underline{-4 \text{ V}}$

By Ohm's Law, $i_x = \frac{v_x}{10} = \underline{-400 \text{ mA}}$

Example 3.3

- For the circuit below, determine (a) v_{R2} and (b) v_x.



$$4 - 36 + v_{R2} = 0$$

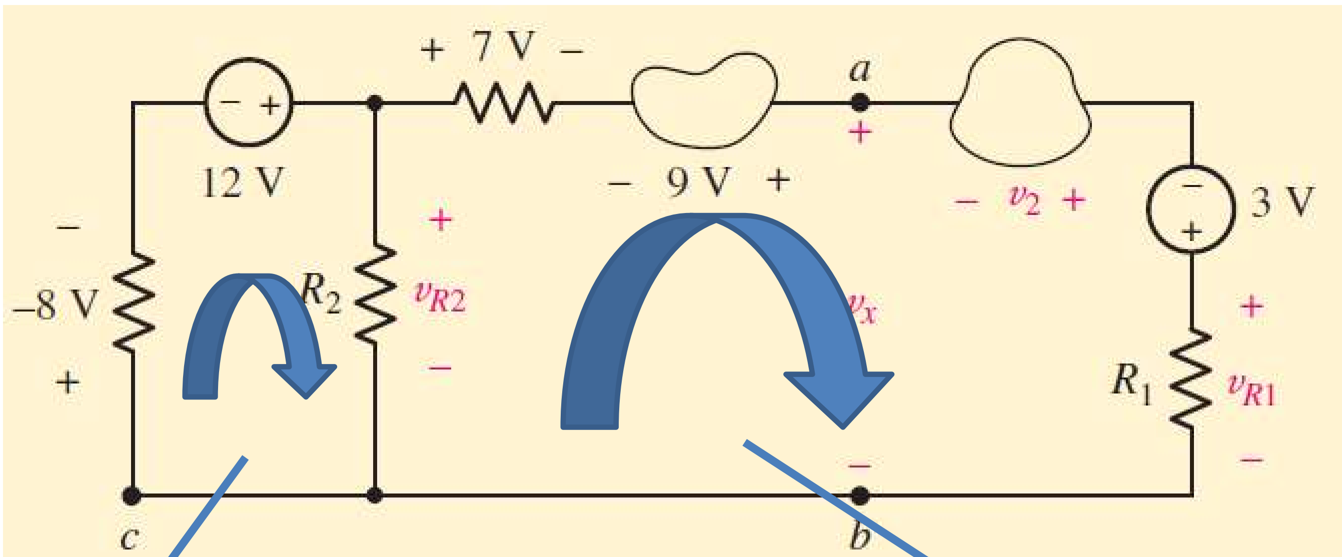
$$v_{R2} = 32 \text{ V}$$

$$-32 + 12 + 14 + v_x = 0$$

$$v_x = 6 \text{ V}$$

Practice 3.3

- For the circuit, determine (*a*) v_{R2} and (*b*) v_2, if $v_{R1} = 1$ V.



KVL yields $-8 - 12 + v_{R2} = 0$

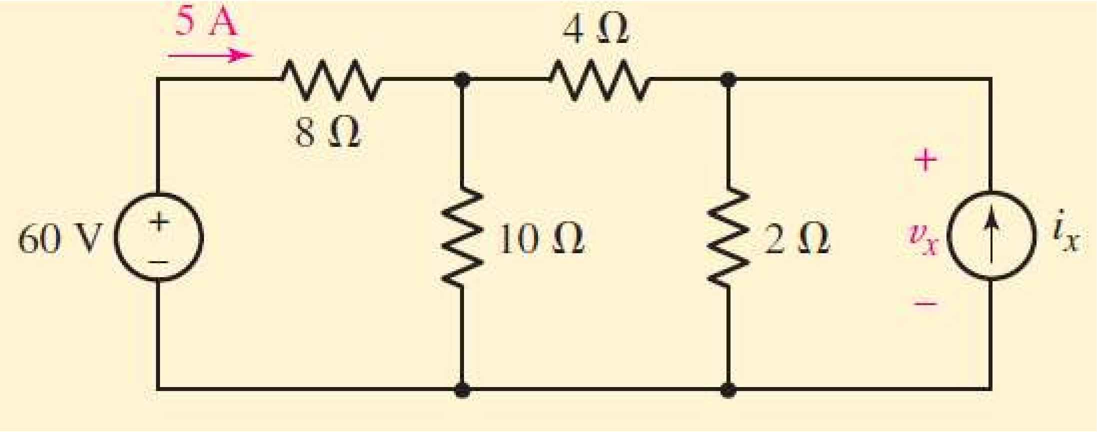
$v_{R2} = 20$ V

KVL yields $-20 + 7 - 9 - v_2 - 3 + v_{R1}$

where $v_{R1} = 1$ V. Thus, $v_2 = -24$ V

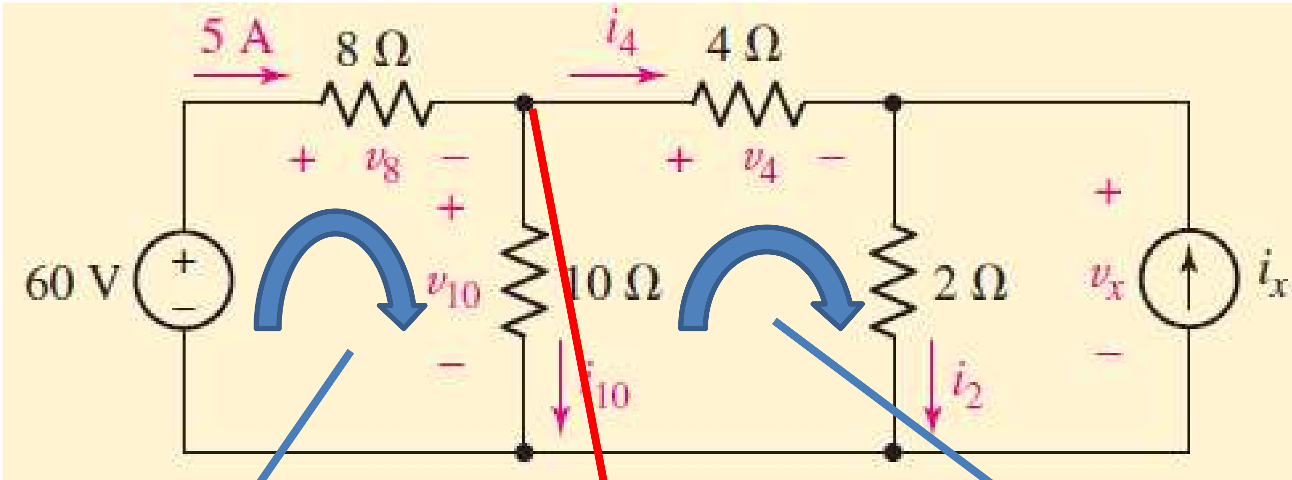
Example 3.4

- Determine v_x in the circuit.



Example 3.4

- Determine v_x in the circuit.



$$-60 + v_8 + v_{10} = 0$$

$$v_{10} = 0 + 60 - 40 = 20\text{ V}$$

$$-v_{10} + v_4 + v_x = 0$$

$$v_x = 20 - v_4$$

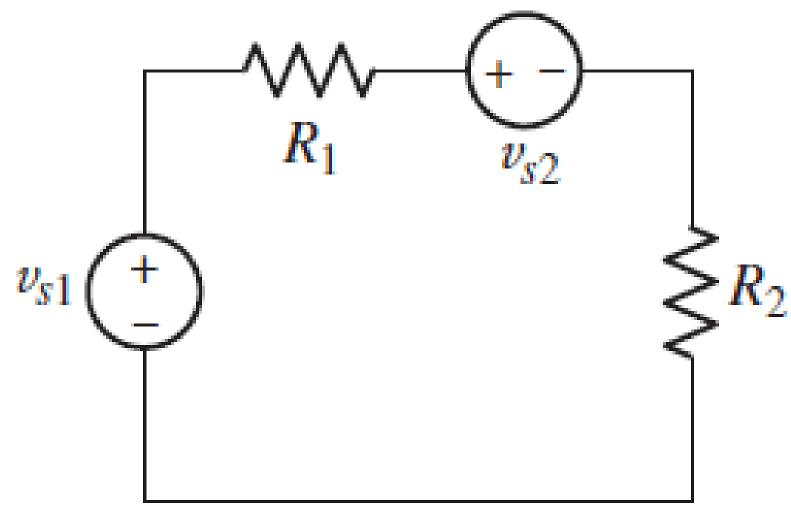
$$i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10} = 5 - \frac{20}{10} = 3$$

$$v_4 = (4)(3) = 12\text{ V}$$

$$v_x = 20 - 12 = 8\text{ V}$$

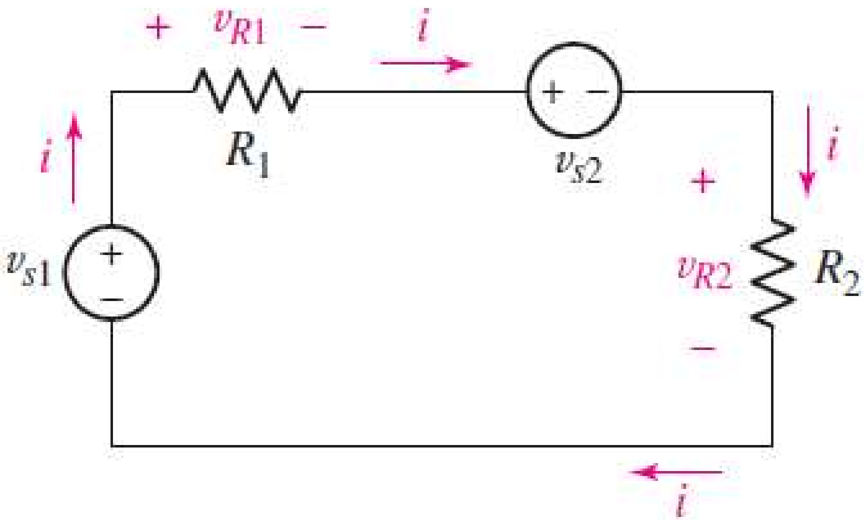
The Single-Loop Circuit

- All of the elements in a circuit that carry the same current are said to be connected in ***series.***



- We seek the **current** through each element, the **voltage** across each element, and the **power** absorbed by each element.

The Single-Loop Circuit



$$-v_{s1} + v_{R1} + v_{s2} + v_{R2} = 0$$

$$v_{R1} = R_1 i \quad \text{and} \quad v_{R2} = R_2 i$$

$$-v_{s1} + R_1 i + v_{s2} + R_2 i = 0$$

$$i = \frac{v_{s1} - v_{s2}}{R_1 + R_2}$$

- First step in the analysis is the assumption of **reference directions** for the **unknown currents**.
- Second step in the analysis is a choice of the **voltage reference** for each of the two resistors.
- The third step is the application of **Kirchhoff's voltage law** to the only closed path.

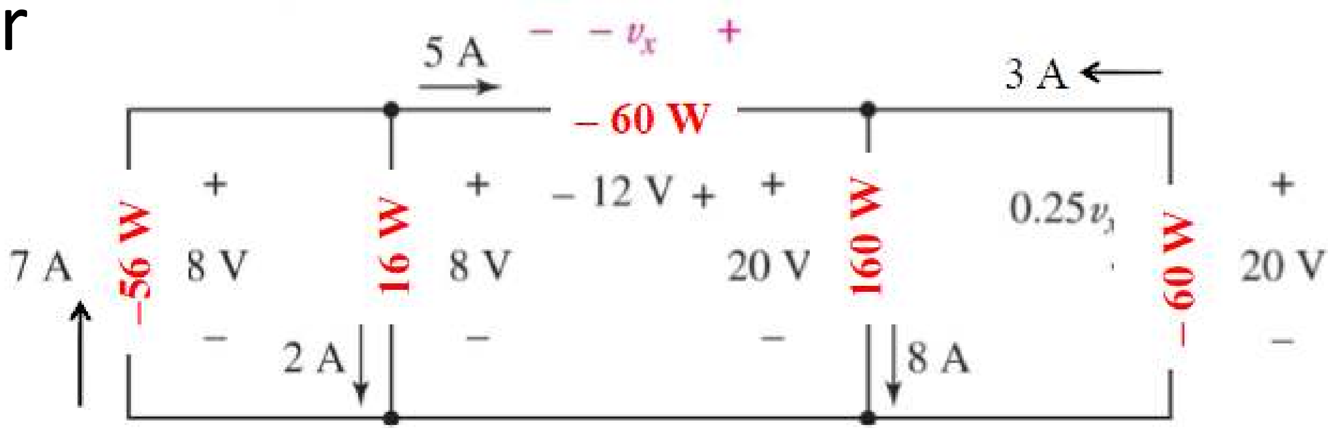
Conservation of Energy

- The sum of the absorbed power for each element of a circuit is zero.

$$\sum_{\text{all elements}} p_{\text{absorbed}} = 0$$

- The sum of the absorbed power equals the sum of the supplied power

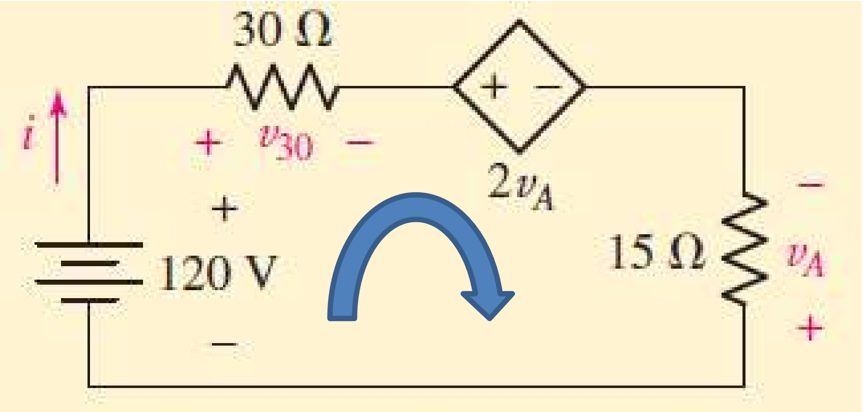
$$\sum p_{\text{absorbed}} = \sum p_{\text{supplied}}$$



$$\sum p_{\text{abs}} = -56 + 16 - 60 + 160 - 60 = -176\text{ W} + 176\text{ W} = 0$$

Example 3.5

- Compute the power absorbed in each element for the circuit shown in below Figure.



$$-120 + v_{30} + 2v_A - v_A = 0$$

$$v_{30} = 30i \quad \text{and} \quad v_A = -15i$$

$$-120 + 30i - 30i + 15i = 0$$

$$i = 8 \text{ A}$$

power *absorbed by each element*

$$p_{120\text{V}} = (120)(-8) = -960 \text{ W}$$

$$p_{30\Omega} = (8)^2(30) \quad = 1920 \text{ W}$$

$$p_{\text{dep}} = (2v_A)(8) \quad = 2[(-15)(8)](8) = -1920 \text{ W}$$

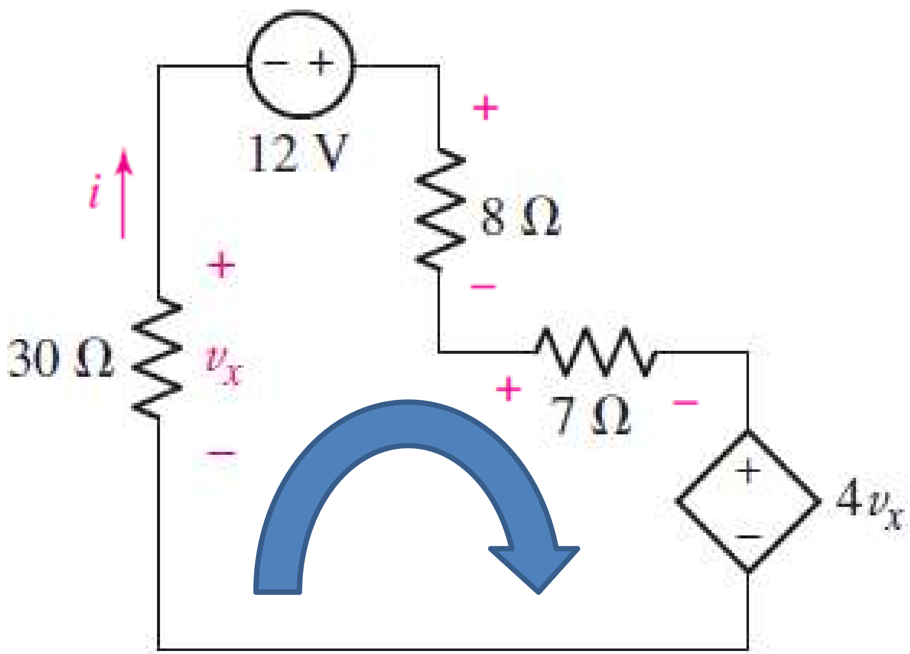
$$p_{15\Omega} = (8)^2(15) \quad = 960 \text{ W}$$

Practice 3.6

- In the circuit of below Figure, find the power absorbed by each of the five elements in the circuit.

$$-v_x - 12 + (8+7)i + 4v_x = 0$$

$$i = {}^{-v_x}\!/_{30} \qquad v_x = {}^{24}\!/_{5}\ \text{V} \qquad i = -{}^{4}\!/_{25}\ \text{A}$$



$$P_{abs}\big|_{30\Omega} = \frac{24}{5}^2 \times \frac{1}{30} = \underline{768\ \text{mW}}$$

$$P_{abs}\big|_{12v} = +\frac{4}{25} \times 12 = \underline{1.92\ \text{W}}$$

$$P_{abs}\big|_{8\Omega} = -\frac{4}{25}^2 \times 8 = \underline{204.8\ \text{mW}}$$

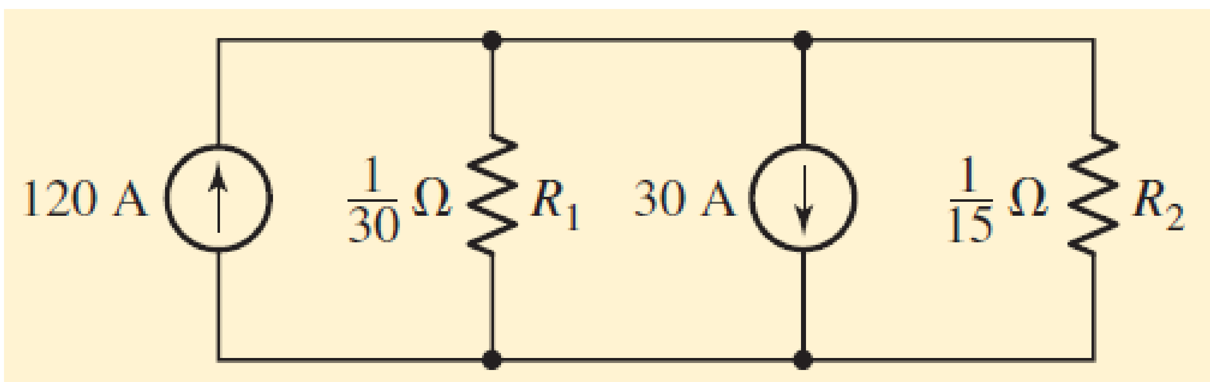
$$P_{abs}\big|_{7\Omega} = -\frac{4}{25}^2 \times 7 = \underline{179.2\ \text{mW}}$$

$$P_{abs}\big|_{4v_x} = -\frac{4}{25} \times 4v_x = \frac{-4}{25} \times 4 \times \frac{24}{5} = \underline{-3.072\ \text{W}}$$

(Check: 768 + 1920 + 204.8 + 179.2 – 3072 = 0 mW)

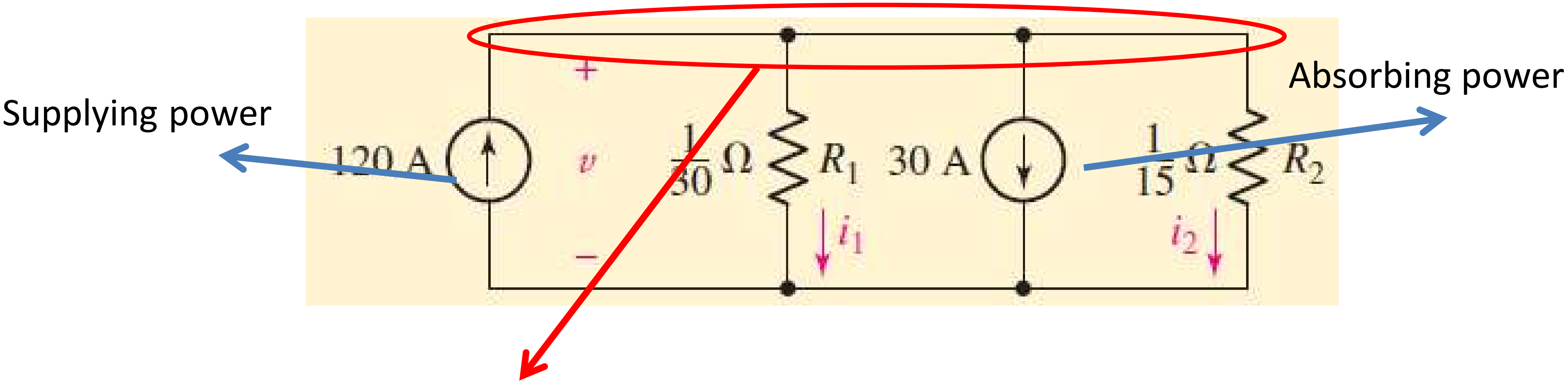
The Single-Node-Pair Circuit

- KVL forces us to recognize that the **voltage across each branch** is the same as that across any other branch.
- *Elements in a circuit having a common voltage across them are said to be connected in **parallel.***



Example 3.6

- Find the voltage, current, and power associated with each element in the circuit of below Figure.



$$-120 + i_1 + 30 + i_2 = 0$$

$$i_1 = 30v \quad \text{and} \quad i_2 = 15v$$

$$-120 + 30v + 30 + 15v = 0$$

$$v = 2\ \text{V}$$

$$i_1 = 60\ \text{A} \quad \text{and} \quad i_2 = 30\ \text{A}$$

$$p_{R1} = 30(2)^2 = 120\ \text{W}$$

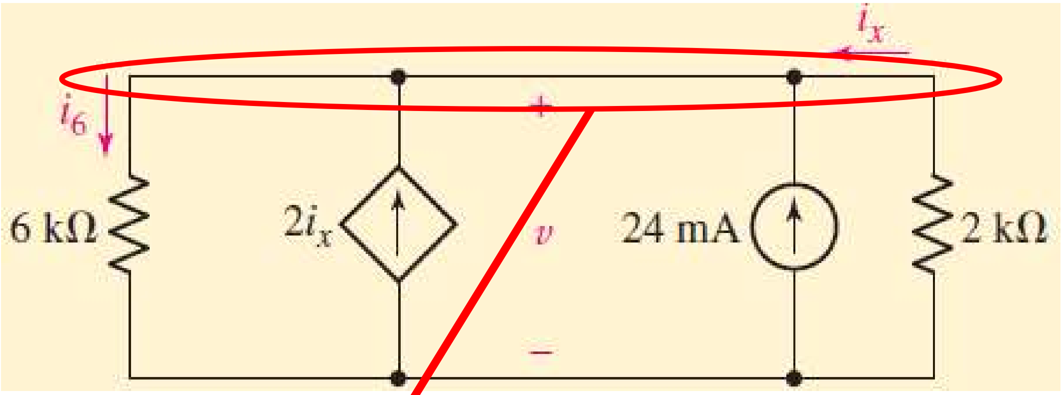
$$p_{R2} = 15(2)^2 = 60\ \text{W}$$

$$p_{120\text{A}} = 120(-2) = -240\ \text{W}$$

$$p_{30\text{A}} = 30(2) = 60\ \text{W}$$

Example 3.7

- Determine the value of v and the power absorbed by the independent current source in below Figure.



$$i_6 - 2i_x - 0.024 - i_x = 0$$

$$i_6 = \frac{v}{6000} \quad \text{and} \quad i_x = \frac{-v}{2000}$$

$$\frac{v}{6000} - 2\left(\frac{-v}{2000}\right) - 0.024 - \left(\frac{-v}{2000}\right) = 0$$

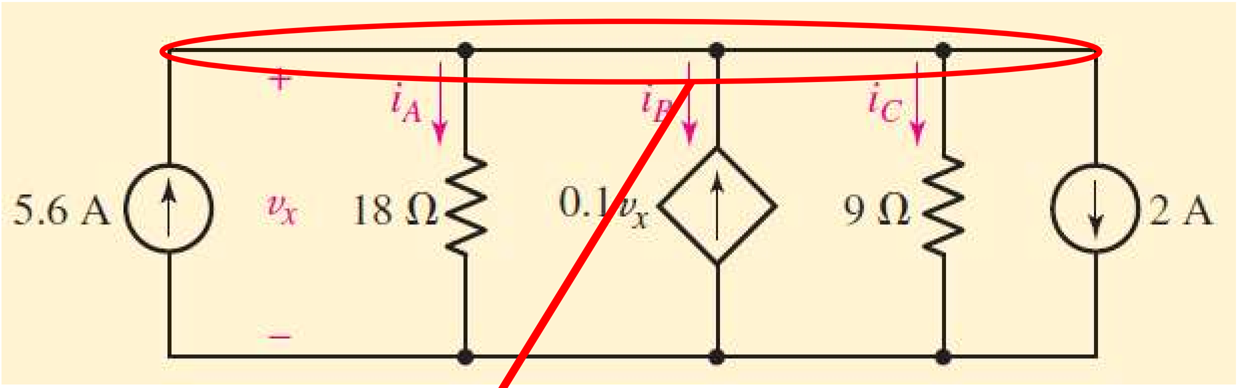
$$v = (600)(0.024) = 14.4 \text{ V}$$

$$p_{24} = -14.4(0.024) = -0.3456 \text{ W } (-345.6 \text{ mW})$$

Actually 345.6 mW is supplied

Practice 3.8

- For the single-node-pair circuit, find i_A, i_B and i_C.



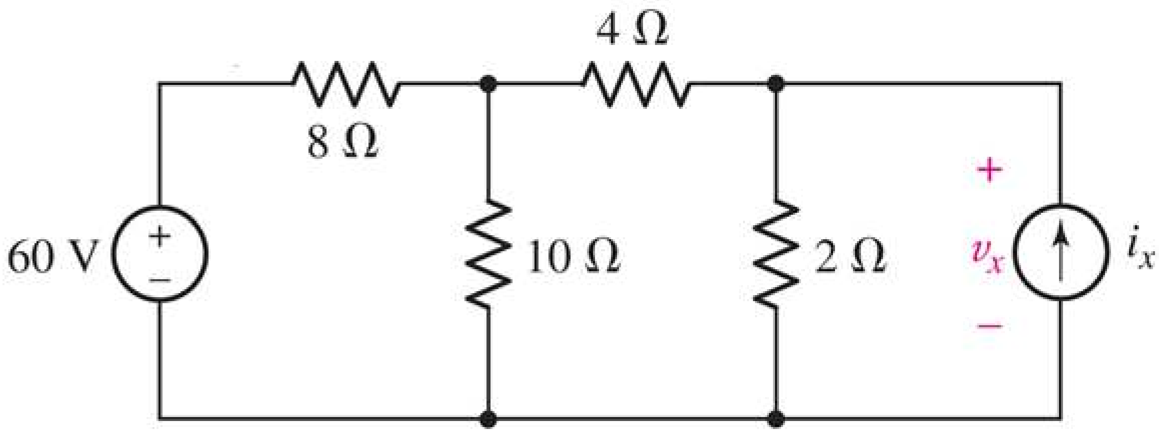
$$5.6 - \frac{v_x}{18} + 0.1v_x - \frac{v_x}{9} - 2 = 0$$

$$v_x = 54 \text{ V}.$$

$$i_A = \frac{v_x}{18} = \underline{3 \text{ A}}, \quad i_B = -0.1v_x = \underline{-5.4 \text{ A}}, \quad i_C = \frac{v_x}{9} = \underline{6 \text{ A}}$$

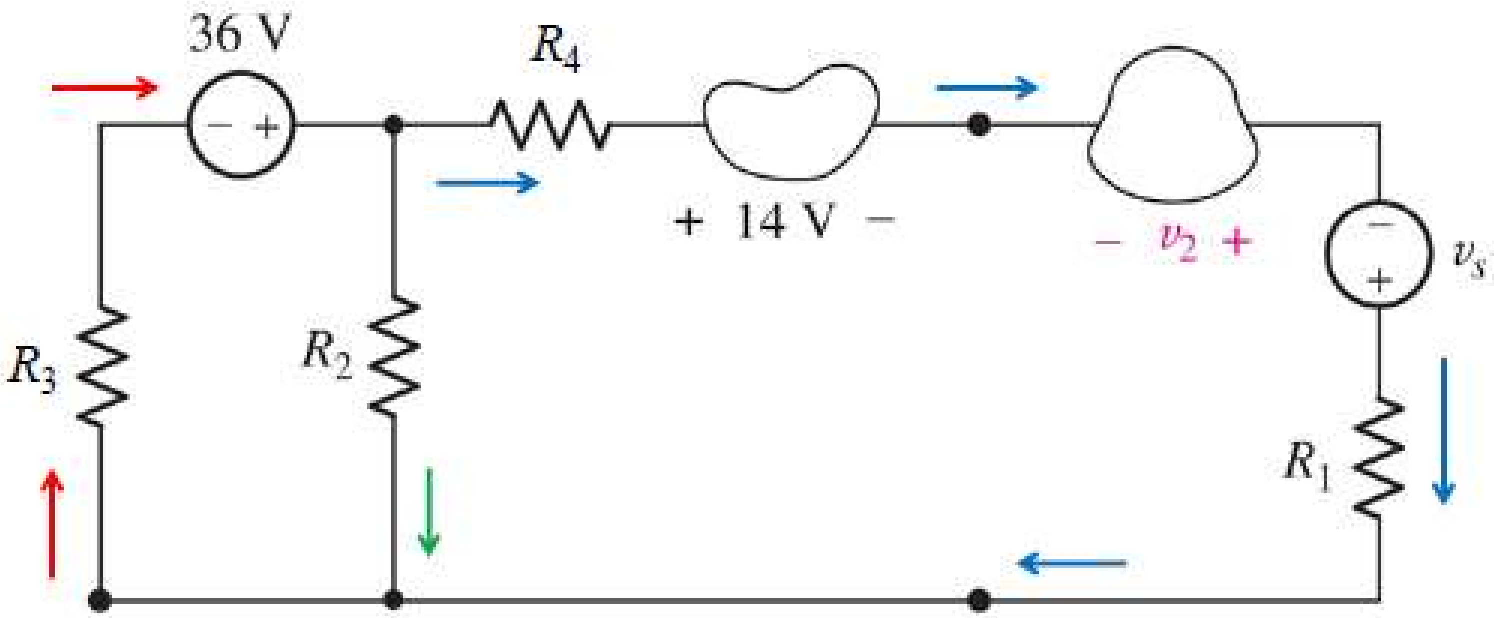
Series Circuits

- **series**: all elements in a circuit (loop) that carry the same current



  – The 60-V source and the 8-Ω resistor *are* in series.
  – The 8-Ω resistor and 4-Ω resistor are *not* in series.

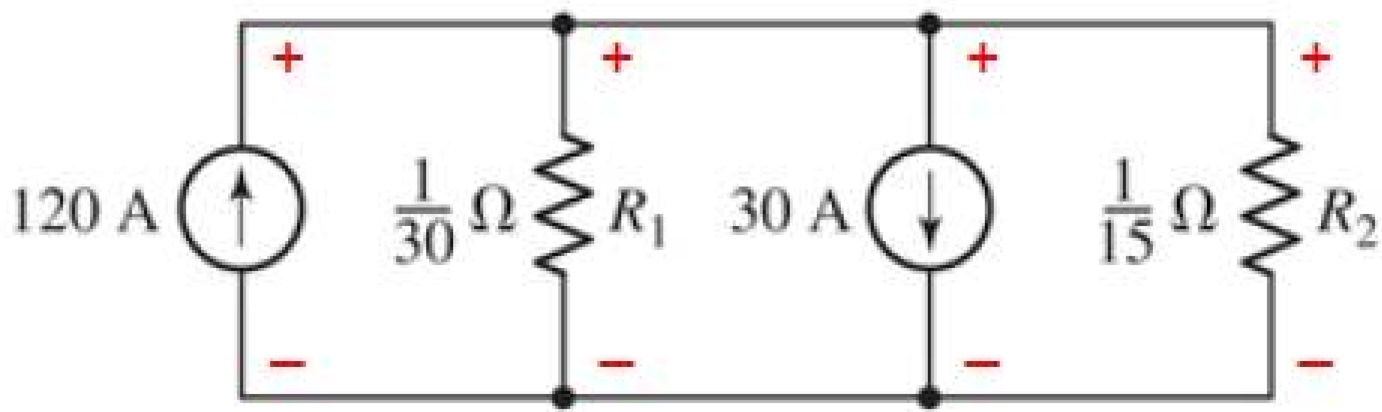
Series Circuits



- R_3 is in series with the 36-V source.
- R_4, the 14-V element, the v_2 element, the v_{s1} source, and R_1 are in series.
- No element is in series with R_2.

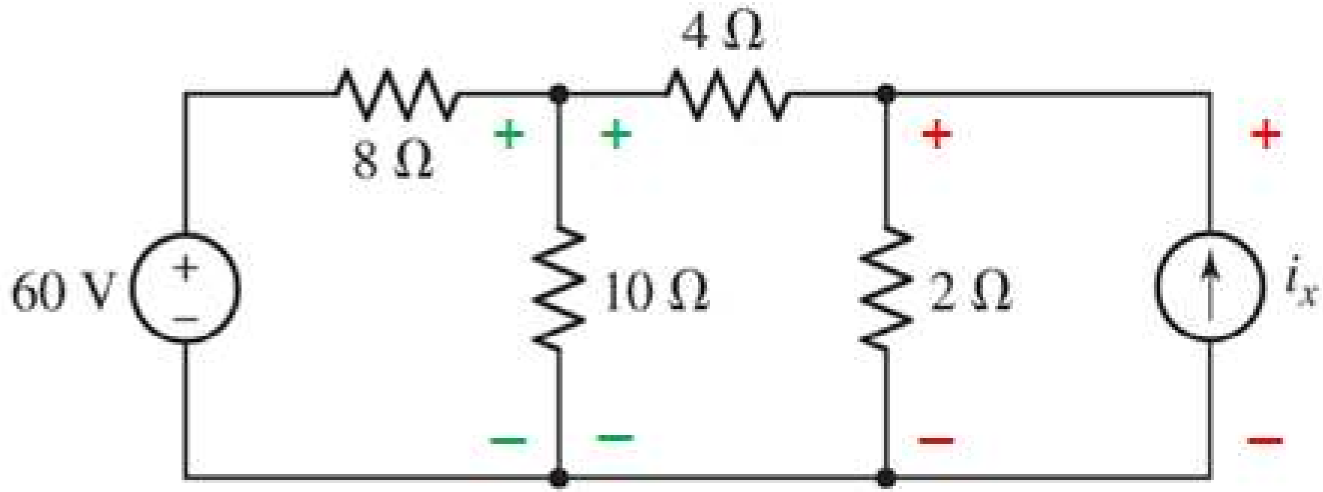
Parallel Circuits

- **parallel**: all elements in a circuit that have a common voltage across them (i.e. elements that share the same 2 nodes)



  – The 120-A source, 1/30- Ω resistor, 30-A source, and 1/15- Ω resistor are in parallel.

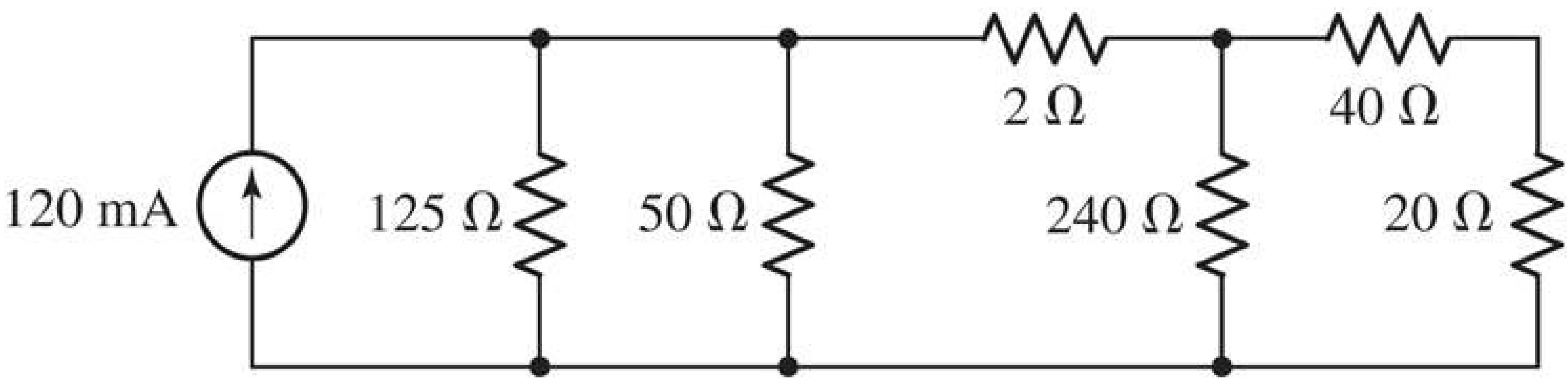
Parallel Circuits



- The current source and the 2-Ω resistor are in parallel. No other single elements are in parallel with each other.
- The 60-V-source-and-8-Ω-resistor branch is in parallel with the 10-Ω resistor.

Example

- (a) Which *individual* elements are in series? in parallel?
- (b) Which *groups* of elements are in series? in parallel?



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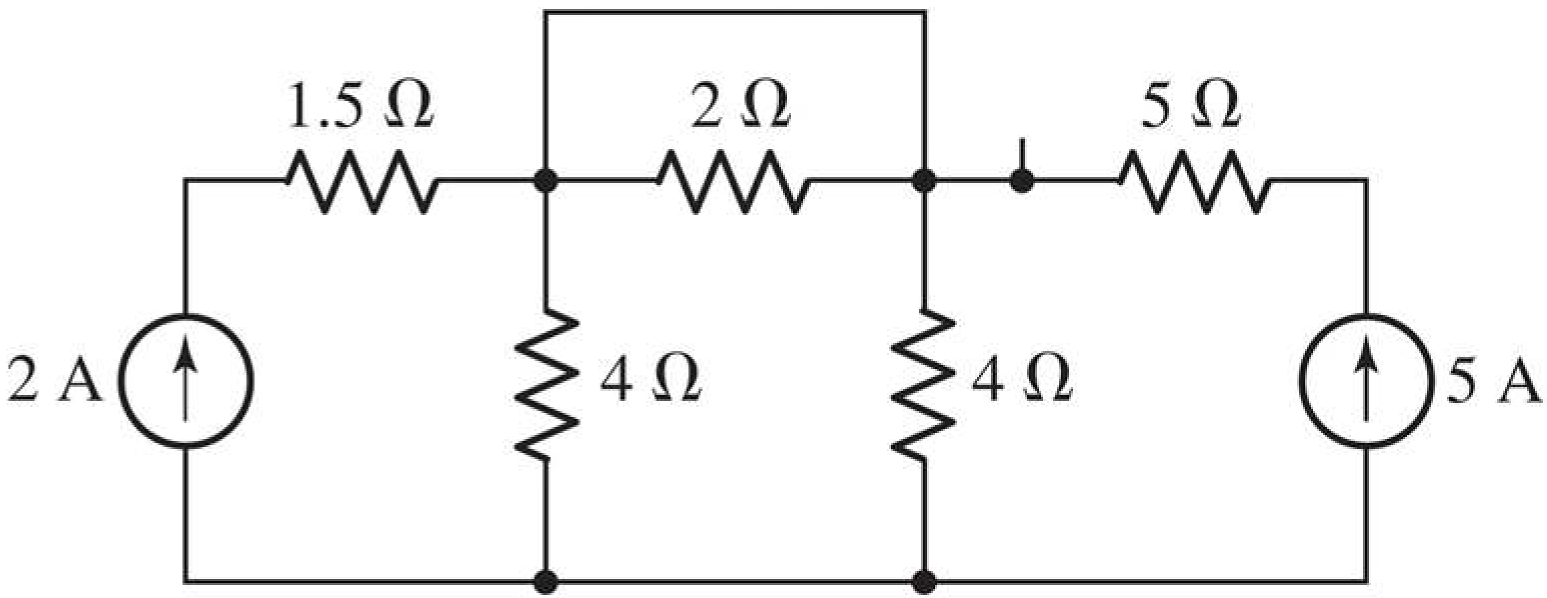

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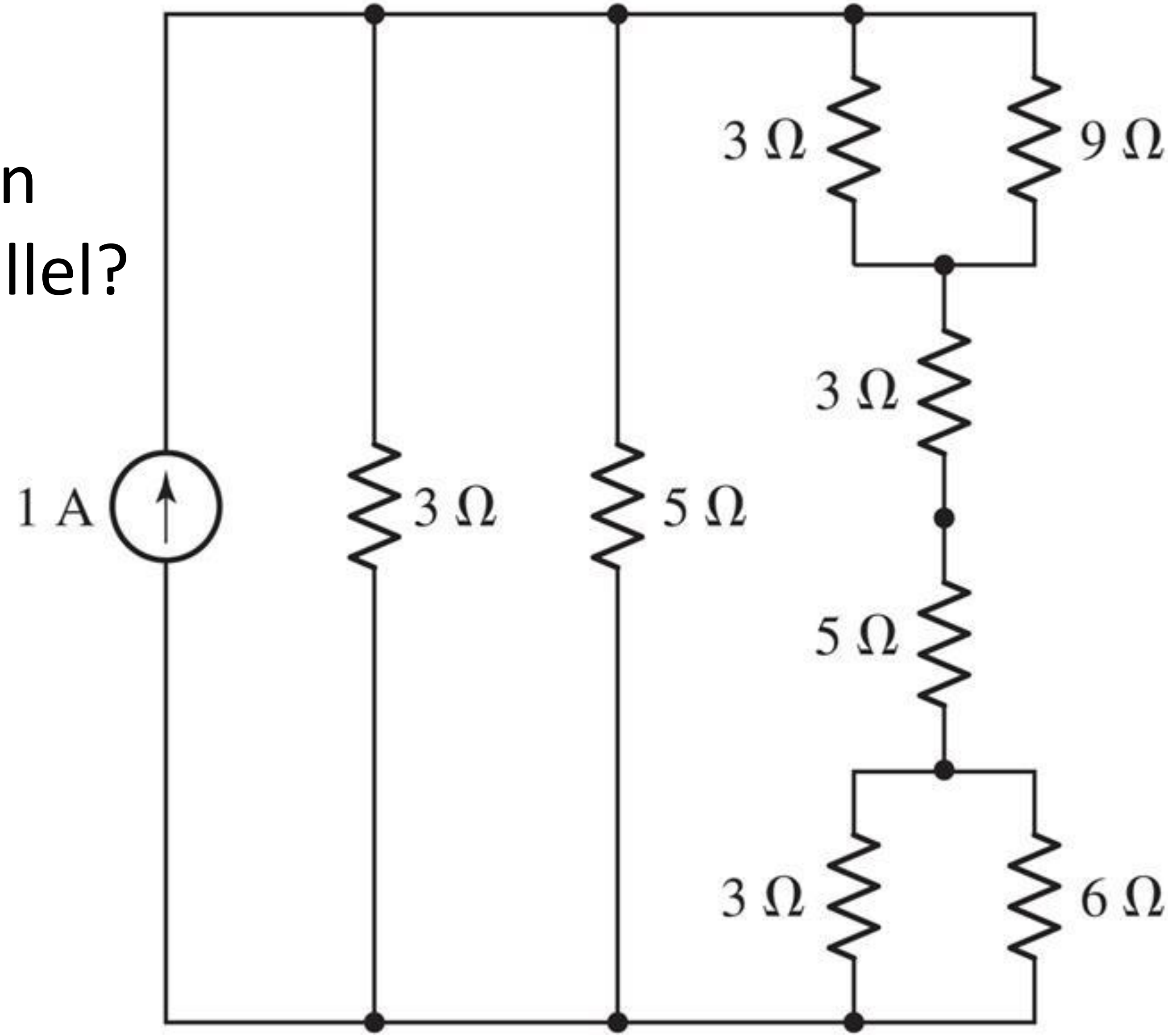
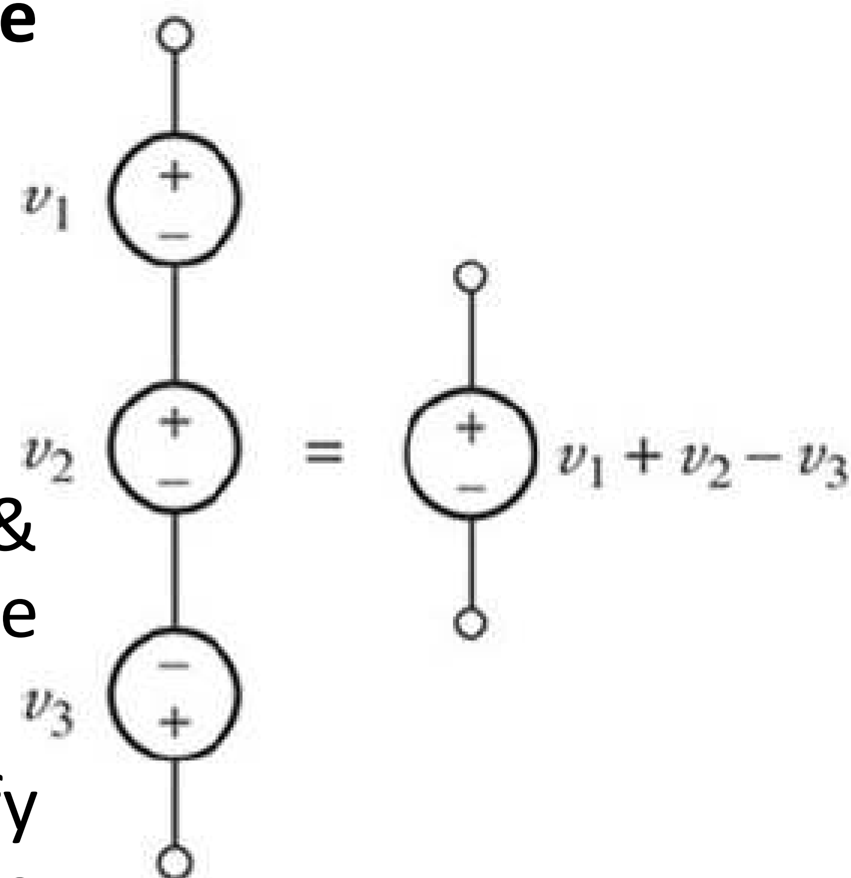
- (a) Which *individual* elements are in series? in parallel?



Voltage Sources in Series

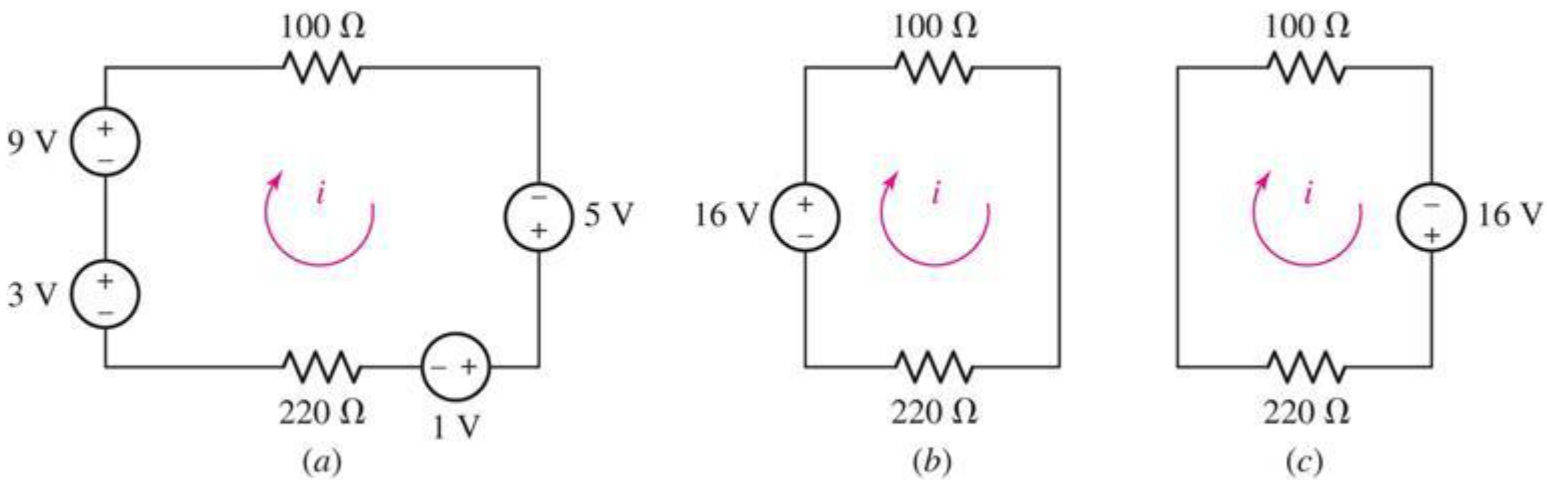
- can replace **voltage** sources in **series** with a **single equivalent source**

$$v_{\text{equivalent}}^{\text{series}} = \sum_{n=1}^{N} v_n$$



- all other voltage, current, & power relationships in the circuit remain **unchanged**
- might greatly simplify analysis of an otherwise complicated circuit

Example 3.8



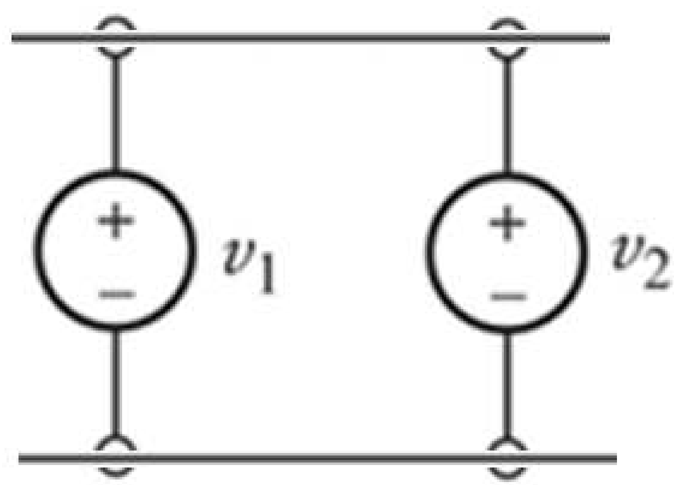
(a) $-3-9+100i-5+1+220i=0 \Rightarrow i=16/320=50\text{ mA}$

(b,c) $-16+100i+220i=0 \Rightarrow i=16/320=50\text{ mA}$

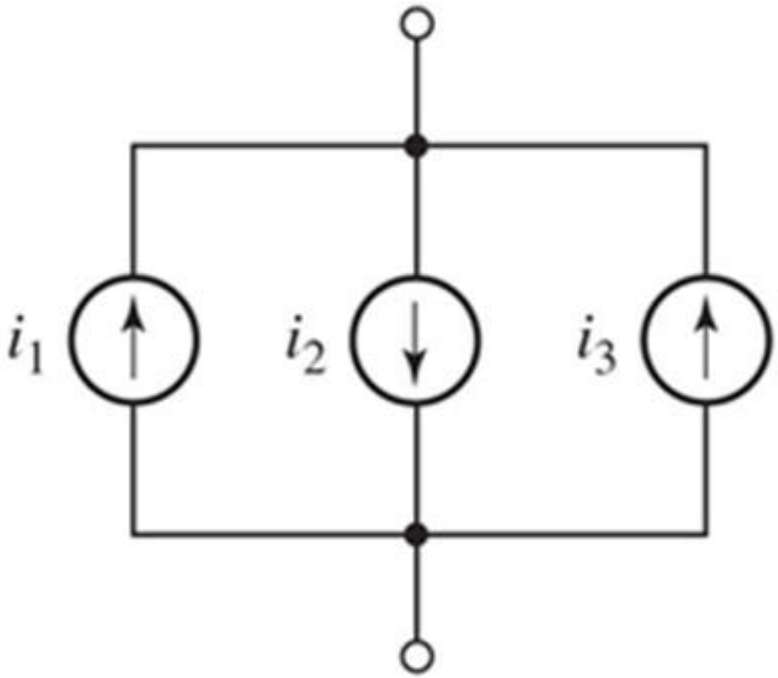
Note: The current and the power consumed by the resistors is the same in (a,b,c). However, the voltage sources must be broken out from the equivalent to solve for their individual powers delivered.

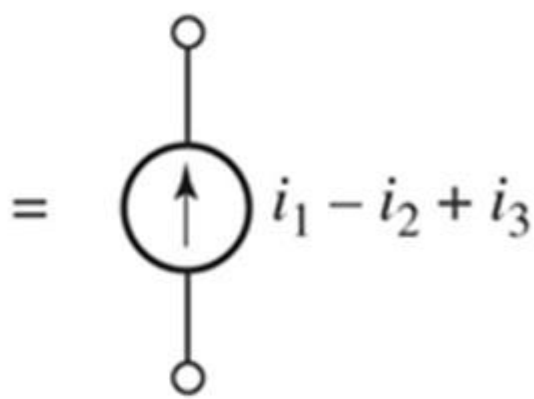
Voltage Sources in Parallel

- Unless $v_1 = v_2 = \ldots$, this circuit is **not** valid for *ideal* sources.
- All real voltage sources have internal resistance and are usually not exactly equal.
- Current will flow from the higher source to the lower source until equilibrium is reached (e.g. dangerously).
- Properly designed, a bank of equal voltage sources can deliver many times the current of a single source.



Current Sources in Parallel



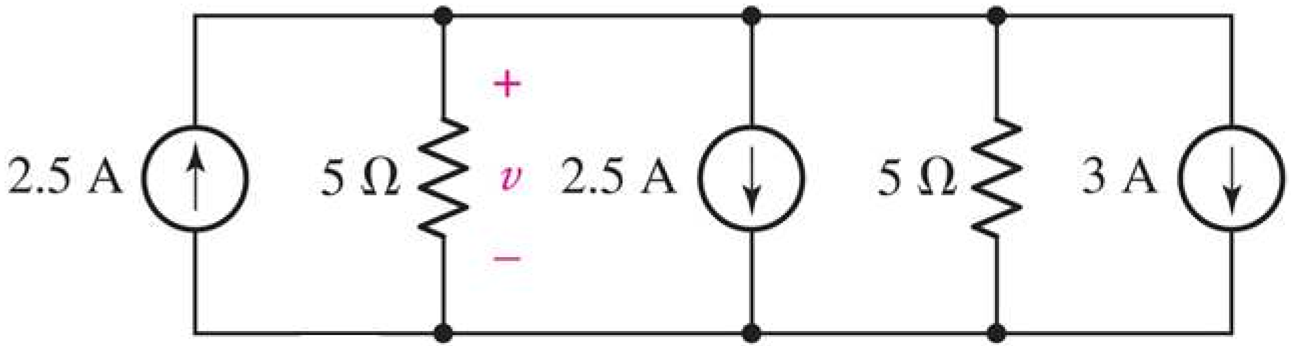


- can replace **current** sources in **parallel** with a **single equivalent source**

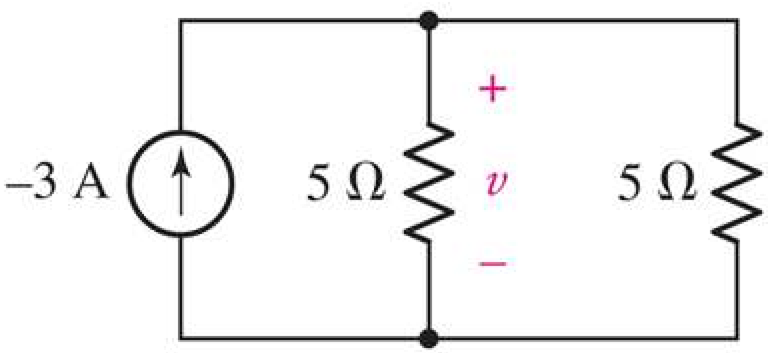
$$i_{\text{equivalent}}^{\text{parallel}} = \sum_{n=1}^{N} i_n$$

- all other voltage, current, & power relationships in the circuit remain **unchanged**
- as with voltage sources, this technique may simplify circuit analyses

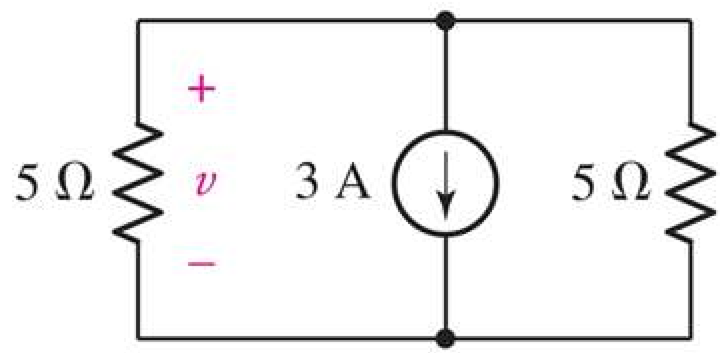
Example 3.9



(*a*)



(*b*)



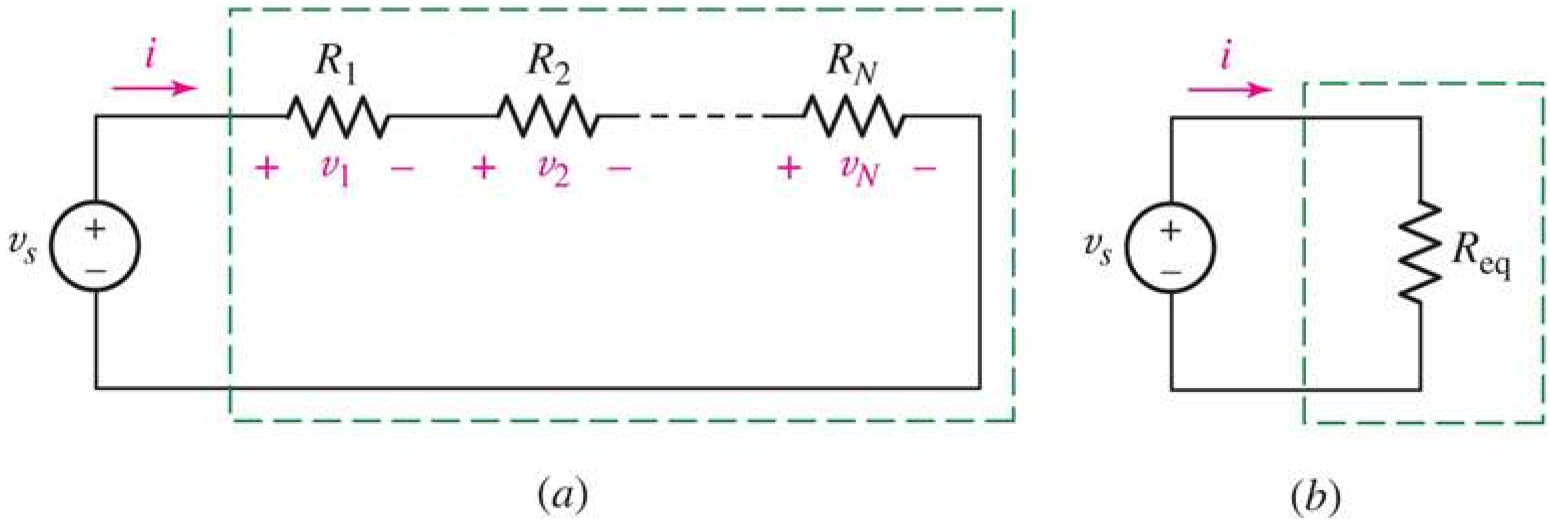
(*c*)

(a) $$2.5 - v/5 - 2.5 - v/5 - 3 = 0 \;\Rightarrow\; v = -7.5\ \mathrm{V}$$

(b,c) $$-3 - v/5 - v/5 = 0 \;\Rightarrow\; v = -7.5\ \mathrm{V}$$

Resistors in Series

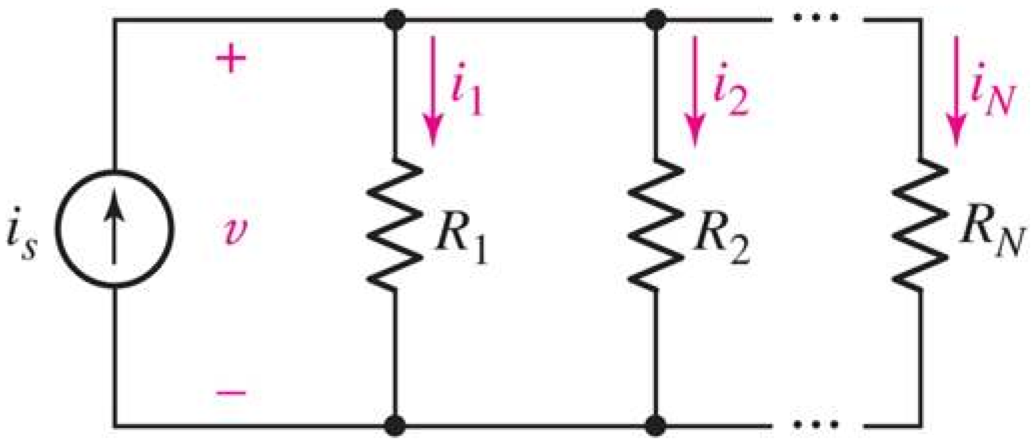
- As with voltage/current sources, resistors may also be replaced with equivalents. In *series*, resistances are added.



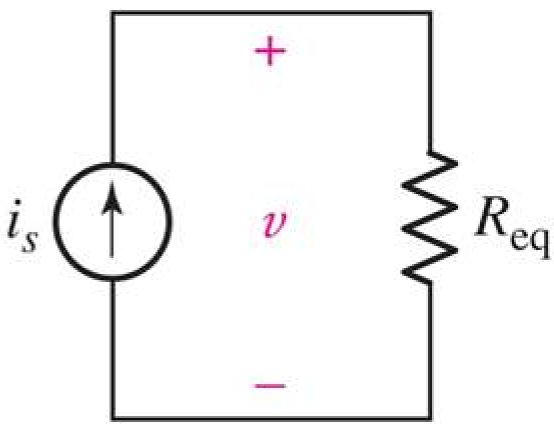
$$-v_s + v_1 + v_2 + \ldots + v_N = 0$$
$$-v_s + iR_1 + iR_2 + \ldots + iR_N = 0$$
$$-v_s + i\left[R_1 + R_2 + \ldots + R_N\right] = 0$$

$$\boxed{R_{\text{equivalent}}^{\text{series}} = \sum_{n=1}^{N} R_n}$$

Resistors in Parallel



(a)



(b)

For resistors in parallel, the *reciprocals* of the resistances sum to 1 / (the equivalent).

$$-i_s + i_1 + i_2 + \ldots + i_N = 0$$

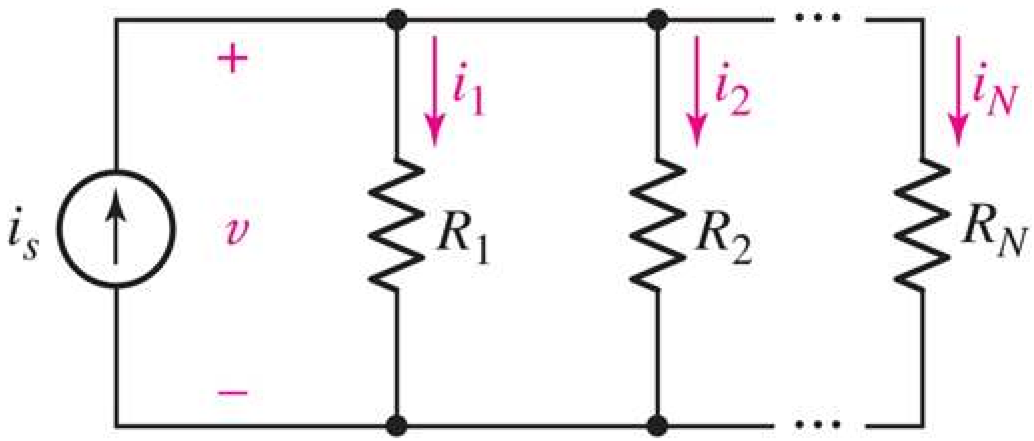
$$-i_s + v/R_1 + v/R_2 + \ldots + v/R_N = 0$$

$$-i_s + v\left[1/R_1 + 1/R_2 + \ldots + 1/R_N\right] = 0$$

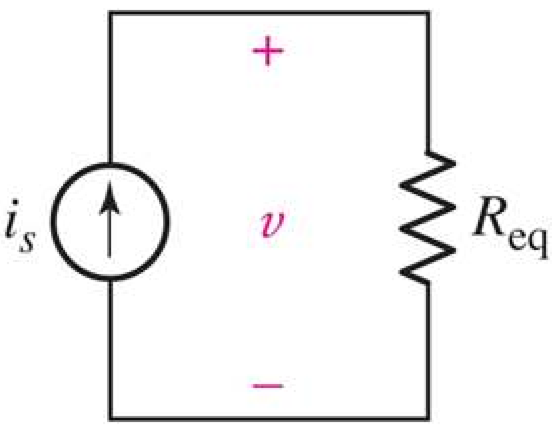
$$-i_s + v\left[1/R_{\text{equivalent}}^{\text{parallel}}\right] = 0$$

$$\boxed{1/R_{\text{equivalent}}^{\text{parallel}} = \sum_{n=1}^{N} 1/R_n}$$

Conductances in Parallel



(a)



(b)

Conductances in parallel add (mathematically) like resistances in series.

$$-i_s + i_1 + i_2 + \ldots + i_N = 0$$

$$-i_s + v \cdot G_1 + v \cdot G_2 + \ldots + v \cdot G_N = 0$$

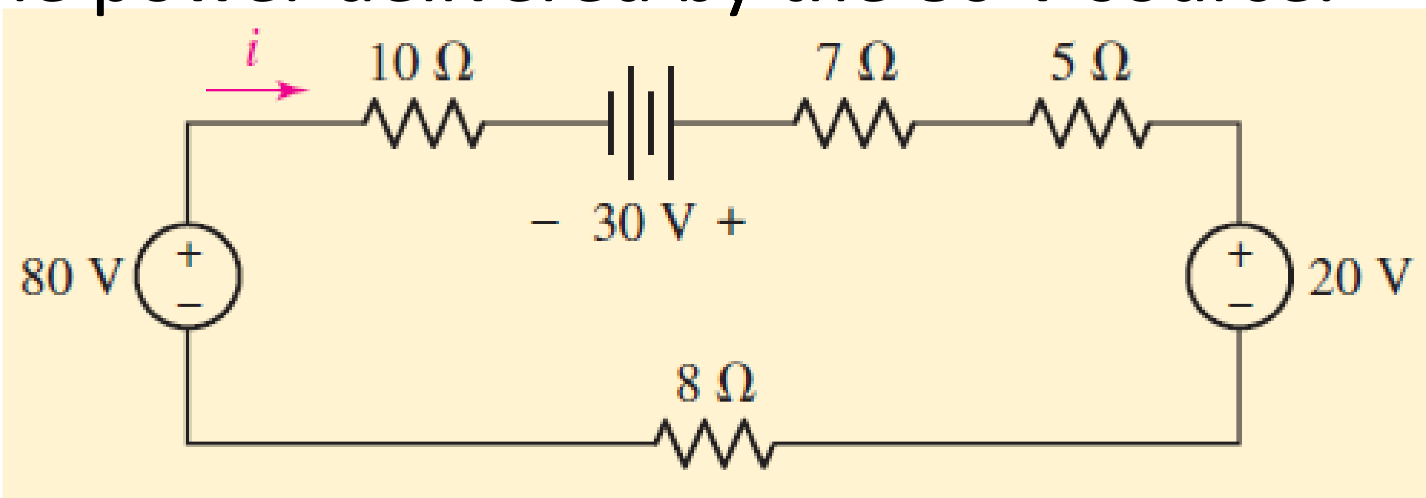
$$-i_s + v\left[G_1 + G_2 + \ldots + G_N\right] = 0$$

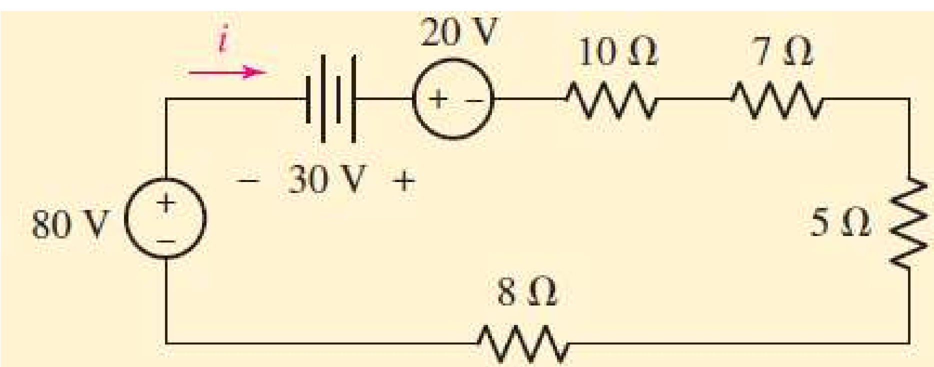
$$-i_s + v\left[G_{\text{equivalent}}^{\text{parallel}}\right] = 0$$

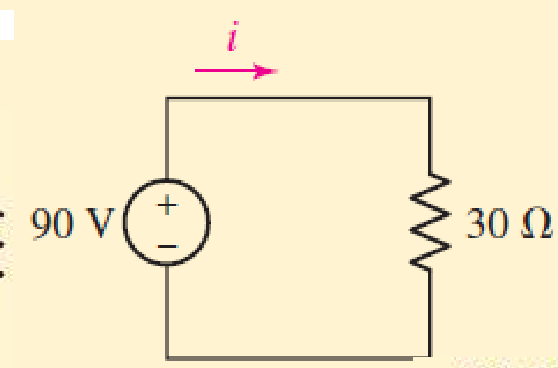
$$\boxed{G_{\text{equivalent}}^{\text{parallel}} = \sum_{n=1}^{N} G_n}$$

Example 3.11

- Use resistance and source combinations to determine the current *i* in below Figure and the power delivered by the 80 V source.







$$-90 + 30i = 0$$

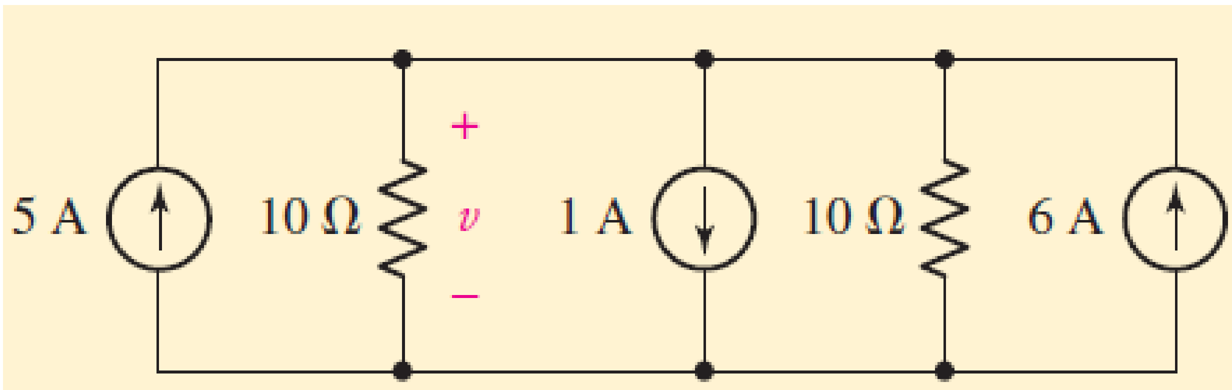
$$i = 3\ \text{A}$$

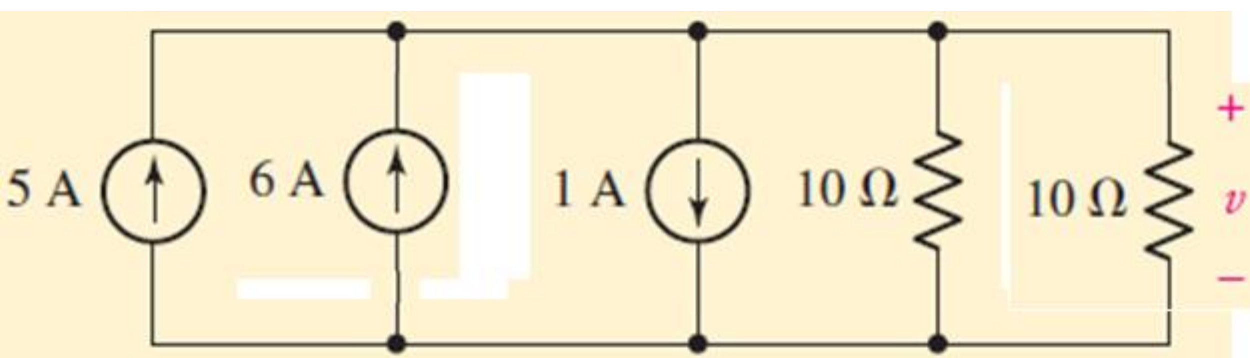
$$-80\ \text{V} \times 3\ \text{A} = -240\ \text{W}$$

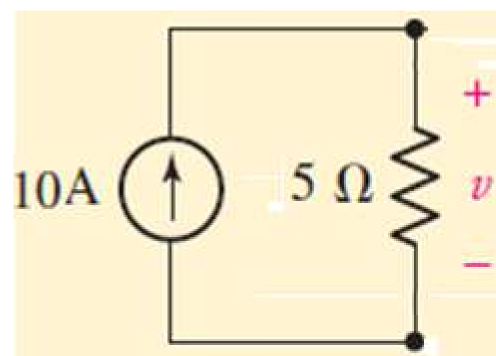
Actually 240 W is supplied

Practice 3.13

- Determine v in the circuit of below Figure by first combining the three current sources, and then the two 10 ohm resistors.



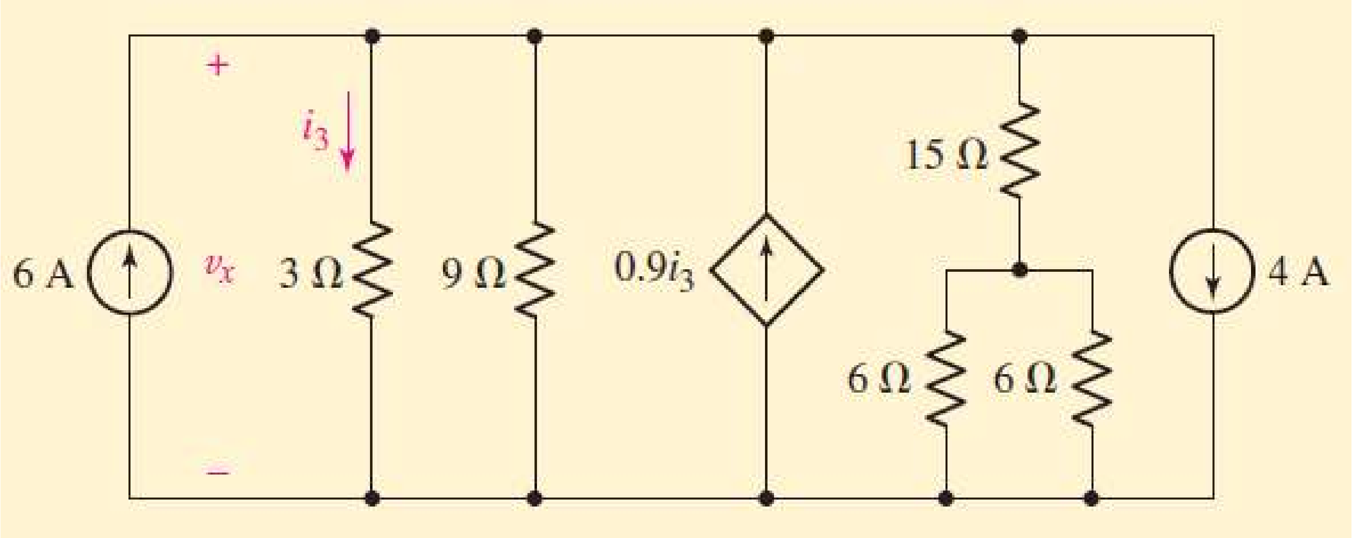


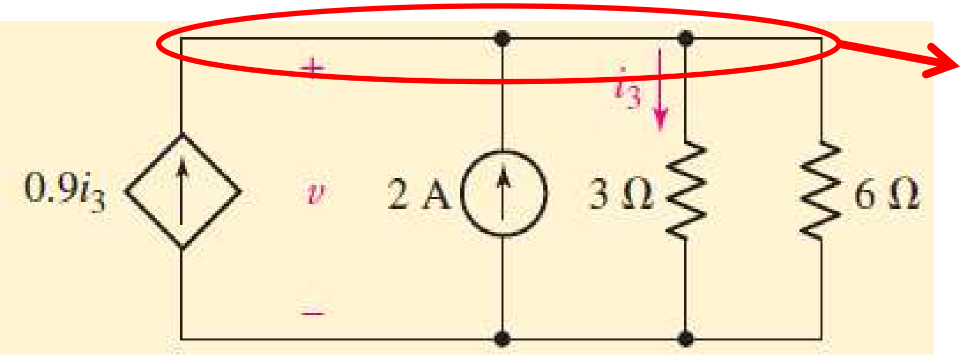


$$v = (5 - 1 + 6)10 // 10 = 10 \times 5 = \underline{50\ \text{V}}$$

Example 3.12

- Calculate the power and voltage of the dependent source in below Figure.





$$-0.9i_3 - 2 + i_3 + \frac{v}{6} = 0$$

$$v = 3i_3$$

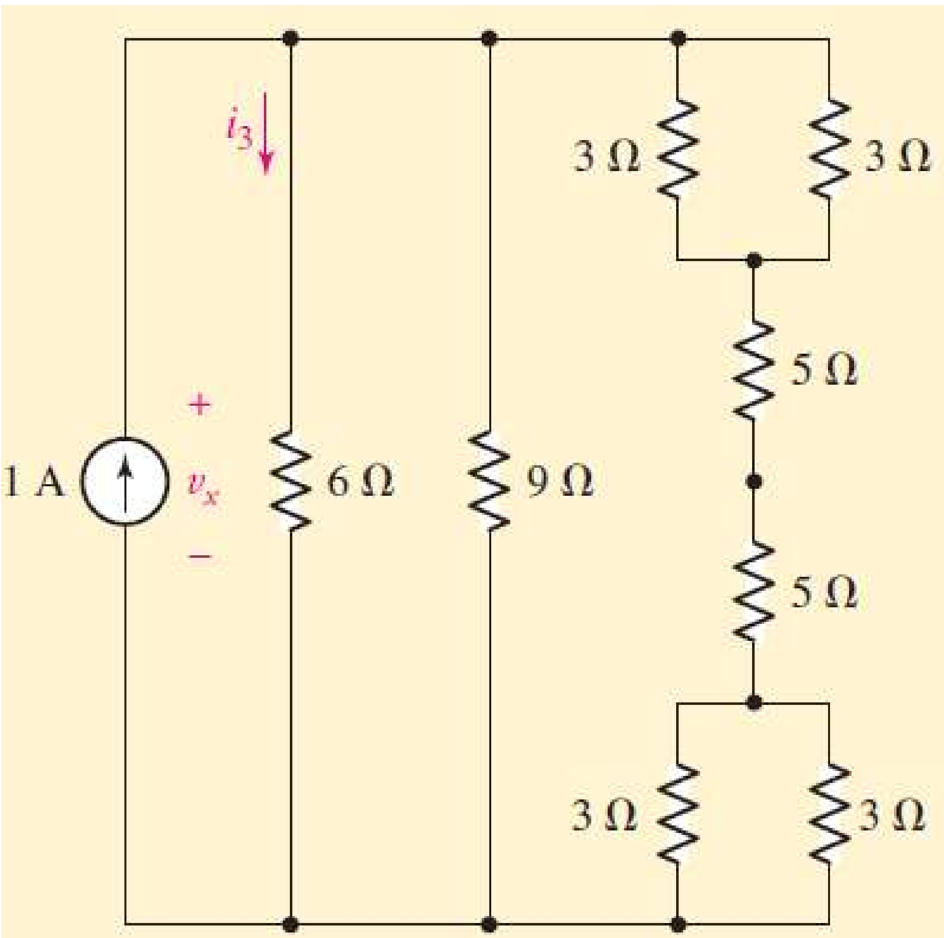
$$i_3 = \frac{10}{3}\ \text{A}$$

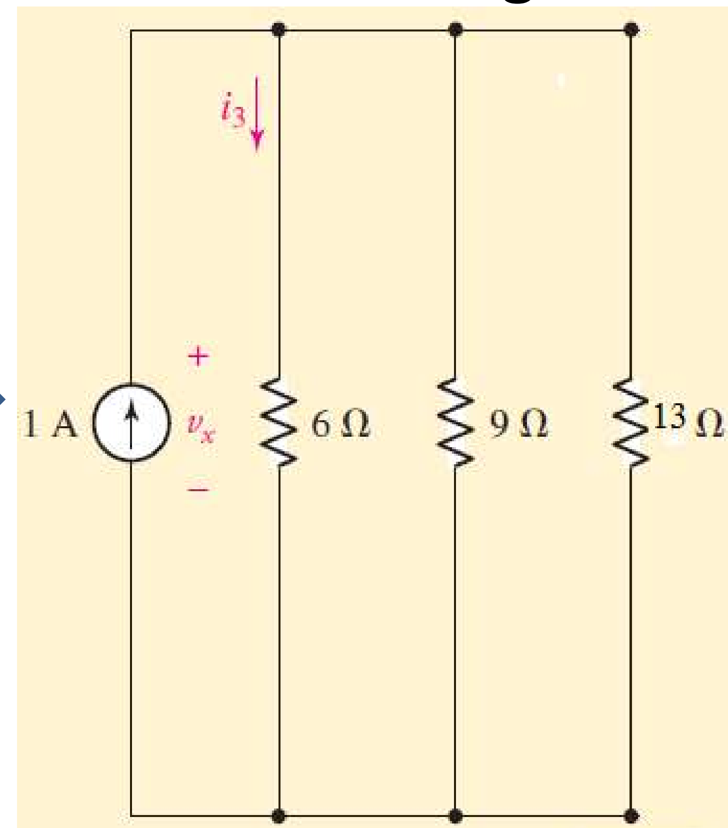
$$-v \times 0.9i_3 = -10(0.9)(10/3) = -30\ \text{W}$$

Actually 30 W is supplied

Practice 3.14

- For the circuit of below Figure, calculate the voltage *vx*



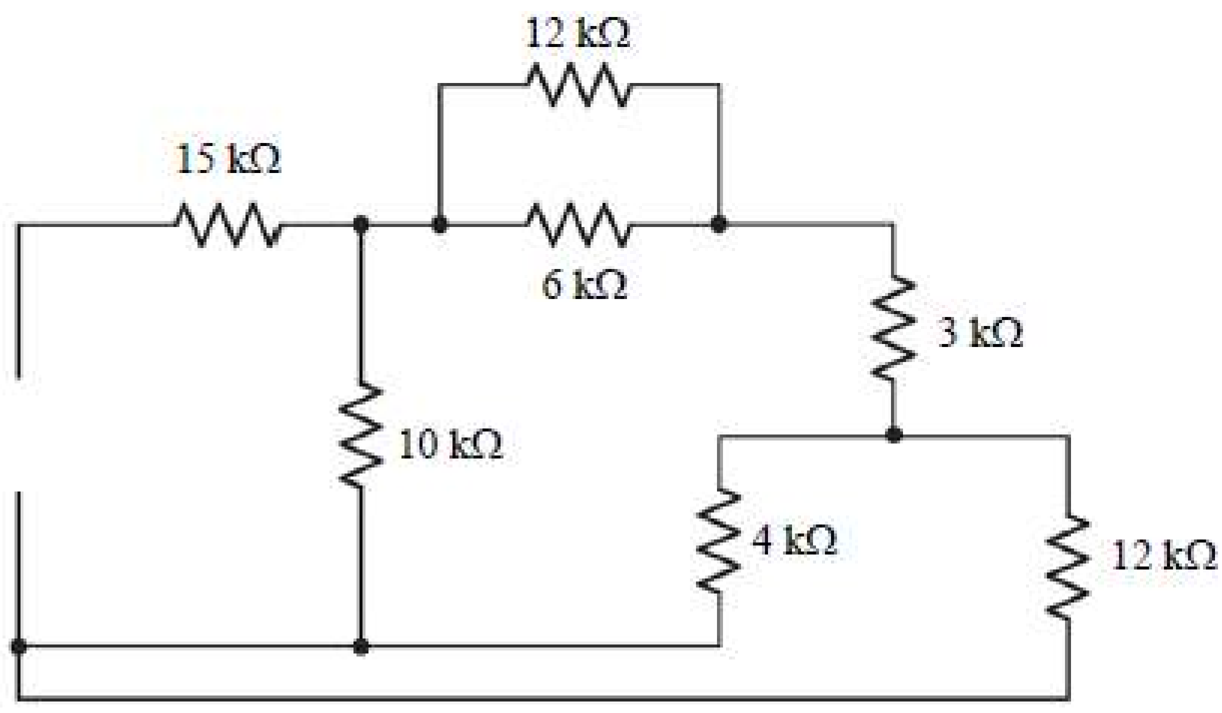


KCL yields $1 = v_x/6 + v_x/9 + v_x/13$

Solving, $v_x = 2.819$ V

Example

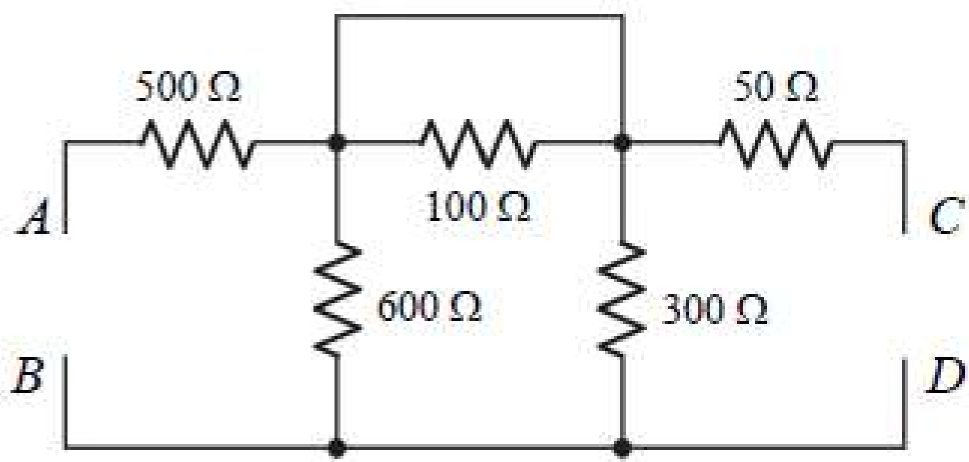
- Determine the equivalent resistance of this network between the open-circuit terminals.



20 kohms

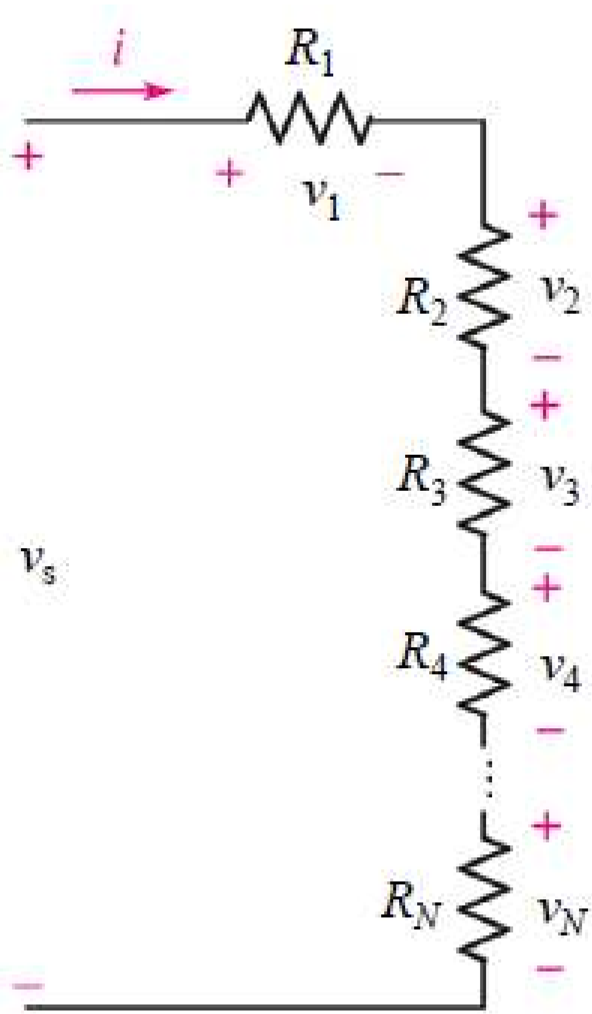
Example

- Determine the equivalent resistance of this network at terminals A-B.



700 ohms

Voltage Division



- another useful circuit simplification
- The voltage across a **single** resistor in a **series network** is equal to the **total voltage** across the network, **scaled by the *single resistance* divided by the *total resistance***.

$$v_s = v_1 + v_2 + v_3 + \ldots + v_N$$
$$= i\left[R_1 + R_2 + \ldots + R_N\right]$$
$$= i \cdot \sum_{k=1}^{N} R_k$$

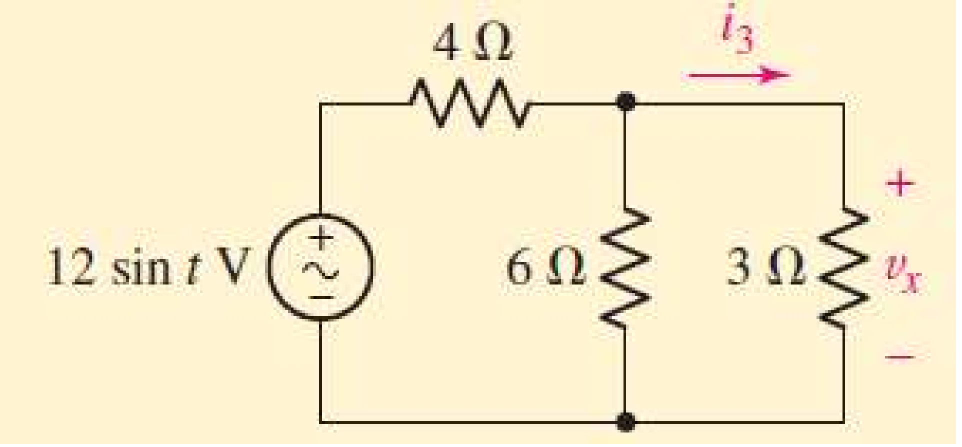
$$i = \frac{v_s}{\sum_{k=1}^{N} R_k}$$

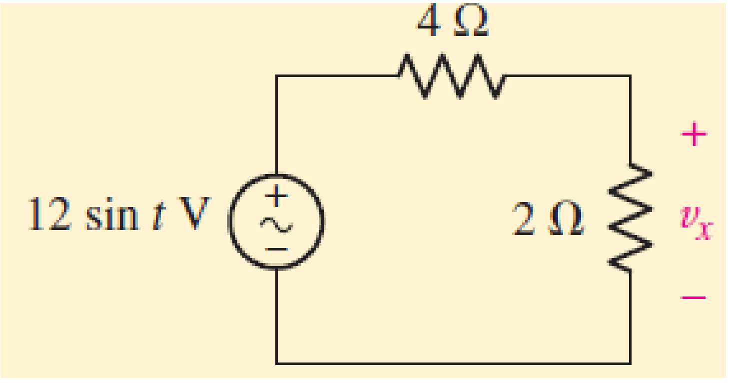
equivalent R in series

$$v_k = i \cdot R_k = \frac{v_s}{\sum_{k=1}^{N} R_k} \cdot R_k = \boxed{\left[\frac{R_k}{\sum_{k=1}^{N} R_k}\right] \cdot v_s}$$

Example (pg 62, #3.13)

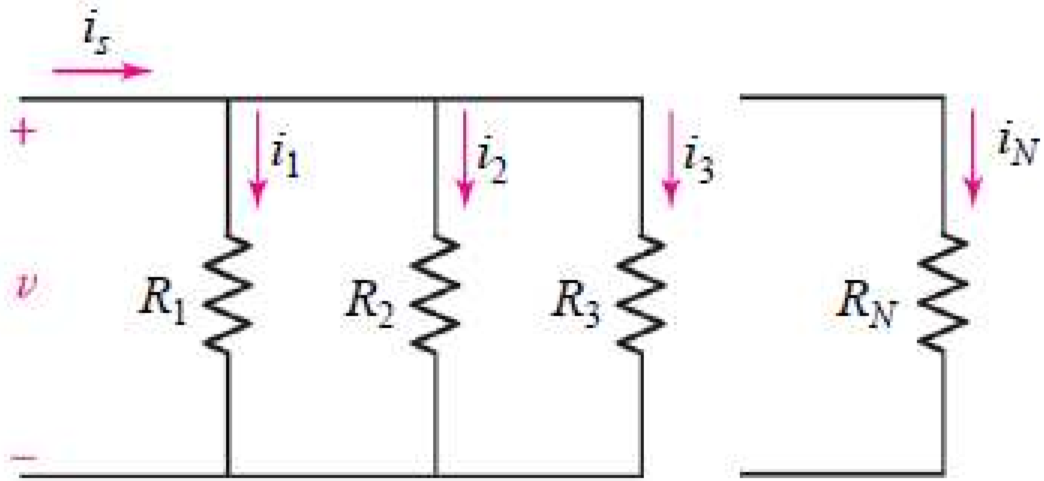
- Determine v_x in this circuit:





$$v_x = (12 \sin t)\frac{2}{4+2} = 4 \sin t$$

Current Division



$$i_s = i_1 + i_2 + i_3 + \ldots + i_N$$
$$= v\left[1/R_1 + 1/R_2 + \ldots + 1/R_N\right]$$
$$= v \cdot \sum_{k=1}^{N} 1/R_k$$

$$v = \frac{i_s}{\sum_{k=1}^{N} 1/R_k}$$

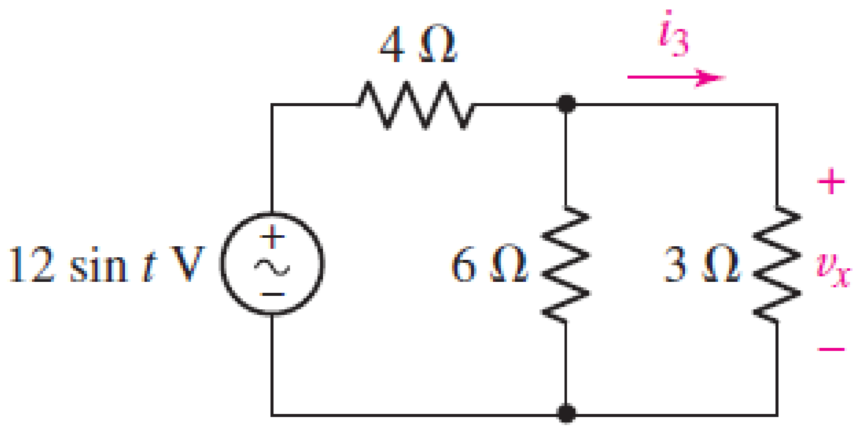
equivalent conductance; parallel circuit

The current through a **single** resistor in a **parallel** network is equal to the **total current** through the network, **scaled by the *conductance* of the resistor divided by the *total conductance***.

$$i_k = v \cdot \frac{1}{R_k} = \frac{i_s}{\sum_{k=1}^{N} 1/R_k} \cdot \frac{1}{R_k} = \boxed{\left[\frac{1/R_k}{\sum_{k=1}^{N} 1/R_k}\right] \cdot i_s}$$

Example (pg 64, #3.14)

- Determine i_3 for this circuit:



The total current flowing into the 3 Ω–6 Ω combination is

$$i(t) = \frac{12 \sin t}{4 + 3\|6} = \frac{12 \sin t}{4 + 2} = 2 \sin t \qquad \text{A}$$

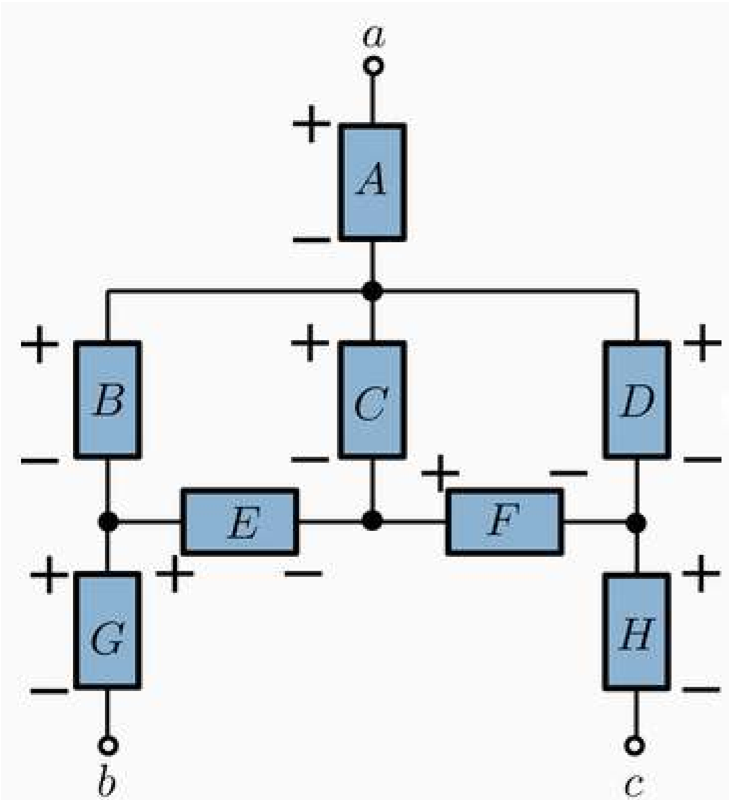
and thus the desired current is given by current division:

$$i_3(t) = (2 \sin t)\left(\frac{6}{6+3}\right) = \frac{4}{3} \sin t \qquad \text{A}$$

Chapter 3 Summary & Review

- Kirchhoff's Current Law: $$\sum i_{\text{in}} = \sum i_{\text{out}} = 0$$
  Kirchhoff's Voltage Law: $$\sum_{\text{loop}} v_k = 0$$
- networks in series: current through one must pass through the next
- networks in parallel: share the same two nodes (common voltage)
- voltage sources in series may be replaced by a single equivalent source; current sources in parallel may be replaced by a single equivalent source
- series resistors may be add to form an equivalent: $R_{\text{equivalent}}^{\text{series}} = \sum_{n=1}^{N} R_n$
- parallel resistors may be combined when *conductances* are added: $1/R_{\text{equivalent}}^{\text{parallel}} = \sum_{n=1}^{N} 1/R_n$
- voltage/current division allow us to calculate what *fraction* of voltage/current is associated with a single resistor

Examples



$$V_A + V_C - V_E + V_G = V_{ab}$$

$$\Rightarrow -2V + V_C - 3V + 4V = 10V$$

$$V_C = \boxed{11V}$$

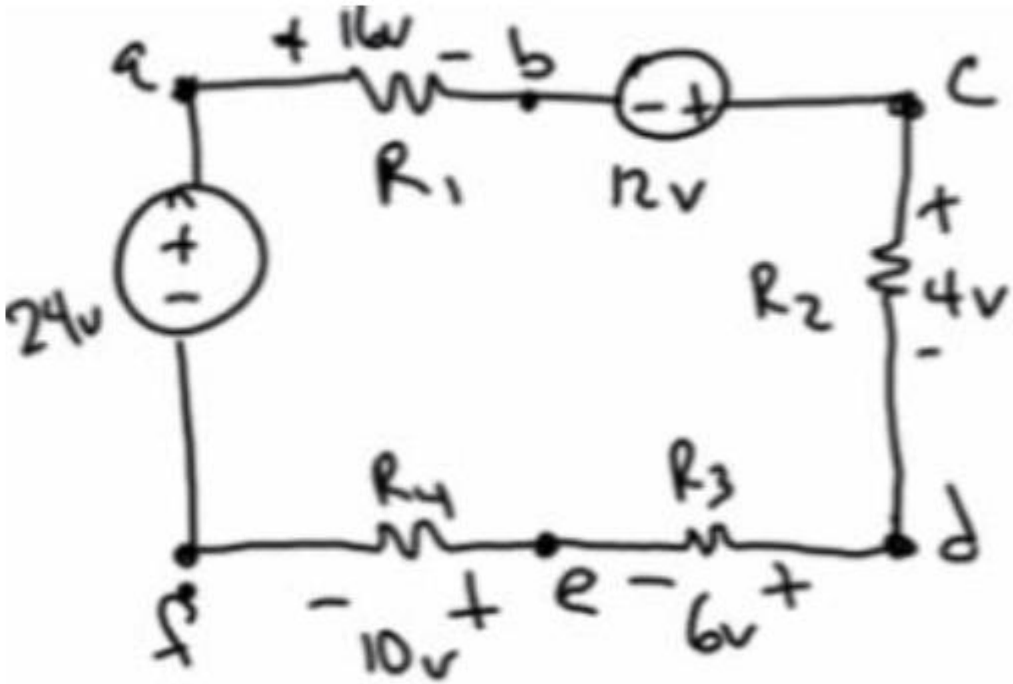
For the diagram above, the following voltages are specified:

$v_A = -2V$, $v_D = 2V$, $v_E = 3V$, $v_G = 4V$, $v_H = 2V$, $v_{ab} = 10V$

Find the value of v_C in volts

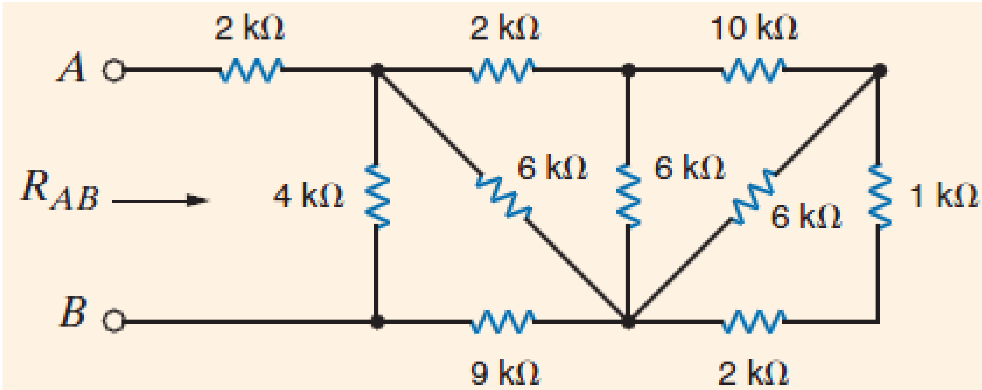
Examples

Write the loop equations. Find V_{ae} and V_{ec}

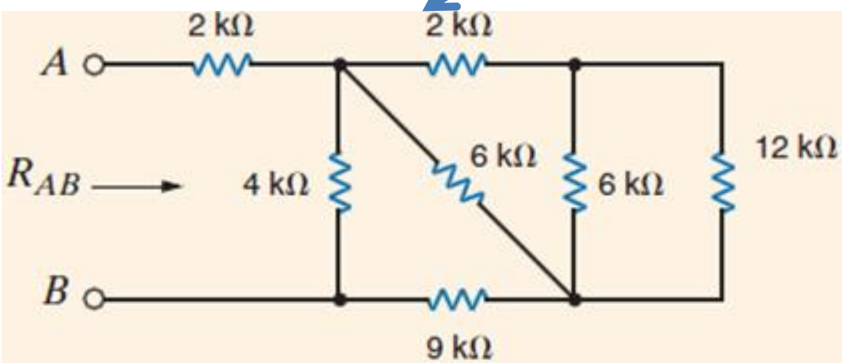


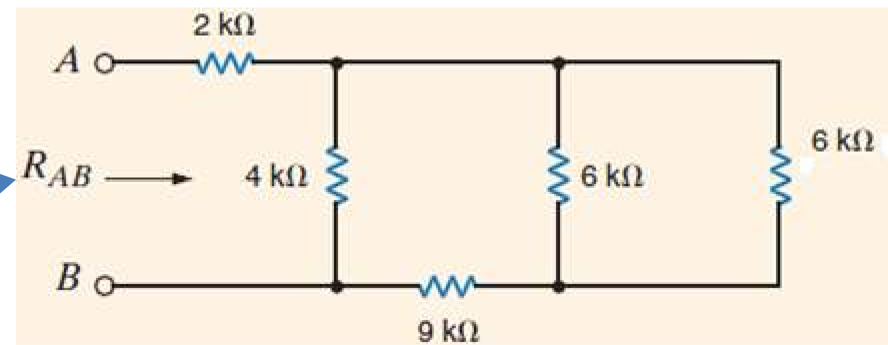
V_{ae} = 14V and V_{ec} = -10V

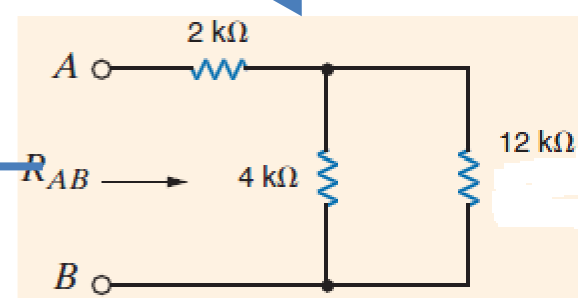
Examples

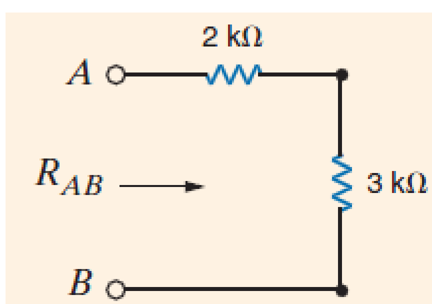


$R_{AB} = ?$



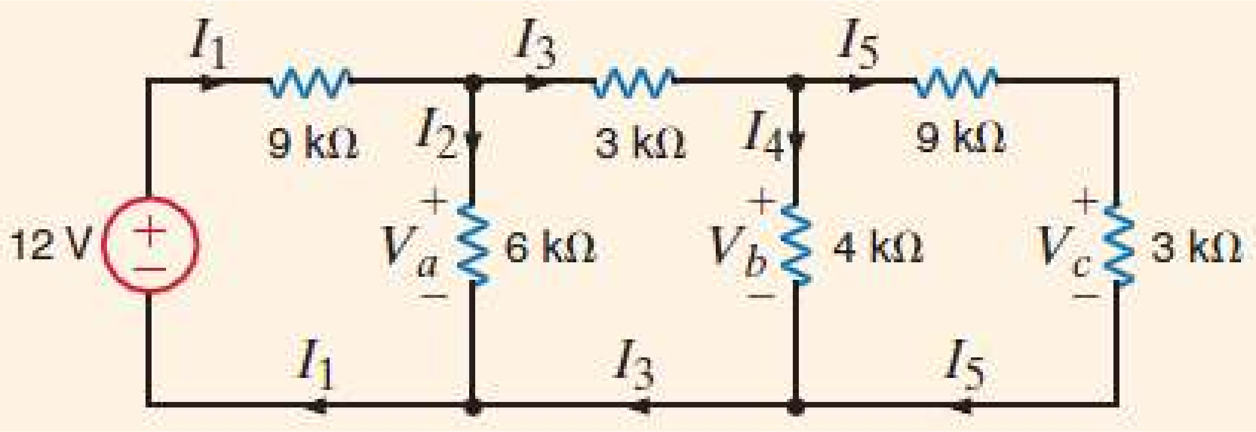


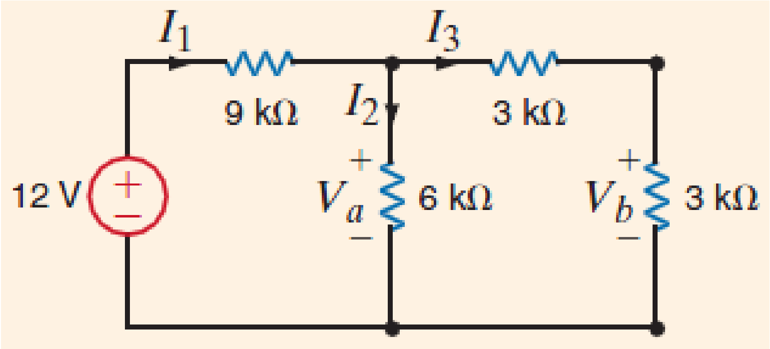


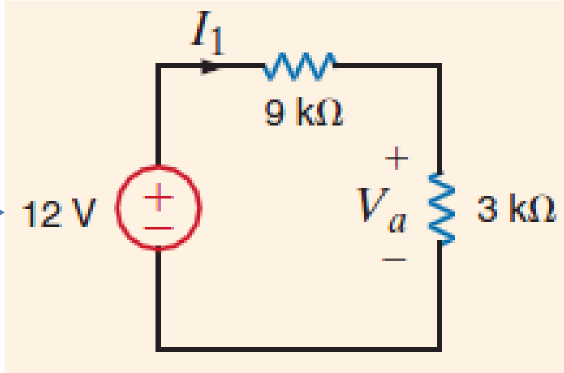


Examples

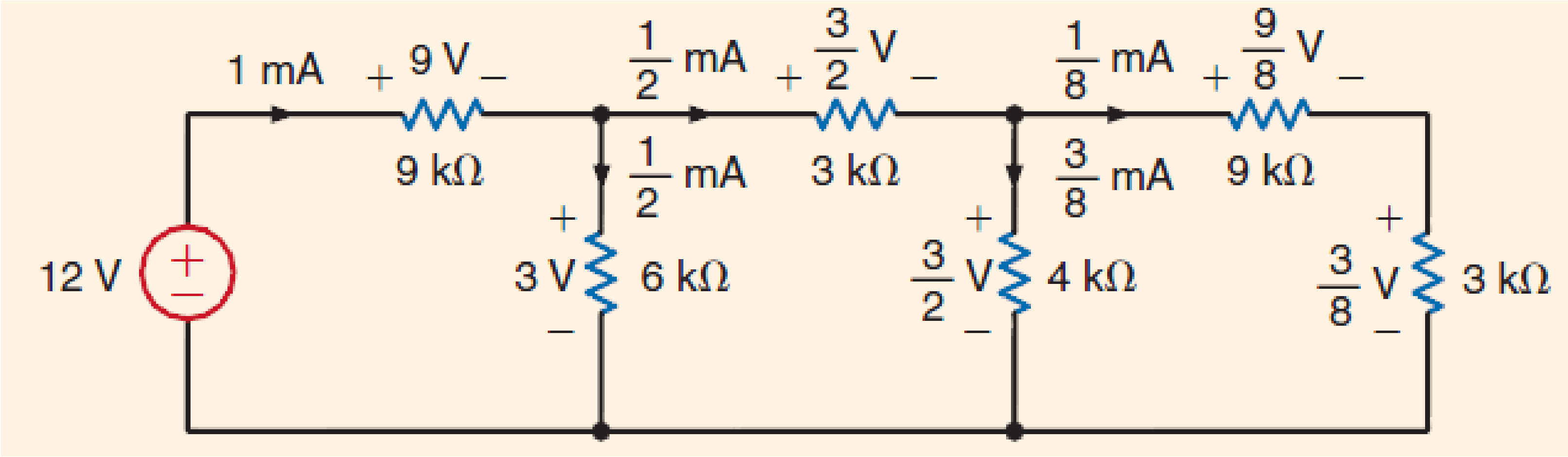
- find all the currents and voltages labeled in the ladder network







Solution



1 mA
+ 9 V −
9 kΩ
1/2 mA
+ 3/2 V −
3 kΩ
1/2 mA
1/8 mA
+ 9/8 V −
9 kΩ
3/8 mA
12 V
+ 3 V −
6 kΩ
+ 3/2 V −
4 kΩ
+ 3/8 V −
3 kΩ