



## BLM3620 Digital Signal Processing

Dr. Ali Can KARACA

*[ackaraca@yildiz.edu.tr](mailto:ackaraca@yildiz.edu.tr)*

Yıldız Technical University – Computer Engineering

## ***Important Materials:***

- James H. McClellan, R. W. Schafer, M. A. Yoder, *DSP First Second Edition*, Pearson, 2015.
- Lizhe Tan, Jean Jiang, *Digital Signal Processing: Fundamentals and Applications*, Third Edition, Academic Press, 2019.

## ***Auxiliary Materials:***

- Prof. Sarp Ertürk, *Sayısal İşaret İşleme*, Birsen Yayınevi.
- Prof. Nizamettin Aydın, DSP Lecture Notes.
- J. G. Proakis, D. K. Manolakis, *Digital Signal Processing Fourth Edition*, Pearson, 2014.
- J. K. Perin, *Digital Signal Processing, Lecture Notes*, Stanford University, 2018.

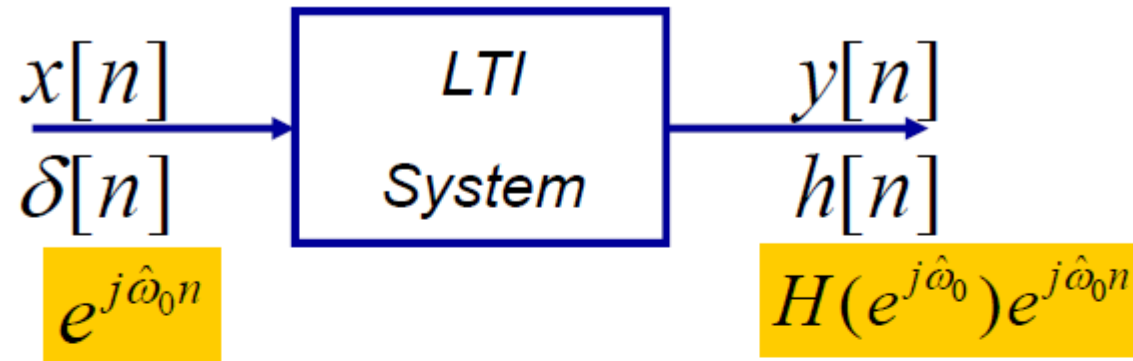
# Syllabus



Week	Lectures
1	Introduction to DSP and MATLAB
2	Sinuzoids and Complex Exponentials
3	Spectrum Representation
4	Sampling and Aliasing
5	Discrete Time Signal Properties and Convolution
6	Convolution and FIR Filters
7	Frequency Response of FIR Filters
8	Midterm Exam
9	Discrete Time Fourier Transform and Properties
10	Discrete Fourier Transform and Properties
11	Fast Fourier Transform and Windowing
12	z- Transforms
13	FIR Filter Design and Applications
14	IIR Filter Design and Applications
15	Final Exam

For more details -> Bologna page: <http://www.bologna.yildiz.edu.tr/index.php?r=course/view&id=5730&aid=3>

# Recap: Frequency Response $H(e^{j\hat{\omega}})$

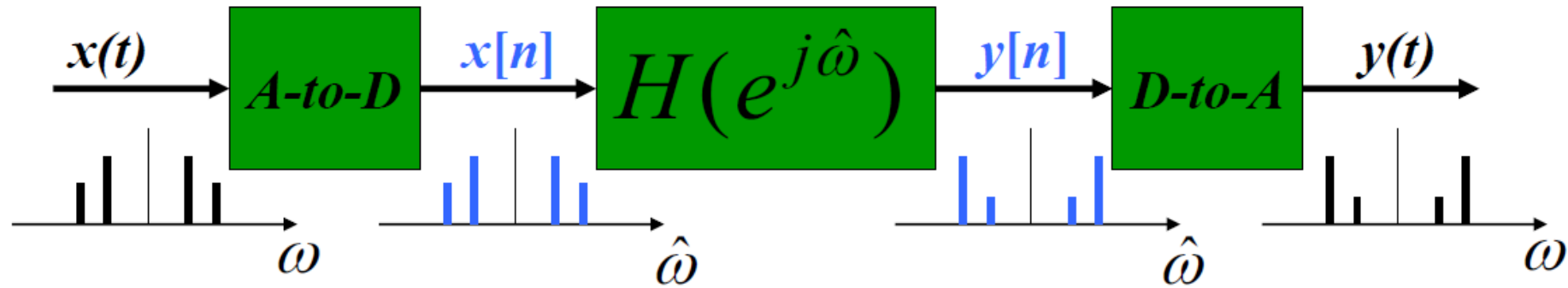


$$H(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\hat{\omega}n}$$

$$\text{Periodic : } H(e^{j(\hat{\omega}+2\pi)}) = H(e^{j\hat{\omega}})$$

$$y[n] = A \cdot |H(e^{j\hat{\omega}_0})| \cos(\hat{\omega}_0 n + \varphi + \angle H(e^{j\hat{\omega}_0}))$$

# Recap: Digital Filtering



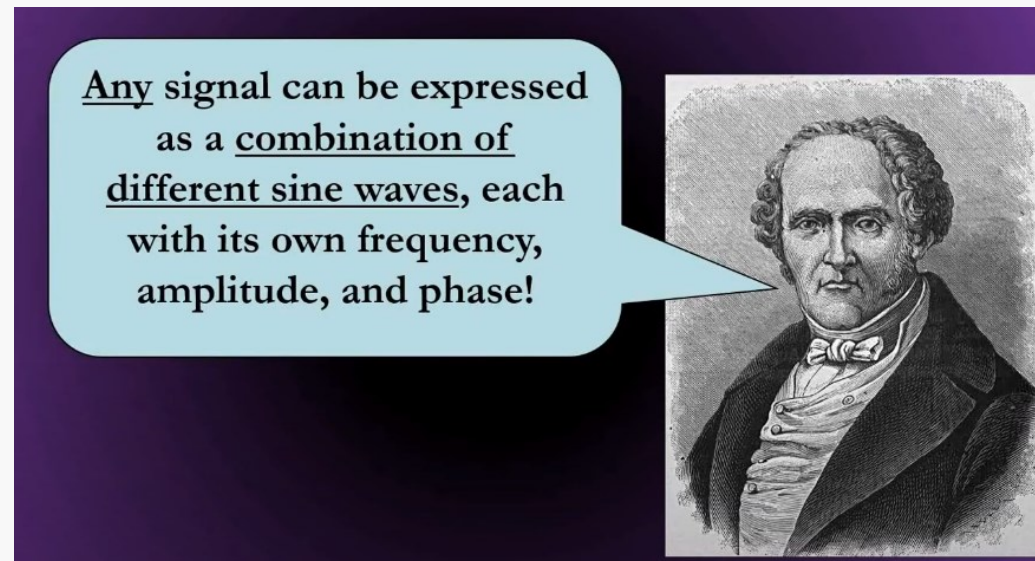
$\omega$  – SPECTRUM of  $x(t)$  (SUM of SINUSOIDS)

$\hat{\omega}$  – SPECTRUM of  $x[n]$   
• Is ALIASING a PROBLEM ?  
– SPECTRUM  $y[n]$  (FIR Gain or Nulls)

$\omega$  – Then, OUTPUT  $y(t)$  = SUM of SINUSOIDS

# Lecture #9 – Discrete Time Fourier Transform and Properties

- Discrete Time Fourier Transform
- Examples
- Solution with Properties
- MATLAB Applications
- Exercises



Credit by Mike X Cohen, @Youtube

# Discrete Time Fourier Transform



- It is a Generalized version of Frequency Response

$$H(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\hat{\omega}n}$$


- Definition of the **DTFT**:

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\hat{\omega}n}$$

- Forward DTFT

$$x[n] = \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\hat{\omega}}) e^{j\hat{\omega}n} d\hat{\omega}$$

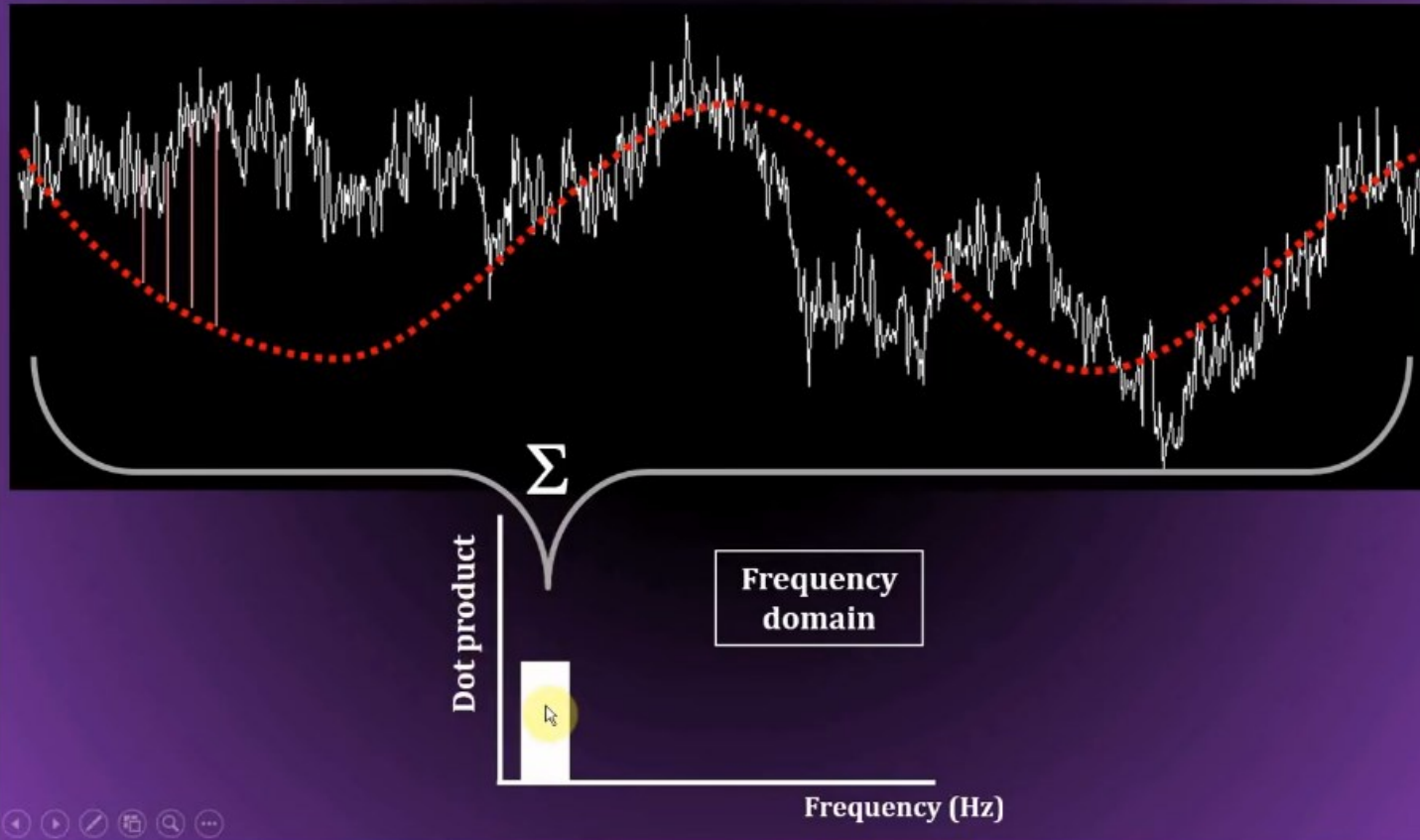
- **Inverse DTFT**


$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\hat{\omega}}) e^{j\hat{\omega}n} d\hat{\omega}$$

- Always periodic with a period of  $2\pi$

$$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$$

## DTFT: all about the dot products



$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\hat{\omega}n}$$

Credit by Mike X Cohen, @Youtube



# Periodicity of DTFT



- For any integer m:  $X(e^{j\hat{\omega}}) = X(e^{j(\hat{\omega}+2m\pi)})$

$$\begin{aligned} X(e^{j(\hat{\omega}+2\pi)}) &= \sum_{n=-\infty}^{\infty} x[n] e^{-j(\hat{\omega}+2m\pi)n} \\ &= \sum_{n=-\infty}^{\infty} x[n] e^{-j\hat{\omega}n} e^{-j2\pi mn} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\hat{\omega}n} \end{aligned}$$

1

- Discrete-time Fourier transform (DTFT) exists – provided that the sequence is **absolutely-summable**

$$\begin{aligned} |X(e^{j\hat{\omega}})| &= \left| \sum_{n=-\infty}^{\infty} x[n]e^{-j\hat{\omega}n} \right| \\ &\leq \sum_{n=-\infty}^{\infty} |x[n]e^{-j\hat{\omega}n}| = \sum_{n=-\infty}^{\infty} |x[n]| < \infty \end{aligned}$$

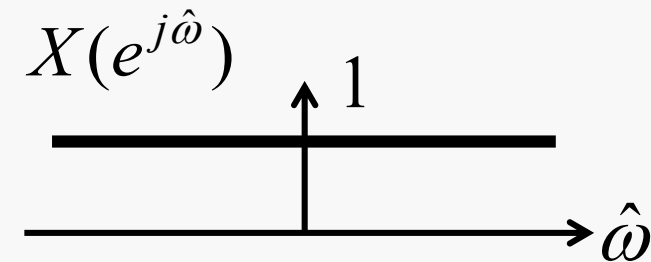
- DTFT applies to discrete time sequences,  $x[n]$ , regardless of length (if  $x[n]$  is **absolute summable**)

# DTFT of a Single Sample



$$x[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{elsewhere} \end{cases} = \delta[n]$$

Unit Impulse function



$$\begin{aligned} X(e^{j\hat{\omega}}) &= \sum_{n=-\infty}^{\infty} \delta[n] e^{-j\hat{\omega}n} \\ &= \sum_{n=0}^0 e^{-j\hat{\omega}n} = 1 \end{aligned}$$

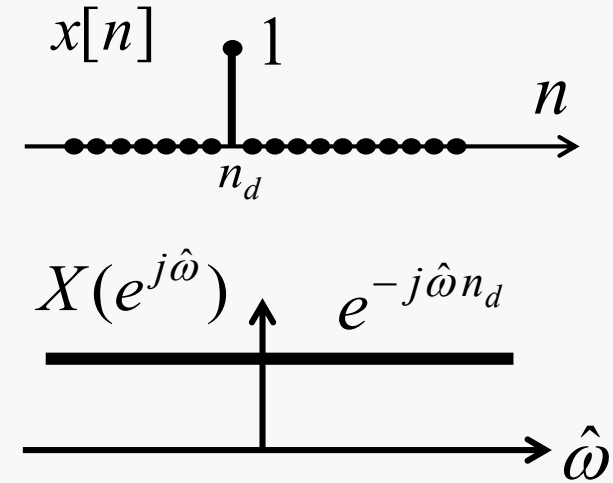
$$x[n] = \delta[n] \stackrel{\mathcal{F}}{\Leftrightarrow} X(e^{j\hat{\omega}}) = 1$$

# Delayed Unit Impulse



$$x_d[n] = \delta[n - n_d] = \begin{cases} 1, & n = n_d \\ 0, & \text{elsewhere} \end{cases}$$

$$X_d(e^{j\hat{\omega}}) = \sum_{n=n_d}^{n_d} e^{-j\hat{\omega} n} = e^{-j\hat{\omega} n_d}$$



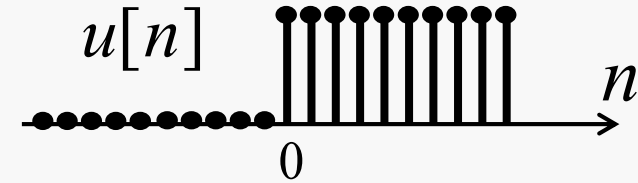
Generalizes to the delay property

$$x_d[n] = x[n - n_d] \Leftrightarrow$$
$$X_d(e^{j\hat{\omega}}) = X(e^{j\hat{\omega}}) e^{-j\hat{\omega} n_d} = e^{-j\hat{\omega} n_d}$$

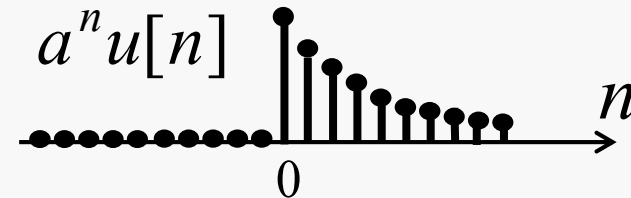
# DTFT of Right-Sided Exponential



$$\text{Unit Step Function: } u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$



$$x[n] = a^n u[n], \quad |a| < 1$$



$$\begin{aligned} X(e^{j\hat{\omega}}) &= \sum_{n=0}^{\infty} a^n e^{-j\hat{\omega}n} \\ &= \sum_{n=0}^{\infty} (ae^{-j\hat{\omega}})^n = \frac{1}{1 - ae^{-j\hat{\omega}}} \quad \text{if } |a| < 1 \end{aligned}$$

# Plotting: Magnitude and Angle Form



$$x[n] = a^n u[n] \Leftrightarrow X(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}}$$

$$X(e^{j\hat{\omega}}) = |X(e^{j\hat{\omega}})| e^{j\angle X(e^{j\hat{\omega}})}$$

$$|X(e^{j\hat{\omega}})|^2 = X(e^{j\hat{\omega}})X^*(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}} \cdot \frac{1}{1 - ae^{j\hat{\omega}}}$$

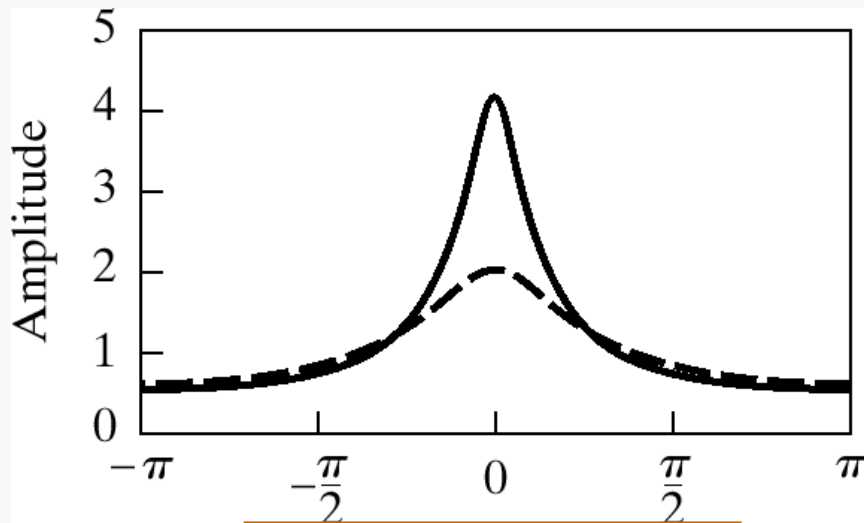
$$= \frac{1}{1 + a^2 - 2a \cos(\hat{\omega})}$$

$$\angle X(e^{j\hat{\omega}}) = \arctan\left(\frac{-a \sin(\hat{\omega})}{1 - a \cos(\hat{\omega})}\right)$$

# Magnitude and Angle Plots

## EVEN Function

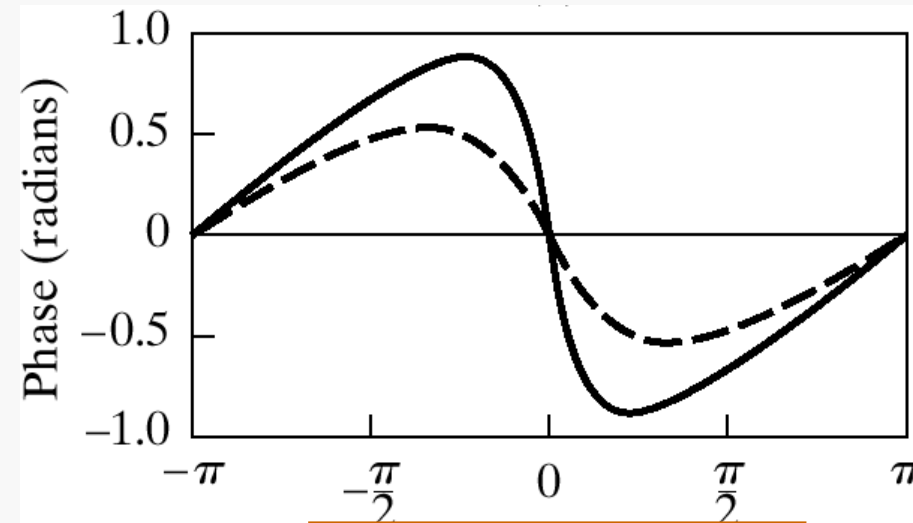
$$|X(e^{j\hat{\omega}})| = \frac{1}{(1 + a^2 - 2a \cos(\hat{\omega}))^{1/2}}$$
$$|X(e^{-j\hat{\omega}})| = |X(e^{j\hat{\omega}})|$$



Radian Frequency ( $\hat{\omega}$ )

## ODD Function

$$\angle X(e^{j\hat{\omega}}) = \arctan\left(\frac{-a \sin(\hat{\omega})}{1 - a \cos(\hat{\omega})}\right)$$
$$\angle X(e^{-j\hat{\omega}}) = -\angle X(e^{j\hat{\omega}})$$



Radian Frequency ( $\hat{\omega}$ )

# Inverse DTFT ?



$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\hat{\omega}}) e^{j\hat{\omega}n} d\hat{\omega}$$

$$X(e^{j\hat{\omega}}) = \frac{1}{1 + 0.3e^{-j\hat{\omega}}} \Rightarrow x[n] = ?$$

$$x[n] = \int_{-\pi}^{\pi} \frac{1}{1 + 0.3e^{-j\hat{\omega}}} e^{j\hat{\omega}n} d\hat{\omega} \quad ??$$

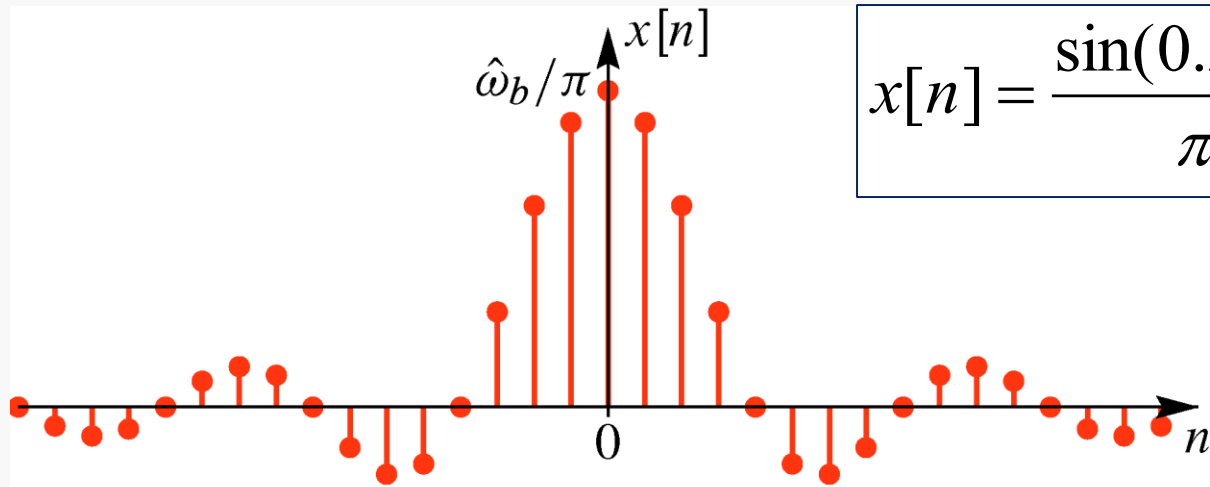
$$x[n] = a^n u[n] \Leftrightarrow X(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}}$$



# SINC Function:



- A “**sinc**” function or sequence



$$x[n] = \frac{\sin(0.25\pi n)}{\pi n}, \quad -\infty < n < \infty$$

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} \frac{\sin(0.25\pi n)}{\pi n} e^{-j\hat{\omega}n} = ??$$

# SINC Function from the inverse DTFT integral

Given a “**sinc**” function or sequence

$$x[n] = \frac{\sin(0.2\pi n)}{\pi n}, \quad -\infty < n < \infty$$

Consider an ideal band-limited signal:

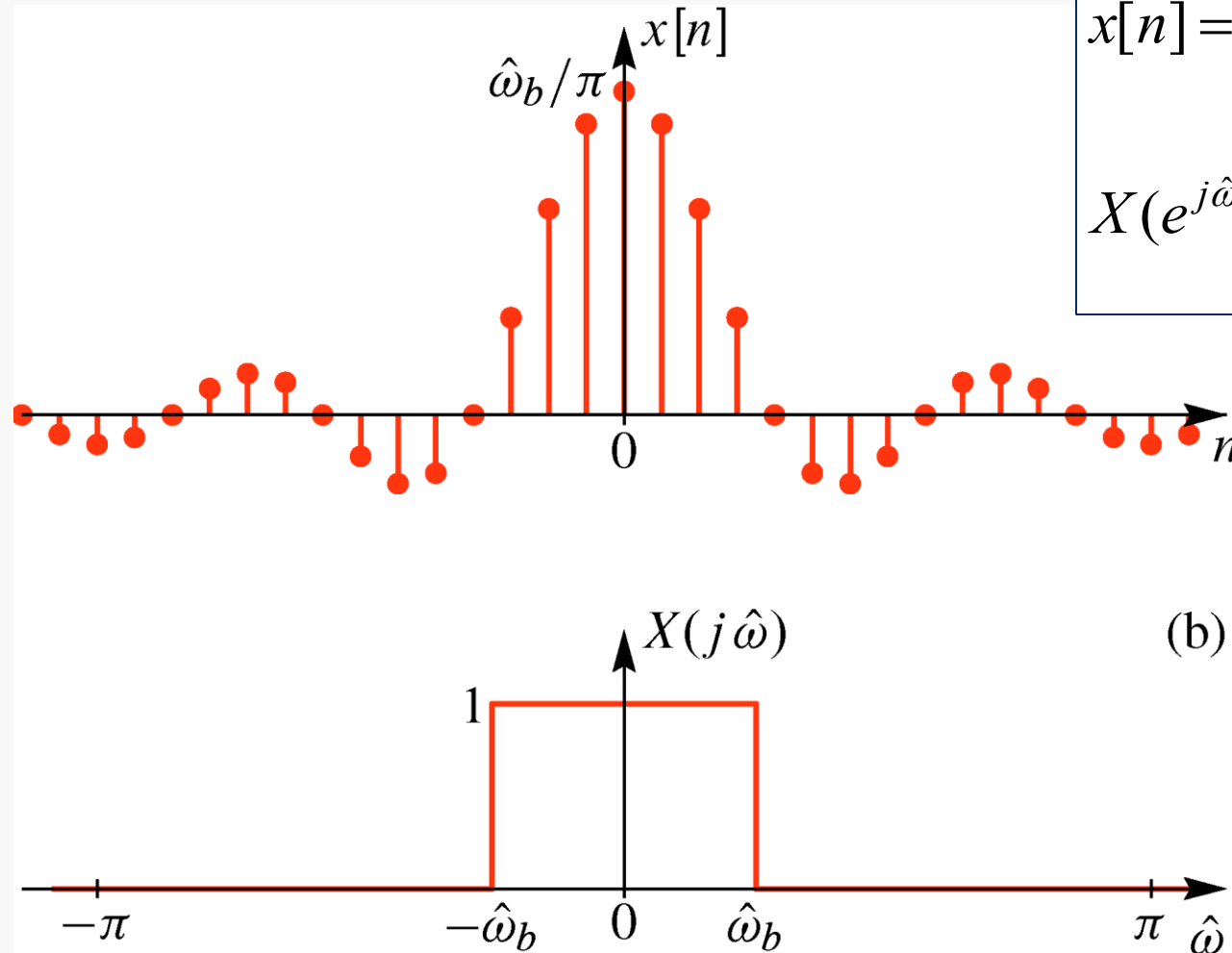
$$X(e^{j\hat{\omega}}) = \begin{cases} 1, & |\hat{\omega}| \leq 0.2\pi \\ 0, & 0.2\pi < |\hat{\omega}| \leq \pi \end{cases}$$

**Discrete-time  
Fourier Transform  
Pair**

$$\begin{aligned} x[n] &= \frac{1}{2\pi} \int_{-0.2\pi}^{0.2\pi} e^{j\hat{\omega}n} d\hat{\omega} = \frac{e^{j\hat{\omega}n}}{2\pi jn} \Big|_{-0.2\pi}^{0.2\pi} \\ &= \frac{e^{j0.2\pi n} - e^{-j0.2\pi n}}{2\pi jn} = \frac{\sin(0.2\pi n)}{\pi n} \end{aligned}$$

$$\begin{aligned} x[n] &= \frac{\sin(\hat{\omega}_b n)}{\pi n} && \xleftrightarrow{DTFT} \\ X(e^{j\hat{\omega}}) &= \begin{cases} 1, & |\hat{\omega}| \leq \hat{\omega}_b \\ 0, & \hat{\omega}_b < |\hat{\omega}| \leq \pi \end{cases} \end{aligned}$$

# SINC Function – Rectangle DTFT pair



$$x[n] = \frac{\sin(\hat{\omega}_b n)}{\pi n} \quad \begin{matrix} DTFT \\ \Leftrightarrow \end{matrix}$$
$$X(e^{j\hat{\omega}}) = \begin{cases} 1, & |\hat{\omega}| \leq \hat{\omega}_b \\ 0, & \hat{\omega}_b < |\hat{\omega}| \leq \pi \end{cases}$$

# Summary of DTFT Pairs



$$x[n] = \delta[n - n_d] \Leftrightarrow X(e^{j\hat{\omega}}) = e^{-j\hat{\omega}n_d}$$

Delayed Impulse

$$x[n] = a^n u[n] \Leftrightarrow X(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}}$$

Right-sided  
Exponential

$$x[n] = \frac{\sin(\hat{\omega}_c n)}{\pi n} \Leftrightarrow X(e^{j\hat{\omega}}) = \begin{cases} 1 & |\hat{\omega}| \leq \hat{\omega}_c \\ 0 & \hat{\omega}_c < |\hat{\omega}| < \pi \end{cases}$$

sinc function  
is **Bandlimited**

$$x[n] = \begin{cases} 1 & 0 \leq n < L \\ 0 & \text{otherwise} \end{cases} \Leftrightarrow X(e^{j\hat{\omega}}) = \frac{\sin(\frac{1}{2} L \hat{\omega})}{\sin(\frac{1}{2} \hat{\omega})} e^{-j\hat{\omega}(L-1)/2}$$

- The DTFT provides a **frequency-domain** representation that is invaluable for thinking about signals and solving DSP problems.
- To use it effectively you must
  - know **PAIRS**: the Fourier transforms of certain important signals
  - know **properties** and certain key **theorems**
  - be able to combine time-domain and frequency domain methods appropriately

# Property of DTFT



**Table 7-2** Basic discrete-time Fourier transform properties.

<b>Table of DTFT Properties</b>		
<i>Property Name</i>	<i>Time-Domain: <math>x[n]</math></i>	<i>Frequency-Domain: <math>X(e^{j\hat{\omega}})</math></i>
Periodic in $\hat{\omega}$		$X(e^{j(\hat{\omega}+2\pi)}) = X(e^{j\hat{\omega}})$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$
Conjugate Symmetry	$x[n]$ is real	$X(e^{-j\hat{\omega}}) = X^*(e^{j\hat{\omega}})$
Conjugation	$x^*[n]$	$X^*(e^{-j\hat{\omega}})$
Time-Reversal	$x[-n]$	$X(e^{-j\hat{\omega}})$
Delay	$x[n - n_d]$	$e^{-j\hat{\omega}n_d} X(e^{j\hat{\omega}})$
Frequency Shift	$x[n]e^{j\hat{\omega}_0 n}$	$X(e^{j(\hat{\omega}-\hat{\omega}_0)})$
Modulation	$x[n] \cos(\hat{\omega}_0 n)$	$\frac{1}{2} X(e^{j(\hat{\omega}-\hat{\omega}_0)}) + \frac{1}{2} X(e^{j(\hat{\omega}+\hat{\omega}_0)})$
Convolution	$x[n] * h[n]$	$X(e^{j\hat{\omega}}) H(e^{j\hat{\omega}})$
Autocorrelation	$x[-n] * x[n]$	$ X(e^{j\hat{\omega}}) ^2$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty}  x[n] ^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi}  X(e^{j\hat{\omega}}) ^2 d\hat{\omega}$	

- Linearity

$$x[n] = ax_1[n] + bx_2[n] \Leftrightarrow X(e^{j\hat{\omega}}) = aX_1(e^{j\hat{\omega}}) + bX_2(e^{j\hat{\omega}})$$

- Time-Delay  $\leftrightarrow$  phase shift

$$y[n] = x[n - n_d] \Leftrightarrow Y(e^{j\hat{\omega}}) = X(e^{j\hat{\omega}})e^{-j\hat{\omega}n_d}$$

- Frequency-Shift  $\leftrightarrow$  multiply by sinusoid

$$y[n] = e^{j\hat{\omega}_c n} x[n] \Leftrightarrow Y(e^{j\hat{\omega}}) = X(e^{j(\hat{\omega} - \hat{\omega}_c)})$$

## Example-1:



$x[n] = 0.2^n u[n - 4] + 0.4^{n-1} u[n]$  -> Find the DTFT of this signal?

$$x[n] = a^n u[n] \xleftrightarrow{\text{DTFT}} X(e^{j\Omega}) = \frac{1}{1 - ae^{-j\hat{\omega}}}, \quad |a| < 1$$

Using Linearity and Shifting Properties:

$$x[n] = 0.2^4 0.2^{n-4} u[n - 4] + 0.4^{-1} 0.4^n u[n]$$



# Example-1:



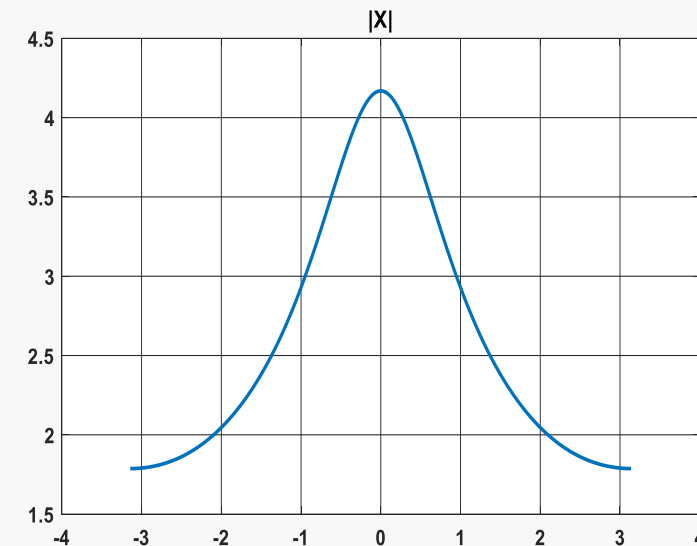
Let's plot the magnitude of DTFT

$$X(e^{j\Omega}) = \frac{0.2^4 e^{-j4\Omega}}{1 - 0.2e^{-j\Omega}} + \frac{2.5}{1 - 0.4e^{-j\Omega}}$$

```
clc; clear all;
what = -pi:0.01:pi;
N = 6;
firstPart = (0.2^4)*(exp(-j*4*omega))./(1-0.2*exp(-j*omega));
secondPart = (2.5)./(1-0.4*exp(-j*omega));

DTFT = firstPart + secondPart ;

figure(1);
plot(what,abs(DTFT));
title('|X|');
```



## Example-2:



Given DT Fourier Transform  $X(e^{j\hat{\omega}}) = \frac{1}{(1 - 0.3e^{-j\hat{\omega}})(1 - 0.4e^{-j\hat{\omega}})}$ , find  $x[n]$  signal.

$$x[n] = a^n u[n] \xleftrightarrow{\text{DTFT}} X(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}}, |a| < 1$$

$$X(e^{j\hat{\omega}}) = \frac{1}{(1 - 0.3e^{-j\hat{\omega}})(1 - 0.4e^{-j\hat{\omega}})} = \frac{A}{(1 - 0.3e^{-j\hat{\omega}})} + \frac{B}{(1 - 0.4e^{-j\hat{\omega}})}$$

$$A = X(e^{j\hat{\omega}}) (1 - 0.3e^{-j\hat{\omega}}) \xrightarrow{e^{j\hat{\omega}}=1/0.3} \frac{1}{(1 - 0.4/0.3)} = -3$$

$$B = X(e^{j\hat{\omega}}) (1 - 0.4e^{-j\hat{\omega}}) \xrightarrow{e^{j\hat{\omega}}=1/0.4} \frac{1}{(1 - 0.3/0.4)} = 4$$

## Example-2: Örnek-2:



Given DT Fourier Transform  $X(e^{j\hat{\omega}}) = \frac{1}{(1 - 0.3e^{-j\hat{\omega}})(1 - 0.4e^{-j\hat{\omega}})}$

$$x[n] = a^n u[n] \xleftrightarrow{\text{DTFT}} X(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}}, |a| < 1$$

$$X(e^{j\hat{\omega}}) = \frac{1}{(1 - 0.3e^{-j\hat{\omega}})(1 - 0.4e^{-j\hat{\omega}})} = \frac{-3}{(1 - 0.3e^{-j\hat{\omega}})} + \frac{4}{(1 - 0.4e^{-j\hat{\omega}})}$$

*-3. (0.3)^n u[n]  
+ 4. (0.4)^n u[n]*

Using Linearity Property:

$$x[n] = -3 \times 0.3^n u[n] + 4 \times 0.4^n u[n]$$

## Example-3:



$$y[n] - 0.2y[n - 1] = 0.5x[n] - 0.3x[n - 1] + 0.1x[n - 2]$$

Find frequency response and impulse response of the system given above.

Using shifting property:

$$y[n] = x[n - n_d] \Leftrightarrow Y(e^{j\hat{\omega}}) = X(e^{j\hat{\omega}})e^{-j\hat{\omega}n_d}$$

Using linearity property:

$$Y(e^{j\hat{\omega}}) - 0.2e^{-j\hat{\omega}}Y(e^{j\hat{\omega}}) = 0.5X(e^{j\hat{\omega}}) - 0.3e^{-j\hat{\omega}}X(e^{j\hat{\omega}}) + 0.1e^{-j2\hat{\omega}}X(e^{j\hat{\omega}})$$

$$Y(e^{j\hat{\omega}}) = X(e^{j\hat{\omega}})H(e^{j\hat{\omega}}) \rightarrow$$

$$H(e^{j\hat{\omega}}) = \frac{Y(e^{j\hat{\omega}})}{X(e^{j\hat{\omega}})} = \frac{0.5 - 0.3e^{-j\hat{\omega}} + 0.1e^{-j2\hat{\omega}}}{1 - 0.2e^{-j\hat{\omega}}} = \frac{0.5}{1 - 0.2e^{-j\hat{\omega}}} - \frac{0.3e^{-j\hat{\omega}}}{1 - 0.2e^{-j\hat{\omega}}} + \frac{0.1e^{-j2\hat{\omega}}}{1 - 0.2e^{-j\hat{\omega}}}$$

$0.5 \cdot (e^{j\hat{\omega}})^0$   
 $0.3 \cdot (e^{j\hat{\omega}})^{-1}$   
 $0.1 \cdot (e^{j\hat{\omega}})^{-2}$

## Example-3:



$$H(e^{j\hat{\omega}}) = \frac{Y(e^{j\hat{\omega}})}{X(e^{j\hat{\omega}})} = \frac{0.5 - 0.3e^{-j\hat{\omega}} + 0.1e^{-j2\hat{\omega}}}{1 - 0.2e^{-j\hat{\omega}}} = \frac{0.5}{1 - 0.2e^{-j\hat{\omega}}} - \frac{0.3e^{-j\hat{\omega}}}{1 - 0.2e^{-j\hat{\omega}}} + \frac{0.1e^{-j2\hat{\omega}}}{1 - 0.2e^{-j\hat{\omega}}}$$

$$x[n] = a^n u[n] \xleftrightarrow{\text{DTFT}} X(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}}, |a| < 1$$

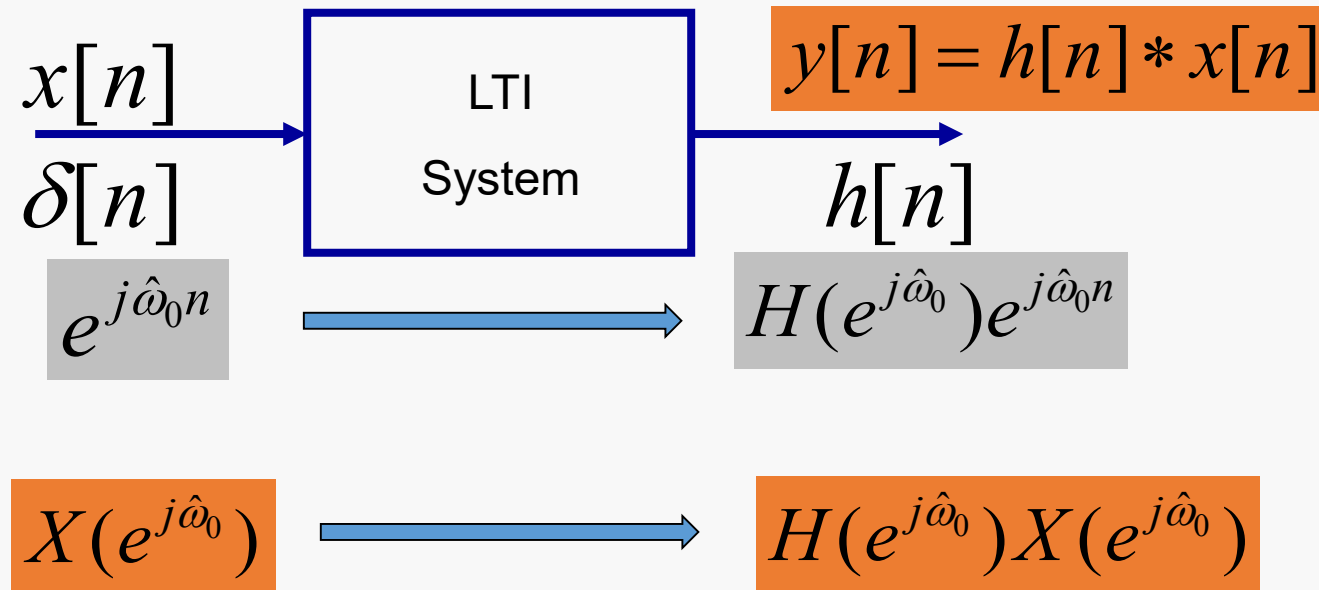
Impulse Response:

$$h[n] = 0.5 \times 0.2^n u[n] - 0.3 \times 0.2^{n-1} u[n-1] + 0.1 \times 0.2^{n-2} u[n-2]$$

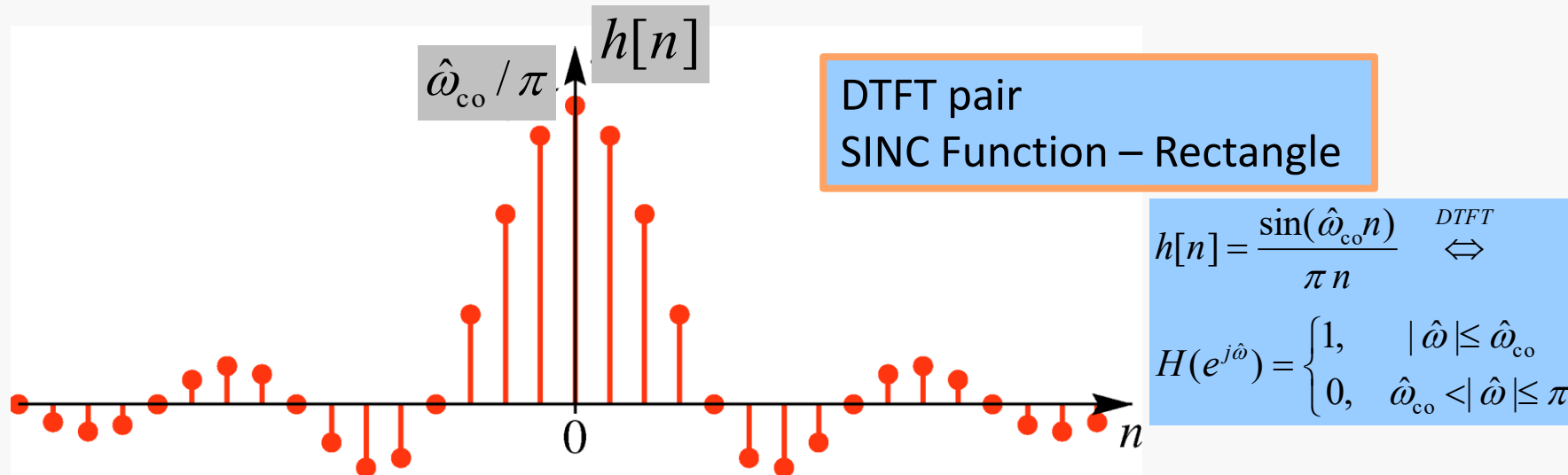
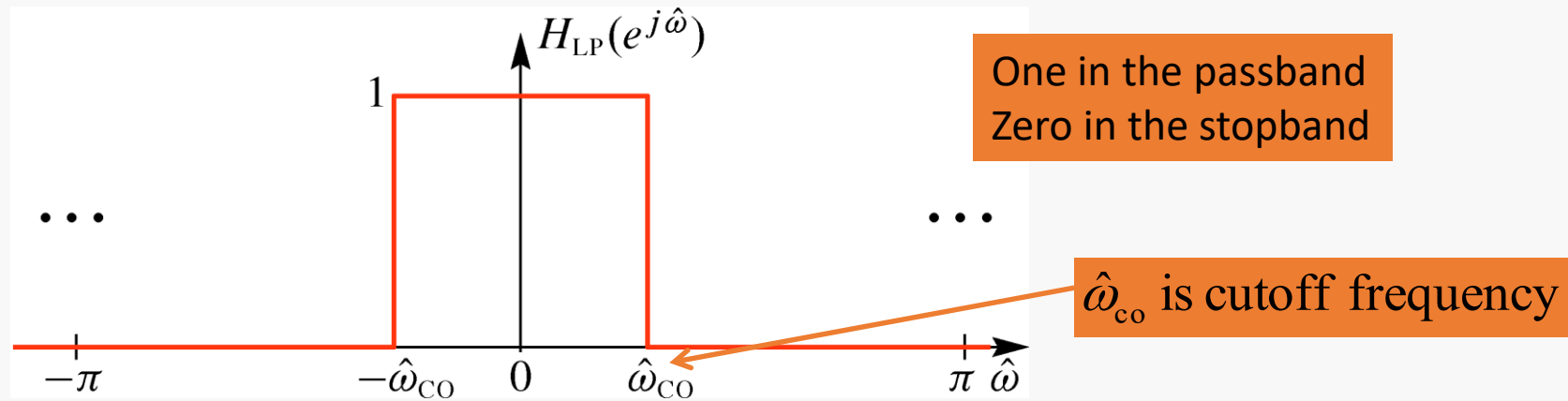
# DTFT maps Convolution to Multiplication



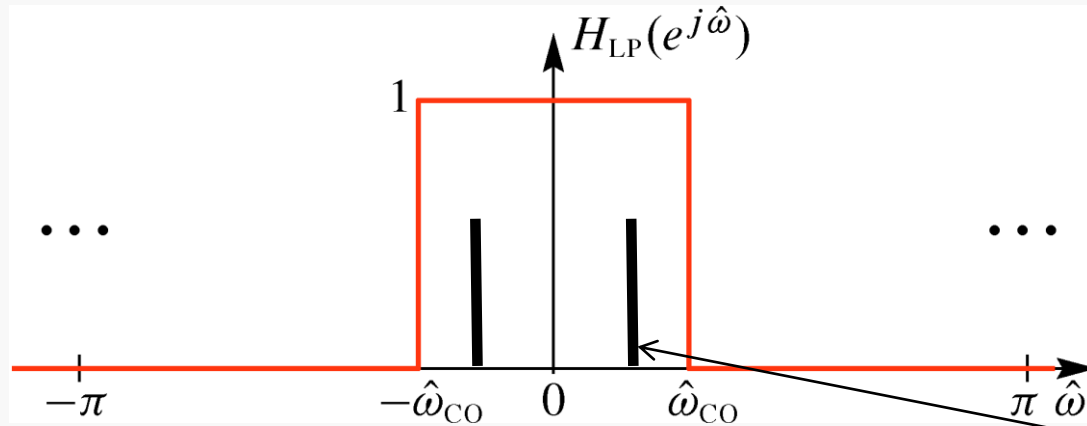
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \Leftrightarrow Y(e^{j\hat{\omega}}) = H(e^{j\hat{\omega}})X(e^{j\hat{\omega}})$$



# IDEAL LowPass Filter (LPF)



# Filtering with the IDEAL LPF



$$h[n] = \frac{\sin(\hat{\omega}_{co}n)}{\pi n} \stackrel{DTFT}{\Leftrightarrow} H(e^{j\hat{\omega}}) = \begin{cases} 1, & |\hat{\omega}| \leq \hat{\omega}_{co} \\ 0, & \hat{\omega}_{co} < |\hat{\omega}| \leq \pi \end{cases}$$

Find the output when the input is a sinusoid:

$$x[n] = \cos(\hat{\omega}_0 n)$$

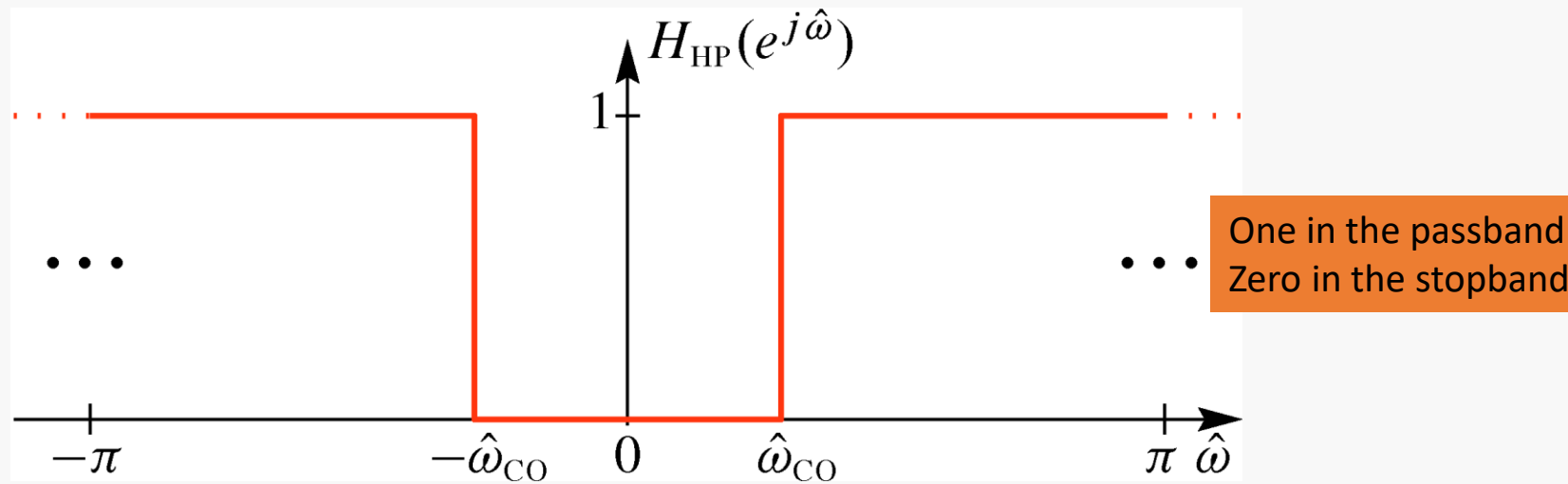
$$y[n] = h[n] * x[n] = \frac{\sin(\hat{\omega}_{co}n)}{\pi n} * \cos(\hat{\omega}_0 n) \quad ??$$

**Multiply** the spectrum of the input times the DTFT of the filter to get

$$y[n] = \begin{cases} \cos(\hat{\omega}_0 n) & \hat{\omega}_0 \leq \hat{\omega}_{co} \\ 0 & \hat{\omega}_0 > \hat{\omega}_{co} \end{cases}$$



# IDEAL HighPass Filter (HPF)



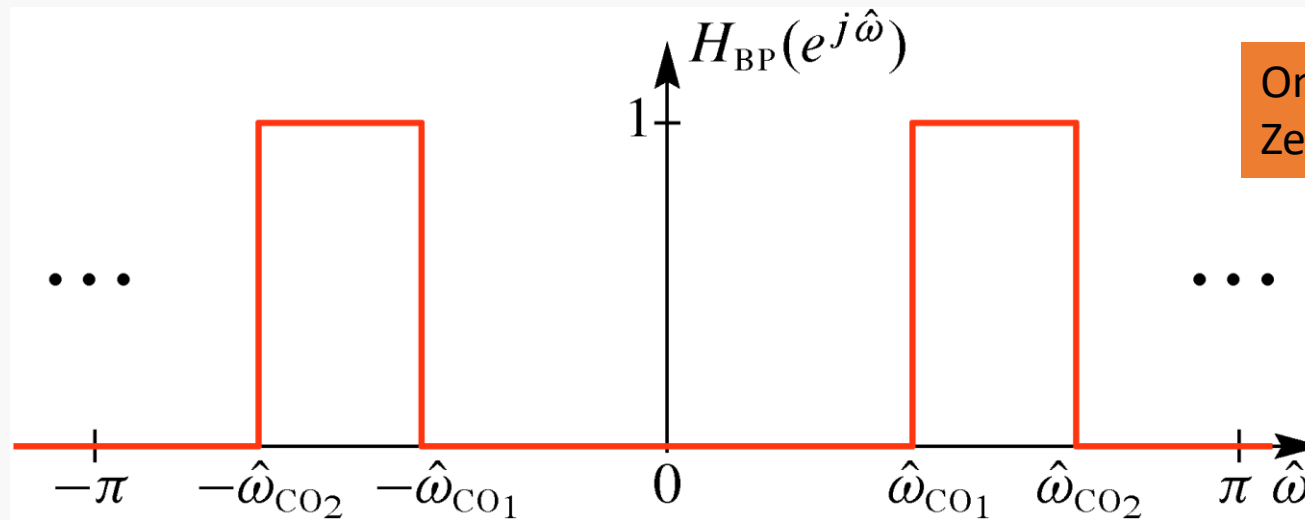
HPF is 1 minus LPF

Inverse DTFT of 1  
is a delta

$$h_{\text{HP}}[n] = \delta[n] - \frac{\sin(\hat{\omega}_{\text{co}}n)}{\pi n} \quad \overset{\text{DTFT}}{\Leftrightarrow}$$

$$H_{\text{HP}}(e^{j\hat{\omega}}) = \begin{cases} 0, & |\hat{\omega}| \leq \hat{\omega}_{\text{co}} \\ 1, & \hat{\omega}_{\text{co}} < |\hat{\omega}| \leq \pi \end{cases}$$

# IDEAL BandPass Filter (BPF)



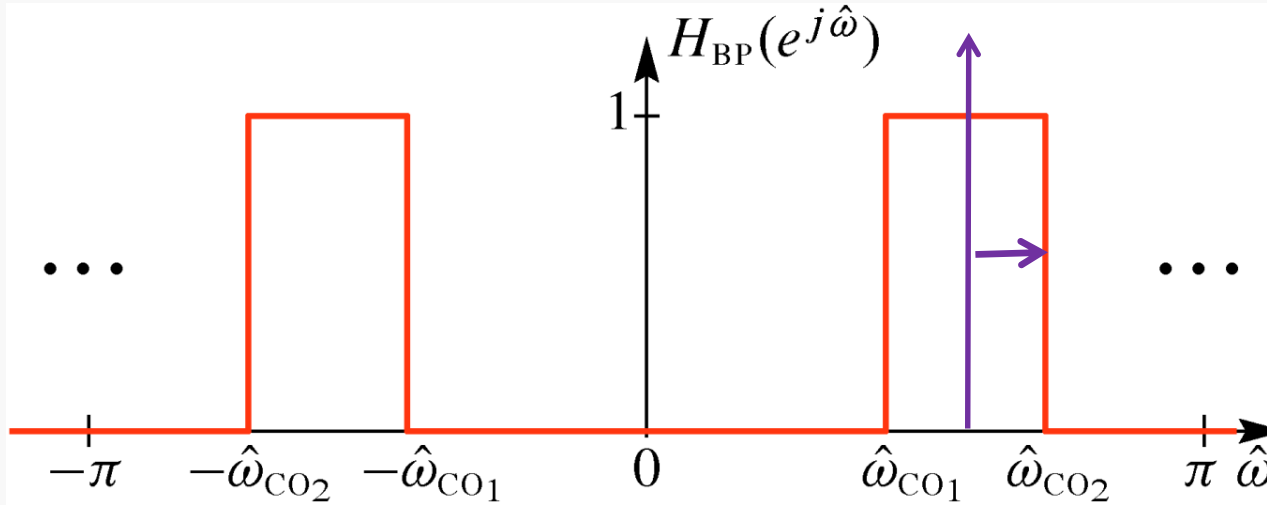
One in the passband  
Zero in the 2 stopbands

BPF has two stopbands

Band Reject Filter has  
one stopband and two  
passbands.  
It is one minus BPF

$$H_{BP}(e^{j\hat{\omega}}) = \begin{cases} 0 & |\hat{\omega}| \leq \hat{\omega}_{co1} \\ 1 & \hat{\omega}_{co1} < |\hat{\omega}| \leq \hat{\omega}_{co2} \\ 0 & \hat{\omega}_{co2} < |\hat{\omega}| \leq \pi \end{cases}$$

# Make IDEAL BPF from LPF



BPF is frequency shifted version of LPF

Frequency shifting **up and down** is done by cosine multiplication in the time domain

$$h_{BP}[n] = 2 \cos(\hat{\omega}_{mid} n) \frac{\sin(\frac{1}{2} \hat{\omega}_{diff} n)}{\pi n}$$

$$\stackrel{DTFT}{\Leftrightarrow} H_{BP}(e^{j\hat{\omega}}) = \begin{cases} 0 & |\hat{\omega}| \leq \hat{\omega}_{CO1} \\ 1 & \hat{\omega}_{CO1} < |\hat{\omega}| \leq \hat{\omega}_{CO2} \\ 0 & \hat{\omega}_{CO2} < |\hat{\omega}| \leq \pi \end{cases}$$

# Exercise-1



**EXERCISE 7.3:** Use the linearity of the DTFT and (7.6) to determine the DTFT of the following sum of two right-sided exponential signals:  $x[n] = (0.8)^n u[n] + 2(-0.5)^n u[n]$ .

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.  
Prentice Hall, Upper Saddle River, NJ 07458. ©2016 Pearson Education, Inc.



## SOLUTION to EXERCISE 7.3:

DSP First 2e

Equation (7.6) states in general

$$x[n] = a^n u[n] \longleftrightarrow X(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}}$$

Now if  $x[n] = (0.8)^n u[n] + 2(-0.5)^n u[n]$  we can use (7.6) with  $a = 0.8$  and  $-0.5$  and the linearity property to write down by inspection, the result

$$X(e^{j\hat{\omega}}) = \frac{1}{1 - 0.8e^{-j\hat{\omega}}} + \frac{2}{1 + 0.5e^{-j\hat{\omega}}}$$

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.  
Prentice Hall, Upper Saddle River, NJ 07458. ©2016 Pearson Education, Inc.



# Exercise-2



**EXERCISE 7.10:** Using the results of Exercise 7.9, show that the impulse response of the ideal HPF is

$$h_{\text{HP}}[n] = \delta[n] - \frac{\sin(\hat{\omega}_{co}n)}{\pi n}$$

**SOLUTION to EXERCISE 7.10:**

**DSP First 2e**

Given that  $H_{\text{HP}}(e^{j\hat{\omega}}) = 1 - H_{\text{LP}}(e^{j\hat{\omega}})$  we can use linearity of the DTFT to find  $h_{\text{HP}}[n]$ . We have the following DTFT pairs:

$$\begin{aligned} 1 &\longleftrightarrow \delta[n] \\ h_{\text{HP}}[n] = \frac{\sin(\hat{\omega}_{co}n)}{\pi n} &\longleftrightarrow H_{\text{LP}}(e^{j\hat{\omega}}) = \begin{cases} 1 & |\hat{\omega}| < \hat{\omega}_{co} \\ 0 & \hat{\omega}_{co} < |\hat{\omega}| \leq \pi \end{cases} \end{aligned}$$

Therefore,

$$h_{\text{HP}}[n] = \delta[n] - \frac{\sin(\hat{\omega}_{co}n)}{\pi n}$$

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.  
Prentice Hall, Upper Saddle River, NJ 07458. ©2016 Pearson Education, Inc.



# Exercise-3



**Example 7-1:** The following FIR filter

$$y[n] = 5x[n - 1] - 4x[n - 3] + 3x[n - 5]$$

has a finite-length impulse response:

$$h[n] = 5\delta[n - 1] - 4\delta[n - 3] + 3\delta[n - 5]$$

Each impulse in  $h[n]$  is transformed using (??), and then combined according to the linearity property of the DTFT which gives

$$H(e^{j\hat{\omega}}) = 5e^{-j\hat{\omega}} - 4e^{-j3\hat{\omega}} + 3e^{-j5\hat{\omega}}$$

McClellan, Schafer, and Yoder, *DSP First, 2e*, ISBN 0-13-065562-7.  
Prentice Hall, Upper Saddle River, NJ 07458. ©2016 Pearson Education, Inc.

