1. Determine the fundamental period of $x[n]$.

$$x[n] = \cos(\frac{n\pi}{10}) + \sin(\frac{n\pi}{15})$$

2. Determine whether or not the signal $x[n]$ is periodic.

$$x[n] = \sin(\sqrt{2} + 0.2n)$$

3. Given that real valued signal $x_1[n]$ is even by definition $x_1[n] = x_1[-n]$, and real valued signal $x_2[n]$ is odd by definition $x_2[n] = -x_2[-n]$, determine symmetricality (even/odd) of $y[n]$.

$$y[n] = x_1[n] \cdot x_2[n]$$

4. Given that the power of real valued signal $x[n]$ is defined as $P = \sum_{n=-\infty}^{\infty} x^2[n]$, compute the power in $y[n]$.

$$y[n] = 2^n \cdot u[-n]$$

5. Given that $x[n]$ is the system input and $y[n]$ is the system output, determine whether or not the following systems is time (shift)-invariant.

$$y[n] = x[n] \cdot u[n]$$

6. Given that $x[n]$ is the system input and $y[n]$ is the system output, determine whether or not the following systems is linear.

$$y[n] = \mathrm{Im}\left(x[n]\right)$$

7. Given that $x[n]$ is the system input and $y[n]$ is the system output, determine whether or not the following systems is casual.

$$y[n] = x[|n|]$$

8. Given that $x[n]$ is the system input and $y[n]$ is the system output, determine unit sample response $h[n]$ of the system.

$$y[n] = 0.5y[n-1] + 4x[n-2]$$

9. The responses of a linear time (shift)-invariant system to specified inputs are defined as follows:

input		reponse
name	symbol	
unit sample	$\delta[n]$	$h[n]$
unit step	$u[n]$	$s[n]$

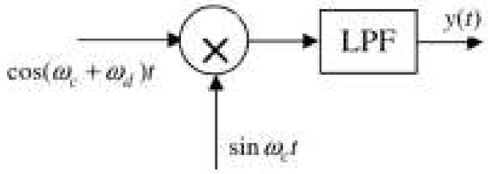
calculate $h[n]$ for the system, given that

$$s[n] = u[n] - u[n-5].$$

10. Find the Fourier transform of $x(t)$.

$$x(t) = \begin{cases} \frac{1}{2}, & -T < t < T \\ 0, & other \end{cases}$$

11. Given that the cut-off frequency of the low pass filter (LPF) is w_c evaluate the output $y(t)$. Note: LPF allows frequency values between $-w_c < w < w_c$.



$$x[n] = 2\delta[n+2] - \delta[n+1] + 3\delta[n] - \delta[n-1] + 2\delta[n-2]$$

12. Given that $x[n] \stackrel{DTFT}{\Longleftrightarrow} X(e^{jw})$ is a DTFT pair, evaluate $X(e^{jw})|_{w=\pi}$ without explicitly finding $X(e^{jw})$.

$$x[n] = 2\delta[n+2] - \delta[n+1] + 3\delta[n] - \delta[n-1] + 2\delta[n-2]$$

13. Given that $x[n] \xleftrightarrow{DTFT} X(e^{jw})$ and $y[n] \xleftrightarrow{DTFT} Y(e^{jw})$ are DTFT pairs, prove the convolution theorem.

$$x[n] * y[n] \xleftrightarrow{DTFT} X(e^{jw})Y(e^{jw})$$

14. Find the inverse DTFT of $X(e^{jw})$.

$$X(e^{jw}) = \cos^2(w)$$

15. Given the 6-point sequence $x[n] = [4, -1, 4, -1, 4, -1]$, determine its 6-point DFT sequence $X[k]$.

16. If the 4-point DFT an unknown length-4 sequence $v[n]$ is $V[k] = [1, 4+j, -1, 4-j]$, determine $v[n]$.

17. Find z-transforms of $x[n]$.

$$x[n] = 6\delta[n] - 7\delta[n-3] - 2\delta[n] - 9\delta[n-5]$$

18. If the region of convergence (ROC) for any $x[n] \overset{ZT}{\Longleftrightarrow} X(z)$ z-transform pair includes the unit cirle in the complex plane then, $x[n] \overset{DTFT}{\Longleftrightarrow} X(e^{jw})$ DTFT pair can also be calculated (converges). Given that the following $X(z)$ includes the unit circle in its region of convergence, evaluate DTFT of $x[n]$ at $w = \pi$.

$$X(z) = \frac{z + 2z^{-2} + z^{-3}}{1 - 3z^{-4} + z^{-5}}$$

19. Evaluate $h[n] * x[n]$ using the convolution property of z-transform.

$$h[n] = (0.5)^n u[n]$$
$$x[n] = 3^n u[-n]$$

20. Given that $x[n] \overset{ZT}{\Longleftrightarrow} X(z)$ is a z-transform pair find $x[n]$.

$$X(z) = 2 + 5(z^2 + z^{-2})$$

21. Given that $x[n] \overset{ZT}{\Longleftrightarrow} X(z)$ is a z-transform pair find $x[n]$ for $|z| > 2$.

$$X(z) = \frac{1}{1 + 3z^{-1} + 2z^{-2}}$$

22. A continuous-time sinusoid $a_1(t) = \cos(w_1 t + 0.1\pi)$ is sampled at $f_{s_1} = 40Hz$ to give $a_1[n]$, and a second continuous-time sinusoid $a_2(t) = \cos(w_2 t + 0.1\pi)$ is sampled at $f_{s_2} = 50\ Hz$ to give $a_2[n]$. If $w_2 = 30\pi\ rad/s$, determine w_1 so that $a_1[n] = a_2[n]$. Assume there is no aliasing when sampling $a_1(t)$.

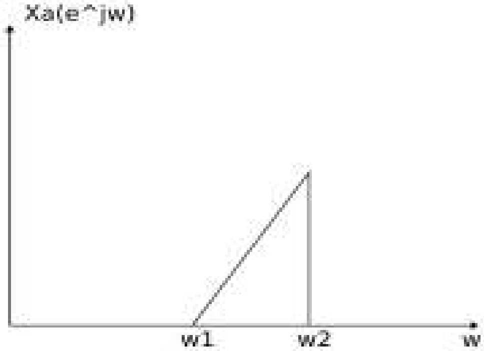
23. A complex bandpass filter is constructed by frequency shifting a running sum filter. evaluate and plot the magnitude frequency response of the complex bandpass filter $\left|H_B(e^{jw})\right|$.

$$h[n] = \sum_{k=0}^{4} \delta[n-k]$$

$$h_B[n] = h[n]e^{jw_0 n}$$

$$h_B[n] \overset{DTFT}{\Longleftrightarrow} H_B(e^{jw})$$

24. A complex bandpass analog signal $x_a(t)$ has Fourier transform that is non-zero over the range of $[w_1, w_2]$. The signal is sampled to produce the sequence $x[n] = x_a(nT_s)$. What is the smallest sampling frequency that can be used so that $x_a(t)$ may be recovered from its samples $x[n]$.



25. Plot STFT (Short Time Fourier Transform) representation of a 1D chirp signal with different window sizes. Compare the results with FT of the same signal. Comment on what might be an optimal window size.