Counting:

$P(A) = \frac{|A|}{\underbrace{|S|}_{\text{the number of elements}}}$ } Equally likely outcome

Coffe example S M L in how many ways can you order your coffee?

→ Filter
→ Decot $3 \times 3 = 9$ ways
→ light

Multiplication Principle (Prensibi)
(r deneyini yaptığımızı varsayalım)
Suppose that we perform r experiments. There are n possible outcomes

for each experiment $1 \le k \le r$

$1 \to n_1$
$2 \to n_2$
⋮
$k \to n_k$

There are a total of $n_1 \times n_2 \dots n_r$ possible outcomes

(seçme)
Choosing a password

→ begin with 2 lower case letters followed by
(büyük harf)
→ 1 capital letter followed by
→ 4 digits

# of total passwords

→ multiplication principle

$\underset{26}{\underline{e}} \times \underset{26}{\underline{e}} \times \underset{26}{\underline{c}} \times \underset{10}{\underline{1}} \times \underset{10}{\underline{1}} \times \underset{10}{\underline{1}} \times \underset{10}{\underline{1}} \to 26^3 \cdot 10^4$

Sampling

- Sampling from a set: randomly choosing an element from a set.

Replacement (torbaya geri atılıyor)
with replacement: put the sampled element back

without replacement → don't put the sampled element back

Ordering If ordering matters $(a_1, a_2, a_3) \neq (a_3, a_2, a_1)$ → sıralı seçim

unordered sampling → order is not important $(a_1, a_2, a_3) = (a_3, a_1, a_2)$

don't matter → önemli değil

types of sampling
- ordered
  - with replacement — 1
  - without replacement — 2
- unordered
  - with r. — 4
  - without r. — 3

Ordered sampling with replacement:

ex $A = \{1,2,3\}$ draw $k=2$ samples

How many ways the sampling can be done?

$(1,2), (1,3), (2,3), (1,1), (2,2), (3,3), (2,1), (3,2), (3,1)$

$\underline{3} \times \underline{3} \to 9 \to n^k$ "k" is number of sampling
n is number of outcomes

→ number of subsets =? $A = \{\overbrace{\dots}^{n}\}$ n elements. What is the number of diffrent subset for A. → $\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \dots}_{n} \to 2^n$

Multiplication principle

exist → mevcut

$\overbrace{\underset{✓\ \times}{2} \quad \underset{✓\ \times}{2} \quad \dots \quad \underset{✓\ \times}{2}}^{n} = 2^n$

Ordered sampling without raplacement

↳ Permutation

- No repetition

ex: $A = \{1,2,3\}$

$(1,2), (1,3), (3,1), (2,1), (2,3), (3,2) \to \underline{3} \times \underline{2} = 6$ $\frac{n!}{(n-k)!}$

Ex: k people in a class. what is the probability that $\overset{\text{minimum}}{\text{at least}}$ two of them have same bhirthday

$\to n = 365$ 1 person $= \frac{1}{365}$

A = event that 2 people have same bhirtday.

A^c = no 2 people have the same bhirtday.

$P(A) = 1 - \frac{|A^c|}{|\Omega|} \to \frac{1 \quad 2 \quad 3 \ \dots\dots \ k}{365 \quad 364 \quad 363 \ \dots\dots \ 366-k} \to 1 - \frac{365!}{(365-k)! \cdot (365)^n}$

$|\Omega| \to \underbrace{365 \cdot 365 \dots 365}_{k \text{ times}}$

$K > 365$ → at least 2 of them are born at same day → Peagenhole principle

Unordered Sampling without replecament

ex $A = \{1,2,3\}$ $K=2$ unordered sampling without replecement

$(1,2), (1,3)\ (2,3)$ → combination $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$\binom{3}{2} = \frac{3!}{2!(1)!} = 3$ $P^n_k = \binom{n}{k} \cdot k!$

ex: Choose 3 cards from a deck (52 cards)

What is the probability that these cards contain at least one ACE

4 ACE in a deck

$P(A^c)$ = None of 3 cards with contain ACE $= \binom{52-4}{3}$

$P(\Omega) = \binom{52}{3}$

$P(A) = 1 - \frac{\binom{48}{3}}{\binom{52}{3}} \to 1 - \frac{\overset{12}{\cancel{48}} \cdot 47 \cdot \overset{23}{\cancel{46}}}{\underset{13}{\cancel{52}} \cdot 51 \cdot \underset{25}{\cancel{50}}} = 1 - \frac{12 \cdot 23 \cdot 47}{51 \cdot 13 \cdot 25}$

Binomial Coefficient (Binom açılımı)

$\binom{n}{k}$ is called Binomal Coefficient (katsayı)

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k \cdot b^{n-k}$ ex/ $(a+b)^2 = a^2 + 2ab + b^2 = a^0 \cdot b^2 \cdot \binom{2}{0} + a^1 b^1 \binom{2}{1} + a^2 b^0 \binom{2}{2}$

Dividing sets into two is like "n choose k"

n elements → K elements, n-k elements

$\binom{n}{k} = \binom{n}{n-k}$

Ex: Unfair coin $P(H) = p$ $0 \le p \le 1$ $p(T) = 1-p$

we toss the coin 5 times.

① What is the prob. that the output is H H T H H
↳ Multiplication rule
$p \cdot p \cdot (1-p) \cdot p \cdot p = p^4 \cdot (1-p)$

② Prob 4 heads 1 tail

$\to \binom{5}{4} \cdot p^4 \cdot (1-p)$

③ If I toss the coin n times, what is the prob. that I observe k head and n-k tails.

$p^k \cdot (1-p)^{(n-k)} \cdot \binom{n}{k}$

ex 10 people .... to a resturant

5 ← main dish.

3 ← drinks

2 ← dessert In how many ways can these people be selected?

$\binom{10}{5} \cdot \binom{5}{3} \binom{2}{2} = \frac{10!}{5!\,5!} \cdot \frac{5!}{3!\,2!} \cdot \frac{2!}{2!\,0!}$

Unordered sampling with Replacement

$A = \{1,2,3\}$

$(1,1), (1,2)\ (1,3)\ (2,3)\ (2,2)\ (3,3)$ 6 possible lists

↳ 2 kişiyi 3 otele yerleştiriyoruz gibi düşün

unordered with replace

$\binom{n+k-1}{k} \to \binom{4}{2} = \frac{4 \cdot 3}{2} = 6$

$(1,2,3) = x_1 + x_2 + x_3 =$

x_1	x_2	x_3
2	0	0
1	1	0
1	0	1
0	2	0
0	1	1
0	0	2

ex: 10 passengers get on an airport shuttle. The shuttle route includes 5 hotels and each passengers gets off his hotel. The driver records how many passengers leave the shuttle at each hotel. How many diffrent possible lists.

x_1	x_2	x_3	x_4	x_5
10	0	0	0	0
9	1	0	0	0
⋮				

$\binom{5+10-1}{10} = \frac{14!}{4! \cdot 10!}$