

Parametrik Denklemler / Kutupsal Koordinatlar

① $x=4\sin t$, $y=2\cos t$ eğrisinin $t=\frac{\pi}{4}$ daki teğetinin denklemini bulunuz.

$$m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin t}{4\cos t} \Big|_{t=\frac{\pi}{4}} = -\frac{1}{2}$$

$$t = \frac{\pi}{4} \Rightarrow \begin{aligned} x_0 &= 2\sqrt{2} \\ y_0 &= \sqrt{2} \end{aligned}$$

$$\text{Denklem: } y - y_0 = m(x - x_0) \Rightarrow y - \sqrt{2} = -\frac{1}{2}(x - 2\sqrt{2}) \Rightarrow y = -\frac{1}{2}x + 2\sqrt{2}$$

② $x=x(t)$, $y=y(t)$ olarak tanımlandıklarını kabul ederek,

$\left. \begin{aligned} t^2 \sin x + x^3 &= e^t \\ y &= t \sin t - 2t \end{aligned} \right\}$ parametrik denklemleri ile verilen eğrinin $t=0$ daki teğet doğrusunun eğimini bulunuz.

$$t=0 \Rightarrow (x, y) = (1, 0)$$

$$t^2 \sin x + x^3 = e^t \Rightarrow 2t \sin x + t^2 \cos x \frac{dx}{dt} + 3x^2 \frac{dx}{dt} = e^t$$

$$\left. \begin{aligned} t=0 \\ x=1 \end{aligned} \right\} 0 + 0 + \frac{3dx}{dt} = 1 \Rightarrow \frac{dx}{dt} \Big|_{t=0} = \frac{1}{3}$$

$$y = t \sin t - 2t \Rightarrow \frac{dy}{dt} = \sin t + t \cos t - 2 \Rightarrow \frac{dy}{dt} \Big|_{t=0} = -2$$

$$\frac{dy}{dx} \Big|_{t=0} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2}{\frac{1}{3}} = -6$$

③ $\left. \begin{aligned} x &= 2t^2 + 3 \\ y &= t^4 \end{aligned} \right\}$ parametrik denklemleri ile verilen eğrinin $t=-1$ noktasındaki teğet doğrusunun denklemini yazınız.

$$t=-1 \Rightarrow \begin{aligned} x &= 5 \\ y &= 1 \end{aligned} \left\{ \begin{aligned} y-1 &= m \cdot (x-5) \end{aligned} \right.$$

$$m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{4t} = t^2 \Rightarrow m \Big|_{t=-1} = 1 \Rightarrow y-1 = x-5 \Rightarrow y = x-4$$

④ $\begin{cases} x = 8\cos t + 8t\sin t \\ y = 8\sin t - 8t\cos t \\ 0 \leq t \leq \frac{\pi}{2} \end{cases}$ parametrisasyonu ile verilen eğrinin uzunluğunu bulunuz.

$$L = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$\frac{dx}{dt} = -8\sin t + 8\sin t + 8t\cos t \Rightarrow \left(\frac{dx}{dt}\right)^2 = 64t^2\cos^2 t$$

$$\frac{dy}{dt} = 8\cos t - 8\cos t + 8t\sin t \Rightarrow \left(\frac{dy}{dt}\right)^2 = 64t^2\sin^2 t$$

$$L = \int_0^{\pi/2} \sqrt{64t^2} = \int_0^{\pi/2} 8t dt = 4t^2 \Big|_0^{\pi/2} = \pi^2$$

⑤ $-1 \leq t \leq 0$ olmak üzere $x(t) = t^2$, $y(t) = 1 - t^2$ ile çizilmiş yolun uzunluğunu bulunuz.

$$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = -2t$$

$$L = \int_{-1}^0 \sqrt{(2t)^2 + (-2t)^2} dt = 2\sqrt{2} \int_{-1}^0 |t| dt = -2\sqrt{2} \int_{-1}^0 t dt = \sqrt{2}$$

⑥ $\begin{cases} x = 2t^2 + 3 \\ y = t^4 \end{cases}$ parametrik denklemleri ile verilen eğrinin $t = -1$ deki normal doğrusunun denklemini bulunuz.

$t = -1 \begin{cases} x = 5 \\ y = 1 \end{cases} \Rightarrow (5, 1)$ noktasından geçen normal doğrusunun denklemi:

$$y - 1 = m_N(x - 5)$$

$$m_T = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{4t} = t^2 \Rightarrow m_T \Big|_{t=-1} = (-1)^2 = 1$$

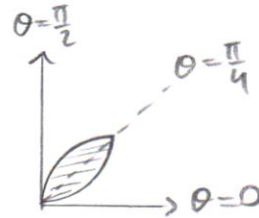
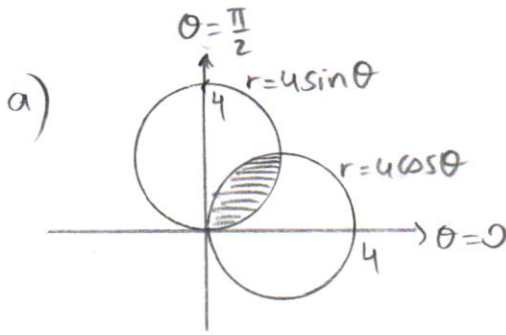
$$m_T \cdot m_N = -1 \Rightarrow m_N = -1$$

$$y - 1 = -1(x - 5) \Rightarrow y = 6 - x$$

7) a) $r=4\cos\theta$
 $r=4\sin\theta$ } ekrillerinin sinirladigi ortak alanı

b) $r=4\cos\theta$ iinde
 $r=4\sin\theta$ diında } kalan alanı

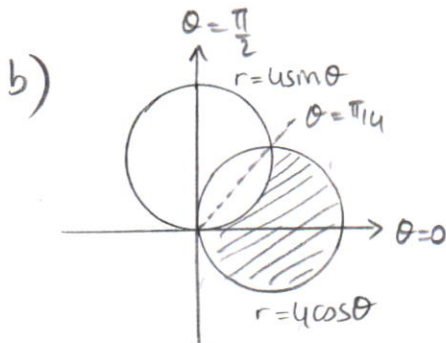
c) $r=4\cos\theta$ diında
 $r=4\sin\theta$ iinde } kalan alanı bulunuz. (integralleri hesaplamayın)



$$4\cos\theta = 4\sin\theta$$

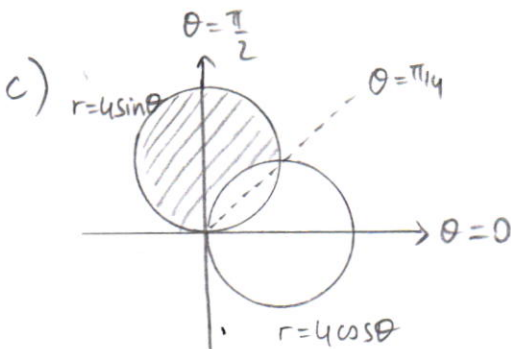
$$\theta = \frac{\pi}{4}$$

$$A = \frac{1}{2} \int_0^{\pi/4} (4\sin\theta)^2 d\theta + \frac{1}{2} \int_{\pi/4}^{\pi/2} (4\cos\theta)^2 d\theta$$



1.yol: $A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} (4\cos\theta)^2 d\theta - \frac{1}{2} \left[\int_0^{\pi/4} (4\sin\theta)^2 d\theta + \int_{\pi/4}^{\pi/2} (4\cos\theta)^2 d\theta \right]$

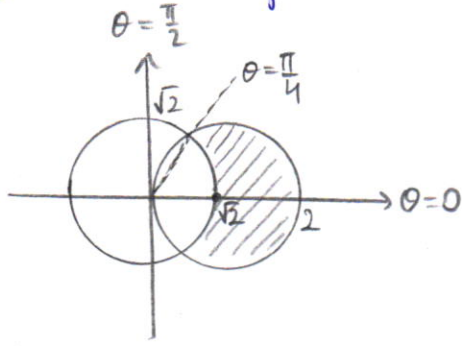
2.yol: $A = \frac{1}{2} \int_{-\pi/2}^{\pi/4} (4\cos\theta)^2 d\theta - \frac{1}{2} \int_0^{\pi/4} (4\sin\theta)^2 d\theta$



1.yol: $A = \frac{1}{2} \int_0^{\pi} (4\sin\theta)^2 d\theta - \frac{1}{2} \left[\int_0^{\pi/4} (4\sin\theta)^2 d\theta + \int_{\pi/4}^{\pi/2} (4\cos\theta)^2 d\theta \right]$

2.yol: $A = \frac{1}{2} \int_{\pi/4}^{\pi} (4\sin\theta)^2 d\theta - \frac{1}{2} \int_{\pi/4}^{\pi/2} (4\cos\theta)^2 d\theta$

8) $r=2\cos\theta$ eğrisinin içinde, $r=\sqrt{2}$ eğrisinin dışında kalan alanı bulunuz.



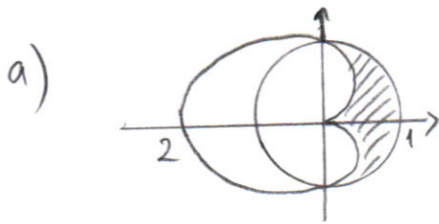
$$2\cos\theta = \sqrt{2} \Rightarrow \cos\theta = \frac{\sqrt{2}}{2} \Rightarrow \theta = \frac{\pi}{4}$$

$$\begin{aligned} \frac{A}{2} &= \frac{1}{2} \int_0^{\pi/4} (2\cos\theta)^2 d\theta - \frac{1}{2} \int_0^{\pi/4} (\sqrt{2})^2 d\theta \\ &= \frac{1}{2} \left(\int_0^{\pi/4} (4\cos^2\theta - 2) d\theta \right) = \frac{1}{2} \int_0^{\pi/4} 2\cos 2\theta d\theta \\ &= \frac{\sin 2\theta}{2} \Big|_0^{\pi/4} = \frac{1}{2} \Rightarrow A = 1 \end{aligned}$$

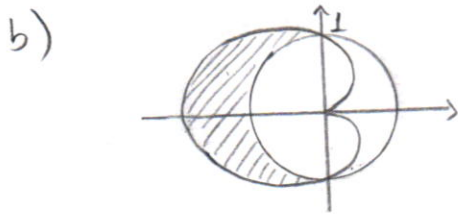
9) a) $r=1$ eğrisinin içinde $r=1-\cos\theta$ eğrisinin dışında kalan

b) $r=1$ " dışında $r=1-\cos\theta$ " içinde "

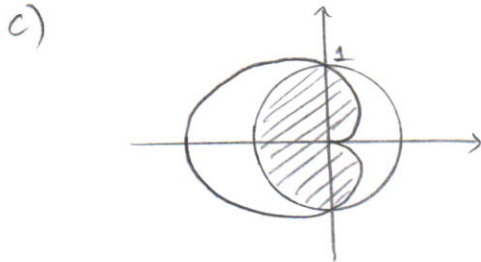
c) $r=1$ ve $r=1-\cos\theta$ eğrilerinin sınırladığı ortak alanı bulunuz.
(Int. hesaplamayın)



$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/2} 1^2 d\theta - \frac{1}{2} \int_0^{\pi/2} (1-\cos\theta)^2 d\theta$$

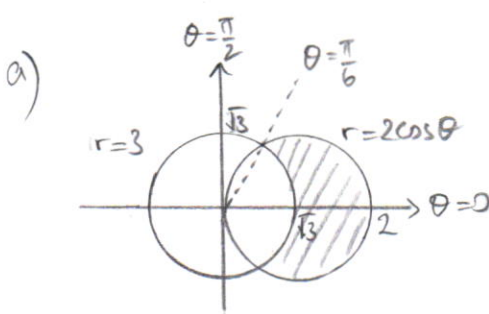


$$\frac{A}{2} = \frac{1}{2} \int_{\pi/2}^{\pi} [(1-\cos\theta)^2 - 1^2] d\theta$$



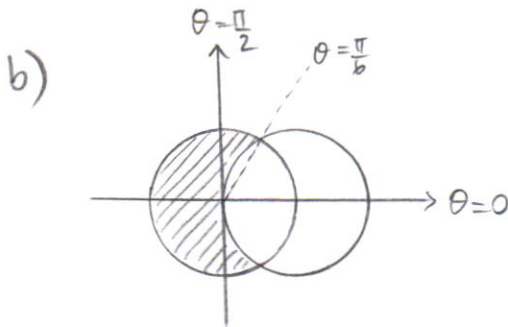
$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/2} (1-\cos\theta)^2 d\theta + \frac{1}{2} \int_{\pi/2}^{\pi} 1^2 d\theta$$

- 10) a) $r=2\cos\theta$ içinde, $r=\sqrt{3}$ dışında kalan,
 b) $r=2\cos\theta$ dışında, $r=\sqrt{3}$ içinde kalan,
 c) $r=2\cos\theta$ ve $r=\sqrt{3}$ sınırladığı ortak alanı bulunuz.

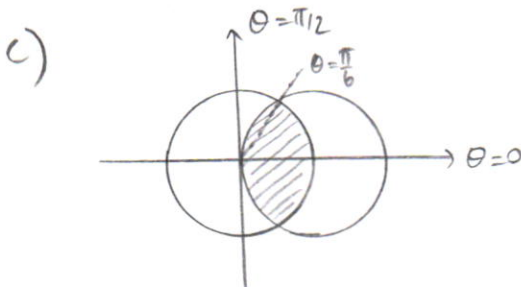


$$2\cos\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{6}$$

$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/6} (2\cos\theta)^2 - 3) d\theta$$

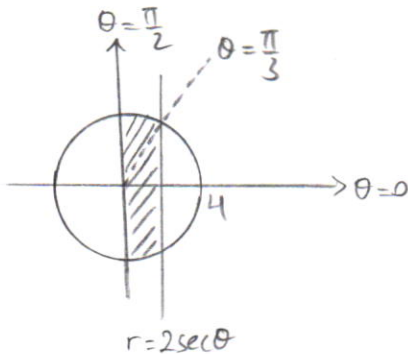


$$\frac{A}{2} = \frac{1}{2} \int_{\pi/6}^{\pi/2} 3 d\theta - \int_{\pi/6}^{\pi/2} (2\cos\theta)^2 d\theta$$



$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/6} 3 d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (2\cos\theta)^2 d\theta$$

- 11) $r=4$, $\theta=\frac{\pi}{2}$, $r=2\sec\theta$ arasında kalan alanı hesaplayınız.

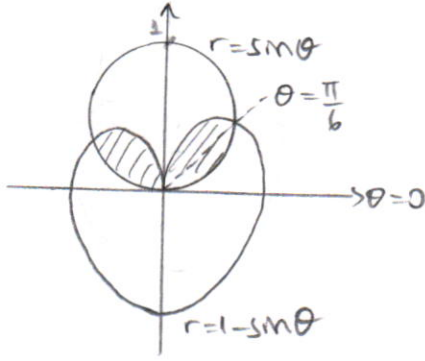


$$r=2\sec\theta = \frac{2}{\cos\theta} \Rightarrow r\cos\theta = 2 \Rightarrow x=2 \text{ doğrusu}$$

$$\left. \begin{array}{l} r=4 \\ r=\frac{2}{\cos\theta} \end{array} \right\} \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/3} (2\sec\theta)^2 d\theta + \frac{1}{2} \int_{\pi/3}^{\pi/2} 4^2 d\theta \Rightarrow A = 4\tan\theta \Big|_0^{\pi/3} + 16\theta \Big|_{\pi/3}^{\pi/2} = 4\sqrt{3} + \frac{8\pi}{3}$$

(12) $r = 1 - \sin \theta$ ve $r = \sin \theta$ eğrilerinin sınırladığı bölgenin alanını hesaplayın.



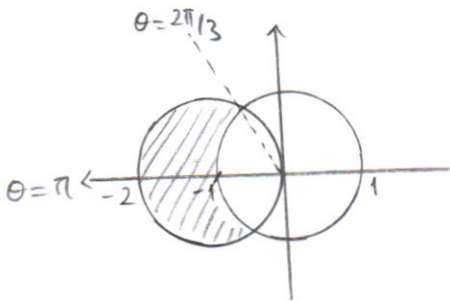
$$1 - \sin \theta = \sin \theta \Rightarrow \theta = \frac{\pi}{6} \text{ (ve } \frac{5\pi}{6})$$

$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/6} (\sin \theta)^2 d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (1 - \sin \theta)^2 d\theta$$

$$A = \int_0^{\pi/6} \frac{1 - \cos 2\theta}{2} d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2} (3 - 4\sin \theta - \cos 2\theta) d\theta$$

$$= \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8} \right) + \left(\frac{\pi}{2} - \frac{7\sqrt{3}}{8} \right) = \frac{7\pi}{12} - \sqrt{3}$$

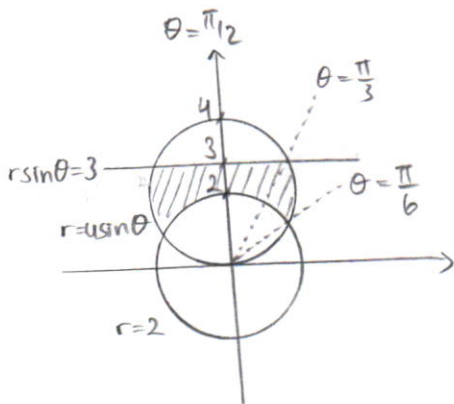
(13) $r = -2\cos \theta$ çemberinin içinde, $r = 1$ çemberinin dışında kalan alanı bulunuz.



$$-2\cos \theta = 1 \Rightarrow \theta = \frac{2\pi}{3}$$

$$\frac{A}{2} = \frac{1}{2} \int_{\frac{2\pi}{3}}^{\pi} (-2\cos \theta)^2 d\theta - \frac{1}{2} \int_{\frac{2\pi}{3}}^{\pi} 1^2 d\theta$$

(14) $r = 2$, $r = 4\sin \theta$ ve $r \sin \theta = 3$ ile sınırlı bölgenin alanını veren belirli integrali yazınız.



$$r \sin \theta = 3 \Rightarrow y = 3 \text{ doğrusu}$$

$$4\sin \theta = \frac{3}{\sin \theta} \Rightarrow \sin^2 \theta = \frac{3}{4} \Rightarrow \sin \theta = \pm \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

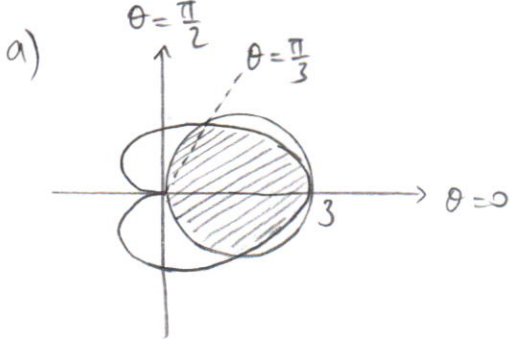
$$4\sin \theta = 2 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$\frac{A}{2} = \frac{1}{2} \int_{\pi/6}^{\pi/3} [(4\sin \theta)^2 - 2^2] d\theta + \frac{1}{2} \int_{\pi/3}^{\pi/2} [(3/\sin \theta)^2 - 2^2] d\theta$$

15) a) $r=3\cos\theta$ ve $r=1+\cos\theta$ eğrilerinin sınırladığı ortak alanı

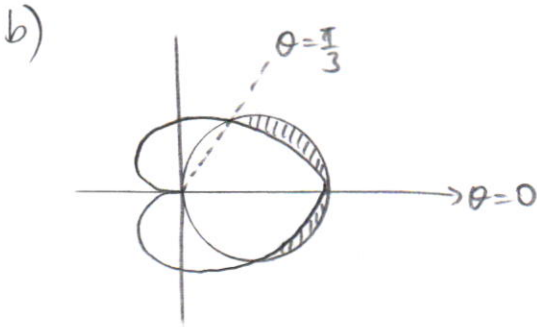
b) $r=3\cos\theta$ nin içinde $r=1+\cos\theta$ nin dışında kalan alanı

c) $r=3\cos\theta$ nin dışında $r=1+\cos\theta$ nin içinde kalan alanı
veren integralleri yazın.

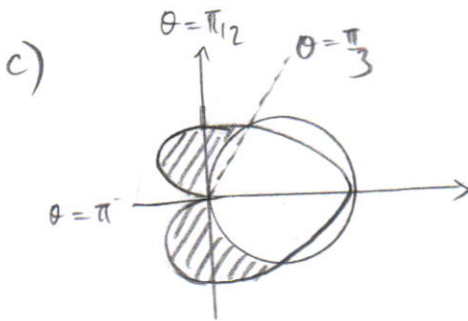


$$1+\cos\theta = 3\cos\theta \Rightarrow \theta = \frac{\pi}{3}$$

$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/3} (1+\cos\theta)^2 d\theta + \frac{1}{2} \int_{\pi/3}^{\pi/2} (3\cos\theta)^2 d\theta$$

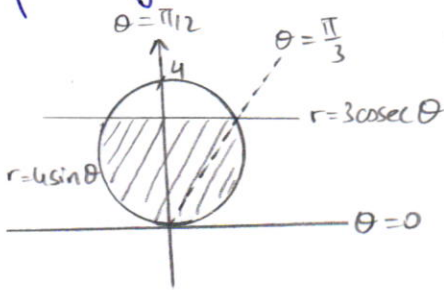


$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/3} [(3\cos\theta)^2 - (1+\cos\theta)^2] d\theta$$



$$\frac{A}{2} = \frac{1}{2} \int_{\pi/3}^{\pi} (1+\cos\theta)^2 d\theta - \frac{1}{2} \int_{\pi/3}^{\pi/2} (3\cos\theta)^2 d\theta$$

16) $r=4\sin\theta$, $r=3\csc\theta$ ve $\theta=0$ ile sınırlı bölgenin alanını veren integrali yazınız.



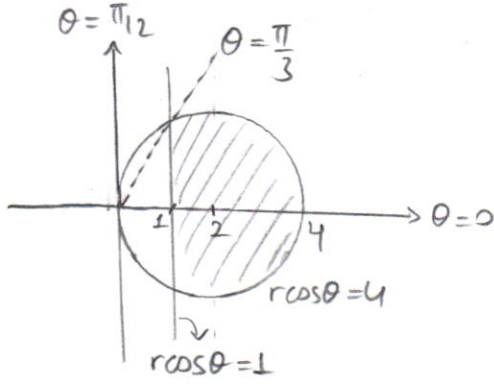
$$r=3\csc\theta = \frac{3}{\sin\theta} \Rightarrow y=3 \text{ doğrusu}$$

$$4\sin\theta = \frac{3}{\sin\theta} \Rightarrow \theta = \frac{\pi}{3}$$

$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/3} (3\sin\theta)^2 d\theta + \frac{1}{2} \int_{\pi/3}^{\pi/2} (3\csc\theta)^2 d\theta$$

17) $r = 4\cos\theta$ ile $r\cos\theta \geq 1$

sınırladığı ortak alanı bulunuz.



$$r\cos\theta \geq 1 \Rightarrow x \geq 1$$

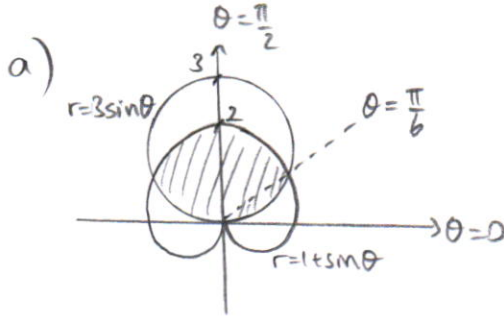
$$\begin{cases} r = 4\cos\theta \\ r = \frac{1}{\cos\theta} \end{cases} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/3} (4\cos\theta)^2 d\theta - \frac{1}{2} \int_0^{\pi/3} \sec^2\theta d\theta$$

18) a) $r = 3\sin\theta$ ve $r = 1 + \sin\theta$ eğrilerinin sınırladığı ortak alanı

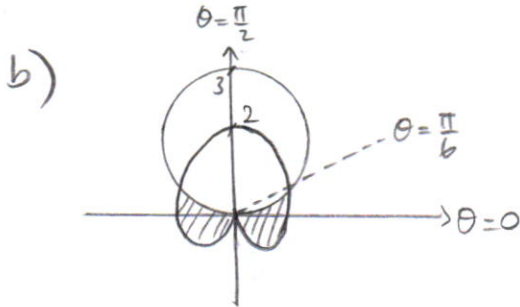
b) $r = 3\sin\theta$ dışında, $r = 1 + \sin\theta$ içinde kalan alanı

c) $r = 3\sin\theta$ içinde, $r = 1 + \sin\theta$ dışında kalan alanı bulunuz.

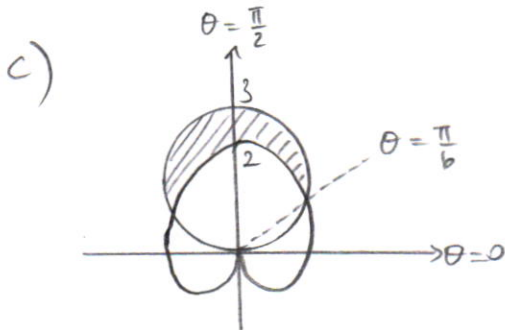


$$1 + \sin\theta = 3\sin\theta \Rightarrow \theta = \frac{\pi}{6}$$

$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/6} (3\sin\theta)^2 d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (1 + \sin\theta)^2 d\theta$$



$$\frac{A}{2} = \frac{1}{2} \int_{-\pi/2}^{\pi/6} (1 + \sin\theta)^2 d\theta - \frac{1}{2} \int_0^{\pi/6} (3\sin\theta)^2 d\theta$$



$$\frac{A}{2} = \frac{1}{2} \int_{\pi/6}^{\pi/2} [(3\sin\theta)^2 - (1 + \sin\theta)^2] d\theta$$