BLM1612 - Circuit Theory

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Impedance and Ohm's Law

- Objective of Lecture
  - Describe the mathematical relationships between ac voltage and ac current for a resistor, capacitor, and inductor .
    - Discuss the phase relationship between the ac voltage and current.
  - Explain how Ohm's Law can be adapted for inductors and capacitors when an ac signal is applied to the components.
    - Derive the mathematical formulas for the impedance and admittance of a resistor, inductor, and capacitor.

Resistors

- Ohm's Law

$$v(t) = Ri(t) = R\, I_m \cos(\omega t + \theta)$$

$$\mathbf{V} = RI_m \angle \theta = R\mathbf{I} \quad \text{where} \quad \theta = \phi$$

- The voltage and current through a resistor are in phase as there is no change in the phase angle between them.

Capacitors

$i(t) = C\, dv(t)/dt$ where $v(t) = V_m \cos(\omega t)$

$i(t) = -C\omega\, V_m \sin(\omega t)$

$i(t) = \omega C V_m \sin(\omega t + 180^o)$

$i(t) = \omega C V_m \cos(\omega t + 180^o - 90^o)$

$i(t) = \omega C V_m \cos(\omega t + 90^o)$

Capacitors

$$\mathbf{V} = V_m \angle 0^o$$

$$\mathbf{I} = \omega C V_m \cos(\omega t + 90^o)$$

$$V_m \cos(\omega t + 90^o) = \mathbf{V}\, e^{j90^\circ} = \mathbf{V} \angle 90^o = j\mathbf{V}$$

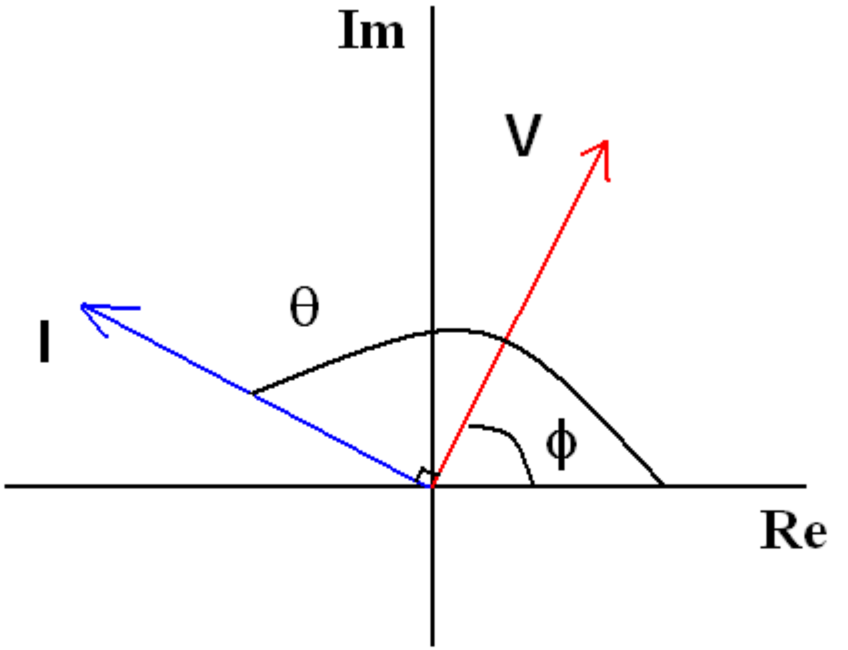
$$\mathbf{I} = j\omega C \mathbf{V}$$

or

$$\mathbf{V} = (1/j\omega C)\, \mathbf{I} = -\,(j/\omega C)\, \mathbf{I}$$

Capacitors

- 90^o phase difference between the voltage and current through a capacitor.
  - Current needs to flow first to place charge on the electrodes of a capacitor, which then induce a voltage across the capacitor
- Current leads the voltage (or the voltage lags the current) in a capacitor.



Inductors

$$v(t) = L\, d\, i(t)/dt \quad \text{where } i(t) = I_m \cos(\omega t)$$

$$v(t) = -\, L\omega\, I_m \sin(\omega t) = \omega L I_m \cos(\omega t + 90^o)$$

$$\mathbf{V} = \omega L I_m \angle 90^o$$

$$\mathbf{I} = I_m \cos(\omega t)$$

$$I_m \cos(\omega t + 90^o) = \mathbf{I}\, e^{j90^\circ} = \mathbf{I} \angle 90^o = j\mathbf{I}$$

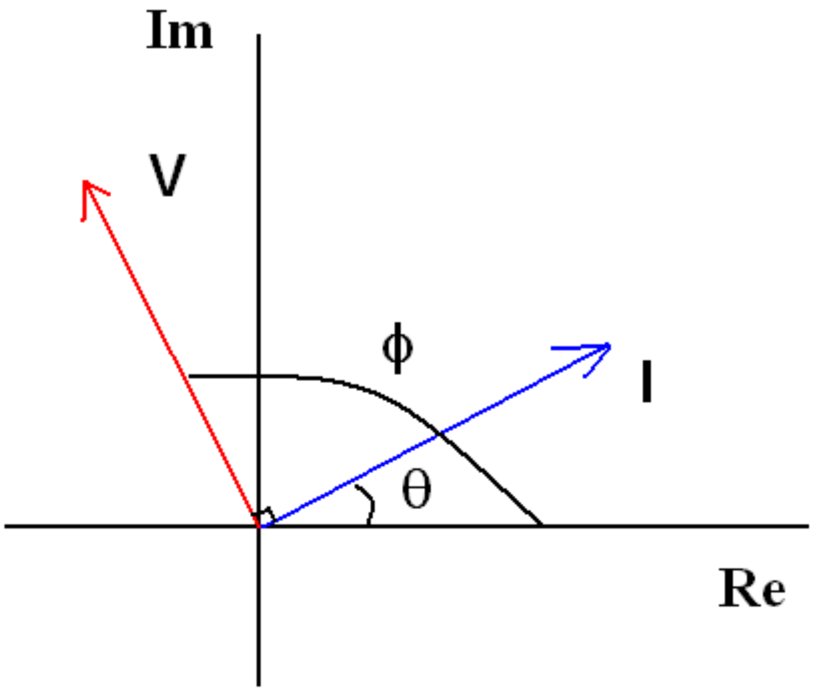
$$\mathbf{V} = j\omega L\mathbf{I}$$

or

$$\mathbf{I} = (1/j\omega L)\, \mathbf{V} = -\, (j/\omega L)\, \mathbf{V}$$

Inductors

- 90^o phase difference between the voltage and current through an inductor.
- The voltage leads the current (or the current lags the voltage).



Impedance

- If we try to force all components to following Ohm's Law, **V** = **Z I**, where **Z** is the impedance of the component.

Resistor:	$\mathbf{Z_R}$	=	R	$R\angle 0^o$
Capacitor:	$\mathbf{Z_C}$	=	$-j/(\omega C)$	$1/\omega C\angle -90^o$
Inductor:	$\mathbf{Z_L}$	=	$j\omega L$	$\omega L\angle 90^o$

Admittance

- If we rewrite Ohm's Law:
- **I** = **Y V (Y = 1/Z)**, where **Y** is admittance of the component

Resistor:	$\mathbf{Y_R} =$	$1/R = G$	$G\angle 0^o$
Capacitor:	$\mathbf{Y_C} =$	$j\omega C$	$\omega C\angle 90^o$
Inductor:	$\mathbf{Y_L} =$	$-j/(\omega L)$	$1/\omega L\angle -90^o$

Impedances-Admittances

Impedances	Value at ω =		Admittances	Value at ω =	
	0 rad/s	∞ rad/s		0 rad/s	∞ rad/s
$Z_R = R = 1/G$	R	R	$Y_R = 1/R = G$	G	G
$Z_L = j\omega L$	$0\ \Omega$	$\infty\ \Omega$	$Y_L = -j/(\omega L)$	$\infty\ \Omega$	$0\ \Omega$
$Z_C = -j/(\omega C)$	$\infty\ \Omega$	$0\ \Omega$	$Y_C = j\omega C$	$0\ \Omega$	$\infty\ \Omega$

- Inductors act like short circuits under d.c. conditions and like open circuits at very high frequencies.
- Capacitors act like open circuits under d.c. conditions and like short circuits at very high frequencies.

Impedance

Generic component that represents a resistor, inductor, or capacitor.

$$\mathbf{Z} = |Z|\angle\phi$$

$$\mathbf{Z} = R + jX$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\phi = \tan^{-1}(X/R)$$

$$R = |Z|\cos(\phi)$$

$$X = |Z|\sin(\phi)$$

Admittance

$$\mathbf{Y} = 1/\mathbf{Z} = \frac{1}{R + jX}$$

$$G = \frac{R}{R^2 + X^2}$$

$$B = \frac{-X}{R^2 + X^2}$$

$$\mathbf{Y} = |Y| \angle \gamma$$

$$\mathbf{Y} = G + jB$$

$$|Y| = \sqrt{G^2 + B^2}$$

$$\gamma = \tan^{-1}(B/G)$$

$$G = |Y| \cos(\gamma)$$

$$B = |Y| \sin(\gamma)$$

Summary

- Ohm's Law can be used to determine the ac voltages and currents in a circuit.
  - Voltage leads current through an inductor.
  - Current leads voltage through a capacitor.

Component		Impedance		Admittance	
Resistor	Z_R	R	$R\angle 0^o$	G	$G\angle 0^o$
Capacitor	Z_C	$-j/\omega C$	$1/\omega C\angle -90^o$	$j\omega C$	$\omega C\angle +90$
Inductor	Z_L	$j\omega L$	$\omega L\angle +90$	$-j/\omega L$	$1/\omega L\angle -90^o$

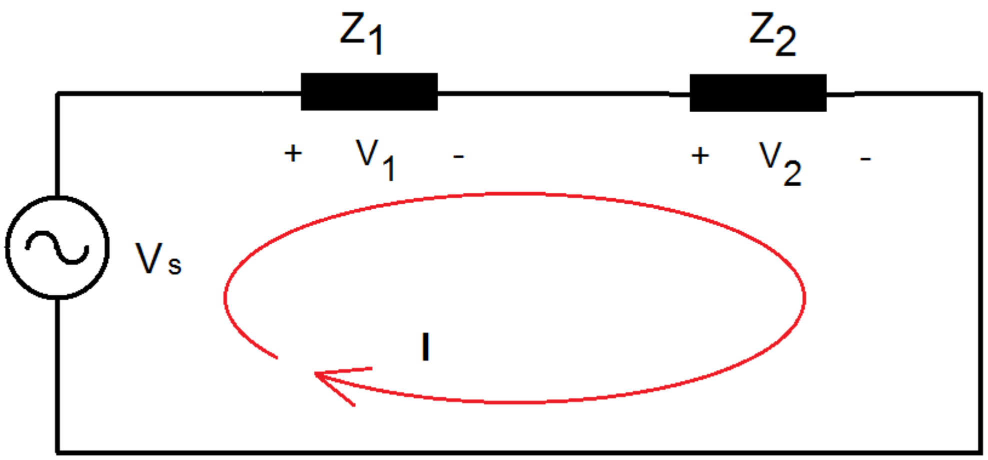
Ohm's Law with Series and Parallel Combinations

- Objective of Lecture
  - Derive the equations for equivalent impedance and equivalent admittance for a series combination of components.
  - Derive the equations for equivalent impedance and equivalent admittance for a parallel combination of components.
- Ohm's Law in Phasor Notation

$$\mathbf{V} = \mathbf{I}\,\mathbf{Z} \qquad\qquad \mathbf{V} = \mathbf{I}/\mathbf{Y}$$

$$\mathbf{I} = \mathbf{V}/\mathbf{Z} \qquad\qquad \mathbf{I} = \mathbf{V}\,\mathbf{Y}$$

Series Connections



Using Kirchhoff's Voltage Law:

$$\mathbf{V_1} + \mathbf{V_2} - \mathbf{V_s} = 0$$

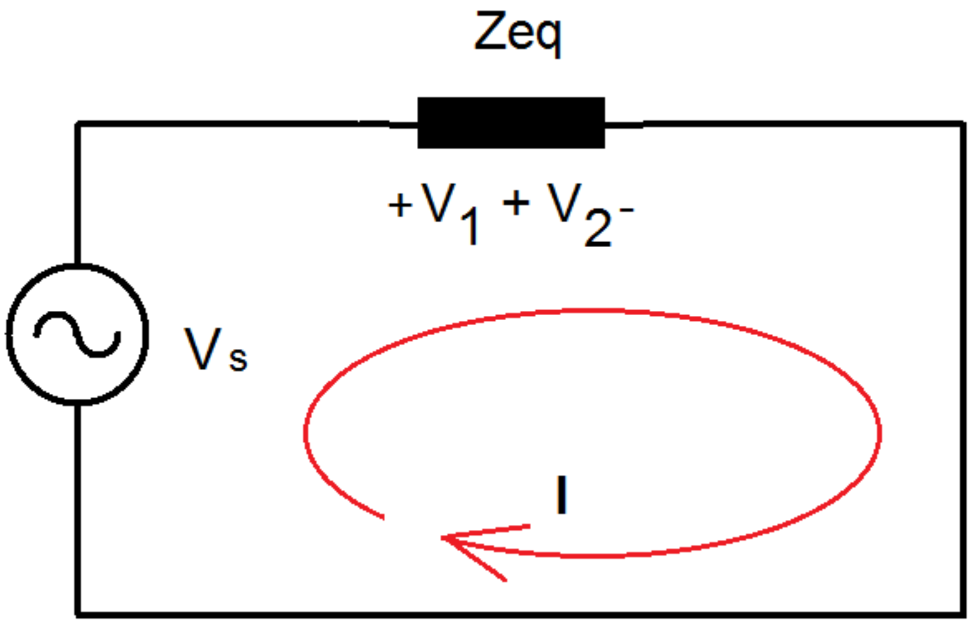
Since $\mathbf{Z_1}$, $\mathbf{Z_2}$, and $\mathbf{V_s}$ are in series, the current flowing through each component is the same.

Using Ohm's Law: $\mathbf{V_1 = I\,Z_1}$ and $\mathbf{V_2 = I\,Z_2}$

Substituting into the equation from KVL:

$$\mathbf{I\,Z_1} + \mathbf{I\,Z_2} - \mathbf{V_s} = 0\text{V}$$
$$\mathbf{I}\,(\mathbf{Z_1} + \mathbf{Z_2}) = \mathbf{V_s}$$

Equivalent Impedance: Series Connections

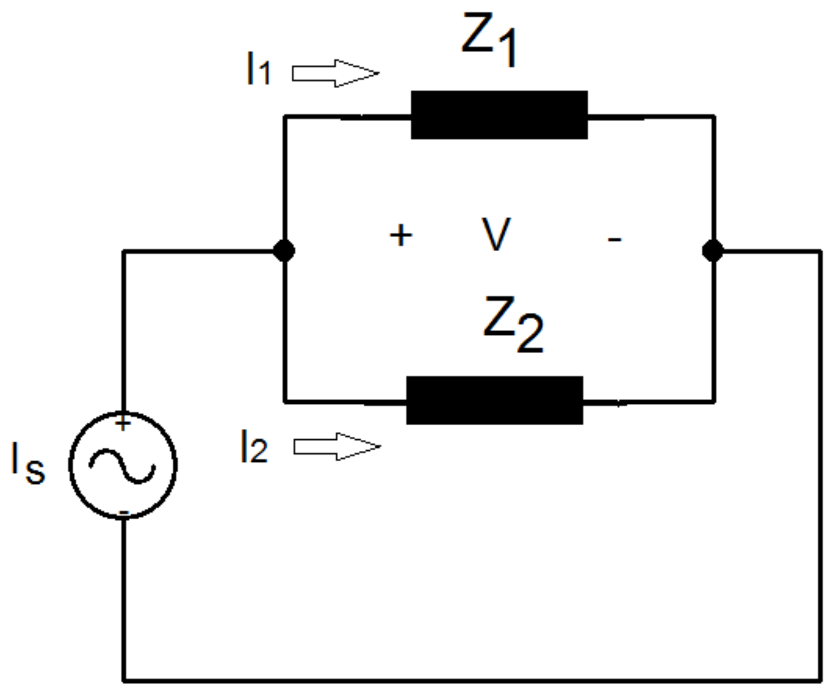


We can replace the two impedances in series with one equivalent impedance, $\mathbf{Z_{eq}}$, which is equal to the sum of the impedances in series.

$$\mathbf{Z_{eq} = Z_1 + Z_2}$$

$$\mathbf{V_s = Z_{eq}\, I}$$

Parallel Connections



Using Kirchoff's Current Law,

$\mathbf{I_1} + \mathbf{I_2} - \mathbf{I_S} = 0$

Since $\mathbf{Z_1}$ and $\mathbf{Z_1}$ are in parallel, the voltage across each component , **V**, is the same.

Using Ohm's Law:

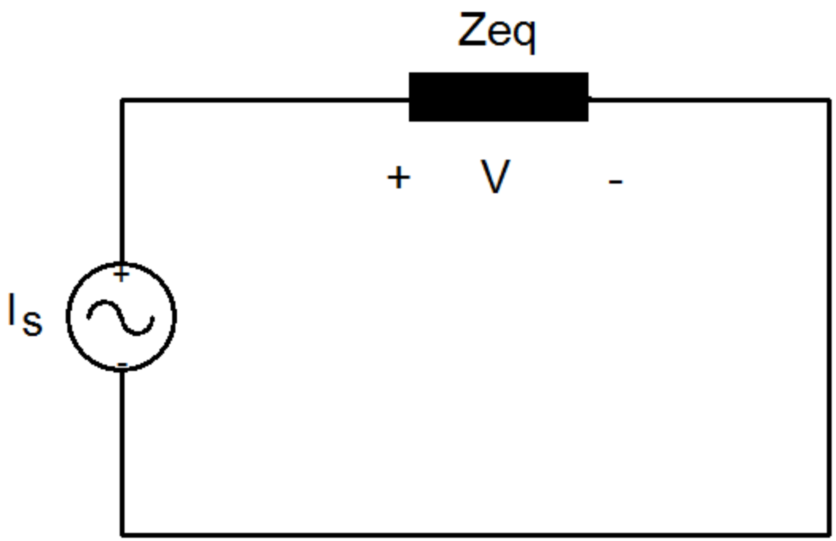
$\mathbf{V = I_1\, Z_1}$

$\mathbf{V = I_2\, Z_2}$

$\mathbf{V/\, Z_1} + \mathbf{V/\, Z_2} = \mathbf{I_S}$

$\mathbf{I_S}\ (1/\mathbf{Z_1} + 1/\mathbf{Z_2})^{-1} = \mathbf{V}$

Equivalent Impedance: Parallel Connections



We can replace the two impedances in series with one equivalent impedance, $\mathbf{Z_{eq}}$, where $1/\mathbf{Z_{eq}}$ is equal to the sum of the inverse of each of the impedances in parallel.

$$1/\mathbf{Z_{eq}} = 1/\mathbf{Z_1} + 1/\mathbf{Z_2}$$

Simplifying

(only for 2 impedances in parallel)

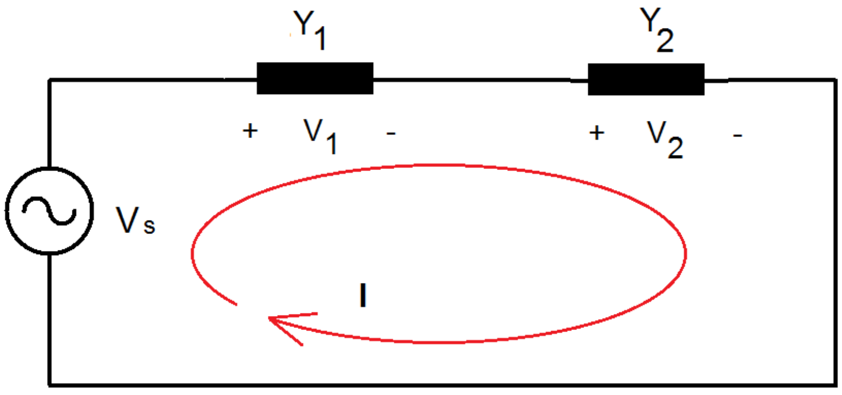
$$\mathbf{Z_{eq}} = \mathbf{Z_1 Z_2}/(\mathbf{Z_1} + \mathbf{Z_2})$$

- An abbreviated means to show that Z_1 is in parallel with Z_2 is to write $Z_1 \parallel Z_2$.

If you used Y instead of Z

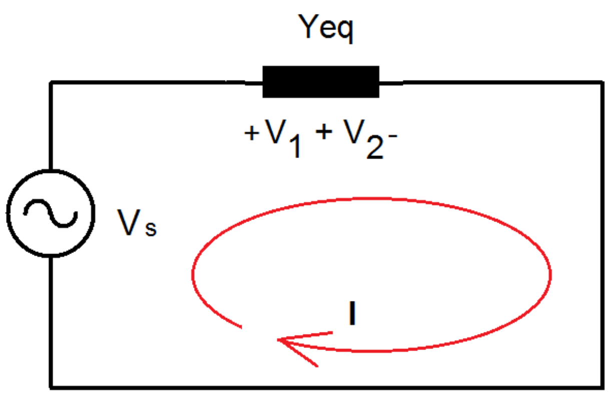
In series:

The reciprocal of the equivalent admittance is equal to the sum of the reciprocal of each of the admittances in series



In this example

$$1/\mathbf{Y}_{\mathbf{eq}} = 1/\mathbf{Y}_{\mathbf{1}} + 1/\mathbf{Y}_{\mathbf{2}}$$



Simplifying

(only for 2 admittances in series)

$$\mathbf{Y}_{\mathbf{eq}} = \mathbf{Y}_{\mathbf{1}}\mathbf{Y}_{\mathbf{2}} /(\mathbf{Y}_{\mathbf{1}} + \mathbf{Y}_{\mathbf{2}})$$

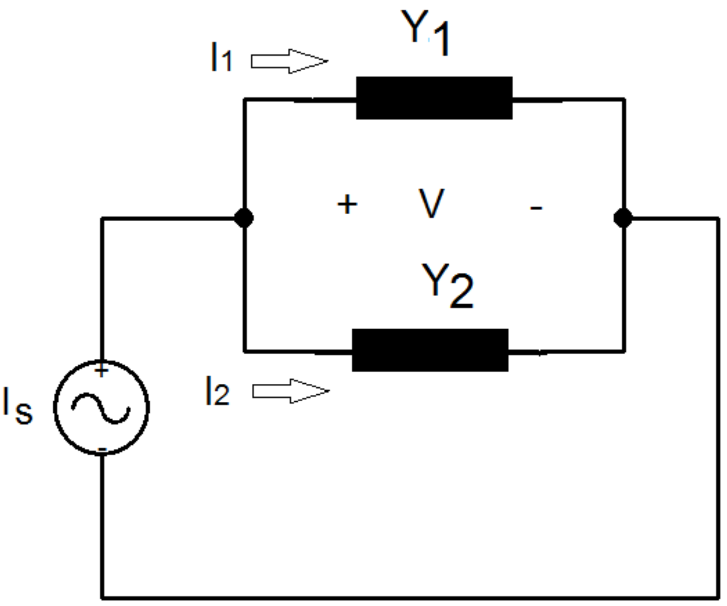
If you used Y instead of Z

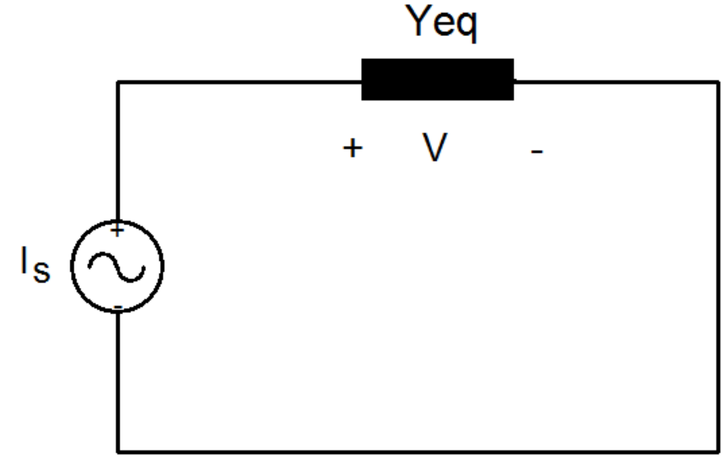
- In parallel:

  The equivalent admittance is equal to the sum of all of the admittance in parallel

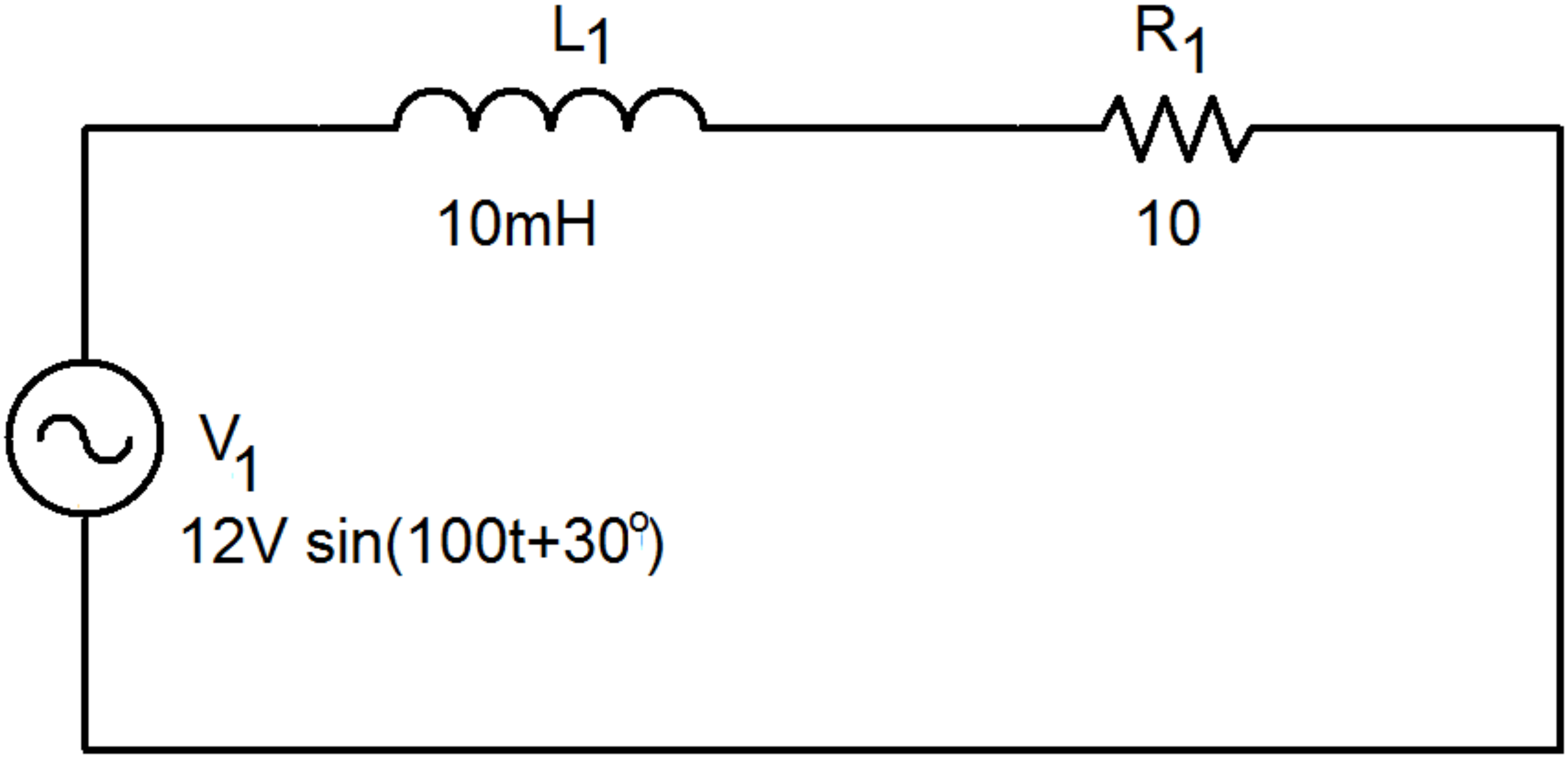
  In this example:

$$Y_{eq} = Y_1 + Y_2$$





Example 03…



…Example 03…

- Impedance

$Z_R = 10\ \Omega$

$Z_L = j\omega L = j(100)(10mH) = 1j\ \Omega$

$Z_{eq} = Z_R + Z_L = 10 + 1j\ \Omega$ (rectangular coordinates)

In Phasor notation:

$\mathbf{Z_{eq}} = (Z_R^2 + Z_L^2)^{½} \angle \tan^{-1}(Im/Re)$

$\mathbf{Z_{eq}} = (100 + 1)^{½} \angle \tan^{-1}(1/10) = 10.05 \angle 5.7^o\ \Omega$

$\mathbf{Z_{eq}} = 10.1 \angle 5.7^o\ \Omega$

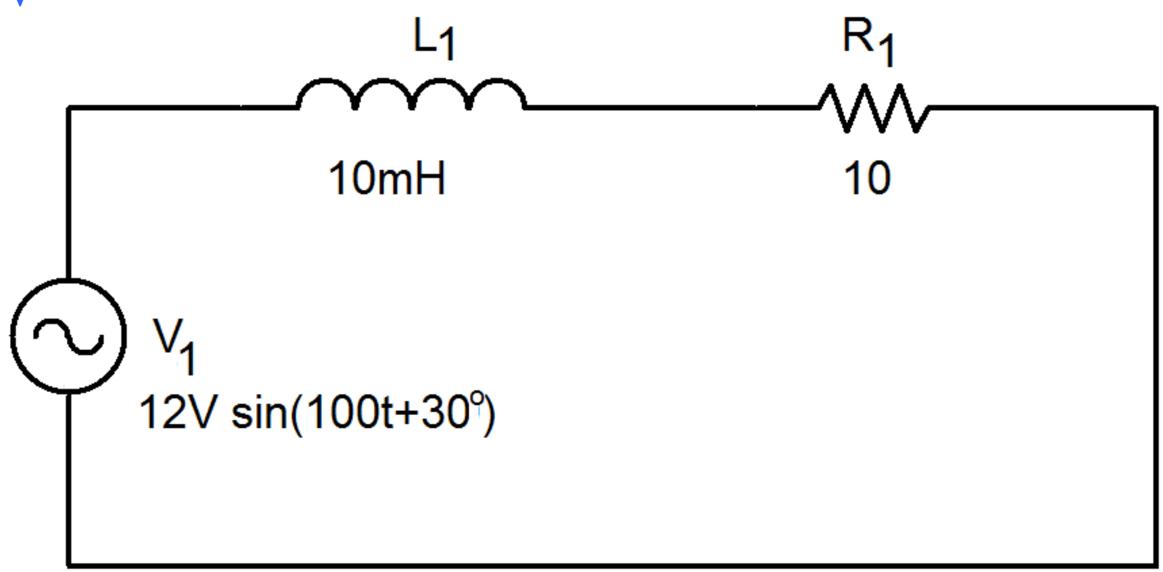
Impedances are easier than admittances to use when combining components in series.

…Example 03…

- Solve for Current
  - Express voltage into cosine and then convert a phasor.

$V1 = 12V \cos(100t + 30^o - 90^o) = 12V \cos(100t - 60^o)$

$\mathbf{V1} = 12 \angle{-60^o}\ V$



…Example 03…

- Solve for Current

  $\mathbf{I} = \mathbf{V}/\mathbf{Z}_{\mathbf{eq}} = (12 \angle -60^o \text{ V})/ (10.1 \angle 5.7^o\ \Omega)$

  $\mathbf{V} = 12 \angle -60^o \text{ V} = 12\text{V } e^{-j60}$ (exponential form)

  $\mathbf{Z}_{\mathbf{eq}} = 10.1 \angle 5.7^o\ \Omega = 10.1\ \Omega\ e^{j5.7}$ (exponential form)

  $\mathbf{I} = \mathbf{V}/\mathbf{Z}_{\mathbf{eq}} = 12\text{V } e^{-j60}/(10.1\ e^{j5.7}) = 1.19\text{A } e^{-j65.7}$

  $\mathbf{I} = 1.19\text{A} \angle -65.7^o$

  $\mathbf{I} = V_m/Z_m \angle (\theta_V - \theta_Z)$

…Example 03

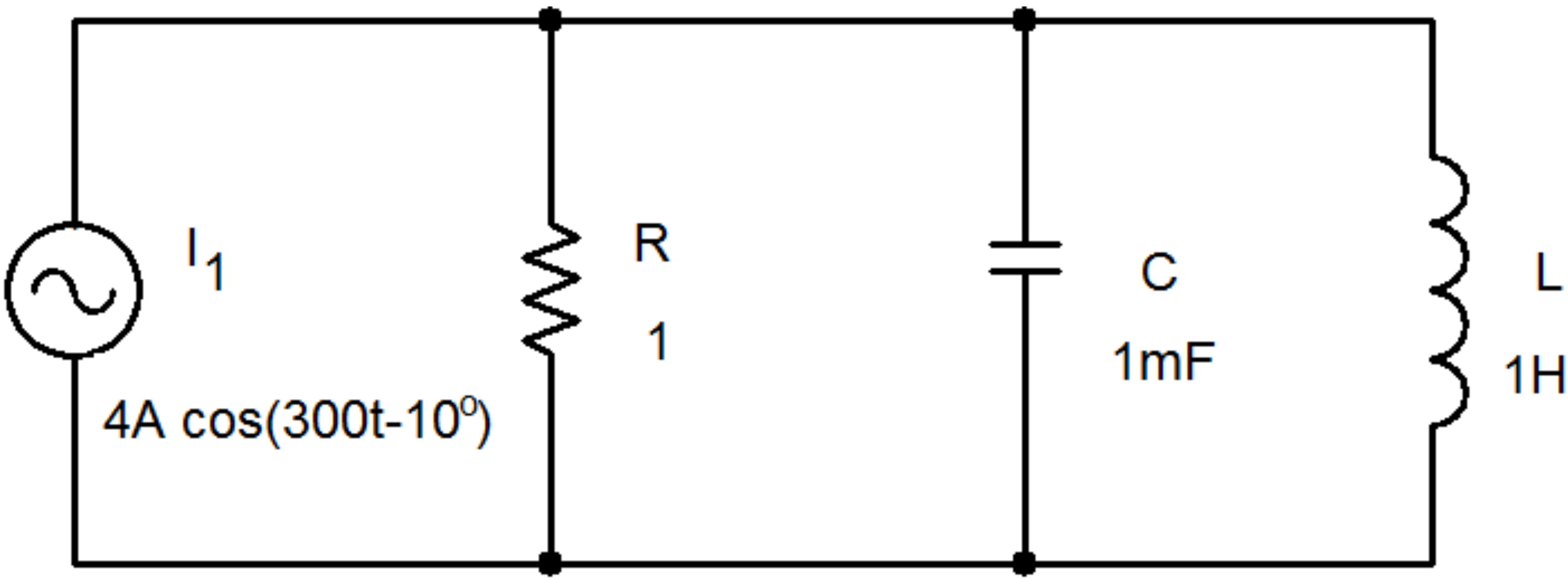
- Leading/Lagging

$$\mathbf{I} = 1.19\text{A}\ e^{-j65.7} = 1.19 \angle -65.7^{o}\ \text{A}$$

$$\mathbf{V} = 12\text{V}\ e^{-j60} = 12 \angle -60^{o}\ \text{V}$$

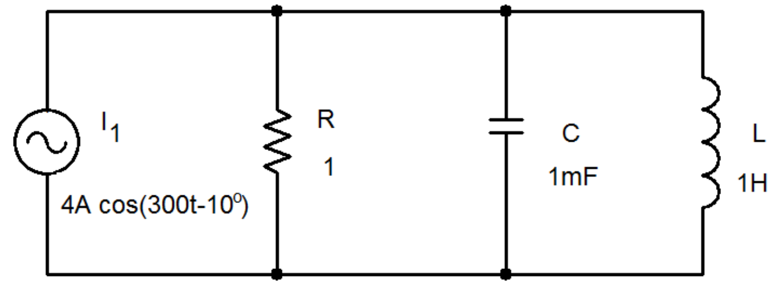
The voltage has a more positive angle, voltage leads the current.

Example 04…



…Example 04…

- Admittance



$Y_R = 1/R = 1\ \Omega^{-1}$

$Y_L = -j/(\omega L) = -j/[(300)(1H)] = -j\ 3.33\ m\Omega^{-1}$

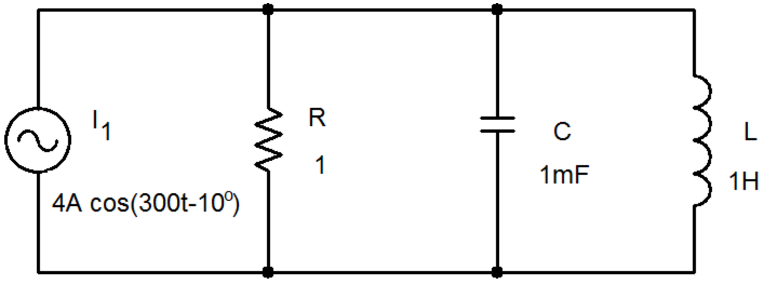
$Y_C = j\omega C = j(300)(1mF) = 0.3j\ \Omega^{-1}$

$Y_{eq} = Y_R + Y_L + Y_C = 1 + 0.297j\ \Omega^{-1}$

Admittances are easier than impedances to use when combining components in parallel.

…Example 04…

- Admittances:
  - In Phasor notation:



$$\mathbf{Y_{eq}} = (Y_{Re}^2 + Y_{Im}^2)^{1/2} \angle \tan^{-1}(Im/Re)$$
$$\mathbf{Y_{eq}} = (1^2 + (.297)^2)^{1/2} \angle \tan^{-1}(.297/1)$$
$$\mathbf{Y_{eq}} = 1.04 \angle 16.5^{o}\ \Omega^{-1}$$

It is relatively easy to calculate the equivalent impedance of the components in parallel at this point as $\mathbf{Z_{eq}} = \mathbf{Y_{eq}}^{-1}$.

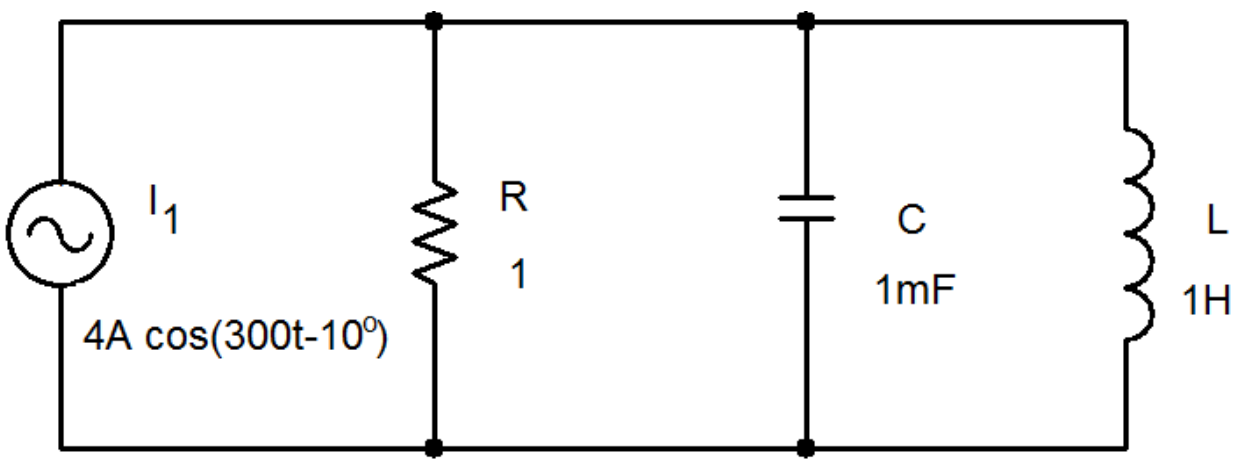
$$\mathbf{Z_{eq}} = \mathbf{Y_{eq}}^{-1} = 1/1.04 \angle 0\text{-}16.5^{o}\ \Omega = 0.959 \angle -16.5^{o}\ \Omega$$

…Example 04…

- Solve for Voltage
  – Convert a phasor since it is already expressed as a cosine.

$$I = 4A\cos(300t - 10^o)$$

$$\mathbf{I} = 4\angle -10^o\ A$$



…Example 04…

- Solve for Voltage

$$\mathbf{V} = \mathbf{I}/\mathbf{Y}_{\mathbf{eq}}$$

$$\mathbf{V} = I_m/Y_m \angle (\theta_I - \theta_Y)$$

$$\mathbf{V} = (4 \angle -10^o\ A)/\ (1.04 \angle 16.5^o\ \Omega^{-1})$$

$$\mathbf{V} = 3.84V \angle -26.5^o$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{\mathbf{eq}}$$

$$\mathbf{V} = I_m Z_m \angle (\theta_I + \theta_Z)$$

$$\mathbf{V} = (4 \angle -10^o\ A)(0.959 \angle -16.5^o\ \Omega^{-1})$$

$$\mathbf{V} = 3.84V \angle -26.5^o$$

…Example 04

- Leading/Lagging

$\mathbf{I} = 4 \angle -10^o$ A

$\mathbf{V} = 3.84V \angle -26.5^o$

Current has a more positive angle than voltage so current leads the voltage.

Equations

Equivalent Impedances	Equivalent Admittances
In Series:	In Series:
$Z_{eq} = Z_1 + Z_2 + Z_3 \ldots. + Z_n$	$Y_{eq} = [1/Y_1 + 1/Y_2 + 1/Y_3 \ldots. + 1/Y_n]^{-1}$
In Parallel:	In Parallel:
$Z_{eq} = [1/Z_1 + 1/Z_2 + 1/Z_3 \ldots. + 1/Z_n]^{-1}$	$Y_{eq} = Y_1 + Y_2 + Y_3 \ldots. + Y_n$

Summary

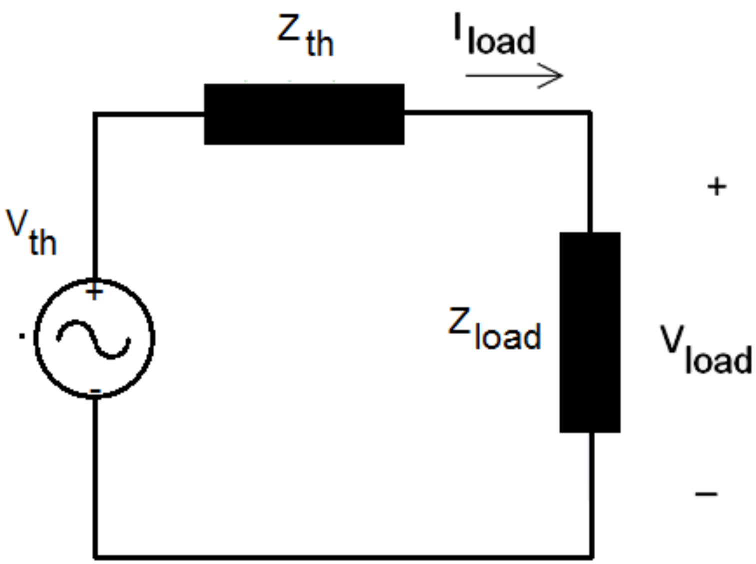
- The equations for equivalent impedance are similar in form to those used to calculate equivalent resistance and the equations for equivalent admittance are similar to the equations for equivalent conductance.
  - The equations for the equivalent impedance for components in series and the equations for the equivalent admittance of components in parallel tend to be easier to use.
  - The equivalent impedance is the inverse of the equivalent admittance.

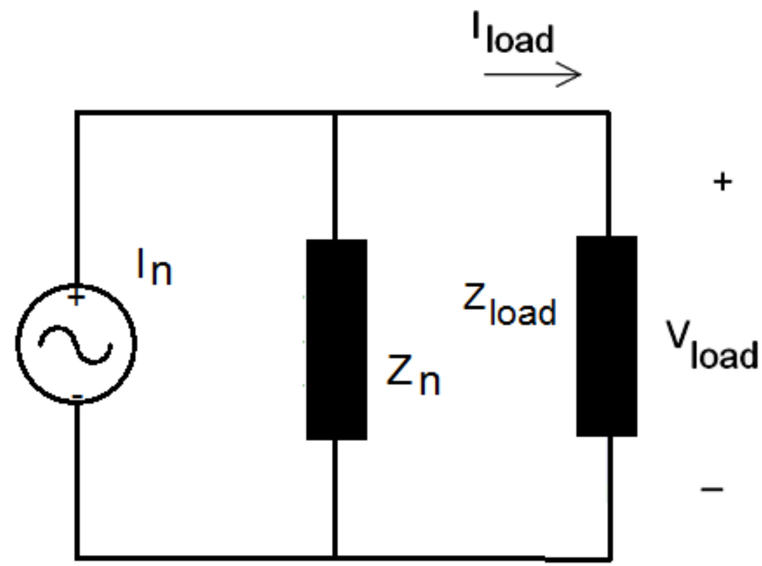
Thévenin and Norton Transformation

- Objective of Lecture
  - Demonstrate how to apply Thévenin and Norton transformations to simplify circuits that contain one or more ac sources, resistors, capacitors, and/or inductors.

- Source Transformation
  - A voltage source plus one impedance in series is said to be equivalent to a current source plus one impedance in parallel when the current into the load and the voltage across the load are the same.

Equivalent Circuits





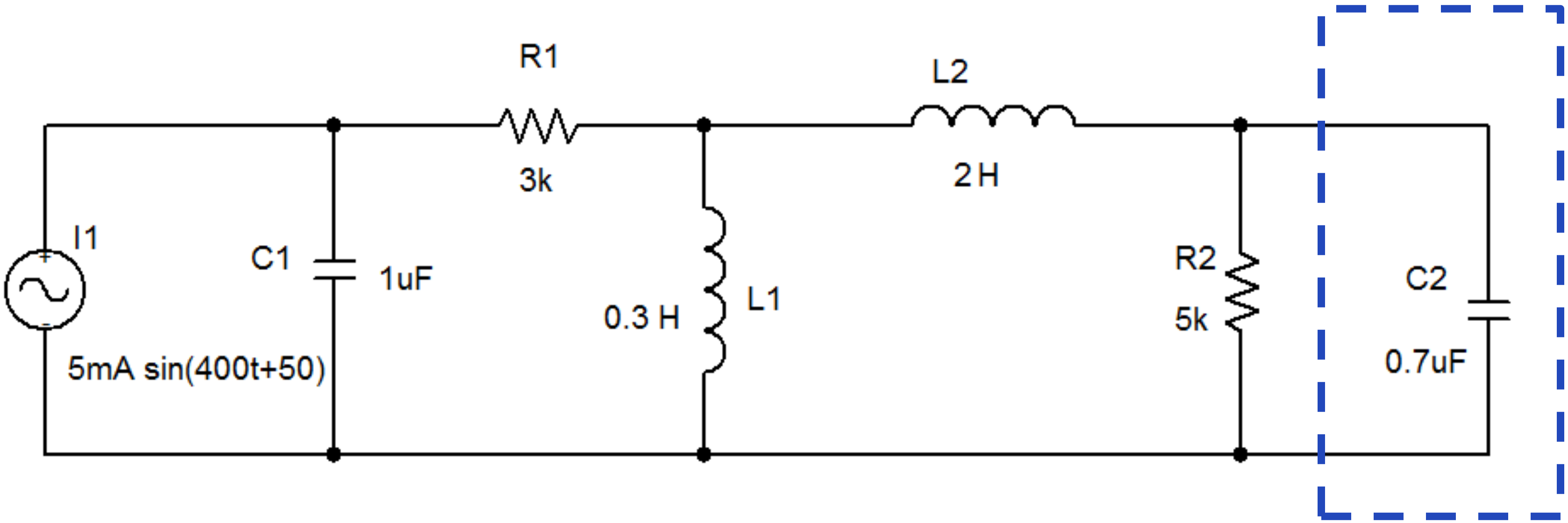
Thévenin

$V_{th} = I_n Z_n$

Norton

$I_n = V_{th}/Z_{th}$

Example 05…



First, convert the current source to a cosine function and then to a phasor.

$I_1 = 5\text{mA}\sin(400t+50^\circ) = 5\text{mA}\cos(400t+50^\circ-90^\circ) = 5\text{mA}\cos(400t-40^\circ)$

$\mathbf{I_1} = 5\text{mA}\ \angle -40^\circ$

…Example 05…

- Determine the impedance of all of the components when ω = 400 rad/s.
  - In rectangular coordinates

$$Z_{C_1} = -j/(\omega C_1) = -j/[(400 rad/s)1\mu F] = -j2.5k\Omega$$

$$Z_{R_1} = R_1 = 3k\Omega$$

$$Z_{L_1} = j\omega L_1 = j(400 rad/s)(0.3H) = j120\Omega$$

$$Z_{L_2} = j\omega L_2 = j(400 rad/s)(2H) = j800\Omega$$

$$Z_{R_2} = R_2 = 5k\Omega$$

$$Z_{C_2} = -j/(\omega C_2) = -j/[(400 rad/s)0.7\mu F] = -j3.57k\Omega$$

…Example 05…

- Convert to phasor notation

$$Z_{C_1} = 2.5k\Omega\angle -90^o$$

$$Z_{R_1} = 3k\Omega\angle 0^o$$

$$Z_{L_1} = 120\Omega\angle 90^o$$

$$Z_{L_2} = 800\Omega\angle 90^o$$

$$Z_{R_2} = 5k\Omega\angle 0^o$$

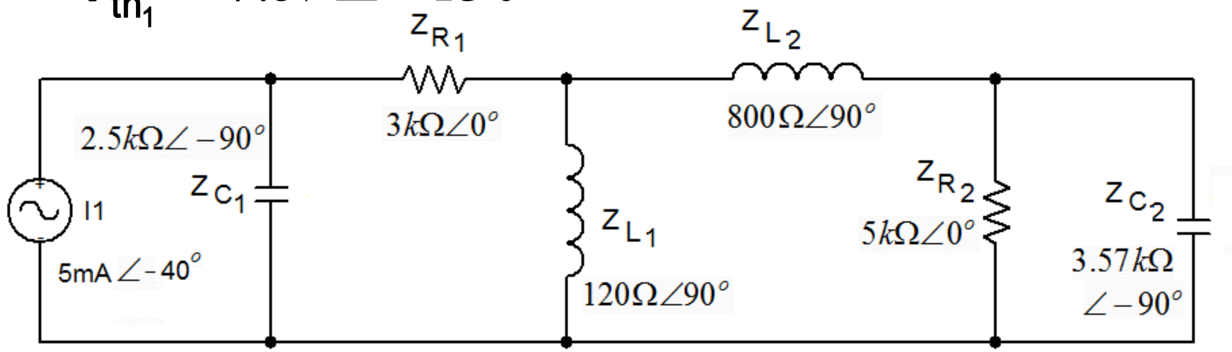
$$Z_{C_2} = 3.57k\Omega\angle -90^o$$

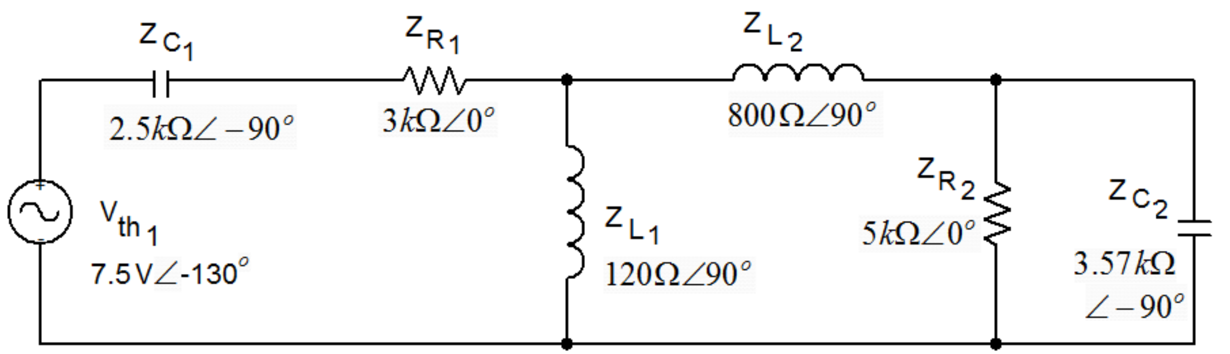
…Example 05…

$$\mathbf{V}_{\mathbf{th}_1} = \mathbf{I}_1 \mathbf{Z}_{\mathbf{C}_1} = \left(5mA\angle -40^o\right)\left(2.5k\Omega\angle -90^o\right)$$

$$\mathbf{V}_{\mathbf{th}_1} = (5mA)\left(2.5k\Omega\right)\angle\left(-40^o + \left(-90^o\right)\right)$$

$$\mathbf{V}_{\mathbf{th}_1} = 7.5V\angle -130^o$$





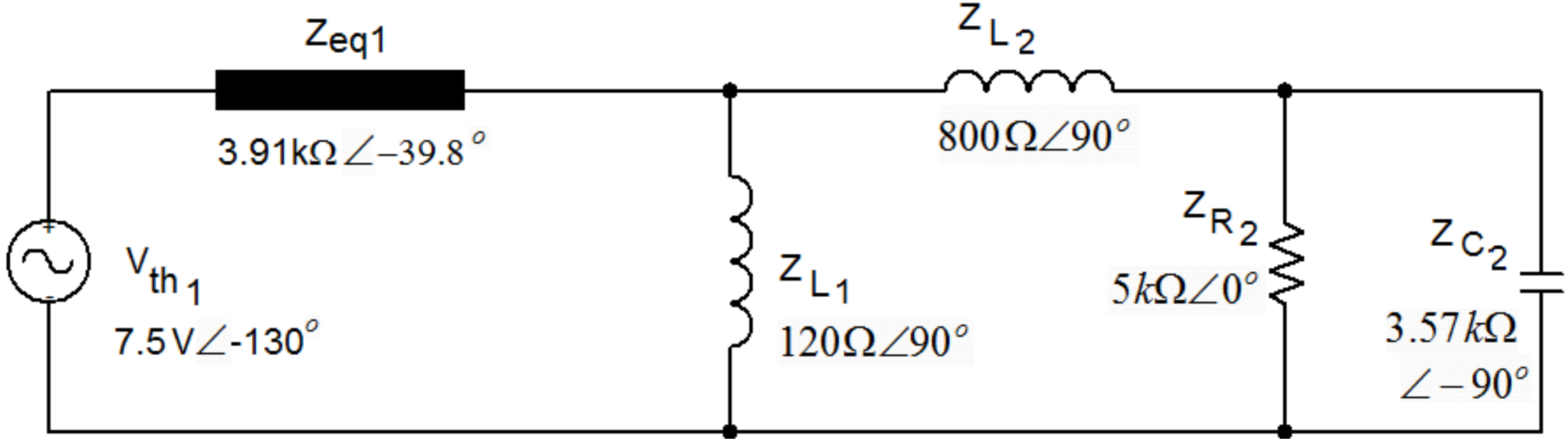
…Example 05…

- Find the equivalent impedance for Z_{C1} and Z_{R1} in series.
  - This is best done by using rectangular coordinates for the impedances.

$$Z_{eq_1} = Z_{R_1} + Z_{C_1} = 3k\Omega - j2.5k\Omega$$

$$\mathbf{Z}_{eq_1} = \sqrt{(3k\Omega)^2 + (-2.5k\Omega)^2} \angle \tan^{-1}(-2.5k/3k)$$

$$\mathbf{Z}_{eq_1} = 3.91k\Omega \angle -39.8^o$$



….Example 05…

- Perform a Norton transformation.

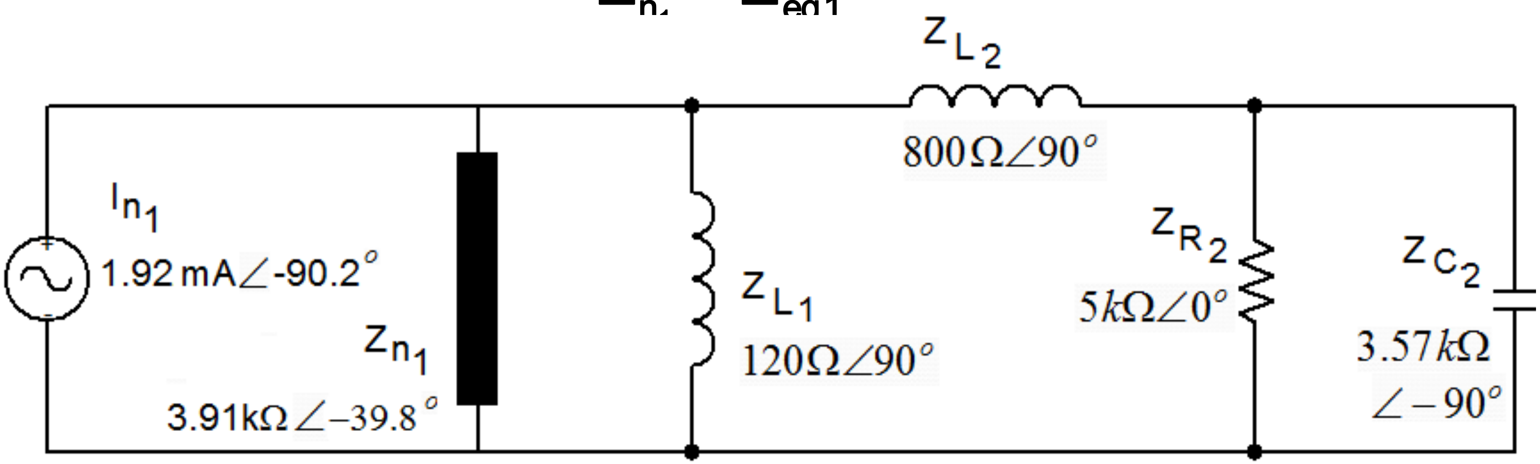
$$\mathbf{I}_{n_1} = \mathbf{V}_{th_1}/\mathbf{Z}_{eq1}$$

$$\mathbf{I}_{n_1} = \left(7.5V\angle -130^o\right)/\left(3.91k\Omega\angle -39.8^o\right)$$

$$\mathbf{I}_{n_1} = \left(7.5V/3.91k\Omega\right)\angle\left[-130^o - \left(-39.8^o\right)\right]$$

$$\mathbf{I}_{n_1} = 1.92mA\angle -90.2^o$$

$$\mathbf{Z}_{n_1} = \mathbf{Z}_{eq1}$$



…Example 05…

- Since it is easier to combine admittances in parallel than impedances, convert Z_{n1} to Y_{n1} and Z_{L1} to Y_{L1}.

- As Y_{eq2} is equal to $Y_{L1} + Y_{n1}$, the admittances should be written in rectangular coordinates, added together, and then the result should be converted to phasor notation.

…Example 05…

$$\mathbf{Y_{n1}} = 1/\mathbf{Z_{n1}} = 0.256 m\Omega^{-1}\angle 39.8^{o}$$

$$Y_{n1} = 0.256 m\Omega^{-1}\left[\cos\left(39.8^{o}\right) + j\sin\left(39.8^{o}\right)\right]$$

$$Y_{n1} = \left(0.198 + j0.164\right)m\Omega^{-1}$$

$$\mathbf{Y_{L1}} = 1/\mathbf{Z_{L1}} = 8.33 m\Omega^{-1}\angle -90^{0}$$

$$Y_{L1} = -j8.33 m\Omega^{-1}$$

$$Y_{eq2} = \left(0.198 + j0.164\right)m\Omega^{-1} - j8.33 m\Omega^{-1}$$

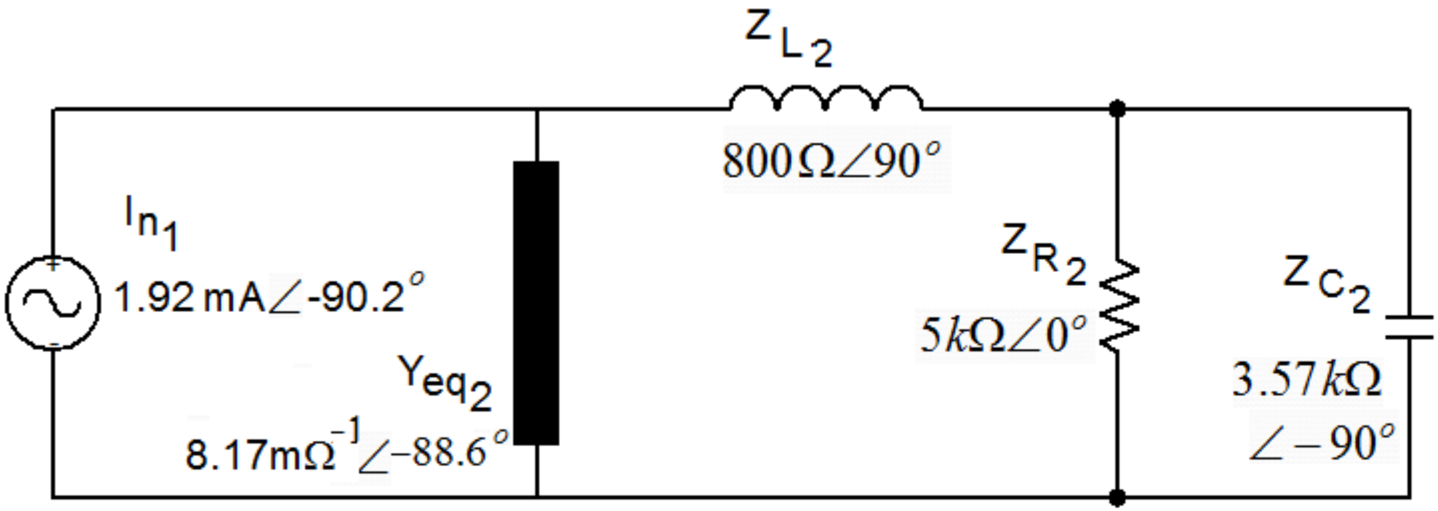
$$Y_{eq2} = \left(0.198 - j8.17\right)m\Omega^{-1}$$

$$\mathbf{Y_{eq2}} = \sqrt{\left(0.198\right)^{2} + \left(-8.17\right)^{2}}\, m\Omega^{-1}\angle \tan^{-1}\left(-8.17/0.198\right)$$

$$\mathbf{Y_{eq2}} = 0.817 m\Omega^{-1}\angle -88.6^{0}$$

…Example 05…

- Next, a Thévenin transformation will allow $\mathbf{Y_{eq2}}$ to be combined with $\mathbf{Z_{L2}}$.

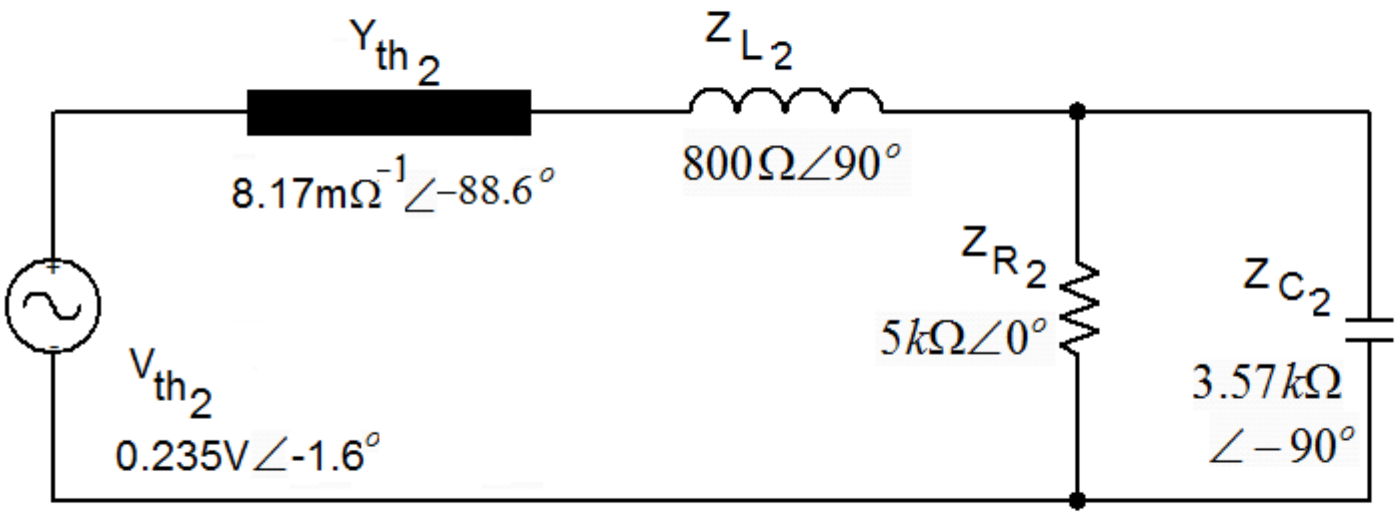


…Example 05…

$$V_{th_2} = I_{n_1} / Y_{th_2}$$

$$V_{th_2} = \frac{1.92mA\angle -90.2^o}{8.17m\Omega^{-1}\angle -88.6^o}$$

$$V_{th_2} = 0.235V\angle -1.6^o$$



…Example 05…

$$\mathbf{Z}_{\mathbf{th}_2} = 1/\mathbf{Y}_{\mathbf{th}_2} = 122\Omega\angle 88.6^o$$

$$Z_{th_2} = 122\Omega\left[\cos\left(88.6^o\right) + j\sin\left(88.6^o\right)\right]$$

$$Z_{th_2} = \left(2.98 + j122\right)\Omega$$

$$Z_{L_2} = j800\Omega$$

$$Z_{eq_3} = Z_{th_2} + Z_{L_2}$$

$$Z_{eq_3} = \left(2.98 + j122\right)\Omega + j800\Omega$$

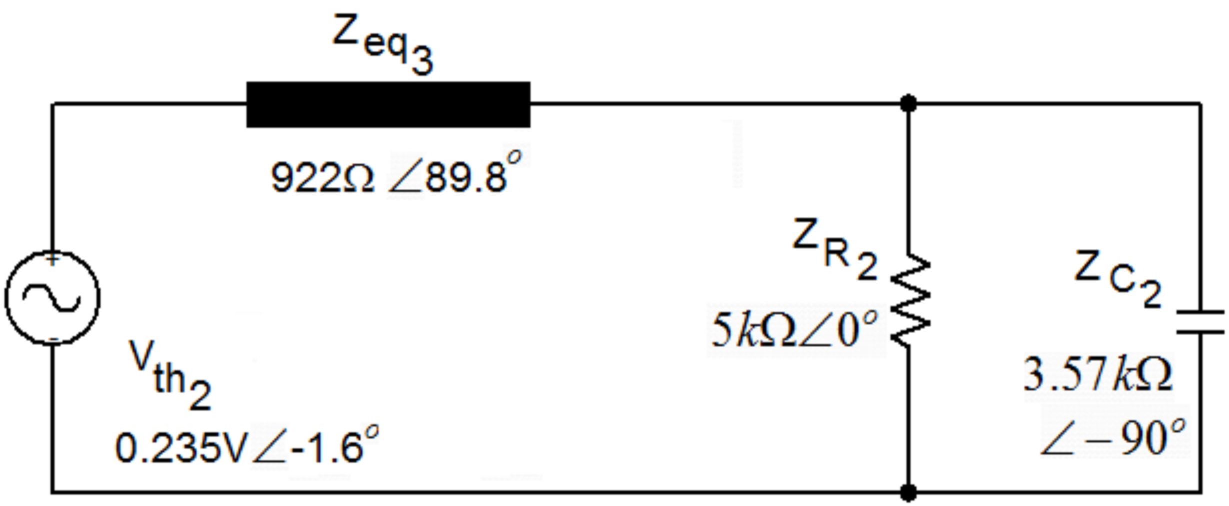
$$Z_{eq_3} = \left(2.98 + j922\right)\Omega$$

$$\mathbf{Z}_{\mathbf{eq}_3} = \sqrt{\left(2.98\right)^2 + \left(922\right)^2}\,\Omega\angle\left[\tan^{-1}\left(922/2.98\right)\right]$$

$$\mathbf{Z}_{\mathbf{eq}_3} = 922\Omega\angle 89.8^o$$

…Example 05…

- Perform a Norton transformation after which Z_{eq3} can be combined with Z_{R2}.

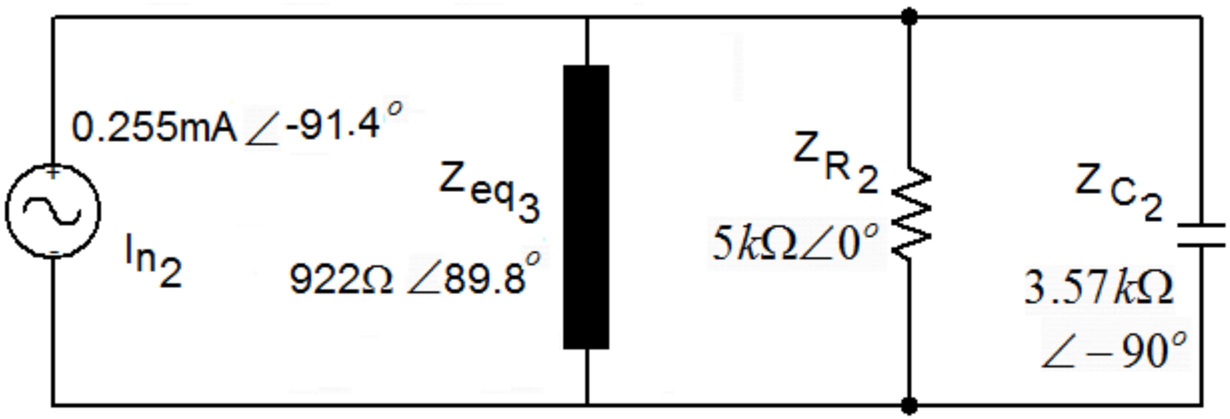


…Example 05…

$$\mathbf{I}_{n2} = \mathbf{V}_{th2} / \mathbf{Z}_{eq3}$$

$$\mathbf{I}_{n2} = \frac{0.235V\angle -1.6^0}{922\Omega\angle 89.8^0}$$

$$\mathbf{I}_{n2} = 0.255mA\angle -91.4^0$$



…Example 05…

$$\mathbf{Y}_{eq4} = 1/\mathbf{Z}_{eq3} + 1/\mathbf{Z}_{R_2}$$

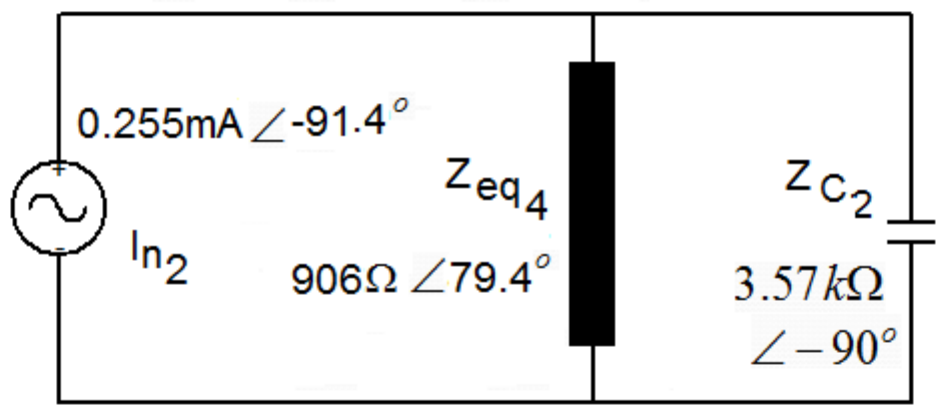
$$\mathbf{Y}_{eq4} = 1.08m\Omega^{-1}\angle -89.8^o + 0.2m\Omega^{-1}\angle 0^o$$

$$Y_{eq4} = 1.08m\Omega^{-1}\left[\cos(-89.8^o) + j\sin(-89.8^o)\right] + 0.2m\Omega^{-1}$$

$$Y_{eq4} = (0.204 - j1.08)m\Omega^{-1}$$

$$\mathbf{Y}_{eq4} = 1.10m\Omega^{-1}\angle -79.4^o$$

$$\mathbf{Z}_{eq4} = 906\Omega\angle 79.4^o$$



…Example 05…

Use the equation for current division to find the current flowing through Z_{C2} and Z_{eq4}.

$$\mathbf{I}_{C_2} = \frac{\mathbf{Y}_{C_2}}{\mathbf{Y}_{C_2} + \mathbf{Y}_{eq4}} \mathbf{I}_{n2}$$

$$\mathbf{Y}_{C_2} = 1/\mathbf{Z}_{C_2} = 0.280 m\Omega \angle 90^o$$

$$Y_{C_2} = j0.280 m\Omega$$

$$Y_{eq4} = (0.204 - j1.08) m\Omega$$

$$\mathbf{I}_{C_2} = \frac{0.280 m\Omega \angle 90^o}{j0.280 m\Omega + (0.204 - j1.08) m\Omega} \left(0.255 mA \angle -91.4^0\right)$$

$$\mathbf{I}_{C_2} = \frac{0.280 m\Omega \angle 90^o}{(0.204 - j0.8) m\Omega} \left(0.255 mA \angle -91.4^0\right)$$

$$\mathbf{I}_{C_2} = \frac{0.280 m\Omega \angle 90^o}{0.826 m\Omega \angle -75.7^o} \left(0.255 mA \angle -91.4^0\right)$$

$$\mathbf{I}_{C_2} = 86.0 \mu A \angle 74.3^o$$

…Example 05…

- Then, use Ohm's Law to find the voltage across $\mathbf{Z_{C2}}$ and then the current through $\mathbf{Z_{eq4}}$.

$$\mathbf{V}_{C_2} = \mathbf{I}_{C_2}\mathbf{Z}_{C_2} = \left(86.0\mu A\angle 74.3^o\right)\left(3.57k\Omega\angle -90^o\right)$$

$$\mathbf{V}_{C_2} = 0.309V\angle -15.7^o$$

$$\mathbf{V}_{C_2} = \mathbf{V}_{eq_4}$$

$$\mathbf{I}_{eq_4} = \frac{\mathbf{V}_{eq_4}}{\mathbf{Z}_{eq_4}} = \frac{0.309V\angle -15.7^o}{906\Omega\angle 79.4^o}$$

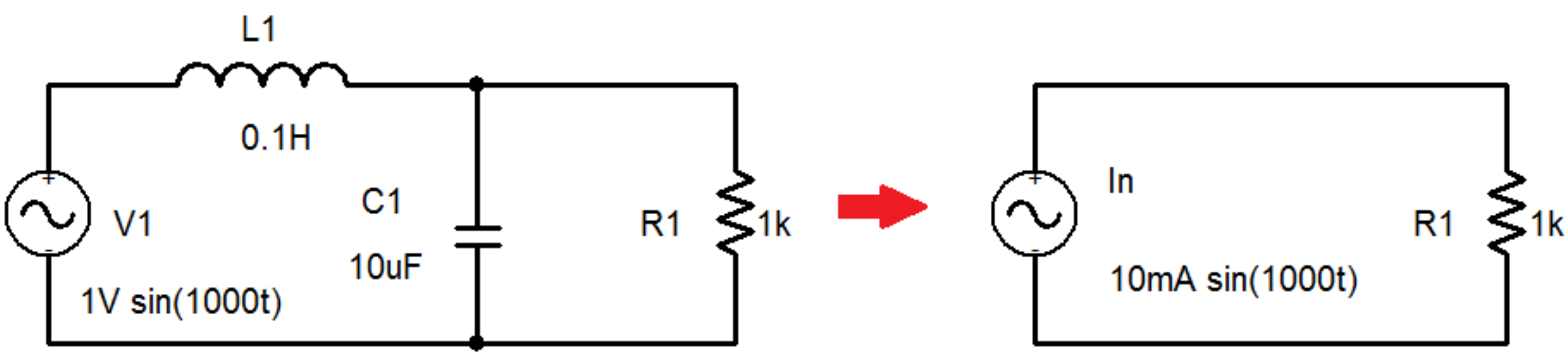
$$\mathbf{I}_{eq_4} = 0.341mA\angle -95.1^o$$

…Example 05

- Note that the phase angles of I_{n2}, I_{eq4}, and I_{C2} are all different because of the imaginary components of Z_{eq4} and Z_{C2}.
  - The current through $\mathbf{Z_{C2}}$ leads the voltage, which is as expected for a capacitor.
  - The voltage through $\mathbf{Z_{eq4}}$ leads the current.
  - Since the phase angle of $\mathbf{Z_{eq4}}$ is positive, it has an inductive part to its impedance.
    - Thus, it should be expected that the voltage would lead the current.

Example

- Explain why the circuit on the right is the result of a Norton transformation of the circuit on the left.
- Also, calculate the natural frequency ω_o of the RLC network.



Summary

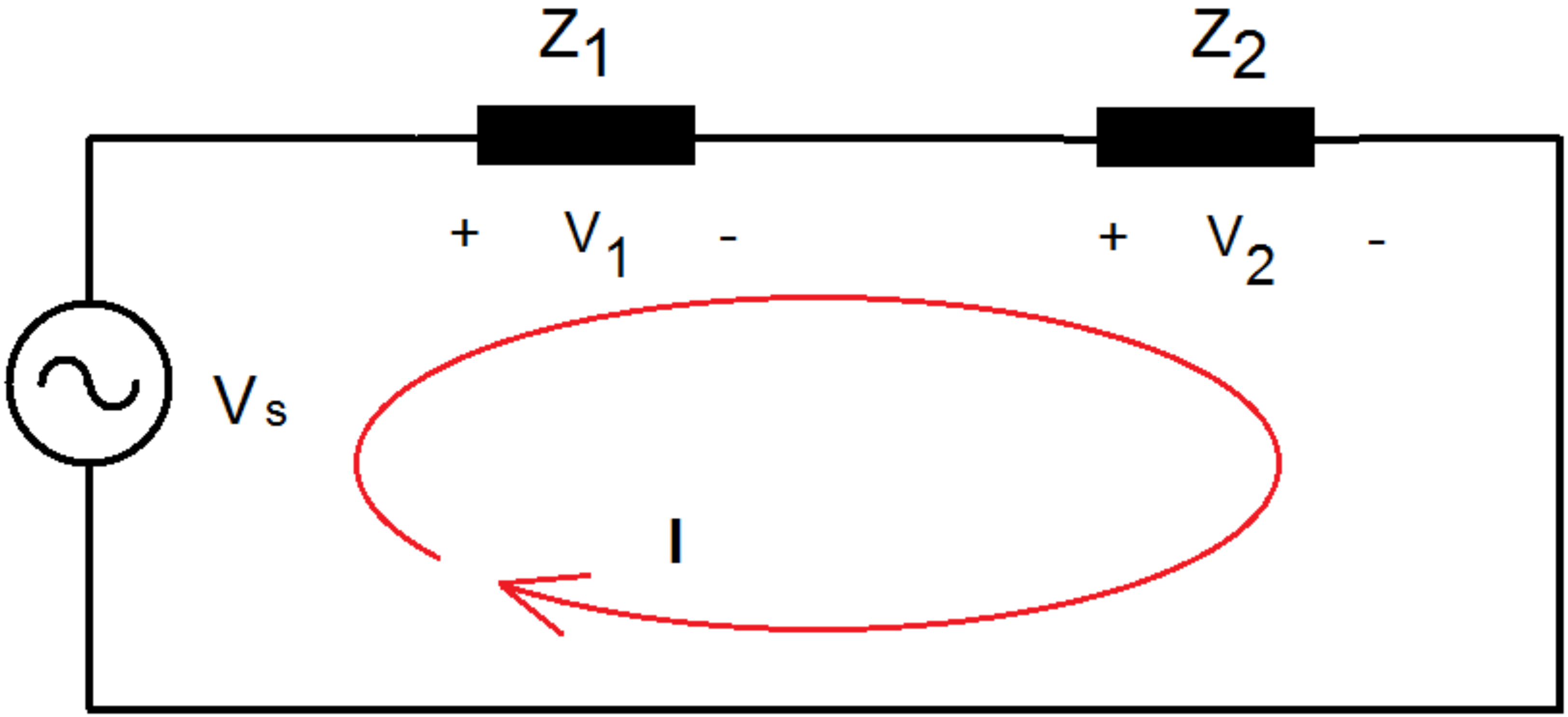
- Circuits containing resistors, inductors, and/or capacitors can simplified by applying the Thévenin and Norton Theorems.
  - Transformations can easily be performed using currents, voltages, impedances, and admittances written in phasor notation.
  - Calculation of equivalent impedances and admittances requires the conversion of phasors into rectangular coordinates.
  - Use of the current and voltage division equations also requires the conversion of phasors into rectangular coordinates.

Voltage and Current Division

- Objective of Lecture
  - Explain mathematically how a voltage that is applied to components in series and how a current that enters the a node shared by components in parallel are distributed among the components.

Voltage Dividers

- Impedances in series share the same current



Voltage Dividers

- From Kirchhoff's Voltage Law and Ohm's Law

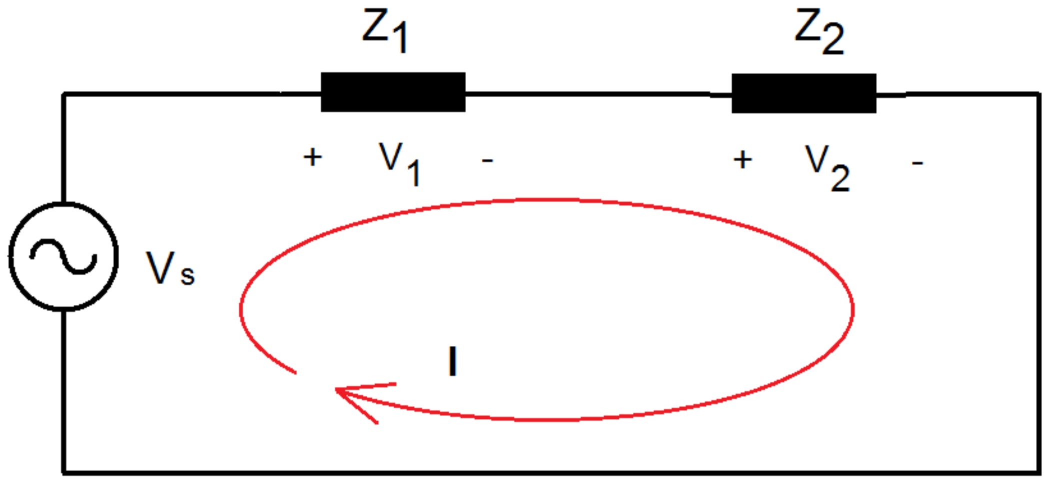
$$0 = -V_S + V_1 + V_2$$

$$V_1 = IZ_1 \quad \text{and} \quad V_2 = IZ_2$$

$$\text{Therefore, } V_2 = \frac{V_1}{Z_1} Z_2$$

$$V_1 = \frac{Z_1}{Z_1 + Z_2} V_s$$

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_s$$



Voltage Division

The voltage associated with one impedance $\mathbf{Z_n}$ in a chain of multiple impedances in series is:

$$V_n = \left[\frac{Z_n}{\sum_{s=1}^{S} Z_s} \right] V_{total} \quad \text{or} \quad V_n = \left[\frac{Z_n}{Z_{eq}} \right] V_{total}$$

where V_{total} is the total of the voltages applied across the impedances.

Voltage Division

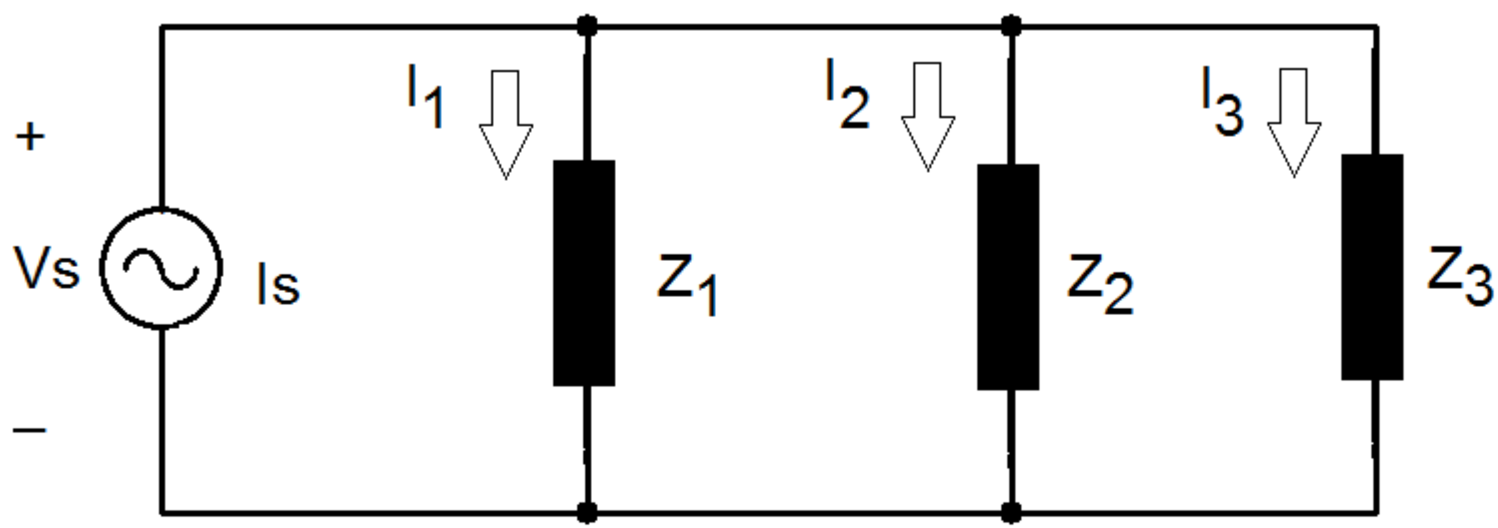
- Because of changes in phase angle of the voltage that occur with inductors and capacitors, the calculation of the percentage of the total voltage associated with a particular impedance, $\mathbf{Z_n}$, is not directly related to the percentage of the magnitude of that particular impedance, Z_n, relative to the total equivalent impedance, Z_{eq}.

$$\mathbf{Z_n} = Z_n \angle \varphi_n$$

$$\mathbf{Z_{eq}} = Z_{eq} \angle \varphi_{eq}$$

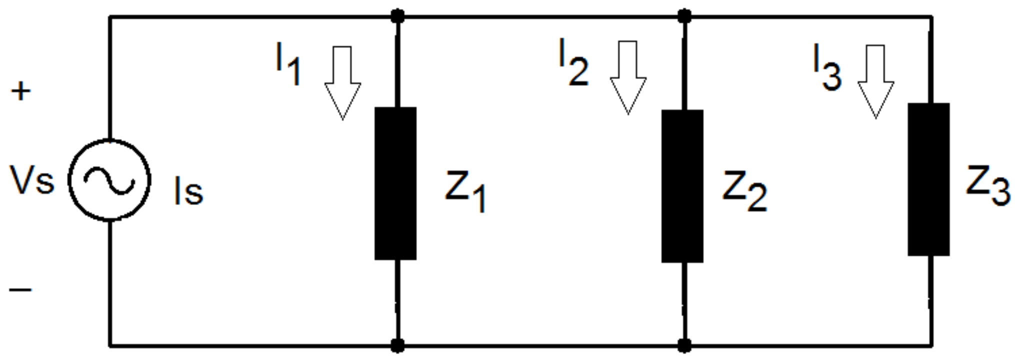
Current Division

- All components in parallel share the same voltage



Current Division

- From Kirchhoff's Current Law and Ohm's Law



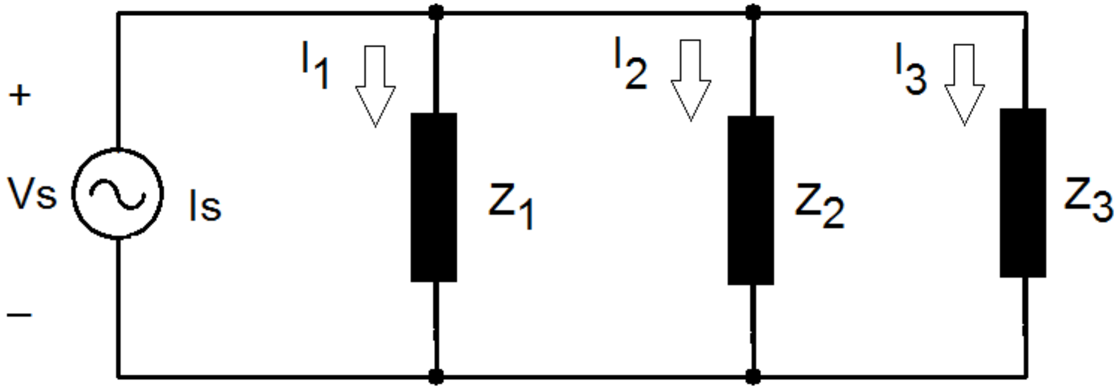
$$0 = -I_S + I_1 + I_2 + I_3$$

$$V_S = I_1 Z_1$$

$$V_S = I_2 Z_2$$

$$V_s = I_3 Z_3$$

Current Division

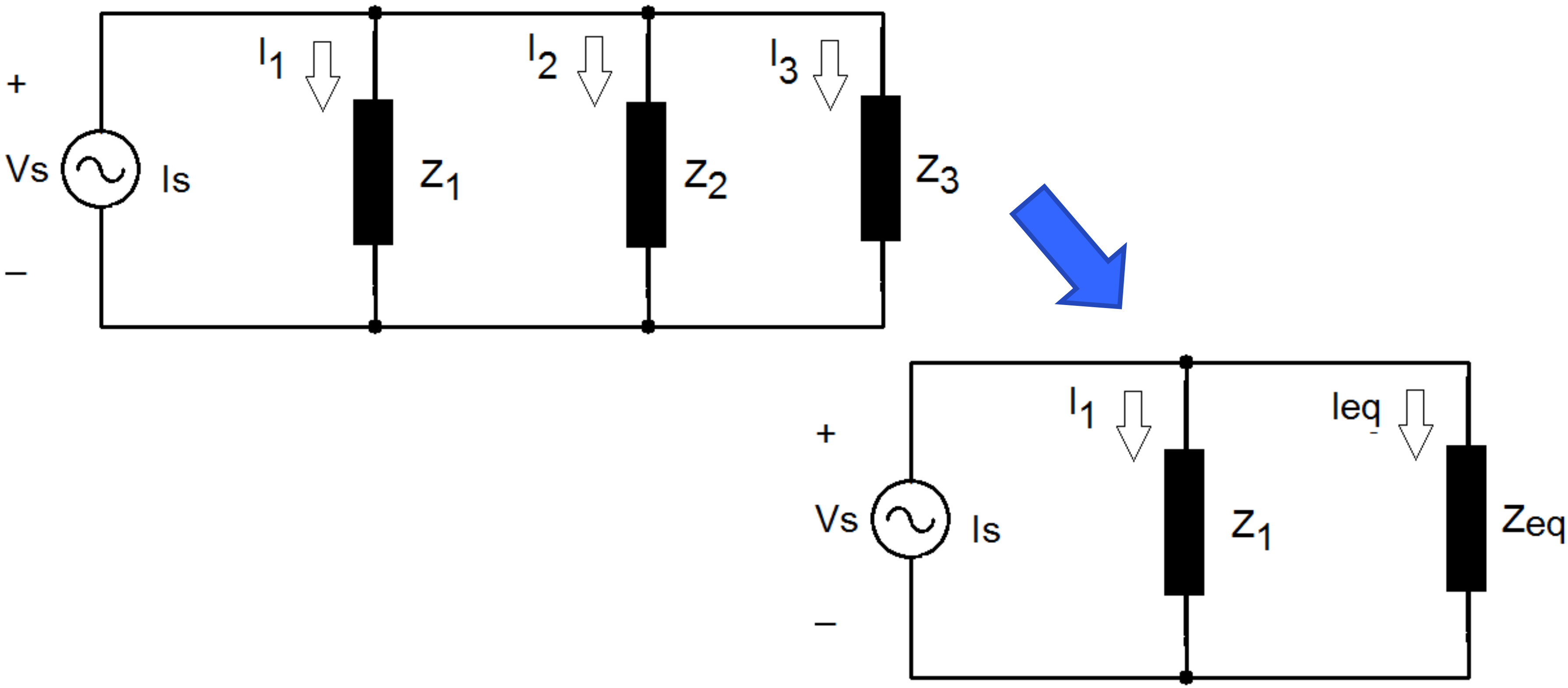


$$I_1 = \frac{Z_2 \| Z_3}{Z_1 + Z_2 \| Z_3} I_S$$

$$I_2 = \frac{Z_1 \| Z_3}{Z_2 + Z_1 \| Z_3} I_S$$

$$I_3 = \frac{Z_1 \| Z_2}{Z_3 + Z_1 \| Z_2} I_S$$

Current Division



where $\mathbf{Z_{eq}} = \mathbf{Z_2} \| \mathbf{Z_3} = \dfrac{\mathbf{Z_2 Z_3}}{\mathbf{Z_2} + \mathbf{Z_3}}$ and $\mathbf{I_1} = \dfrac{\mathbf{Z_{eq}}}{\mathbf{Z_1} + \mathbf{Z_{eq}}} \mathbf{I_S}$

Current Division

The current associated with one component Z_1 in parallel with one other component is:

$$I_1 = \left[\frac{Z_2}{Z_1 + Z_2}\right] I_{total}$$

The current associated with one component Z_m in parallel with two or more components is:

$$I_m = \left[\frac{Z_{eq}}{Z_m}\right] I_{total}$$

where I_{total} is the total of the currents entering the node shared by the components in parallel.

Summary

- The equations used to calculate the voltage across a specific component Z_n in a set of components in series are:

$$V_n = \left[\frac{Z_n}{Z_{eq}}\right] V_{total}$$

$$V_n = \left[\frac{Y_{eq}}{Y_n}\right] V_{total}$$

- The equations used to calculate the current flowing through a specific component Z_m in a set of components in parallel are:

$$I_m = \frac{Z_{eq}}{Z_m} I_{total}$$

$$I_m = \frac{Y_m}{Y_{eq}} I_{total}$$