

Counting:

$$P(A) = \frac{|A|}{|S|} \left\{ \begin{array}{l} \text{Equally likely outcome} \\ \text{the number of elements} \end{array} \right.$$

Coffee example S M L in how many ways can you order your coffee?

→ Flavor
→ Decoe 3x3 = 9 ways
→ Light

Multiplication Principle (Prinzip der Multiplikation)

(r dependent, unabhängige Versuche)

Suppose that we perform r experiments. There are n possible outcomes

for each experiment

$$1 \leq k \leq r$$

$$\begin{array}{l} 1 \rightarrow n_1 \\ 2 \rightarrow n_2 \\ \vdots \\ k \rightarrow n_k \end{array}$$

There are a total of $n_1 \times n_2 \times \dots \times n_r$ possible outcomes

(sequence)

Choosing a password

→ begin with 2 lower case letters followed by

(biggie harp)

→ 1 capital letter followed by

→ 4 digits

of total passwords

→ multiplication principle

$$\frac{26}{26} \times \frac{26}{26} \times \frac{26}{26} \times \frac{10}{10} \times \frac{10}{10} \times \frac{10}{10} \times \frac{10}{10} \rightarrow 26^3 \cdot 10^4$$

Sampling

- Sampling from a set randomly choosing an element from a set.

Replacement

(combination, permutation)

with replacement: put the sampled element back

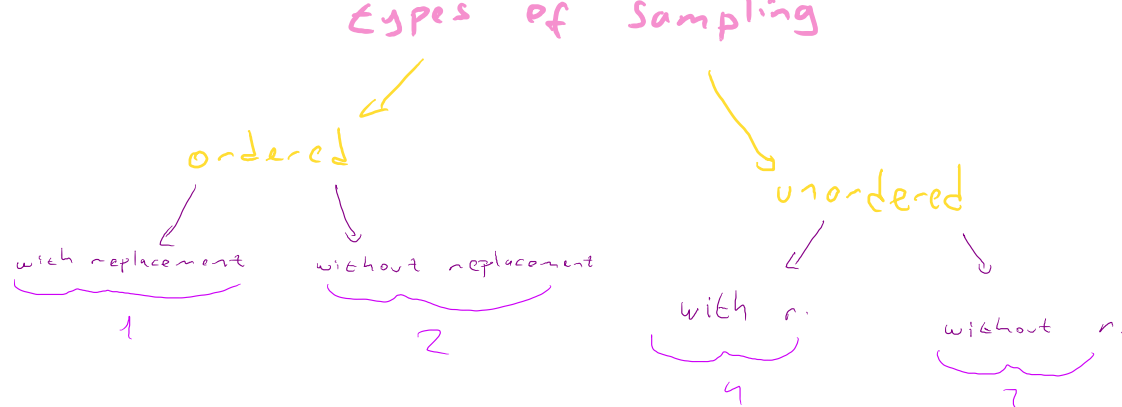
without replacement → don't put the sampled element back

Ordering If ordering matters $(a_1, a_2, a_3) \neq (a_3, a_2, a_1)$ → serial, sequ.

Unordered sampling → order is not important $(a_1, a_2, a_3) = (a_3, a_2, a_1)$

don't matter → nicht! egal

Types of sampling



Ordered sampling with replacement:

ex $A = \{1, 2, 3\}$ draw $k=2$ samples

How many ways the sampling can be done?

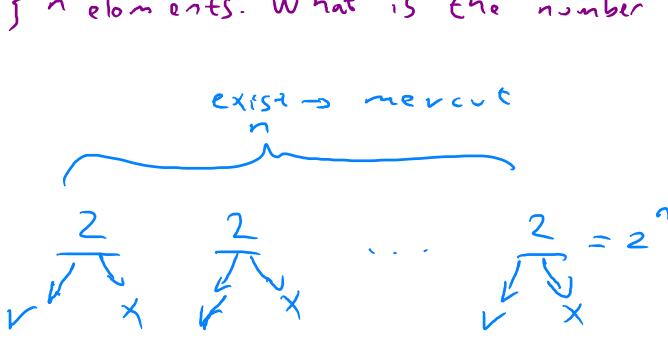
$(1,1), (1,2), (2,3), (1,1), (2,2), (3,3), (2,1), (3,2), (3,1)$

$3 \times 3 \rightarrow 9 \rightarrow n^k$ k is number of sampling n is number of outcomes

number of subsets? $A = \{1, \dots, n\}$ elements. What is the number of different

subset for $A \rightarrow \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \dots}_n \rightarrow 2^n$

Multiplication principle



Ordered sampling without replacement

- No repetitions

↳ Permutation

ex: $A = \{1, 2, 3\}$

$(1,2), (1,3), (3,1), (2,1), (2,3), (3,2) \rightarrow 3 \times 2 = 6$

$$\frac{n!}{(n-k)!}$$

Ex: k people in a class. What is the probability that at least two of them have same birthday

→ $n=365$ 1 person = $\frac{1}{365}$

A = event that 2 people have same birthday.

A^c = no 2 people have the same birthday.

$$P(A) = 1 - \frac{|A^c|}{|S|} \rightarrow 1 - \frac{1}{365 \cdot 364 \cdot 363 \dots 365-k} \rightarrow 1 - \frac{365!}{(365-k)! \cdot (365)^n}$$

$k \geq 365 \rightarrow$ at least 2 of them are born at same day → Pigeonhole principle

Unordered sampling without replacement

ex $A = \{1, 2, 3\}$ $k=2$ unordered sampling without replacement

$(1,2), (1,3), (2,3) \rightarrow$ combination $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$$\binom{3}{2} = \frac{3!}{2!(1)!} = 3 \quad P_k^n = \binom{n}{k} \cdot k!$$

Ex: Choose 3 cards from a deck (52 cards)

What is the probability that these cards contain at least one ACE

4 ACE in a deck

$P(A^c) =$ None of 3 cards with contain ACE $= \binom{52-4}{3}$

$$P(A) = 1 - \binom{52-4}{3}$$

$$P(A) = 1 - \frac{\binom{48}{3}}{\binom{52}{3}} \rightarrow 1 - \frac{17 \cdot 47 \cdot 46}{52 \cdot 51 \cdot 50} = 1 - \frac{12,2347}{51,13,25}$$

Binomial Coefficient (Binomialkoeffizient)

$\binom{n}{k}$ is called Binomial Coefficient (Kombi.)

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k} \quad \text{ex/ } (a+b)^2 = a^2 + 2ab + b^2 = a \cdot b \cdot \binom{2}{0} + a^1 b^1 \binom{2}{1} + a^2 b^0 \binom{2}{2}$$

Dividing sets into two is like "n choose k"



$$\binom{n}{k} = \binom{n}{n-k}$$

Ex: Unfair coin $P(H)=p$ $0 \leq p \leq 1$ $P(T)=1-p$

we toss the coin 5 times.

What is the prob. that the output is H H T H H

↳ Multiplication rule $p \cdot p \cdot (1-p) \cdot p \cdot p = p^4(1-p)$

Prob 4 heads 1 tail

$$\rightarrow \binom{5}{4} \cdot p^4(1-p)$$

If I toss the coin n times, what is the prob. that I observe

k head and $n-k$ tails.

$$p^k \cdot (1-p)^{n-k} \cdot \binom{n}{k}$$

Ex 10 people ... to a restaurant

5 ← main dish.

3 ← drinks

2 ← dessert In how many ways can these people be selected?

$$\binom{10}{5} \cdot \binom{3}{2} \cdot \binom{2}{2} = \frac{10!}{5!5!} \cdot \frac{3!}{2!1!} \cdot \frac{2!}{2!0!}$$

Unordered sampling with Replacement

$A = \{1, 2, 3\}$

$(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)$ 6 possible lists

↳ 2 things: 3 hotels, 3! ways

unordered with replace

$$\binom{n+k-1}{k} \rightarrow \binom{4}{2} = \frac{4 \cdot 3}{2} = 6$$

| x_1 | x_2 | x_3 |
|-------|-------|-------|
| 2 | 0 | 0 |
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 2 | 0 |
| 0 | 1 | 1 |
| 0 | 0 | 2 |

ex: 10 passengers get on an airport shuttle. The shuttle route includes

5 hotels and each passengers gets off his hotel. The driver

records how many passengers leave the shuttle at each hotel. How many different possible lists.

| x_1 | x_2 | x_3 | x_4 | x_5 |
|-------|-------|-------|-------|-------|
| 10 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 0 |

$$\binom{5+10-1}{10} = \frac{14!}{4! \cdot 10!}$$