Introduction to Digital Logic

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Course Outline

1. Digital Computers, Number Systems, Arithmetic Operations, Decimal, Alphanumeric, and Gray Codes
2. Binary Logic, Gates, Boolean Algebra, Standard Forms
3. Circuit Optimization, Two-Level Optimization, Map Manipulation, Multi-Level Circuit Optimization
4. Additional Gates and Circuits, Other Gate Types, Exclusive-OR Operator and Gates, High-Impedance Outputs
5. Implementation Technology and Logic Design, Design Concepts and Automation, The Design Space, Design Procedure, The major design steps
6. Programmable Implementation Technologies: Read-Only Memories, Programmable Logic Arrays, Programmable Array Logic,Technology mapping to programmable logic devices
7. Combinational Functions and Circuits
8. Arithmetic Functions and Circuits
9. Sequential Circuits Storage Elements and Sequential Circuit Analysis
10. Sequential Circuits, Sequential Circuit Design State Diagrams, State Tables
11. Counters, register cells, buses, & serial operations
12. Sequencing and Control, Datapath and Control, Algorithmic State Machines (ASM)
13. Memory Basics

Introduction to Digital Logic

Lecture 2

Gate Circuits and Boolean Equations

- Binary Logic and Gates
- Boolean Algebra
- Standard Forms

Binary Logic and Gates

- **Binary variables** take on one of two values.
- **Logical operators** operate on binary values and binary variables.
- Basic logical operators are the **logic functions** **AND, OR** and **NOT**.
- **Logic gates** implement logic functions.
- **Boolean Algebra**: a useful mathematical system for specifying and transforming logic functions.
- We study Boolean algebra as foundation for designing and analyzing **digital systems**!

Binary Variables

- **Recall that the two binary values have different names:**
  - True/False
  - On/Off
  - Yes/No
  - 1/0
- **We use 1 and 0 to denote the two values.**
- **Variable identifier examples:**
  - A, B, y, z, or X_1 for now
  - RESET, START_IT, or ADD1 later

Logical Operations

- The three basic logical operations are:
  - AND
  - OR
  - NOT
- AND is denoted by a dot ($\cdot$)
- OR is denoted by a plus ($+$)
- NOT is denoted by an overbar ($\overline{\ }$), a single quote mark (') after, or (~) before the variable

Notation Examples

- **Examples:**
  - **Y=A.B is read "Y is equal to A AND B."**
  - **z=x+y is read "z is equal to x OR y."**
  - **X=$\bar{A}$ is read "X is equal to NOT A."**
- **Note: The statement:**

  1 + 1 = 2 (read "one <u>plus</u> one equals two")

  is not the same as

  1 + 1 = 1 (read "1 <u>or</u> 1 equals 1").

Operator Definitions

- Operations are defined on the values "0" and "1" for each operator:

AND	OR	NOT
$0 \cdot 0 = 0$	$0 + 0 = 0$	$\overline{0} = 1$
$0 \cdot 1 = 0$	$0 + 1 = 1$	$\overline{1} = 0$
$1 \cdot 0 = 0$	$1 + 0 = 1$	
$1 \cdot 1 = 1$	$1 + 1 = 1$	

Truth Tables

- ***Truth table*** **– a tabular listing of the values of a function for all possible combinations of values on its arguments**
- **Example: Truth tables for the basic logic operations:**

AND		
X	Y	$Z = X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

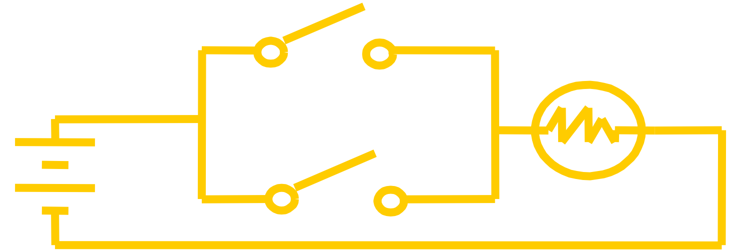
OR		
X	Y	$Z = X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

NOT	
X	$Z = \overline{X}$
0	1
1	0

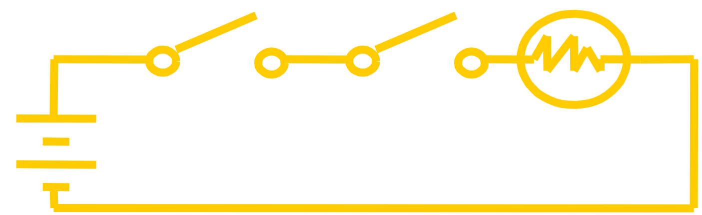
Logic Function Implementation

- Using Switches
  - For inputs:
    - logic 1 is <u>switch closed</u>
    - logic 0 is <u>switch open</u>
  - For outputs:
    - logic 1 is <u>light on</u>
    - logic 0 is <u>light off</u>.
- NOT uses a switch such that:
    - logic 1 is <u>switch open</u>
    - logic 0 is <u>switch closed</u>

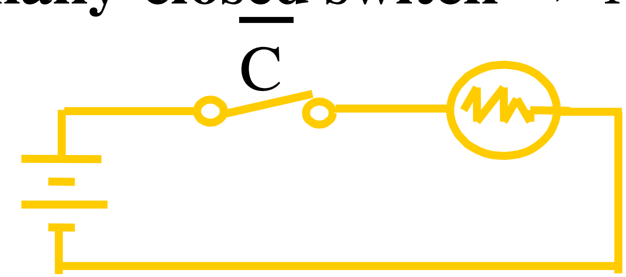
Switches in parallel => OR



Switches in series => AND

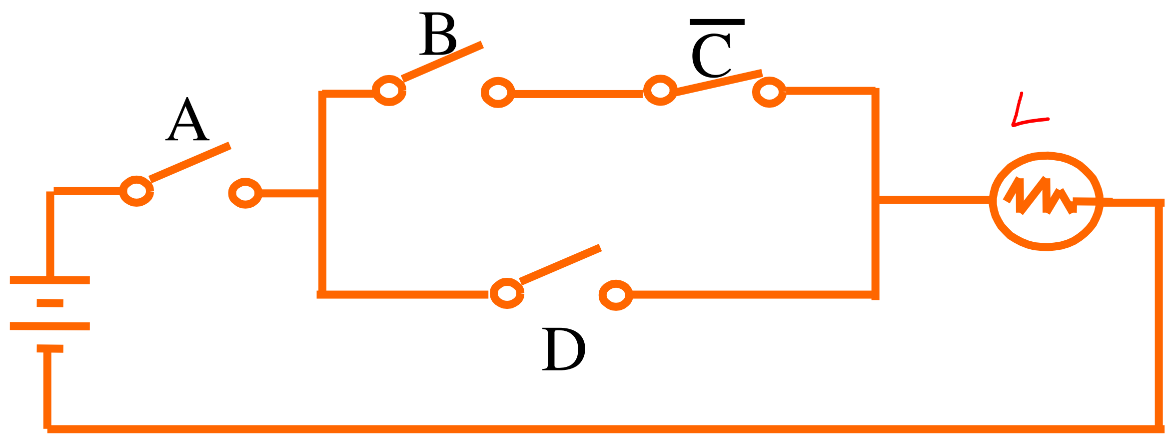


Normally-closed switch => NOT



Logic Function Implementation (Continued)

- **Example: Logic Using Switches**



- **Light is on (L = 1) for**

  L(A, B, C, D) = A . ((B . C')+D)

  and off (L = 0), otherwise.

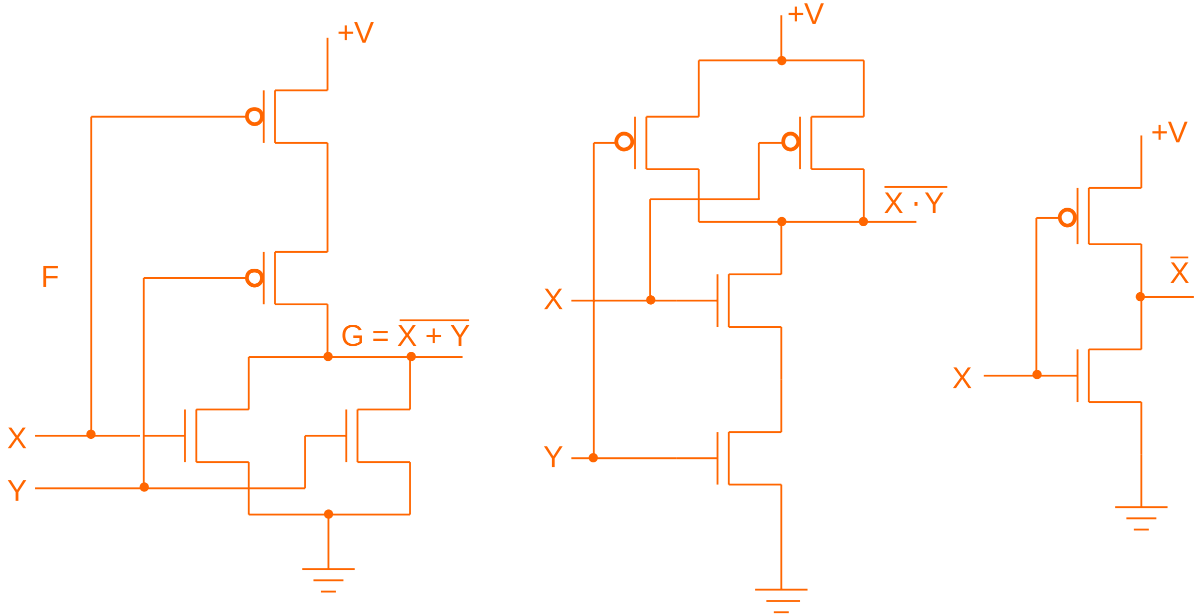
- **Useful model for relay circuits and for CMOS gate circuits, the foundation of current digital logic technology**

Logic Gates

- **In the earliest computers, switches were opened and closed by magnetic fields produced by energizing coils in *relays*. The switches in turn opened and closed the current paths.**
- **Later, *vacuum tubes* that open and close current paths electronically replaced relays.**
- **Today, *transistors* are used as electronic switches that open and close current paths.**

Logic Gates (continued)

- **Implementation of logic gates with transistors (See Reading Supplement – CMOS Circuits)**

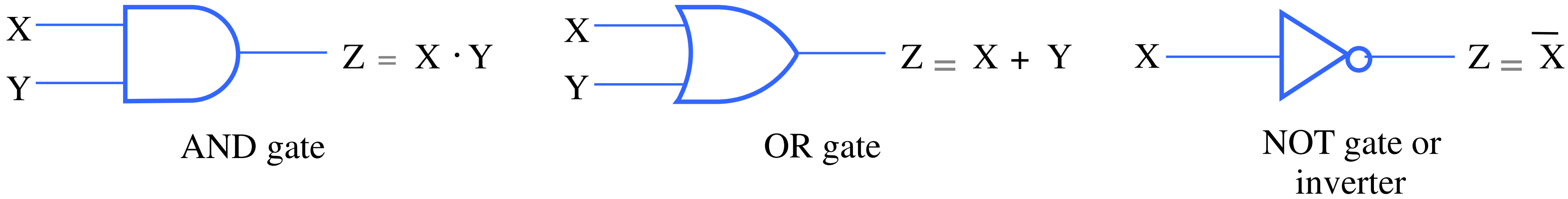


(a) NOR (b) NAND (c) NOT

- **Transistor or tube implementations of logic functions are called <u>logic gates</u> or just <u>gates</u>**
- **Transistor gate circuits can be modeled by switch circuits**

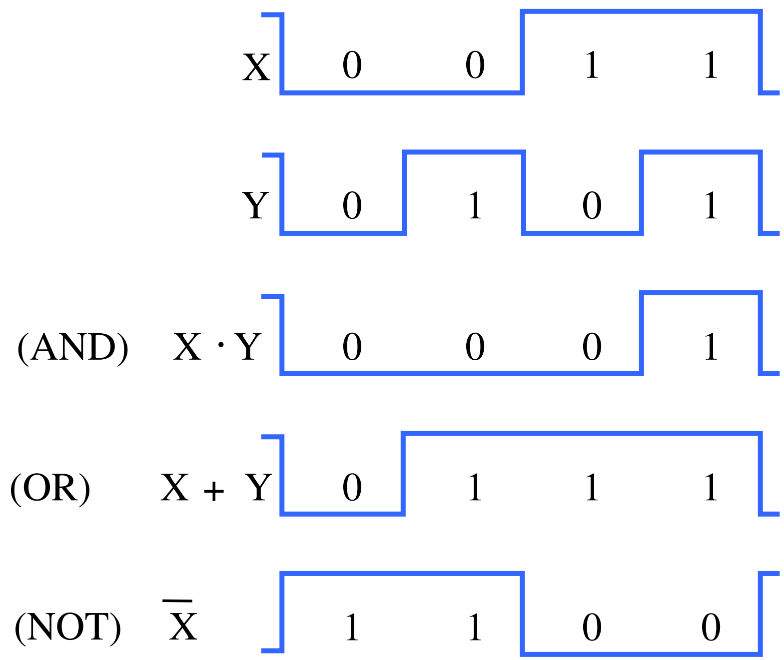
Logic Gate Symbols and Behavior

- **Logic gates have special symbols:**



(a) Graphic symbols

- **And waveform behavior in time as follows:**



(b) Timing diagram

Logic Diagrams and Expressions

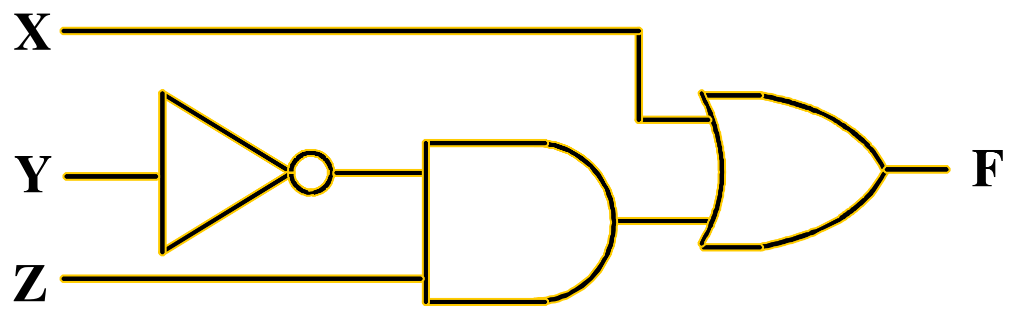
Truth Table

X Y Z	$F = X + \overline{Y} \cdot Z$
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

Equation

$$F = X + \overline{Y}\, Z$$

Logic Diagram



- **Boolean equations, truth tables and logic diagrams describe the same function!**
- **Truth tables are unique; expressions and logic diagrams are not. This gives flexibility in implementing functions.**

Boolean Algebra

- An algebraic structure defined on a set of at least two elements, B, together with three binary operators (denoted $+$, $\cdot$ and $\overline{}$) that satisfies the following basic identities:

1. $X + 0 = X$	2. $X \cdot 1 = X$	Existence of 0 and1
3. $X + 1 = 1$	4. $X \cdot 0 = 0$	
5. $X + X = X$	6. $X \cdot X = X$	Idempotence
7. $X + \overline{X} = 1$	8. $X \cdot \overline{X} = 0$	Existence of complement
9. $\overline{\overline{X}} = X$		Involution
10. $X + Y = Y + X$	11. $XY = YX$	Commutative
12. $(X + Y) + Z = X + (Y + Z)$	13. $(XY)Z = X(YZ)$	Associative
14. $X(Y + Z) = XY + XZ$	15. $X + YZ = (X + Y)(X + Z)$	Distributive
16. $\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17. $\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

Boolean Operator Precedence

- **The order of evaluation in a Boolean expression is:**
  1. Parentheses
  2. NOT
  3. AND
  4. OR
- **Consequence: Parentheses appear around OR expressions**
- **Example:** $F = A(B + C)(C + \overline{D})$

Example 1: Boolean Algebraic Proof

- $A + A \cdot B = A$ (Absorption Theorem)

Proof Steps	Justification (identity or theorem)
$A + A \cdot B$	
$= A \cdot 1 + A \cdot B$	$X = X \cdot 1$
$= A \cdot (1 + B)$	$X \cdot Y + X \cdot Z = X \cdot (Y + Z)$ (Distributive Law)
$= A \cdot 1$	$1 + X = 1$
$= A$	$X \cdot 1 = X$

- **Our primary reason for doing proofs is to learn:**
  - **Careful and efficient use of the identities and theorems of Boolean algebra, and**
  - **How to choose the appropriate identity or theorem to apply to make forward progress, irrespective of the application.**

Example 2: Boolean Algebraic Proofs

- $AB + \overline{A}C + BC = AB + \overline{A}C$ (Consensus Theorem)

Proof Steps: **Justification (identity or theorem)**

$AB + \overline{A}C + BC$

$= AB + \overline{A}C + 1 \cdot BC$

$= AB + \overline{A}C + (A + \overline{A}) \cdot BC$

$= AB + \overline{A}C + ABC + \overline{A}BC$

$= AB\,(1+C) + \overline{A}C\,(1+B)$

$= AB \cdot 1 + \overline{A}C \cdot 1$

$= AB + \overline{A}C$

Example 3: Boolean Algebraic Proofs

- $(\overline{X + Y})Z + X\overline{Y} = \overline{Y}(X + Z)$

Proof Steps **Justification (identity or theorem)**

$$(\overline{X + Y})Z + X\overline{Y}$$

$=$

Useful Theorems

$$x \cdot y + \bar{x} \cdot y = y \qquad (x + y)(\bar{x} + y) = y$$ **Minimization**

$$x + x \cdot y = x \qquad x \cdot (x + y) = x$$ **Absorption**

$$x + \bar{x} \cdot y = x + y \qquad x \cdot (\bar{x} + y) = x \cdot y$$ **Simplification**

$$x \cdot y + \bar{x} \cdot z + y \cdot z = x \cdot y + \bar{x} \cdot z$$ **Consensus**

$$(x + y) \cdot (\bar{x} + z) \cdot (y + z) = (x + y) \cdot (\bar{x} + z)$$

$$\overline{x + y} = \bar{x} \cdot \bar{y} \qquad \overline{x \cdot y} = \bar{x} + \bar{y}$$ **DeMorgan's Laws**

Proof of Simplification

$$\mathbf{x} \cdot \mathbf{y} + \bar{\mathbf{x}} \cdot \mathbf{y} = \mathbf{y}$$

$$(\mathbf{x} + \mathbf{y})(\bar{\mathbf{x}} + \mathbf{y}) = \mathbf{y}$$

Proof of DeMorgan's Laws

$$\overline{x+y} = \bar{x} \cdot \bar{y}$$

$$\overline{x \cdot y} = \bar{x} + \bar{y}$$

Boolean Function Evaluation

$F1 = xy\overline{z}$

$F2 = x + \overline{y}z$

$F3 = \overline{x}\,\overline{y}\,\overline{z} + \overline{x}\,y\,z + x\overline{y}$

$F4 = x\overline{y} + \overline{x}\,z$

x	y	z	F1	F2	F3	F4
0	0	0	0	0	1	0
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0

Expression Simplification

- **An application of Boolean algebra**
- **Simplify to contain the smallest number of <u>literals</u> (complemented and uncomplemented variables):**

$$A\,B + \overline{A}\,C\,D + \overline{A}\,B\,D + \overline{A}\,C\,\overline{D} + A\,B\,C\,D$$

$$= AB + ABCD + \overline{A}\,C\,D + \overline{A}\,C\,\overline{D} + \overline{A}\,B\,D$$

$$= AB + AB(CD) + \overline{A}\,C\,(D + \overline{D}) + \overline{A}\,B\,D$$

$$= AB + \overline{A}\,C + \overline{A}\,B\,D = B(A + \overline{A}\overline{D}) + \overline{A}C$$

$$= B\,(A + D) + \overline{A}\,C \quad \textbf{5 literals}$$

Complementing Functions

- Use DeMorgan's Theorem to complement a function:
  1. Interchange AND and OR operators
  2. Complement each constant value and literal
- Example: Complement $F = \overline{x}y\overline{z} + x\overline{y}\overline{z}$

$$\overline{F} = (x + \overline{y} + z)(\overline{x} + y + z)$$

- Example: Complement $G = (\overline{a} + bc)\overline{d} + e$

$$\overline{G} = ?$$

Overview – Canonical Forms

- What are Canonical Forms?
- Minterms and Maxterms
- Index Representation of Minterms and Maxterms
- Sum-of-Minterm (SOM) Representations
- Product-of-Maxterm (POM) Representations
- Representation of Complements of Functions
- Conversions between Representations

Canonical Forms

- **It is useful to specify Boolean functions in a form that:**
  - **Allows comparison for equality.**
  - **Has a correspondence to the truth tables**
- **Canonical Forms in common usage:**
  - **Sum of Minterms (SOM)**
  - **Product of Maxterms (POM)**

Minterms

- **<u>Minterms</u> are AND terms with every variable present in either true or complemented form.**
- **Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n minterms for n variables.**
- **<u>Example:</u> Two variables (X and Y)produce 2 x 2 = 4 combinations:**

  XY (both normal)
  $X\overline{Y}$ (X normal, Y complemented)
  $\overline{X}Y$ (X complemented, Y normal)
  $\overline{X}\,\overline{Y}$ (both complemented)
- **Thus there are <u>four minterms</u> of two variables.**

Maxterms

- **<u>Maxterms</u> are OR terms with every variable in true or complemented form.**
- **Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n maxterms for *n* variables.**
- **<u>Example:</u> Two variables (X and Y) produce 2 x 2 = 4 combinations:**

$X+Y$	(both normal)
$X+\overline{Y}$	(X normal, Y complemented)
$\overline{X}+Y$	(X complemented, Y normal)
$\overline{X}+\overline{Y}$	(both complemented)

Maxterms and Minterms

- **Examples: Two variable minterms and maxterms.**

Index	Minterm	Maxterm
0	$\bar{x}\,\bar{y}$	$x + y$
1	$\bar{x}\,y$	$x + \bar{y}$
2	$x\,\bar{y}$	$\bar{x} + y$
3	$x\,y$	$\bar{x} + \bar{y}$

- **The index above is important for describing which variables in the terms are true and which are complemented.**

Standard Order

- **Minterms and maxterms are designated with a subscript**
- **The subscript is a number, corresponding to a binary pattern**
- **The bits in the pattern represent the complemented or normal state of each variable listed in a standard order.**
- **All variables will be present in a minterm or maxterm and will be listed in the same order (usually alphabetically)**
- **Example: For variables a, b, c:**
  - **Maxterms: $(a + b + \bar{c})$, $(a + b + c)$**
  - **Terms: $(b + a + c)$, $a\,\bar{c}\,b$, and $(c + b + a)$ are NOT in standard order.**
  - **Minterms: $a\,\bar{b}\,c$, $a\,b\,c$, $\bar{a}\,\bar{b}\,c$**
  - **Terms: $(a + c)$, $\bar{b}\,c$, and $(\bar{a} + b)$ do not contain all variables**

Purpose of the Index

- **The <u>index</u> for the minterm or maxterm, expressed as a binary number, is used to determine whether the variable is shown in the true form or complemented form.**
- **For Minterms:**
  - **"1" means the variable is "Not Complemented" and**
  - **"0" means the variable is "Complemented".**
- **For Maxterms:**
  - **"0" means the variable is "Not Complemented" and**
  - **"1" means the variable is "Complemented".**

Index Example in Three Variables

- **Example: (for three variables)**
- **Assume the variables are called X, Y, and Z.**
- **The standard order is X, then Y, then Z.**
- **The <u>Index 0</u> (base 10) = 000 (base 2) for three variables). All three variables are complemented for <u>minterm 0</u> ($\overline{X}, \overline{Y}, \overline{Z}$) and no variables are complemented for <u>Maxterm 0</u> (X,Y,Z).**
  - **Minterm 0, called m_0 is $\overline{X}\overline{Y}\overline{Z}$.**
  - **Maxterm 0, called M_0 is $(X + Y + Z)$.**
  - **Minterm 6 ?**
  - **Maxterm 6 ?**

Index Examples – Four Variables

Index i	Binary Pattern	Minterm m_i	Maxterm M_i
0	0000	$\bar{a}\bar{b}\bar{c}\bar{d}$	$a+b+c+d$
1	0001	$\bar{a}\bar{b}\bar{c}d$	?
3	0011	?	$a+b+\bar{c}+\bar{d}$
5	0101	$\bar{a}b\bar{c}d$	$a+\bar{b}+c+\bar{d}$
7	0111	?	$a+\bar{b}+\bar{c}+\bar{d}$
10	1010	$a\bar{b}c\bar{d}$	$\bar{a}+b+\bar{c}+d$
13	1101	$ab\bar{c}d$	?
15	1111	$abcd$	$\bar{a}+\bar{b}+\bar{c}+\bar{d}$

Minterm and Maxterm Relationship

- **Review: DeMorgan's Theorem**
  $\overline{x \cdot y} = \overline{x} + \overline{y}$ **and** $\overline{x + y} = \overline{x} \cdot \overline{y}$
- **Two-variable example:**
  $M_2 = \overline{x} + y$ **and** $m_2 = x \cdot \overline{y}$
  Thus M_2 is the complement of m_2 and vice-versa.
- **Since DeMorgan's Theorem holds for *n* variables, the above holds for terms of *n* variables**
- **giving:**

$$M_i = \overline{m_i} \text{ and } m_i = \overline{M_i}$$

Thus M_i is the complement of m_i.

Function Tables for Both

- **Minterms of 2 variables**

x y	m_0	m_1	m_2	m_3
0 0	1	0	0	0
0 1	0	1	0	0
1 0	0	0	1	0
1 1	0	0	0	1

Maxterms of 2 variables

x y	M_0	M_1	M_2	M_3
0 0	0	1	1	1
0 1	1	0	1	1
1 0	1	1	0	1
1 1	1	1	1	0

- **Each column in the maxterm function table is the complement of the column in the minterm function table since M_i is the complement of m_i.**

Observations

- **In the function tables:**
  - **Each <u>min</u>term has one and only one 1 present in the 2^n terms (a <u>minimum</u> of 1s). All other entries are 0.**
  - **Each <u>max</u>term has one and only one 0 present in the 2^n terms All other entries are 1 (a <u>max</u>imum of 1s).**
- **We can implement any function by "ORing" the minterms corresponding to "1" entries in the function table. These are called the minterms of the function.**
- **We can implement any function by "ANDing" the maxterms corresponding to "0" entries in the function table. These are called the maxterms of the function.**
- **This gives us two <u>canonical forms</u>:**
  - **<u>Sum of Minterms (SOM)</u>**
  - **<u>Product of Maxterms (POM)</u>**

for stating any Boolean function.

Minterm Function Example

- **Example: Find $F_1 = m_1 + m_4 + m_7$**
- **$F1 = \overline{x}\ \overline{y}\ z + x\ \overline{y}\ \overline{z} + x\ y\ z$**

x y z	index	m_1	+	m_4	+	m_7	$= F_1$
0 0 0	0	0	+	0	+	0	= 0
0 0 1	1	1	+	0	+	0	= 1
0 1 0	2	0	+	0	+	0	= 0
0 1 1	3	0	+	0	+	0	= 0
1 0 0	4	0	+	1	+	0	= 1
1 0 1	5	0	+	0	+	0	= 0
1 1 0	6	0	+	0	+	0	= 0
1 1 1	7	0	+	0	+	1	= 1

Minterm Function Example

- $F(A, B, C, D, E) = m_2 + m_9 + m_{17} + m_{23}$
- $F(A, B, C, D, E) =$

Maxterm Function Example

- **Example: Implement F1 in maxterms:**

$$F_1 = M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$$

$$F_1 = (x + y + z) \cdot (x + \overline{y} + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + z)$$

x y z	i	$M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$ = F1
0 0 0	0	$0 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 0$
0 0 1	1	$1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$
0 1 0	2	$1 \cdot 0 \cdot 1 \cdot 1 \cdot 1 = 0$
0 1 1	3	$1 \cdot 1 \cdot 0 \cdot 1 \cdot 1 = 0$
1 0 0	4	$1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$
1 0 1	5	$1 \cdot 1 \cdot 1 \cdot 0 \cdot 1 = 0$
1 1 0	6	$1 \cdot 1 \cdot 1 \cdot 1 \cdot 0 = 0$
1 1 1	7	$1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$

Maxterm Function Example

- $\mathbf{F(A,B,C,D) = M_3 \cdot M_8 \cdot M_{11} \cdot M_{14}}$
- **F(A, B,C,D) =**

Canonical Sum of Minterms

- **Any Boolean function can be expressed as a <u>Sum of Minterms</u>.**
  - For the function table, the <u>minterms</u> used are the terms corresponding to the 1's
  - For expressions, <u>expand</u> all terms first to explicitly list all minterms. Do this by "ANDing" any term missing a variable v with a term $(v + \bar{v})$.
- **Example: Implement $f = x + \bar{x}\,\bar{y}$ as a sum of minterms.**

  First expand terms: $f = x(y + \bar{y}) + \bar{x}\,\bar{y}$

  Then distribute terms: $f = xy + x\bar{y} + \bar{x}\,\bar{y}$

  Express as sum of minterms: $f = m_3 + m_2 + m_0$

Another SOM Example

- Example: $F = A + \overline{B}\,C$
- There are three variables, A, B, and C which we take to be the standard order.
- Expanding the terms with missing variables:

- Collect terms (removing all but one of duplicate terms):
- Express as SOM:

Shorthand SOM Form

- **From the previous example, we started with:**

$$F = A + \overline{B}\,C$$

- **We ended up with:**

$$F = m_1 + m_4 + m_5 + m_6 + m_7$$

- **This can be denoted in the formal shorthand:**

$$F(A,B,C) = \Sigma m(1,4,5,6,7)$$

- **Note that we explicitly show the standard variables in order and drop the "m" designators.**

Canonical Product of Maxterms

- **Any Boolean Function can be expressed as a <u>Product of Maxterms (POM)</u>.**
  - **For the function table, the maxterms used are the terms corresponding to the 0's.**
  - **For an expression, expand all terms first to explicitly list all maxterms. Do this by first applying the second distributive law , "ORing" terms missing variable V with a term equal to $V \cdot \overline{V}$ and then applying the distributive law again.**
- **Example: Convert to product of maxterms:**

$$f(x, y, z) = x + \overline{x}\,\overline{y}$$

Apply the distributive law:

$$x + \overline{x}\,\overline{y} = (x + \overline{x})(x + \overline{y}) = 1 \cdot (x + \overline{y}) = x + \overline{y}$$

Add missing variable z:

$$x + \overline{y} + z \cdot \overline{z} = (x + \overline{y} + z)(x + \overline{y} + \overline{z})$$

Express as POM: $f = M_2 \cdot M_3$

Another POM Example

- **Convert to Product of Maxterms:**

$$f(A,B,C) = A\,\overline{C} + B\,C + \overline{A}\,\overline{B}$$

- **Use $x + y\,z = (x+y)\cdot(x+z)$ with $x = (A\,\overline{C} + B\,C)$, $y = \overline{A}$, and $z = \overline{B}$ to get:**

$$f = (A\,\overline{C} + B\,C + \overline{A})(A\,\overline{C} + B\,C + \overline{B})$$

- **Then use $x + \overline{x}\,y = x + y$ to get:**

$$f = (\overline{C} + BC + \overline{A})(A\overline{C} + C + \overline{B})$$

and a second time to get:

$$f = (\overline{C} + B + \overline{A})(A + C + \overline{B})$$

- **Rearrange to standard order,**

$$f = (\overline{A} + B + \overline{C})(A + \overline{B} + C)$$ **to give $f = M_5 \cdot M_2$**

Function Complements

- **The complement of a function expressed as a sum of minterms is constructed by selecting the minterms missing in the sum-of-minterms canonical forms.**
- **Alternatively, the complement of a function expressed by a Sum of Minterms form is simply the Product of Maxterms with the same indices.**
- **Example: Given** $F(x, y, z) = \Sigma_m(1,3,5,7)$

$$\bar{F}(x, y, z) = \Sigma_m(0,2,4,6)$$

$$\bar{F}(x, y, z) = \Pi_M(1,3,5,7)$$

Conversion Between Forms

- To convert between sum-of-minterms and product-of-maxterms form (or vice-versa) we follow these steps:
  - Find the function complement by swapping terms in the list with terms not in the list.
  - Change from products to sums, or vice versa.
- Example: Given F as before: $F(x, y, z) = \Sigma_m(1,3,5,7)$
- Form the Complement: $\overline{F}(x, y, z) = \Sigma_m(0,2,4,6)$
- Then use the other form with the same indices – this forms the complement again, giving the other form of the original function: $F(x, y, z) = \Pi_M(0,2,4,6)$

Standard Forms

- **Standard Sum-of-Products (SOP) form:** equations are written as an OR of AND terms
- **Standard Product-of-Sums (POS) form:** equations are written as an AND of OR terms
- **Examples:**
  - SOP: $A\,B\,C + \overline{A}\,\overline{B}\,C + B$
  - POS: $(A + B)\cdot(A + \overline{B} + \overline{C})\cdot C$
- These "mixed" forms are **neither SOP nor POS**
  - $(A\,B + C)\,(A + C)$
  - $A\,B\,\overline{C} + A\,C\,(A + B)$

Standard Sum-of-Products (SOP)

- **A sum of minterms form for *n* variables can be written down directly from a truth table.**
  - **Implementation of this form is a two-level network of gates such that:**
  - **The first level consists of *n*-input AND gates, and**
  - **The second level is a single OR gate (with fewer than 2^n inputs).**
- **This form often can be simplified so that the corresponding circuit is simpler.**

Standard Sum-of-Products (SOP)

- **A Simplification Example:**
- $F(A,B,C) = \Sigma m(1,4,5,6,7)$
- **Writing the minterm expression:**

$$F = \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,\overline{B}\,C + AB\overline{C} + ABC$$

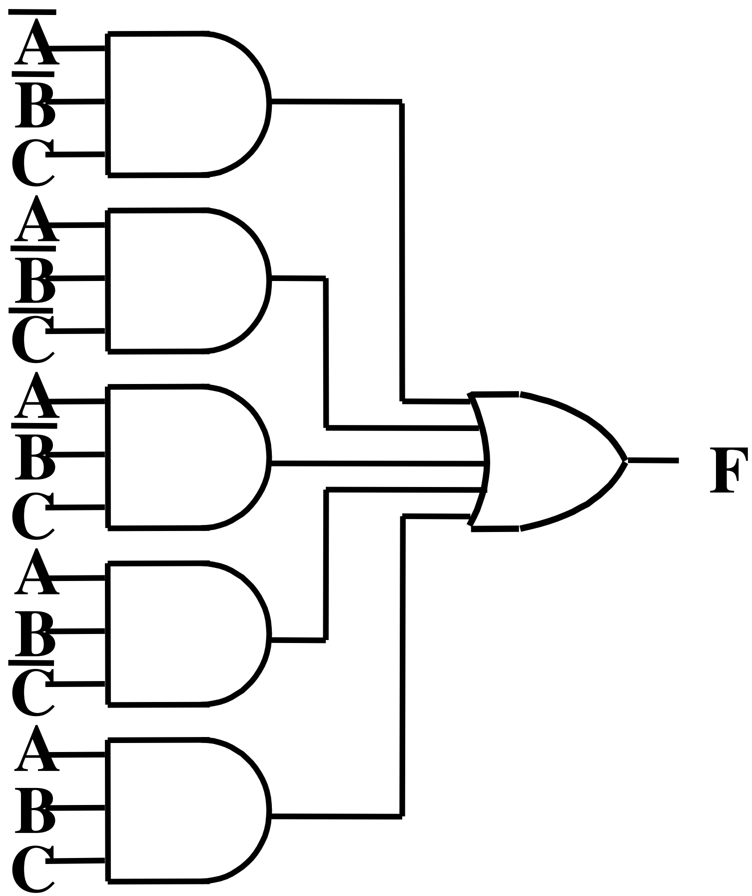
- **Simplifying:**

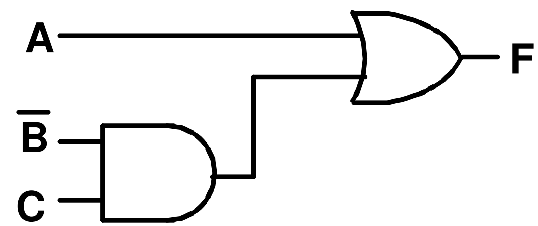
$$F = A + \overline{B}C$$

- **Simplified F contains 3 literals compared to 15 in minterm F**

AND/OR Two-level Implementation of SOP Expression

- **The two implementations for F are shown below – it is quite apparent which is simpler!**





- **The previous examples show that:**
  - Canonical Forms (Sum-of-minterms, Product-of-Maxterms), or other standard forms (SOP, POS) differ in complexity
  - Boolean algebra can be used to manipulate equations into simpler forms.
  - Simpler equations lead to simpler two-level implementations
- **Questions:**
  - How can we attain a “simplest” expression?
  - Is there only one minimum cost circuit?
  - The next part will deal with these issues.