

radio receiver radyo sinyali  $\rightarrow$  ses

audio amplifier ses  $\rightarrow$  ses

microphone ses  $\rightarrow$  elektrisel sinyal

Signal in  $\rightarrow$  **system**  $\rightarrow$  signal out

\* Sistemler girdiyi

çıkıya döndürd. için

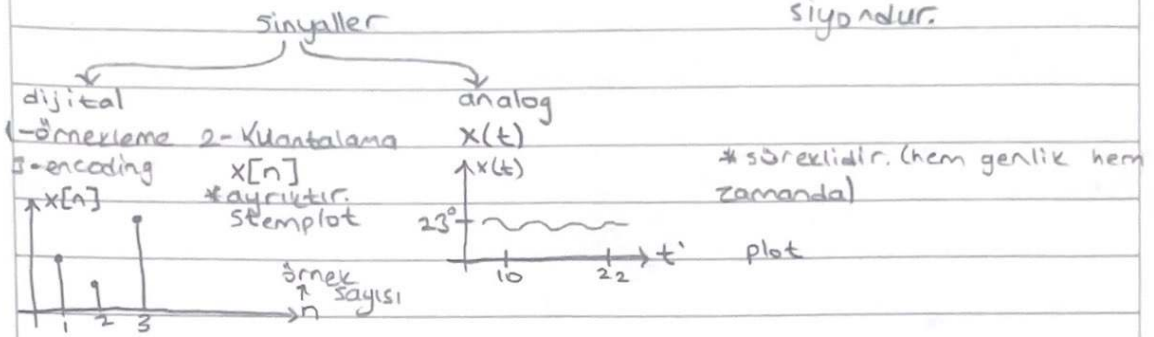
fonk. gibi davranır.

\* Sinyal herhangi bir

fiziksel olay hakkında

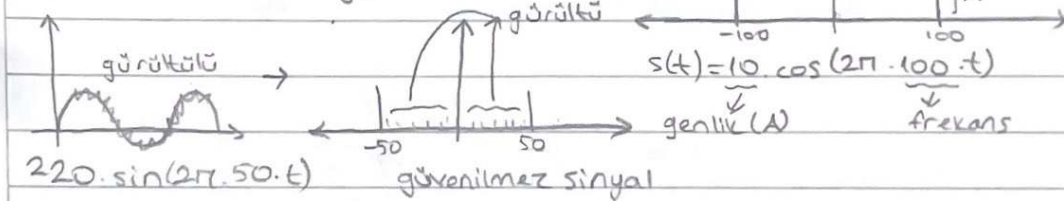
büyüklik taşıyan fonk.

siyondur.

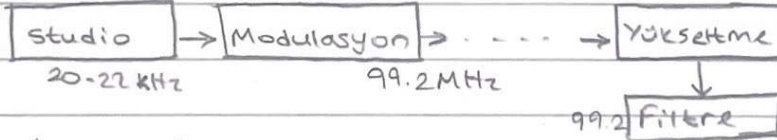


**Frequency Domain:** Herhangi bir sinyali sinüzoid sinyallerin toplamı olarak yazabiliriz.

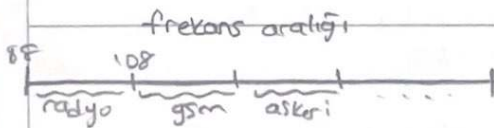
Frekans: Sinyalin içindeki oranlarla ilgilendir. (limonata örneği)



A.M. / F.M.



$\lambda = \frac{c}{f}$  \* frekans artarsa dalga boyu azalır ve daha küçük antenlerle yakalanabilir.



\* Gösterim basit ve kolay sistemin karakteristiğini analiz etmeye yarar.

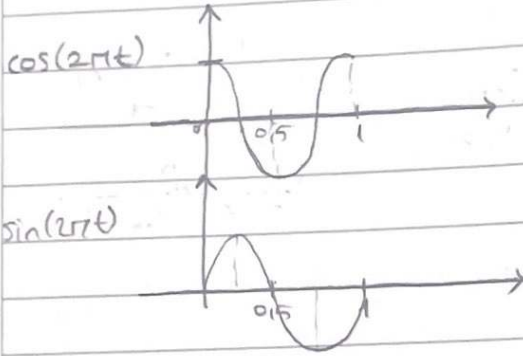
## Sinyallerin Sınıflandırılması:

- 1-Digital - Analog
- 2-Discrete - Continuous
- 3-Periyodik - Aperiodyik
- 4-causal, Anti-causal non-causal
- 5-odd-even signals
- 6-Deterministic random
- 7-Enerji-güç

## Discrete - Continuous:

$$x(t) = A \cdot \sin(2\pi f \cdot t + \phi)$$

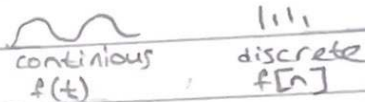
$\downarrow$  genlik       $\downarrow$   $\omega = \text{angular frek. (radyan/s)}$        $\downarrow$   $\phi = \text{phase angle (radyan)}$   
 $\downarrow$   $f(\text{Hz})$        $\downarrow$   $\text{frekans}$        $\downarrow$   $\text{eksende ötelenme miktarı}$



\*  $\cos$  0.25 sn sağa kaydırılarak  $\sin$  elde edilir. ( $0.25 \text{ s} = \pi/2$ ) ( $2\pi \times 0.25 \text{ s}$ )

$$\cos(2\pi t - \pi/2) = \sin(2\pi t)$$

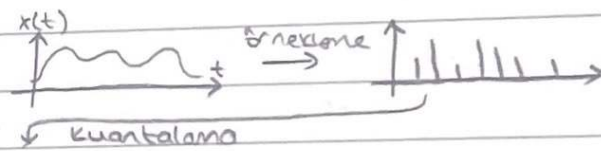
$\sin, \cos \rightarrow \text{sinüzoidal signal}$



\* Discrete örneklenmiş sinyaldir.

## Analog - Digital:

- \* Hem genlikte hem zamanda continuous olan analog sinyaldir.
- \* Örnekleme zamanda discrete olmasını sağlar. Kuantalama genlikte discrete olmasını sağlar.



## Periyodik - Aperiodyik:

- \* Kendini T zaman sonra tekrar eden sinyal periyodiktir.

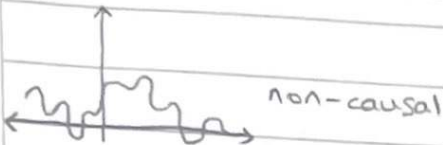
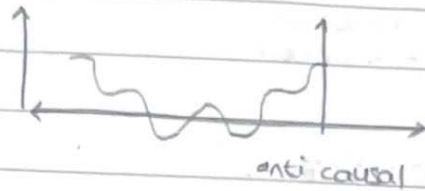
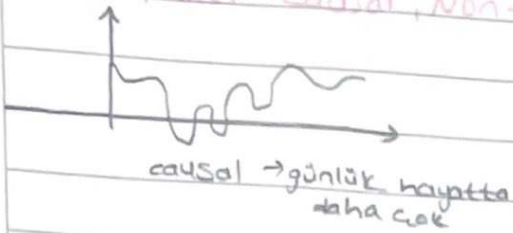
$$f(t) = 10 \cdot \sin(2\pi \cdot 10 \cdot t) \quad T = 1/10$$

$$f(t) = f(t+T) \quad 10 \cdot \sin(2\pi \cdot 10 \cdot t) = 10 \cdot \sin(2\pi \cdot 10 \cdot (t + 1/10))$$

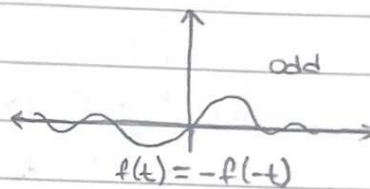
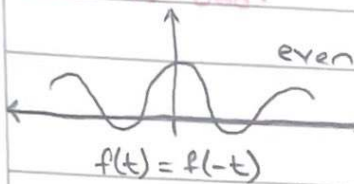
$$\sin(20t) = \sin(20t + 2\pi) = \sin(20t)$$

\* Periyodu yüksek olanın periyodu ortak periyod kabul edilir.

## Causal, Anti-Causal, Non-Causal:



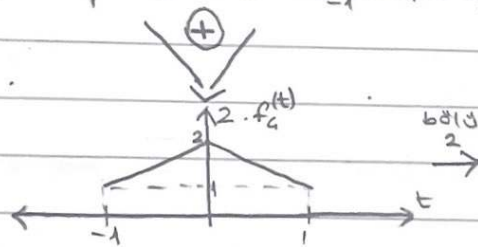
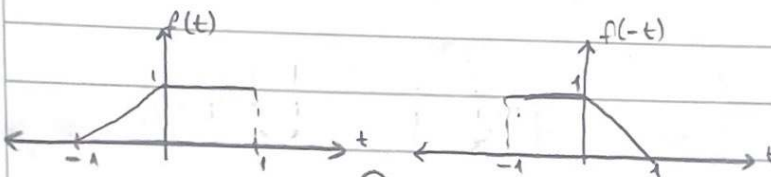
## Even vs. Odd:



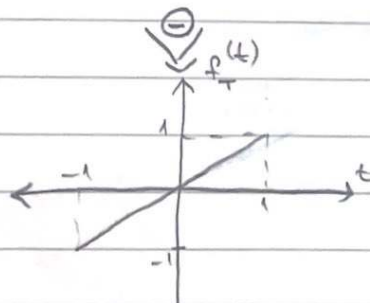
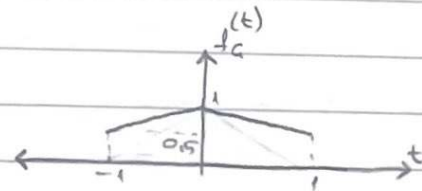
$$f(t) = f_e(t) + f_o(t)$$

$$f_e(t) = \frac{f(t) + f(-t)}{2}$$

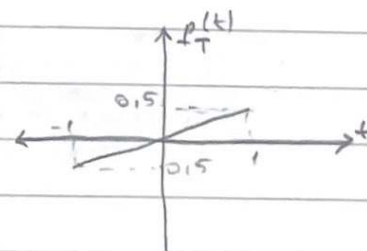
$$f_o(t) = \frac{f(t) - f(-t)}{2}$$



bölü 2



bölü 2

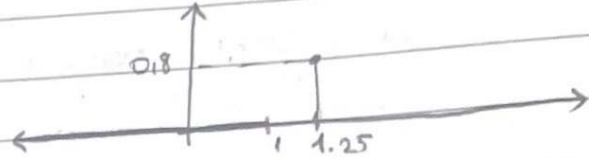


## Deterministic Random:

\*Çıkışı bir fonksiyona bağlı sinyal deterministiktir.

\*Deterministic sinyale gürültü binerse random sinyal olur.

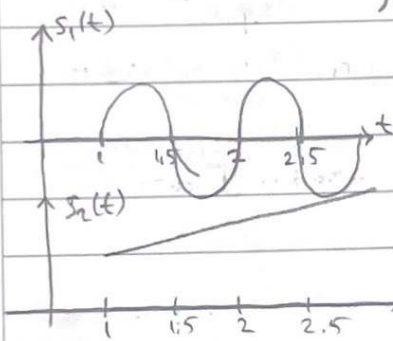
$$f(t) = \begin{cases} \sin(2\pi t)/t & t > 1 \\ 0 & t < 1 \end{cases}$$



$$f(1.25) = \frac{\sin(2 \cdot 5\pi)}{1.25} = 0.8$$

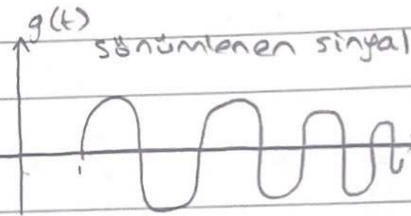
bu şekilde ifade edilebilir.

$$g(t) = \frac{\sin(2\pi t)}{t}, t > 1 \quad \left\{ \quad g(t) = \frac{s_1(t)}{s_2(t)}, t > 1 \right.$$



$$s_1(t) = \sin(2\pi t), t > 1$$

$$s_2(t) = t, t > 1$$

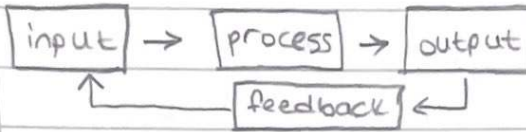




amaç gerçekleştirmek

**Sistem:** Genel olarak tanımlı bir nesneyi oluşturmak için bir araya getirilmiş elementler olarak tanımlanabilir. Ama tanımı kullanım alanına göre değişir. Budaş için;

**Sistem:** multiple parts working together for a common purpose.  
 - input, process, outputs vardır. - model kullanılarak açıklanır.  
 - feedback önemlidir.



**Sinyal:** Fiziksel miktarla ifade edilen, bilgi saklayan değişkenlerdir.

**Transducers:** Enerjiyi bir formdan başka forma çeviren cihazdır. Sinyal işlemede enerjiyi değil bilgi transferinde kullanılır.

input transducer } non-electrical signal → electrical signal } output da tam tersini yapar  
 transducer → sensor (input)  
 ↳ actuator (output)

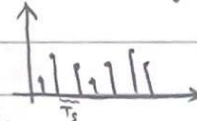
**Analog → Dijital Dönüşümü:**

- 1- Filtering
- 2- Sampling
- 3- Quantization
- 4- Binary encoding

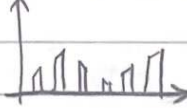
**Filtering:** Örneklem oranında etki edecek max frekans sınırı koymak için yapılır.

**Sampling:** Eşit zaman aralıklarıyla alınmış, Sinyali temsil eden bloşım sonuçlarıdır.

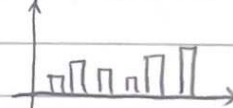
Her  $T_s$  saniyede örneklenir.  $f_s$  → Örnekleme frekansı



ideal sampling



natural sampling



flat-top sampling

\*Örnekleme yaparken küçük ve hızlı değişimleri kayırabiliriz, zaman belirsizliği yaşayabiliriz.

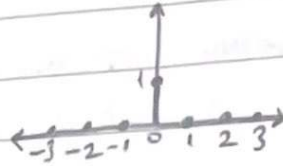
**NOT:** According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal

$$f_s \geq 2f_m$$

06/03/2024

## Sinyaller ve Sistemler

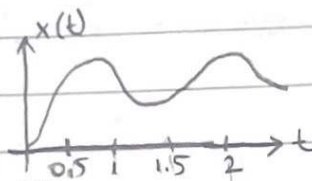
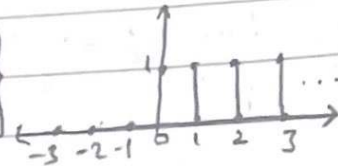
$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & \text{diğer} \end{cases}$$



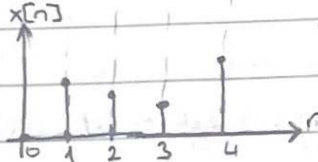
\*n temel discrete timesinyali dir. Bütün fonk-lar (discrete) An cinsinden ifade edilebilir.

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{diğer} \end{cases}$$

(unitstep)



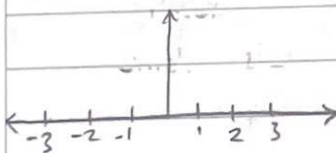
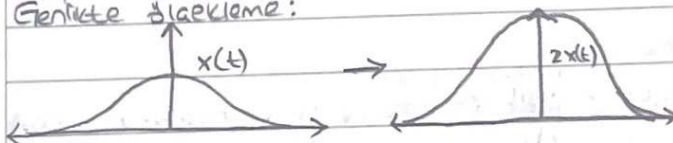
NOT:  $x[n] = x(n \cdot T_s)$



$T_s = 0.5 \rightarrow x[0] = x(0)$   
 $x[1] = x(T_s)$   
 $x[2] = x(2T_s)$   
 $x[3] = x(3T_s)$   
 $x[4] = x(4T_s)$

## Sinyallerde Operasyonlar:

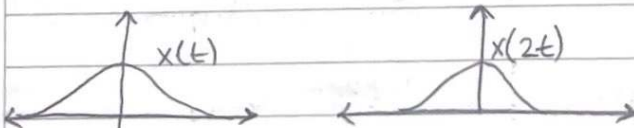
(Amplitude Scaling)  
 Genlikte Ölçme:



Zamanda Ölçme: Verilmiş  $x[n]$  veya  $x(t)$ 'den  $z[n]$  veya  $z(t)$

elde edilir.  $x(bt) = z(t)$   $0 < b < 1$  veya  $b > 1$

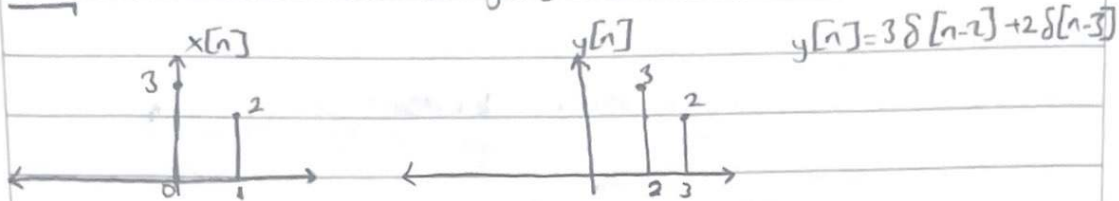
upsampling, interpolation      downsampling, decimation



Time Reversal:  $t$ 'yi  $x(t)$  eksenine göre aynılamaktır. ( $t \rightarrow -t$ )

Time shift: Sinyali  $t$  ekseninde öteleme.  $x(t-t_1) \rightarrow$  sağa öteleme  
 $x(t+t_2) \rightarrow$  sola öteleme  
 Sağa ötelenmiş sinyal gecikmiş sinyal, sola ötelenmiş sinyal önde sinyaldir.

Ör:  $x[n] = 3\delta[n] + 2\delta[n-1]$   $y[n] = x[n-2]$  çiziniz.



Periyodik Sinyaller:

$x(t) = x(t_0 + t)$  olan  $T_0 > 0$  için  $x[n+N_0] = x[n]$ , en küçük  $N_0$  için periyodiktir.

Periodic Extension:

\* Periyodik olmayan bir sinyali periyodik hale getirmektir.

Complex Signals:

\* Tek sinyal içinde 2 farklı sinyal göndermek istediğimizde kullanılır.

$z = x + jy$   
 $|z| = \sqrt{x^2 + y^2}$   
 $\theta = \tan^{-1}(y/x)$   
 $z = r \cdot e^{j\theta} \rightarrow$  polar gösterim  
 Euler denk.  $\rightarrow e^{j\theta} = \cos\theta + j\sin\theta$   
 $r(\cos\theta + j\sin\theta) \rightarrow z = \underbrace{r\cos\theta}_x + j\underbrace{r\sin\theta}_y$

Signal Energy and Power:

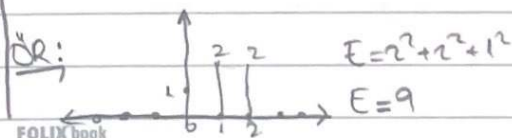
$E_x = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$  } enerji denklemi  
 $P = i^2 R$   $R = 1\Omega$  alınır.  
 $E = Pt$

Ör:  $x(t) = \begin{cases} 0, & t < 0 \\ e^{-t}, & t \geq 0 \end{cases}$

$E = \int_0^{\infty} |e^{-t}|^2 dt = \frac{1}{2}$   
 $= \left. \frac{e^{-2t}}{-2} \right|_0^{\infty} = \frac{0}{-2} - \frac{1}{-2} = \frac{1}{2}$

$P_x = \lim_{T \rightarrow \infty} \frac{E}{T}$

$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$  } Ayırık zamanda enerji formülü



Complex Sinusoids:

$$A \cdot e^{j(\omega t + \theta)} = A [\cos(\omega t + \theta) + j \sin(\omega t + \theta)]$$

$$e^{j\theta} = \cos \theta + j \sin \theta$$

complex sinusoid

$$x(t) = A \cos(2\pi f t + \theta) \quad | \quad x(t) = \cos(\omega t + \theta)$$

Damped or Growing Sinusoids:

↳ gittikçe azalan

↳ gittikçe artan

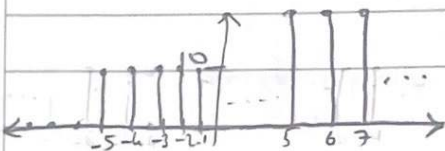
\* exponential signal 'in sin ya da cos ile carpımı ile olur.

$$x(t) = e^{\sigma t} \cos(\omega t + \theta)$$

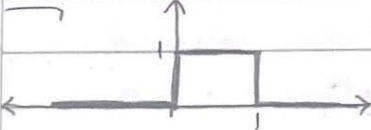
Unit Step Signal:

$$g[n] = 10u[n-5] + 10u[n+5]$$

$$y[n] = \begin{cases} 10, & -5 \leq n \leq 5 \\ 20, & n \geq 5 \\ 0, & \text{diğer} \end{cases}$$



$$g_r: u(t) - u(t-1)$$

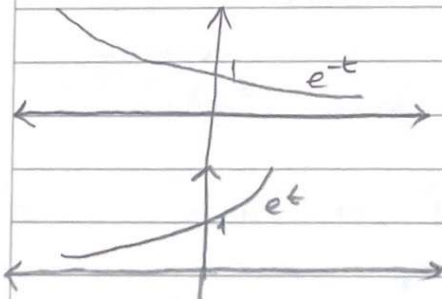


Unit Ramp:

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases} = \int_{-\infty}^t \underbrace{u(\tau)}_{\text{unit step}} d\tau$$

Exponential Signals:

$$x(t) = e^{\sigma t} \quad \begin{matrix} \sigma < 0 \text{ ise azalan} \\ \sigma > 0 \text{ ise artan} \end{matrix}$$



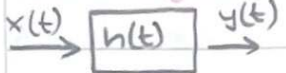
Unit Triangle

$$\Delta(t) = \begin{cases} 1-t, & 0 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$$



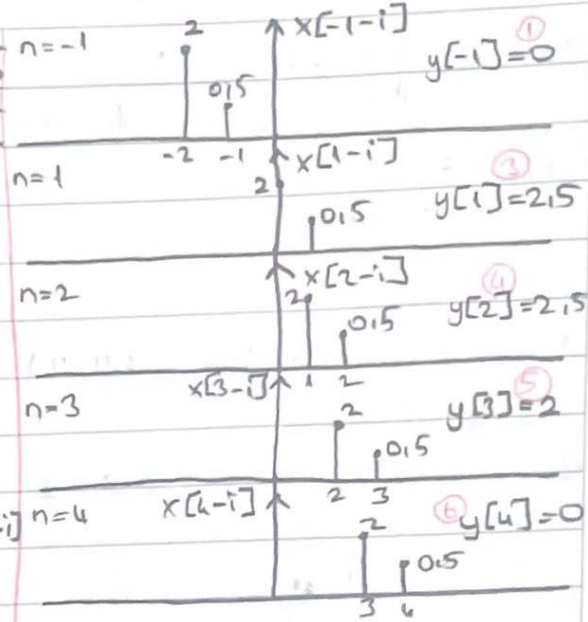


## Korrelasyon



konv. işareti değil  
 $y(t) = h(t) * x(t)$   
 $= x(t) * h(t)$   
 $= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$

\* girişini ve işle-  
 yişini bildiğimiz  
 sistemlerin çıktı-  
 larını bulabiliriz.

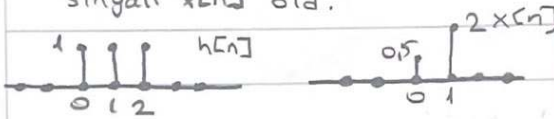


Ayrık Zamanlıda:

$x[n] \rightarrow [h[n]] \rightarrow y[n] = x[n] * h[n]$   
 $y[n] = \sum_{i=-\infty}^{\infty} x[i] \cdot h[n-i]$

\* Soruda lineer zamanda değişmeyen sistem demeli.

ÖR: Birim çıktı yanıtı  $h[n]$  ve giriş sinyali  $x[n]$  old.



Pratik Yol:

\* Sabit tuttuğumuz  $n=0$ 'den itibaren yazılır.

a) işaretlerin mat. ifadesini yazınız.

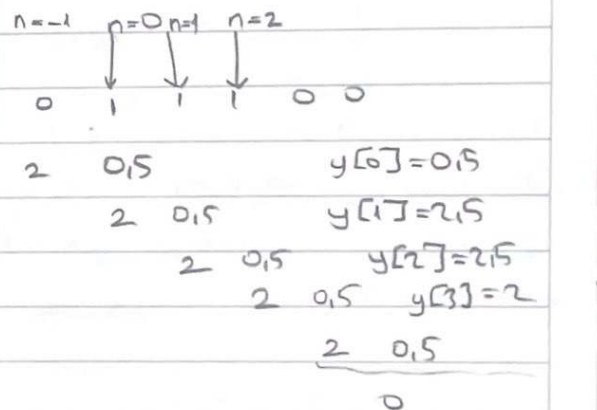
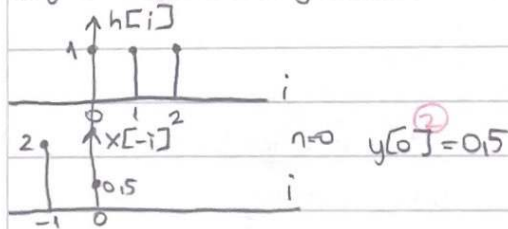
$x[n] = 0.5\delta[n] + 2\delta[n-1]$   
 $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

b)  $y[n]$  hesaplayınız.

①  $h[i]$ ,  $x[-i] \rightarrow$  kısa olan kaydırılır.

②  $n \rightarrow x[n-i] \rightarrow n$  kadar ötelenir.

③  $y[n] = \sum x[i] \cdot h[n-i] \rightarrow y$  bulunur.



ÖR:  $x(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & \text{aksi halde} \end{cases}$   $h(t) = \begin{cases} t, & 0 \leq t \leq T \\ 0, & \text{aksi halde} \end{cases}$  Dörtü Yanıtının Nedenselliği

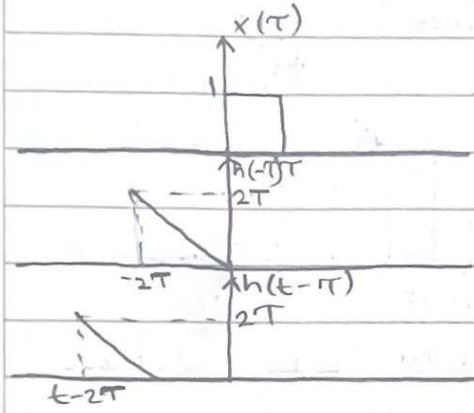
$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

①  $x(\tau)$ ,  $h(-\tau)$

②  $h$ 'in alt indislerini bairde.  $h(t-\tau)$

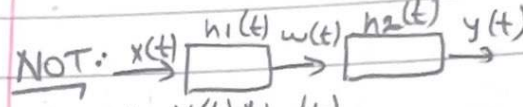
③ bölgeleri belirle

④ her bölge için integral hesabı yap.



$\int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$   
 $\hookrightarrow t-\tau \geq 0, h=0$   
 $t-\tau \leq T, h=t-\tau$

$\int_{-\infty}^t x(\tau) h(t-\tau) d\tau$  nedensellikte bu formül kullanılır.

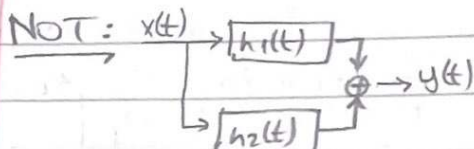


$$w(t) = x(t) * h_1(t)$$

$$y(t) = w(t) * h_2(t)$$

$$= (x(t) * h_1(t)) * h_2(t)$$

$$= x(t) * \underbrace{(h_1(t) * h_2(t))}_{h_T(t)}$$



$$x(t) * (h_1(t) + h_2(t)) = y(t)$$

Bölgeler:

B1 - hiç kesismeme:  $-\infty < t < 0, y(t) = 0$

B2 - kısmen kesisme:  $0 \leq t < T$

B3 - tam " :  $T \leq t < 2T$

B4 - kısmi " :  $2T \leq t < 3T$

B5 - hiç kesismeme:  $t \geq 3T, y(t) = 0$

NOT: Yukarıdaki özellikler ayrık zamanda da geçerlidir.

$$\text{NOT: } x[n] * \delta[n] = x[n]$$

$$x(t) * \delta(t) = x(t)$$

$$\text{NOT: } x(t) * \delta(t-a) = x(t-a)$$

$$x[n] * \delta[n-a] = x[n-a]$$

$$\text{ÖR: } x(t) = \cos(2\pi 100t)$$

$$h(t) = 0,5 \delta(t-2) + 0,5 \delta(t+2)$$

$$y(t) = ?$$

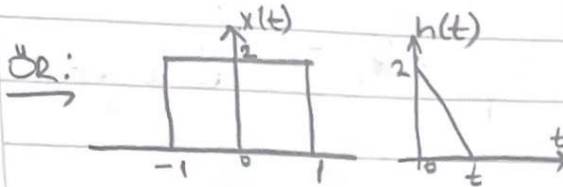
$$\cos(2\pi 100t) * 0,5 \delta(t-2) + \cos(2\pi 100t) * 0,5 \delta(t+2)$$

$$\text{FOLIX book } 0,5 \cos(2\pi 100(t-2)) + 0,5 \cos(2\pi 100(t+2))$$

Ör:  $x[n] = 2\delta[n] + 3\delta[n-1]$

$h[n] = \delta[n-2]$   $y[n] = ?$

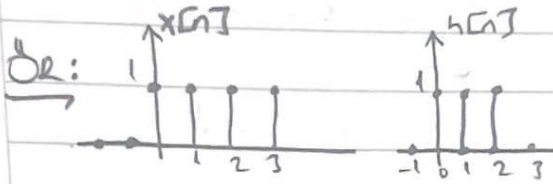
$y[n] = 2\delta[n-2] + 3\delta[n-3]$



\* Analog sistemler sabit katsayılı diferansiyel denklemler ile ifade edilir.

$$\frac{d^k y(t)}{dt^k} = D^k y(t)$$

zero state: 0 anındaki değer  
zero input: homojen denklemin kuma ya yazar.



Ör:  $\frac{d^2 y(t)}{dt^2} - 3\frac{dy(t)}{dt} + 2y(t) = \frac{d^2 f(t)}{dt^2}$

IC ver:  $y(0) = 0, y'(0) = -5$

NOT: Ölevde 1. soruda n ve m fonksiyonun 0 noktasının indis değerinin kaçınıcı indiste olduğunu tutar.

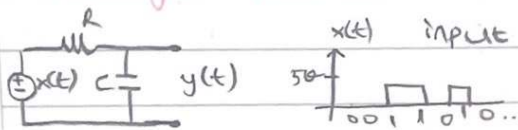
$D^2 y(t) + 3D y(t) + 2y(t) = D^2 f(t)$

$y(t) [D^2 + 3D + 2] = D^2 f(t)$

Karakteristik denklemin zero input:

$\lambda^2 + 3\lambda + 2 = 0 \Rightarrow (\lambda + 2)(\lambda + 1) = 0$   
 $\lambda_1 = -2, \lambda_2 = -1$   
karakt. kök

Diferansiyel Denklemler:



$y(t) = y_{zi}(t) + y_{zs}(t)$

$x(t) = v_R(t) + y(t)$

$v_R(t) = Ri(t)$   $i(t) = C \frac{dy(t)}{dt}$

$y_{zi}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$   
 $= C_1 e^{-2t} + C_2 e^{-t}$

başlangıç koşulları:  $C_1 + C_2 = 0$   $+C_1 + 2C_2 = +5$

$C_2 = 5$   $C_1 = -5$

$y_{zi}(t) = -5e^{-2t} + 5e^{-t}$



\* A sistemin nasıl davranacağını gösterir. kökün sıfırdan küçük olması daha iyidir.

\* Output inputten smoother versiyonu



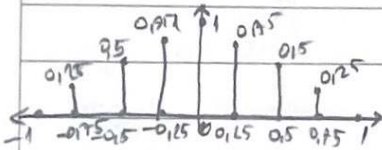
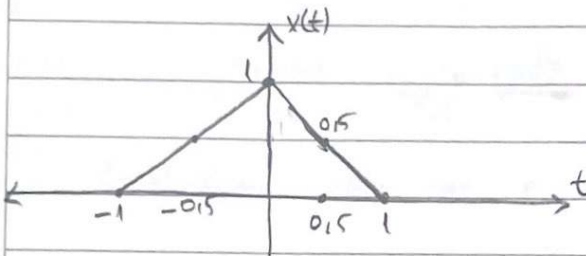
$$\text{ÖR: } x(t) = \begin{cases} 1-|t|, & -1 \leq t \leq 1 \\ 0, & \text{diğer} \end{cases}$$

a)  $T_s = 0.25s$  ( $f_s = 4Hz$ )   b)  $T_s = 0.5s$

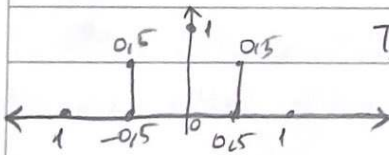
c)  $T_s = 1s$

iken  $x[n]$ 'i elde ediniz.

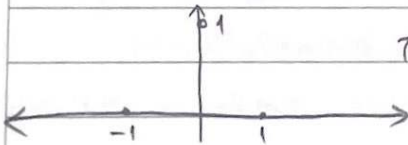
$$\begin{cases} t \geq 0, & 1-t \\ t < 0, & 1+t \end{cases}$$



$T_s = 0.25s$



$T_s = 0.5s$



$T_s = 1s$

ÖR:  $x(t) = e^{jt}$  çift ve tek bi-  
leşenlerini bulunuz.

$$x_e(t) = \frac{x(t) + x(-t)}{2} \quad x_t(t) = \frac{x(t) - x(-t)}{2}$$

$$x_e = \frac{e^{jt} + e^{-jt}}{2} \quad x_t = \frac{e^{jt} - e^{-jt}}{2}$$

Euler'e göre:

$$x_e = \cos t \quad x_t = j \sin t$$

$$e^{jt} = \cos t + j \sin t$$

NOT: sonsuza gitmiyorsa karar-  
lı (stable) olur.

NOT: cos'un veya sin'inin  
deki değer  $2\pi$ 'nin katı  
değilse periyodik olmaz.