BLM3620 Digital Signal Processing

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Yıldız Technical University – Computer Engineering

Course Materials

Important Materials:

- James H. McClellan, R. W. Schafer, M. A. Yoder, *DSP First Second Edition*, Pearson, 2015.

- Lizhe Tan, Jean Jiang, *Digital Signal Processing: Fundamentals and Applications*, Third Edition, Academic Press, 2019.

Auxilary Materials:

- Prof. Sarp Ertürk, *Sayısal İşaret İşleme*, Birsen Yayınevi.

- Prof. Nizamettin Aydin, DSP Lecture Notes.

- J. G. Proakis, D. K. Manolakis, *Digital Signal Processing Fourth Edition*, Peason, 2014.

- J. K. Perin, *Digital Signal Processing*, *Lecture Notes,* Standford University, 2018.

Syllabus

Week	Lectures
1	Introduction to DSP and MATLAB
2	Sinuzoids and Complex Exponentials
3	Spectrum Representation
4	Sampling and Aliasing
5	Discrete Time Signal Properties and Convolution
6	Convolution and FIR Filters
7	Frequency Response of FIR Filters
8	Midterm Exam
9	Discrete Time Fourier Transform and Properties
10	Discrete Fourier Transform and Properties
11	Fast Fourier Transform and Windowing
12	z- Transforms
13	FIR Filter Design and Applications
14	IIR Filter Design and Applications
15	Final Exam

For more details -> Bologna page: http://www.bologna.yildiz.edu.tr/index.php?r=course/view&id=5730&aid=3

Remember: DT Convolution

$h[n]$

$x[n] \longrightarrow$ System $\longrightarrow$ $y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

CONVOLUTION SUM

Or

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

There are three approaches to calculate convolution:

1) Mathematical Approach
2) Table Approach (Polynomial Multiplication)
3) Graphical Approach

Lecture #6 – Convolution and FIR Filters

- Convolution Example
- Graphical Convolution
- MATLAB demo
- FIR Filter
- FIR Filter Application

$x[n] = (0.5)^n u[n]$

One Example to Mathematical Approach (Study it at home)

Given two signals $a[n] = (0.2)^n u[n]$ ve $b[n] = (0.6)^n u[n]$ find the convolution results $c[n] = a[n] * b[n]$ using mathematical approach.

$$c[n] = \sum_{k=-\infty}^{\infty} a[k]b[n-k]$$

$$c[n] = \sum_{k=-\infty}^{\infty} (0.2)^k u[k](0.6)^{n-k} u[n-k]$$

$k<0$ için $k>n$ için

0   0

$$c[n] = \sum_{k=0}^{n} (0.2)^k u[k](0.6)^{n-k} u[n-k]$$

$$c[n] = \sum_{k=0}^{n} (0.2)^k (0.6)^{n-k}$$

$$\sum_{k=0}^{n} (0.2)^k (0.6)^{-k} (0.6)^n = (0.6)^n u[n] \sum_{k=0}^{n} (1/3)^k$$

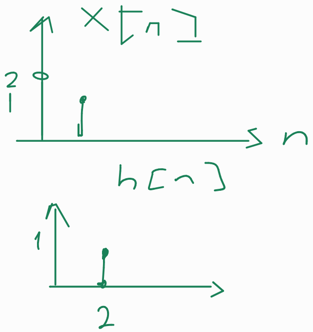
$$c[n] = (0.6)^n u[n] \sum_{k=0}^{n} \frac{(1/3)^{n+1} - (1/3)^0}{(1/3) - 1}$$

$$c[n] = 2.5(0.6^{n+1} - 0.2^{n+1})u[n]$$

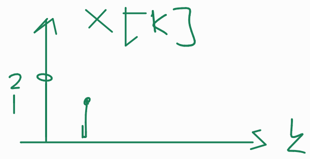
Geometric Serial Sum

$$\sum_{n=M}^{N} r^k = \frac{r^{N+1} - r^M}{r-1}$$

Convolution Method – 3: Graphical Approach



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



For n=0,

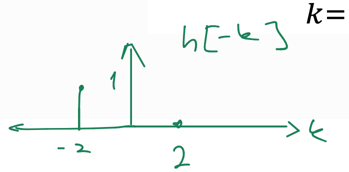
$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k]$$

For n=-5,

$$y[-5] = \sum_{k=-\infty}^{\infty} x[k]h[-5-k]$$

For n=5,

$$y[5] = \sum_{k=-\infty}^{\infty} x[k]h[5-k]$$



Step 1. Obtain the reversed sequence $h(-k)$.

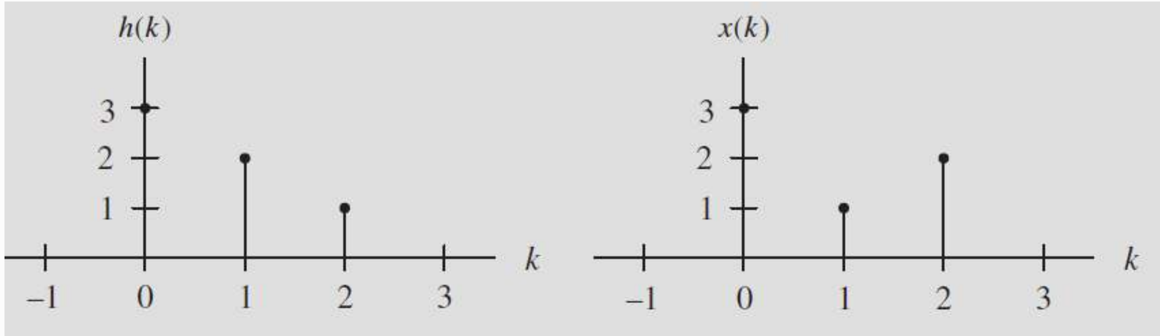
Step 2. Shift $h(-k)$ by $|n|$ samples to get $h(n-k)$. If $n \geq 0$, $h(-k)$ will be shifted to right by n samples; but if $n < 0$, $h(-k)$ will be shifted to the left by $|n|$ samples.

Step 3. Perform the convolution sum that is the sum of products of two sequences $x(k)$ and $h(n-k)$ to get $y(n)$.

Step 4. Repeat steps (1)–(3) for the next convolution value $y(n)$.

3 1 2 1 2 3 y[0]=9 y[1]=9 y[-1]=7 y[2]=11 y[3]=5

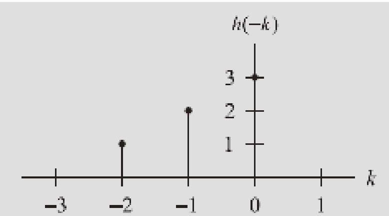
Example for Graphical Approach

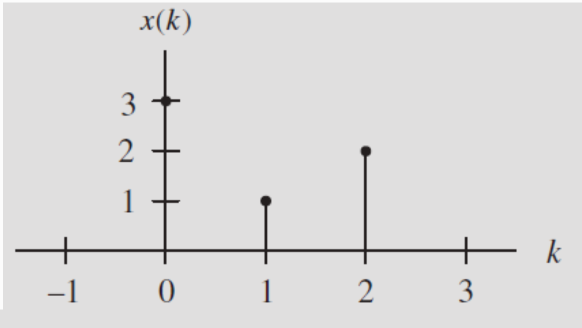


$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

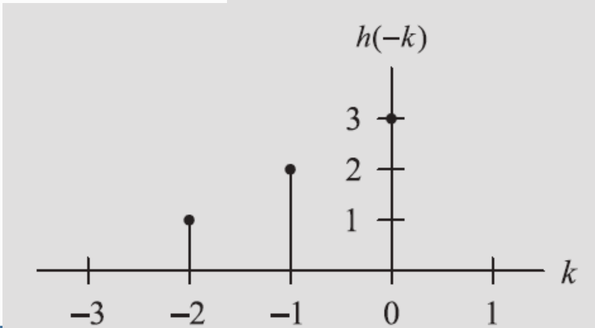
$$y[n] = \sum_{k=0}^{2} x[k]h[n-k]$$

1) Obtain h(-k)





2) Shift it by *0* and get h(0-k)



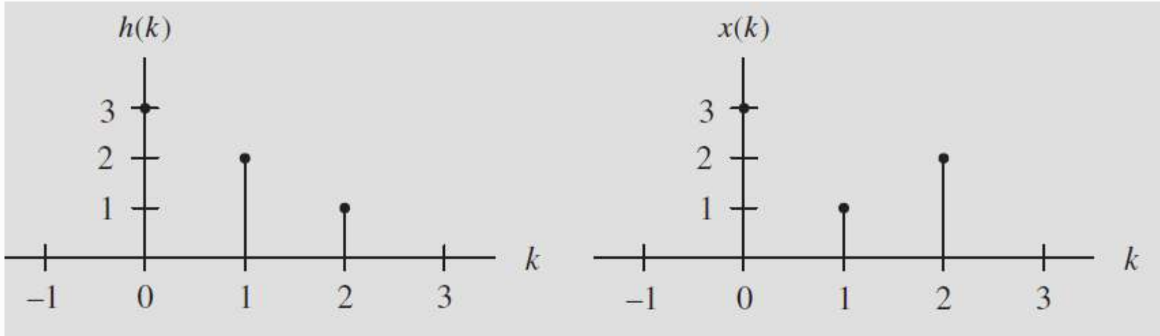
3) Perform conv. sum

Equal to:

$$y[0] = \sum_{k=0}^{2} x[k]h[-k]$$

y[0]=9

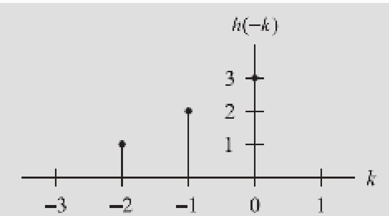
Example for Graphical Approach



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

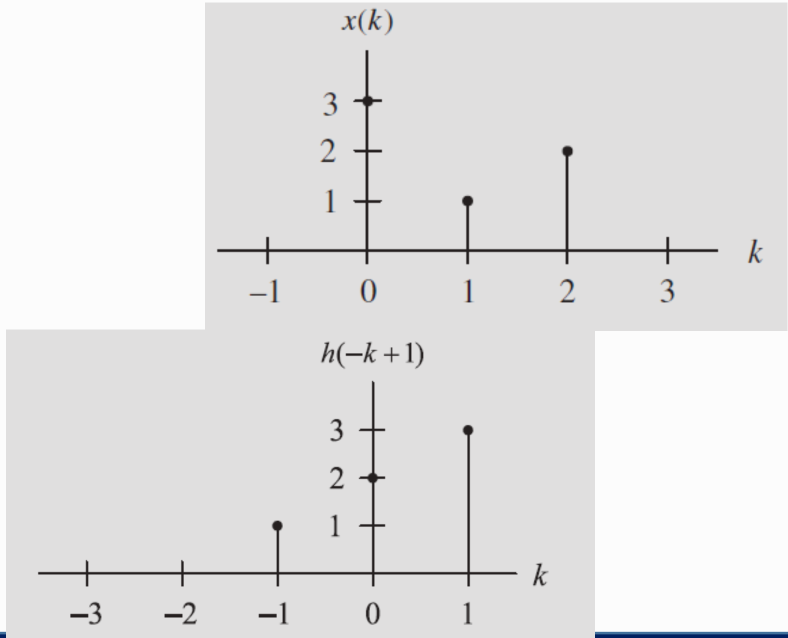
$$y[n] = \sum_{k=0}^{3} x[k]h[n-k]$$

1) Obtain h(-k)



2) Shift it by *1* and get h(1-k)

3) Perform conv. sum

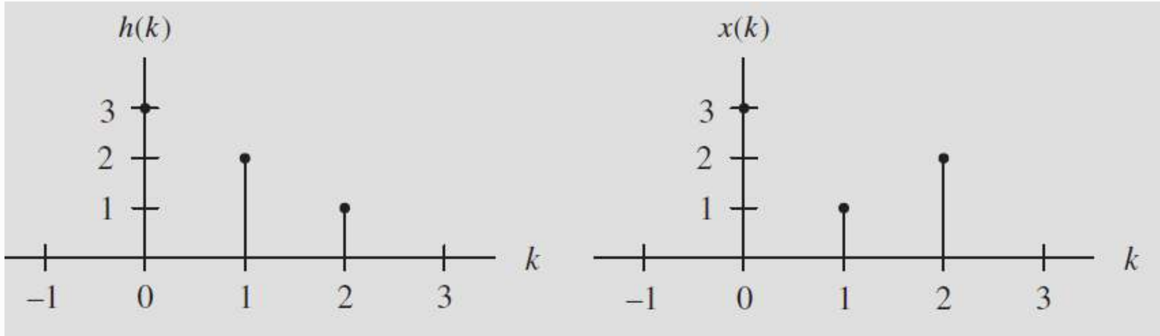


Equal to:

$$y[1] = \sum_{k=0}^{3} x[k]h[1-k]$$

y[1]=2*3+1*3=9

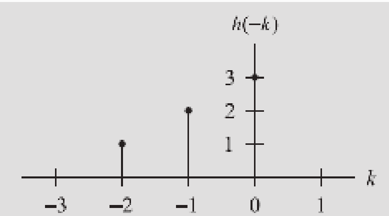
Example for Graphical Approach



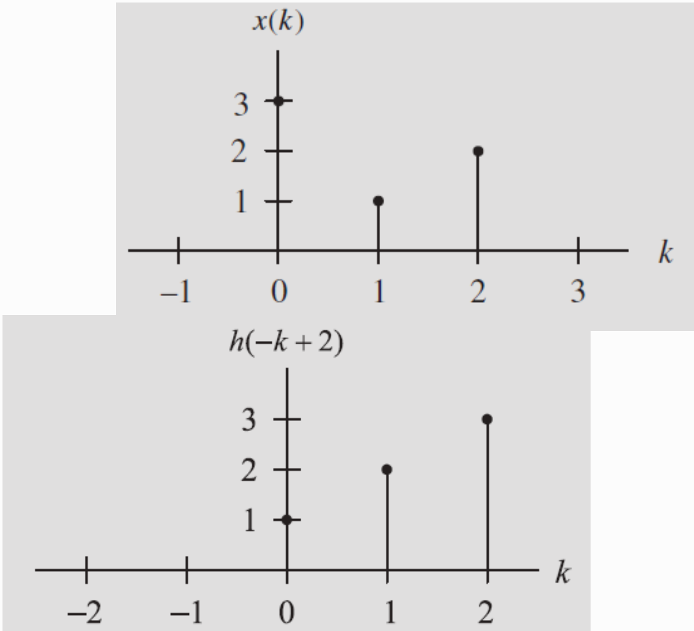
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=0}^{3} x[k]h[n-k]$$

1) Obtain h(-k)



2) Shift it by 2 and get h(2-k)



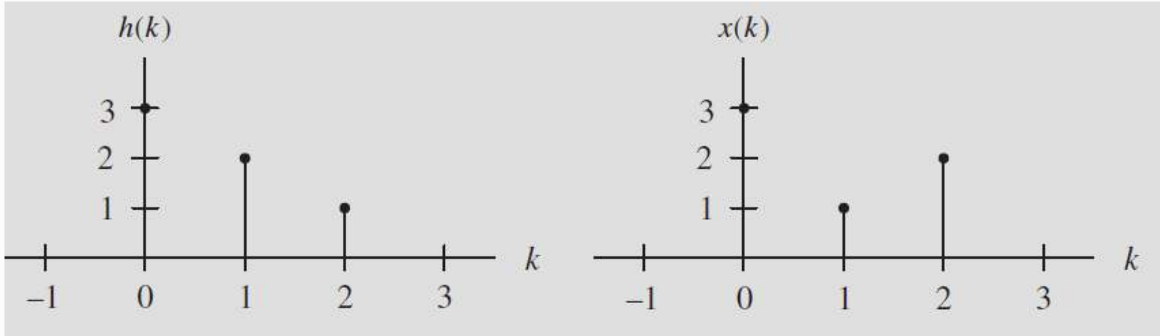
3) Perform conv. sum

Equal to:

$$y[2] = \sum_{k=0}^{3} x[k]h[2-k]$$

y[2]=1*3+2*1+3*2=11

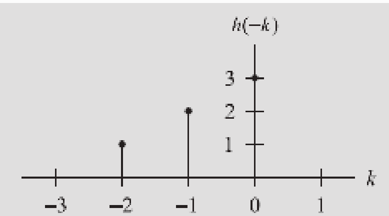
Example for Graphical Approach



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

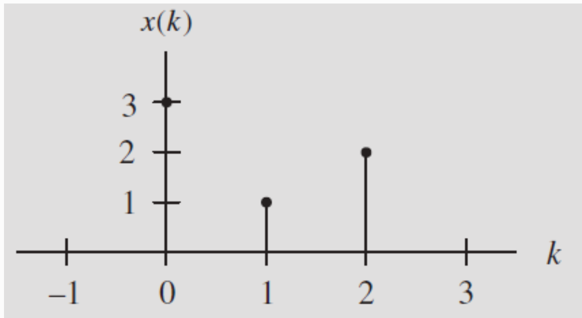
$$y[n] = \sum_{k=0}^{3} x[k]h[n-k]$$

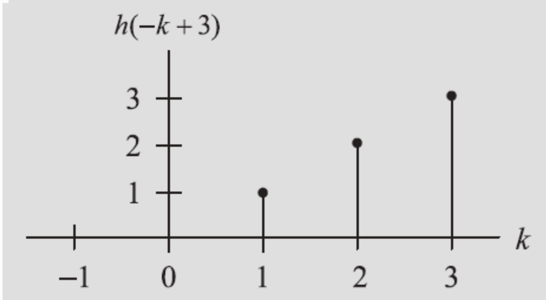
1) Obtain h(-k)



2) Shift it by *3* and get h(3-k)

3) Perform conv. sum



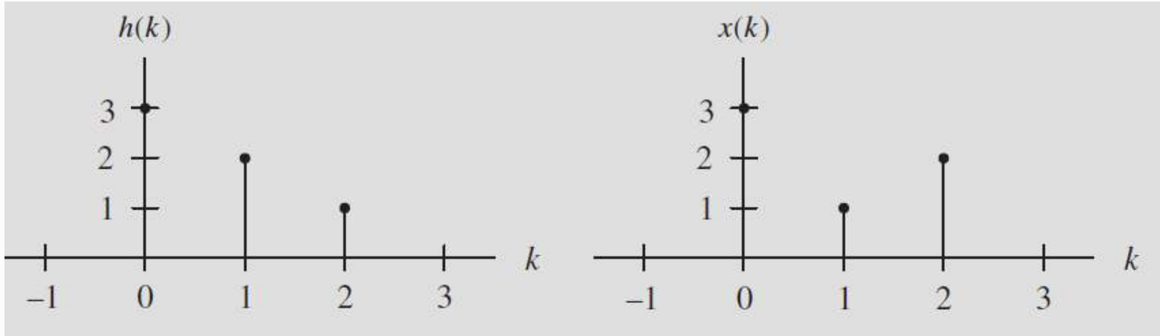


Equal to:

$$y[3] = \sum_{k=0}^{3} x[k]h[3-k]$$

y[3]=1*1+2*2=5

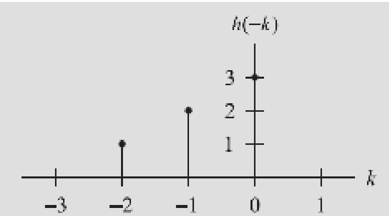
Example for Graphical Approach



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

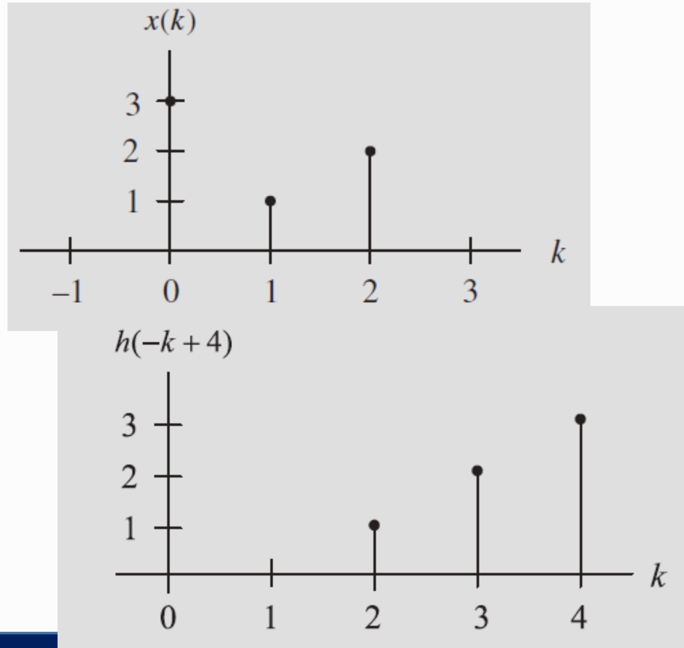
$$y[n] = \sum_{k=0}^{3} x[k]h[n-k]$$

1) Obtain h(-k)



2) Shift it by *4* and get h(4-k)

3) Perform conv. sum

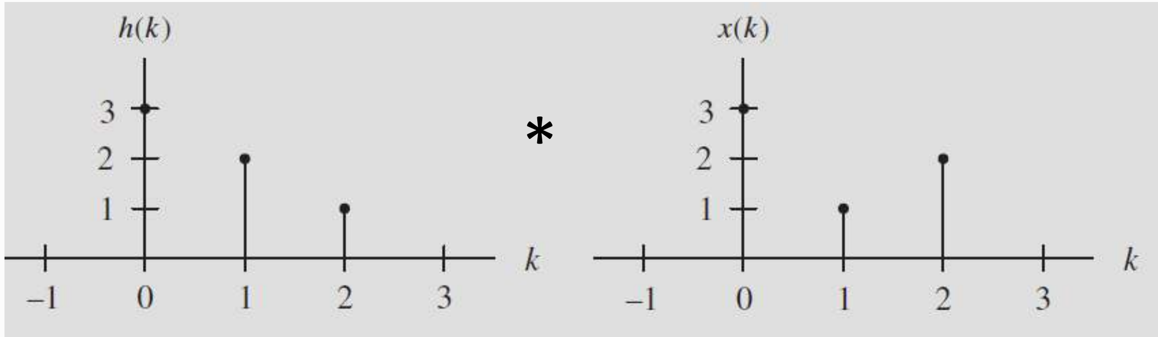


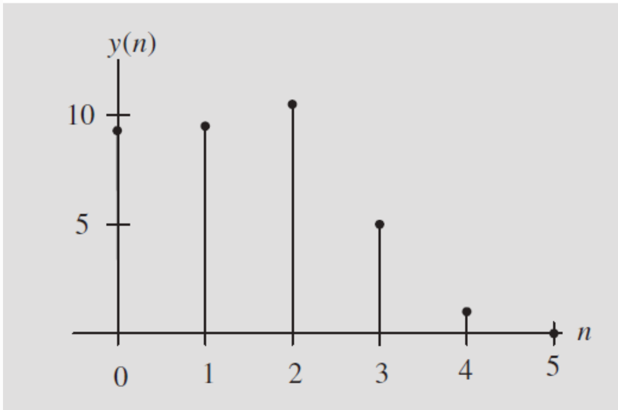
Equal to:

$$y[4] = \sum_{k=0}^{3} x[k]h[4-k]$$

$\boxed{y[4]=2}$

Example for Graphical Approach





$x \rightarrow 3\ 1\ 2$ $h \rightarrow 3\ 2\ 1$

$y[0] = 9$

3 1 2

1 2 3

$y[1] = 9$

3 1 2

1 2 3

$y[2] = 11$

3 1 2

1 2 3

$y[3] = 5$

3 1 2

1 2 3

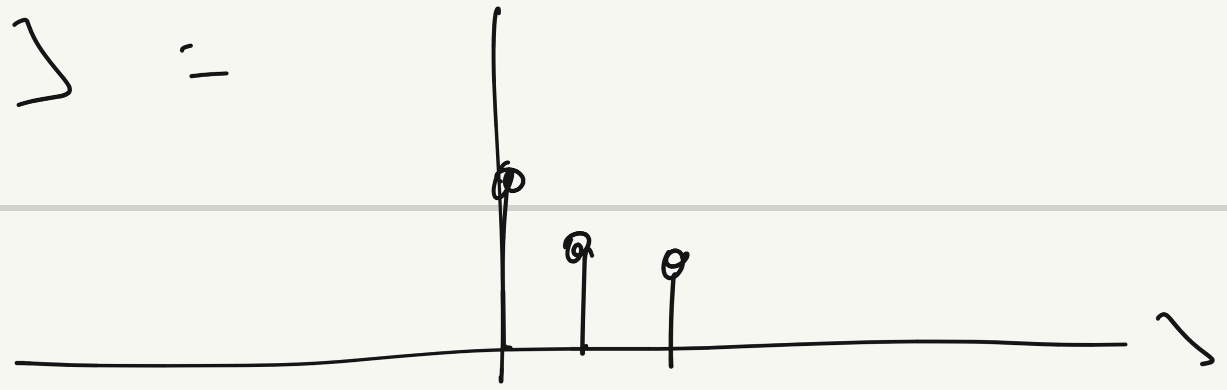
$y[4] = 2$

3 1 2

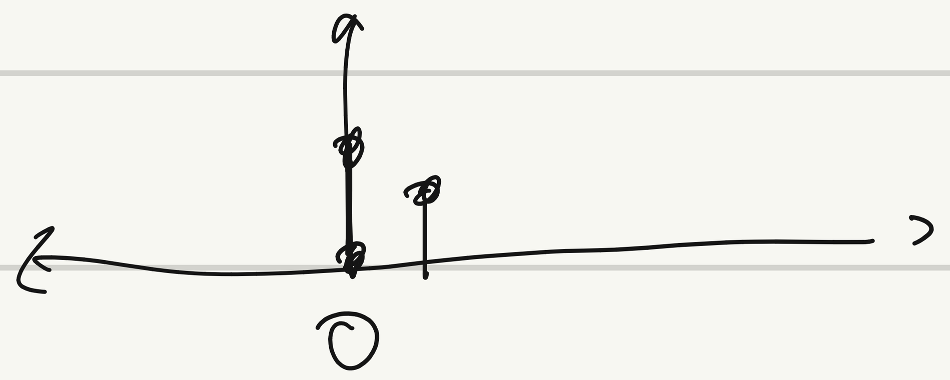
1 2 3

$h[-k]$

$x[k] =$

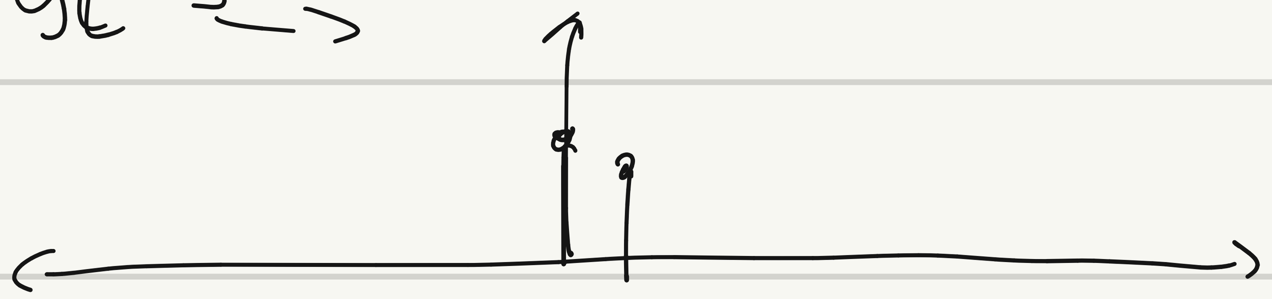


$h[-k]$



$y[0] \rightarrow$

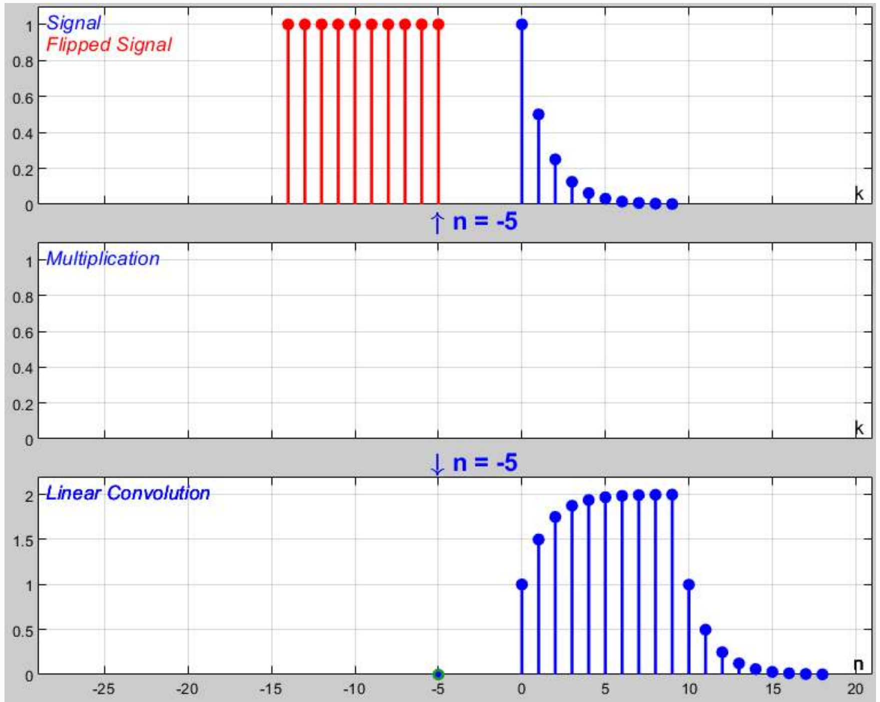
$h[-k]$

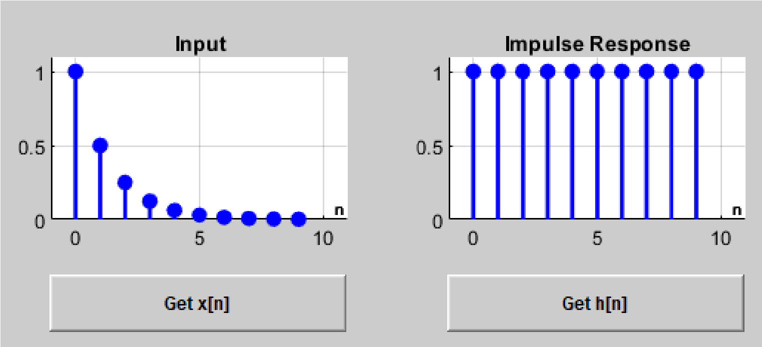


→ bundan dolayı $y[-1]$'den başlar.

MATLAB dconv Demo

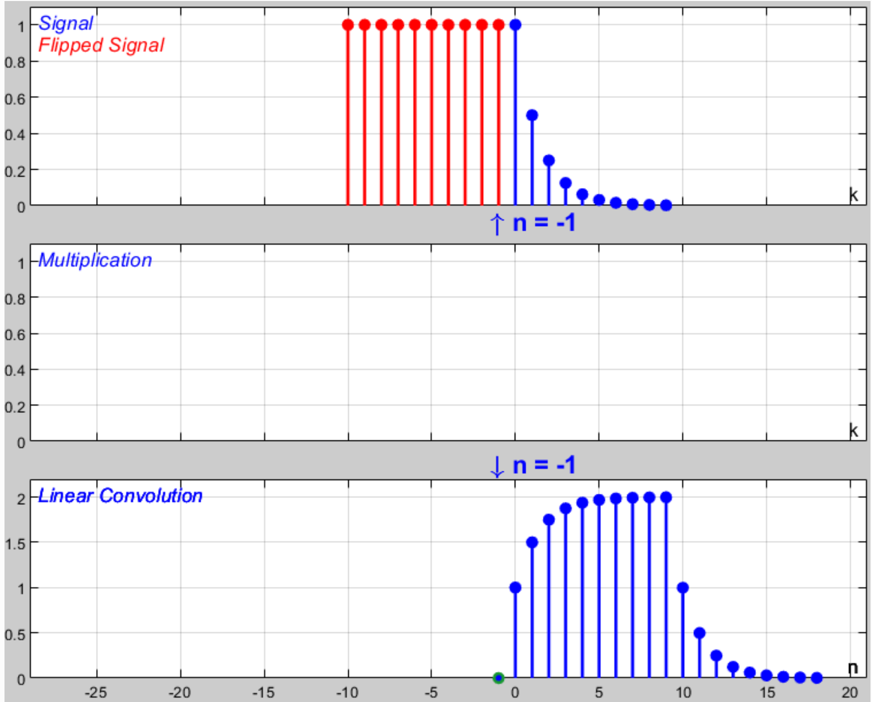
https://dspfirst.gatech.edu/matlab/#dconvdemo

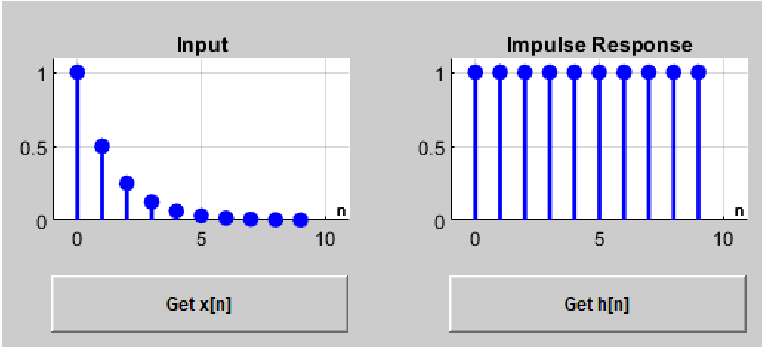




Ref. Discrete Conv. Demo v 3.15 @GitHub

MATLAB dconv Demo





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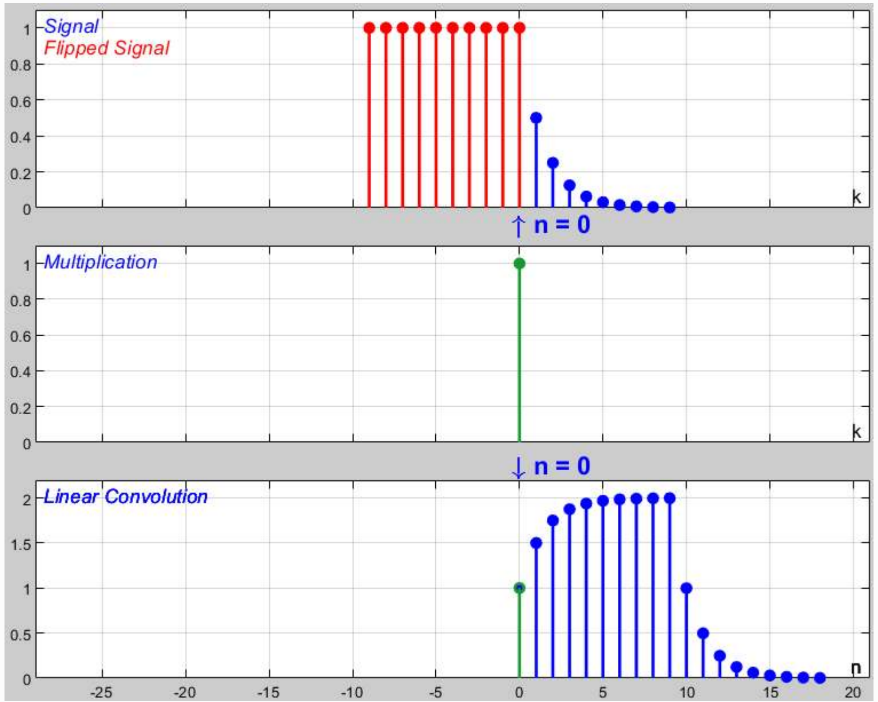

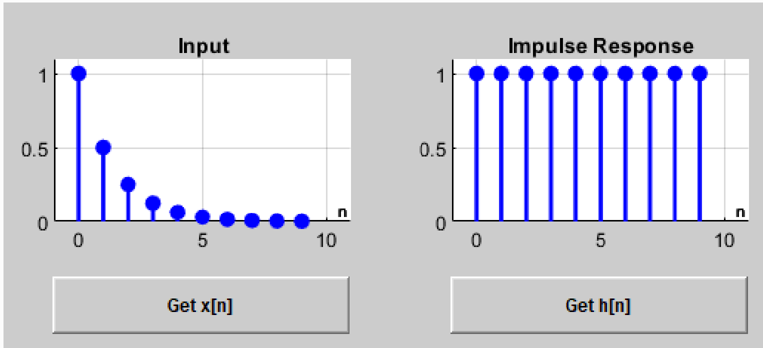

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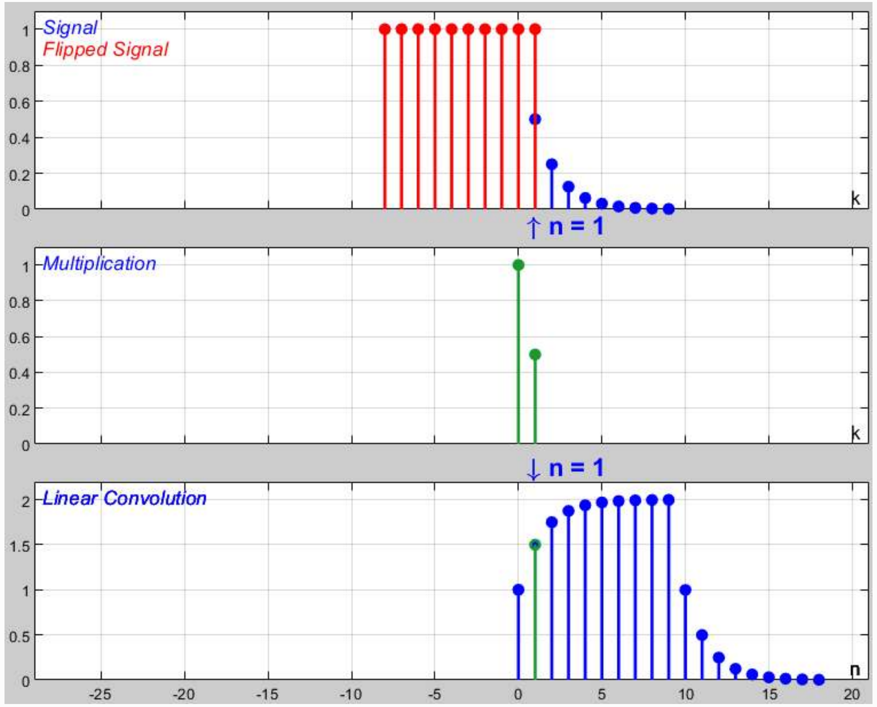

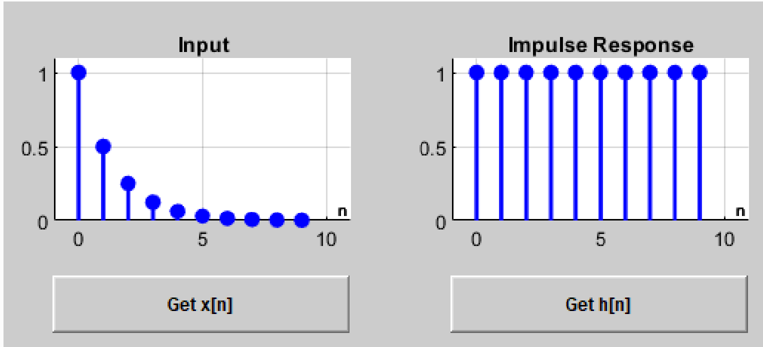

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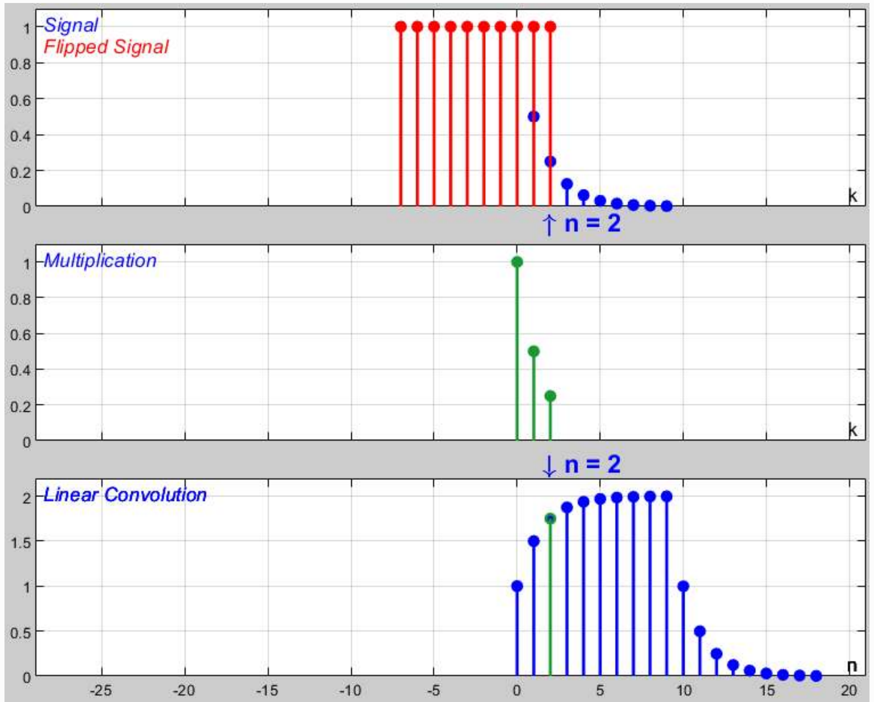

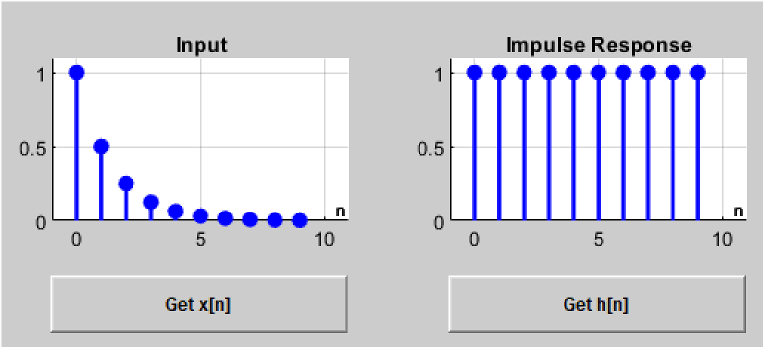

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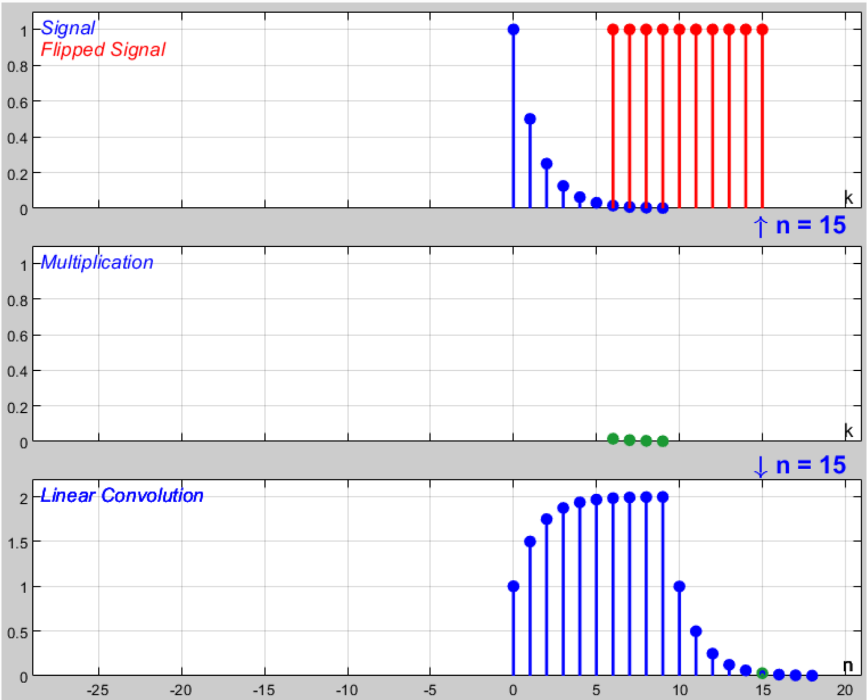

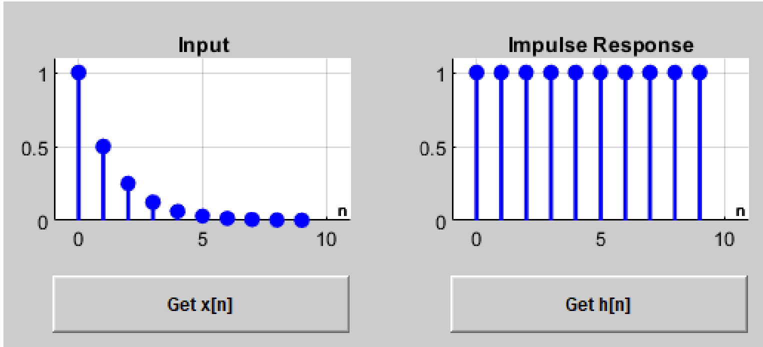

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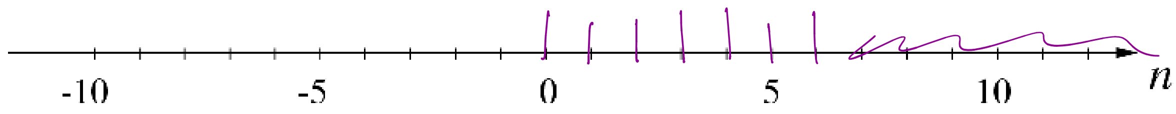
Exercise - 1

PROBLEM:

Let $x[n] = u[n] - u[n-7]$ and $h[n] = \begin{cases} (\frac{1}{2})^n & 0 \leq n \leq 3 \\ 0 & \text{otherwise.} \end{cases}$

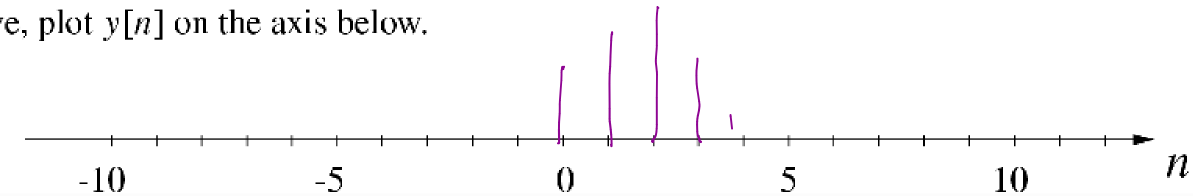
(a) Plot $x[n]$.



Plot $h[n]$.

Label the amplitudes for each sample.

(b) If we now assume $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ and $y[n] = x[n] * h[n]$, where $h[n]$ is as defined above, plot $y[n]$ on the axis below.



Exercise – 1

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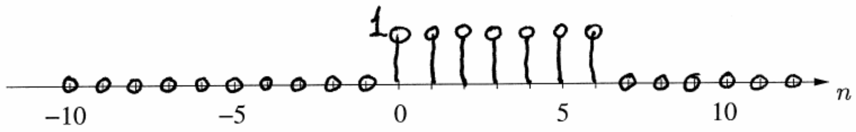
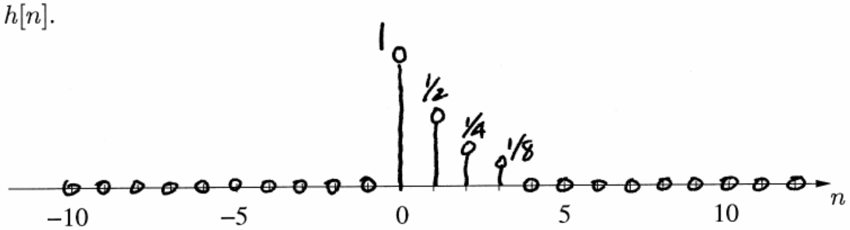


Plot $h[n]$.



(b) If we now assume $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ and $y[n] = x[n] * h[n]$, where $h[n]$ is as defined above, plot $y[n]$ on the axis below.

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$

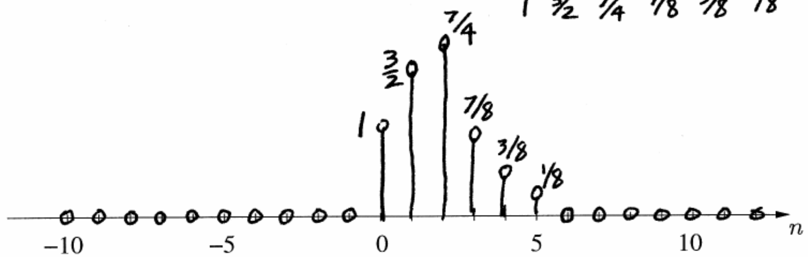
$1 \quad 1 \quad 1$

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$

$\quad 1 \quad \frac{1}{2} \quad 1/4 \quad 1/8$

$\quad\quad 1 \quad 1/2 \quad 1/4 \quad 1/8$

$1 \quad 3/2 \quad 7/4 \quad 7/8 \quad 3/8 \quad 1/8$



A linear, time-invariant system can be described by a difference equation having the following general form:

$$y(n)+a_1y(n-1)+\cdots+a_Ny(n-N)=b_0x(n)+b_1x(n-1)+\cdots+b_Mx(n-M)$$

$$y(n)=-a_1y(n-1)-\cdots-a_Ny(n-N)+b_0x(n)+b_1x(n-1)+\cdots+b_Mx(n-M)$$

Bütün ayrık sistemleri gösterir

$$\boxed{y(n)=-\sum_{i=1}^{N}a_iy(n-i)+\sum_{j=0}^{M}b_jx(n-j)}$$

$a=0$

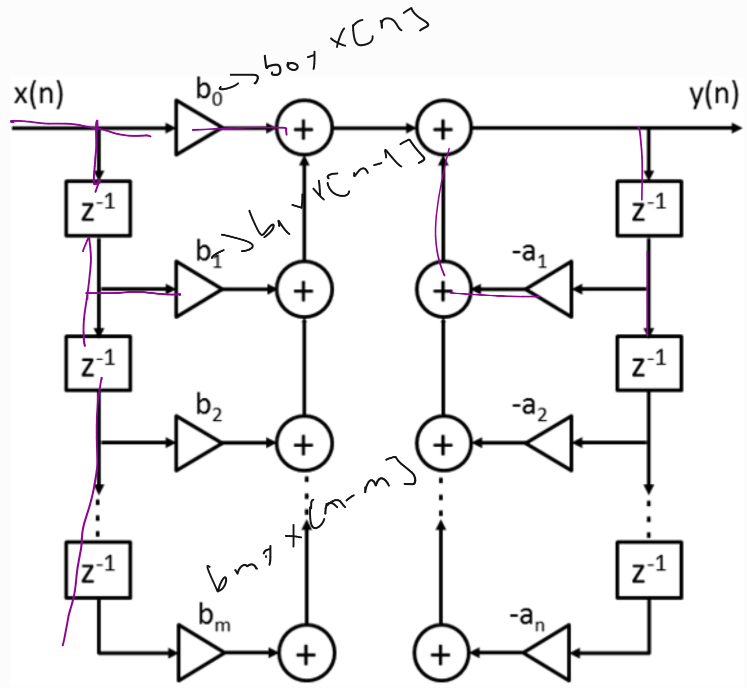
$b=5$

$y[n]=5x[n]$

Linear Constant Coefficient Difference Equation

Block Diagram Representation of LCCDE

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$



It is easy to implement the filters to hardware using block diagrams!

Example

Given the following difference equation:

$$y(n) = 0.25y(n-1) + x(n),$$

identify the nonzero system coefficients.

$$b_0 = 1$$

$$-a_1 = 0.25$$

Classification of Impulse Response h[n]

FIR – Finite Impulse Response:

- Number of impulses are limited.
- Always stable.

For example: $h[n] = \delta[n-1] + 5\delta[n-5]$

IIR – Infinite Impulse Response:

- Number of impulses are infinite.
- Sometimes these systems are not stable.

For example: $h[n] = u[n-1] + 5u[n-5]$

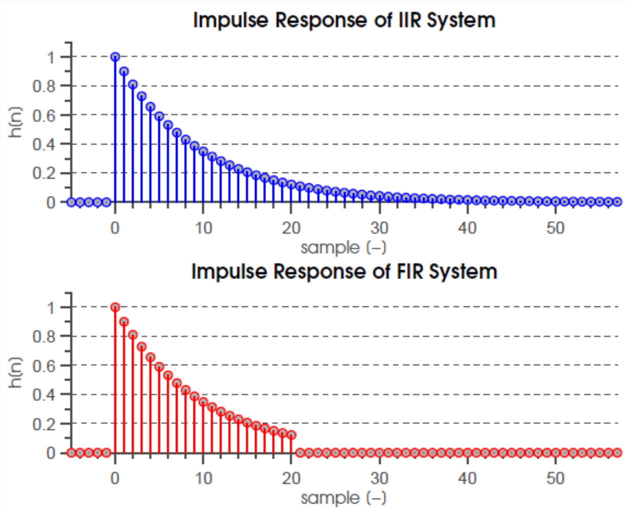
Finite impulse response (FIR):

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

Infinite impulse response (IIR):

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

Another example:



If N = 0, the system has FIR

$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{j=0}^{M} b_j x(n-j)$$

$$y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

M th order FIR filter

Finite impulse response (FIR):

$$y(n) = \sum_{k=0}^{M} h(k) x(n-k)$$

- Direct Form

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

SIGNAL FLOW GRAPH

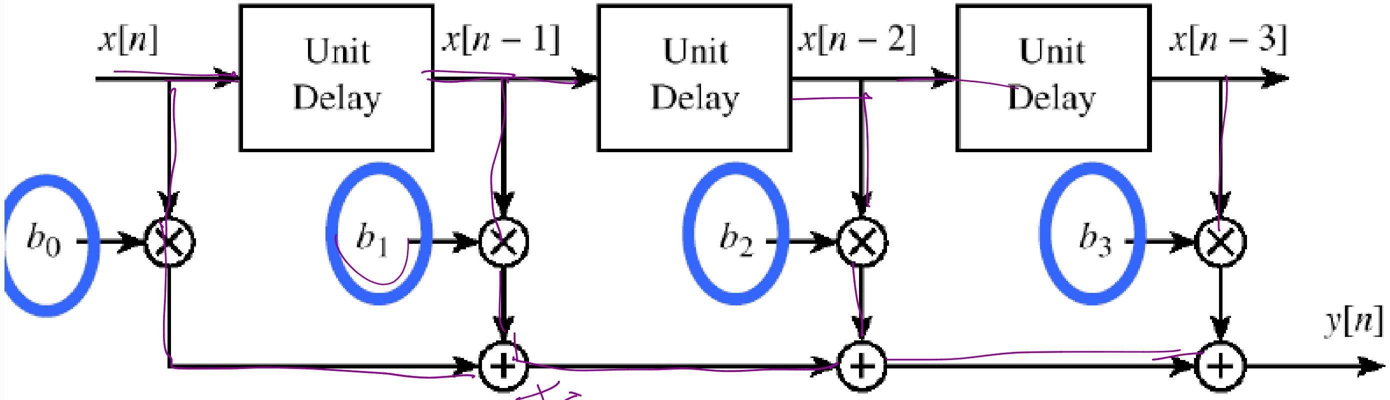
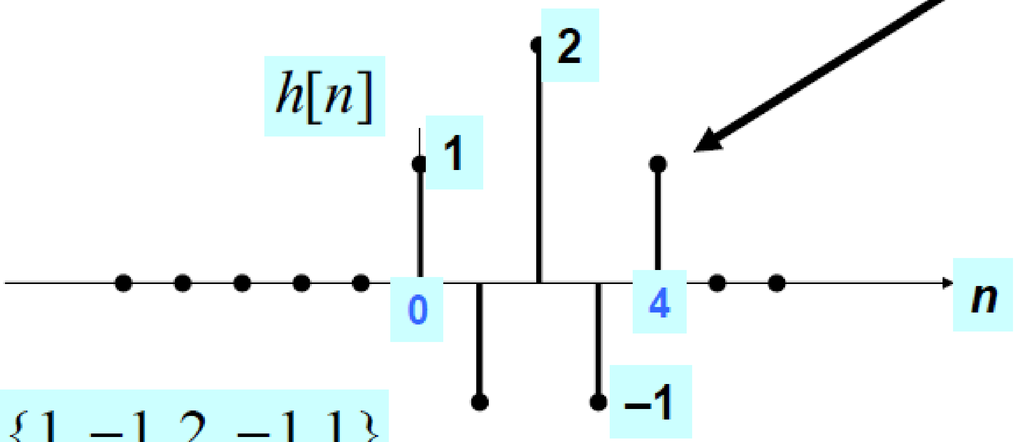


Figure 5.13 Block-diagram structure for the Mth order FIR filter.

Math Formula of h[n] : FIR example

$$h[n]=\delta[n]-\delta[n-1]+2\delta[n-2]-\delta[n-3]+\delta[n-4]$$



$$b_k=\{1,-1,2,-1,1\}$$

FINITE LIMITS

$$y[n]=\sum_{k=0}^{M}h[k]x[n-k]$$

Same as b_k

FINITE LIMITS

FIR Filter conv. - Table Method (Study it at home)

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$
$$x[n] = u[n]$$

n	-1	0	1	2	3	4	5	6	7
$x[n]$	0	1	1	1	1	1	1	1	...
$h[n]$	0	1	-1	2	-1	1	0	0	0
$h[0]x[n]$	0	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	0	-1	-1	-1	-1	-1	-1	-1
$h[2]x[n-2]$	0	0	0	2	2	2	2	2	2
$h[3]x[n-3]$	0	0	0	0	-1	-1	-1	-1	-1
$h[4]x[n-4]$	0	0	0	0	0	1	1	1	1
$y[n]$	0	1	0	2	1	2	2	2	...

3-point Average Filter

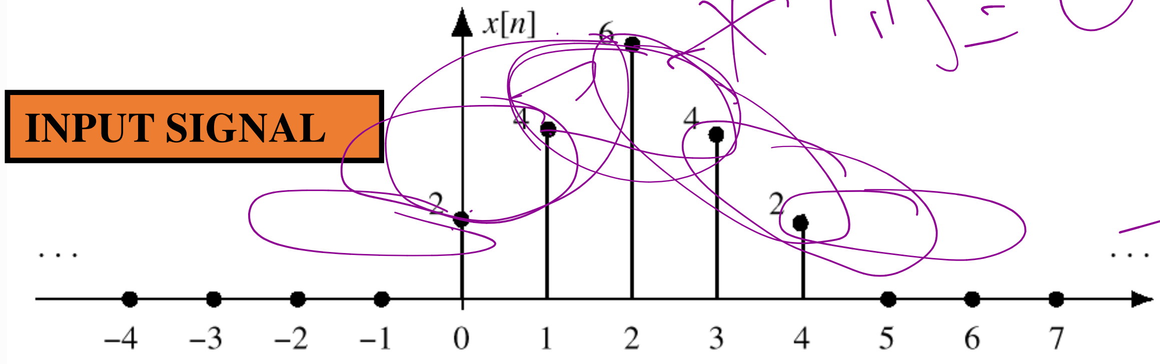


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

$$\boxed{y[n] = \tfrac{1}{3}\left(x[n] + x[n+1] + x[n+2]\right)}$$

non causal → real time değil

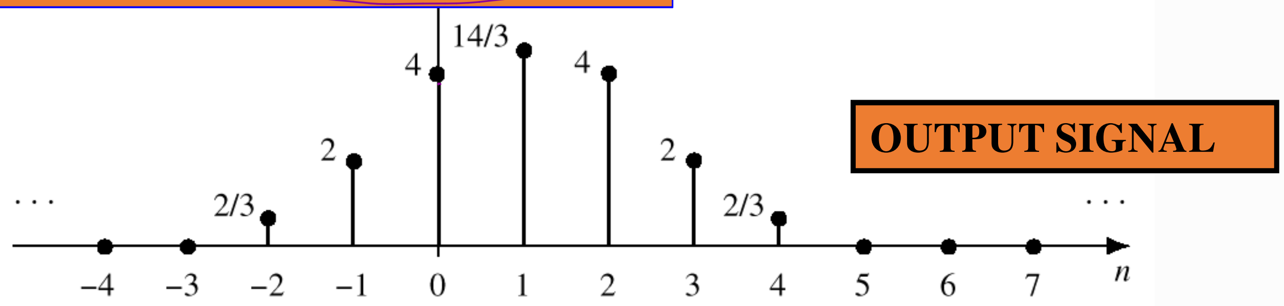


Figure 5.3 Output of running average, $y[n]$.

Is this system causal?

Do this system has FIR ?

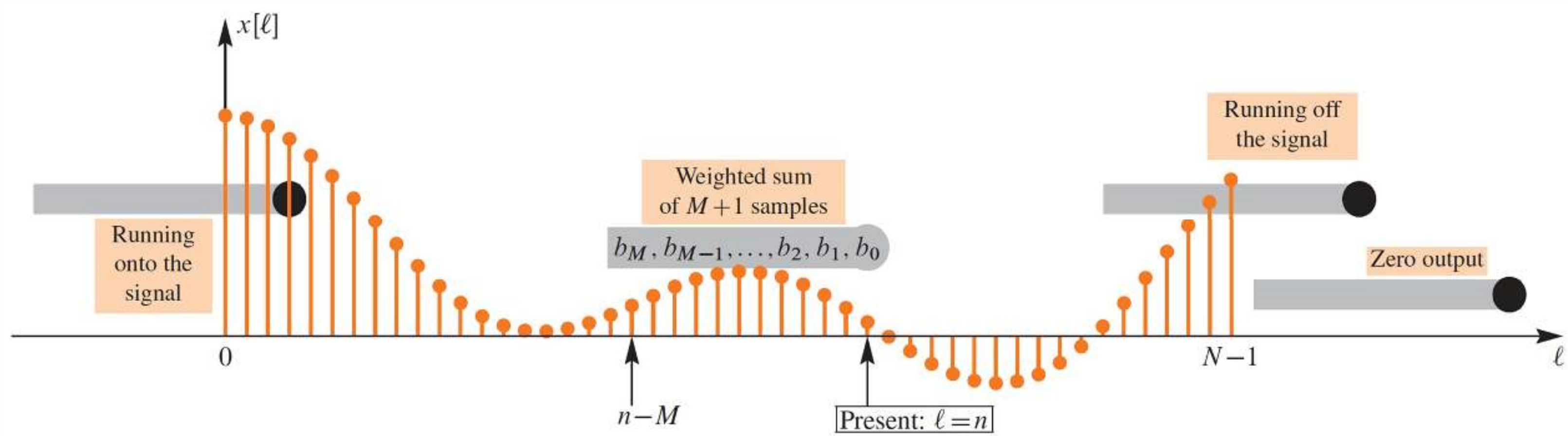
$$y[n] = \sum_{k=0}^{M} h[k]\,x[n-k]$$

FINITE LIMITS

FINITE LIMITS

Same as b_k

Recall: FIR Filter of a causal system



***x*[*n-M*]**

***x*[*n*]**

FINITE LIMITS

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

Same as b_k

FINITE LIMITS

4-point Average FIR Filter

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

Find impulse response:

$$x[n] = \delta[n]$$
$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

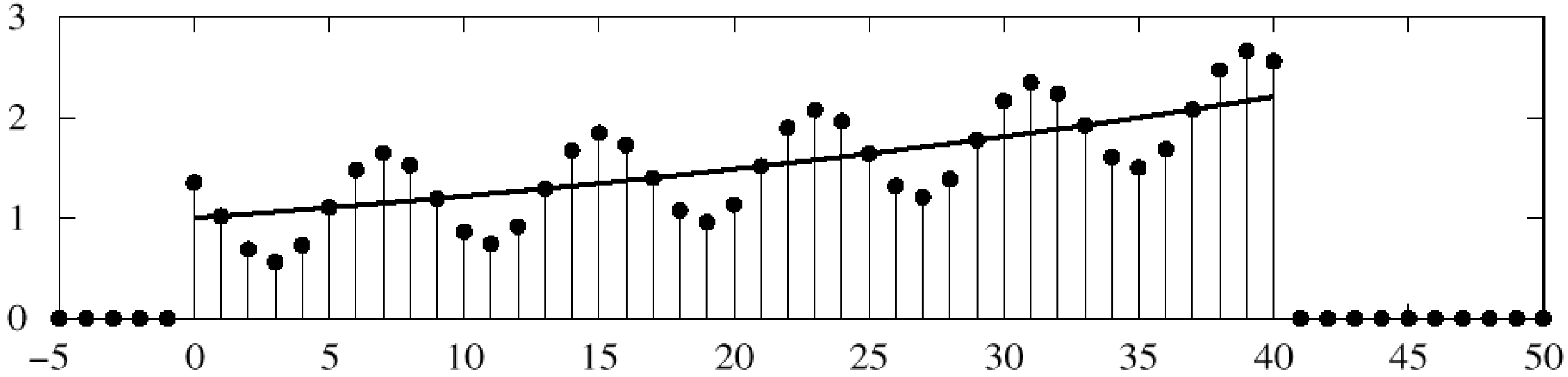
n	-3	-2	-1	0	1	2	3	4	5
$x[n]$	0	0	0	1	0	0	0	0	0
$y[n]$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0

$$h[n] = \{\ldots, 0, 0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, 0, 0, \ldots\}$$

n=0

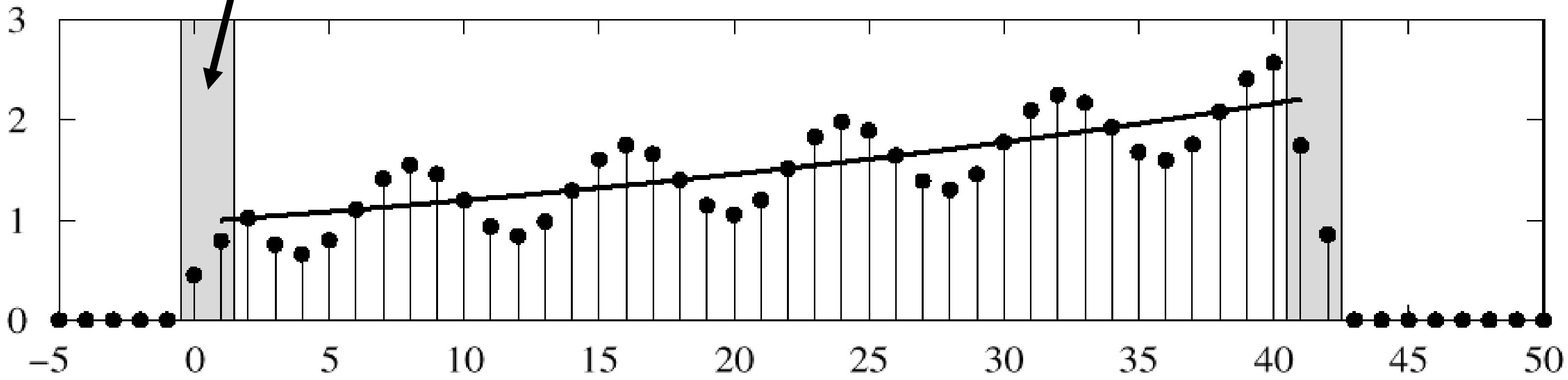
3-pt AVG EXAMPLE

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



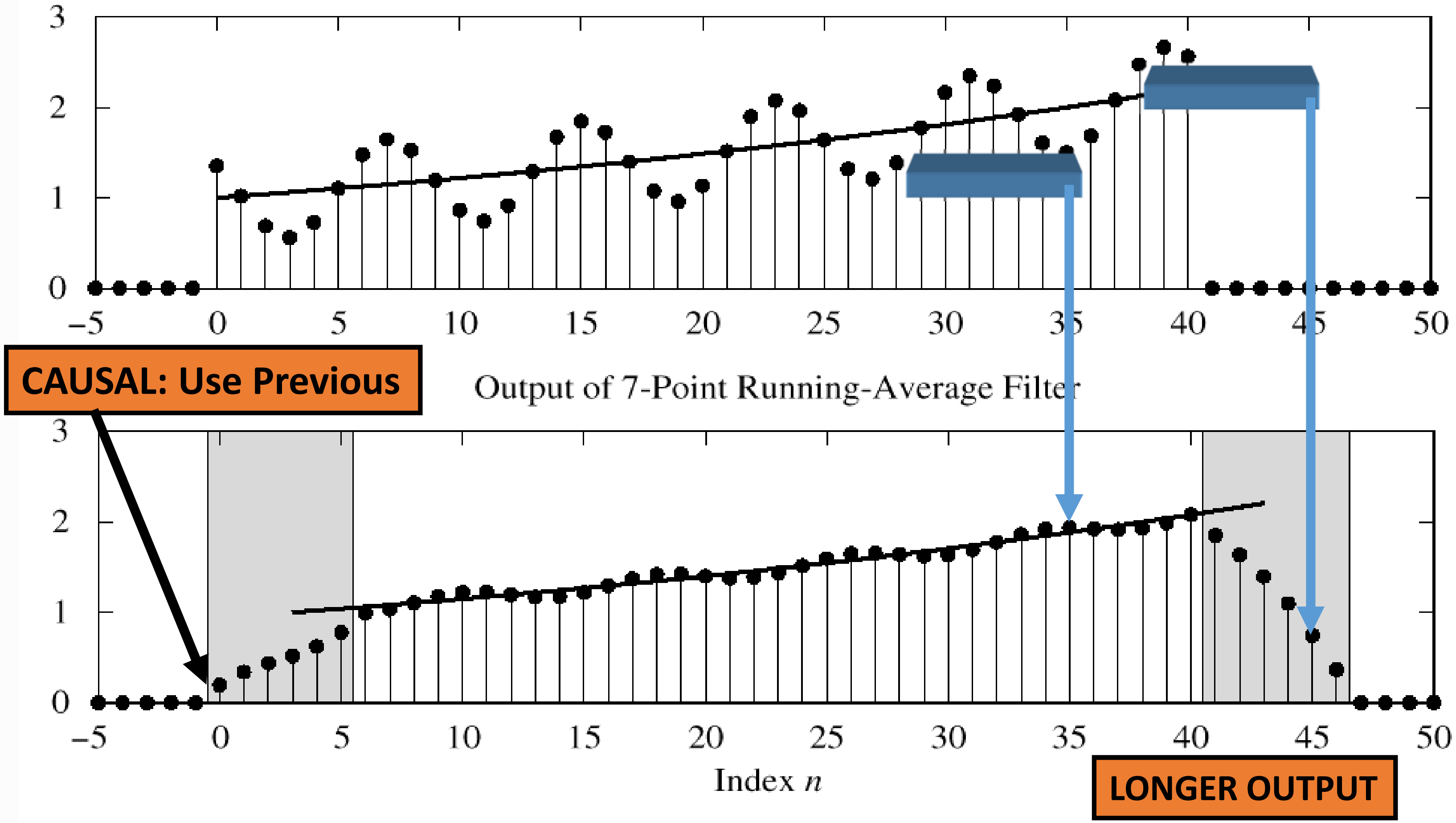
USE PAST VALUES

Output of 3-Point Running-Average Filter

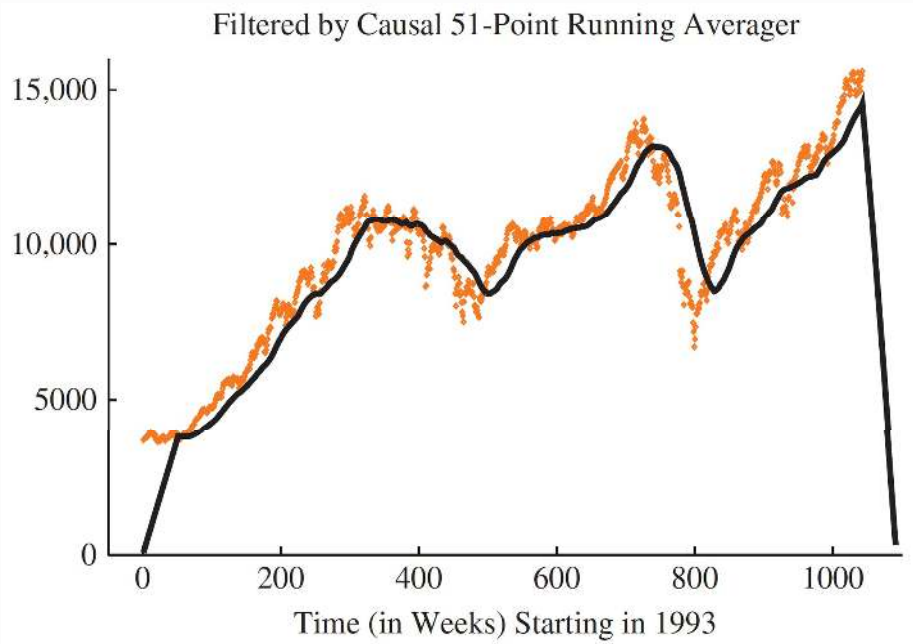


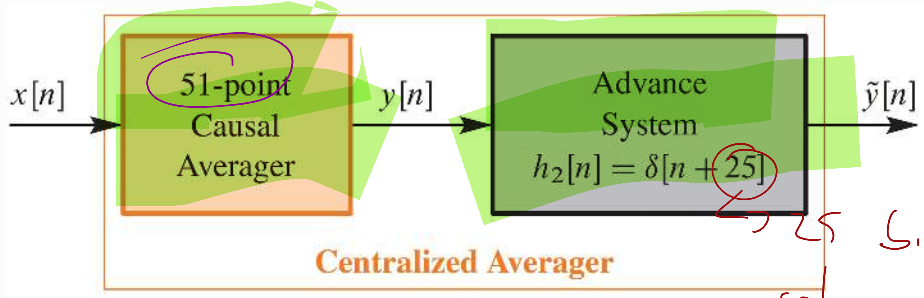
7-pt FIR EXAMPLE (AVG)

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$

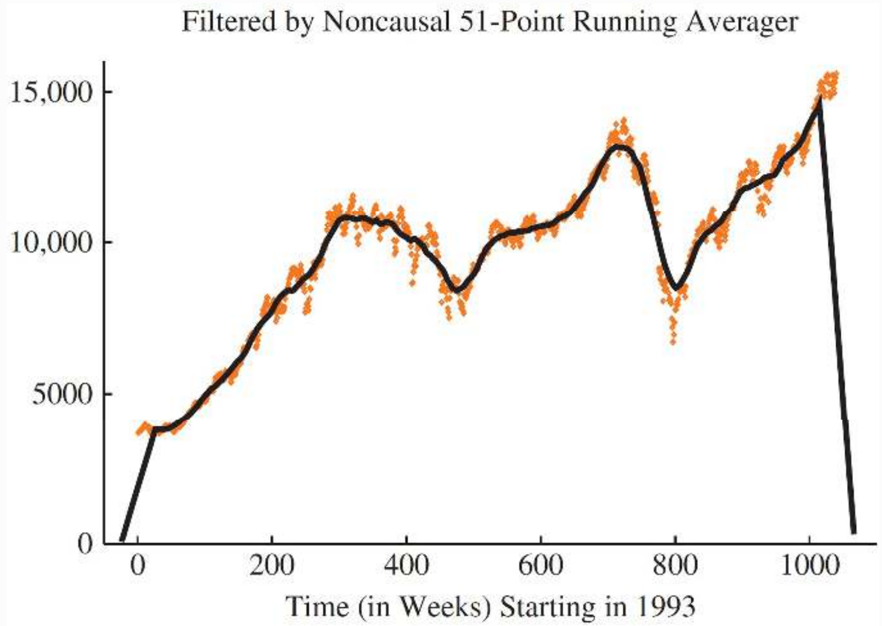


FILTER STOCK PRICES - CAUSAL VS ANTICAUSAL





25 birim
sola shift



Let's apply 17-pt Centralized Average filter to Noisy Audio

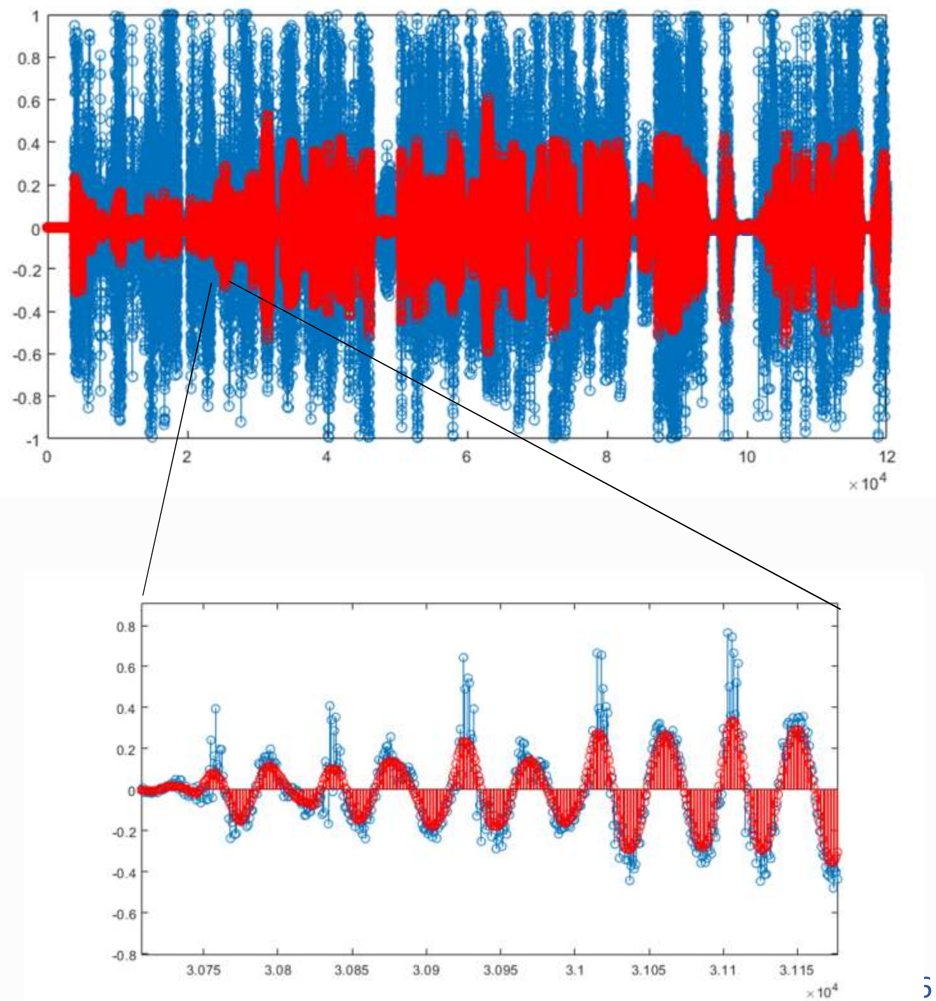
```
clc; clear all;

%% Load Sound
load ('piano2.mat');
x = x(1:16000);
soundsc(x,Fs);

%% Add noise
K = awgn(x,40);
soundsc(K,Fs);

%%  Filter
N = 17;
h = 1/N*ones(1,N);

%% Apply Convolution
y = conv(K,h,'same');
soundsc(y,Fs);
%%
plot(x,'r'); hold on; plot(y,'b');
```

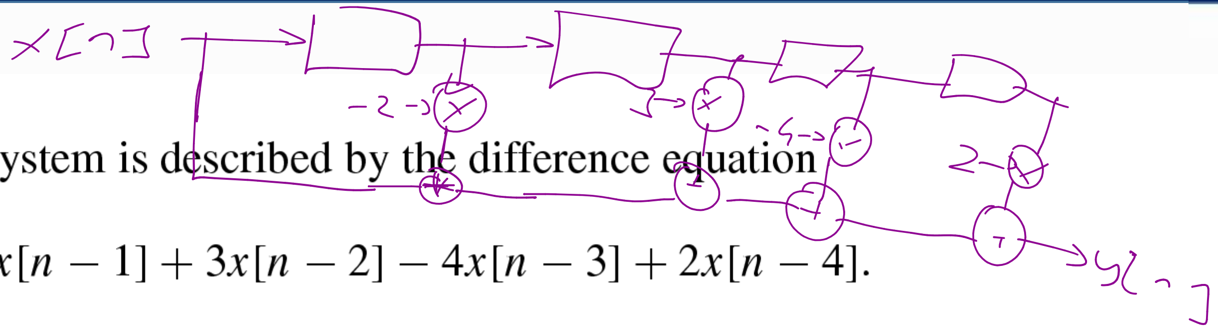


Apply Average Filter to An Image

```
clc; clear all;
I = imread('eight.tif');
I_noise =
imnoise(I,'gaussian',0,0.001);
%%
H = (1/9)*ones(3,3);
ortalamaSonucu =
conv2(I_noise,H,'same');

%%
figure(1), imshow(I_noise,[]);
figure(2), imshow(ortalamaSonucu,[]);
```

Exercise -1



PROBLEM:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] - 2x[n-1] + 3x[n-2] - 4x[n-3] + 2x[n-4].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders as in Figure 5.13 in the *SP First*.

1 -2 3 -4 2

1 -1 1

(b) Determine the impulse response $h[n]$ for this system.

(c) Use convolution to determine the output due to the input

y[0]= 2   y[2]=9   y[4]=6

y[1]=-6   y[3]=-9   y[5]=-3

y[6]=1

1 -1 1

1 -2 3 -4 2

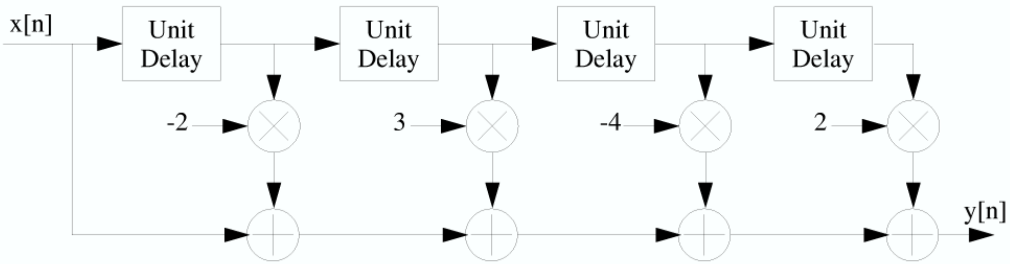
$$x[n] = \delta[n] - \delta[n-1] + \delta[n-2] = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Plot the output sequence $y[n]$ for $-3 \le n \le 10$.

Exercise -1

$$y[n] = x[n] - 2x[n-1] + 3x[n-2] - 4x[n-3] + 2x[n-4]$$

a) The block diagram for $y[n]$ is as follows.



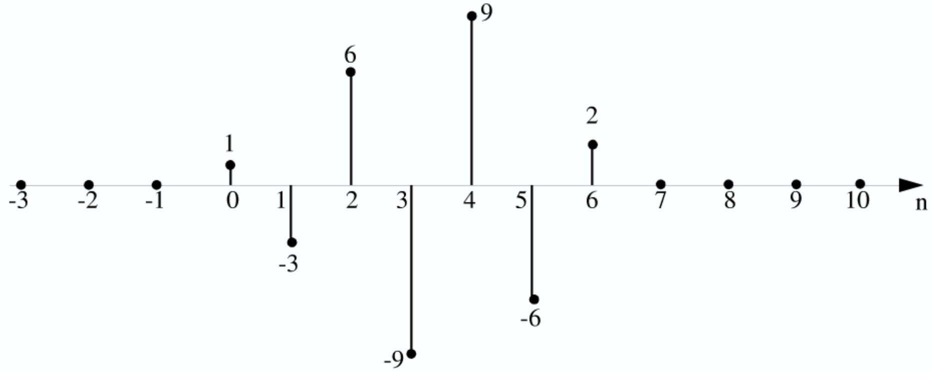
b) The impulse response for $y[n]$ can be found by using $x[n] = \delta[n]$ which results in

$$y[n] = h[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-2] - 4\delta[n-3] + 2\delta[n-4]$$

c) $y[n]$ can be tabulated as follows.

n	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
$h[n]$				1	-2	3	-4	2						
$x[n]$				1	-1	1								
				1	-2	3	-4	2						
					-1	2	-3	4	-2					
						1	-2	3	-4	2				
$y[n]$				1	-3	6	-9	9	-6	2				

Plotting $y[n]$ gives



Exercise -2

PROBLEM:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$b_0 \to 1$ $\quad b_4 \to 5$

$b_1 \to 2$

$b_2 \to 3$

$b_3 \to 4$

$$y[n] = \sum_{k=0}^{4} (k+1)x[n-k]$$

$n = 5$

$-] + 2 \cdot x[n-1] -] + 4 \cdot x[n-3] + \cdot x[n-4]$

$\delta[n] + 2 \cdot \delta[n-1] + 3 \cdot \delta[n- \ldots$

The input to this system is *unit step* signal, denoted by $u[n]$, i.e., $\quad x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Determine the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Use convolution to compute $y[n]$, over the range $-5 \leq n \leq \infty$, when the input is $u[n]$. Make a plot of $y[n]$ vs. n. (Hint: you might find it useful to check your results with MATLAB's `conv()` function.)

Exercise -2

a) $y[n] = 1x[n] + 2x[n-1] + 3x[n-2] + 4x[n-3] + 5x[n-4]$

Filter coefficients $\boxed{b_0 = 1 \quad b_1 = 2 \quad b_2 = 3 \quad b_3 = 4 \quad b_4 = 5}$

($b_n = 0$ for $n < 0$ and $n > 4$)

b) $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 5\delta[n-4]$