BLM1612 Circuit Theory

Basic Concepts

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Units

- numbers & units: used to express some measurable quantity
  - numbers: we typically use base-10 (numerals 0 through 9)
  - units: we typically use either English or Metric (SI)

TABLE 1.1

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

TABLE 1.2

The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

- SI units are used by all modern engineering textbooks

Units

Name	Symbol	Quantity	Expressed in terms of other SI units	Expressed in terms of SI base units
hertz	Hz	frequency		s^{-1}
newton	N	force		$kg \cdot m \cdot s^{-2}$
joule	J	energy, work	N·m	$kg \cdot m^2 \cdot s^{-2}$
watt	W	power	J/s	$kg \cdot m^2 \cdot s^{-3}$
coulomb	C	electric charge		$s \cdot A$
volt	V	voltage	W/A or J/C	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$
ohm	Ω	electric resistance	V/A	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$
farad	F	electric capacitance	C/V	$kg^{-1} \cdot m^{-2} \cdot s^4 \cdot A^2$
henry	H	inductance	Wb/A	$kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}$
weber	Wb	magnetic flux	V·s	$kg \cdot m^2 \cdot s^{-2} \cdot A^{-1}$

joule: fundamental unit of ***work or energy***

1 J = 2.78×10^{-7} kW·h = 2.39×10^{-4} kcal

- Since the **joule** is also a **watt-second** and the common unit for electricity sales to homes is the kW·h (kilowatt-hour), a kW·h is thus 1000 (kilo) × 3600 seconds = 3.6 MJ (megajoules).

Examples

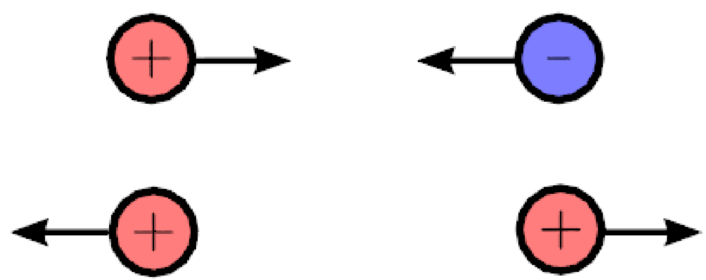
- A 1.2-kWh toaster takes roughly 4 minutes to heat four slices of bread. Find the cost of operating the toaster once per day for 1 month (30 days). Assume energy costs 9 cents/kWh. (answer = 21.6 cents)

- The clock speed of your computer is 2.6 GHz. This clock speed is equal to…
  - …………….MHz
  - …………….KHz
  - …………….HZ

Charge

Charge	
Units	coulomb (C)
Variable	q, Q

- is the fundamental property of matter that causes it to experience a force when placed in electro magnetic field and refers to electrons & protons
  - particles that attract each other (opposite “charge”) or repel each other (same “charge”)



  - fundamental unit of charge (SI system) = coulomb

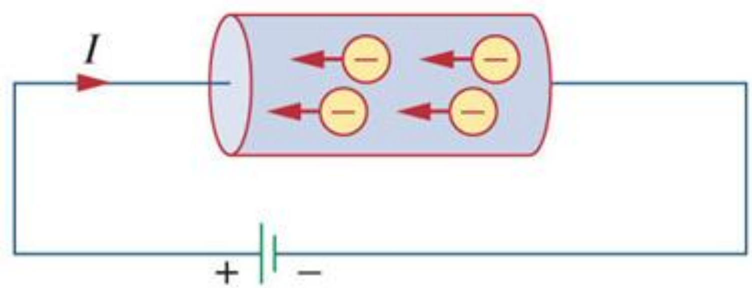
p^+ $\quad$ e^-

1.602×10^{-19} C $\quad$ -1.602×10^{-19} C

Current

Current	
Units	ampere ($A = \frac{C}{s}$)
Variable	i

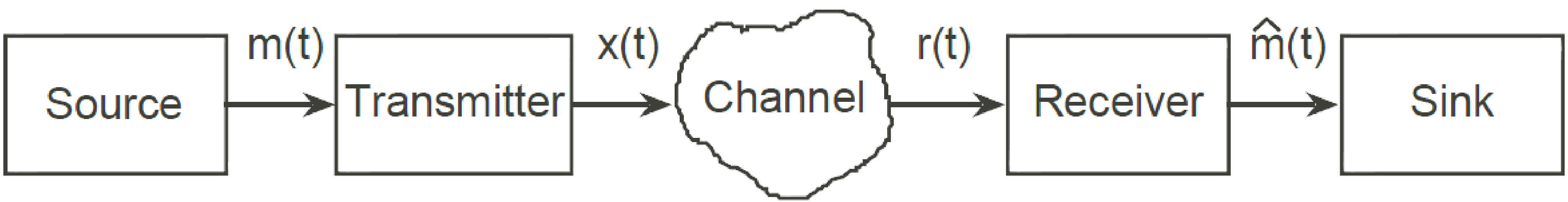
- The movement of charge is called a current
- Historically the moving charges were thought to be positive



- The mechanism by which electrical energy is transferred

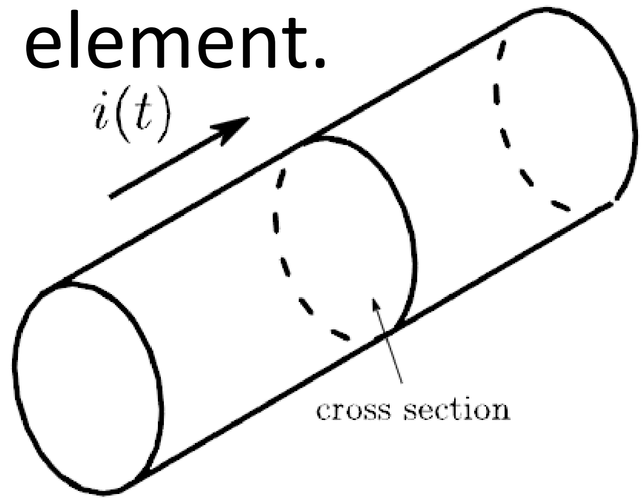
  - Send power from generation point to consumption point

  - Send signals from source to sink



Current

- Current, *i*, is measured as charge moved per unit time through an element.



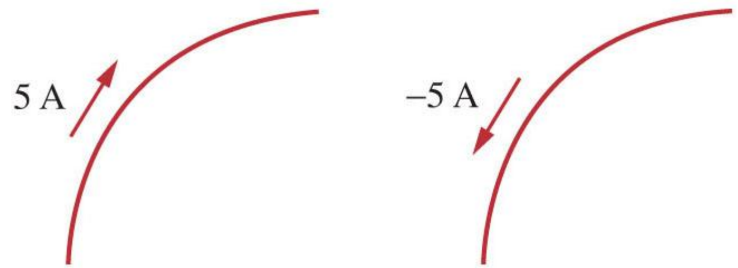
$$i = \frac{dq}{dt}$$

- Amount of charge that has passed a given point:

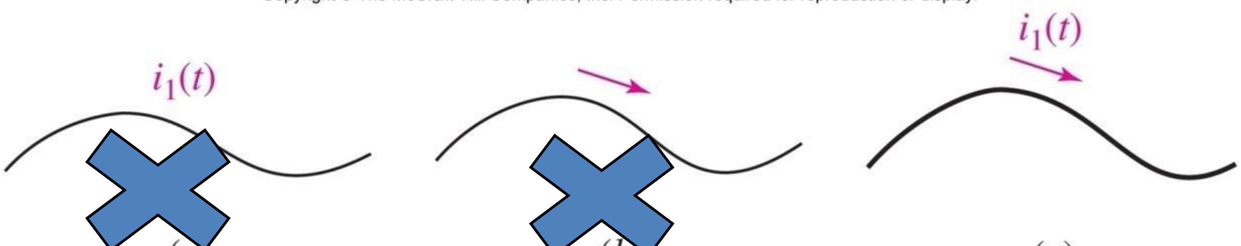
$$q(t) = \int_{t_0}^{t} i(\tau)d\tau + q(t_0)$$

Direction of Current

- defined as the flow of *positive* charge in a conductor (i.e. in reality, a positive forward current means the electrons are flowing backwards)
- when written, current must be labeled with **direction** & **value**:



(a) (b)

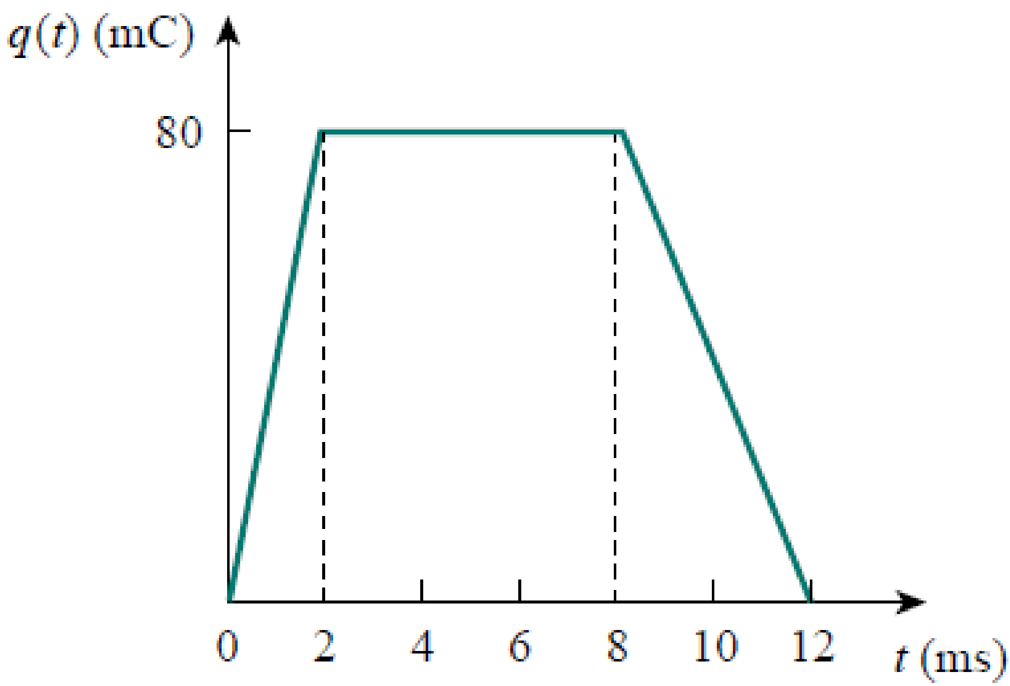


(a) (b) (c)

Examples

- The charge entering a certain element is shown in below Figure. Find the current at:

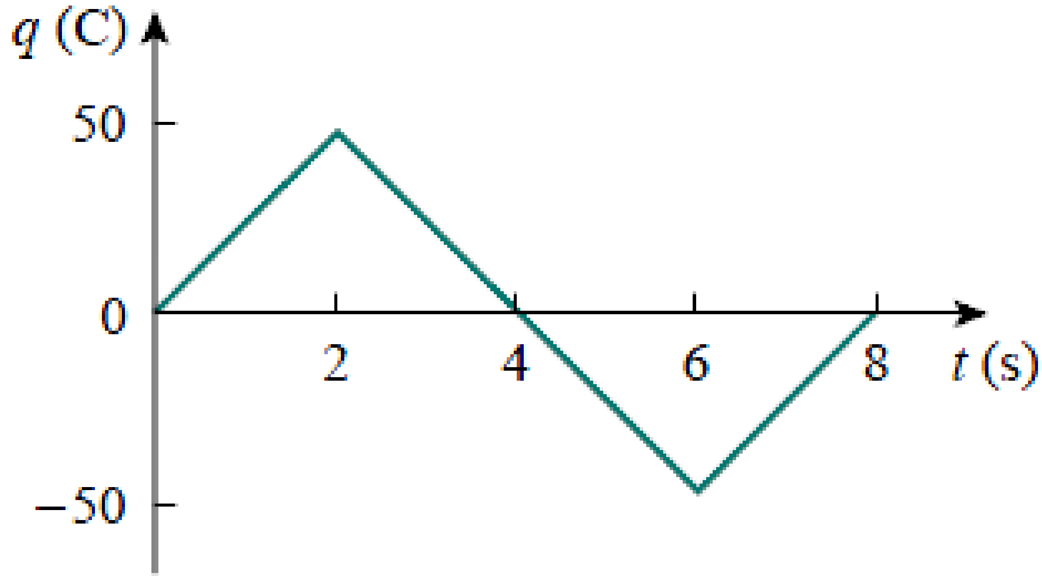
  (a) $t = 1\ ms$ (b) $t = 6\ ms$ (c) $t = 10\ ms$



- The slope is defined as the ratio of the vertical change between two points, to the horizontal change between the same two points.

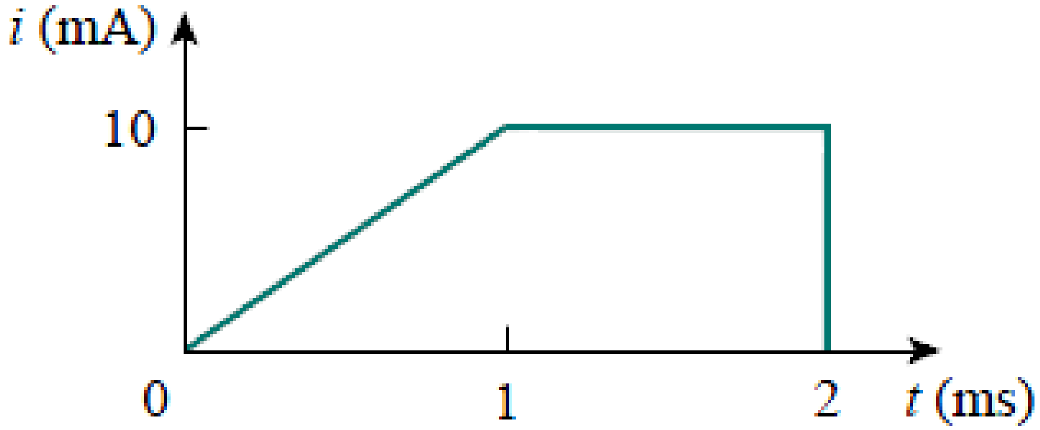
Examples

- The charge flowing in a wire is plotted in below Figure. Sketch the corresponding current.



Examples

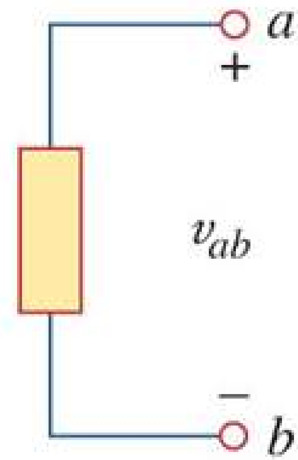
- The current flowing past a point in a device is shown in below Figure. Calculate the total charge through the point.



Voltage

Voltage	
Units	volt ($\mathrm{V}=\frac{\mathrm{J}}{\mathrm{C}}$)
Variable	v

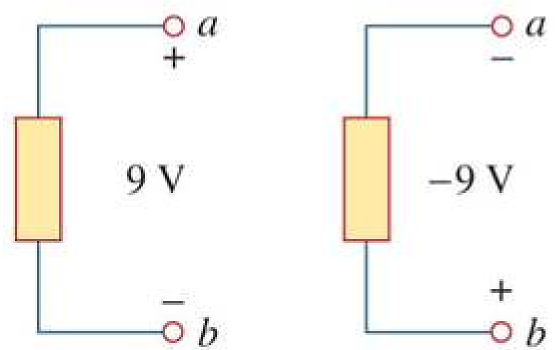
- Electrons move when there is a difference in charge between two locations.
- Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in **volts** (V) (from *a* to *b*).



- v_{ab}

- point a is at *a* potential of v_{ab} volts higher than point *b*

$$v_{ab} = -v_{ba}.$$



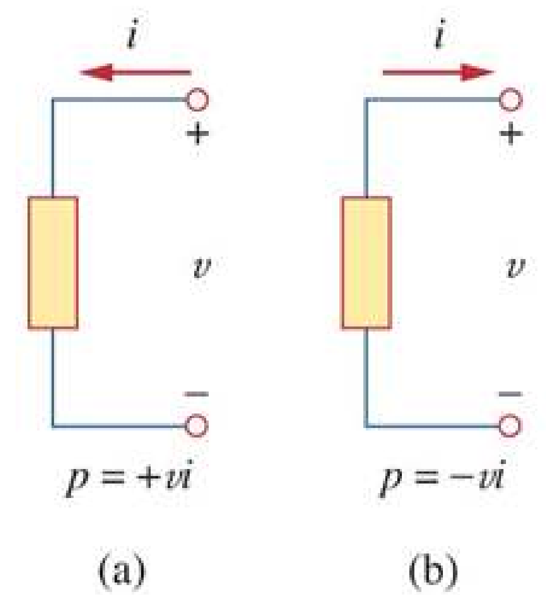
Power

Power	
Units	watt ($W = \frac{J}{s}$)
Variable	p

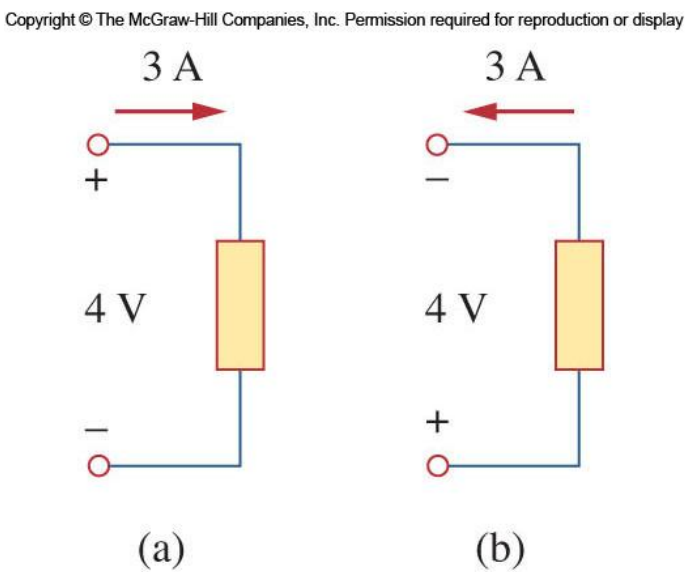
- Power is the product of voltage and current

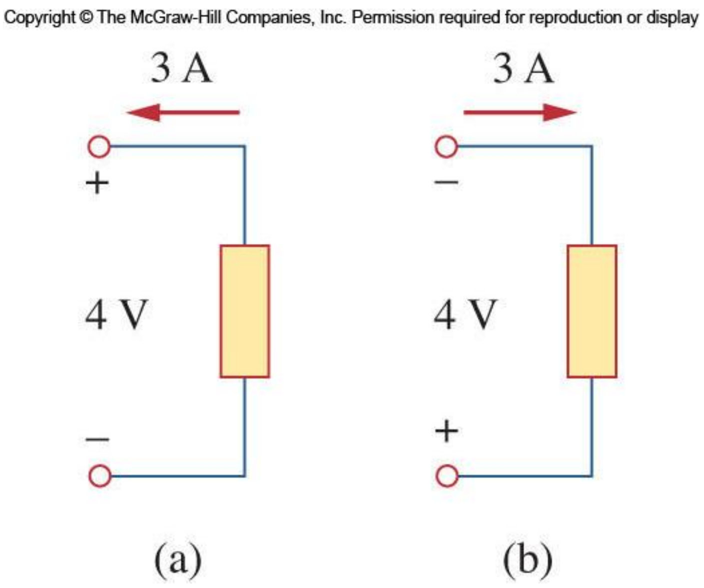
$$p = vi$$

- It is equal to the rate of energy provided or consumed per unit time.
- It is measured in Watts (W)
- Passive sign convention
  - current into positive terminal
  - positive for power *absorbed*
  - negative for power *supplied*



Power





Two cases of an element with an absorbing power of 12 W

Two cases of an element with an supplying power of 12 W

Conservation of Energy (Tellegen's Theorem)

- The sum of all power supplied must be absorbed by the other elements.
- For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero

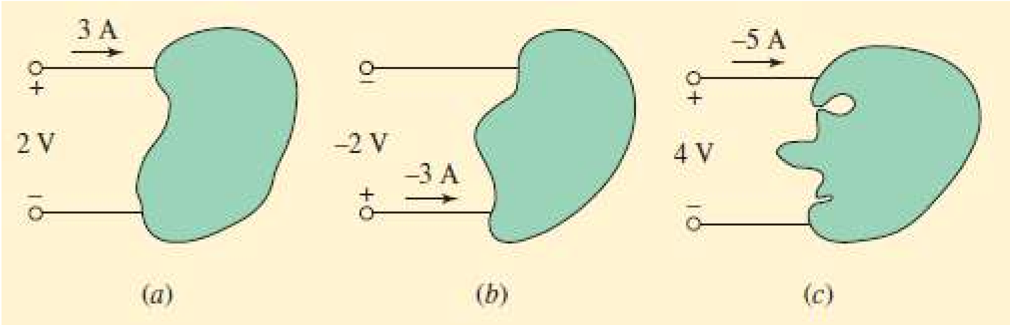
$$\sum p = 0$$

- The energy absorbed or supplied by an element from time t_0 *to time* t *is*

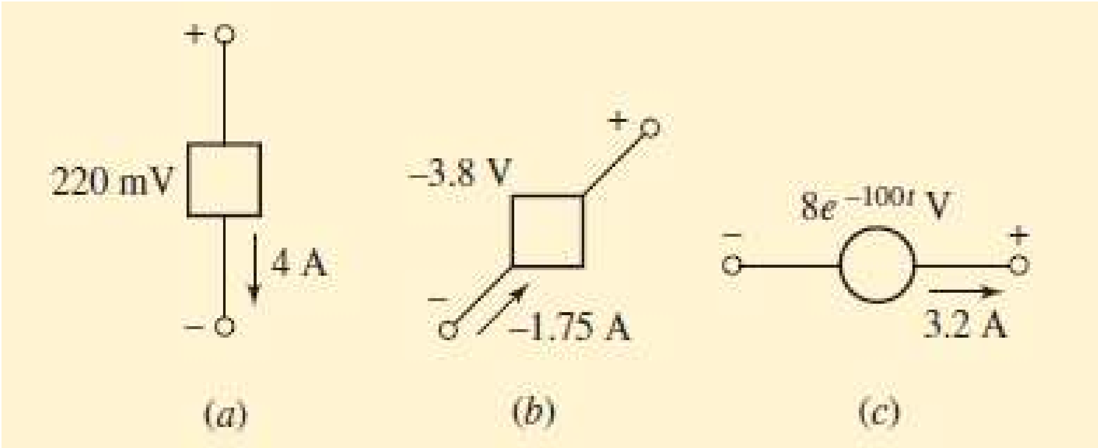
$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$$

Examples

- Compute the power absorbed by each part



- Determine the power absorbed by each element (a,b,c). (t = 5 ms for the element in(c))

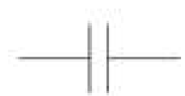


Circuit Elements

- circuit element: mathematical model for the voltage/current behavior of a component (or collection of components)

$$V = I \cdot R \qquad V = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

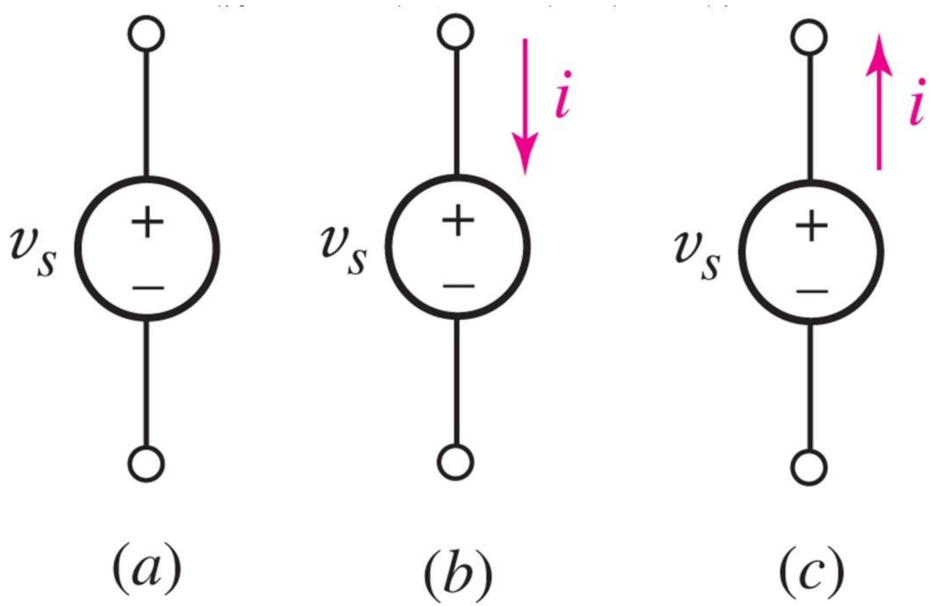




- sources-- independent vs. dependent
  – **independent**: value of the source voltage/current is *not* affected by the rest of the circuit
  – **dependent**: value of the source voltage/current is *controlled* by the rest of the circuit

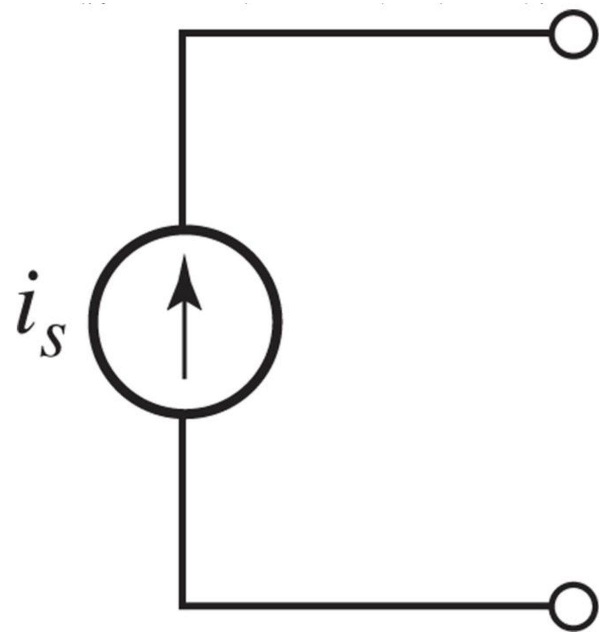
Independent Voltage Source

- An ideal voltage source is a circuit element that will maintain the specified voltage v_s across its terminals.
- The current will be determined by other circuit elements.



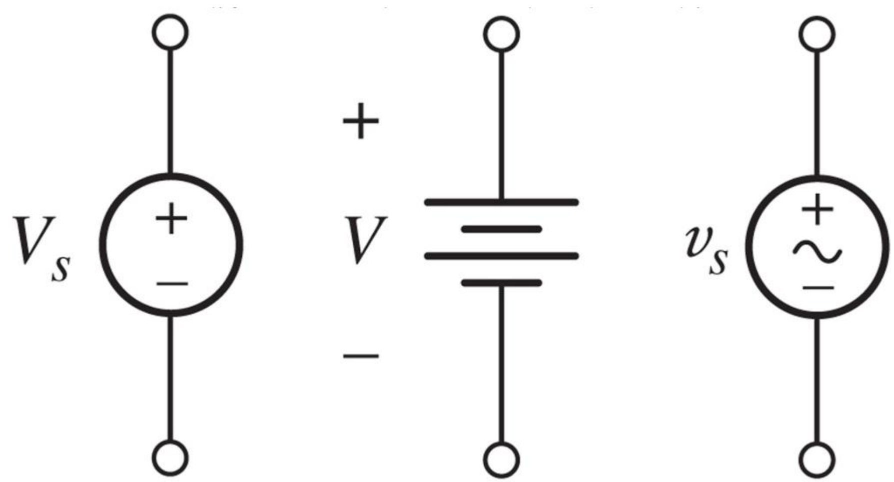
Independent Current Source

- An ideal current source is a circuit element that maintains the specified current flow i_s through its terminals.
- The voltage is determined by other circuit elements.



Battery as Voltage Source

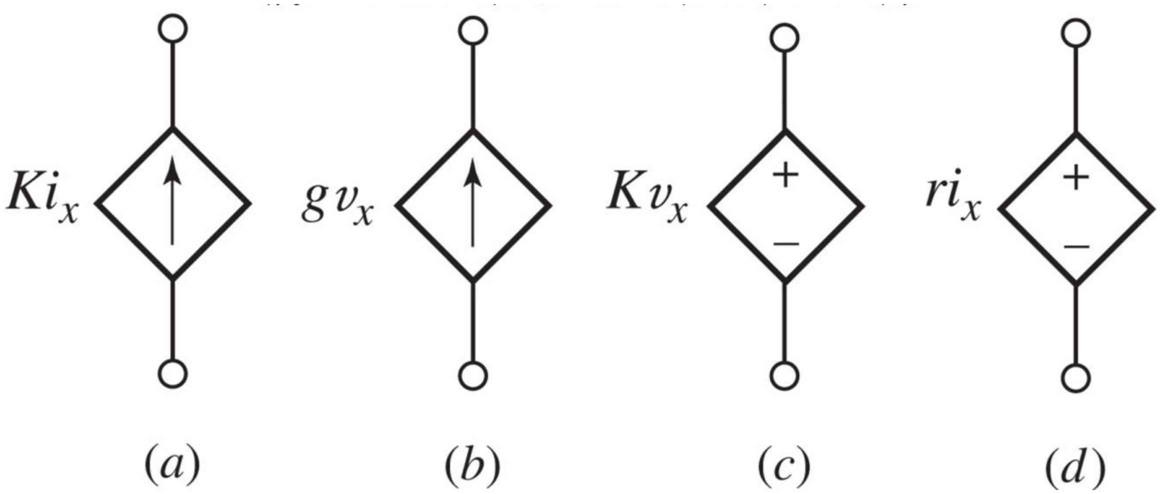
- A voltage source is an idealization (no limit on current) and generalization (voltage can be time-varying) of a battery.



(a) (b) (c)

Dependent Sources

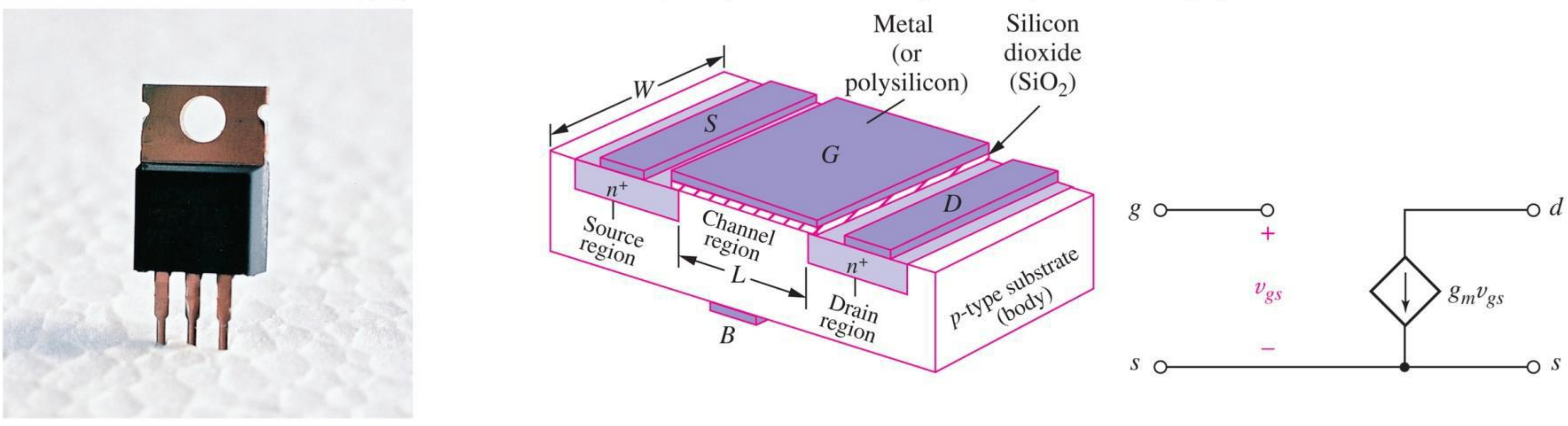
- **dependent V/C source:** source whose V/C is dependent upon or *controlled by* some V/C at another point



The four different types of dependent sources: (*a*) current-controlled current source; (*b*) voltage-controlled current source; (*c*) voltage-controlled voltage source; (*d*) current-controlled voltage source.

Dependent Sources

- useful for analyzing (simplifying) the behavior of complicated circuit elements (e.g. transistors, operational amplifiers)



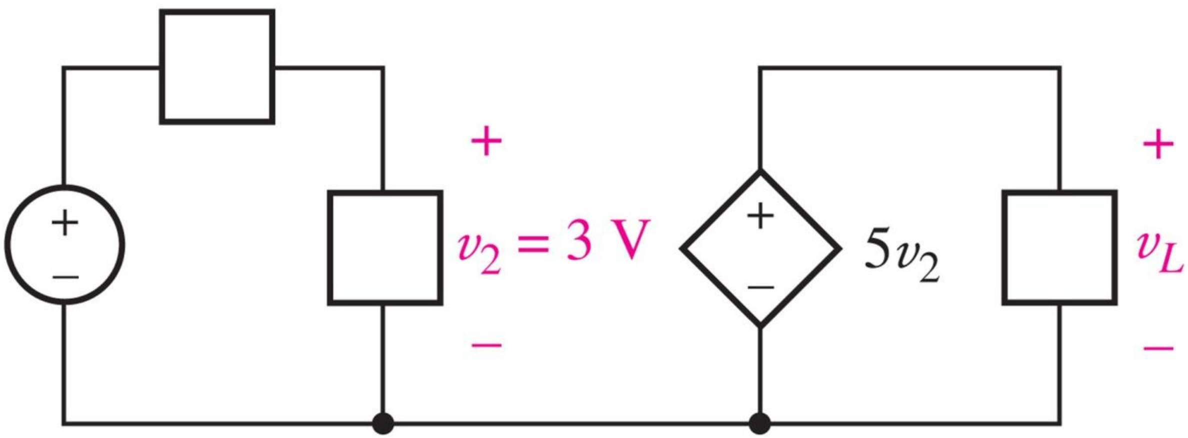
(a) (b) (c)

Metal Oxide Semiconductor Field Effect Transistor (MOSFET)

Example Dependent Sources

- If v_2 = 3 V, Find the voltage v_L in the circuit below.

Assume that all points along a wire that do not cross a circuit element have the same voltage.



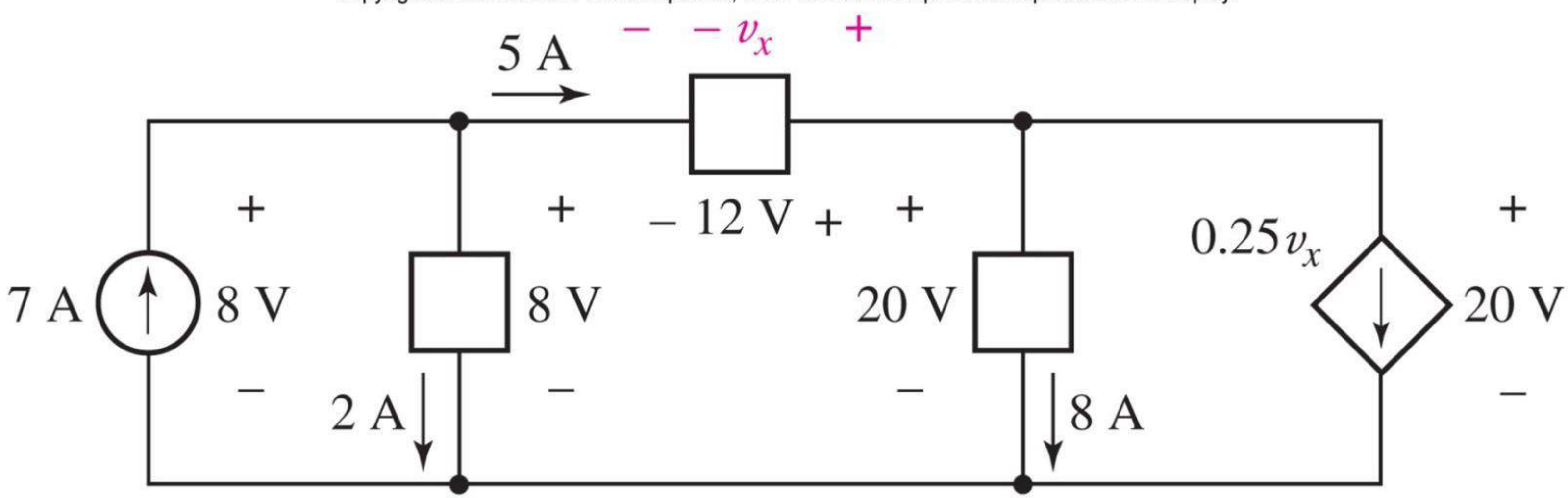
$v_2 = 3$ V

$v_L = 5v_2$

$v_L = 15$ V

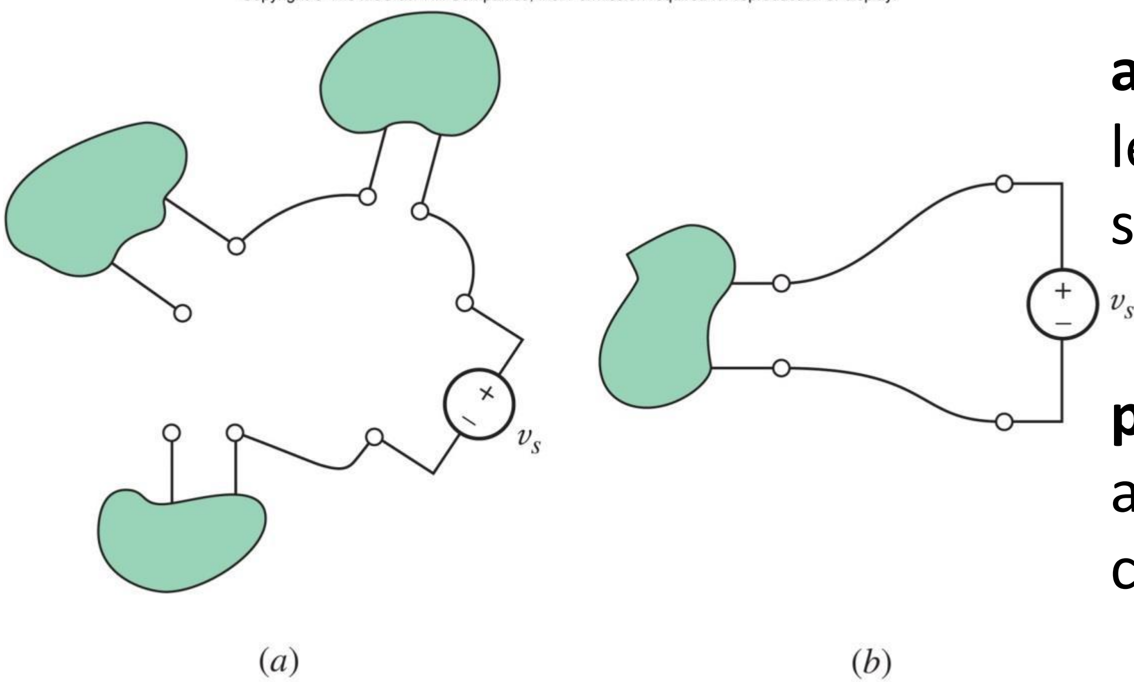
Example Dependent Sources

- Determine the power *absorbed* by each element in the circuit



Networks vs. Circuits

- **network:** interconnection of 2+ elements
- **circuit:** a network that contains at least 1 closed path



network that is **not** a circuit

network that **is** a circuit

active network: contains at least 1 active device (that supplies energy)

passive network: contains no active devices (only consumes energy)

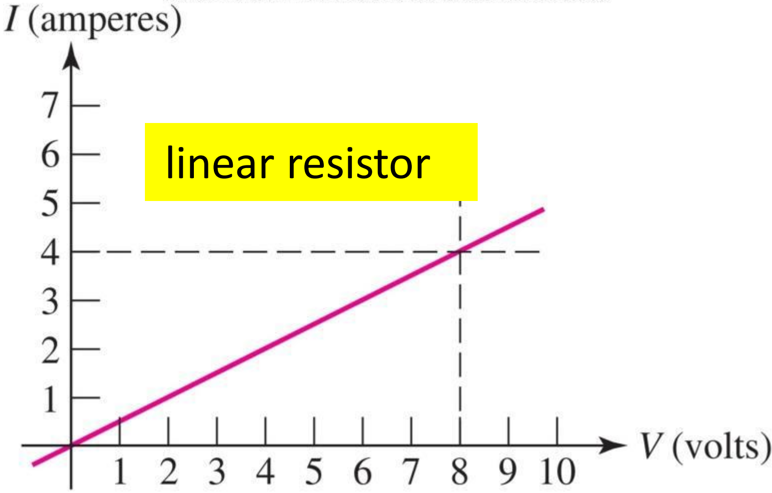
Ohm's Law

- first discussed by Georg Simon Ohm (German physicist) in a pamphlet describing voltage & current measurements

$$V = I \cdot R \qquad v = i \cdot r$$

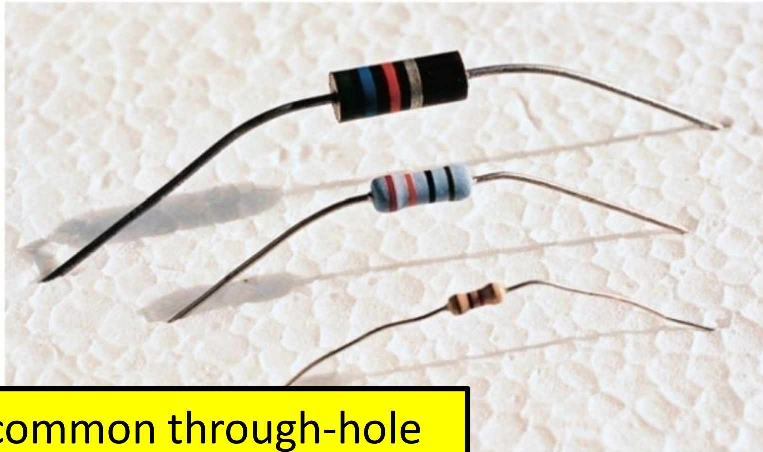
- the *voltage* across a conducting material is *linearly proportional* to the *current* flowing through that material
  - constant of proportionality
    = the *resistance* of the material
- unit of resistance = the ***ohm***

$$1\ \Omega = 1\ \text{V/A}$$

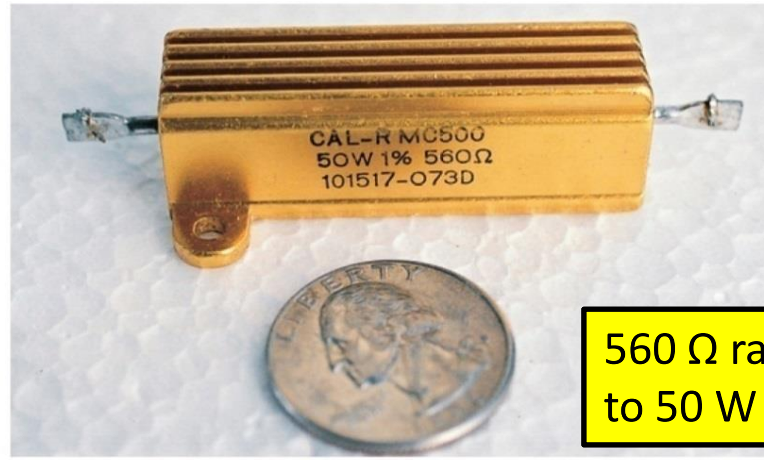


another idealization, but still a good approximation for many elements (over certain ranges of voltage, current)

Resistors

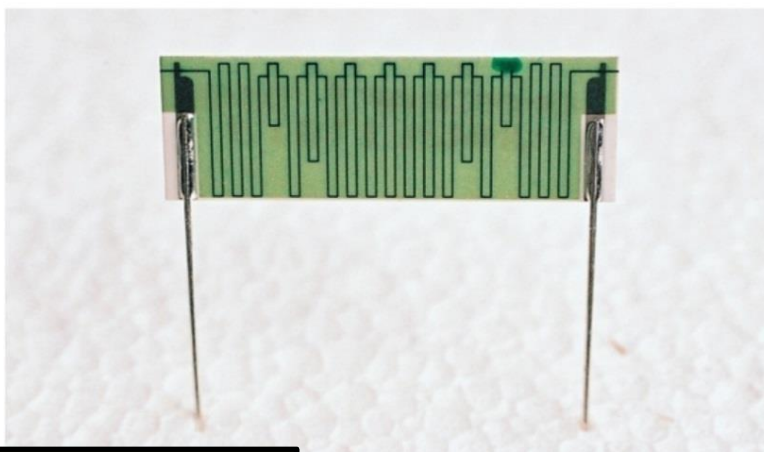


common through-hole components (for breadboarding)



560 Ω rated up to 50 W

(*b*)



10 teraohms (10 x 10^{12} Ω)

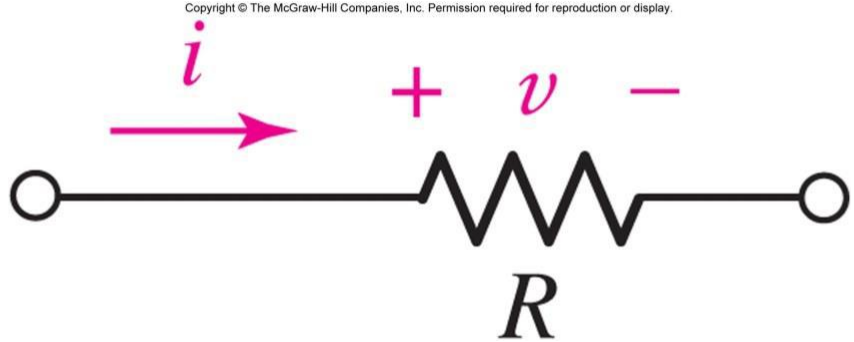
(*c*)

circuit symbol for a resistor

(*d*)

Example

- Let R = 560 Ω and i = 42.4 mA. Calculate the voltage across the resistor and power it is dissipating?



Conductance & Open/Short Circuits

- for a linear resistor, the ratio of current : voltage is also a constant

$$\frac{i}{v} = \frac{1}{R} = G$$

where *G* is called the *conductance*

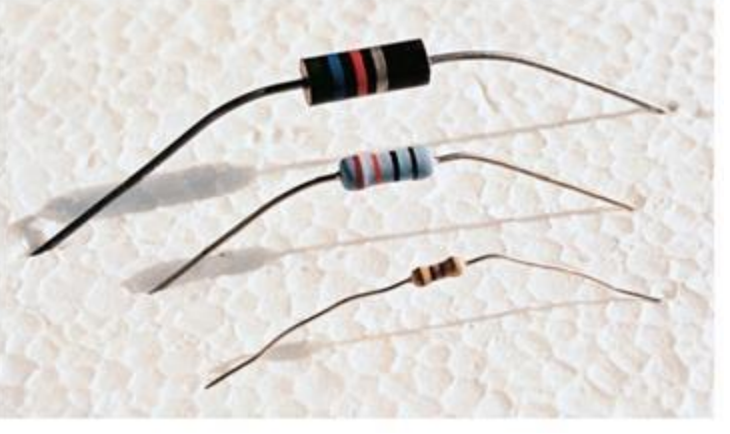
units = ***siemens*** (S), 1 S = 1 A/V = Ω^{-1}

- “**open circuit**”: $R = \infty$, and $i = 0$ for *any* voltage across the open terminals
- “**short circuit**”: $R = 0$, and $v = 0$ for *any* current through the short
  - For all of our circuits, wires are assumed to be perfect short circuits.

  i.e. Since $v = i R$ and wires have $R = 0$, all neighboring points along a wire have the same voltage.

Resistors & Tolerance

- Real resistors are manufactured within a specific *tolerance* (5%, 10%, 20%).
- Given a 50 Ω resistor with a tolerance of 10%, what is the maximum voltage across the resistor when a current of exactly 2 mA flows through it?

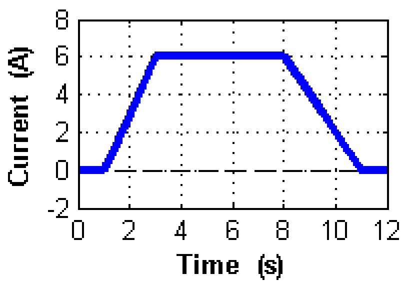


Chapter 2 Summary & Review

- metric units for electricity: coulomb, volt, amp(ere), joule, Watt, ohm (Ω)
- **charge** = polarization of particles; current = flow of charge
- **current** is defined with *value* and *direction* **voltage** is defined with value and polarity (+ terminal, – terminal)
- electrical **power**: *consumed* (current *enters* the + terminal; voltage drops) or *supplied* (current *leaves* the + terminal; voltage rises) is equal to V x I and is an instantaneous or average quantity
- sources: **independent** vs. **dependent** (controlled or not controlled by the rest of the circuit)
- **Ohm's** Law: $V = I R$, a linear relationship that holds for resistors and other circuit elements for certain ranges of V, I

Examples

(a) The current through an element is given in the figure below (bold line). What is the net charge that passes through the element between $t = 4$ and $t = 8$ seconds?

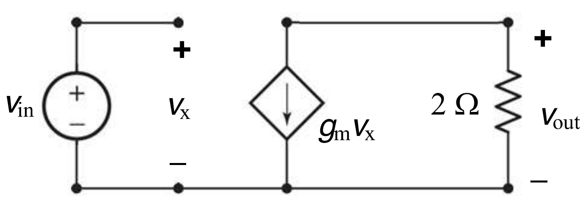


$$q = \int i \, dt$$

$= i \cdot \Delta t$ for constant current

$$= (6\,A)(8-4\,s) = \boxed{24\,C}$$

(b) In the circuit below, $v_{in} = 3\sin(\omega \cdot t)$ mV and $g_m = 10$ A/V. Determine v_{out}.

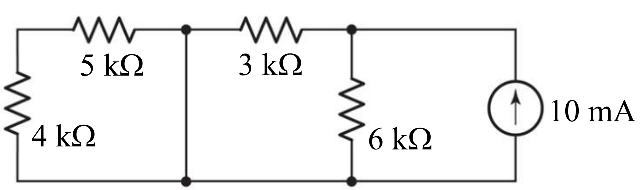


$$v_{out} = -g_m v_x \cdot 2$$

$$= -(10)(3\sin(\omega t))(2)$$

$$= \boxed{-60\sin(\omega t)\ mV}$$

(c) In the circuit below, determine the power absorbed by the 5-kΩ resistc



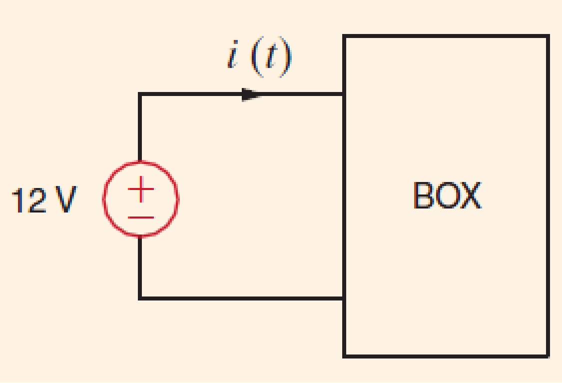
short circuit across 4 kΩ and 5 kΩ

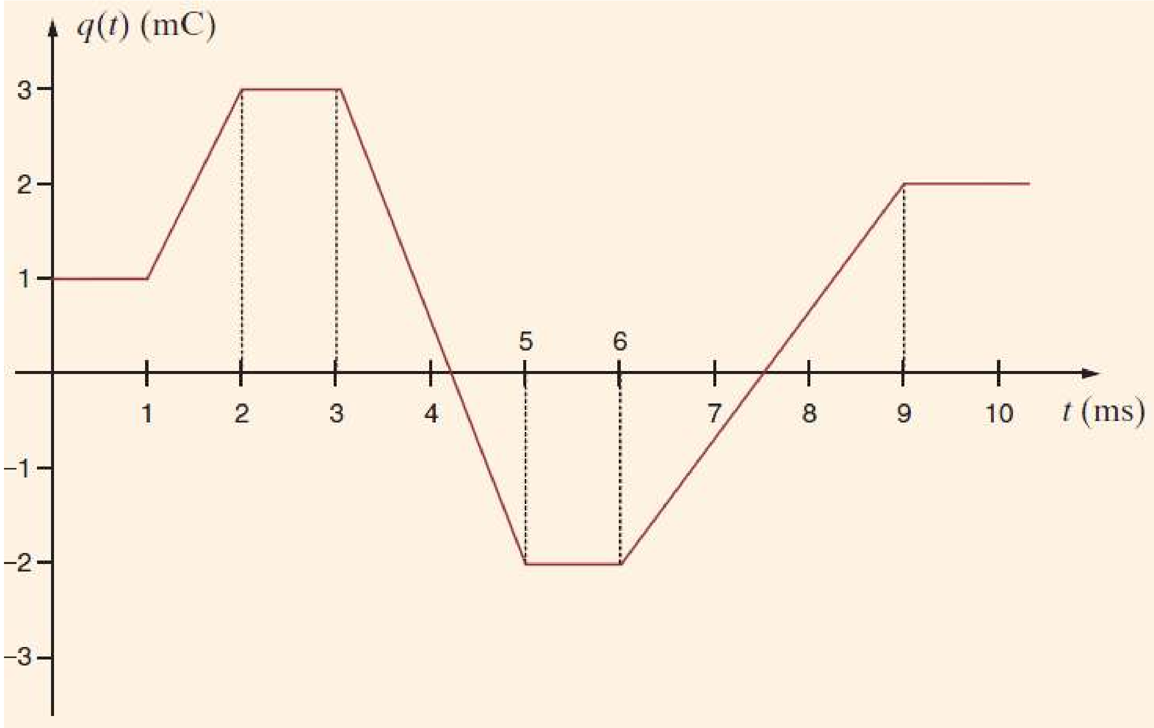
$\Rightarrow$ no current through 5 kΩ

$\Rightarrow$ $\boxed{P_{abs} = 0}$

Examples

- The charge that enters the BOX is shown in below Figure. Calculate and sketch the current flowing into and the power absorbed by the BOX between 0 and 10 milliseconds.





Solution

Recall that current is related to charge by $i(t) = \dfrac{dq(t)}{dt}$. The current is equal to the slope of the charge waveform.

$$i(t) = 0 \qquad 0 \le t \le 1 \text{ ms}$$

$$i(t) = \frac{3 \times 10^{-3} - 1 \times 10^{-3}}{2 \times 10^{-3} - 1 \times 10^{-3}} = 2\text{A} \qquad 1 \le t \le 2 \text{ ms}$$

$$i(t) = 0 \qquad 2 \le t \le 3 \text{ ms}$$

$$i(t) = \frac{-2 \times 10^{-3} - 3 \times 10^{-3}}{5 \times 10^{-3} - 3 \times 10^{-3}} = -2.5\text{ A} \qquad 3 \le t \le 5 \text{ ms}$$

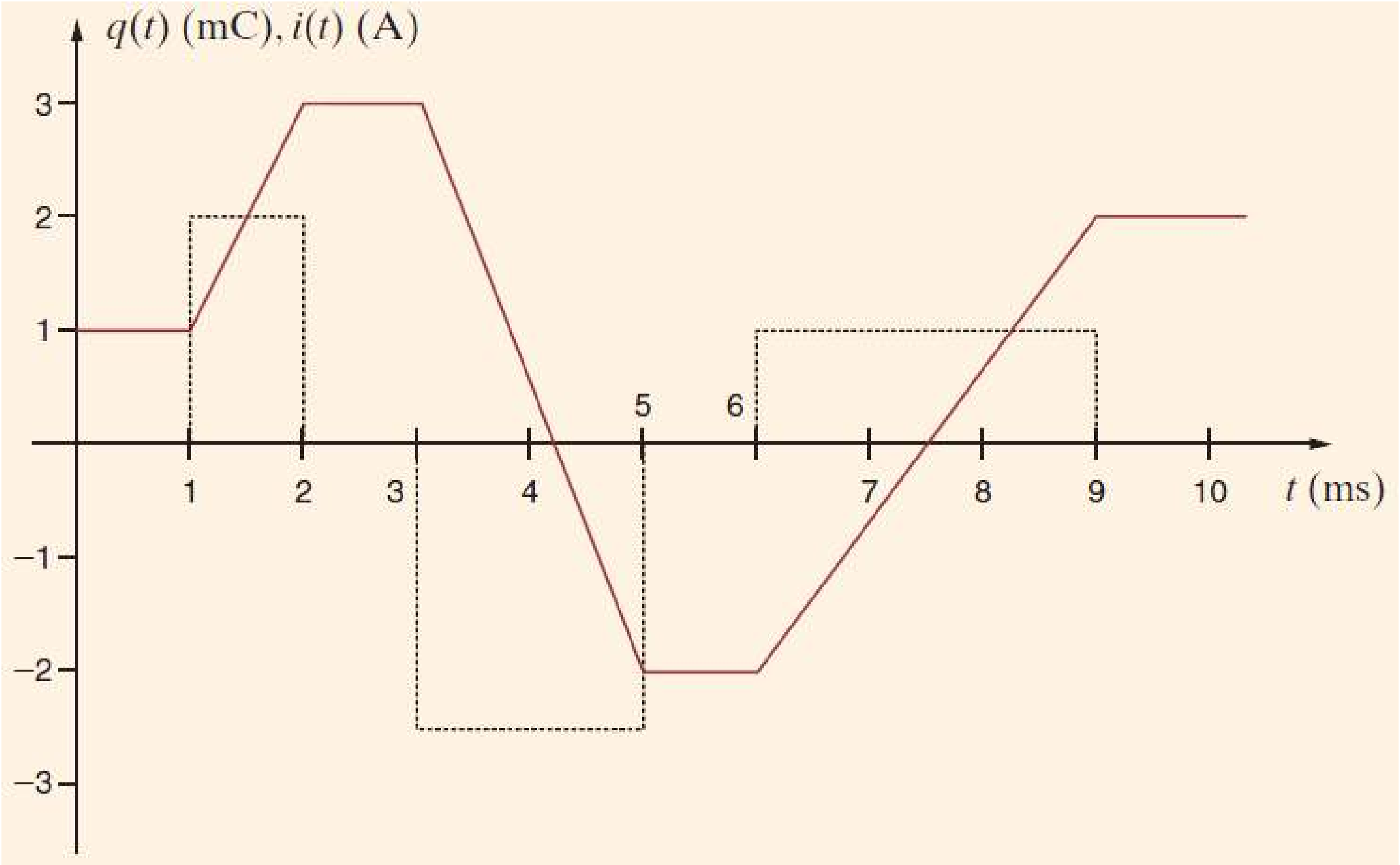
$$i(t) = 0 \qquad 5 \le t \le 6 \text{ ms}$$

$$i(t) = \frac{2 \times 10^{-3} - (-2 \times 10^{-3})}{9 \times 10^{-3} - 6 \times 10^{-3}} = 1.33\text{ A} \qquad 6 \le t \le 9 \text{ ms}$$

$$i(t) = 0 \qquad t \ge 9 \text{ ms}$$

The current is plotted with the charge waveform in Fig. 1.21. Note that the current is zero during times when the charge is a constant value. When the charge is increasing, the current is positive, and when the charge is decreasing, the current is negative.

Solution

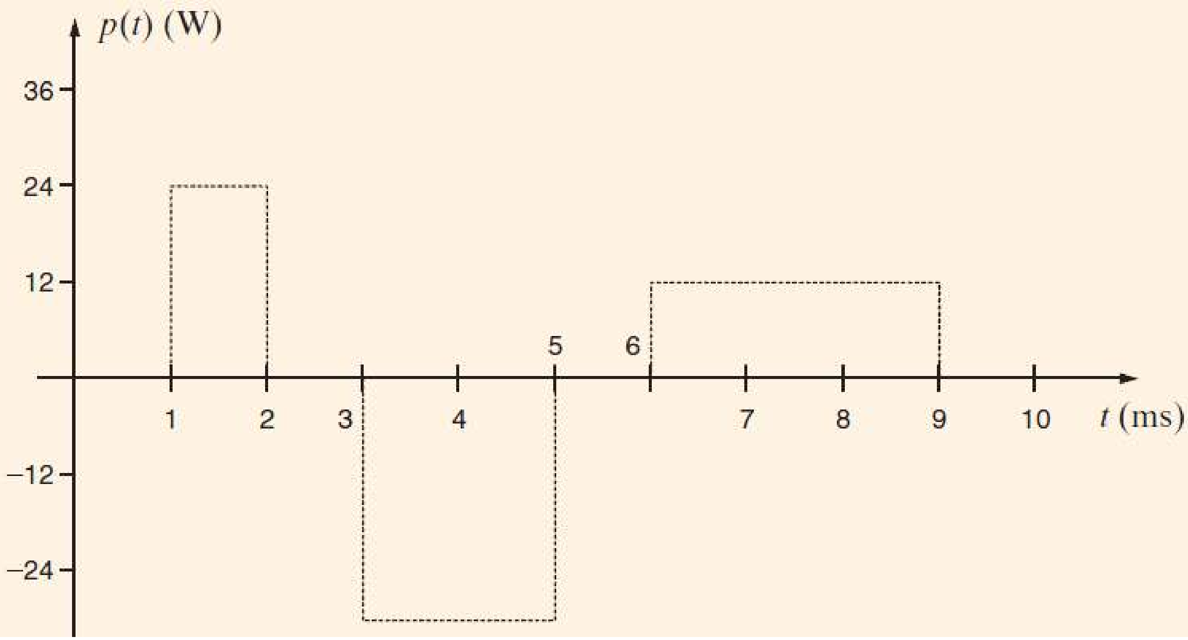


Solution

The power absorbed by the BOX is $12 \cdot i(t)$.

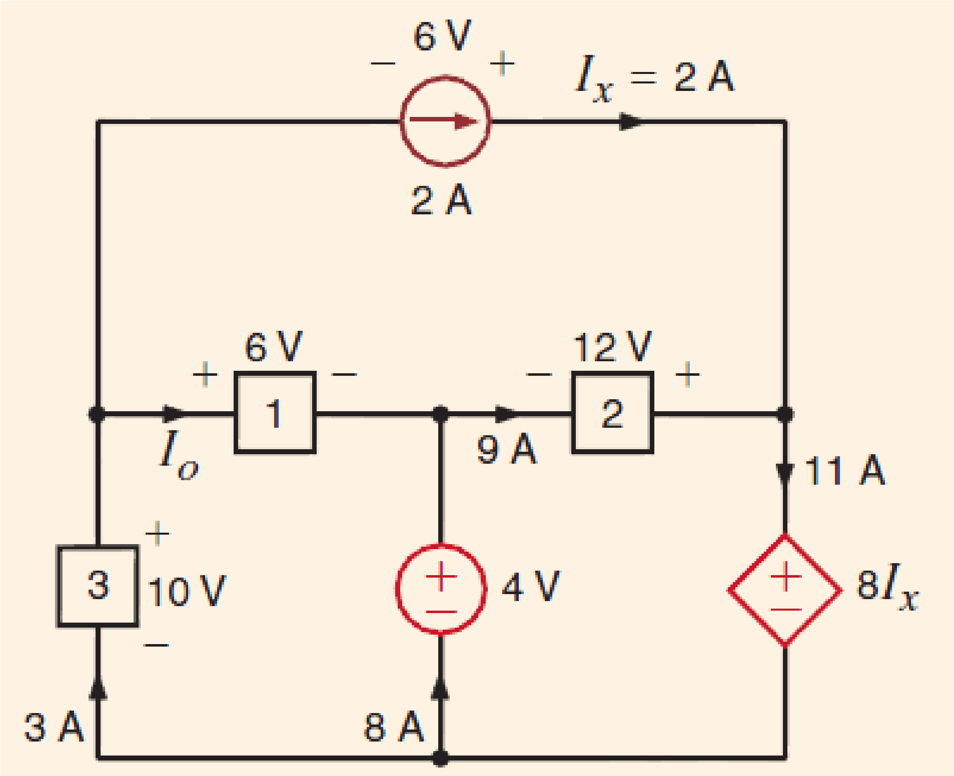
$$
\begin{aligned}
p(t) &= 12*0 = 0 && 0 \le t \le 1 \text{ ms} \\
p(t) &= 12*2 = 24 \text{ W} && 1 \le t \le 2 \text{ ms} \\
p(t) &= 12*0 = 0 && 2 \le t \le 3 \text{ ms} \\
p(t) &= 12*(-2.5) = -30 \text{ W} && 3 \le t \le 5 \text{ ms} \\
p(t) &= 12*0 = 0 && 5 \le t \le 6 \text{ ms} \\
p(t) &= 12*1.33 = 16 \text{ W} && 6 \le t \le 9 \text{ ms} \\
p(t) &= 12*0 = 0 && t \ge 9 \text{ ms}
\end{aligned}
$$

The power absorbed by the BOX is plotted in Fig. 1.22. For the time intervals, $1 \le t \le 2$ ms and $6 \le t \le 9$ ms, the BOX is absorbing power. During the time interval $3 \le t \le 5$ ms, the power absorbed by the BOX is negative, which indicates that the BOX is supplying power to the 12-V source.



Example

- Use Tellegen's theorem to find the current Io in the network in below Figure.



Solution

First, we must determine the power absorbed by each element in the network. Using the sign convention for power, we find

$$
\begin{aligned}
P_{2\,\mathrm{A}} &= (6)(-2) = -12\ \mathrm{W} \\
P_1 &= (6)(I_o) = 6I_o\ \mathrm{W} \\
P_2 &= (12)(-9) = -108\ \mathrm{W} \\
P_3 &= (10)(-3) = -30\ \mathrm{W} \\
P_{4\,\mathrm{V}} &= (4)(-8) = -32\ \mathrm{W} \\
P_{DS} &= (8I_x)(11) = (16)(11) = 176\ \mathrm{W}
\end{aligned}
$$

Applying Tellegen's theorem yields

$$-12 + 6I_o - 108 - 30 - 32 + 176 = 0$$

or

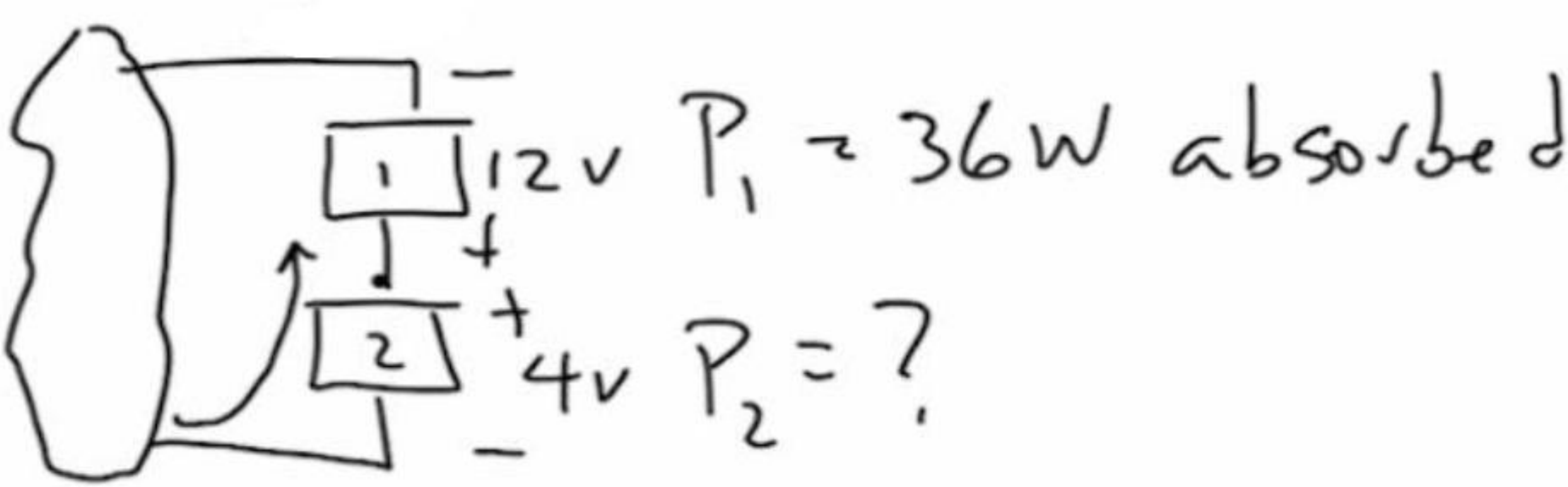
$$6I_o + 176 = 12 + 108 + 30 + 32$$

Hence,

$$I_o = 1\mathrm{A}$$

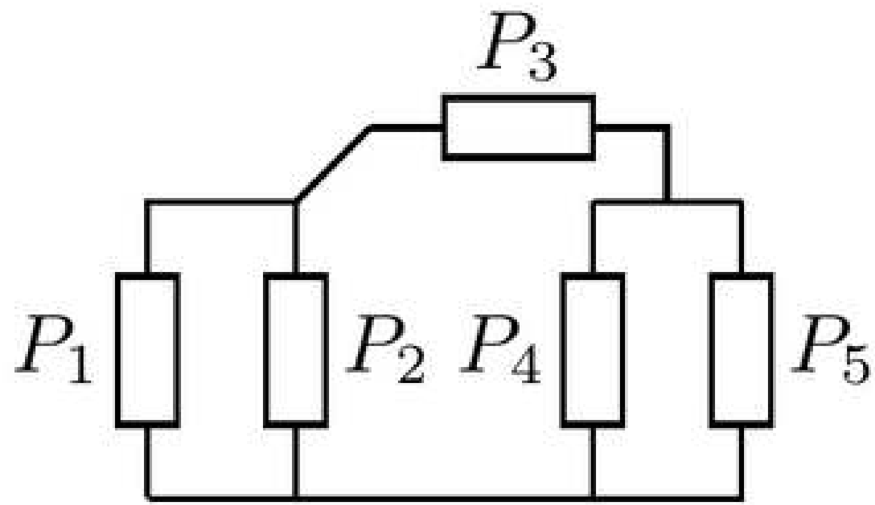
Example

- Determine the missing quantity (power).



Example

Below is a schematic of unknown elements.



We measure the following powers:

$P_1 = -50W$
$P_2 = 40W$
$P_3 = -10W$
$P_4 = 5W$

What is P_5?

$$P_1 + P_2 + P_3 + P_4 + P_5 = 0$$

$$\Longrightarrow P_5 = -(-50 + 40 - 10 + 5)\,W = \boxed{15\,W}$$

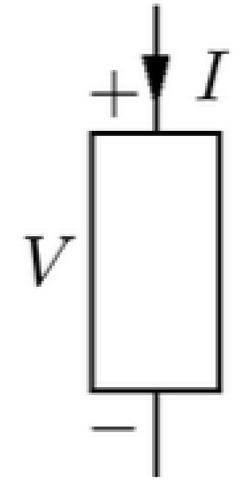
Example

What is the power in Watts across the element if $V = 0.4V$ and $I = 2A$?

$P = +IV$

$= +(2A)(0.4V)$

$= \boxed{0.8W}$

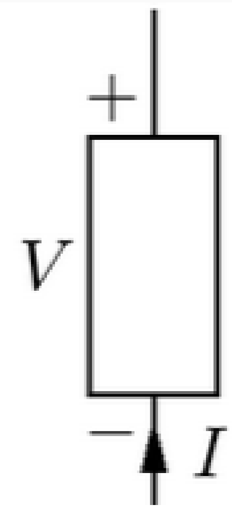


What is the power across the element if $V = 5V$ and $I = 0.1A$?

Opposite of convention:

$P = -IV$

$= -(0.1A)(5V) = \boxed{-0.5W}$



Example

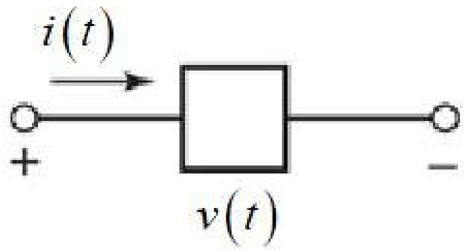
The current through a circuit element, $i(t)$, is plotted below. Assume $v(t) = 2$ V .

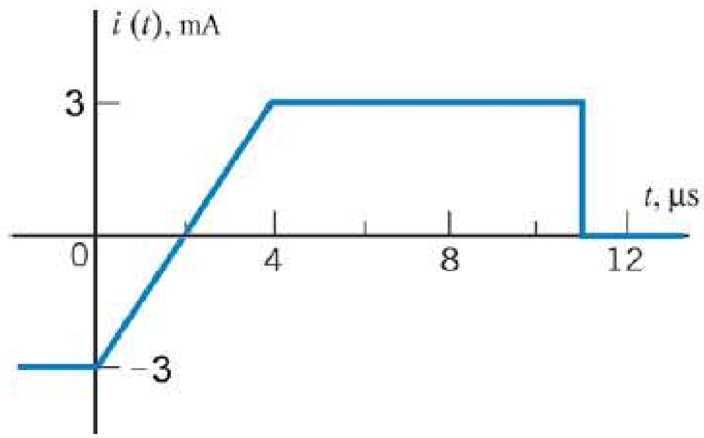
(a) Compute the net charge that passes through the element between $t = 2$ μs and $t = 12$ μs.

(b) Determine if the element absorbs and/or delivers electrical energy.

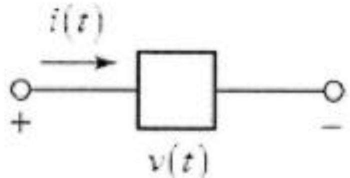
If the element absorbs electrical energy, when does this happen?
If the element delivers electrical energy, when does this happen?

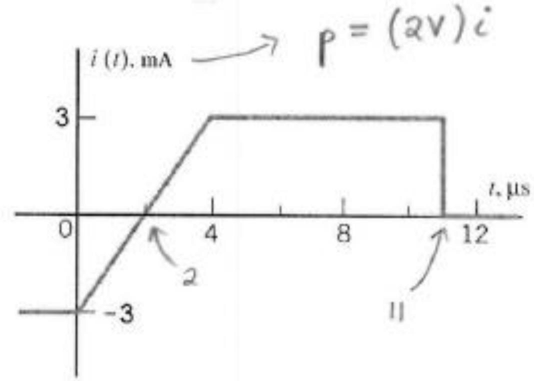
(c) Compute the net energy absorbed by the element between $t = 0$ μs and $t = 9$ μs.





Solution





(a) $$q = \int_{2\mu s}^{12\mu s} i(t)\,d\tau = \left(\tfrac{1}{2}\right)(2\mu s)(3mA) + (7\mu s)(3mA)$$
$$= 3\ nC + 21\ nC = \boxed{24\ nC}$$

(b) if $v \cdot i > 0 \Rightarrow$ absorbs; $v \cdot i < 0 \Rightarrow$ delivers

absorbs from $t = 2 \rightarrow t = 11\ \mu s$
delivers before $t = 2\ \mu s$

(c) $$w = \int_{0\mu s}^{8\mu s} p(t)\,d\tau\ , \quad p(t) = i(t)\,v(t)$$

$$= \cancel{\left(\tfrac{1}{2}\right)(2\mu s)(-6\ mW)} + \cancel{\left(\tfrac{1}{2}\right)(2\mu s)(+6\ mW)}$$
$$+ (5\mu s)(6\ mW) = \boxed{30\ nJ}$$