integral - 1

$(3\sec^2 2x)' = \frac{3\tan 2x}{2}$ $\quad \sec^2 2x$

① $\int \left(\cos 4x + \sin 2x - 2x^3 + \frac{3}{\cos^2 2x} + 2\right) dx = ?$

$\frac{\sin 4x}{4} - \frac{\cos 2x}{2} - \frac{2x^4}{4} + \frac{3\tan 2x}{2} + 2x + c$

② $\int \left(\frac{2}{1+x^2} - \frac{3}{\sqrt{1-x^2}} + \frac{1}{x} - \frac{2}{x^2} + e^{4x} + x + x^2\right) dx = ?$

x^{-1} $\quad -2x^{-2}$

$2\arctan x - 3\arcsin x + \ln|x| + \frac{2}{x} + \frac{e^{4x}}{4} + \frac{x^2}{2} + \frac{x^3}{3} + c$

③ $\int x^2 \cdot \cos(x^3+2)\, dx = ?$

$x^3+2 = u \qquad 3x^2 dx = du$

$\int \frac{\cos u}{3} du = \frac{\sin u}{3} + c$

$\frac{\sin(x^3+2)}{3} + c$

④ $\int_0^2 \frac{dx}{\sqrt{x+2} \cdot \cos^2\sqrt{x+2}} = ?$

$\sqrt{x+2} = u$

$\frac{1}{2\sqrt{x+2}} dx = du$

$\int_0^2 \frac{2}{\cos^2 u} du = \int_0^2 2\sec^2 u\, du = 2\tan u \Big|_0^2 = 2\tan\sqrt{x+2} \Big|_0^2 = 2\tan 2 - 2\tan\sqrt{2}$

⑤ $\int (\tan 4x + \cot 3x)\, dx = ?$

$\int \tan x\, dx = -\ln|\cos x| + c$

$\int \cot x\, dx = \ln|\sin x| + c$

$-\frac{\ln|\cos 4x|}{4} + \frac{\ln|\sin 3x|}{3} + c$

⑥ $\int_0^{\pi/2} \cos x \cdot e^{2\sin x + 3}\, dx = ?$

$2\sin x + 3 = u \qquad 2\cos x\, dx = du$

$\frac{1}{2}\int_0^{\pi/2} e^u du = \frac{1}{2}\left(e^{2\sin x+3} \Big|_0^{\pi/2}\right)$

$\frac{1}{2}\left(e^5 - e^3\right)$

⑦ $\int_0^1 \frac{x+1}{\cos(x^2+2x+1)} dx = ?$

$x^2+2x+1 = u \qquad 2x+2\, dx = du$

$\frac{1}{2}\int_0^1 \frac{1}{\cos u} du = \frac{1}{2}\int_0^1 \sec u\, du = \frac{1}{2}\left(\ln|\sec u + \tan u|\right)\Big|_0^1$

$\frac{1}{2}\left(\ln|\sec 4 + \tan 4| - \ln|\sec 1 + \tan 1|\right)$

⑧ $\int \frac{dx}{x \cdot \tan(\ln x)} = ?$

$\ln x = u \qquad \frac{1}{x} dx = du$

$\int \cot u\, du = \ln|\sin u| + c \quad = \ln|\sin(\ln x)| + c$

⑨ ✗ $\int_0^{\pi/4} \frac{\tan x - 2\ln(\cos x)}{e^{\tan x}} dx = ?$

$e^{\tan x} = u$

$e^{\tan x} \cdot \sec^2 x\, dx = du$

$\int_0^{\pi/4} e^{\tan x} \cdot \frac{1}{\cos^2 x} dx \qquad \int_0^{\pi/4} du$

e-1

Soruldugu sene cok az ögrencinin dogru cözebildigi basit bir soru aslında :)

Cevaplar

① $\int \left(\cos 4x + \sin 2x - 2x^3 + \underbrace{\frac{3}{\cos^2 2x}}_{3\sec^2 2x} + 2 \right) dx = ?$

$$= \frac{\sin 4x}{4} - \frac{\cos 2x}{2} - 2 \cdot \frac{x^4}{4} + 3 \cdot \frac{\tan 2x}{2} + 2x + C$$

② $\int \left(\frac{2}{1+x^2} - \frac{3}{\sqrt{1-x^2}} + \frac{1}{x} - \frac{2}{x^2} + e^{4x} + x + x^2 \right) dx = ?$

$$= 2\operatorname{Arctan} x - 3\operatorname{Arc}\sin x + \ln|x| + \frac{2}{x} + \frac{e^{4x}}{4} + \frac{x^2}{2} + \frac{x^3}{3} + C$$

Bunun yerine $-2\operatorname{Arccot} x$ de yazabiliriz

Bunun yerine $+3\operatorname{Arccos} x$ yazabiliriz

③ $\int x^2 \cdot \cos(x^3+2)\, dx = ?$ — Türevi burda ☺

$u = x^3 + 2$

$du = 3x^2 dx \rightarrow x^2 dx = \frac{du}{3}$

$$= \int \frac{1}{3} \cdot \cos u \, du = \frac{1}{3} \sin u + C = \frac{1}{3} \sin(x^3+2) + C$$

④ $\int_0^2 \frac{dx}{\sqrt{x+2} \cdot \cos^2 \sqrt{x+2}} = ?$

$u = \sqrt{x+2}$ $\quad du = \frac{1}{2\sqrt{x+2}} dx$

$u = \sqrt{x+2}$

E.S	→	4.S
$x = 0$	→	$u = \sqrt{2}$
$x = 2$	→	$u = 2$

$\frac{dx}{\sqrt{x+2}} = 2du$

$$= \int_{\sqrt{2}}^{2} \frac{2\,du}{\cos^2 u}$$

$$= 2\tan u \Big|_{\sqrt{2}}^{2} = 2\tan 2 - 2\tan\sqrt{2}$$

⑤ $\int (\tan 4x + \cot 3x)\,dx = ?$

$\int \tan x\,dx = -\ln|\cos x| + c$

$\int \cot x\,dx = \ln|\sin x| + c$

$= -\frac{1}{4}\cdot \ln|\cos 4x| + \frac{1}{3}\ln|\sin x| + c$

⑥ $\int_0^{\pi/2} \cos x \cdot e^{2\sin x + 3}\,dx \Rightarrow$ Türevi burda

$u = 2\sin x + 3 \qquad du = 2\cos x\,dx$

$\cos x\,dx = \frac{du}{2}$

E.S	U.S
$x = 0$	$\rightarrow u = 3$
$x = \frac{\pi}{2}$	$\rightarrow u = 5$

$= \int_3^5 \frac{1}{2}\cdot e^u\,du = \frac{1}{2}e^u\Big|_3^5 = \frac{1}{2}(e^5 - e^3)$

⑦ $\int_0^1 \frac{x+1}{\cos(x^2+2x+1)}\,dx$

$x^2 + 2x + 1 = u \rightarrow du = (2x+2)\,dx$

$\frac{du}{2} = (x+1)\,dx$

E.S.	U.S
$x = 0$	$\rightarrow u = 1$
$x = 1$	$\rightarrow u = 4$

$= \int_1^4 \frac{1}{2}\cdot \frac{du}{\cos u} = \frac{1}{2}\int_1^4 \sec u\,du = \frac{1}{2}\ln|\sec u + \tan u|\Big|_1^4$

$= \frac{1}{2}\ln|\sec 4 + \tan 4| - \frac{1}{2}\ln|\sec 1 + \tan 1|$

(8) $\int \frac{dx}{x \cdot \tan(\ln x)} = ?$ $\quad u = \ln x \quad du = \frac{dx}{x}$

$$= \int \frac{du}{\tan u} = \int \cot u \, du = \ln|\sin u| + C = \ln|\sin(\ln x)| + C$$

(9) $I = \int_0^{\pi/4} e^{\tan x - 2\ln(\cos x)} \, dx = ?$ $\quad *\left(e^{\ln f(x)} = f(x)\right)*$

$$e^{\tan x - 2\ln(\cos x)} = e^{\tan x} \cdot e^{-2\ln(\cos x)} = e^{\tan x} \cdot e^{\ln(\cos x)^{-2}}$$

$$= e^{\tan x} \cdot (\cos x)^{-2}$$

$$= e^{\tan x} \cdot \sec^2 x$$

$$I = \int_0^{\pi/4} \underbrace{e^{\tan x}}_{e^u} \cdot \underbrace{\sec^2 x \, dx}_{du}$$

$u = \tan x \rightarrow du = \sec^2 x \, dx$

E.S	U.S
$x = 0$	$u = 0$
$x = \frac{\pi}{4}$	$u = 1$

$$= \int_0^1 e^u \, du = e^u \Big|_0^1$$

$$= e - 1$$