

# Robot Teknolojisine Giriş

## BLM4830



Öğr. Grv. Furkan ÇAKMAK

### Ders Tanıtım Formu ve Konular

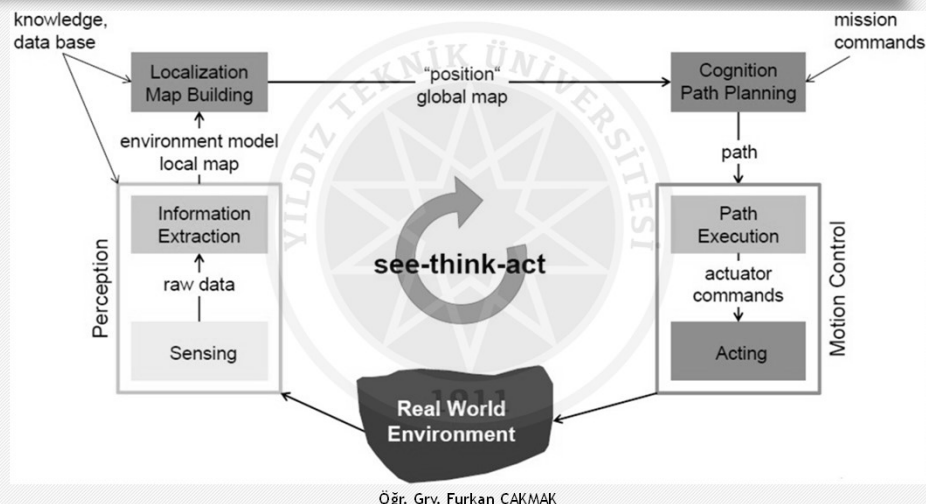
BLM4830  
Robot  
Teknolojisine  
Giriş  
Hafta 2

Hafta	Tarih	Konular
1	2.03.2022	Ders Tanıtımı, ROS ve Platform Tanıtımı, Robot Çeşitleri ve Robotik Konuları Başlangıcı
2	9.03.2022	Kinematik - Genel Tanımlar - Diferansiyel Sürürlü Robot İçin Hesaplama Örnekleri
3	16.03.2022	Sensörler - Çeşitleri ve Çalışma Sistem atikleri ve Uygulamaları
4	23.03.2022	Odometri ve Lokalizasyon Kavramları
5	30.03.2022	Uygulama 1 (Laboratuvar)
6	6.04.2022	Haritalama Yöntemleri ve Uygulamaları
7	13.04.2022	Navigasyon ve Keşif Yaklaşımları ve Uygulamaları (Ödev Teslimi)
8	20.04.2022	Ara Sınav
9	27.04.2022	Uygulama 2 (Laboratuvar)
10	4.05.2022	Tatil - Ramazan Bayramı Arifesi
11	11.05.2022	Robot Üzerinden Görüntü İşleme Teknikleri
12	18.05.2022	Robot Üzerinden Görüntü İşleme Teknikleri (Devam)
13	25.05.2022	3B Haritalama Yöntemleri
14	1.06.2022	Proje Sunumları

Öğr. Grv. Furkan ÇAKMAK

## See-Think-Act Cycle

BLM4830  
Robot  
Teknolojisine  
Giriş  
Hafta 2



## Wheeled Mobile Robots

BLM4830  
Robot  
Teknolojisine  
Giriş  
Hafta 2

- Combination of various physical (hardware) and computational (software) components
- **A collection of subsystems:**
  - **Locomotion:** How the robot moves through its environment.
  - **Sensing:** How the robot measures properties of itself and its environment.
  - **Control:** How the robot generate physical actions.
  - **Reasoning:** How the robot maps measurements into actions.
  - **Communication:** How the robots communicate with each other or with an outside operator.

# Wheeled Mobile Robots

1

## Sabırla Dinlediğiniz İçin Teşekkürler



# Wheeled Mobile Robots

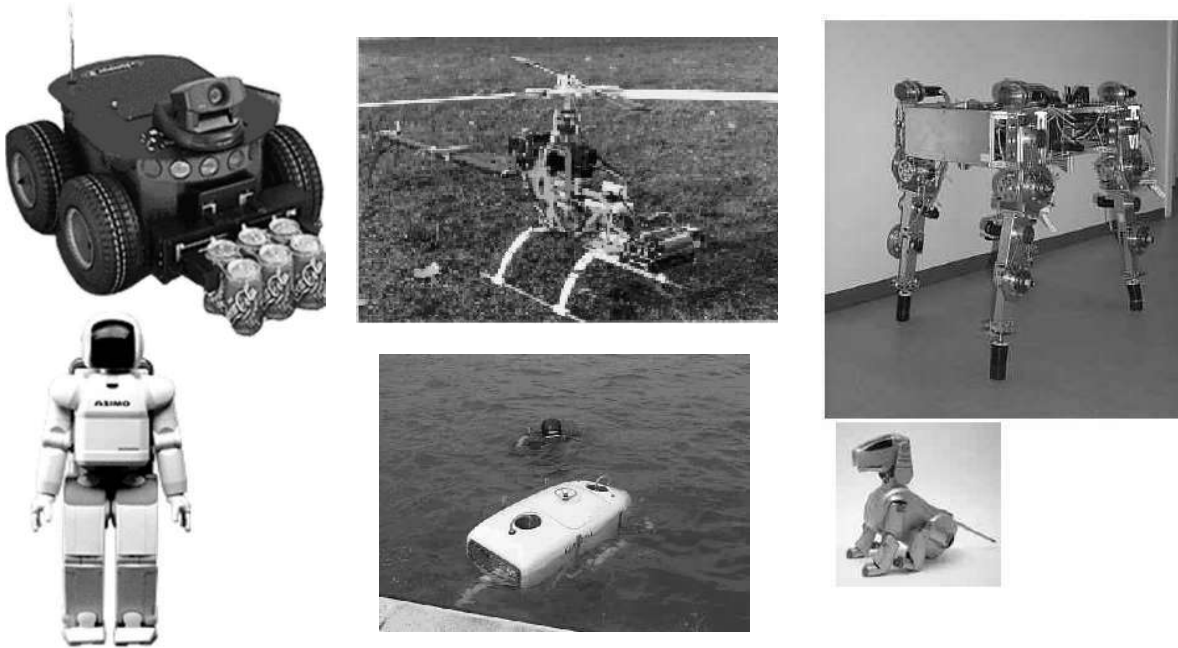
1

## Contents

- Introduction
- Classification of wheels
  - Fixed wheel
  - Centered orientable wheel
  - Off-centered orientable wheel
  - Swedish wheel
- Mobile Robot Locomotion
  - Differential Drive
  - Tricycle
  - Synchronous Drive
  - Omni-directional
  - Ackerman Steering
- Kinematics models of WMR
- Summary

2

# Locomotion



- **Locomotion** is the process of causing an autonomous robot to move
  - In order to produce motion, forces must be applied to the vehicle

3

## Wheeled Mobile Robots (WMR)



4

# Wheeled Mobile Robots

- **Combination of various physical (hardware) and computational (software) components**
- **A collection of subsystems:**
  - **Locomotion:** how the robot moves through its environment
  - **Sensing:** how the robot measures properties of itself and its environment
  - **Control:** how the robot generate physical actions
  - **Reasoning:** how the robot maps measurements into actions
  - **Communication:** how the robots communicate with each other or with an outside operator

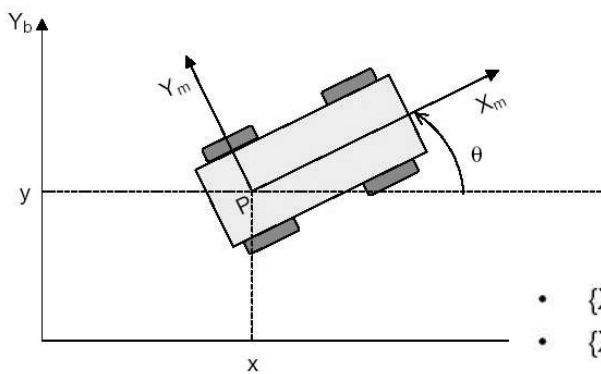
5

# Wheeled Mobile Robots

- **Locomotion** — the process of causing an robot to move.
  - In order to produce motion, forces must be applied to the robot
  - Motor output, payload
- **Kinematics** – study of the mathematics of motion without considering the forces that affect the motion.
  - Deals with the geometric relationships that govern the system
  - Deals with the relationship between control parameters and the behavior of a system.
- **Dynamics** – study of motion in which these forces are modeled
  - Deals with the relationship between force and motions.

6

# Notation



Pose/Posture: position(x, y)  
and orientation  $\theta$

- $\{X_m, Y_m\}$  – moving frame
- $\{X_b, Y_b\}$  – base frame

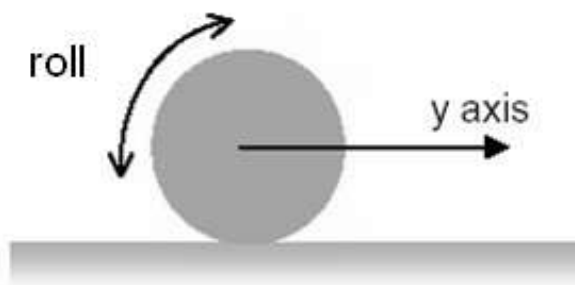
$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad \text{robot posture in base frame}$$

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

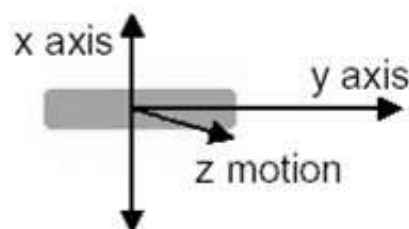
Rotation matrix expressing  
the orientation of the base  
frame with respect to the  
moving frame

7

# Wheels



Rolling motion



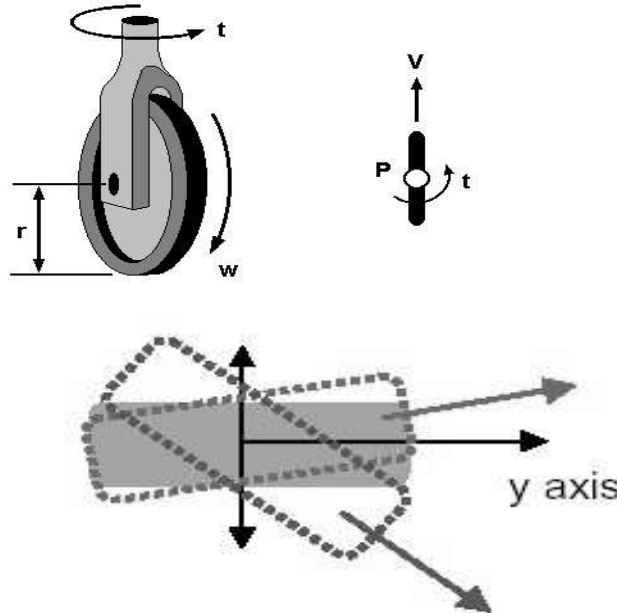
Lateral slip

8

# Steered Wheel

- **Steered wheel**

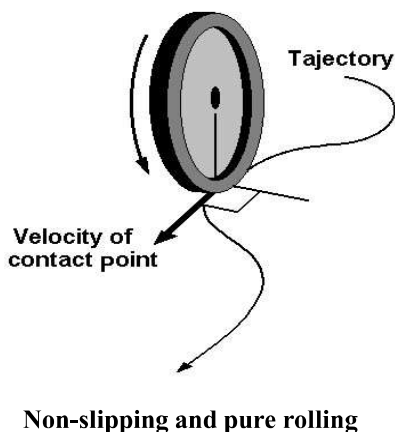
- The orientation of the rotation axis can be controlled



9

# Idealized Rolling Wheel

- **Assumptions**



1. The robot is built from rigid mechanisms.
2. No slip occurs in the orthogonal direction of rolling (non-slipping).
3. No translational slip occurs between the wheel and the floor (pure rolling).
4. The robot contains at most one steering link per wheel.
5. All steering axes are perpendicular to the floor.

10



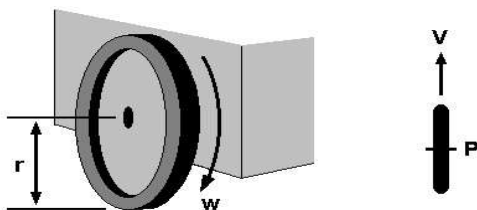
# Robot wheel parameters

- For low velocities, rolling is a reasonable wheel model.
  - This is the model that will be considered in the kinematics models of WMR
- Wheel parameters:
  - $r$  = wheel radius
  - $v$  = wheel linear velocity
  - $w$  = wheel angular velocity
  - $t$  = steering velocity

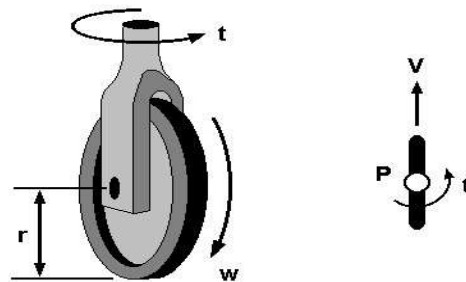
11

## Wheel Types

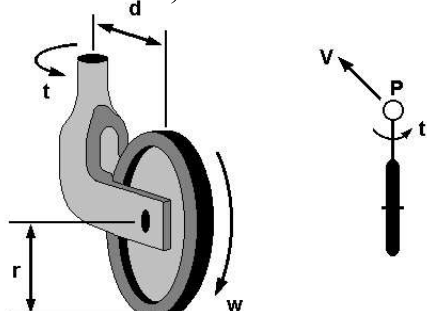
Fixed wheel



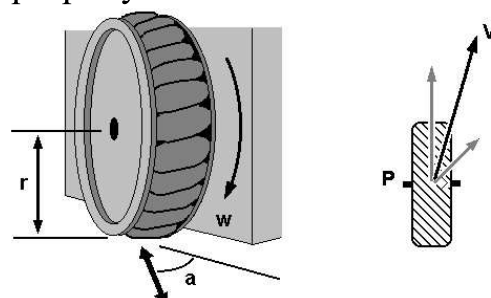
Centered orientable wheel



Off-centered orientable wheel  
(Caster wheel)



Swedish wheel: omnidirectional property



12

# Fixed wheel

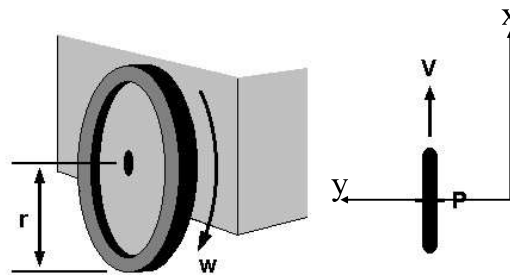
- Velocity of point **P**

$$\mathbf{V} = (\mathbf{r} \times \mathbf{w}) \mathbf{a}_x$$

where,  $\mathbf{a}_x$  : A unit vector to X axis

- Restriction to the robot mobility

Point **P** cannot move to the direction perpendicular to plane of the wheel.



13

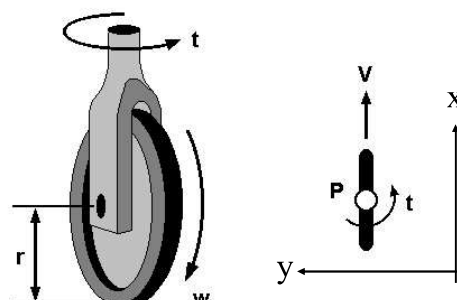
# Centered orientable wheels

- Velocity of point **P**

$$\mathbf{V} = (\mathbf{r} \times \mathbf{w}) \mathbf{a}_x$$

where,  $\mathbf{a}_x$  : A unit vector of x axis  
 $\mathbf{a}_y$  : A unit vector of y axis

- Restriction to the robot mobility



14

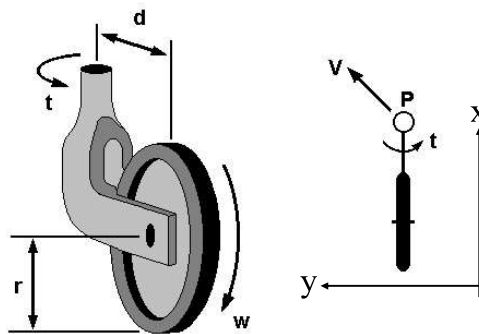
# Off-Centered Orientable Wheels

- Velocity of point **P**

$$\mathbf{v} = (\mathbf{r} \times \mathbf{w})\mathbf{a}_x + (\mathbf{d} \times \mathbf{t})\mathbf{a}_y$$

where,  $\mathbf{a}_x$  : A unit vector of x axis  
 $\mathbf{a}_y$  : A unit vector of y axis

- Restriction to the robot mobility



15

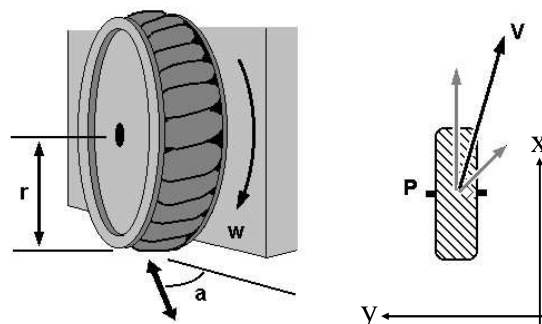
# Swedish wheel

- Velocity of point **P**

$$\mathbf{v} = (\mathbf{r} \times \mathbf{w})\mathbf{a}_x + U\mathbf{a}_s$$

where,  $\mathbf{a}_x$  : A unit vector of x axis  
 $\mathbf{a}_s$  : A unit vector to the motion of roller

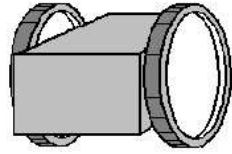
- Omnidirectional property



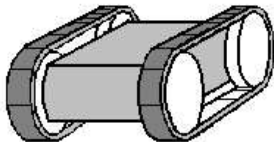
16

# Examples of WMR

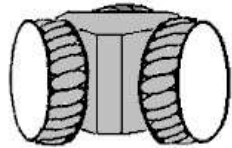
## Example



Bi-wheel type robot



Caterpillar type robot



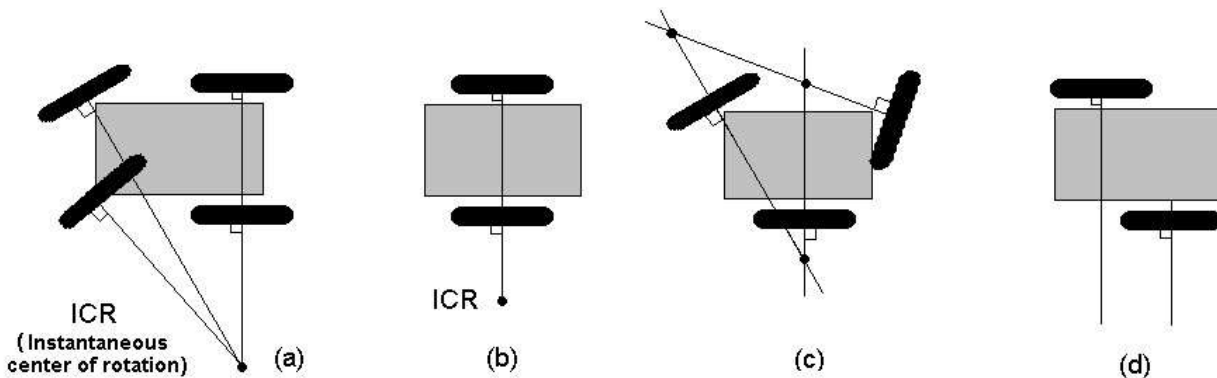
Omnidirectional robot

- Smooth motion
- Risk of slipping
- Some times use roller-ball to make balance
- Exact straight motion
- Robust to slipping
- Inexact modeling of turning
- Free motion
- Complex structure
- Weakness of the frame

17

## Mobile Robot Locomotion

- Instantaneous center of rotation (ICR) or Instantaneous center of curvature (ICC)
  - A cross point of all axes of the wheels



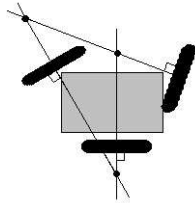
*We talk about the instantaneous center, because we'll analyze this at each instant- the curve may, and probably will, change in the next moment.*

18

# Degree of Mobility

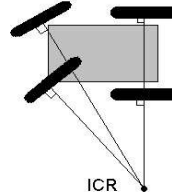
- Degree of mobility**

The degree of freedom of the robot motion



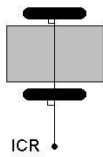
Cannot move anywhere (No ICR)

- Degree of mobility : 0



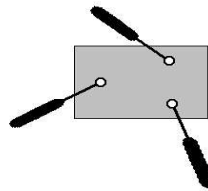
Fixed arc motion (Only one ICR)

- Degree of mobility : 1



Variable arc motion (line of ICRs)

- Degree of mobility : 2



Fully free motion (ICR can be located at any position)

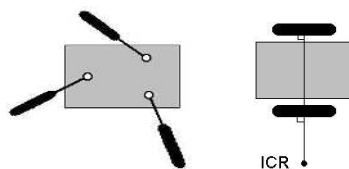
- Degree of mobility : 3

19

# Degree of Steerability

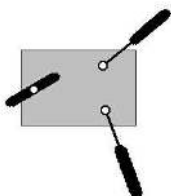
- Degree of steerability**

The number of centered orientable wheels that can be steered independently in order to steer the robot

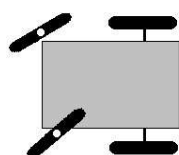


No centered orientable wheels

- Degree of steerability : 0

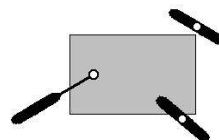


One centered orientable wheel



Two mutually dependent centered orientable wheels

- Degree of steerability : 1



Two mutually independent centered orientable wheels

- Degree of steerability : 2

20

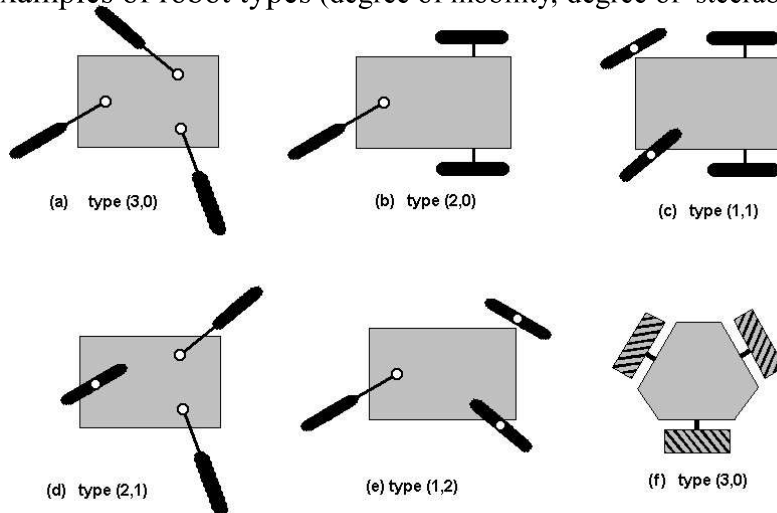
# Degree of Maneuverability

- The overall degrees of freedom that a robot can manipulate:

$$\delta_M = \delta_m + \delta_s$$

Degree of Mobility	3	2	2	1	1
Degree of Steerability	0	0	1	1	2

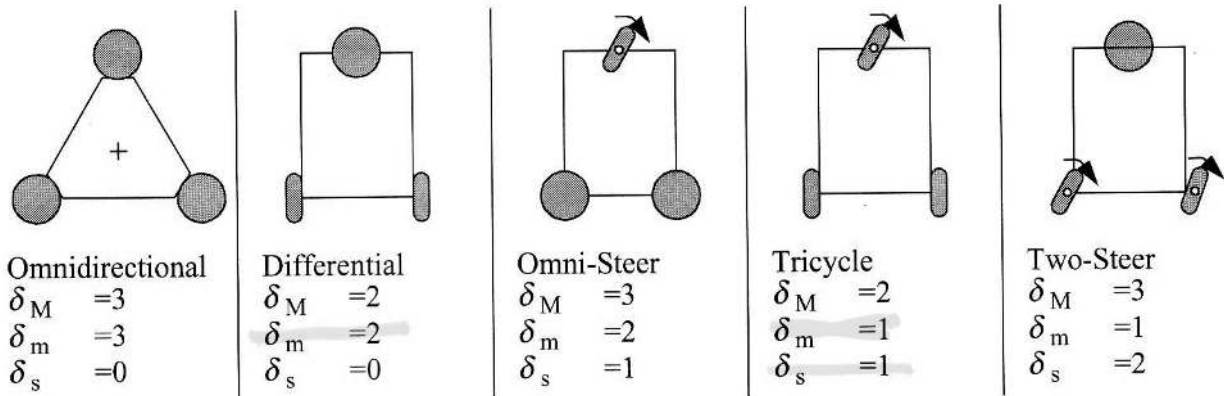
- Examples of robot types (degree of mobility, degree of steerability)



21

# Degree of Maneuverability

$$\delta_M = \delta_m + \delta_s$$



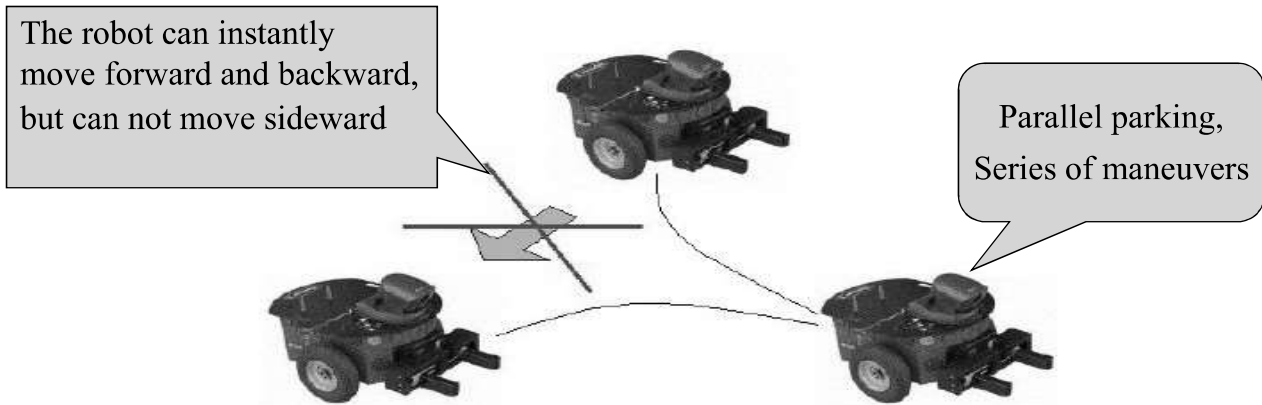
22

# Non-holonomic constraint

A non-holonomic constraint is a constraint on the feasible velocities of a body

*So what does that mean?*

**Your robot can move in some directions (forward and backward), but not others (sideward).**



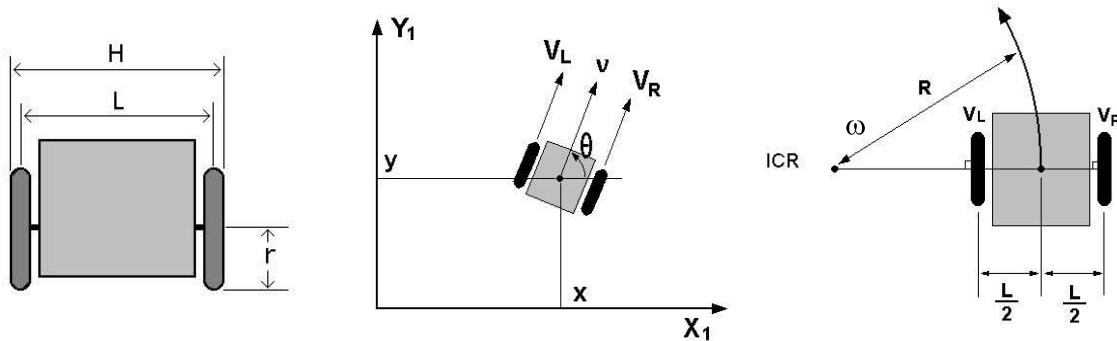
23

# Mobile Robot Locomotion

- Differential Drive
  - two driving wheels (plus roller-ball for balance)
  - simplest drive mechanism
  - sensitive to the relative velocity of the two wheels (small error result in different trajectories, not just speed)
- Steered wheels (tricycle, bicycles, wagon)
  - Steering wheel + rear wheels
  - cannot turn  $\pm 90^\circ$
  - limited radius of curvature
- Synchronous Drive
- Omni-directional
- Car Drive (Ackerman Steering)

24

# Differential Drive



- Posture of the robot

$$P = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \quad \begin{matrix} (x,y) : \text{Position of the robot} \\ \theta : \text{Orientation of the robot} \end{matrix}$$

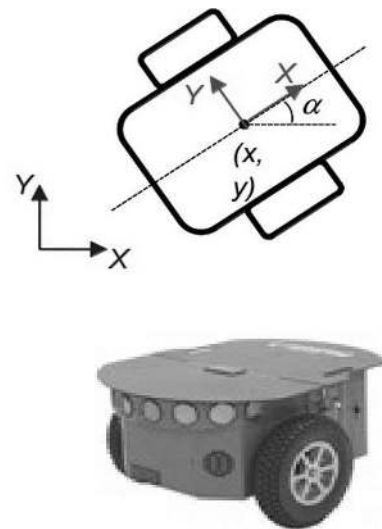
- Control input

$$U = \begin{pmatrix} v \\ w \end{pmatrix} \quad \begin{matrix} v : \text{Linear velocity of the robot} \\ w : \text{Angular velocity of the robot} \\ \text{(notice: not for each wheel)} \end{matrix}$$

25

# Differential Drive

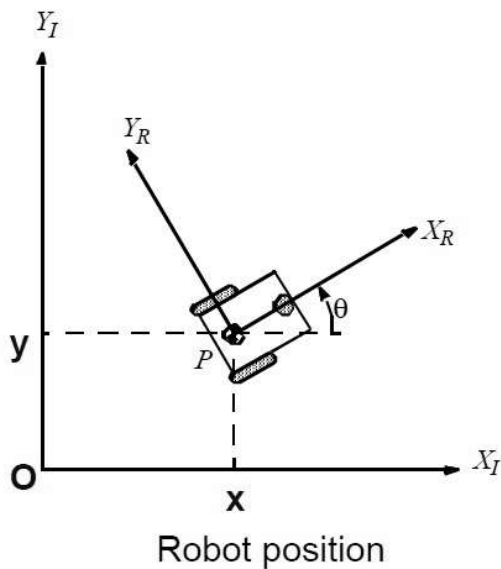
- Two wheels, either side of the robot driven independently
- Steering is achieved by driving the wheels at different speeds
- Two degrees of freedom -> 2 controllable values:  $(V_l, V_r)$



26



# Robot Reference Frame

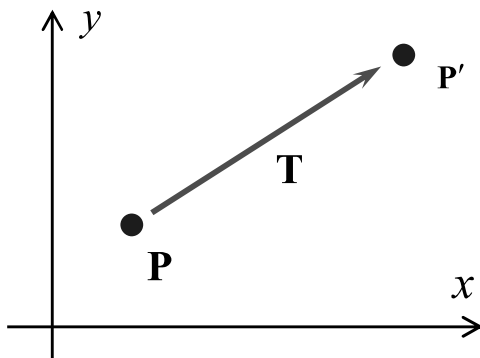


- The robot's reference frame is three dimensional including position on the plane and the orientation,  $\{X_R, Y_R, \theta\}$
- The axes  $\{X_I, Y_I\}$ , define inertial global reference frame with origin,  $O$
- The angular difference between the global and reference frames is  $\theta$
- Point  $P$  on the robot chassis in the global reference frame is specified by coordinates  $(x, y)$

$$\xi_I = [x \quad y \quad \theta]^T$$

27

## 2D Translation

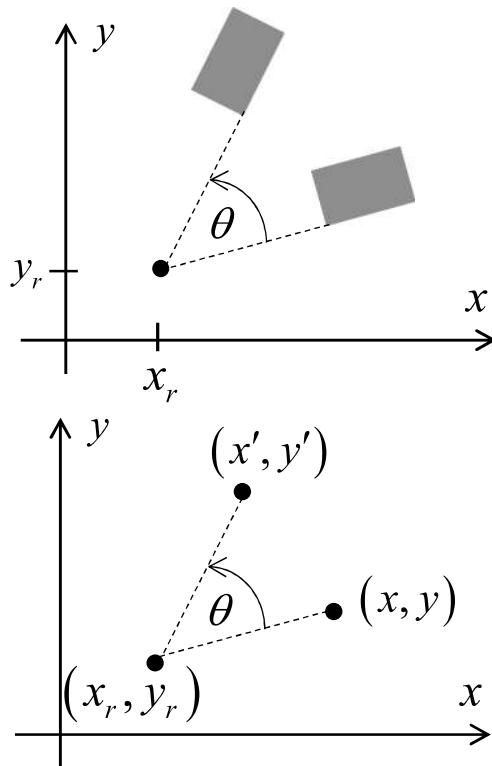


$$x' = x + t_x, \quad y' = y + t_y$$

$$\mathbf{P} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{P}' = \begin{bmatrix} x' \\ y' \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{P} + \mathbf{T}$$

## 2D Rotation



Rotation in angle  $\theta$  about a pivot (rotation) point  $(x_r, y_r)$ .

$$x' = x_r + (x - x_r) \cos \theta - (y - y_r) \sin \theta$$

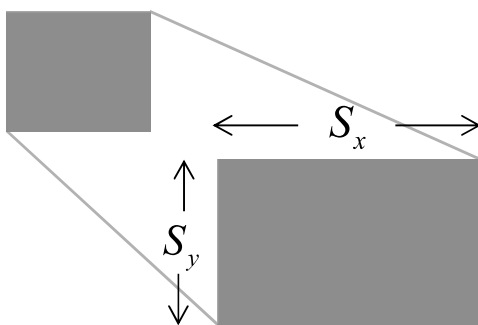
$$y' = y_r + (x - x_r) \sin \theta + (y - y_r) \cos \theta$$

$$\mathbf{P}' = \mathbf{P}_r + \mathbf{R} \cdot (\mathbf{P} - \mathbf{P}_r)$$

$$\mathbf{R} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

29  
April 2010

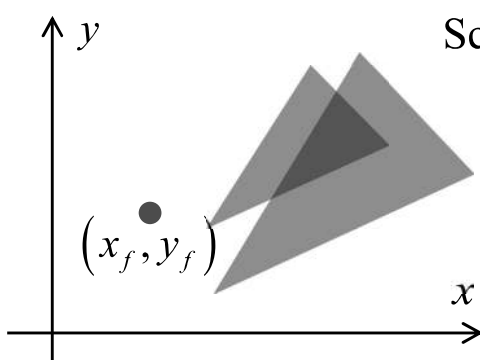
## 2D Scaling



$$x' = x \cdot s_x, \quad y' = y \cdot s_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P}$$



Scaling about a fixed point  $(x_f, y_f)$

$$x' = x \cdot s_x + x_f (1 - s_x)$$

$$y' = y \cdot s_y + y_f (1 - s_y)$$

$$\mathbf{P}' = \mathbf{P} \cdot \mathbf{S} + \mathbf{P}_f \cdot (1 - \mathbf{S})$$

30  
April 2010

# Homogeneous Coordinates

Rotate and then displace a point  $\mathbf{P}$  :  $\mathbf{P}' = \mathbf{M}_1 \cdot \mathbf{P} + \mathbf{M}_2$

$\mathbf{M}_1$ :  $2 \times 2$  rotation matrix.  $\mathbf{M}_2$ :  $2 \times 1$  displacement vector.

Displacement is unfortunately a non linear operation.

Make displacement linear with **Homogeneous Coordinates**.

$(x, y) \Rightarrow (x, y, 1)$ . Transformations turn into  $3 \times 3$  matrices.

Very big advantage. All transformations are concatenated by matrix multiplication.

31  
April 2010

$$\text{2D Translation} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{T}(t_x, t_y) \cdot \mathbf{P}$$

$$\text{2D Rotation} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{R}(\theta) \cdot \mathbf{P}$$

$$\text{2D Scaling} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{S}(S_x, S_y) \cdot \mathbf{P}$$

32  
April 2010

# Orthogonal Rotation Matrix

The **orthogonal rotation matrix** is used to map motion in the global reference  $\{X_I, Y_I\}$  frame to motion in the robot's local reference frame  $\{X_R, Y_R\}$

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The orthogonal rotation matrix is used to convert robot velocity in the global reference frame to components of motion along the robot's local axes  $\{X_R, Y_R\}$

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = R(\theta) \cdot \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$$

33

## How to convert Degrees to Radians

- One degree is equal 0.01745329252 radians:  
 $1^\circ = \pi/180^\circ = 0.005555556\pi = 0.01745329252$  rad
- The angle  $\alpha$  in radians is equal to the angle  $\alpha$  in degrees times pi constant divided by 180 degrees:

$$\alpha(\text{radians}) = \alpha(\text{degrees}) \times \pi / 180^\circ$$

- or radians = degrees  $\times \pi / 180^\circ$

34