Story 1: Pipe Flow Regulation

$\frac{dy}{dt}+2y=e^{t}$ $\quad\quad$ $\mu=e^{\int 2dt}=e^{2t}$ (Integrating Factor)

$e^{2t}\left(\frac{dy}{dt}+2y\right)=e^{2t}\cdot e^{t}$

$\frac{d}{dt}\left(e^{2t}\cdot y\right)=e^{3t}$

$\int \frac{d}{dt}\left(e^{2t}\cdot y\right)dt=\int e^{3t}dt$

$e^{2t}\cdot y=\frac{1}{3}e^{3t}+c$

$y(t)=\frac{1}{3}e^{t}+c.e^{-2t}$

Story 2: Sediment Accumulation in a River

$\frac{dy}{dx}=xy^{2}$ (Separable Variables)

$\frac{1}{y^{2}}dy=x\,dx$

$\int\frac{1}{y^{2}}dy=\int x\,dx$

$-\frac{1}{y}=\frac{x^{2}}{2}+c' \longrightarrow -\frac{1}{y}=\frac{x^{2}+2c'}{2}$ $(2c'=c)$

$y(x)=-\frac{2}{x^{2}+c}$

Story 3: Stability of Retaining Walls

$(x^2+y)\,dx + (y^2+x)\,dy = 0$ (Exact Differential Equations)

$M(x,y) = x^2+y \qquad N(x,y) = y^2+x$

$\frac{\partial M}{\partial y} = 1 \quad = \quad \frac{\partial N}{\partial x} = 1 \rightarrow$ tam dif. denk.

$M(x,y) = \frac{\partial u}{\partial x} \quad , \quad N(x,y) = \frac{\partial u}{\partial y}$

$u(x,y) = \int M(x,y)\,dx = \int N(x,y)\,dy$

$\int (x^2+y)\,dx = \int (y^2+x)\,dy$

$\frac{x^3}{3} + xy + f(y) = \frac{y^3}{3} + xy + f(x)$

$u(x,y) = \frac{x^3}{3} + xy + \frac{y^3}{3} + c$

Story 4: Beam Loading Analysis

$\frac{dy}{dx} + y = xy^2$ (Bernoulli Differential Equations)

$n=2 \ , \ P(x)=1 \ , \ Q(x)=x \ , \ v=\frac{1}{y} \ , \ \frac{dv}{dx} = -\frac{1}{y^2}\cdot\frac{dy}{dx}$

$\frac{1}{y^2}\cdot\frac{dy}{dx} + \frac{1}{y} = x$

$-\frac{dv}{dx} + v = x \rightarrow \frac{dv}{dx} - v = -x \qquad \mu = e^{\int -1dx} = e^{-x}$

$e^{-x}\left(\frac{dv}{dx} - v\right) = e^{-x}\cdot(-x)$

$\frac{d}{dx}\left(e^{-x}\cdot v\right) = -e^{-x}\cdot x$

$u=x \rightarrow du=dx$

$dv' = e^{-x}dx \rightarrow v' = -e^{-x}$

$\int \frac{d}{dx}(e^{-x}\cdot v)\,dx = -\int e^{-x} x\,dx$

$e^{-x}\cdot v = x\cdot(-e^{-x}) - \int -e^{-x}\,dx = -x\cdot e^{-x} - e^{-x} + c' = -e^{-x}(x+1) + c'$

$e^{-x}\cdot v = -e^{-x}(x+1) + c'$

$\frac{1}{y} = v = -(x+1) + c' \quad (c' = -c)$

$y(x) = -\frac{1}{x+1+c}$

Story 5 : Concrete Pouring Rates

$\frac{dy}{dt} = 2t$, $t=0$, $y=3$ (separable Variables)

$\int dy = \int 2t\,dt$

$y = t^2 + c$

$y(t=0) = 3 \longrightarrow 0 + c = c = 3$

$y(t) = t^2 + 3$

Story 6 : Heat Dissipation in a Processor

$\frac{dT}{dt} = -k(T - T_a)$, $k > 0$ constant , $T(0) = T_o$ (Separable Variables)

$\frac{1}{T - T_a}\, dT = -k\, dt$

$\int \frac{1}{T - T_a}\, dT = \int -k\, dt$

$\ln|T - T_a| = -kt + c'$

$T - T_a = e^{-kt + c'}$ $(e^{c'} = c)$

$T - T_a = e^{-kt} . c$

$T(t=0) = T_0 = T_a + e^{-k.0} . c = T_a + c \longrightarrow c = T_0 - T_a$

$T(t) = T_a + (e^{-kt})(T_0 - T_a)$

Story 7: Network Traffic Modeling

$\frac{dP}{dt} + P = e^{-2t}$, $P(0) = P_0$ $\qquad \mu = e^{\int dt} = e^{t}$ (Integrating Factor)

$e^{t}\left(\frac{dP}{dt} + P\right) = e^{t} \cdot e^{-2t}$

$\frac{d}{dt}\left(e^{t} \cdot P\right) = e^{-t}$

$\int \frac{d}{dt}(e^{t} \cdot P)\, dt = \int e^{-t}\, dt$

$e^{t} \cdot P = -e^{-t} + c$

$P = -e^{-2t} + c \cdot e^{-t}$

$P(t=0) = -e^{-2 \cdot 0} + c \cdot e^{-0} = c - 1 = P_0 \rightarrow c = P_0 + 1$

$P(t) = -e^{-2t} + (P_0 + 1)e^{-t}$

Story 8: Gradient Descent for Optimization

$f(w) = w^2 + 2w + 5$, $\quad w_{n+1} = w_n - \alpha \frac{df}{dw}$, $w_0 = 3$, $\alpha = 0{,}1$

$\frac{df}{dw} = \frac{d}{dw}\left(w^2 + 2w + 5\right) = 2w + 2$

$w_{n+1} = w_n - \alpha\,(2w_n + 2)$

$w_{n+1} = w_n - 0{,}1\,(2w_n + 2)$

$w_{n+1} = 0{,}8w_n - 0{,}2$

$n = 0 \rightarrow w_1 = 0{,}8 \cdot 3 - 0{,}2 = 2{,}2$

$n = 1 \rightarrow w_2 = 0{,}8 \cdot 2{,}2 - 0{,}2 = 1{,}56$

$n = 2 \rightarrow w_3 = 0{,}8 \cdot 1{,}56 - 0{,}2 = 1{,}048$

$w_1 = 2{,}2$, $\quad w_2 = 1{,}56$, $\quad w_3 = 1{,}048$