Recursive (özyinelemeli) Fonksiyonlar:

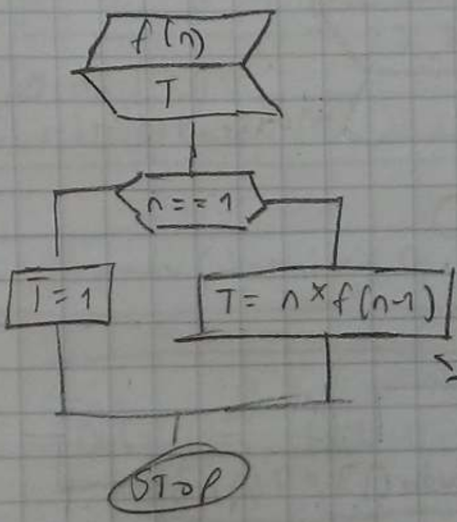
a → a (direkt)

a → b → a (dolaylı)

faktöriyel:

$n = n \times n-1$

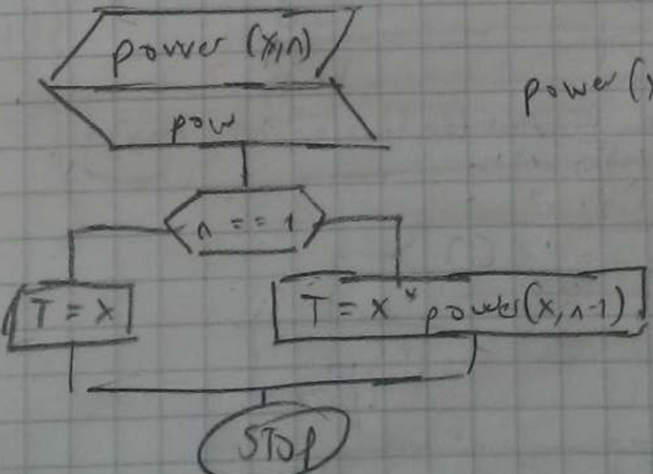
$\rightarrow f(n) = n \times f(n-1)$



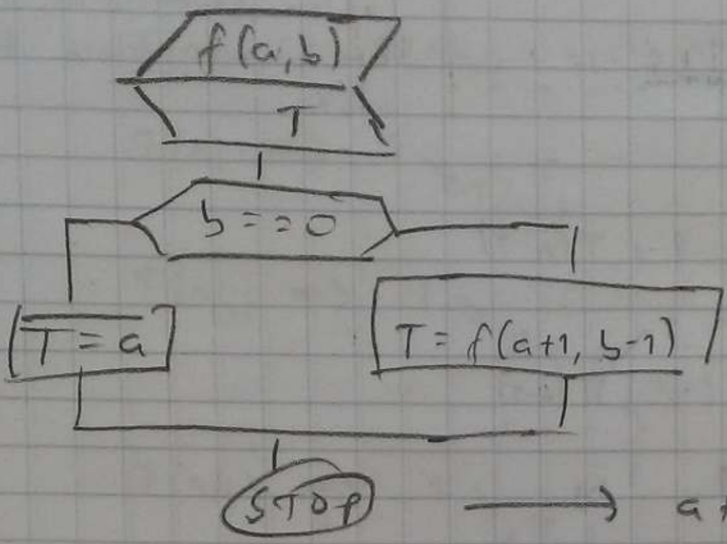
fonksiyon kendisini aynı parametrede çağırmamalı

recursive olmayan kısım

recursive kısım

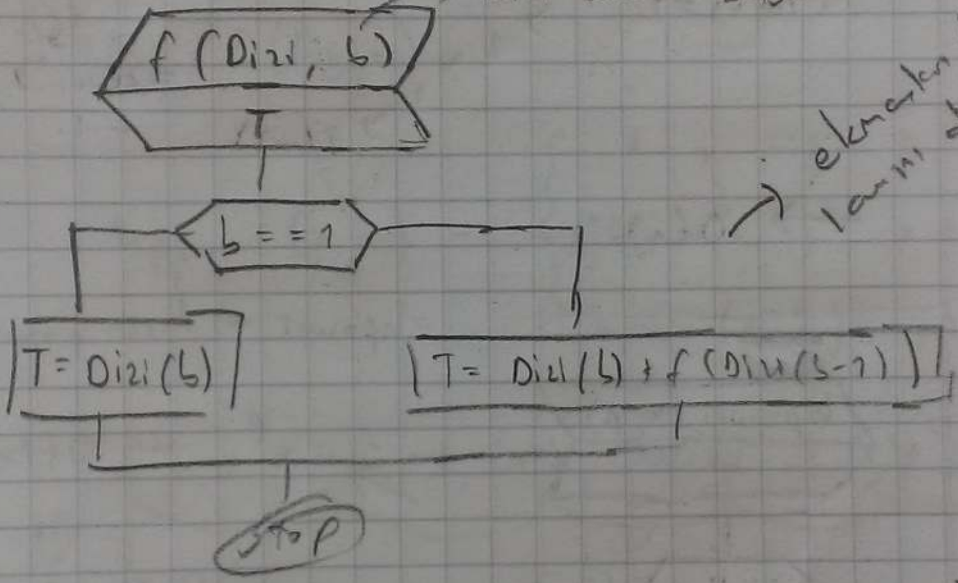


$power(x, n) = x * x^{n-1}$

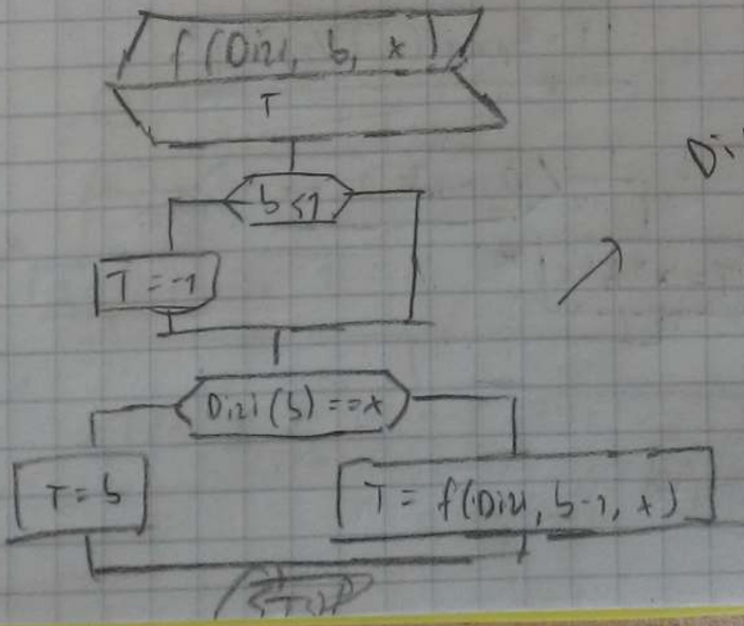


a+b döndürür

dizi eleman sayısı



elemanları toplamını döndürür



Dizinin n. indisini döndürür

Toplam karmaşıklık ağacın tüm elemanlarının toplamı olur.

Recursive Tree

fact(x):

tree ← $T(n) = 1 + T(n-1)$

$T(n) \Rightarrow 1 \searrow T(n-1)$

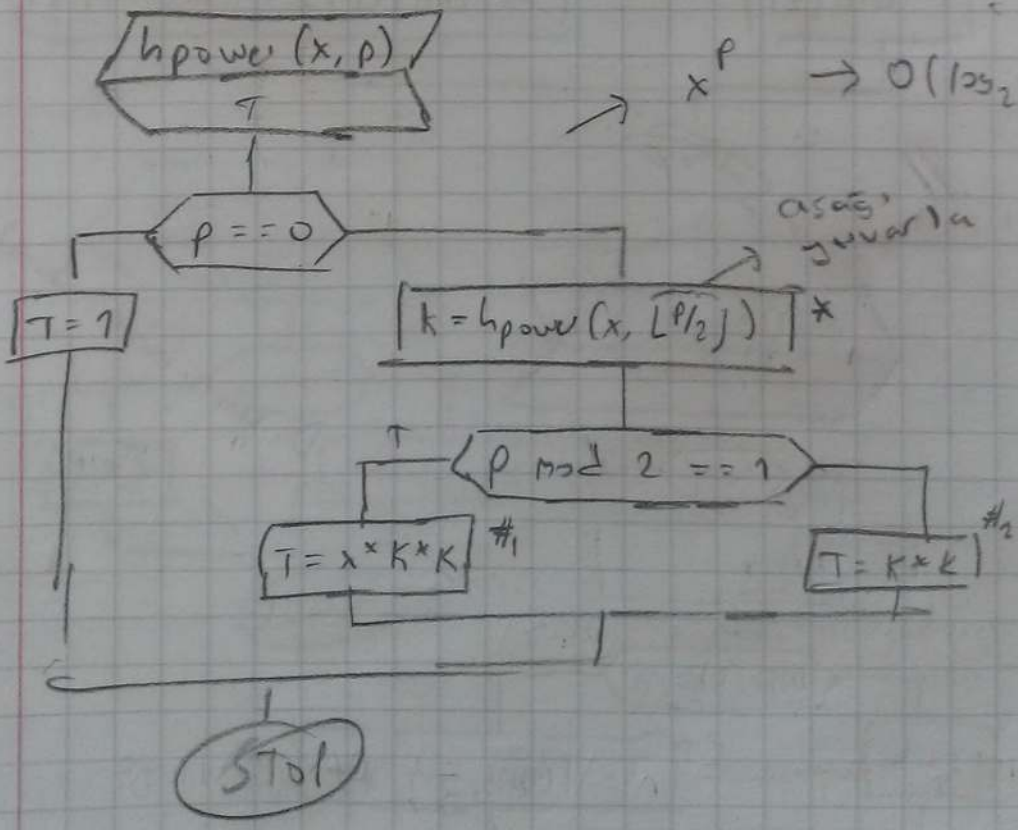
$T(n-1) \Rightarrow 1 \searrow 1 \searrow T(n-2)$

$T(n-2) \Rightarrow 1 \searrow 1 \searrow 1 \searrow T(n-3) \Rightarrow 1 \searrow 1 \searrow \dots \searrow 1 \quad h = n$

$\longrightarrow O(n)$

hpower → high power

$x^p \rightarrow O(\log_2 n)$



$T(n) = 1 + T(n/2)$

$T(n) \Rightarrow 1 \searrow T(n/2)$

$T(n/2) \Rightarrow 1 \searrow 1 \searrow T(n/4)$

$T(n/4) \Rightarrow 1 \searrow 1 \searrow 1 \searrow T(\frac{n}{8})$ $\quad \vdots \quad h = \log_2 n$

$T(n) = O(\log_2 n)$

$$hpower(x,p) = \begin{cases} 1, & p == 0 \\ x * hpower(x, p/2)^2, & p \to \text{tek} \\ hpower(x, p/2)^2, & p \to \text{çift} \end{cases}$$

$x^{32} \to (x^{16})^2 \to (x^8)^2 \to (x^4)^2 \to (x^2)^2 \to (x^1)^2$

$\log_2 32 = 5$ işlem

$$\left.\begin{array}{l} (*) \to k = hpower(x, \lfloor p/2 \rfloor)^2 \\ (\#_1) \to T = X * K \\ (\#_2) \to T = K \end{array}\right\}$$

olsa idi $T(n) = 1 + 2T(n/2)$ olurdu

$h = \log_2 n$

$$T(n) = \sum_{i=0}^{\log_2 n} 2^i = 2 \cdot 2^{\log_2 n} - 1$$
$$= O(n)$$

Ex/ $T(n) = 2T(n/2) + n$ bir ağaç olsun:

quick sort ve merge sort ağacı

$$T(n) = \begin{array}{c} n \\ \swarrow \quad \searrow \\ T(n/2) \quad T(n/2) \end{array} \rightarrow \begin{array}{c} n \\ \swarrow \quad \searrow \\ n/2 \qquad n/2 \\ \swarrow \searrow \qquad \swarrow \searrow \\ T(n/4)\ T(n/4)\ T(n/4)\ T(n/4) \end{array}$$

$$\rightarrow \begin{array}{c} n \\ \swarrow \quad \searrow \\ n/2 \qquad n/2 \\ \swarrow \searrow \qquad \swarrow \searrow \\ n/4 \quad n/4 \quad n/4 \quad n/4 \\ \swarrow \searrow \quad \swarrow \searrow \\ T(n/8)\ T(n/8)\ T(n/8)\ T(n/8) \ \dots \end{array} \qquad \begin{array}{l} \Sigma = n \\ \Sigma = n \\ \Sigma = n \\ \vdots \end{array} \qquad h = \log_2 n$$

$$T(n) = n \cdot \log_2 n$$

$$\boxed{\sum_{i=0}^{n} A^i = \begin{cases} \dfrac{A^{n+1}-1}{A-1} & (A>1) \\ \dfrac{1-A^{n+1}}{1-A} & (A<1) \end{cases}}$$

Ex/ $T(n) = 2T(n/4) + n$

$$\begin{array}{c} n \\ \swarrow \quad \searrow \\ T(n/4) \quad T(n/4) \end{array} \rightarrow \begin{array}{c} n \\ \swarrow \quad \searrow \\ n/4 \qquad n/4 \\ \swarrow \searrow \qquad \swarrow \searrow \\ T(n/16)\ T(n/16)\ T(n/16)\ T(n/16) \end{array}$$

$h = \log_4 n$

$$T(n) = n + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} \dots$$

$$= n\left(1 + \frac{1}{2} + \frac{1}{4} \dots\right) = n \cdot \sum_{i=0}^{\log_4 n} \left(\frac{1}{2}\right)^i$$

$$= n \cdot \frac{1-\left(\frac{1}{2}\right)^{\log_4 n + 1}}{1-\frac{1}{2}} = 2n - \sqrt{n}$$

$O(n) \leftarrow$

c) $T(n) = 8T(n/2) + n$

n

$T(n/2) \quad T(n/2) \quad \ldots \quad \rightarrow 8$ tane $T(n/2)$

n

$n/2 \quad n/2 \quad \ldots \quad \rightarrow 8$ tane $n/2$

$T(n/4) \quad T(n/4) \quad \ldots\ldots \quad \rightarrow 64$ tane $T(n/4)$

$\rightarrow 64$ tane $\frac{n}{4}$

$h = \log_2 n$

$T(n) = n + 8 \cdot \frac{n}{2} + 64 \cdot \frac{n}{4} \cdots$

$= n + 4n + 8n + 16n$

$= n(1 + 4 + 8 + 16 \ldots)$

$$T(n) = n \cdot \sum_{n=0}^{\log_2 n} (4)^{\log_2 n + 1}$$

$$= n \cdot \frac{4^{\log_2 n + 1} - 1}{4 - 1} = n \cdot \frac{4 \cdot n^{\overset{2}{\cancel{\log_2 4}}} - 1}{3}$$

$$= \frac{4}{3} n^3 - \frac{n}{3}$$

$\rightarrow O(n^3)$

Ex/

$$T(n) = 8T(n/2) + n^2$$

n^2

$T(n/2) \quad T(n/2) \ldots \quad 8$ tane $T(n/2)$

n^2

$(n/2)^2 \quad (n/2)^2 \quad \rightarrow 8$ tane $(n/2)^2$

$T((\frac{n}{4})^2) \; T((\frac{n}{4})^2) \ldots \quad \rightarrow 64$ tane $(n/4)^2$

$\rightarrow 64 \cdot 8$ tane $(n/8)^2$

$\ldots$

$h = \log_2 n$

$T(n) = n^2 + 2n^2 + 4n^2 \ldots = n^2(1+2+4 \ldots)$

$$= n^2 \cdot \sum_{i=0}^{\log_2 n} 2^i = n^2 \cdot \frac{2^{\log_2 n + 1} - 1}{2-1} = \frac{2 \cdot 2^{\log_2 n} - 1}{1} n^2$$

$$= (n-1) \cdot n^2 \rightarrow O(n^3)$$

Ex

$$T(n) = \overbrace{8\,T(n/2)}^{\text{recursive kısım}} + \overbrace{\underline{1}}^{\text{recursive olmayan}}$$

$\left(\frac{n}{2}\right) \to 1$

$\downarrow$

$\left(\frac{n}{2}\right)^0$

$$\begin{array}{ccc} & 1 & \\ \swarrow & \downarrow & \\ T(n/2) & T(n/2) & \cdots \end{array} \rightarrow 8 \text{ tane } T(n/2)$$

$$\begin{array}{ccc} & 1 & \\ \swarrow & \downarrow & \\ 1 & 1 & \cdots \end{array} \rightarrow 8 \text{ tane } 1$$

$$\begin{array}{cccc} & & 1 & \\ & \swarrow & \downarrow & \cdots \\ & 1 & 1 & \\ \swarrow & \searrow & & \\ T(n/4) & T(n/4) & \cdots & \end{array} \rightarrow 64 \text{ tane } T(n/4)$$

$$\begin{array}{cccc} & & 1 & \\ & 1 \swarrow & \downarrow 1 & \cdots \\ 1 \swarrow & \downarrow 1 & \cdots & \end{array} \rightarrow 64 \text{ tane } 1$$

$h = \log_2 n$

$T(n) = 1 + 8 + 64 + \cdots$

$$= \sum_{i=0}^{\log_2 n} (8)^i = \frac{8^{\log_2 n + 1} - 1}{8-1}$$

$$= \frac{8 \cdot n^{\log_2 8} - 1}{7} = \frac{8n^3 - 1}{7} \to O(n^3)$$

Ex

$T(n) = 4T(n/3) + 1$

1

$\swarrow$ 4

$T(n/3) \quad T(n/3) \cdots \quad \rightarrow$ 4 tane $T(n/3)$

1

$\swarrow \ \downarrow$

$1 \quad 1 \cdots \quad \rightarrow$ 4 tane 1

$\rightarrow$ 16 tane 1

$h = \log_3 n$

$$T(n) = \sum_{i=0}^{\log_3 n} 4^i = \frac{4^{\log_3 n + 1} - 1}{4 - 1}$$

$$= \frac{4 \cdot n^{\log_3 4} - 1}{3}$$

$O(n^{\log_3 4}) \Leftarrow$

EX

$$T(n) = 9\,T(n/3) + 1$$

1

$T(n/3) \cdots \rightarrow 9$ tane $T(n/3)$

1

$1 \quad 1 \cdots \rightarrow 9$ tane 1

$\rightarrow 81$ tane 1

$$h = \log_3 n$$

$$T(n) = \sum_{i=0}^{\log_3 n} 9^i = \frac{9^{\log_3 n + 1} - 1}{9-1} = \frac{9 \cdot n^{\log_3 9} - 1}{8}$$

$$= \frac{9n^2 - 1}{8} \rightarrow O(n^2)$$

EX

$$T(n) = 4\,T(n/2) + n$$

n

$T(n/3) \quad T(n/3) \cdots \rightarrow 4 \times T(n/3)$

n

$n/2 \quad n/2 \cdots$

$T(n/9) \quad T(n/9) \cdots \rightarrow 16 \times T(n/9)$

$h = \log_3 n$

$$T(n) = n + 4 \cdot \frac{n}{3} + 16 \cdot \frac{n}{9} \cdots$$

$$= n \left(1 + \frac{4}{3} + \frac{16}{9} \cdots\right)$$

$$= n \cdot \sum_{i=0}^{\log_3 n} \left(\frac{4}{3}\right)^i = n \cdot \frac{\left(\frac{4}{3}\right)^{\log_3 n + 1} - 1}{\frac{4}{3} - 1}$$

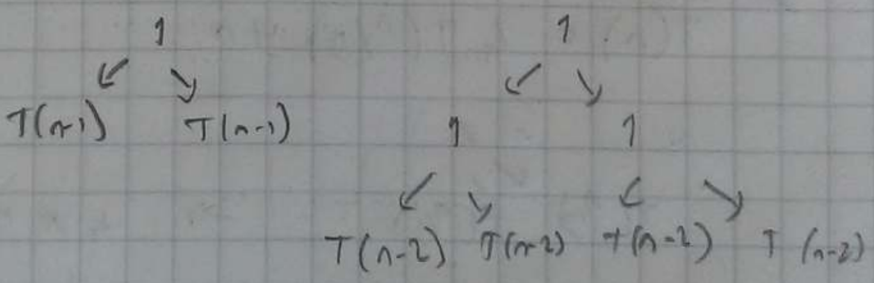
$$= n \cdot \frac{\frac{4}{3} \cdot \frac{4^{\log_3 n}}{3^{\log_3 n}} - 1}{1/3}$$

$$= 3n \left(\frac{4 \cdot 4^{\log_3 n}}{3n} - 1 \right)$$

$$= 4 \cdot 4^{\log_3 n} - 3n \rightarrow O(n^{\log_3 4})$$

$2 . T(n-2) + 1$

Ex $T(n) = 2\,T(n-1) + 1$



$h = n$

$T(n) = 1 + 2 + 4 \ldots$

$$= \sum_{i=0}^{n} 2^i = \frac{2^{n+1}-1}{2-1} = 2.2^n - 1$$

$$\to O(2^n)$$

	n^2	2^n
1	1	2
10	100	1024
20	400	1 M
30	900	1 B ...

$2^n - n^2$ Karşılaştırması

$T(n) = T(n-1) + T(n-2) + 1 \to$ Fibonacci

$$1$$
$$\swarrow \quad \searrow$$
$$T(n-1) \quad T(n-2)$$

$$1$$
$$\swarrow \quad \searrow$$
$$1 \qquad\qquad 1$$
$$\swarrow \ \searrow \qquad \swarrow \ \searrow$$
$$T(n-2) \quad T(n-3) \quad T(n-3) \quad T(n-4) \ \ldots$$

→ Ağacın kolları farklı uzunlukta olduğunda

$T(n) = \underline{2T(n-1)+1}$ varsayalım $\to O(n^2)$

bundan daha küçük olmalı

Gerçekte $\to O(k^n)$, $k = \frac{1+\sqrt{5}}{2}$ (altın oran)

ALTERNATİF YOL

$$F = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

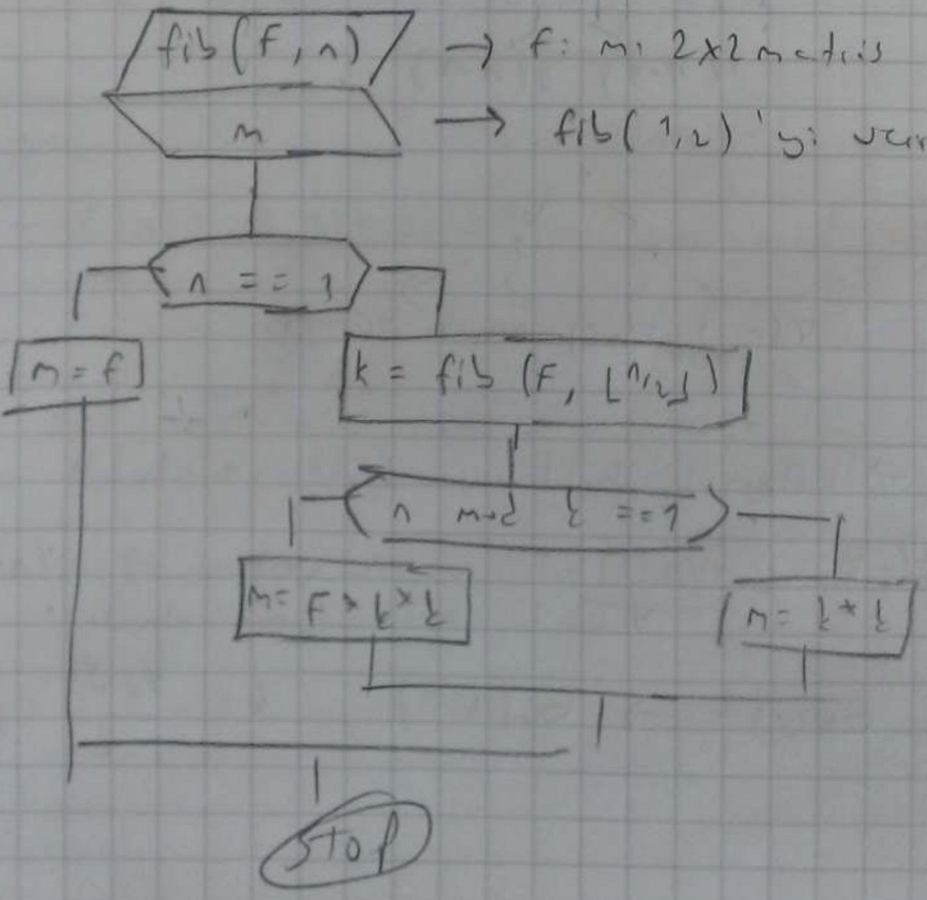
$$F^2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$F^3 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$F^4 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

$$F^5 = \begin{bmatrix} 8 & 5 \\ 5 & 3 \end{bmatrix}$$

$$F^n = \begin{bmatrix} fib(n+1) & fib(n) \\ fib(n) & fib(n-1) \end{bmatrix} \rightarrow$$ 4 power serisi ($O(\log_2 n)$)



$$T(n) = T(n/2) + c$$ → önemsiz işlem sayısı (n'dan bağımsız)

$$= O(\log_2 n).$$

Sepet Algoritması:

S (N×2)

	Sepet ID	Ürün ID
1 →	3	42
2 →	3	15
⋮	3	42
	⋮	⋮
	5	6
N →	5	15

A)

Sepet ID → sepet (M)

$$\begin{array}{c|ccccccccc}
 & \multicolumn{9}{c}{\text{Sepet ID}} \\
\hline
 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
 & & & 1 & & & & & & \\
 & & & 0 & & & & & & \\
\text{ürün ID} & & & 1 & & & & & & \\
 & & & 0 & & & & & & \\
 & & & 0 & & & & & & \\
 & & & 1 & & & & & & \\
 & & & 0 & & & & & &
\end{array}$$

ürün (M)

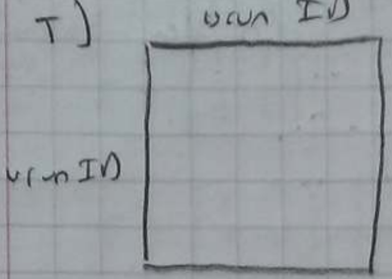
i=1 | 1

M

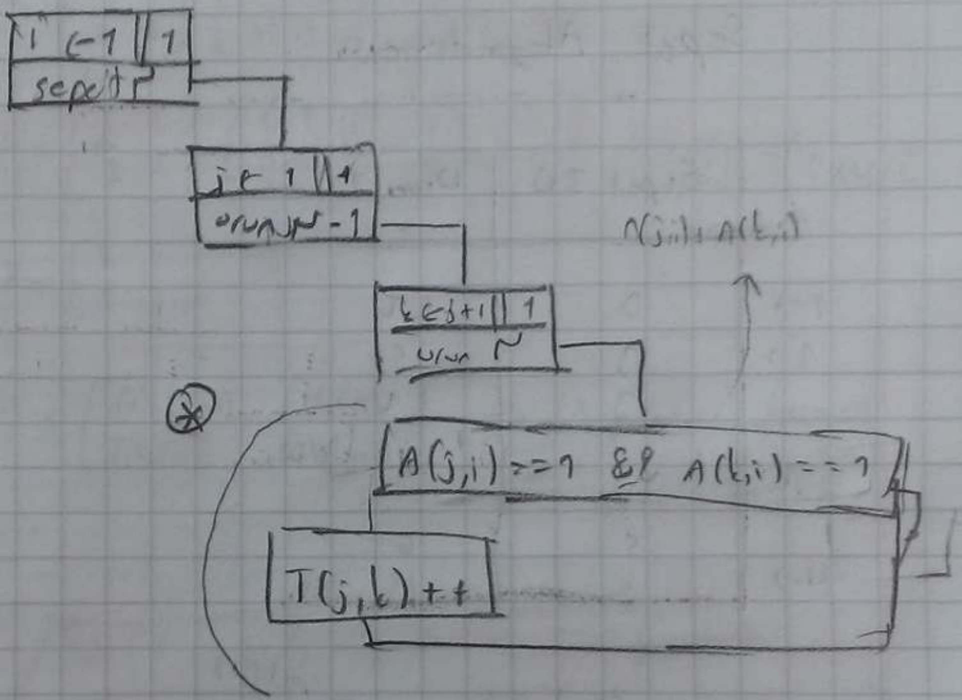
$$\boxed{A(S(i,2), S(i,1)) = 1}$$

S'dan A'ya geçiş?

T)



→ A'dan T'ye geçiş?



⊛ → $T(j,k) \mathrel{+}= A(j,i) * A(k,i)$

(iyileştirme)

Daha da iyisi Apriori algoritması

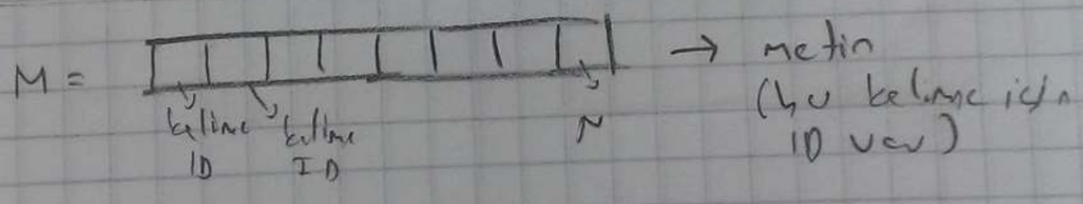
Şifreleme Algo.

	1	2	3	4	5
1	' '	'A'	'B'	'C'	'D'
2	.	.	.	.	.
3	'.'	'.'	.	.	.
4	.		.	.	
5	.	.			.

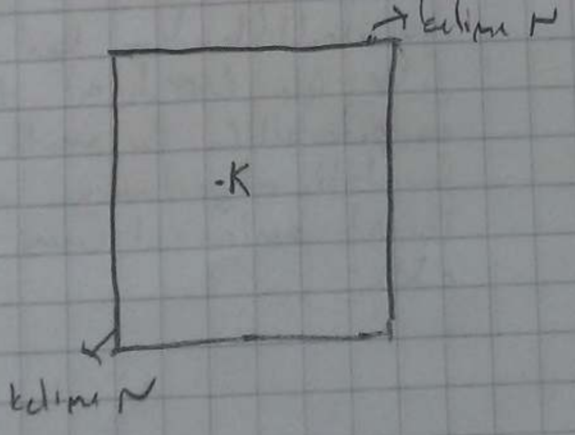
A B - C D
↓ ↓ ↓ ↓ ↓
12 13 32 14 15 ...

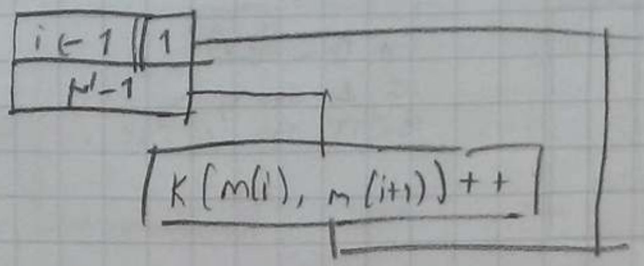
Şifrelenmemiş bir mesajda, "A" harfi genelde en çok kullanılan harf olması mantığıyla mesaj tahmin olarak çözülebilir. Bunun sebebi bir harfin numarasının hep aynı olmasıdır.

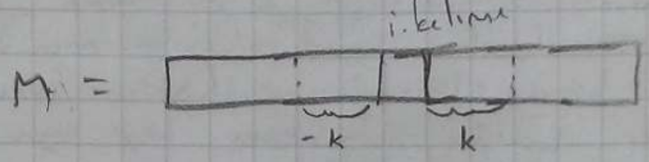
Bunun önüne geçmek için bir harfin bir dilde kullanılma sıklığı kadar farklı numarası olur. Böylece şifrelenecek metnin tüm sayıları benzer frekansta olur, kırılması zorlaşır.



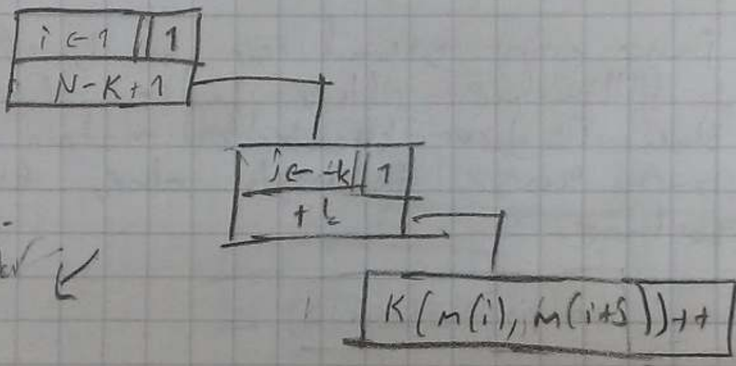
Arka arkaya en sık tekrar eden kelime ikilisi?





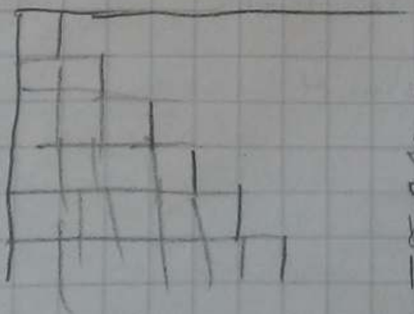


→ ortada i'nin olduğu kelime grupları



kelime grupları içindeki anlamsal yakınlık

HARITA → şehirler arası uzaklık



↓

bu uzaklıklardan her bir şehir için koordinat tahmini yapılabilir. Bu mantıkla yukarıdaki algoritmaya uygularak anlamsal uzay elde edilir.