1.1 Determine whether or not the signals below are periodic and, for each signal that is periodic, determine the fundamental period.

(a) $x(n) = \cos(0.125\pi n)$

(b) $x(n) = \text{Re}\{e^{jn\pi/12}\} + \text{Im}\{e^{jn\pi/18}\}$

(c) $x(n) = \sin(\pi + 0.2n)$

(d) $x(n) = e^{j\frac{\pi}{16}n}\cos(n\pi/17)$

(a) Because $0.125\pi = \pi/8$, and

$$\cos\left(\frac{\pi}{8}n\right) = \cos\left(\frac{\pi}{8}(n+16)\right)$$

$x(n)$ is periodic with period $N = 16$.

(b) Here we have the sum of two periodic signals,

$$x(n) = \cos(n\pi/12) + \sin(n\pi/18)$$

with the period of the first signal being equal to $N_1 = 24$, and the period of the second, $N_2 = 36$. Therefore, the period of the sum is

$$N = \frac{N_1 N_2}{\gcd(N_1, N_2)} = \frac{(24)(36)}{\gcd(24, 36)} = \frac{(24)(36)}{12} = 72$$

(c) In order for this sequence to be periodic, we must be able to find a value for N such that

$$\sin(\pi + 0.2n) = \sin(\pi + 0.2\,(n + N))$$

The sine function is periodic with a period of 2π. Therefore, $0.2N$ must be an integer multiple of 2π. However, because π is an irrational number, no integer value of N exists that will make the equality true. Thus, this sequence is aperiodic.

(d) Here we have the product of two periodic sequences with periods $N_1 = 32$ and $N_2 = 34$. Therefore, the fundamental period is

$$N = \frac{(32)(34)}{\gcd(32, 34)} = \frac{(32)(34)}{2} = 544$$

3.1 Consider the discrete-time sequence

$$x(n) = \cos\left(\frac{n\pi}{8}\right)$$

Find two different continuous-time signals that would produce this sequence when sampled at a frequency of $f_s = 10$ Hz.

A continuous-time sinusoid

$$x_a(t) = \cos(\Omega_0 t) = \cos(2\pi f_0 t)$$

that is sampled with a sampling frequency of f_s results in the discrete-time sequence

$$x(n) = x_a(nT_s) = \cos\left(2\pi\frac{f_0}{f_s}n\right)$$

However, note that for any integer k,

$$\cos\left(2\pi\frac{f_0}{f_s}n\right) = \cos\left(2\pi\frac{f_0 + kf_s}{f_s}n\right)$$

Therefore, any sinusoid with a frequency

$$f = f_0 + kf_s$$

will produce the same sequence when sampled with a sampling frequency f_s. With $x(n) = \cos(n\pi/8)$, we want

$$2\pi\frac{f_0}{f_s} = \frac{\pi}{8}$$

or

$$f_0 = \tfrac{1}{16}f_s = 625 \text{ Hz}$$

Therefore, two signals that produce the given sequence are

$$x_1(t) = \cos(1250\pi t)$$

and

$$x_2(t) = \cos(21250\pi t)$$

Supplementary Problems

A/D and D/A Conversion

3.27 Find two different continuous-time signals that will produce the sequence

$$x(n) = \cos(0.15n\pi)$$

when sampled with a sampling frequency of 8 kHz.

3.28 If the Nyquist rate for $x_a(t)$ is Ω_s, find the Nyquist rate for (*a*) $x^2(2t)$, (*b*) $x(t/3)$, (*c*) $x(t) * x(t)$.

3.29 A continuous-time signal $x_a(t)$ is known to be uniquely recoverable from its samples $x_a(nT_s)$ when $T_s = 1$ ms. What is the highest frequency in $X_a(f)$?

3.30 Suppose that $x_a(t)$ is bandlimited to 8 kHz (that is, $X_a(f) = 0$ for $|f| > 8000$). (*a*) What is the Nyquist rate for $x_a(t)$? (*b*) What is the Nyquist rate for $x_a(t)\cos(2\pi \cdot 1000t)$?

3.31 Let $x_a(t) = \cos(650\pi t) + 2\sin(720\pi t)$. (*a*) What is the Nyquist rate for $x_a(t)$? (*b*) If $x_a(t)$ is sampled at twice the Nyquist rate, what are the frequencies of the sinusoids in the sampled sequence?

3.32 If a continuous-time filter with an impulse response $h_a(t)$ is sampled with a sampling frequency of f_s, what happens to the cutoff frequency ω_c of the discrete-time filter as f_s is increased?

3.33 A complex bandpass signal $x_a(t)$ with $X_a(f)$ nonzero for 10 kHz $< f <$ 12 kHz is sampled at a sampling rate of 2 kHz. The resulting sequence is

$$x(n) = \delta(n)$$

What is $x_a(t)$?

3.34 If the highest frequency in $x_a(t)$ is $f = 8$ kHz, find the minimum sampling frequency for the bandpass signal $y_a(t) = x_a(t)\cos(\Omega_0 t)$ if (*a*) $\Omega_0 = 2\pi \cdot 20 \cdot 10^3$ and (*b*) $\Omega_0 = 2\pi \cdot 24 \cdot 10^3$.

3.35 The continuous-time signal $x_a(t) = 7.25\cos(2000\pi t)$ is sampled at a sampling frequency of 8 kHz and quantized with a resolution $\Delta = 0.02$. How many bits are required in the A/D converter to avoid clipping $x_a(t)$?

Answers to Supplementary Problems

3.27 $x_1(t) = \cos(1200\pi t)$ and $x_2(t) = \cos(17200\pi t)$.

3.28 (*a*) $4\Omega_s$. (*b*) $\Omega_s/3$. (*c*) Ω_s.

3.29 500 Hz.

3.30 (*a*) 16 kHz. (*b*) 18 kHz.

3.31 (*a*) 720 kHz. (*b*) $\omega_1 = 65\pi/142$ and $\omega_2 = \pi/2$.

3.32 ω_c decreases.

3.33 $x_a(t) = \dfrac{1}{2000} \dfrac{\sin(2000\pi t)}{\pi t} e^{j2\pi(11000)t}$.

3.34 (*a*) 56 kHz. (*b*) 32 kHz.

3.35 10 bits.