BLM1612 - Circuit Theory

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1^{st} Order Op Amp Circuits

Objectives of Lecture

- Discuss analog computing and the application of 1st order operational amplifier circuits.
- Derive the equations that relate the output voltage to the input voltage for a differentiator and integrator.
- Explain the source of the phase shift between the output and input voltages.

Subsystems

- Multipliers
  - Inverting and non-inverting amplifiers
    - Typically fixed number, which means fixed resistor values in amplifiers
- Adders and Subtractors
  - Summing and difference amplifiers

- Differentiators
- Integrators

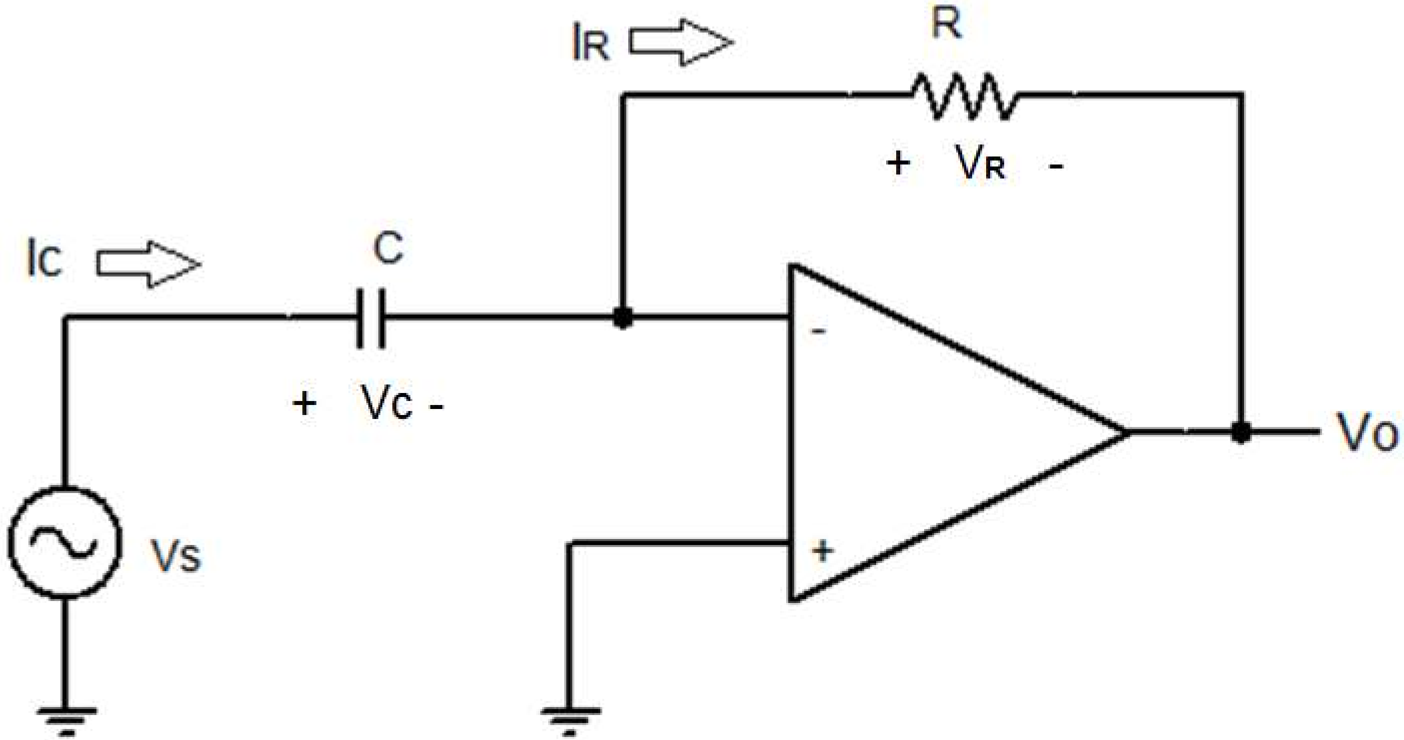
1st order op amp circuits

Capacitors

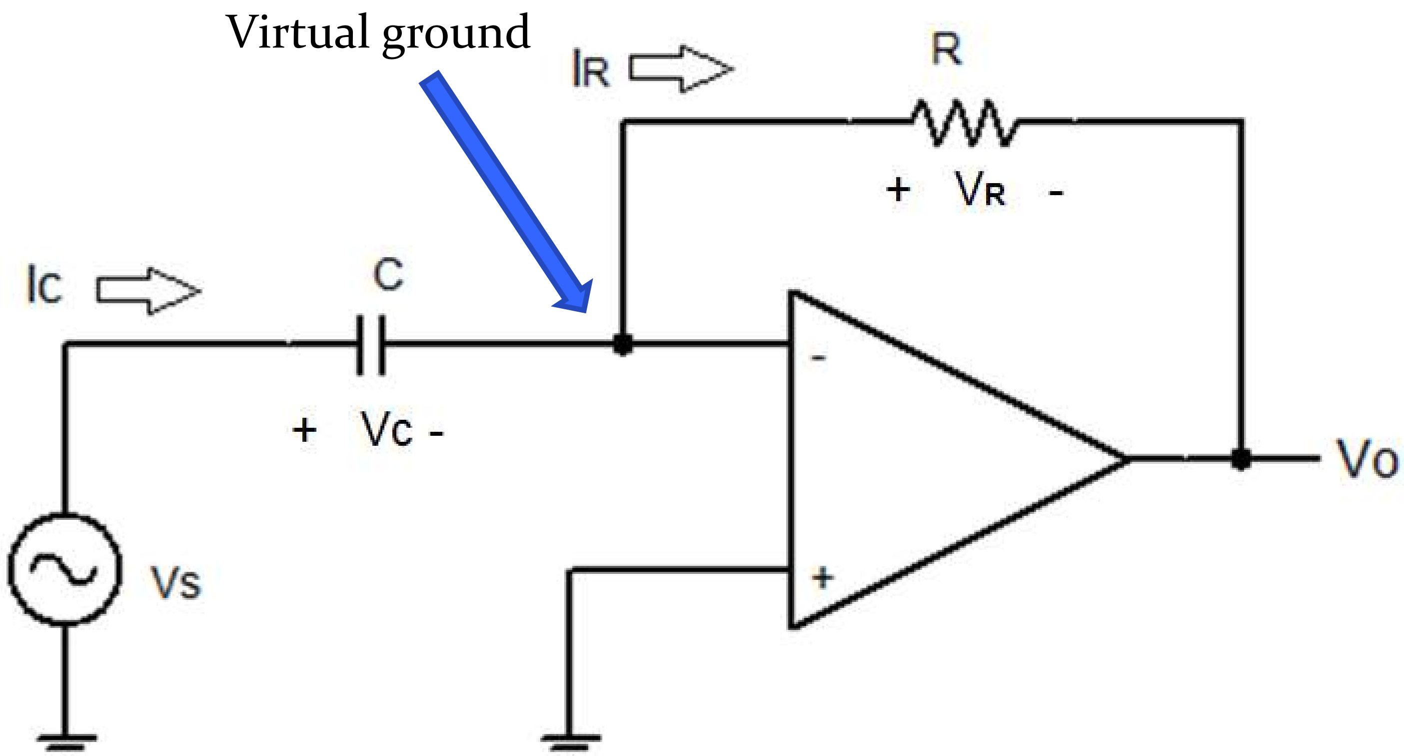
$$i_C(t) = C\frac{dv_C}{dt}$$

$$v_C(t) = \frac{1}{C}\int_{t_o}^{t_1} i_C(t)\,dt + v_C(t_o)$$

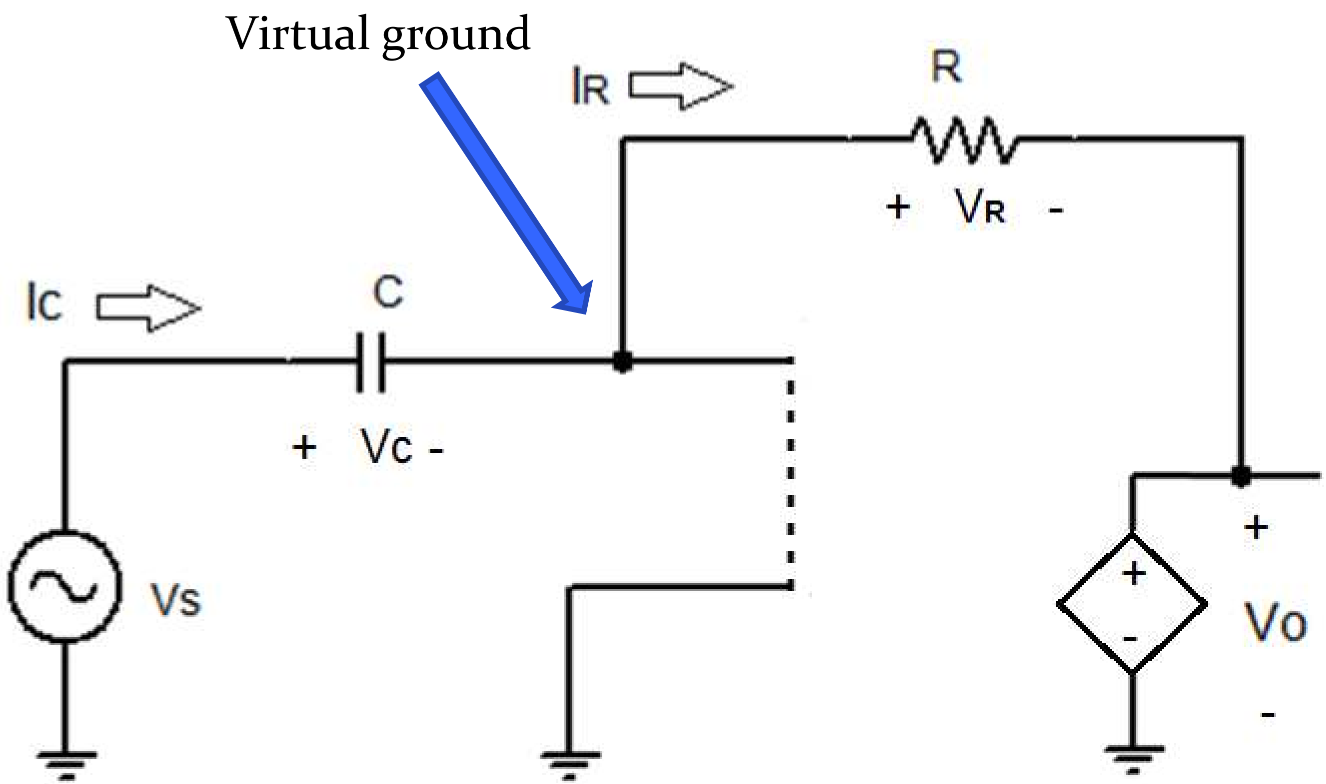
Differentiator



Ideal Op Amp Model



Op Amp Model



Analysis

- Since current is not allowed to enter the input terminals of an ideal op amp.

$$i_C(t) = i_R(t)$$

$$v_C(t) = v_S(t)$$

$$i_C(t) = C\frac{dv_C}{dt} = C\frac{dv_S}{dt}$$

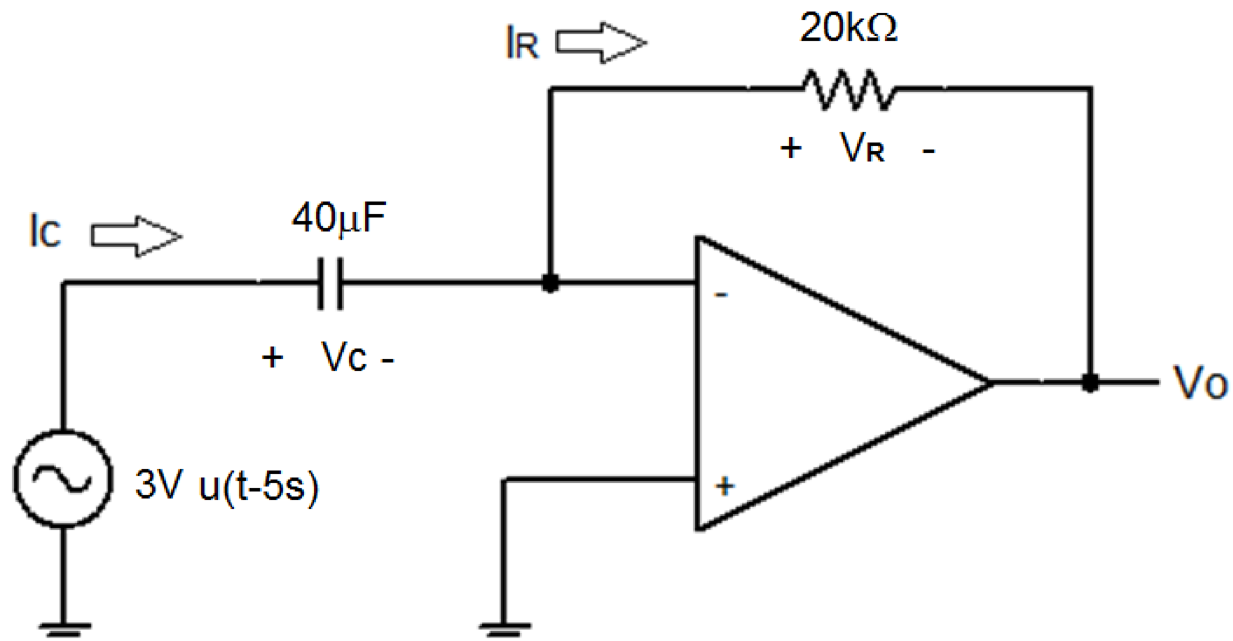
$$i_R(t) = -\frac{v_o}{R}$$

$$-\frac{v_o}{R} = C\frac{dv_S}{dt}$$

$$v_o(t) = -RC\frac{dv_S(t)}{dt}$$

Example 01…

- Suppose $v_S(t) = 3V\ u(t\text{-}5s)$
  - The voltage source changes from 0V to 3V at t = 5s.
    - Initial condition of $V_C = 0V$ when t <5s.
    - Final condition of $V_C = 3V$ when t > 5RC.



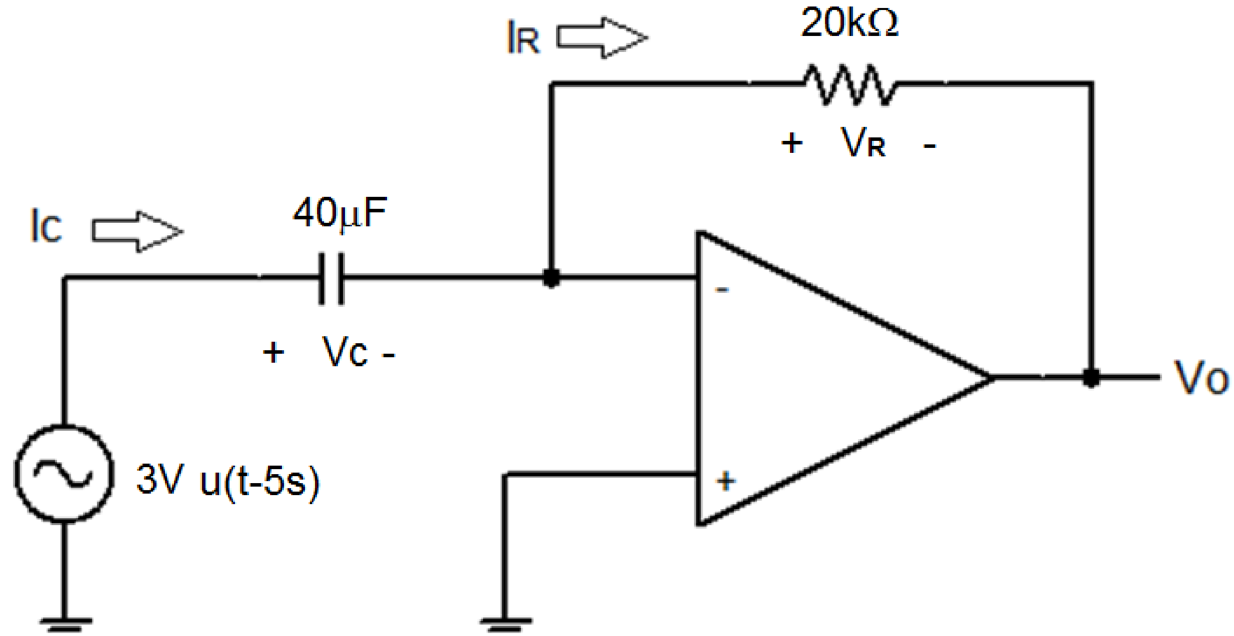
…Example 01…

$$v_C(t) = 0V \quad \text{when t} < \text{t}_\text{o}$$

$$v_C(t) = V_{C_{initial}} + \left(V_{C_{final}} - V_{C_{initial}}\right)e^{-(t-t_o)/\tau} \quad \text{when t} > \text{t}_\text{o}$$

$$v_C(t) = 0V + \left(3V - 0V\right)e^{-(t-5s)/0.8s} \quad \text{when t} > \text{t}_\text{o}$$

$$v_C(t) = 3V\ e^{-(t-5s)/0.8s} \quad \text{when t} > 5s$$



…Example 01

$$v_o(t) = -RC\frac{dv_C(t)}{dt}$$

$$v_o(t) = 0V \quad \text{when } t < 5s$$

$$v_o(t) = 0V \quad \text{when } t > t_o + 5\tau, \text{ where } \tau = RC$$

$$v_o(t) = 0V \quad \text{when } t > 5s + 5(20k\Omega)(40\mu F) = 9s$$

$$v_o(t) = \frac{-1}{0.8s}(-20x10^3\Omega)(40x10^{-6}F)(3V)\ e^{-(t-5s)/0.8s}$$

$$v_o(t) = 3V\ e^{-(t-5s)/0.8s}$$

Example 02

- Let R = 2 kΩ, C = 0.1μF, and $v_S(t)$ = 2V sin(500t) at t = 0s

$$\text{Since} \quad v_C(t) = v_S(t)$$

$$v_o(t) = -RC\,\frac{dv_S}{dt}$$

$$v_o(t) = -(2000\Omega)(10^{-7}F)\,\frac{d\left[2V\sin(500t)\right]}{dt}$$

$$v_o(t) = \left(-0.2ms\right)\left(2V\right)\left(500\right)\cos(500t)$$

$$v_o(t) = -0.2V\cos(500t) \quad \text{when t} > 0\text{s}$$

$$v_o(t) = 0V \quad \text{when t} < 0\text{s}$$

Cosine to Sine Conversion

$$v_o(t) = -0.2V\cos(500t)$$

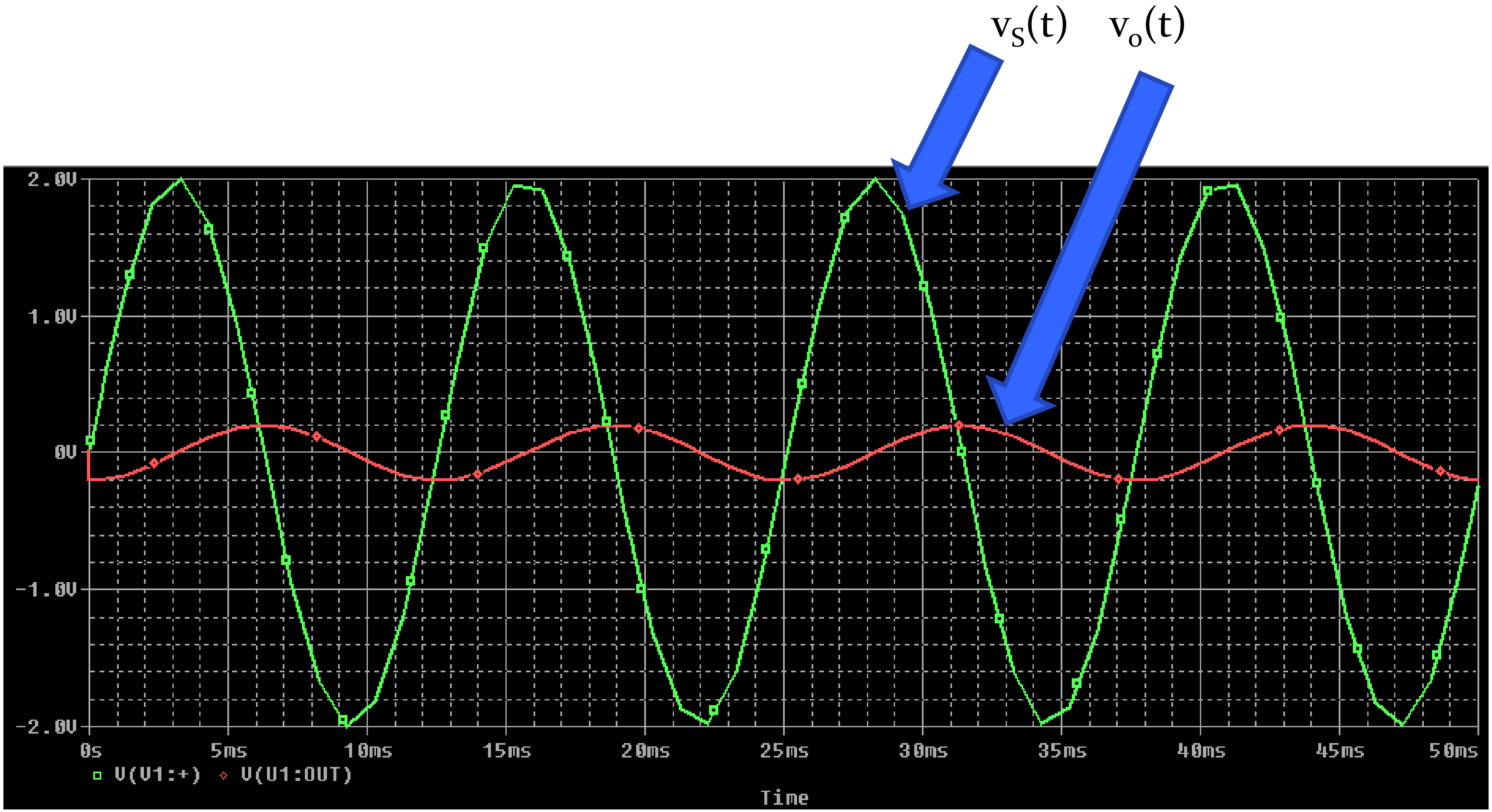
$$v_o(t) = -0.2V\sin(500t + 90^o)$$

$$v_o(t) = 0.2V\sin(500t + 90^o - 180^o)$$

$$v_o(t) = 0.2V\sin(500t - 90^o)$$

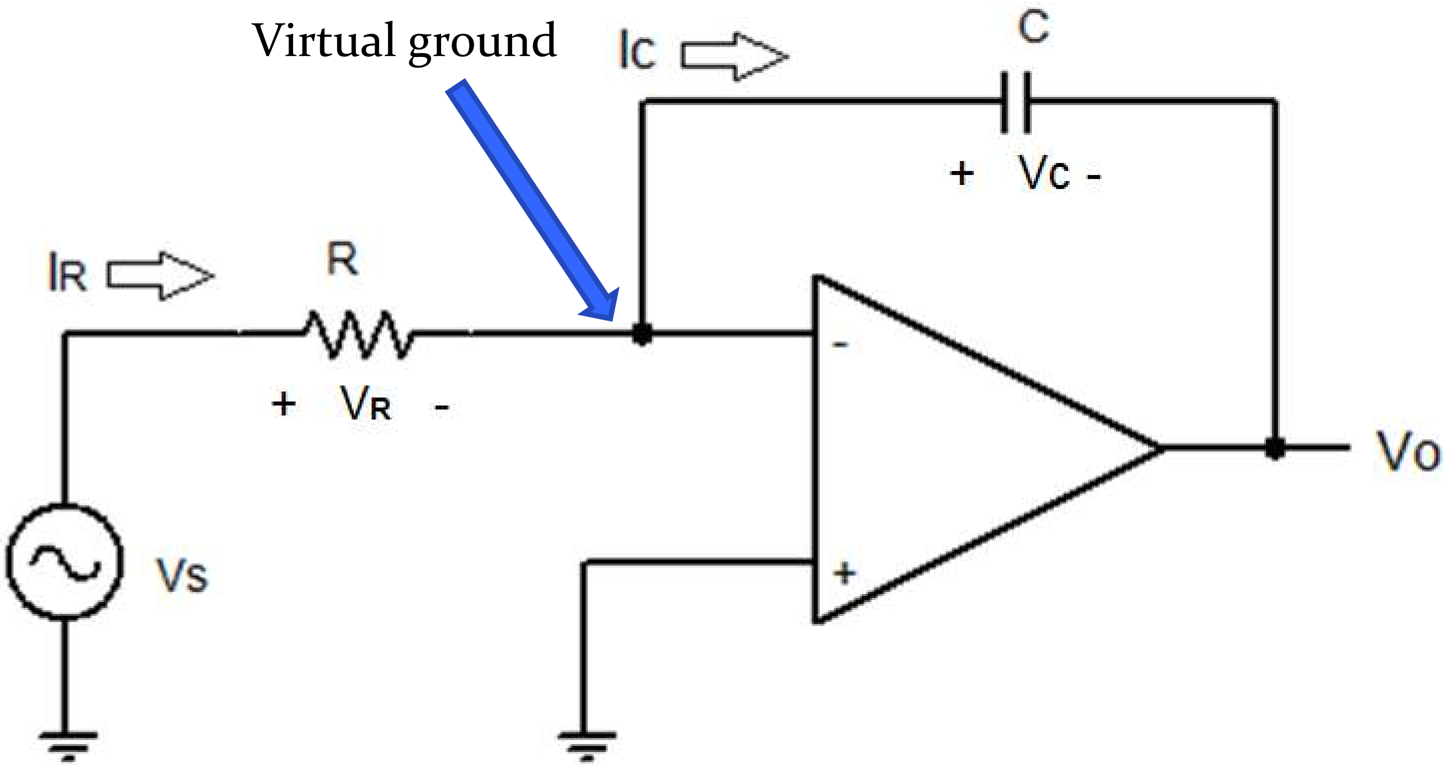
As $v_S(t) = 2V\sin(500t)$, the output voltage lags the input voltage by 90 degrees.

PSpice Simulation

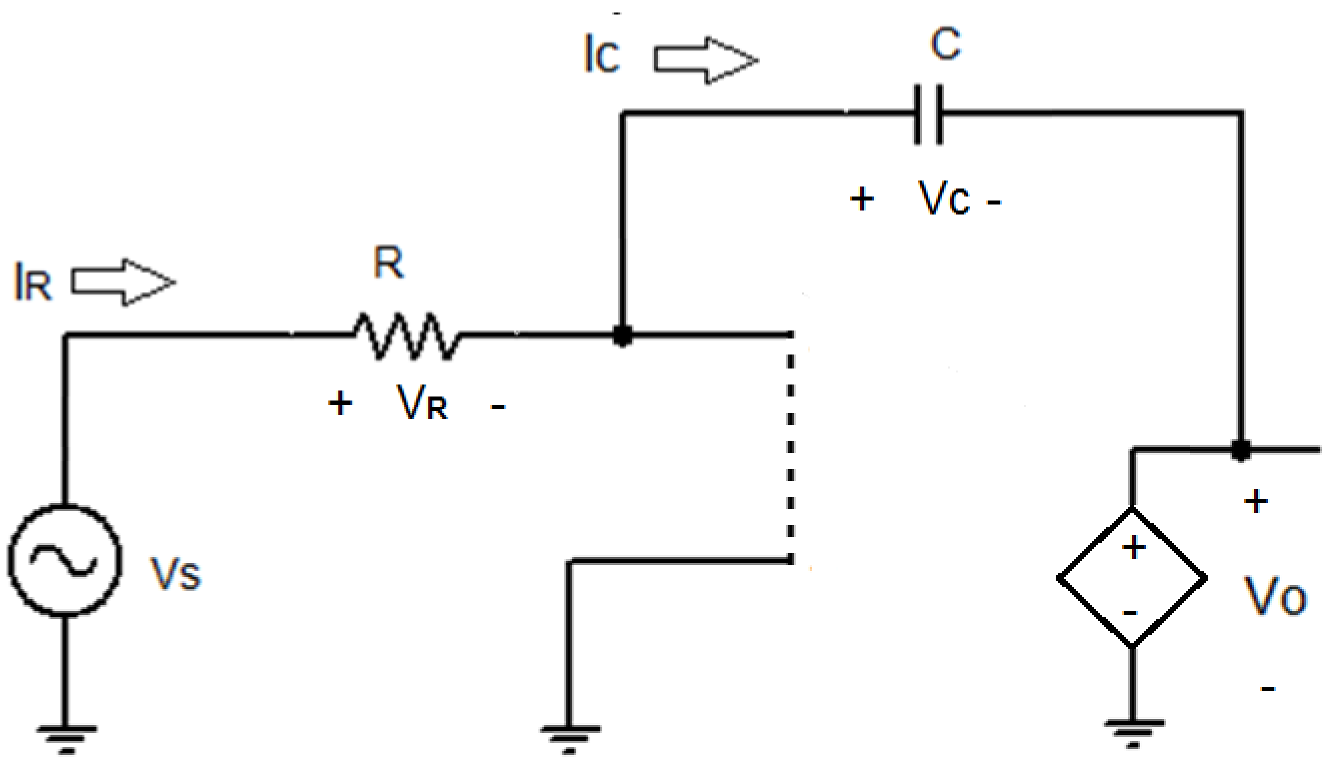


Shows the 90 degree phase shift as well as the attenuation.

Integrator

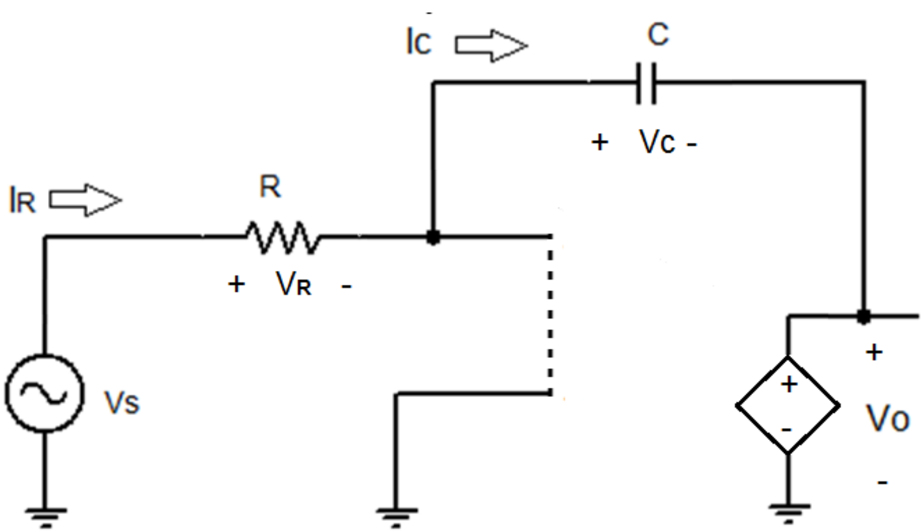


Op Amp Model



Integrator

- Op-Amp Model:



$$i_R = \frac{v_S(t) - v_1}{R} = \frac{v_S(t)}{R}$$

$$i_C = C\frac{dv_C}{dt}$$

$$v_C(t) = v_1 - v_o(t) = -v_o(t)$$

$$i_R - i_C = 0mA$$

$$\frac{v_S(t)}{R} - C\frac{d\left[-v_o(t)\right]}{dt} = 0$$

$$\frac{dv_o(t)}{dt} + \frac{v_S(t)}{RC} = 0$$

$$v_o(t_2) = \frac{-1}{RC}\int_{t_1}^{t_2} v_S(t)dt + v_o(t_1)$$

Example 03

- Let R = 25 kΩ, C = 5nF, $v_S(t) = 3V \sin\left(6.24k\frac{rad}{s}t\right)$ at t=0s

$$V_o(t_2) = \frac{-1}{RC}\int_{t_1}^{t_2} V_{in}(t)dt + V_o(t_1)$$

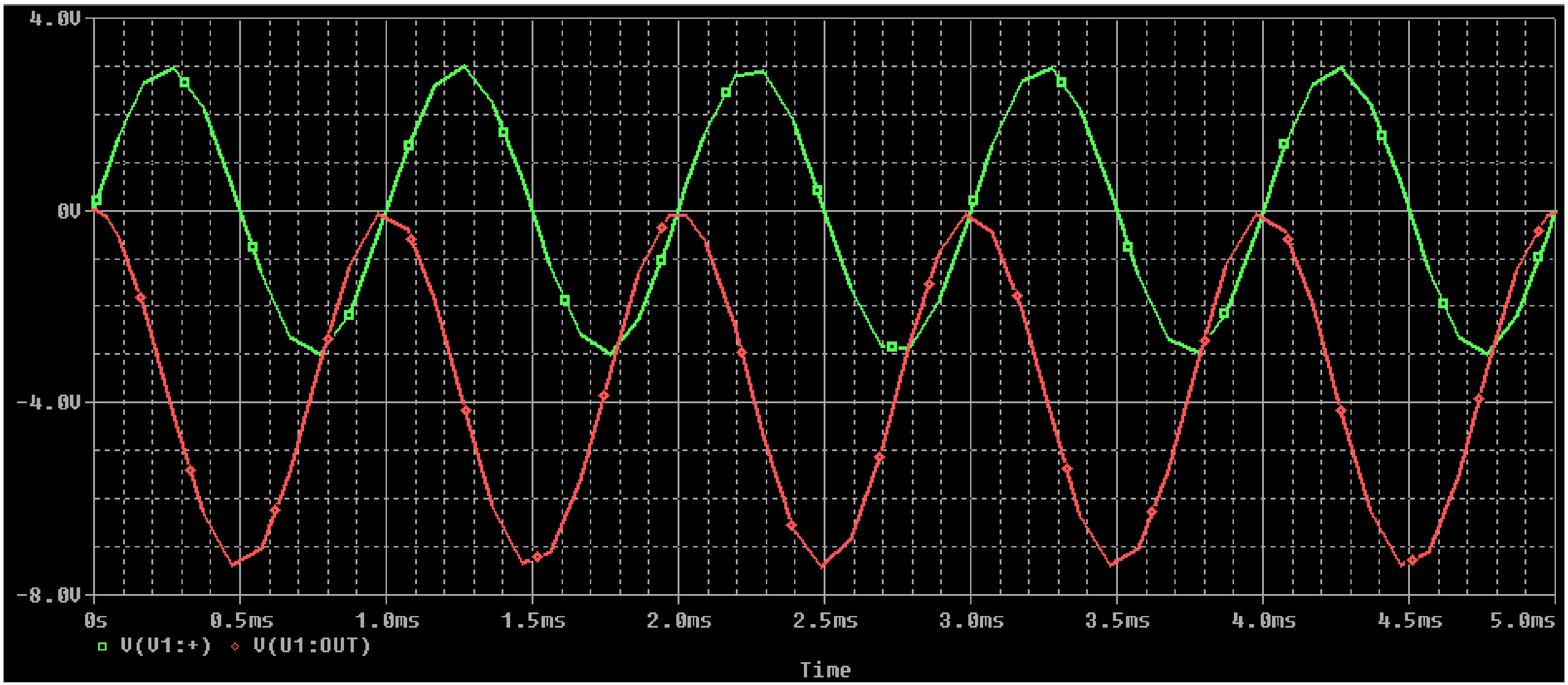
$$V_o(t_2) = \frac{-1}{25k\Omega(5nF)}\int_{t_1}^{t_2} 3V \sin\left(6.24k\frac{rad}{s}t\right)dt$$

$$V_o(t_2) = 3.85V \cos\left(6.24k\frac{rad}{s}t\right)\Bigg|_{t_1}^{t_2} + V_o(t_1)$$

$$V_o(t_2) = 3.85V \sin\left(6.24k\frac{rad}{s}t_2 + 90^o\right) - 3.85V \text{ when } t_1 = 0s$$

since $v_o(t) = -v_C(t)$ and the voltage across a capacitor can't be discontinuous.

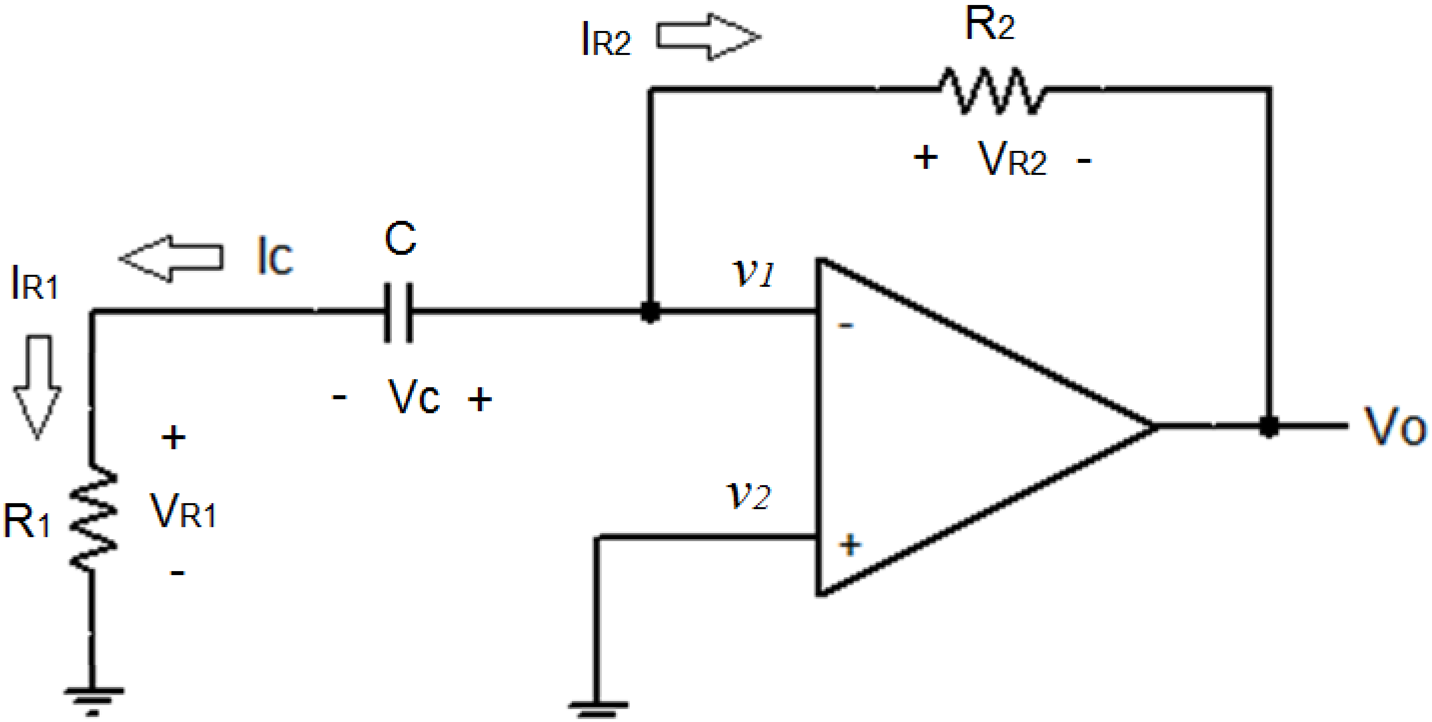
PSpice Simulation



Shows that the output voltage leads the input voltage by +90 degree and the voltage offset due to the $V_o(t_1)$ term.

Example 04…

Initial Charge on Capacitor



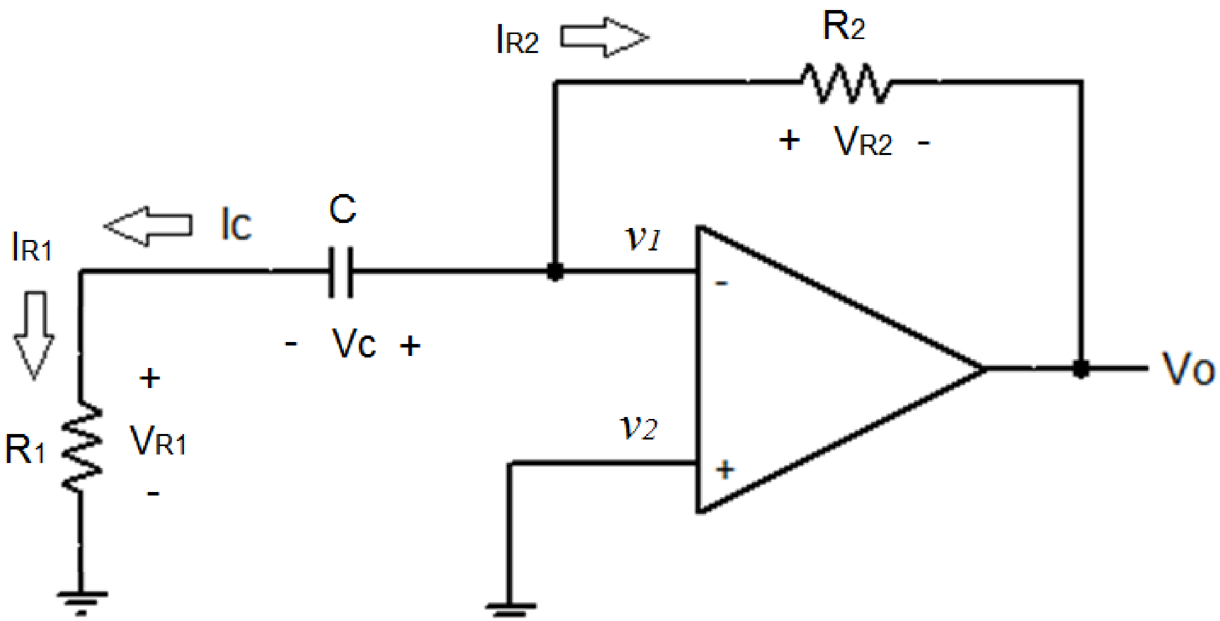
…Example 04…

If there is an initial charge that produces a voltage on the capacitor at some time, t_o:

The voltage on the negative input of the op amp is:

$v_1 = V_C + V_{R1}$

$v_1 = v_2 = 0V$



…Example 04…

The current flowing through R_1 is the same current flowing through C.

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$i_{R1}(t) = \frac{V_{R1}}{R_1} = \frac{[v_1 - v_C(t)]}{R_1} = \frac{[0V - v_C(t)]}{R_1} = -\frac{v_C(t)}{R_1}$$

$$\text{at t} = \text{t}_\text{o}, i_R(t_o) = -\frac{v_C(t_o)}{R_1}$$

$$\text{as t} \rightarrow \infty, v_C(t) \rightarrow 0V, i_C(t) \rightarrow 0mA$$

…Example 04…

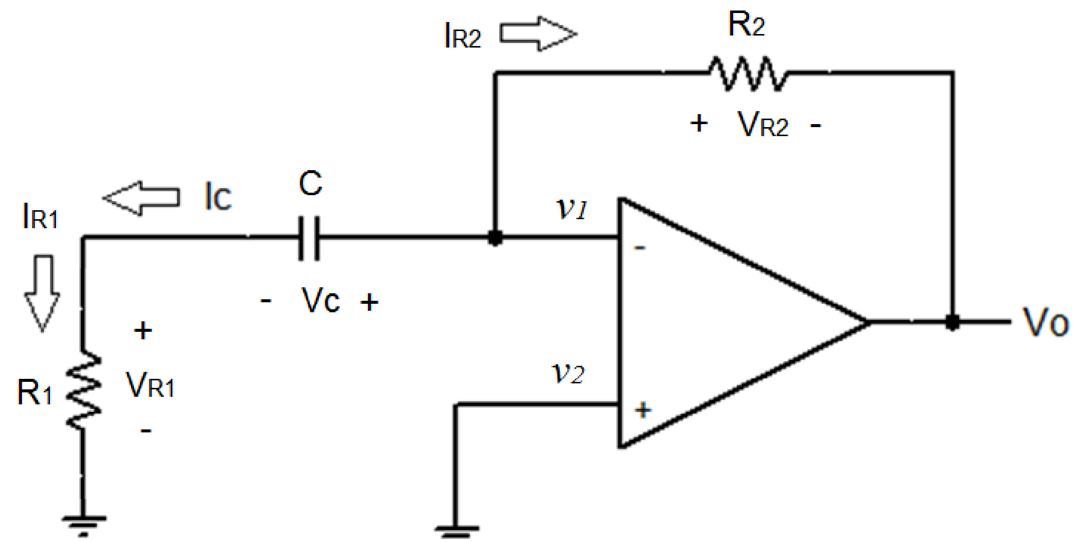
$$i_C(t) - i_{R1}(t) = 0$$

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R_1} = 0$$

$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R_1C} = 0$$

$$v_C(t) = v_C(t_o)e^{-\frac{t-t_o}{R_1C}}$$

$$i_C = C\frac{dv_C(t)}{dt} = -\frac{1}{R_1}v_C(t_o)e^{-\frac{t-t_o}{R_1C}}$$



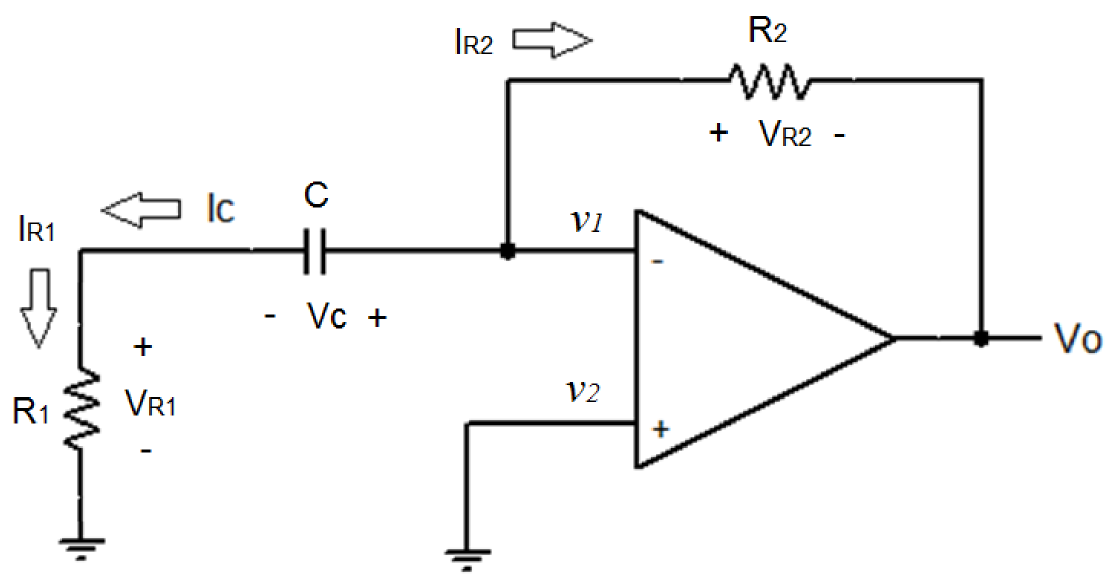
R_1C is the time constant, τ.

…Example 04

$$i_{R2} = -i_C$$

$$i_{R2} = \frac{0V - v_o(t)}{R_2} = -\frac{v_o(t)}{R_2}$$

$$v_o(t) = \frac{R_2}{R_1} v_C(t_o) e^{-\frac{t-t_o}{R_1 C}}$$



Summary

- Differentiator and integrator circuits are 1st order op amp circuits.
  - When the C is connected to the input of the op amp, the circuit is a differentiator.
    - If the input voltage is a sinusoid, the output voltage lags the input voltage by 90 degrees.
      - The output voltage may be discontinuous.
  - When the C is connected between the input and output of the op amp, the circuit is an integrator.
    - If the input voltage is a sinusoid, the output voltage leads the input voltage by 90 degrees.
      - The output voltage must be continuous.