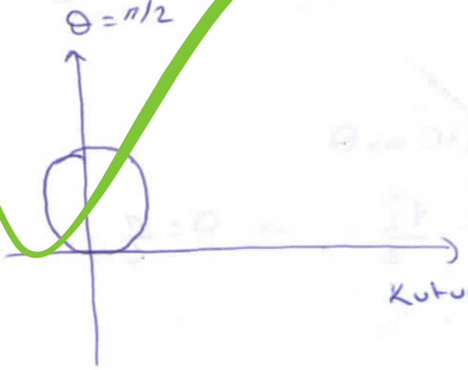
(*) $r=\sqrt{2}\sin\theta$ ile sınırlı bölgenin alanı?

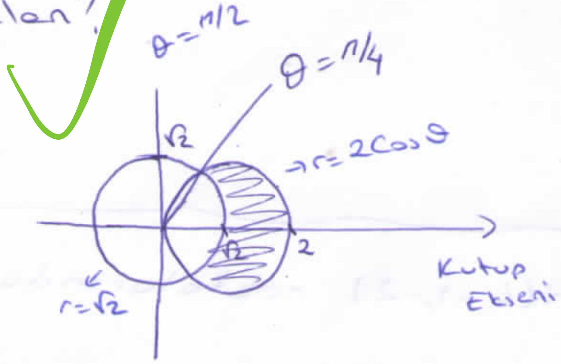


$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/2}(\sqrt{2}\sin\theta)^2\,d\theta$$

$$=\frac{1}{2}\int_0^{\pi/2}2\sin^2\theta\,d\theta$$

$$=\int_0^{\pi/2}\frac{1-\cos 2\theta}{2}\,d\theta=\frac{\pi}{4}\qquad \boxed{A=\frac{\pi}{2}}$$

(*) a) $r=2\cos\theta$ eğrisinin içinde $r=\sqrt{2}$ nin dışında kalan alan?



$$2\cos\theta=\sqrt{2}\ \rightarrow\ \boxed{\theta=\frac{\pi}{4}}$$

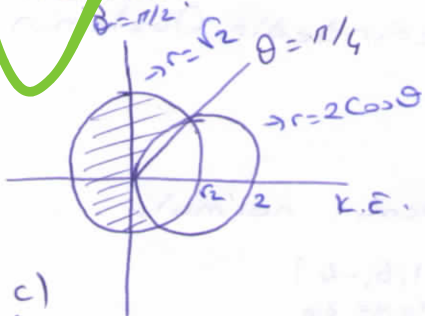
$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/4}(2\cos\theta)^2\,d\theta-\frac{1}{2}\int_0^{\pi/4}(\sqrt{2})^2\,d\theta$$

$$=\frac{1}{2}\left[\int_0^{\pi/4}(4\cos^2\theta-2)\,d\theta\right]$$

$$=\frac{1}{2}\int_0^{\pi/4}2\cos 2\theta\,d\theta=\left.\frac{\sin 2\theta}{2}\right|_0^{\pi/4}=\frac{1}{2}$$

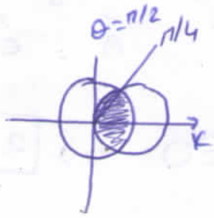
$$\boxed{A=1}$$

b) ☆ $r=2\cos\theta$ nın dışında, $r=\sqrt{2}$ nin içinde kalan alanı veren integral:



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/4}^{\pi}(\sqrt{2})^2\,d\theta-\frac{1}{2}\int_{\pi/4}^{\pi/2}(2\cos\theta)^2\,d\theta$$

c) ☆ Ortak Alanı veren integral:



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/4}(\sqrt{2})^2\,d\theta+\frac{1}{2}\int_{\pi/4}^{\pi/2}(2\cos\theta)^2\,d\theta$$

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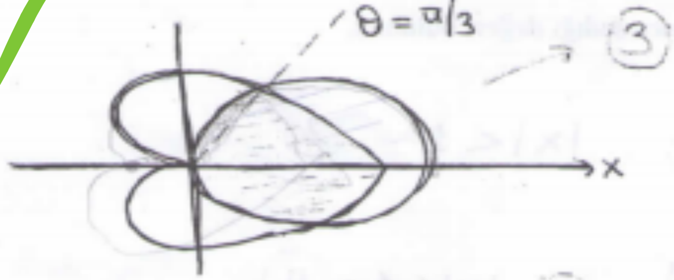
4. a) $r=3\cos\theta$ ve $r=1+\cos\theta$ eğrilerinin içinde kalan ortak bölgenin alanını veren integrali yazınız. (İntegral hesaplanmayacak.)



a)

$$\frac{A}{2}=\frac{1}{2}\int_{0}^{\pi/3}(1+\cos\theta)^2\,d\theta+\frac{1}{2}\int_{\pi/3}^{\pi/2}(3.\cos\theta)^2\,d\theta$$

2 2 2 2 2 2

b) Kardiyoid içi, çember dışı

$$\frac{A}{2}=\frac{1}{2}\int_{\pi/3}^{\pi}(1+\cos\theta)^2\,d\theta-\frac{1}{2}\int_{\pi/3}^{\pi/2}(3\cos\theta)^2\,d\theta$$

c) Çember içi kar. dışı

$$\frac{A}{2}=\frac{1}{2}\int_{0}^{\pi/3}(3\cos\theta)^2-(1+\cos\theta)^2\,d\theta$$

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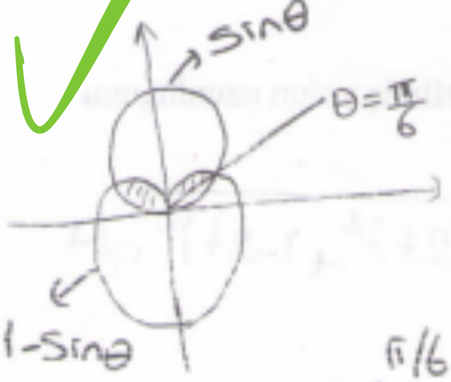
b) $\rho=1-\sin\theta$ kardiyoidi ve $\rho=\sin\theta$ çemberinin her ikisinin de içinde kalan bölgenin alanını bulunuz .(12p)



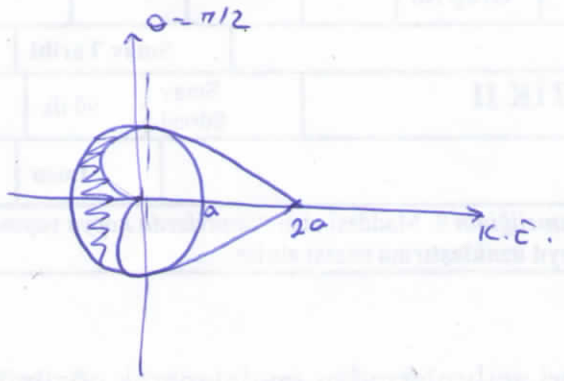
$$1-\sin\theta=\sin\theta \Rightarrow \theta=\frac{\pi}{6},\ \frac{5\pi}{6}$$

$$\frac{A}{2}=\frac{1}{2}\int_{0}^{\pi/6}(\sin\theta)^2\,d\theta+\frac{1}{2}\int_{\pi/6}^{\pi/2}(1-\sin\theta)^2\,d\theta$$

$$A=\int_{0}^{\pi/6}\frac{1-\cos 2\theta}{2}\,d\theta+\int_{\pi/6}^{\pi/2}\frac{1}{2}(3-4\sin\theta-\cos 2\theta)\,d\theta$$

$$A=\left(\frac{\pi}{12}-\frac{\sqrt{3}}{8}\right)+\left(\frac{\pi}{2}-\frac{7\sqrt{3}}{8}\right)=\frac{7\pi}{12}-\sqrt{3}\ \text{br}^2$$

2) $a>0$ olmak üzere $r=a(1+\cos\theta)$ kardiyoidinin dışında, $r=a$ çemberinin içinde kalan bölgenin alanını hesaplayınız. (Şekil çiziniz)



$$\frac{A}{2}=\int_{\pi/2}^{\pi} a^2-(a+a\cos\theta)^2\,d\theta=\int_{\pi/2}^{\pi}\left(-2a^2\cos\theta-a^2\underbrace{\cos^2\theta}_{\frac{1+\cos 2\theta}{2}}\right)d\theta$$

$$=-2a^2\sin\theta-a^2\frac{\theta}{2}-\frac{a^2}{4}\sin 2\theta\Big|_{\pi/2}^{\pi}$$

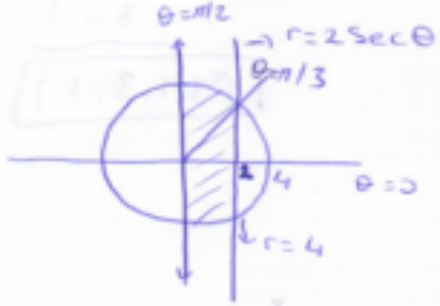
$$=-a^2\frac{\pi}{2}-\left(-2a^2-a^2\frac{\pi}{4}\right)=-\frac{a^2\pi}{2}+2a^2+\frac{a^2\pi}{4}$$

$$=2a^2-\frac{\pi}{4}a^2$$

---

* $r=4$, $\theta=\frac{\pi}{2}$, $r=2\sec\theta$ arasında kalan bölgenin alanını veren integral?

$r=2\sec\theta=\frac{2}{\cos\theta} \Rightarrow \underbrace{r\cos\theta}_{x}=2 \Rightarrow \boxed{x=2}$ doğrusu

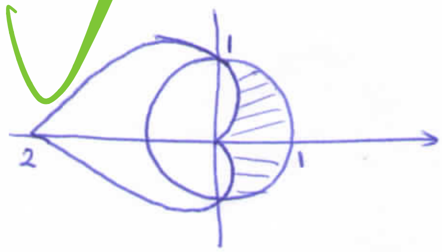


$2\sec\theta=4 \Rightarrow \sec\theta=2 \Rightarrow \cos\theta=\frac{1}{2}$

$\theta=\pi/3$

$$\frac{A}{2}=\frac{1}{2}\int_{0}^{\pi/3}(2\sec\theta)^2\,d\theta+\frac{1}{2}\int_{\pi/3}^{\pi/2}(4)^2\,d\theta$$

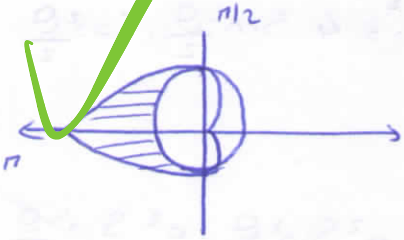
(*) r=1 çemberinin içinde, r=1-Cosθ kardiyoidinin dışında kalan bölgenin alanını veren integral?



$$\frac{A}{2} = \int_0^{\pi/2} \frac{1}{2}\, d\theta - \frac{1}{2}\int_0^{\pi/2} (1-\cos\theta)^2\, d\theta$$

$$A = \int_0^{\pi/2} \left(1 - (1-\cos\theta)^2\right) d\theta$$

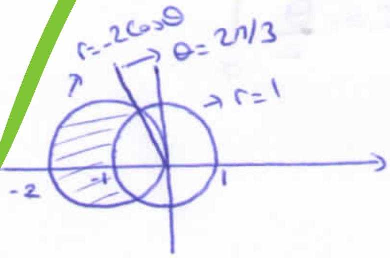
b) çemberin dışı, kardiyoidin içi:



$$\frac{A}{2} = \int_{\pi/2}^{\pi} \frac{1}{2} (1-\cos\theta)^2\, d\theta - \int_{\pi/2}^{\pi} \frac{1}{2}\, d\theta$$

$$A = \int_{\pi/2}^{\pi} \left((1-\cos\theta)^2 - 1\right) d\theta$$

(*) r=-2Cosθ çemberinin içinde, r=1 çemberinin dışında kalan alan?



$$-2\cos\theta = 1 \Rightarrow \theta = \frac{2\pi}{3}$$

$$\frac{A}{2} = \frac{1}{2}\int_{2\pi/3}^{\pi} (-2\cos\theta)^2\, d\theta - \frac{1}{2}\int_{2\pi/3}^{\pi} 1^2\, d\theta$$

$$A = \int_{2\pi/3}^{\pi} (4\overbrace{\cos^2\theta}^{\frac{1+\cos 2\theta}{2}} - 1)\, d\theta = \int_{\frac{2\pi}{3}}^{\pi} (1 + 2\cos 2\theta)\, d\theta$$

$$= \theta + \sin 2\theta \Big|_{2\pi/3}^{\pi} = \pi - \frac{2\pi}{3} - \sin\frac{\pi}{3} = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$

(*) $r = a\sin^2\frac{\theta}{2}$ eğrisinin $0 \le \theta \le \pi$ aralığındaki uzunluğu? $(a>0)$

$$S = \int_0^{\pi} \sqrt{r^2 + (r')^2}\, d\theta$$

$$r^2 = a^2 \sin^4\frac{\theta}{2}$$

$$r' = a.2.\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\frac{1}{2} = a.\sin\frac{\theta}{2}.\cos\frac{\theta}{2}$$

$$(r')^2 = a^2.\sin^2\frac{\theta}{2}.\cos^2\frac{\theta}{2}$$

$$r^2 + (r')^2 = a^2.\sin^4\frac{\theta}{2} + a^2\sin^2\frac{\theta}{2}.\overbrace{\cos^2\frac{\theta}{2}}^{1-\sin^2\frac{\theta}{2}} = a^2\cancel{\sin^4\frac{\theta}{2}} + a^2\sin^2\frac{\theta}{2} - a^2\cancel{\sin^4\frac{\theta}{2}}$$

$$= a^2\sin^2\frac{\theta}{2}$$

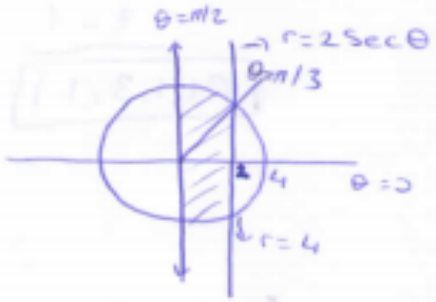
$$\sqrt{r^2+(r')^2} = \sqrt{a^2.\sin^2\frac{\theta}{2}} = a.\left|\sin\frac{\theta}{2}\right|$$

$$S = \int_0^{\pi} a.\left|\sin\frac{\theta}{2}\right| d\theta = \int_0^{\pi} a.\sin\frac{\theta}{2}\, d\theta = -2a\cos\frac{\theta}{2}\Big|_0^{\pi}$$

$$= \boxed{2a}$$

(*) $r=4$, $\theta = \frac{\pi}{2}$, $r = 2\sec\theta$ arasında kalan bölgenin alanını veren integral?

$$r = 2\sec\theta = \frac{2}{\cos\theta} \Rightarrow \underbrace{r\cos\theta}_{x} = 2 \Rightarrow \boxed{x=2} \text{ doğrusu}$$



$$2\sec\theta = 4 \Rightarrow \sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2}$$

$$\theta = \pi/3$$

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/3} (2\sec\theta)^2 d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} (4)^2 d\theta$$

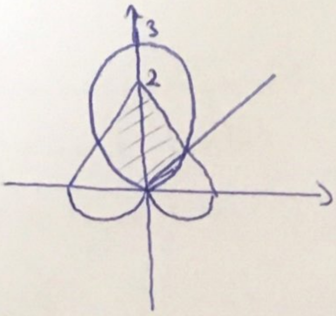
(*) a) r=3Sinθ , r=1+Sinθ ortak alan?

b) r=3Sinθ içi, r=1+Sinθ dışı alan?

c) r=3Sinθ dışı, r=1+Sinθ içi alan?

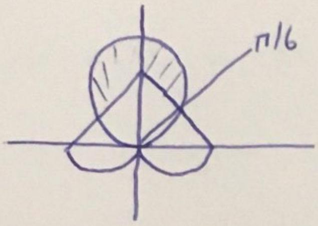
$3\sin\theta = 1+\sin\theta \Rightarrow \sin\theta = \frac{1}{2}$ $\boxed{\theta = \frac{\pi}{6}}$

a)



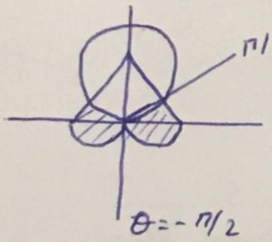
$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/6} (3\sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (1+\sin\theta)^2 d\theta$$

b)



$$\frac{A}{2} = \frac{1}{2}\int_{\pi/6}^{\pi/2} (3\sin\theta)^2 - (1+\sin\theta)^2 \, d\theta$$

c)



$$\frac{A}{2} = \frac{1}{2}\int_{-\pi/2}^{\pi/6} (1+\sin\theta)^2 d\theta - \frac{1}{2}\int_0^{\pi/6} (3\sin\theta)^2 d\theta$$

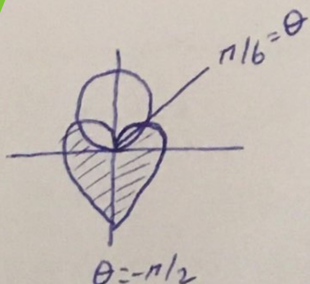
(*) a) r=1-Sinθ içi r=Sinθ dışı alan?

b) r=1-Sinθ dışı r=Sinθ içi alan?

$1-\sin\theta = \sin\theta$

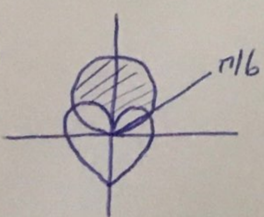
$\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}$

a)



$$\frac{A}{2} = \frac{1}{2}\int_{-\pi/2}^{\pi/6} (1-\sin\theta)^2 d\theta - \frac{1}{2}\int_0^{\pi/6} (\sin\theta)^2 d\theta$$

b)



$$\frac{A}{2} = \frac{1}{2}\int_{\pi/6}^{\pi/2} \left((\sin\theta)^2 - (1-\sin\theta)^2\right) d\theta$$