

Turing's Thesis

Turing's thesis (1930):

Any computation carried out
by mechanical means
can be performed by a Turing Machine

Algorithm:

An algorithm for a problem is a Turing Machine which solves the problem

The algorithm describes the steps of the mechanical means

This is easily translated to computation steps of a Turing machine

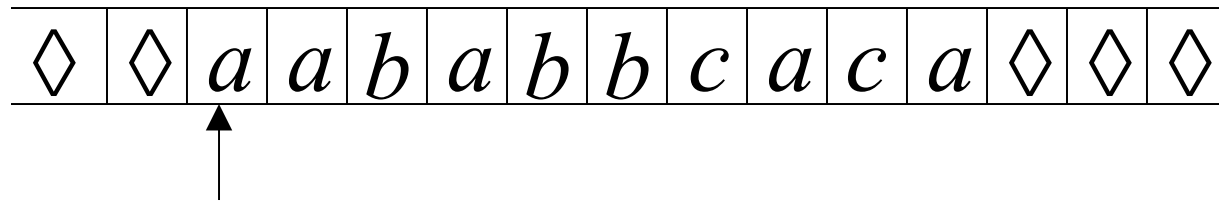
When we say: There exists an algorithm

We mean: There exists a Turing Machine
that executes the algorithm

Variations of the Turing Machine

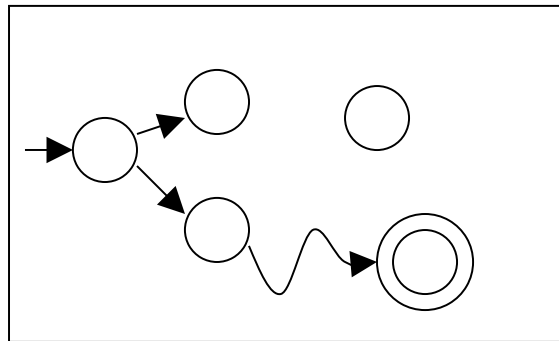
The Standard Model

Infinite Tape



Read-Write Head (Left or Right)

Control Unit



Deterministic

Variations of the Standard Model

- Turing machines with:
- Stay-Option
 - Semi-Infinite Tape
 - Multitape
 - Multidimensional
 - Nondeterministic

Different Turing Machine **Classes**

Same Power of two machine classes:

both classes accept the
same set of languages

We will prove:

each new class has the same power
with Standard Turing Machine

(accept Turing-Recognizable Languages)

Same Power of two classes means:

for any machine M_1 of first class

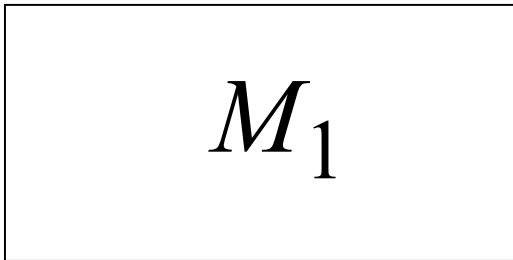
there is a machine M_2 of second class

such that: $L(M_1) = L(M_2)$

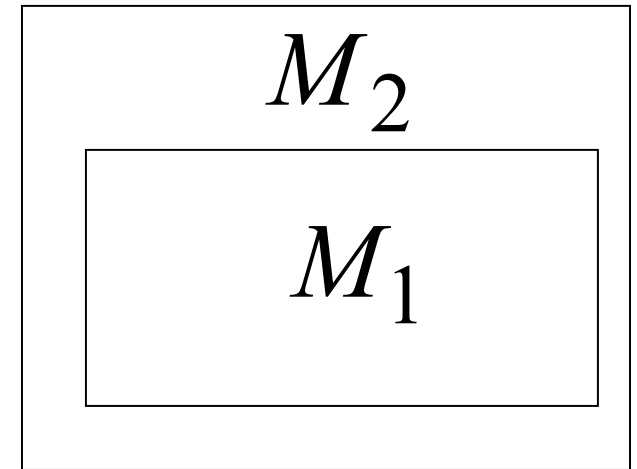
and vice-versa

Simulation: A technique to prove same power.
Simulate the machine of one class
with a machine of the other class

First Class
Original Machine



Second Class
Simulation Machine



simulates M_1

Configurations in the Original Machine M_1
 have corresponding configurations
 in the Simulation Machine M_2

Original Machine: $d_0 \succ d_1 \succ \dots \succ d_n$

M_1

$\begin{array}{ccccc} \updownarrow & & \updownarrow & & \updownarrow \\ & * & & * & \\ & & & & * \end{array}$

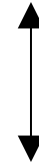
Simulation Machine: $d'_0 \succ d'_1 \succ \dots \succ d'_n$

M_2

Accepting Configuration

Original Machine:

d_f



Simulation Machine:

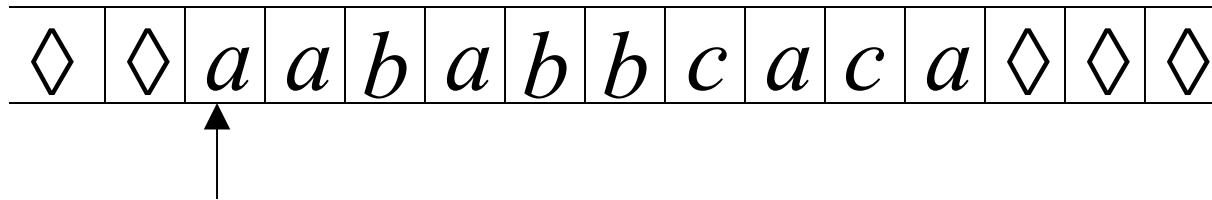
d'_f

the Simulation Machine
and the Original Machine
accept the same strings

$$L(M_1) = L(M_2)$$

Turing Machines with Stay-Option

The head can stay in the same position

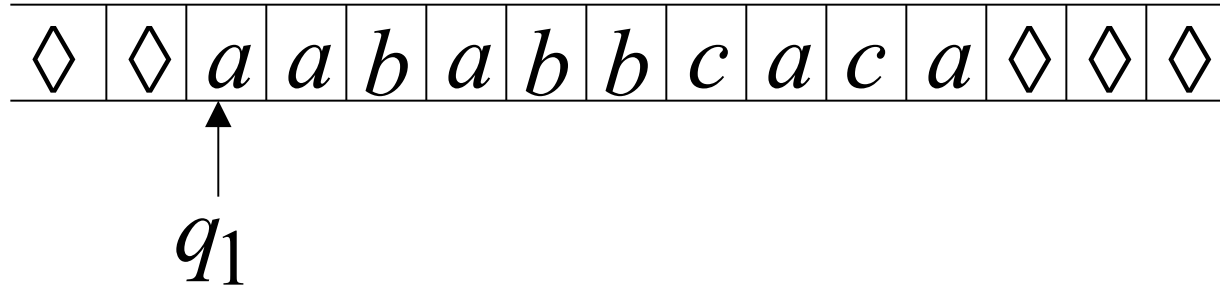


Left, Right, Stay

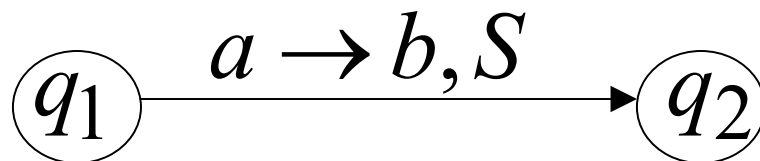
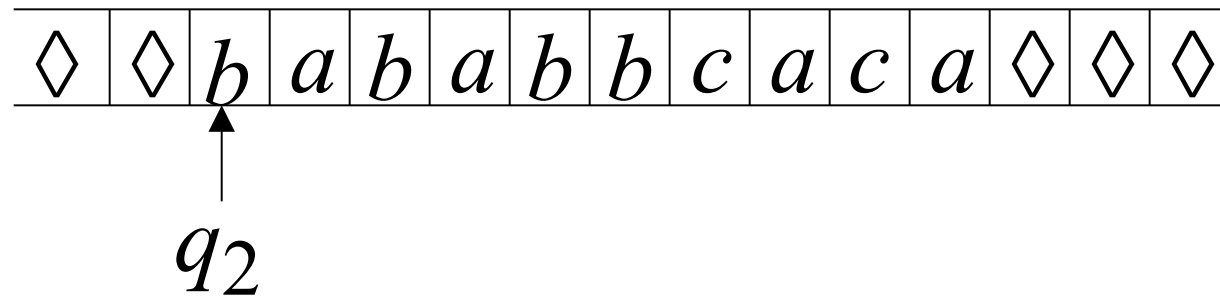
L,R,S: possible head moves

Example:

Time 1



Time 2



Theorem: Stay-Option machines
have the same power with
Standard Turing machines

Proof: 1. Stay-Option Machines
simulate Standard Turing machines

2. Standard Turing machines
simulate Stay-Option machines

1. Stay-Option Machines

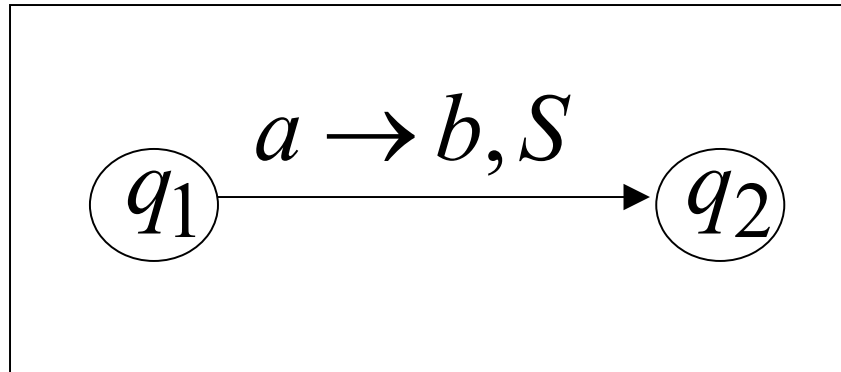
simulate Standard Turing machines

Trivial: any standard Turing machine
is also a Stay-Option machine

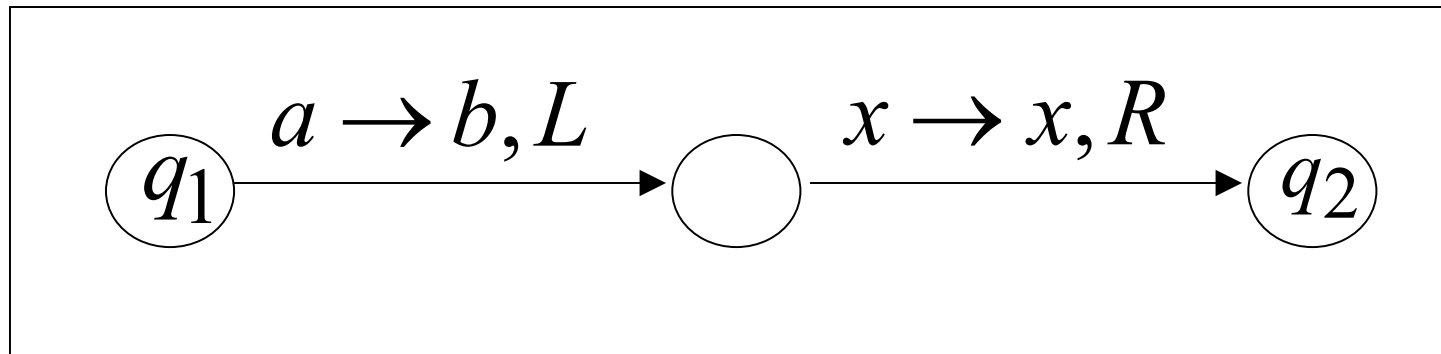
2. Standard Turing machines simulate Stay-Option machines

We need to simulate the **stay** head option
with two head moves, one **left** and one **right**

Stay-Option Machine



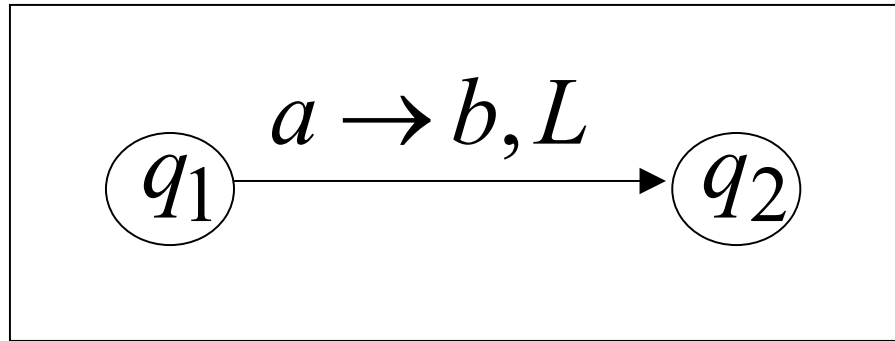
Simulation in Standard Machine



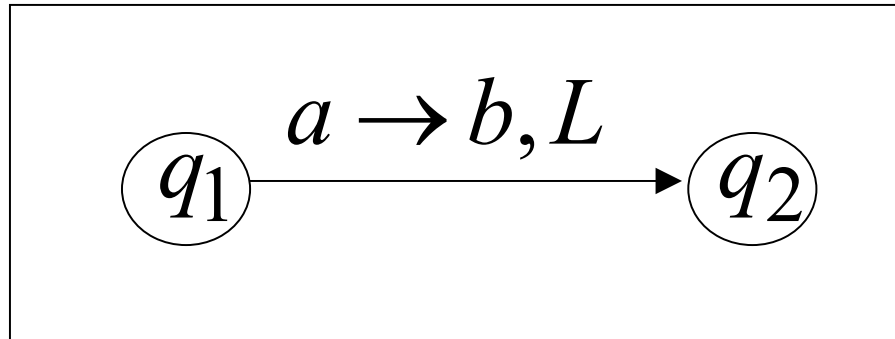
For every possible tape symbol x

For other transitions nothing changes

Stay-Option Machine



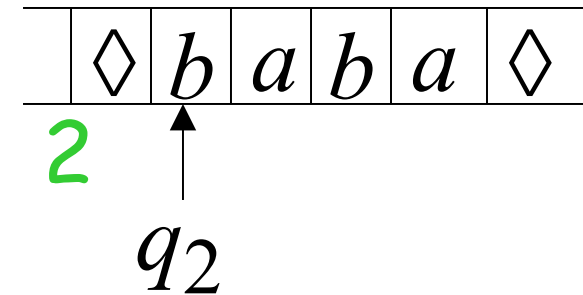
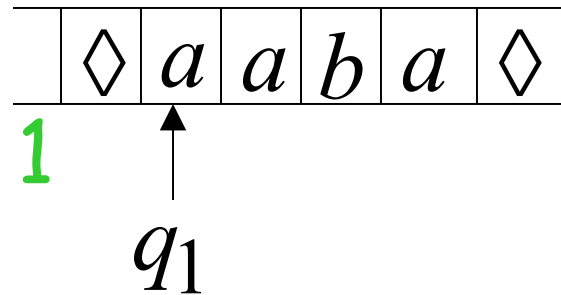
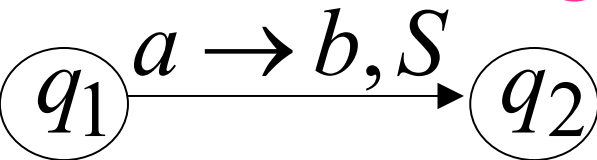
Simulation in Standard Machine



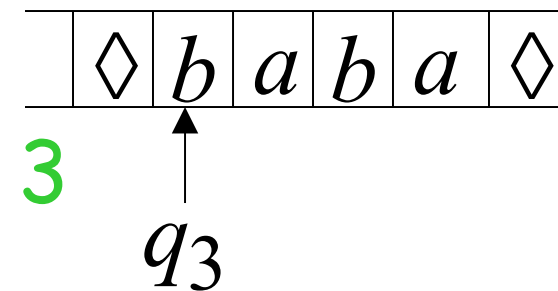
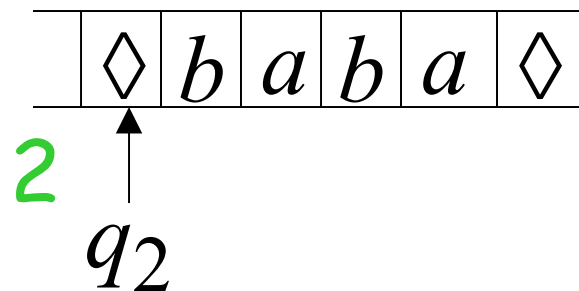
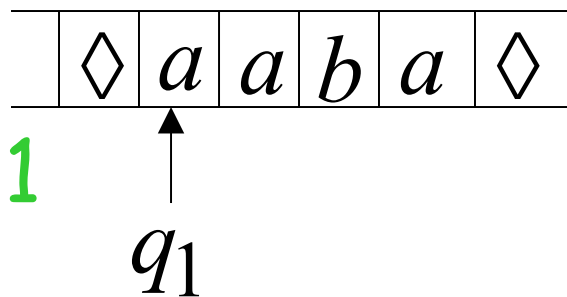
Similar for Right moves

example of simulation

Stay-Option Machine:



Simulation in Standard Machine:



END OF PROOF

A useful trick: Multiple Track Tape

helps for more complicated simulations

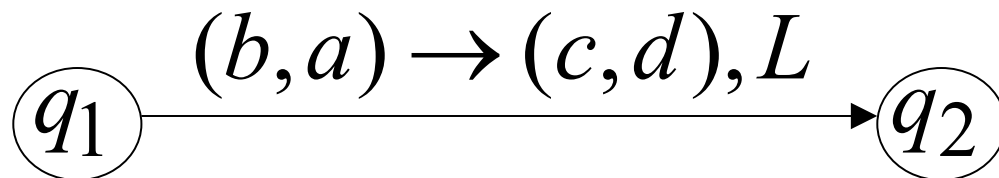
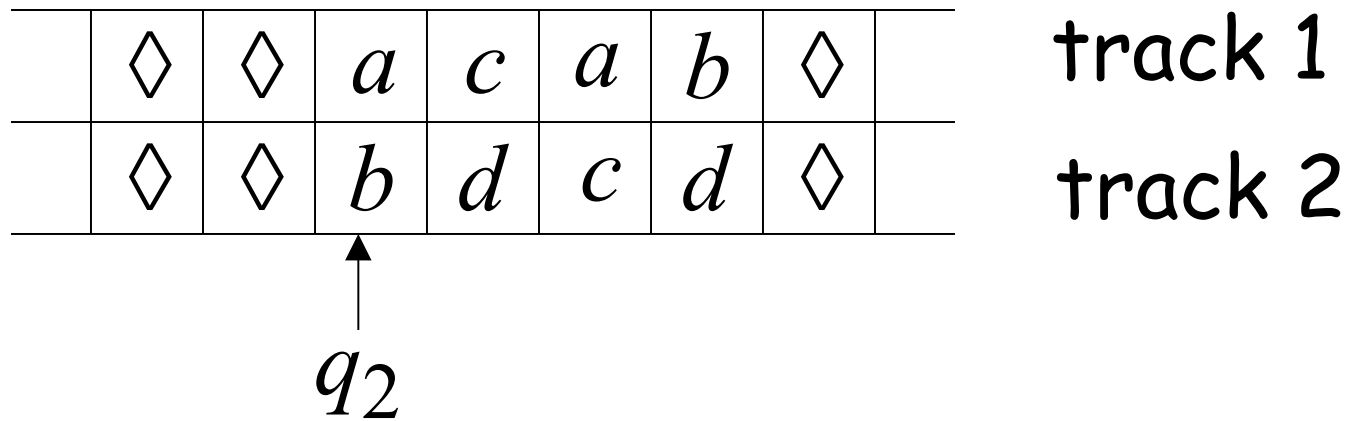
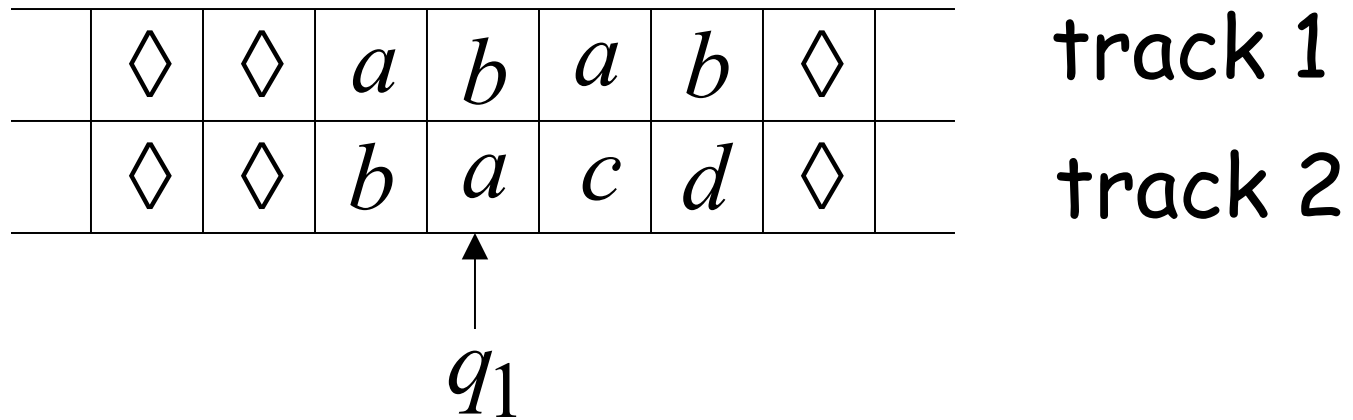
One Tape

	◇	◇	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	◇		track 1
	◇	◇	<i>b</i>	<i>a</i>	<i>c</i>	<i>d</i>	◇		track 2

One head

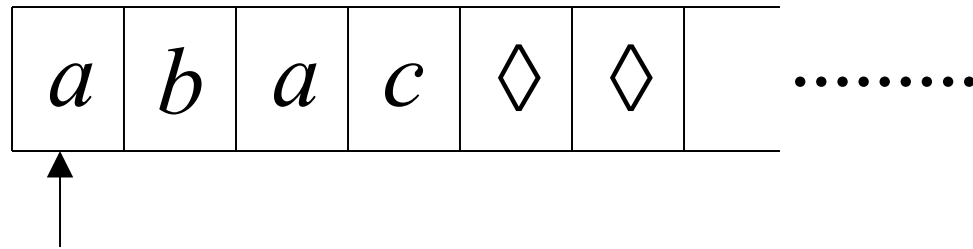
One symbol (*a*, *b*)

It is a standard Turing machine, but each tape alphabet symbol describes a pair of actual useful symbols



Semi-Infinite Tape

The head extends infinitely only to the right



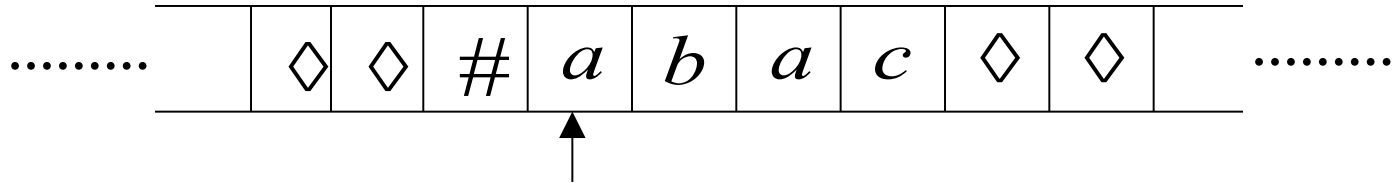
- Initial position is the leftmost cell
- When the head moves left from the border, it returns back to leftmost position

Theorem: Semi-Infinite machines
have the same power with
Standard Turing machines

Proof: 1. Standard Turing machines
simulate Semi-Infinite machines

2. Semi-Infinite Machines
simulate Standard Turing machines

1. Standard Turing machines simulate Semi-Infinite machines:

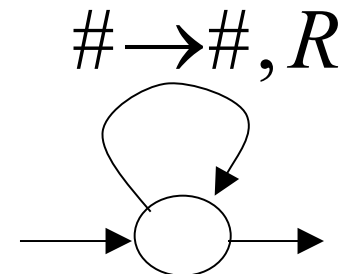


Standard Turing Machine

Semi-Infinite machine modifications

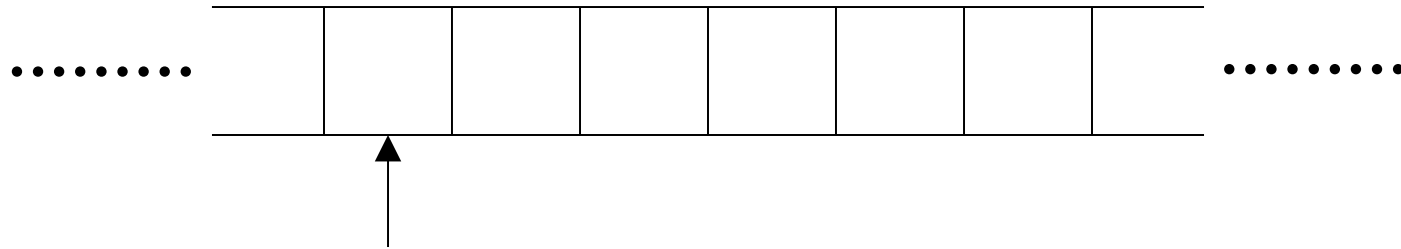
a. insert special symbol $\#$
at left of input string

b. Add a self-loop
to every state
(except states with no
outgoing transitions)

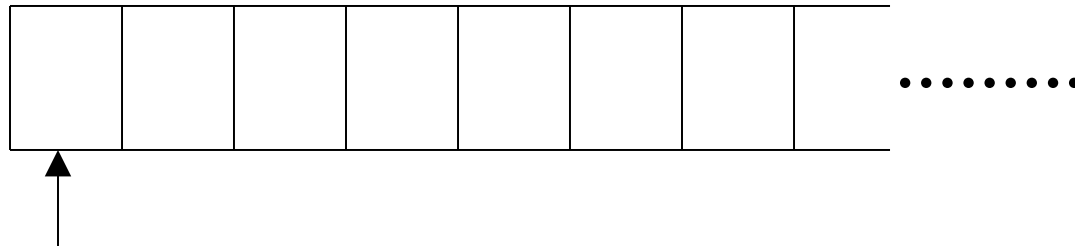


2. Semi-Infinite tape machines simulate Standard Turing machines:

Standard machine

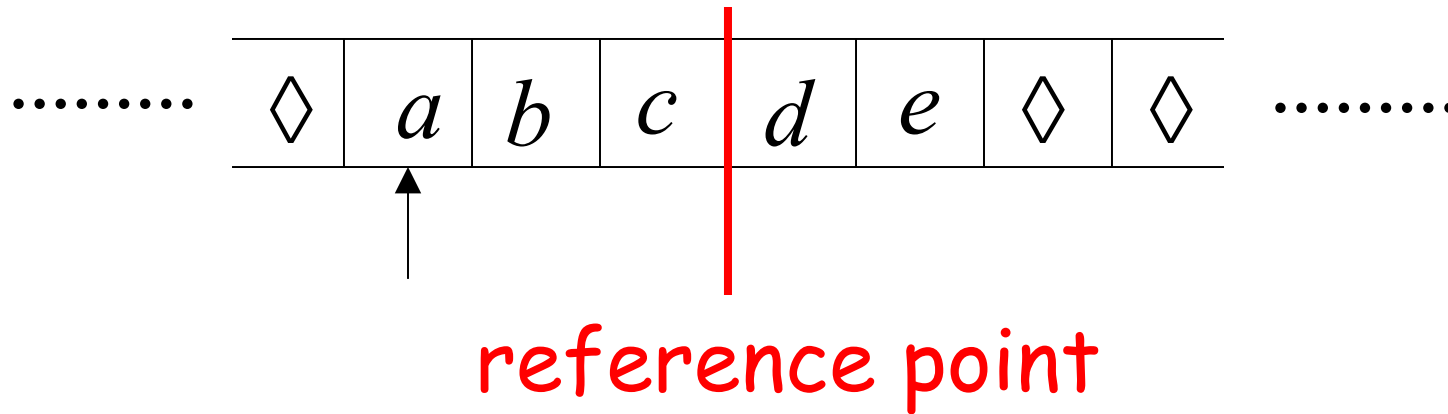


Semi-Infinite tape machine

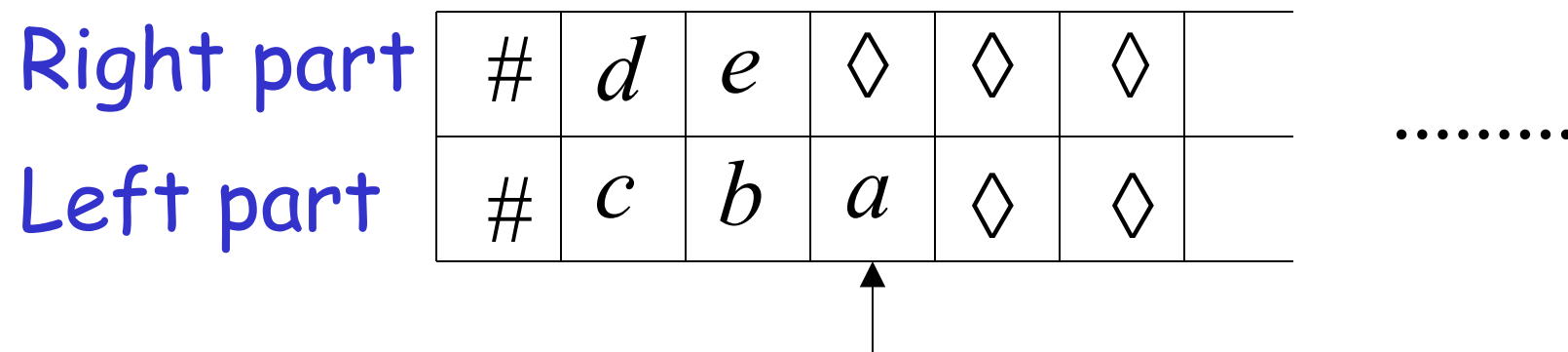


Squeeze infinity of both directions
to one direction

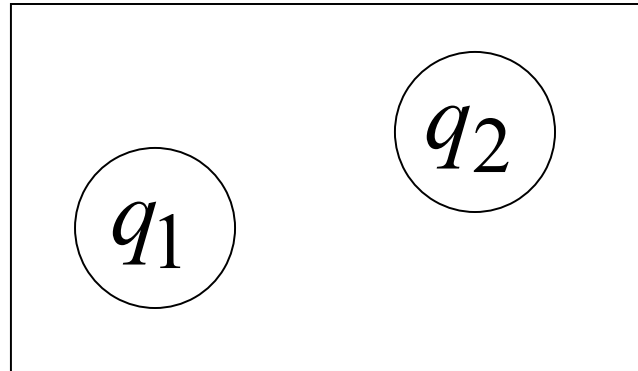
Standard machine



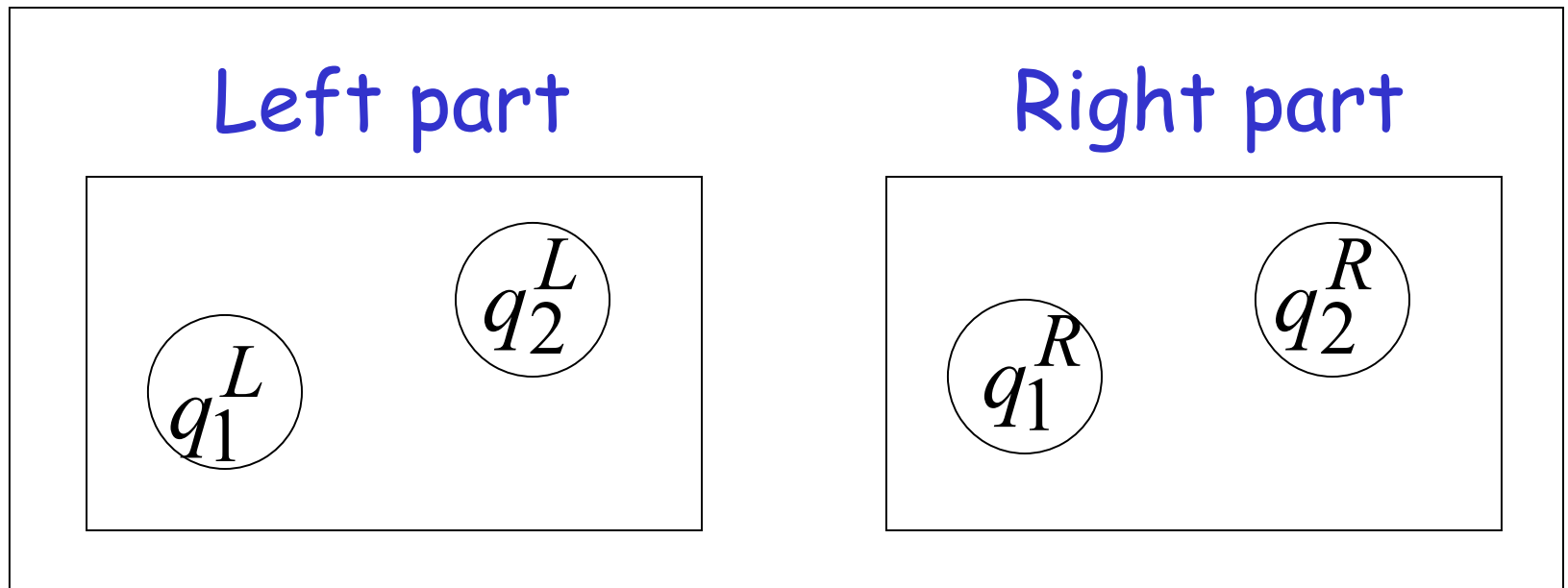
Semi-Infinite tape machine with two tracks



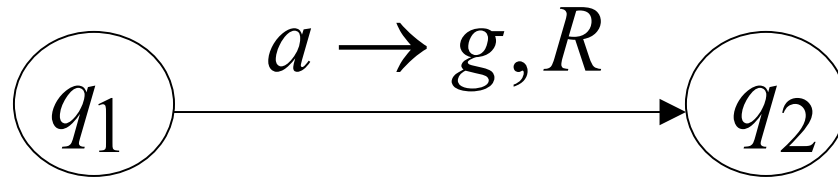
Standard machine



Semi-Infinite tape machine

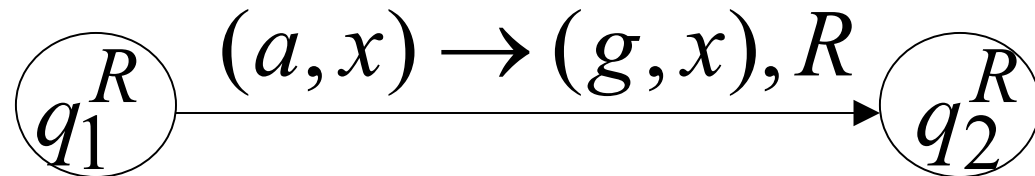


Standard machine

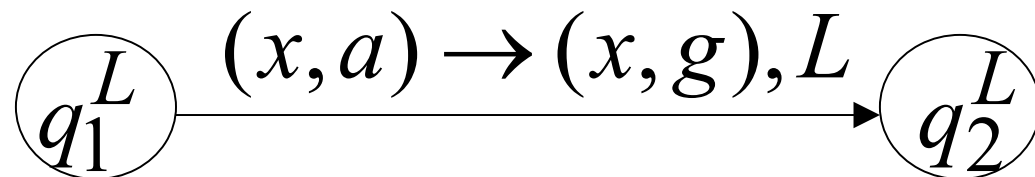


Semi-Infinite tape machine

Right part



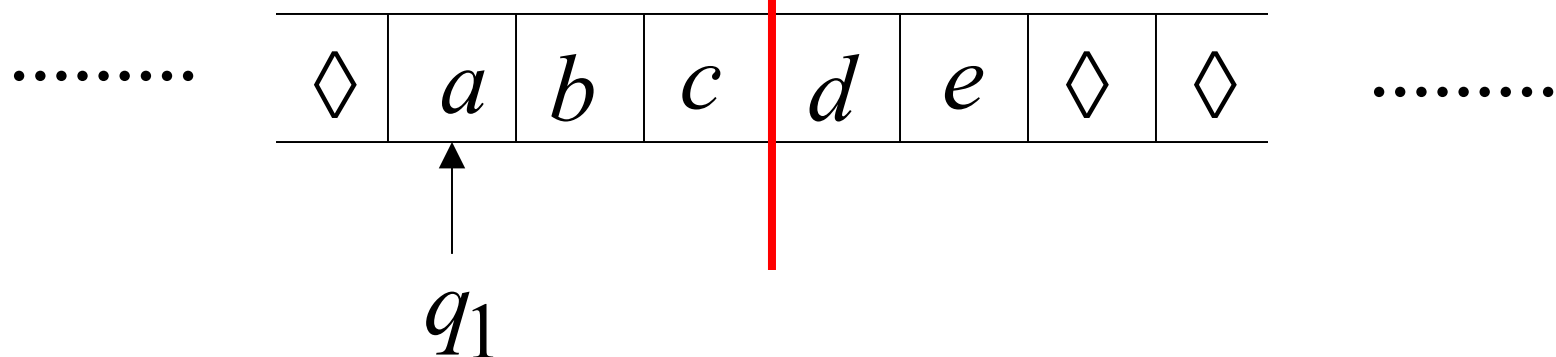
Left part



For all tape symbols x

Time 1

Standard machine



Semi-Infinite tape machine

Right part

#	d	e	\diamond	\diamond	\diamond	
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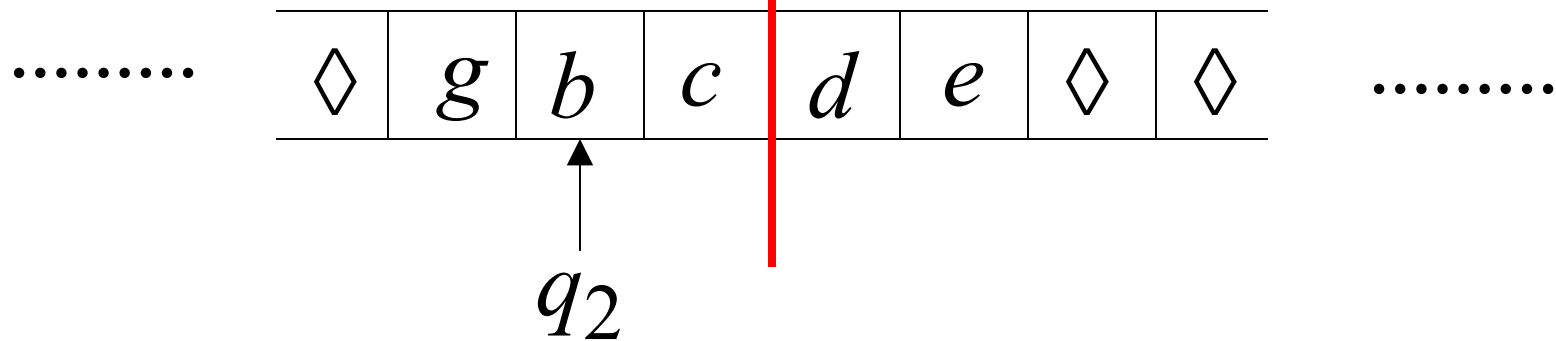
Left part

#	c	b	a	\diamond	\diamond	
---	-----	-----	-----	------------	------------	--

q_1^L

Time 2

Standard machine



Semi-Infinite tape machine

Right part

#	<i>d</i>	<i>e</i>	◇	◇	◇	
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.....

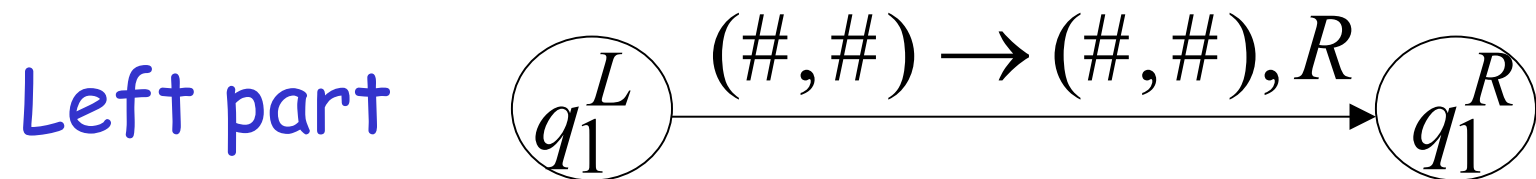
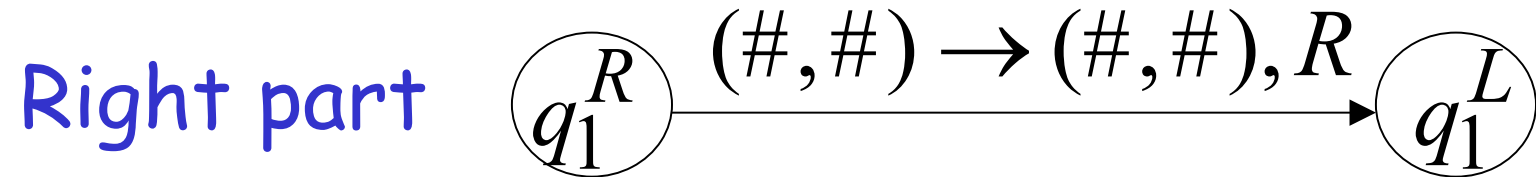
Left part

#	<i>c</i>	<i>b</i>	<i>g</i>	◇	◇	
---	----------	----------	----------	---	---	--

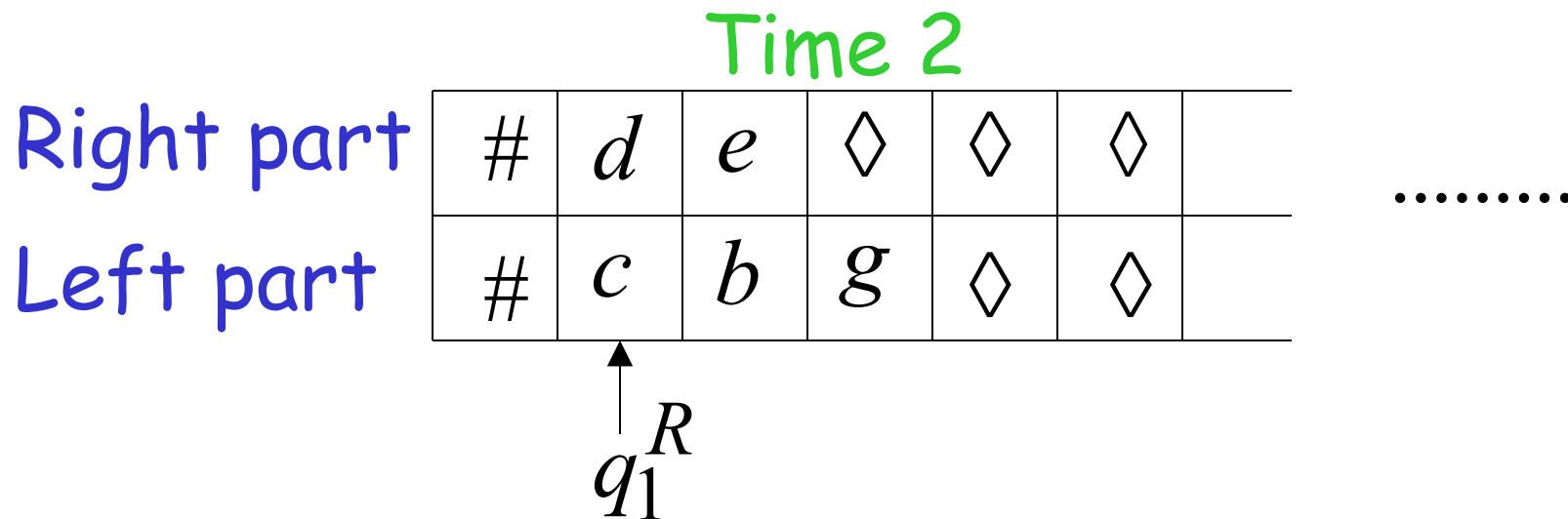
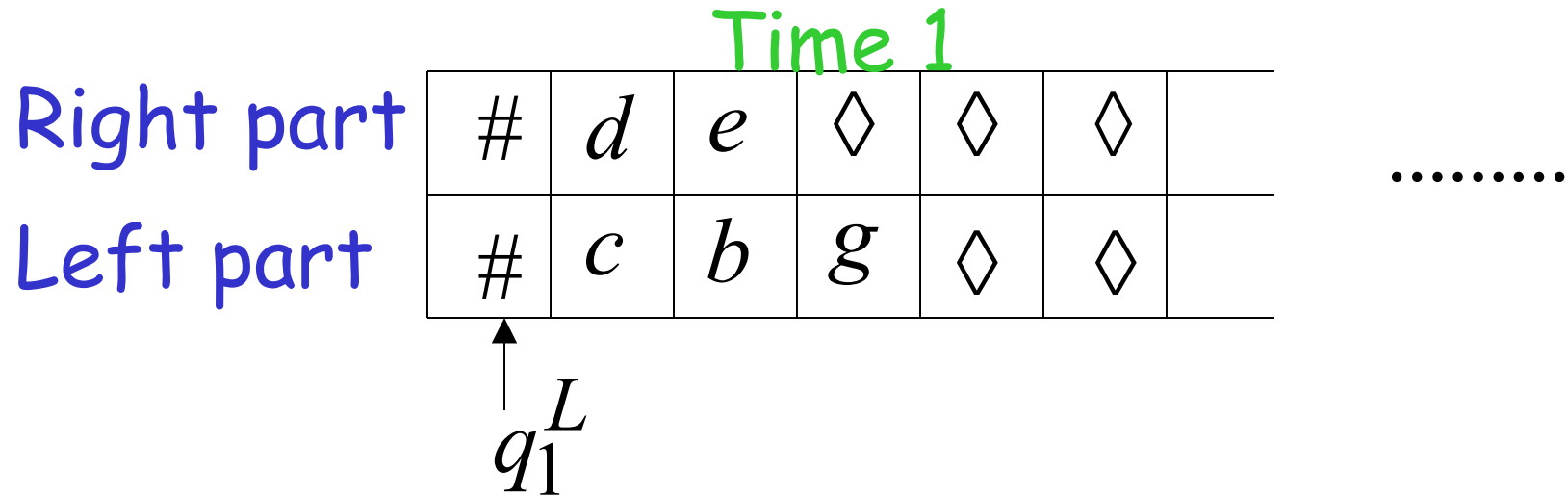
q_2^L

At the border:

Semi-Infinite tape machine

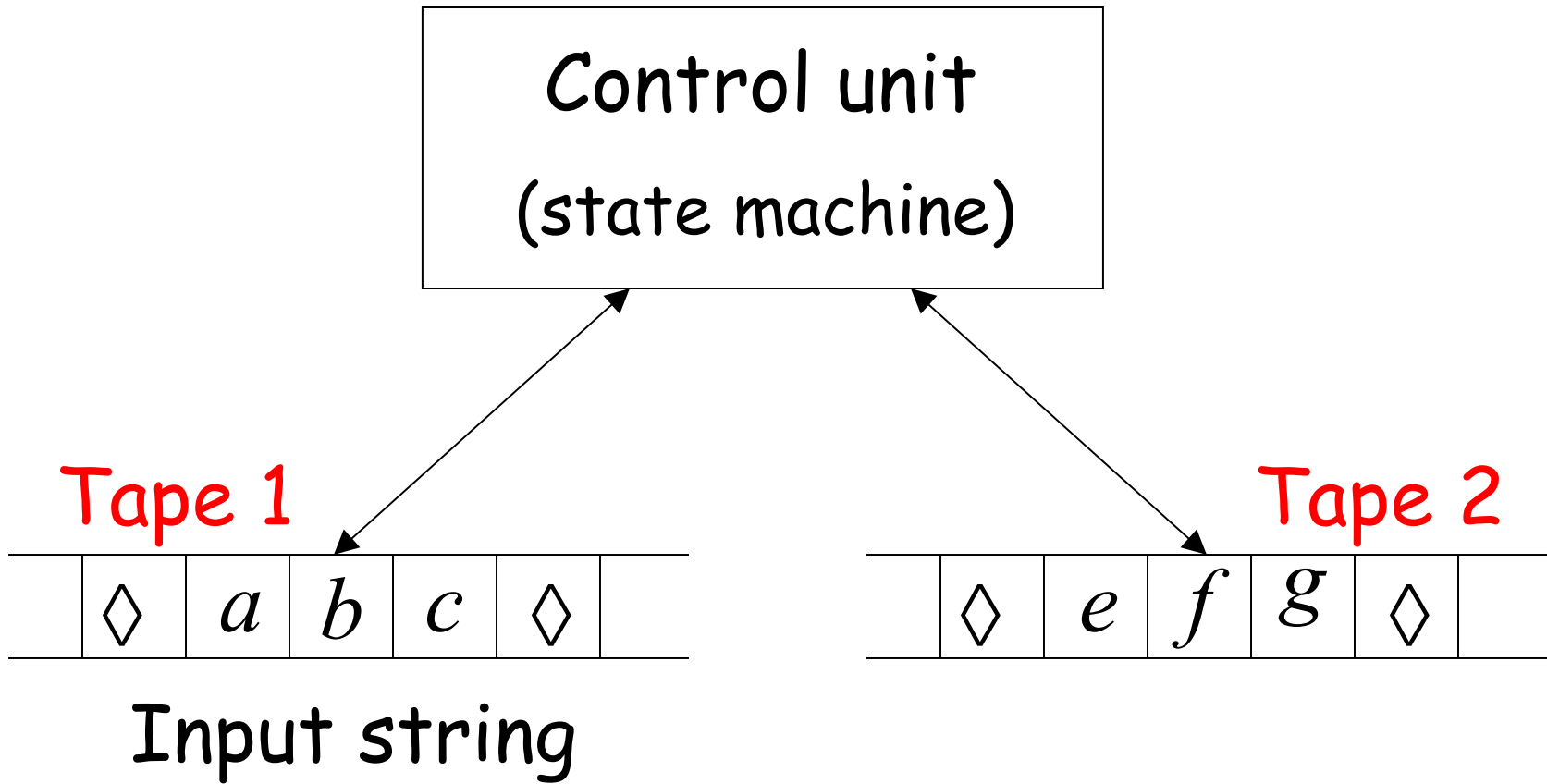


Semi-Infinite tape machine

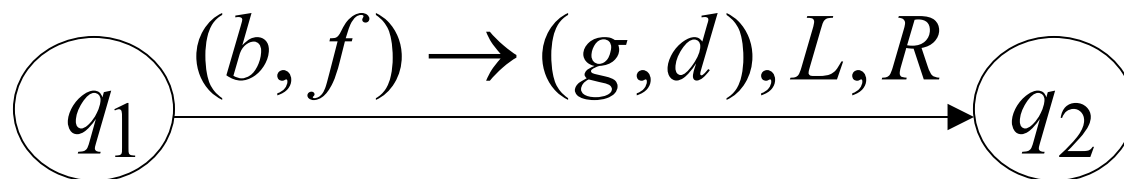
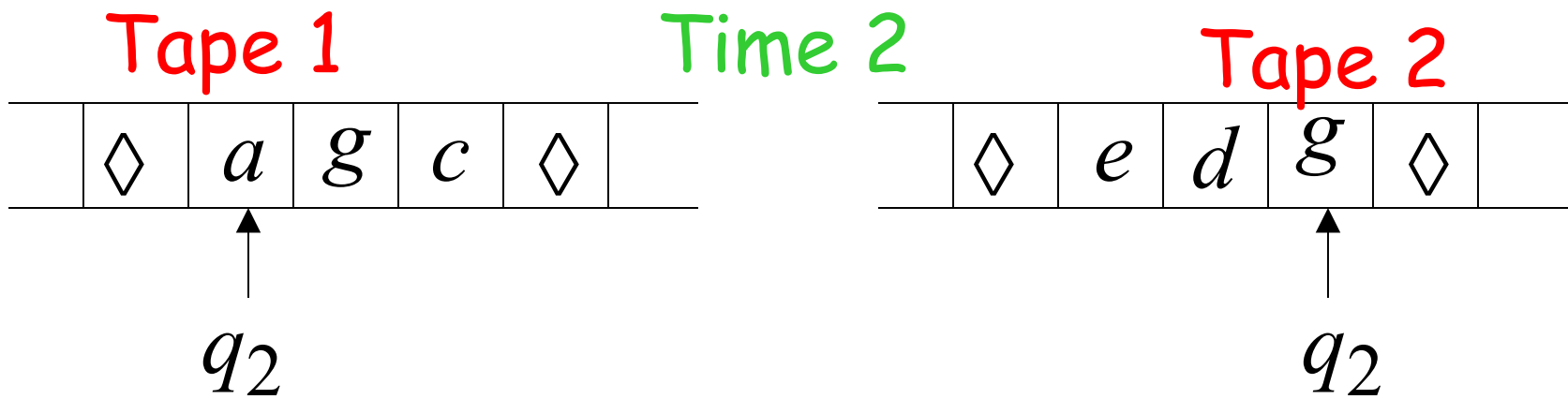
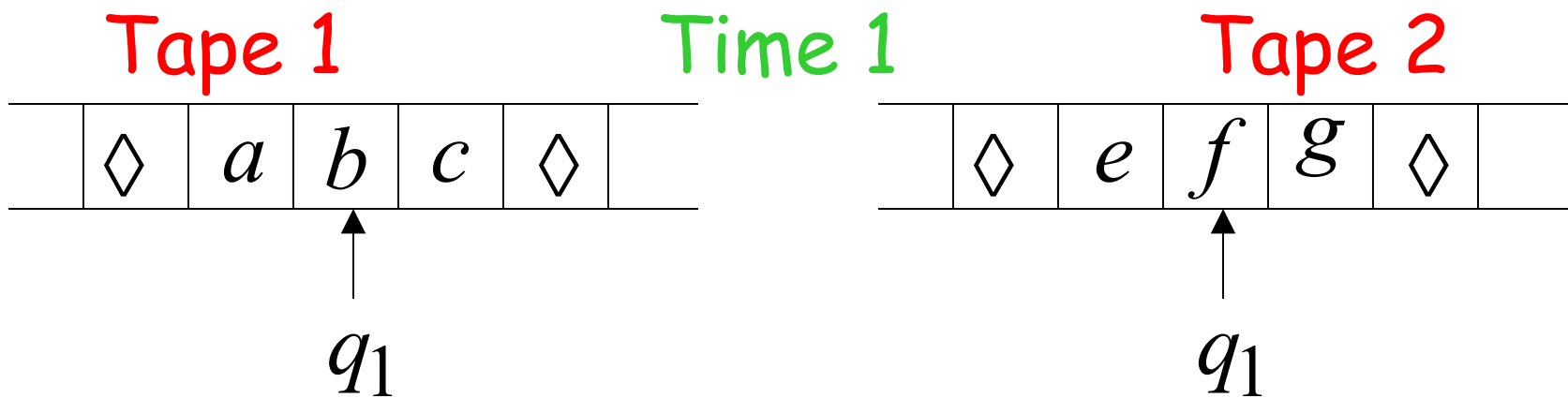


END OF PROOF

Multi-tape Turing Machines



Input string appears on Tape 1



Theorem: Multi-tape machines
have the same power with
Standard Turing machines

Proof: 1. Multi-tape machines
simulate Standard Turing machines

2. Standard Turing machines
simulate Multi-tape machines

1. Multi-tape machines simulate Standard Turing Machines:

Trivial: Use one tape

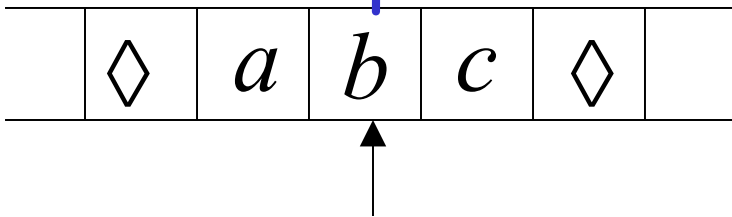
2. Standard Turing machines simulate Multi-tape machines:

Standard machine:

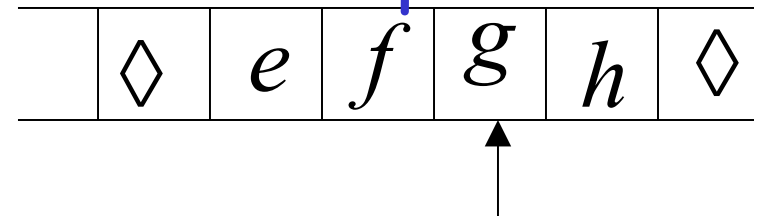
- Uses a multi-track tape to simulate the multiple tapes
- A tape of the Multi-tape machine corresponds to a pair of tracks

Multi-tape Machine

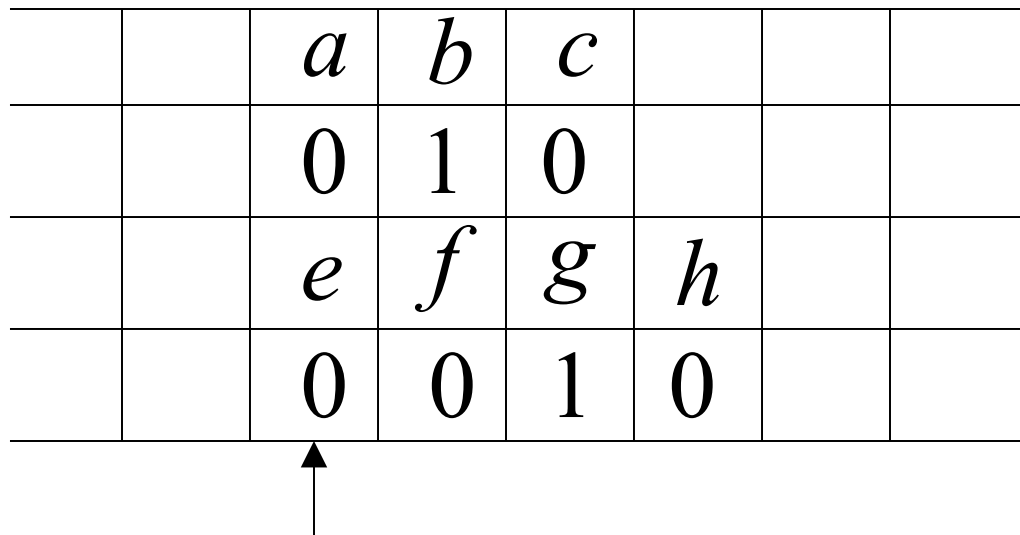
Tape 1



Tape 2



Standard machine with four track tape



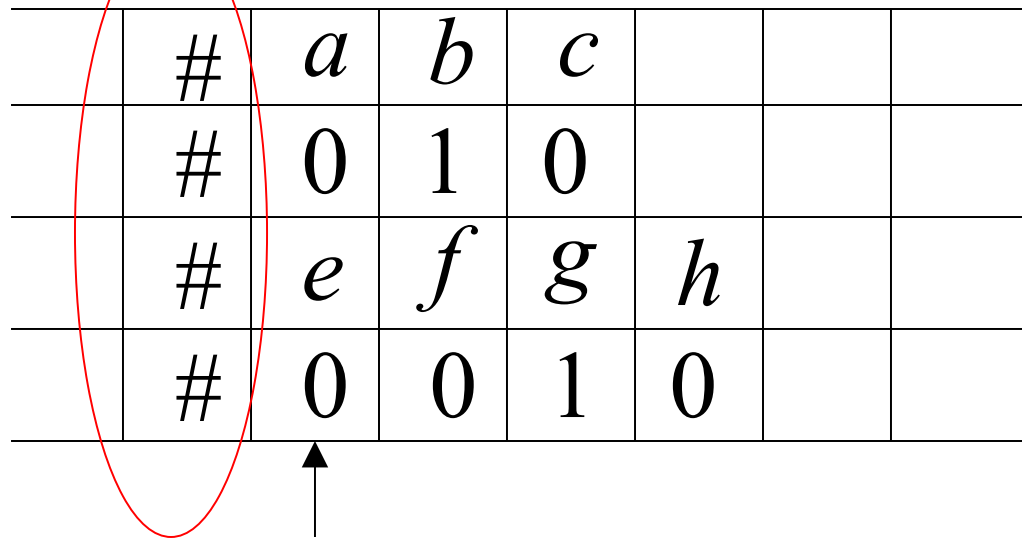
Tape 1

head position

Tape 2

head position

Reference point



#	<i>a</i>	<i>b</i>	<i>c</i>			
#	0	1	0			
#	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>		
#	0	0	1	0		

Tape 1

head position

Tape 2

head position

Repeat for each Multi-tape state transition:

1. Return to reference point
2. Find current symbol in Track 1 and update
3. Return to reference point
4. Find current symbol in Tape 2 and update

END OF PROOF

Same power doesn't imply same speed:

$$L = \{a^n b^n\}$$

Standard Turing machine: $O(n^2)$ time

Go back and forth $O(n^2)$ times
to match the a's with the b's

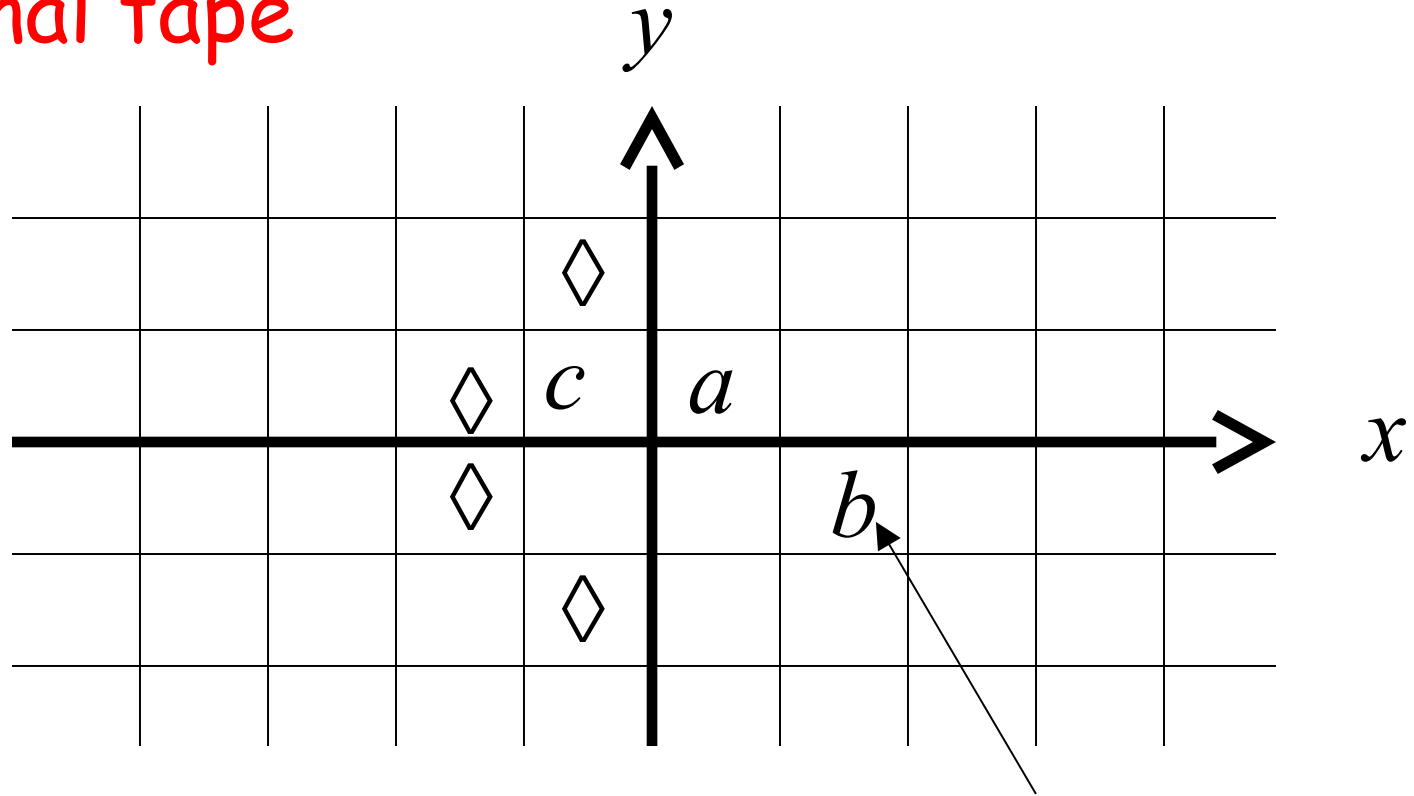
2-tape machine: $O(n)$ time

1. Copy b^n to tape 2 ($O(n)$ steps)

2. Compare a^n on tape 1
and b^n on tape 2 ($O(n)$ steps)

Multidimensional Turing Machines

2-dimensional tape



MOVES: L,R,U,D

U: up D: down

HEAD

Position: +2, -1

Theorem: Multidimensional machines
have the same power with
Standard Turing machines

Proof: 1. Multidimensional machines
simulate Standard Turing machines

2. Standard Turing machines
simulate Multi-Dimensional machines

1. Multidimensional machines simulate Standard Turing machines

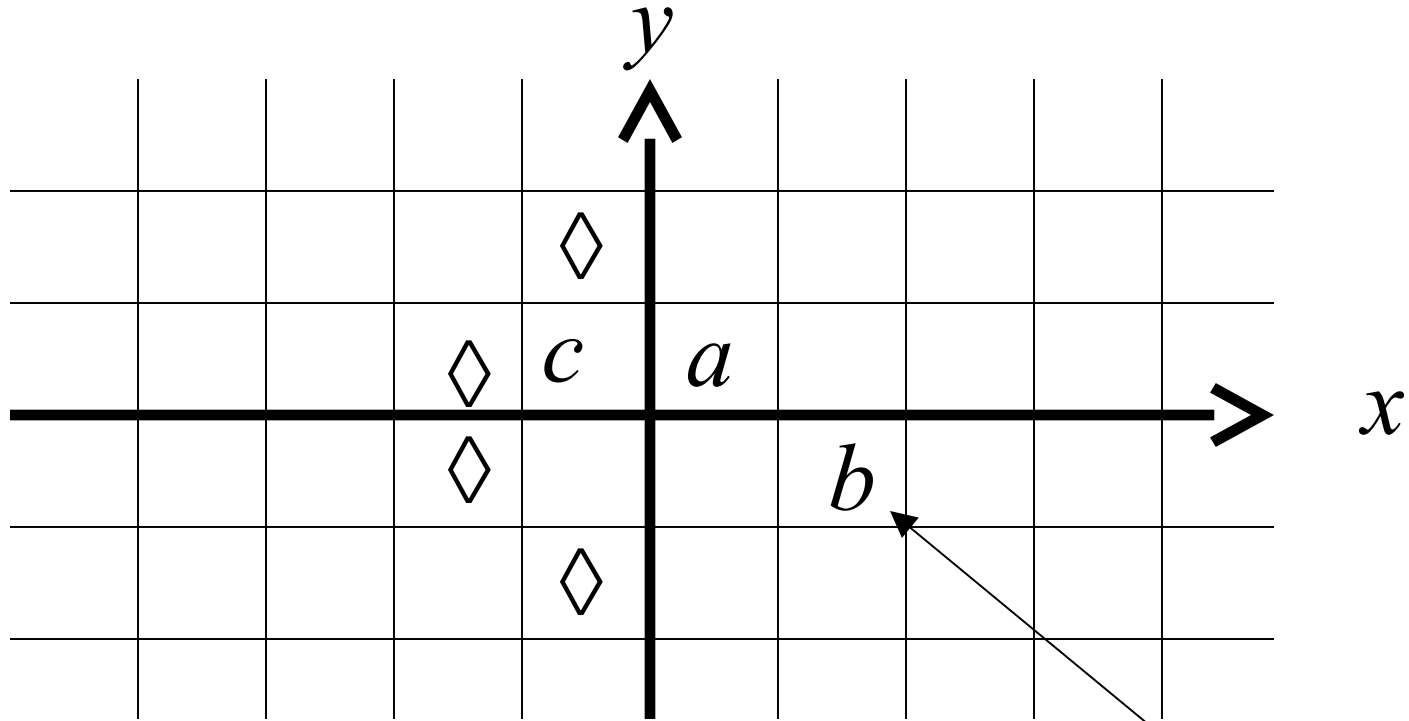
Trivial: Use one dimension

2. Standard Turing machines simulate Multidimensional machines

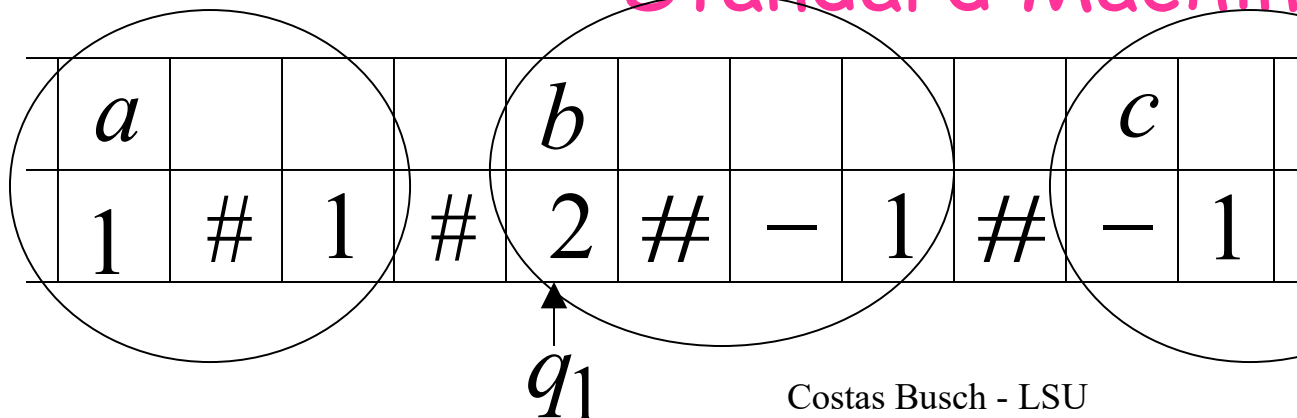
Standard machine:

- Use a two track tape
- Store symbols in track 1
- Store coordinates in track 2

2-dimensional machine



Standard Machine



symbol
coordinates

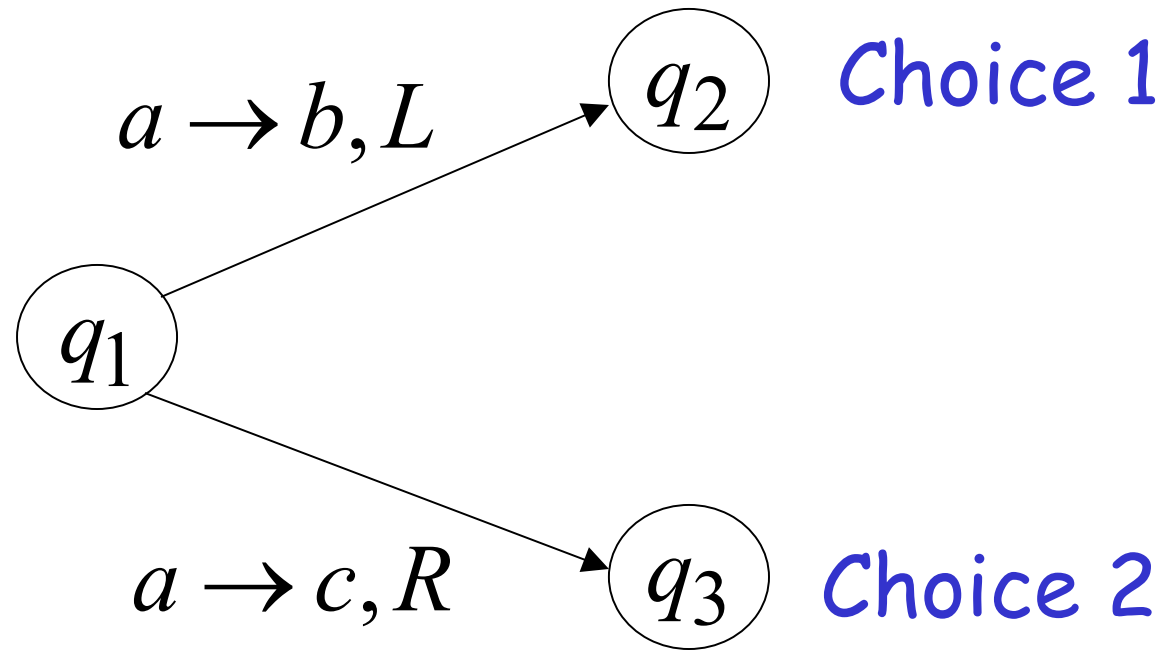
Standard machine:

Repeat for each transition followed in the 2-dimensional machine:

1. Update current symbol
2. Compute coordinates of next position
3. Find next position on tape

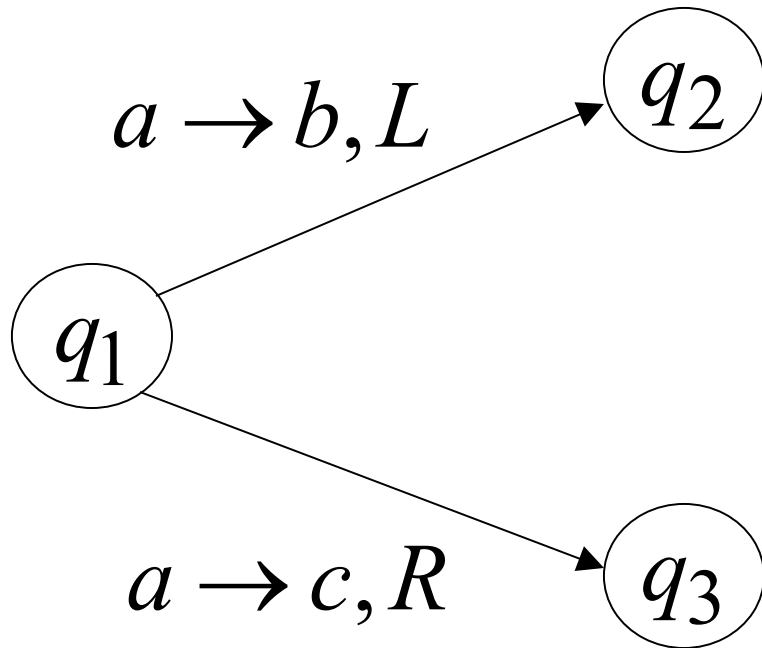
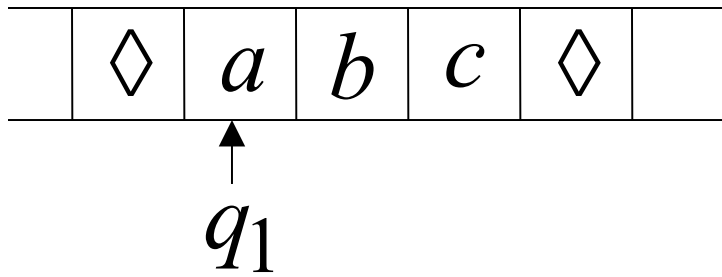
END OF PROOF

Nondeterministic Turing Machines



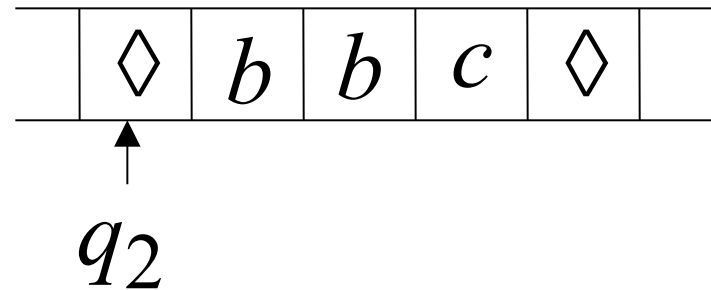
Allows Non Deterministic Choices

Time 0

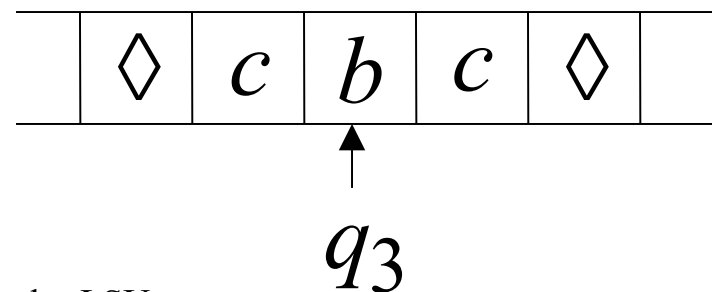


Time 1

Choice 1



Choice 2



Input string w is accepted if
there is a computation:

*

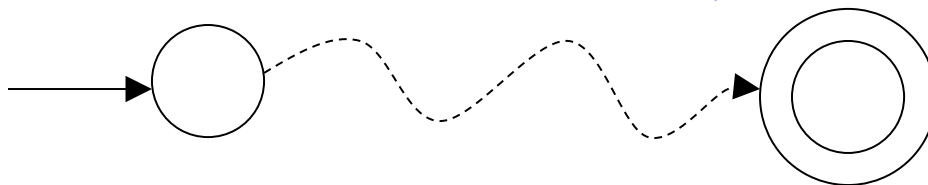
$$q_0 w \succ x q_f y$$

Initial configuration

Final Configuration

Any accept state

There is a computation:



Theorem: Nondeterministic machines
have the same power with
Standard Turing machines

Proof: 1. Nondeterministic machines
simulate Standard Turing machines

2. Standard Turing machines
simulate Nondeterministic machines

1. Nondeterministic Machines simulate Standard (deterministic) Turing Machines

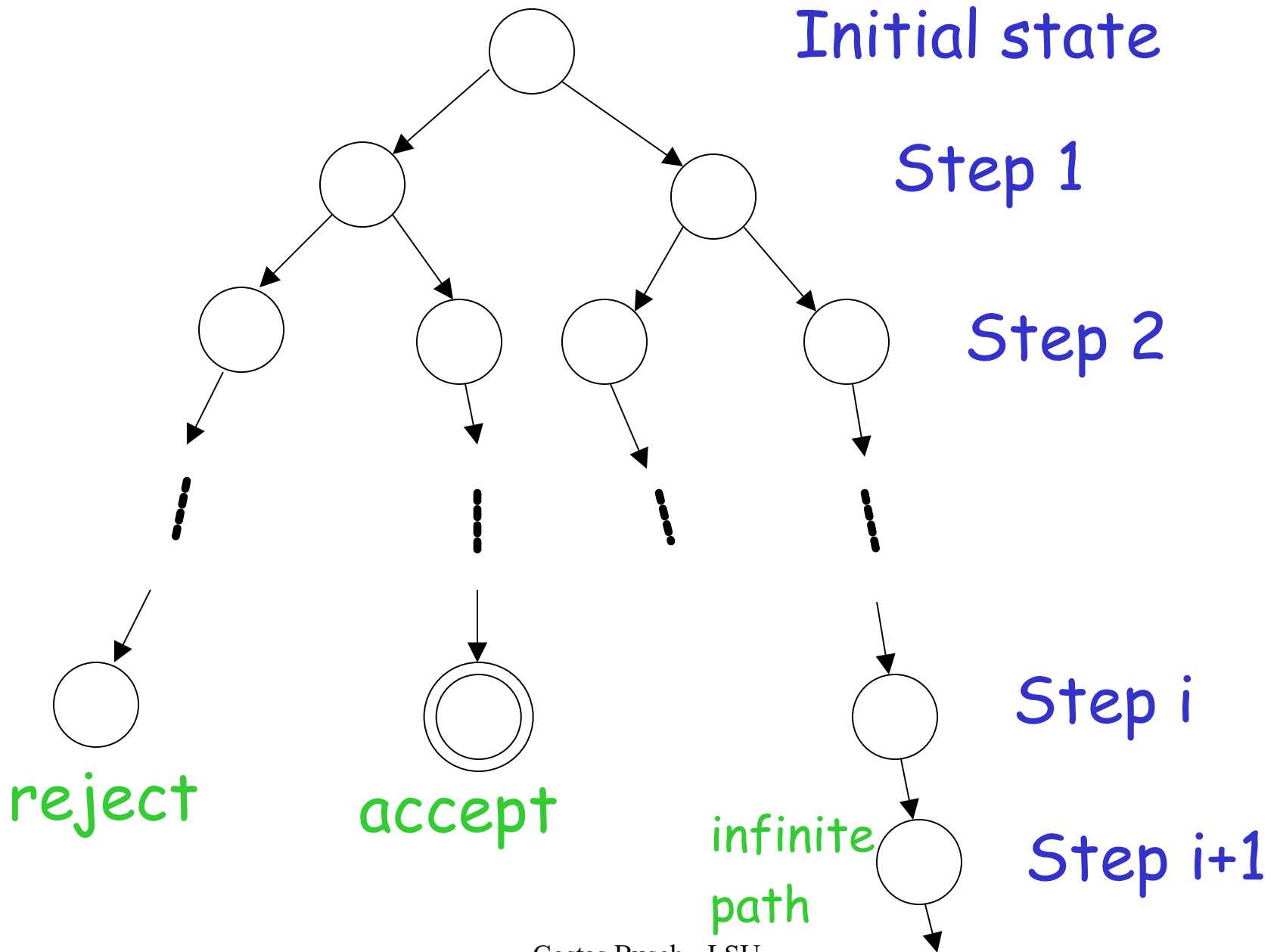
Trivial: every deterministic machine
is also nondeterministic

2. Standard (deterministic) Turing machines simulate Nondeterministic machines:

Deterministic machine:

- Uses a 2-dimensional tape
(equivalent to standard Turing machine with one tape)
- Stores all possible computations of the non-deterministic machine on the 2-dimensional tape

All possible computation paths

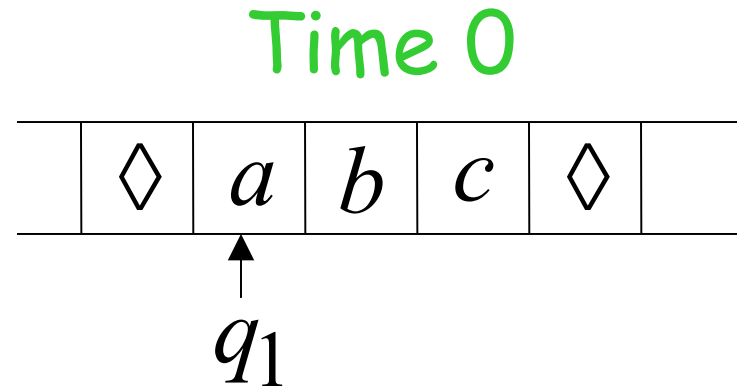
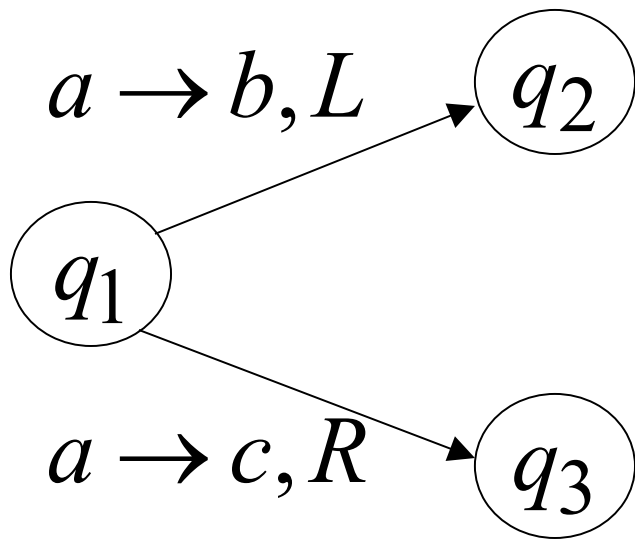


The Deterministic Turing machine
simulates all possible computation paths:

- simultaneously
- step-by-step
- with breadth-first search

depth-first may result getting stuck at exploring
an infinite path before discovering the accepting path

NonDeterministic machine



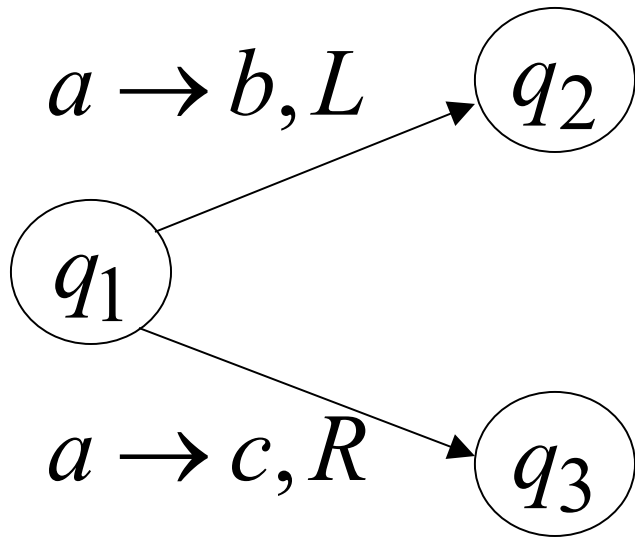
Deterministic machine

	#	#	#	#	#	#	
	#	a	b	c	#		
	#	q_1			#		
	#	#	#	#	#		

current
configuration

NonDeterministic machine

Time 1



	◇	<i>b</i>	<i>b</i>	<i>c</i>	◇	
--	---	----------	----------	----------	---	--

Choice 1

q_2

	◇	<i>c</i>	<i>b</i>	<i>c</i>	◇	
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Choice 2

q_3

Deterministic machine

	#	#	#	#	#	#	
#		<i>b</i>	<i>b</i>	<i>c</i>	#		
#	q_2				#		
#		<i>c</i>	<i>b</i>	<i>c</i>	#		
#			q_3		#		

Computation 1

Computation 2

Deterministic Turing machine

Repeat

For each configuration in current step of non-deterministic machine,
if there are two or more choices:

1. Replicate configuration
2. Change the state in the replicas

Until either the input string is accepted
or rejected in all configurations

If the non-deterministic machine accepts the input string:

The deterministic machine accepts and halts too

The simulation takes in the worst case exponential time compared to the shortest length of an accepting path

If the non-deterministic machine does not accept the input string:

1. The simulation halts if all paths reach a halting state

OR

2. The simulation never terminates if there is a never-ending path (infinite loop)

In either case the deterministic machine rejects too (1. by halting or 2. by simulating the infinite loop)

END OF PROOF