MAT1320 LINEAR ALGEBRA EXERCISES VII

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1. If $A = \begin{bmatrix} 2 & 323 & -1 \\ 1 & 466 & 1 \\ 2 & 889 & 1 \end{bmatrix}$ and $\det(A) = -480$, then which of the followings is the solution of x_2 for the linear system of equations $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$?

a) 0 b) ~~480~~ $\frac{1}{480}$ c) -1 d) -480 $\frac{-1}{480}$ e) ~~240~~ $\frac{1}{240}$

Since $\det(A) = -480 \neq 0$, the system has a solution by Cramer's method. Then,

$$x_2 = \frac{\begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}}{|A|} = \frac{-2}{-480} = \frac{1}{240}$$

$$\frac{-2}{-480} = \frac{1}{240}$$

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix} = (0-1+2)-(0+2+1) = 1-3 = -2$$

2) $A^T x = b \Rightarrow \underbrace{(A^T)^{-1} A^T}_{I_2} x = (A^T)^{-1} b \Rightarrow x = (A^{-1})^T b$

$\Rightarrow x = \begin{pmatrix} 0 & 1 \\ 1 & -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \end{pmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{matrix} -2 \\ 1+8 \end{matrix} \quad \begin{matrix} -2 \\ 9 \end{matrix}$

$\cancel{I_2} x = (A^T)^{-1} b$

2. (A points) Let A be an invertible matrix and $A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix}$. If $b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then which of the followings is the solution of the linear system of equation $(A^T)^{-1} A^T x = (A^T)^{-1} b$?

a) $x = \begin{bmatrix} \cancel{1} \\ \cancel{-3} \end{bmatrix}$ $\begin{matrix} -2 \\ 9 \end{matrix}$ b) $x = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ c) $x = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

d) $x = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ e) $x = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

3. (B points) Let $A = \begin{bmatrix} \clubsuit & \diamondsuit & \heartsuit \\ \spadesuit & \bigstar & \square \\ \triangle & \blacklozenge & \blacksquare \end{bmatrix}$ with $det(A) \neq 0$. Which of the followings is the value $x_1 + x_2 + x_3$ for the solution of the linear system of equations $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \diamondsuit \\ \bigstar \\ \blacklozenge \end{bmatrix}$?

a) 0 b) 1 c) 2 d) 3

e) The system is inconsistent.

The system can be solved by Cramer's rule.

$$x_1 = \frac{\begin{vmatrix} \diamondsuit & \diamondsuit & \heartsuit \\ \bigstar & \bigstar & \square \\ \blacklozenge & \blacklozenge & \blacksquare \end{vmatrix}}{\det A} = \frac{0}{\det A} = 0,$$

$$x_2 = \frac{\det A}{\det A} = 1,$$

$$x_3 = \frac{\begin{vmatrix} \clubsuit & \diamondsuit & \diamondsuit \\ \spadesuit & \bigstar & \bigstar \\ \triangle & \blacklozenge & \blacklozenge \end{vmatrix}}{\det A} = \frac{0}{\det A} = 0$$

$x_1 + x_2 + x_3 = 1.$

$0+1+0=1$

$A^T x = b \Rightarrow \underbrace{(A^T)^{-1} A^T}_{I_2} x = (A^T)^{-1} b$

$\Rightarrow x = (A^{-1})^T b$

$= \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{matrix} 12-10 \\ -9+8 \end{matrix} \quad \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

4. (C points) Let A be an invertible matrix and $A^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$. If $b = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, then which of the followings is the solution of the linear system of equations $(A^T)^{-1} A^T x = (A^T)^{-1} b$?

a) $x = \begin{bmatrix} 7 \\ -6 \end{bmatrix}$ b) $x = \begin{bmatrix} 6 \\ -7 \end{bmatrix}$ c) $x = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

d) $x = \begin{bmatrix} 22 \\ 17 \end{bmatrix}$ e) $x = \begin{bmatrix} 18 \\ 23 \end{bmatrix}$