Teorem:

$$\begin{vmatrix} a_{11} & a_{12} \cdots a_{1(i-1)} & a_{1i}+b_{1i} & a_{1(i+1)} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots a_{2(i-1)} & a_{2i}+b_{2i} & a_{2(i+1)} \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} \cdots a_{n(i-1)} & a_{ni}+b_{ni} & a_{n(i+1)} \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} \cdots a_{1i} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2i} \cdots a_{2n} \\ \vdots \\ a_{n1} & a_{n2} \cdots a_{ni} \cdots a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \cdots a_{1(i-1)} & b_{1i} & a_{1(i+1)} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2(i-1)} & b_{2i} & a_{2(i+1)} \cdots a_{2n} \\ \vdots \\ a_{n1} & a_{n2} \cdots a_{n(i-1)} & b_{ni} & a_{n(i+1)} \cdots a_{nn} \end{vmatrix}$$

Teorem: $|AB| = |A|.|B|$

Teorem: A^{-1} mevcut ise $A.A^{-1} = A^{-1}.A = I \Rightarrow |A||A^{-1}| = |A^{-1}||A| = 1$ mevcut ve

$|A^{-1}| = \dfrac{1}{|A|} = |A|^{-1}$ $|A| \neq 0$ / $\perp$ matrisin tersi olması için gerek ve yeter koşul $\rightarrow |A| \neq 0$

Örnek: $|A| = \begin{vmatrix} a & a+b & b+c \\ b & b+c & c+a \\ c & c+a & a+b \end{vmatrix}$ det. özelliklerinden yararlanarak hesaplayınız.

$$= \begin{vmatrix} a & a+b & b \\ b & b+c & c \\ c & c+a & a \end{vmatrix} + \begin{vmatrix} a & a+b & c \\ b & b+c & a \\ c & c+a & b \end{vmatrix} = \underbrace{\begin{vmatrix} a & a & b \\ b & b & c \\ c & c & a \end{vmatrix}}_{=0} + \underbrace{\begin{vmatrix} a & b & b \\ b & c & c \\ c & a & a \end{vmatrix}}_{=0} + \underbrace{\begin{vmatrix} a & a & c \\ b & b & a \\ c & c & b \end{vmatrix}}_{=0} + \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = (a+b+c)\begin{vmatrix} c-b & a-c \\ a-b & b-c \end{vmatrix}$$

2 ve 3'ü toplayıp 1'e ekledik

$$= (a+b+c)\left[(c-b)(b-c) - (a-c)(a-b)\right] = \_\_\_ \; 3abc - a^3 - b^3 - c^3$$

Sarrus Kuralı

3×3 mertebesinde determinant hesaplarında kullanılır.

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ 1. yol:

$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{matrix}$$

$= [(a_{11}.a_{22}.a_{33}) + (a_{21}.a_{32}.a_{13}) + (a_{31}.a_{12}.a_{23})]$
$- [(a_{13}.a_{22}.a_{31}) + (a_{23}.a_{32}.a_{11}) + (a_{33}.a_{12}.a_{21})]$

2. yol:

$$\begin{matrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{matrix} = \_\_\_\_$$

Örnek:

$$|A| = \begin{vmatrix} -1 & 4 & 2 \\ 2 & -5 & 7 \\ 6 & 2 & 0 \end{vmatrix} = [2^3 + 6.4.7] - [2.(-5).6 + 7.2.(-1)] = 250$$

$$\begin{matrix} -1 & 4 & 2 \\ 2 & -5 & 7 \end{matrix}$$

Bir Matrisin Tersi

Tanım: Bir kare matriste, matrisin elemanlarının yerine eş çarpanlarının yazılmasıyla elde edilen matrisin transpozesine "ek matris" ve "adjoint matris" denir.

$a_{11} \xrightarrow{\text{eş çarpan}} A_{11}$

$a_{21} \longrightarrow A_{21}$

$$Ek(A),\ adj(A),\ A^*$$

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad Ek(A)=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}^T$$

$$=\begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & & & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}$$

Teorem: A bir kare matris olsun. Eğer $|A|\neq 0$ ise $A^{-1}=\dfrac{Ek A}{|A|}$ 'dir.

Örnek: $A=\begin{bmatrix} 2 & 1 & -3 \\ 4 & -2 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ matrisinin tersini ekmatristen yararlanarak hesaplayınız.

$$A=\begin{bmatrix} 2 & 1 & -3 \\ 8 & 0 & -4 \\ 1 & 0 & 3 \end{bmatrix} \qquad |A|=-1\cdot\begin{vmatrix} 8 & -4 \\ 1 & 3 \end{vmatrix}=-(24+4)=-28\neq 0$$

$H_{21}(2)$

$$A_{11}=\begin{vmatrix} -2 & 2 \\ 0 & 3 \end{vmatrix}=-6 \qquad A_{12}=-\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}=-10 \qquad A_{13}=\begin{vmatrix} 4 & -2 \\ 1 & 0 \end{vmatrix}=2$$

$$Ek A=\begin{bmatrix} -6 & -10 & 2 \\ -3 & 9 & 1 \\ -4 & -16 & -8 \end{bmatrix}^T \qquad Ek A=\begin{bmatrix} -6 & -3 & -4 \\ -10 & 9 & -16 \\ 2 & 1 & -8 \end{bmatrix}$$

$$A^{-1}=-\frac{1}{28}\begin{bmatrix} -6 & -3 & -4 \\ -10 & 9 & -16 \\ 2 & 1 & -8 \end{bmatrix}=\begin{bmatrix} \frac{3}{14} & \frac{3}{28} & \frac{1}{7} \\ \frac{5}{14} & \frac{-9}{28} & \frac{4}{7} \\ \frac{-1}{14} & \frac{-1}{28} & \frac{2}{7} \end{bmatrix}$$

Soru: A $n\times n$ mertebesinde ters simetrik matris olsun. n'nin tek tamsayı olması durumunda $|A|$'yı hesaplayınız.

$A=-A^T \rightarrow$ ters simetrik $\qquad$ A $n\times n$ matr. $\quad |kA|=k^n|A|$

$|A|=|A^T|$

$|A|=|-A|$

$|A|=(-1)^n|A|=-|A| \rightarrow n$ tek

$2|A|=0$

$|A|=0$

$$|A| = \begin{vmatrix} 1+x_1 & 1 & 1 & \cdots & 1 \\ 1 & 1+x_2 & 1 & \cdots & 1 \\ - & - & - & & - \\ 1 & 1 & 1 & \cdots & 1+x_n \end{vmatrix}$$ det. hesaplayınız

$$|A| = \begin{vmatrix} x_1(1+\frac{1}{x_1}) & 1 & 1 & \cdots & 1 \\ 1 & x_2(1+\frac{1}{x_2}) & 1 & \cdots & 1 \\ - & - & - & & - \\ 1 & 1 & 1 & \cdots & x_n(1+\frac{1}{x_n}) \end{vmatrix} = x_1 \cdot x_2 \cdots x_n \begin{vmatrix} (1+\frac{1}{x_1}) & \frac{1}{x_2} & \cdots & \frac{1}{x_n} \\ \frac{1}{x_1} & (1+\frac{1}{x_2}) & \cdots & \frac{1}{x_n} \\ \vdots & \vdots & & \vdots \\ \frac{1}{x_1} & \frac{1}{x_2} & \cdots & (1+\frac{1}{x_n}) \end{vmatrix}$$

$$= \begin{vmatrix} (1+\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}) & \frac{1}{x_2} & \cdots & \frac{1}{x_n} \\ (1+\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}) & 1+\frac{1}{x_2} & \cdots & \frac{1}{x_n} \\ \vdots & \vdots & & \vdots \\ (1+\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}) & \frac{1}{x_2} & \cdots & 1+\frac{1}{x_n} \end{vmatrix} = (x_1 \cdot x_2 \cdots x_n)(1+\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}) = \begin{vmatrix} 1 & \frac{1}{x_2} & \cdots & \frac{1}{x_n} \\ 1 & 1+\frac{1}{x_2} & \cdots & \frac{1}{x_n} \\ \vdots & \vdots & & \vdots \\ 1 & \frac{1}{x_2} & \cdots & 1+\frac{1}{x_n} \end{vmatrix} \begin{matrix} -1 \\ -1 \\ \\ -1 \end{matrix}$$

$$= (x_1 \cdots x_n)(1+\frac{1}{x_1}+\frac{1}{x_2}\cdots+\frac{1}{x_n}) \underbrace{\begin{vmatrix} 1 & \frac{1}{x_2} & \cdots & \frac{1}{x_n} \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 \cdots & 0 \\ 0 & 0 & \cdots & 1 \end{vmatrix}}_{=1} = (x_1 \cdot x_2 \cdots x_n)(1+\frac{1}{x_1}+\frac{1}{x_2}\cdots+\frac{1}{x_n})$$

üst üçgen