1. $\lim\limits_{x\to 2} \dfrac{1}{x-2}\displaystyle\int_{2x}^{x^2} \dfrac{1}{1+t^3}\,dt$ limitinin değeri aşağıdakilerden hangisidir?

A) 0 B) 1 C) ∞ D) $\dfrac{2}{65}$ E) $\dfrac{4}{65}$

$$\lim_{x\to 2} \frac{\displaystyle\int_{2x}^{x^2} \frac{dt}{1+t^3}}{x-2} = \lim_{x\to 2} \frac{(2x)\cdot \dfrac{1}{1+(x^2)^3} - 2\cdot \dfrac{1}{1+(2x)^3}}{1}$$

$\hookrightarrow 0/0 \to$ L'H

$$= \lim_{x\to 2} \frac{2x}{1+x^6} - \frac{2}{1+8x^3} = \frac{4}{65} - \frac{2}{65}$$

$$= \frac{2}{65}$$

2) $h(x)=8+\int_1^{x^2}\frac{dt}{\cos^2(t-1)}$ fonksiyonunun $x=1$ noktasında lineer ifadesi aşağıdakilerden hangisidir?

A) $L(x)=2x+6$ B) $L(x)=2x+10$ C) $L(x)=2x+8$

D) $L(x)=x+6$ E) $L(x)=x+8$

$L(x)=h(1)+h'(1).(x-1)$ hesaplamalıyız.

$h(1)=8+\underbrace{\int_1^1\frac{dt}{\cos^2(t-1)}}_{0}=8$

$h'(x)=2x.\frac{1}{\cos^2(x^2-1)} \rightarrow h'(1)=\frac{2}{\underbrace{\cos^2 0}_{1}}=2$

$L(x)=\underbrace{h(1)}_{8}+\underbrace{h'(1)}_{2}.(x-1)=8+2x-2=2x+6$

3) $\int_1^{\ln x} f(t)dt=e^x\sin(\ln x)$ ise $f(0)$ değeri nedir? $(x>0)$

A) 0 B) e C) e^2 D) 1 E) $\sin 1$

Türev alalım

$\frac{1}{x}.f(\ln x)=e^x.\sin(\ln x)+e^x.\frac{1}{x}.\cos(\ln x)$

$f(0)$ bulmak için $x=1$ koyalım

$f(\underbrace{\ln 1}_{0})=e.\underbrace{\sin(\underbrace{\ln 1}_{0})}_{0}+e.\underbrace{\cos(\underbrace{\ln 1}_{0})}_{1} \Rightarrow \boxed{f(0)=e}$

(4) $\lim_{x\to 0}\left(\frac{1}{x^3}\int_0^x \frac{e^{t^4}-e^{-t^4}}{t^2}\,dt\right) = ?$ A) $\frac{1}{3}$ B) 1 C) $\frac{2}{3}$ D) $-\frac{1}{3}$ E) $-\frac{2}{3}$

$$\lim_{x\to 0}\frac{\int_0^x \frac{e^{t^4}-e^{-t^4}}{t^2}\,dt}{x^3} = \lim_{x\to 0}\frac{\frac{e^{x^4}-e^{-x^4}}{x^2}}{3x^2} = \lim_{x\to 0}\frac{e^{x^4}-e^{-x^4}}{3x^4}$$

$0/0 \to$ L'H. $\quad$ $0/0 \to$ L'H

$$= \lim_{x\to 0}\frac{4x^3.e^{x^4}+4x^3.e^{-x^4}}{12x^3}$$

$$= \lim_{x\to 0}\frac{4\cancel{x^3}(\overset{1}{e^{x^4}}+\overset{1}{e^{-x^4}})}{12\cancel{x^3}} = \frac{8}{12} = \frac{2}{3}$$

(5) $\int(tan^2x - cot^2x)dx$ integrali aşağıdakilerden hangisidir?

A) $secx + cosecx + c$ B) $-\tan x + \cot x - 2x + c$ C) $\tan x - \cot x + 2x + c$

D) $\tan x + \cot x + c$ E) $\cot x - \operatorname{cosec}x + c$

1 ekleyip 1 çıkaralım

$$\int(\operatorname{Tan}^2x - \operatorname{Cot}^2x)dx = \int(\underbrace{1+\operatorname{Tan}^2x}_{\operatorname{Sec}^2x} - (\underbrace{1+\operatorname{Cot}^2x}_{\operatorname{Cosec}^2x}))\,dx$$

$$= \int(\operatorname{Sec}^2x - \operatorname{Cosec}^2x)\,dx = \operatorname{Tan}x + \operatorname{Cot}x + c$$

$$\ln\left(\frac{\cancel{x}.\cancel{(x^2-1)}}{x^3\cancel{-x}}\right) = \ln 1 = 0$$

(6) $\int(1 + \underbrace{lnx + ln(x^2-1) - ln(x^3-x)}_{\text{Burası } 0})dx$ integralinin değeri hangisidir?

A) c B) $\frac{1}{x}+\frac{2x}{x^2-1}-\frac{(3x^2-1)}{x^3-x}+c$ C) $x\ln x - x\ln(x^2-1)+c$ D) $x+c$

$$\int dx = x + c$$

7 $\int \left(\dfrac{dx}{4+9x^2}\right)$ integralinin değeri aşağıdakilerden hangisidir?

A) $\frac{1}{3}\ln\left|\frac{9x}{4}\right|+c$

C) $\frac{1}{6}\ln\left|\frac{3x}{2}\right|+c$

E) $\frac{1}{6}\arctan\frac{3x}{2}+c$

B) $\ln|x|+c$

D) $\arcsin\frac{3x}{2}+c$

C) $\frac{1}{6}\ln\left|\frac{3x}{2}\right|+c$

KURAL : $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\text{Arctan}\frac{x}{a}+c$

D) $\arcsin\frac{3x}{2}+c$

$$\int \frac{dx}{4+9x^2} = \frac{1}{9}\int \frac{dx}{\left(\frac{2}{3}\right)^2+x^2} = \frac{1}{9}\cdot\frac{1}{\frac{2}{3}}\text{Arctan}\frac{x}{\frac{2}{3}}+c$$

$\frac{2}{3} = a$

$$= \frac{1}{6}\text{Arctan}\frac{3x}{2}+c$$

8 $\int e^{-2\ln(\sqrt{x})}dx = ?$

A) c B) $\ln x+c$ C) $x+c$ D) $\frac{1}{\sqrt{x}}+c$ E) $\frac{1}{\sqrt[3]{x^2}}+c$

$\bigstar\ e^{\ln f(x)} = f(x)$

$$e^{-2\ln(\sqrt{x})} = e^{\ln(\sqrt{x})^{-2}} = e^{\ln\left(\frac{1}{x}\right)} = \frac{1}{x}$$

$$\int e^{-2\ln\sqrt{x}}dx = \int\frac{dx}{x} = \ln x+c$$

9. $\int_0^3 g(x)dx = 7$, $\int_0^6 g(2x)dx = 5$ $\Rightarrow \int_1^2 x\, g(3x^2)dx = ?$

$\underbrace{\int_0^3 g(x)dx = 7}_{(*)}$, $\underbrace{\int_0^6 g(2x)dx = 5}_{(*)(*)}$

a) 2 b) $\frac{1}{2}$ c) 10 d) 3 e) $\frac{5}{6}$

$$\int_1^2 x \cdot g(3x^2)dx = \frac{1}{6}\int_3^{12} g(u)du = \frac{1}{6}\Big[\underbrace{\int_0^{12} g(u)du}_{(*)(*)(*)} - \underbrace{\int_0^3 g(u)du}_{(*) \text{ dan } 7}\Big]$$

$3x^2 = u$ $6x\,dx = du$

E.S Y.S

$x = 1 \rightarrow u = 3$

$x = 2 \rightarrow u = 12$

(*)(*) dan : $\int_0^6 g(2x)dx = 5$ $\Rightarrow \frac{1}{2}\int_0^{12} g(u)du = 5$

$2x = u \rightarrow 2dx = du$

E.S Y.S

$x = 0 \rightarrow u = 0$

$x = 6 \rightarrow u = 12$

$$\underbrace{\int_0^{12} g(u)du = 10}_{(*)(*)(*)}$$

in sonucu

Sonuç:

$$\int_1^2 x \cdot g(3x^2)dx = \frac{1}{6}\Big[\underbrace{\int_0^{12} g(u)du}_{10} - \underbrace{\int_0^3 g(u)du}_{7}\Big]$$

$$= \frac{1}{6} \cdot 3 = \frac{1}{2}$$

10. $\displaystyle\int \frac{\arctan(\ln x)}{x\left(1+\ln^2 x\right)}\,dx \;=?$

A) $\arctan x + c$ B) $\dfrac{1}{2}\arctan(\ln x)+c$ C) $x\arctan(\ln x)+c$

D) $\dfrac{1}{2}\left(\arctan(\ln x)\right)^2+c$ E) $\arctan(\ln x)+\arctan x+c$

$$\int \frac{\text{Arctan}(\ln x)}{x.(1+\ln^2 x)}\,dx = \int \frac{\text{Arctan}\,u}{1+u^2}\,du = \int t.dt$$

$\ln x = u \quad \dfrac{dx}{x} = du$

$\left\{ \begin{array}{l} t = \text{Arctan}\,u \\ dt = \dfrac{du}{1+u^2} \end{array} \right.$

$$= \frac{t^2}{2} + c$$

$$= \frac{(\text{Arctan}\,u)^2}{2} + c$$

$$= \frac{(\text{Arctan}(\ln x))^2}{2} + c$$

11. Aşağıda grafiği verilen $y = f(x)$ fonksiyonu için

$$\int_{-1}^{1/2} f(x)\,dx + \int_3^6 f(x)\,dx + \int_{1/2}^{3} f(x)\,dx =?$$

$$\int_{-1}^{1/2} + \int_{1/2}^{3} + \int_{3}^{6}$$

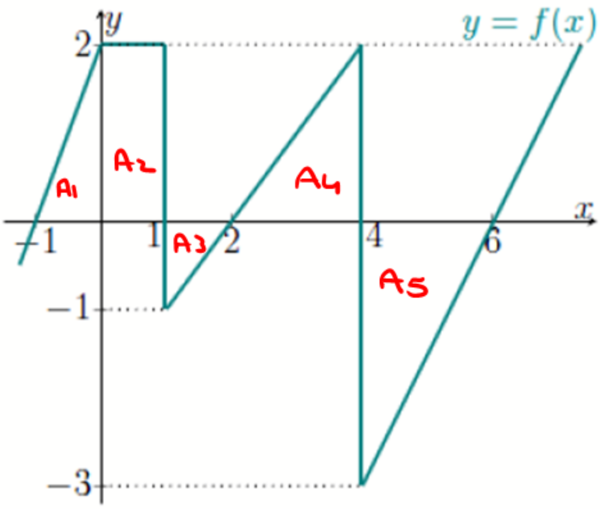
$A_1 = 1$

$A_2 = 2$

$A_3 = \frac{1}{2}$

$A_4 = 2$

$A_5 = 3$



$$= \int_{-1}^{6} f(x)\,dx$$

soruluyor.

x-ekseni üstündeki alanların toplamından x-ekseni altındaki alanları çıkarmalıyız.

A) $\dfrac{3}{2}$ B) $\dfrac{5}{2}$ C) $\dfrac{9}{2}$ D) $\dfrac{11}{2}$ E) $\dfrac{17}{2}$

$$\int_{-1}^{6} f(x)\,dx = A_1 + A_2 + A_4 - (A_3 + A_5)$$

$$= 1 + 2 + 2 - \left(\frac{1}{2} + 3\right)$$

$$= 5 - \frac{7}{2} = \frac{3}{2}$$

12) f fonksiyonu $[0,4]$ aralığında tanımlı sürekli bir fonksiyon ve ters türevi de F olmak üzere $\int_0^4 f(x)dx = 10!$ ve $F(0) = 8!$ ise $F(4) = ?$

A) $10! + 8!$ B) $10! - 8!$ C) 0 D) 2! E) 4!

f in ters türevi F ise:

$$10! = \int_0^4 f(x)dx = F(x)\Big|_0^4 = F(4) - F(0)$$

$$10! = F(4) - \underbrace{F(0)}_{8!} \Rightarrow F(4) = 10! + 8!$$

13) $\begin{cases} F(x) = (fog)(x) \\ g(2) = \frac{\pi}{4}\ ;\ g(-1) = \frac{\pi}{3} \\ f(x) = tanx \end{cases}$ bilgileri veriliyor. F fonksiyonu h fonksiyonunun bir ters türevi olduğuna göre $\int_{-1}^{2} h(x)dx = ?$

A) $\sqrt{2} - \sqrt{3}$ B) $\sqrt{3}$ C) $1 - \sqrt{3}$ D) 1 E) $\sqrt{3} - \sqrt{2}$

f, h in ters türevi ise;

$$\int_{-1}^{2} h(x)dx = F(x)\Big|_{-1}^{2} = F(2) - F(-1)$$

$$= f(\underbrace{g(2)}_{\pi/4}) - f(\underbrace{g(-1)}_{\pi/3})$$

$$= f\left(\frac{\pi}{4}\right) - f\left(\frac{\pi}{3}\right) = \tan\frac{\pi}{4} - \tan\frac{\pi}{3}$$

$$= 1 - \sqrt{3}$$

14) $f(x)=\begin{cases}\sin x\cos x, & 0\le x\le \frac{\pi}{2}\\ \cos^2 x, & \frac{\pi}{2}<x\le \pi\end{cases}$ fonksiyonu için $\int_0^{\pi} f(x)dx$ belirli integralinin değeri kaçtır?

A) 1

B) $\frac{\pi}{2}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{4}+\frac{1}{2}$

E) $\frac{\pi+1}{2}$

$$\int_0^{\pi} f(x)dx = \int_0^{\pi/2} f(x)dx + \int_{\pi/2}^{\pi} f(x)dx = \int_0^{\pi/2} \underbrace{\underbrace{\sin x}_{u}.\underbrace{\cos x\, dx}_{du}}_{} + \int_{\pi/2}^{\pi} \underbrace{\cos^2 x}_{\frac{1+\cos 2x}{2}} dx$$

$$\frac{u^2}{2} \rightarrow \frac{(\sin^2 x)}{2}$$

$$= \underbrace{\frac{\sin^2 x}{2}\Big|_0^{\pi/2}} + \left(\frac{x}{2}+\frac{\sin 2x}{4}\right)\Big|_{\pi/2}^{\pi} =$$

$$= \frac{1}{2} - 0 + \left(\frac{\pi}{2} + 0 - \left(\frac{\pi}{4} - 0\right)\right) = \frac{1}{2} + \frac{\pi}{4}$$

15) $\int_{-\pi/4}^{\pi/4} \left[\tan^3(x^3)+\tan^2 x\right]dx$ belirli integralinin değeri kaçtır?

A) 0

B) $\sqrt{2}-\frac{\pi}{2}$

C) $2-\frac{\pi}{2}$

D) π

E) $1+\frac{\pi}{2}$

$$\int_{-\pi/4}^{\pi/4} \left(\underbrace{(\operatorname{Tan} x^3)^3}_{\text{Tek Fonk.}} + \operatorname{Tan}^2 x\right) dx = \int_{-\pi/4}^{\pi/4} \underbrace{\operatorname{Tan}^2 x}_{T^2 \rightarrow \text{Çift Fonk}} dx$$

$[-\frac{\pi}{4}, \frac{\pi}{4}]$ integrali 0

$$= 2\int_0^{\pi/4} (\operatorname{Sec}^2 x - 1)dx$$

$$= 2\left[\operatorname{Tan} x - x\right]\Big|_0^{\pi/4} = 2\left[1-\frac{\pi}{4}\right]$$

$$= 2-\frac{\pi}{2}$$

16) $I=\int_{-2}^{2}\left(\cancel{x^3\cos^3 x}-\cancel{x}-\cancel{x^5\sin^2 x}+\cancel{x^2\sin x}+3\right)dx=?$

T → Tek Fonk.

Ç → Çift Fonk.

A) 12

B) 0

C) 2

D) 4

E) 8

$I=\int_{-2}^{2}3\,dx=2\int_{0}^{2}3\,dx=2\cdot 3x\Big|_{0}^{2}=12$

17) Aşağıda verilen belirsiz integraller ile çözümleri için yapılabilecek dönüşümler hangi seçenekte doğru olarak eşleştirilmiştir?

1. $\int \frac{dx}{2-\sin x}$	**a.** $x=\frac{\sqrt{7}}{2}\tan t-\frac{1}{2}$
2. $\int \frac{dx}{\sqrt{x^2+x+2}}$	**b.** $x=\frac{\sqrt{2}}{3}\sin t$
3. $\int \sqrt{2-9x^2}\,dx$	**c.** $x=3\sec t+1$
4. $\int \frac{dx}{\sqrt{x^2-2x-8}}$	**d.** $\tan\frac{x}{2}=t$

A) 1 − a, 2 − b, 3 − c, 4 − d

B) 1 − d, 2 − a, 3 − b, 4 − c

C) 1 − c, 2 − b, 3 − d, 4 − a

D) 1 − d, 2 − a, 3 − c, 4 − b

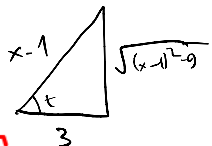
E) 1 − d, 2 − c, 3 − a, 4 − b

1. $\int \frac{dx}{2-\sin x}$ → Sinx, Cosx içeren kesirli int. ⇒ $x=\tan\frac{x}{2}$ → D

2. $\int \frac{dx}{\sqrt{x^2+x+2}}$ → $\sqrt{x^2+x+2}=\sqrt{(x+\frac{1}{2})^2+\underbrace{\frac{7}{4}}_{(\frac{\sqrt{7}}{2})^2}}$ → $x+\frac{1}{2}=\frac{\sqrt{7}}{2}\tan t$

$x=\frac{\sqrt{7}}{2}\tan t-\frac{1}{2}$ → A

3. $\int \sqrt{\underbrace{2}_{(\sqrt{2})^2}-\underbrace{9x^2}_{(3x)^2}}\,dx$ $3x=\sqrt{2}\sin t$ → B



$\cos t=\frac{3}{x-1}$ $\frac{1}{\cos t}=\frac{x-1}{3}$

$3\sec t=x-1$

4. $\int \frac{dx}{\sqrt{x^2-2x-8}}$ → $\sqrt{x^2-2x-8}=\sqrt{(x-1)^2-\underbrace{9}_{(3)^2}}$

$x-1=3\sec t$

$x=1+3\sec t$ → C

18 Aşağıda verilen belirsiz integraller ile çözümleri için yapılabilecek dönüşümler hangi seçenekte doğru olarak eşleştirilmiştir?

1. $\int \frac{dx}{2-\cos x}$ → B

2. $\int \frac{dx}{\sqrt{x^2+x+1}}$ → $\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$ → $x+\frac{1}{2}=\frac{\sqrt{3}}{2}\tan t$

3. $\int \sqrt{\underbrace{4}_{2^2}-\underbrace{3x^2}_{(\sqrt{3}x)^2}}\,dx$ → $\sqrt{3}x=2\sin t$

4. $\int \frac{dx}{\sqrt{x^2-2x-3}}$ → $\sqrt{(x-1)^2-\underbrace{4}_{2^2}}$ → $x-1=2\sec t$

a. $x=\frac{\sqrt{3}}{2}\tan t-\frac{1}{2}$

b. $\tan\frac{x}{2}=t$

c. $x=2\sec t+1$

d. $x=\frac{2\sqrt{3}}{3}\sin t$

A) $1-a$, $2-b$, $3-c$, $4-d$

B) $1-b$, $2-a$, $3-d$, $4-c$

C) $1-c$, $2-b$, $3-d$, $4-a$

D) $1-b$, $2-d$, $3-c$, $4-a$

E) $1-b$, $2-a$, $3-c$, $4-d$