MAT1320 LINEAR ALGEBRA EXERCISES XI-XIV

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1. Let $V = \mathbb{R}^2$ and $A = \{(1,0),(0,1)\}$, $B = \{(2,0),(1,3)\}$, $C = \{(1,-3),(2,4)\}$ be subsets of V. The vector $v = (8,6) \in V$ can be written as the linear combination of .....

   a) Only A b) A and B c) Only C
   d) B and C e) A, B and C

2. Let $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid y = x + z, \text{ where } x,y,z \in \mathbb{R} \right\}$ be the subspace of $\mathbb{R}^3$. What is the dimension of S ?

   a) 1 b) 2 c) 3 d) 4
   e) None of them

3. Let $M_{n\times n}$ denote the vector space of all $n \times n$ real matrices. Consider the subset

$$W = \left\{ \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} \in M_{2\times 2} \mid a+b+c = 0 \text{ where } a,b,c \in \mathbb{R} \right\}$$

   Which of the following statements are always true?

   I. The set W is a subspace of $M_{2\times 2}$.

   II. $B = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix} \right\}$ forms a basis for W.

   III. $\dim(W) = 2$.

   a) Only I b) Only II c) Only III
   d) I and II e) I and III

4. (C points) For what value(s) of t, the set $\{(1,0,2),(0,t,1),(t^2,0,2)\}$ forms a basis for $\mathbb{R}^3$?

   a) $t \in \mathbb{R} - \{0,1\}$ b) $t \in \mathbb{R} - \{0,-1\}$
   c) $t = -1$ d) $t \in \{-1,0,1\}$
   e) $t \in \mathbb{R} - \{-1,0,1\}$

5. Which of the following matrices is the transition matrix $[M]_S^T$ from basis S to basis T of $\mathbb{R}^2$ where

$$S = \{(-3,2),(4,-2)\},\ T = \{(-1,2),(2,-2)\}\ ?$$

a) $\begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$ b) $\begin{bmatrix} -1 & -2 \\ -2 & 3 \end{bmatrix}$ c) $\begin{bmatrix} -1 & 2 \\ -2 & -3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

6. Let $S = \{(1,0,1),(1,1,0),(0,0,1)\}$ and $T = \{w_1, w_2, w_3\}$ be ordered bases for $\mathbb{R}^3$. Suppose that the transition matrix from T to S is $[M]_T^S = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$. Which of the following is T ?

a) $\{(3,2,0),(2,1,0),(3,1,2)\}$
b) $\{(1,0,1),(2,1,3),(3,0,1)\}$
c) $\{(1,1,1),(1,1,3),(3,3,1)\}$
d) $\{(1,2,1),(1,1,2),(2,2,1)\}$
e) $\{(2,0,2),(1,3,0),(3,0,1)\}$

7. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$. Which of the following can be the eigenvector associated with the largest eigenvalue of the matrix A ?

a) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ b) $\begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$ c) $\begin{bmatrix} 15 \\ 6 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$ e) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

8. If $\lambda = 1$ is one of the eigenvalues of the matrix $A = \begin{bmatrix} 3 & a \\ b & -5 \end{bmatrix}$, which of the following might be another eigenvalue for A ?

a) 2 b) 3 c) -1 d) -3 e) -2

9. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $ad - bc = 5$ and $a + d = 6$, which of the following is the characteristic polynomial of A ?

a) $p(\lambda) = \lambda^2 - 6\lambda + 5$ b) $p(\lambda) = 3\lambda^2 - 4\lambda + 6$
c) $p(\lambda) = \lambda^2 - 5\lambda + 6$ d) $p(\lambda) = 2\lambda^2 - 3\lambda + 6$
e) $p(\lambda) = \lambda^2 + 5\lambda - 6$

10. Let B be an invertible matrix with an appropriate size and $A = \begin{bmatrix} -1 & 2 \\ 3 & 3 \end{bmatrix}$. If the equation $A^{-1}B^2 = A^3B$ holds, what is B ? (Hint: Cayley-Hamilton theorem can be used.)

a) $16A + 24I_2$ b) $32A + 34I_2$ c) $44A + 117I_2$
d) $76A + 184I_2$ e) $96A + 196I_2$