

BLM2041 Signals and Systems

Syllabus

The Instructors:

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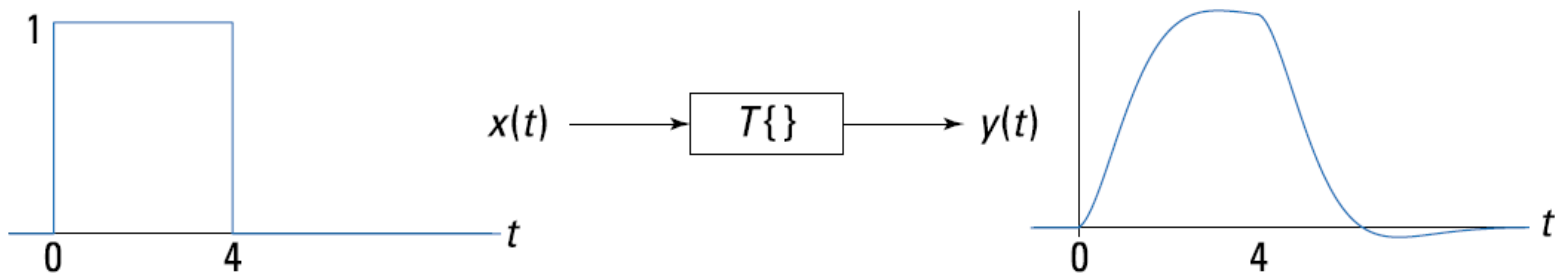
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Responses to arbitrary signals

- Although we have focused on responses to simple signals ($\delta[n], \delta(t)$) we are generally interested in responses to more complicated signals.
- How do we compute responses to a more complicated input signals?



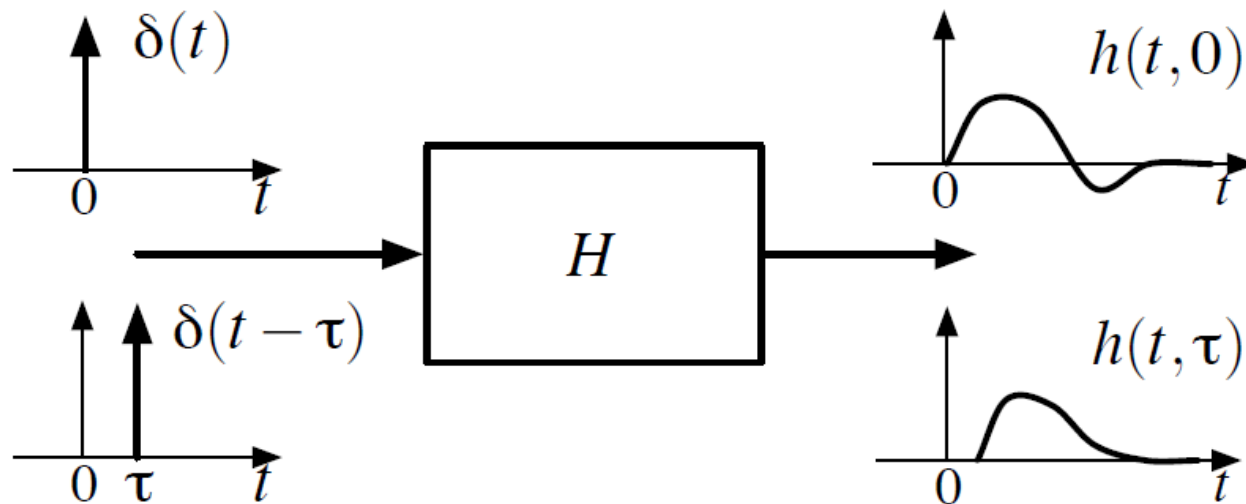
Block diagram depicting a general input/output relationship.

Impulse Response

The *impulse response* of a linear system $h_\tau(t)$ is the output of the system at time t to an impulse at time τ . This can be written as

$$h_\tau = H(\delta_\tau)$$

Care is required in interpreting this expression!



Note: Be aware of potential confusion here:

When you write

$$h_{\tau}(t) = H(\delta_{\tau}(t))$$

the variable t serves different roles on each side of the equation.

- t on the left is a specific value for time, the time at which the output is being sampled.
- t on the right is varying over all real numbers, it is not the same t as on the left.
- The output at time specific time t on the left in general depends on the input at all times t on the right (the entire input waveform).

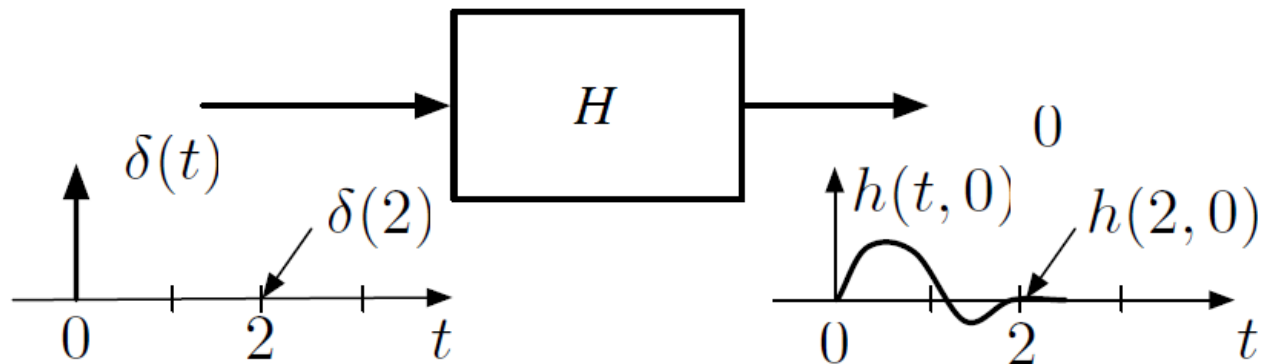
- Assume the input impulse is at $\tau = 0$,

$$h = h_0 = H(\delta_0).$$

We want to know the impulse response at time $t = 2$. It doesn't make any sense to set $t = 2$, and write

$$h(2) = H(\delta(2)) \quad \Leftarrow \text{No!}$$

First, $\delta(2)$ is something like zero, so $H(0)$ would be zero. Second, the value of $h(2)$ depends on the entire input waveform, not just the value at $t = 2$.

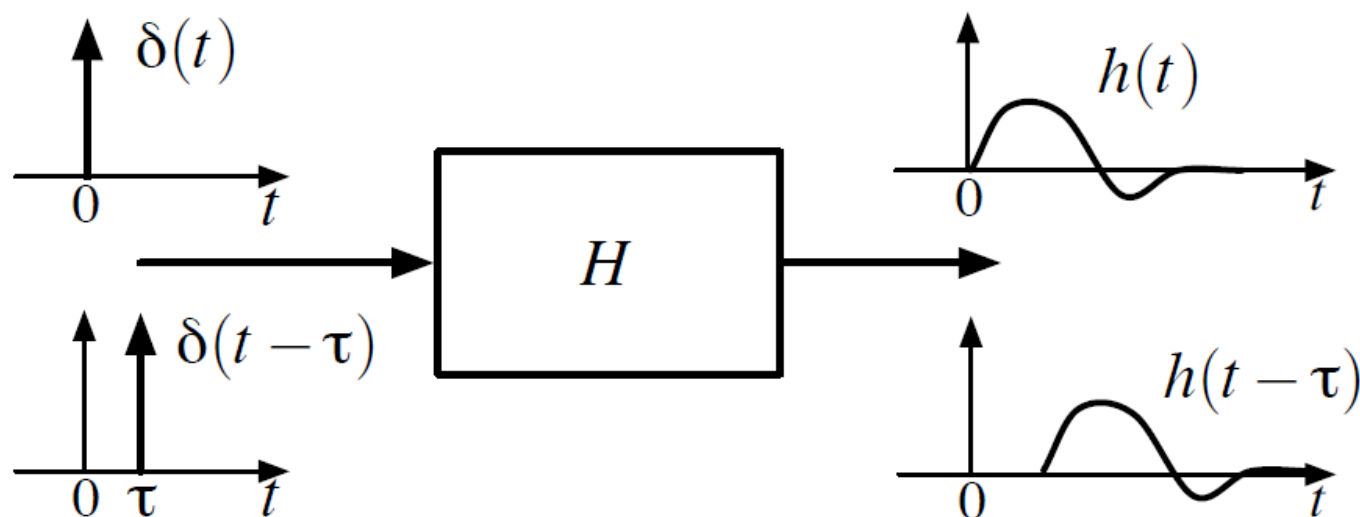


Time-invariance

If H is time invariant, delaying the input and output both by a time τ should produce the same response

$$h_{\tau}(t) = h(t - \tau).$$

In this case, we don't need to worry about h_{τ} because it is just h shifted in time.



Linearity and Extended Linearity

Linearity: A system S is linear if it satisfies both

- *Homogeneity:* If $y = Sx$, and a is a constant then

$$ay = S(ax).$$

- *Superposition:* If $y_1 = Sx_1$ and $y_2 = Sx_2$, then

$$y_1 + y_2 = S(x_1 + x_2).$$

Combined Homogeneity and Superposition:

If $y_1 = Sx_1$ and $y_2 = Sx_2$, and a and b are constants,

$$ay_1 + by_2 = S(ax_1 + bx_2)$$

Extended Linearity

- *Summation*: If $y_n = S(x_n)$ for all n , an integer from $(-\infty < n < \infty)$, and a_n are constants

$$\sum_n a_n y_n = S \left(\sum_n a_n x_n \right)$$

Summation and the system operator commute, and can be interchanged.

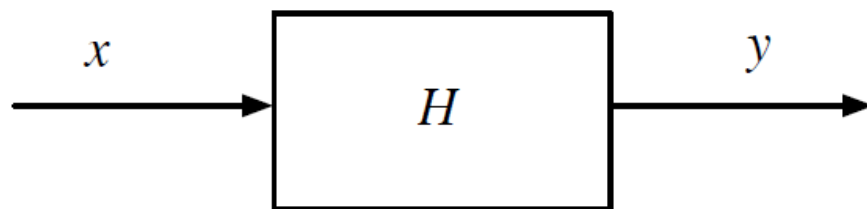
- *Integration (Simple Example)* : If $y = S(x)$,

$$\int_{-\infty}^{\infty} a(\tau) y(t - \tau) d\tau = S \left(\int_{-\infty}^{\infty} a(\tau) x(t - \tau) d\tau \right)$$

Integration and the system operator commute, and can be interchanged.

Output of an LTI System

We would like to determine an expression for the output $y(t)$ of an linear time invariant system, given an input $x(t)$



We can write a signal $x(t)$ as a sample of itself

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta_{\tau}(t) d\tau$$

This means that $x(t)$ can be written as a weighted integral of δ functions.

Applying the system H to the input $x(t)$,

$$\begin{aligned} y(t) &= H(x(t)) \\ &= H\left(\int_{-\infty}^{\infty} x(\tau)\delta_{\tau}(t)d\tau\right) \end{aligned}$$

If the system obeys extended linearity we can interchange the order of the system operator and the integration

$$y(t) = \int_{-\infty}^{\infty} x(\tau)H(\delta_{\tau}(t))d\tau.$$

The impulse response is

$$h_{\tau}(t) = H(\delta_{\tau}(t)).$$

Substituting for the impulse response gives

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h_{\tau}(t)d\tau.$$

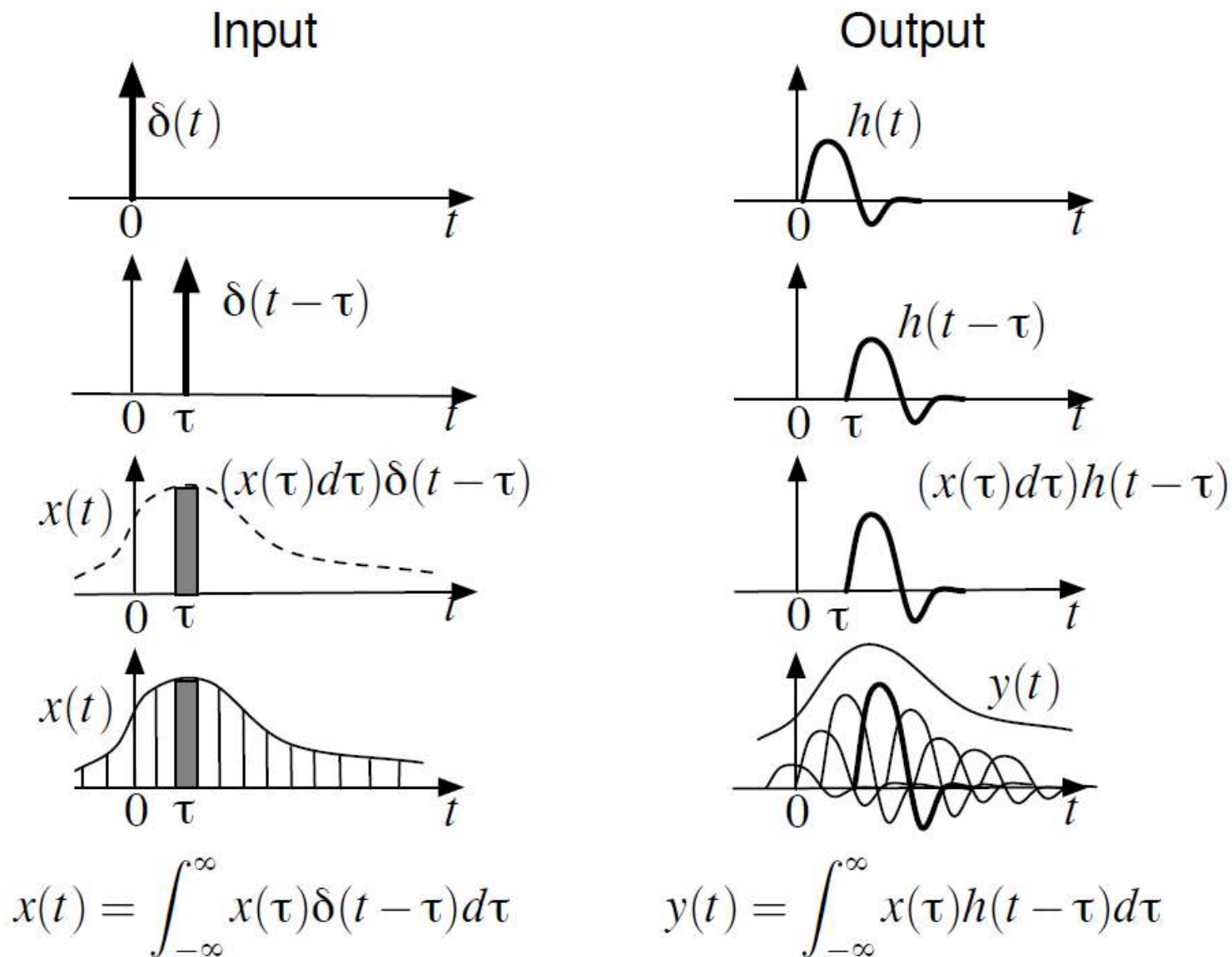
This is a *superposition integral*. The values of $x(\tau)h(t, \tau)d\tau$ are superimposed (added up) for each input time τ .

If H is time invariant, this written more simply as

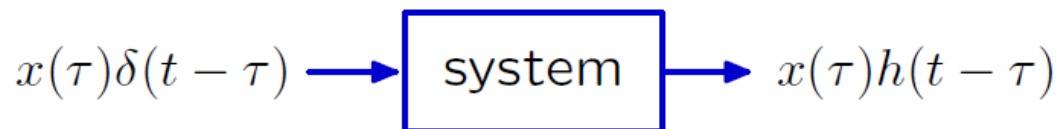
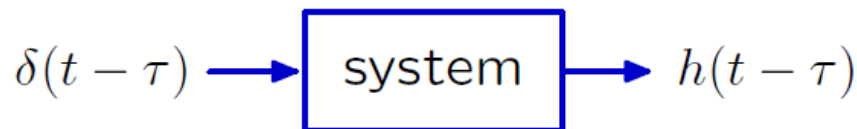
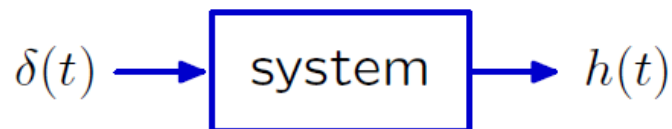
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h_{\tau}(t)d\tau.$$

This is in the form of a *convolution integral*, which will be the subject of the next class.

Graphically, this can be represented as:



If a system is linear and time-invariant (LTI) then its output is the integral of weighted and shifted unit-impulse responses.

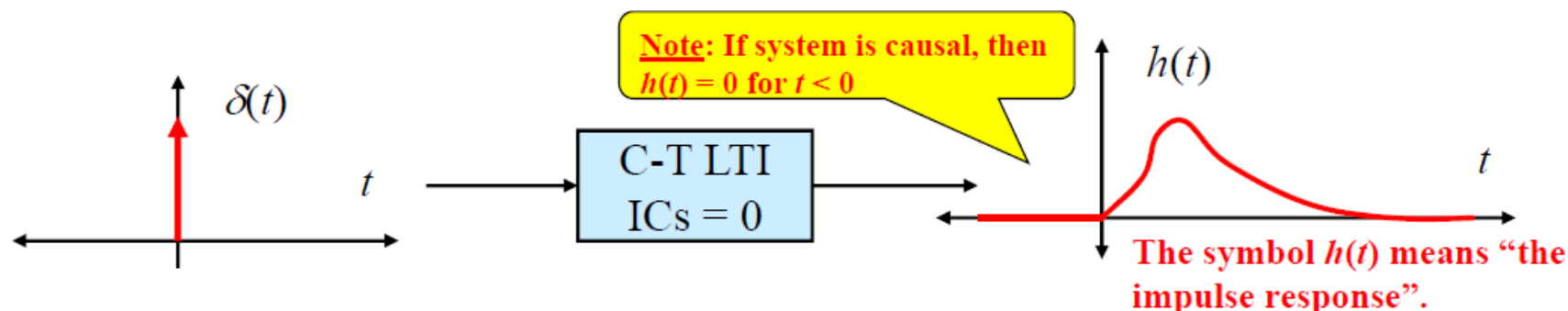


$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau \rightarrow \text{system} \rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

Recall: Impulse Response

Earlier we introduced the concept of impulse response...

...what comes out of a system when the input is an impulse (delta function)



Noting that the LT of $\delta(t) = 1$ and using the properties of the transfer function and the Z transform we said that

$$h(t) = \mathcal{L}^{-1}\{H(s)\mathcal{L}\{\delta(t)\}\}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\}$$

$$h(t) = \mathcal{F}^{-1}\{H(\omega)\}$$

So...once we have either $H(s)$ or $H(\omega)$ we can get the impulse response $h(t)$

Convolution Property and System Output

Let $x(t)$ be a signal with CTFT $X(\omega)$ and LT of $X(s)$

$$x(t) \leftrightarrow X(\omega)$$

$$x(t) \leftrightarrow X(s)$$

Consider a system w/ freq resp $H(\omega)$ & trans func $H(s)$

$$h(t) \leftrightarrow H(\omega)$$

$$h(t) \leftrightarrow H(s)$$

We've spent much time using these tools to analyze system outputs this way:

$$Y(\omega) = H(\omega)X(\omega) \leftrightarrow y(t) = \mathcal{F}^{-1}\{H(\omega)X(\omega)\}$$

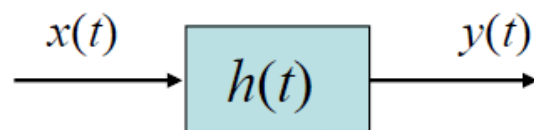
$$Y(s) = H(s)X(s) \leftrightarrow y[n] = \mathcal{L}^{-1}\{H(s)X(s)\}$$

The convolution property of the CTFT and LT gives an alternate way to find $y(t)$:

$$\mathcal{F}^{-1}\{X(\omega)H(\omega)\} = x(t) * h(t)$$

$$\mathcal{L}^{-1}\{X(s)H(s)\} = x(t) * h(t)$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

LTI System with impulse response $h(t)$

**“Convoluting”
input $x(t)$ with the
impulse response
 $h(t)$ gives the
output $y(t)$!**

Convolution for Causal System & with Causal Input

An arbitrary LTI system's output can be found using the general convolution form:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

General LTI System

Convolution Properties

1. Commutativity

$$x(t) * h(t) = h(t) * x(t)$$

2. Associativity

$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$

Associativity together with commutativity says we can interchange the order of two cascaded systems.

3. Distributivity

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

4. Derivative Property:

$$\begin{aligned}\frac{d}{dt}[x(t) * v(t)] &= \dot{x}(t) * v(t) \\ &= x(t) * \dot{v}(t)\end{aligned}$$

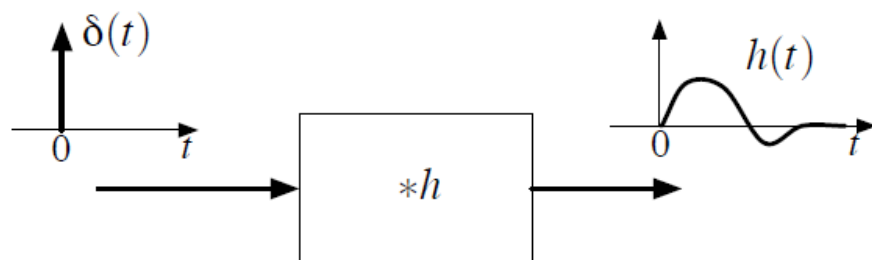
derivative

5. Integration Property Let $y(t) = x(t) * h(t)$, then

$$\int_{-\infty}^t y(\lambda) d\lambda = \left[\int_{-\infty}^t x(\lambda) d\lambda \right] * h(t) = x(t) * \left[\int_{-\infty}^t h(\lambda) d\lambda \right]$$

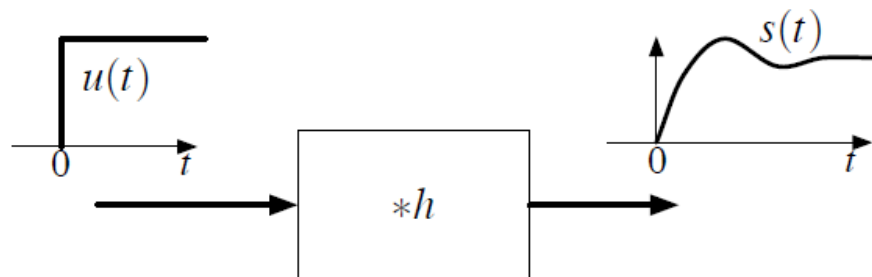
Example: Measuring the impulse response of an LTI system.

We would like to measure the impulse response of an LTI system, described by the impulse response $h(t)$

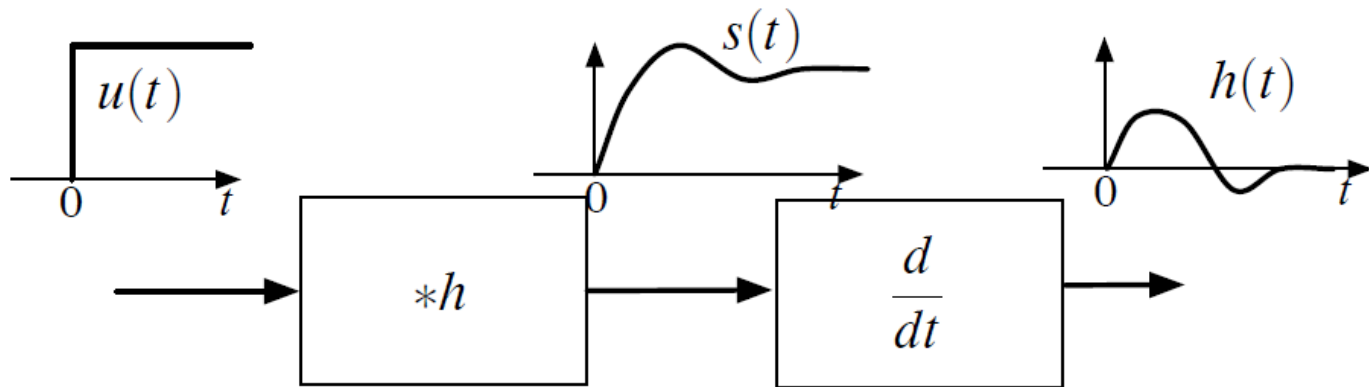


This can be practically difficult because input amplitude is often limited. A very short pulse then has very little energy.

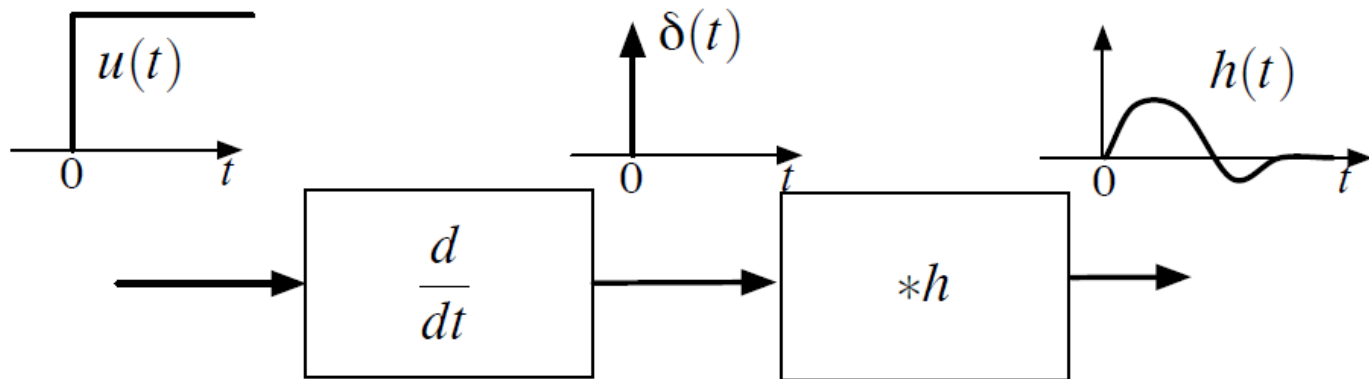
A common alternative is to measure the *step response* $s(t)$, the response to a unit step input $u(t)$



The impulse response is determined by differentiating the step response,



To show this, commute the convolution system and the differentiator to produce a system with the same overall impulse response



Steps for Graphical Convolution $x(t)*h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

1. **Re-Write the signals as functions of τ** : $x(\tau)$ and $h(\tau)$
2. **Flip** just one of the signals around $t = 0$ to get either $x(-\tau)$ or $h(-\tau)$
 - a. It is usually best to flip the signal with shorter duration
 - b. For notational purposes here: we'll flip $h(\tau)$ to get $h(-\tau)$
3. **Find Edges** of the flipped signal
 - a. Find the left-hand-edge τ -value of $h(-\tau)$: **call it $\tau_{L,0}$**
 - b. Find the right-hand-edge τ -value of $h(-\tau)$: **call it $\tau_{R,0}$**
4. **Shift** $h(-\tau)$ by an arbitrary value of t to get $h(t - \tau)$ and **get its edges**
 - a. Find the left-hand-edge τ -value of $h(t - \tau)$ as a function of t : **call it $\tau_{L,t}$**
 - **Important**: It will always be... **$\tau_{L,t} = t + \tau_{L,0}$**
 - b. Find the right-hand-edge τ -value of $h(t - \tau)$ as a function of t : **call it $\tau_{R,t}$**
 - **Important**: It will always be... **$\tau_{R,t} = t + \tau_{R,0}$**

Note: If the signal you flipped is NOT finite duration,
one or both of $\tau_{L,t}$ and $\tau_{R,t}$ will be infinite ($\tau_{L,t} = -\infty$ and/or $\tau_{R,t} = \infty$)

Steps Continued

5. Find Regions of τ -Overlap

- What you are trying to do here is find intervals of t over which the product $x(\tau) h(t - \tau)$ has a single mathematical form in terms of τ
- In each region find: Interval of t that makes the identified overlap happen
- Working examples is the best way to learn how this is done

Tips: Regions should be contiguous with no gaps!!!
Don't worry about $<$ vs. \leq etc.

6. For Each Region: Form the Product $x(\tau) h(t - \tau)$ and Integrate

- Form product $x(\tau) h(t - \tau)$
- Find the Limits of Integration by finding the interval of τ over which the product is nonzero
 - Found by seeing where the edges of $x(\tau)$ and $h(t - \tau)$ lie
 - Recall that the edges of $h(t - \tau)$ are $\tau_{L,t}$ and $\tau_{R,t}$, which often depend on the value of t
 - So... the limits of integration may depend on t
- Integrate the product $x(\tau) h(t - \tau)$ over the limits found in 6b
 - The result is generally a function of t , but is only valid for the interval of t found for the current region
 - Think of the result as a “time-section” of the output $y(t)$

Steps Continued

7. **“Assemble” the output** from the output time-sections for all the regions
 - a. Note: you do NOT add the sections together
 - b. You define the output “piecewise”
 - c. Finally, if possible, look for a way to write the output in a simpler form

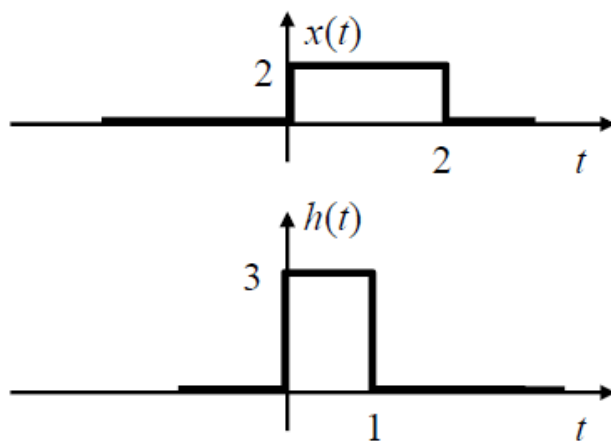
Example: Graphically Convolve Two Signals

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

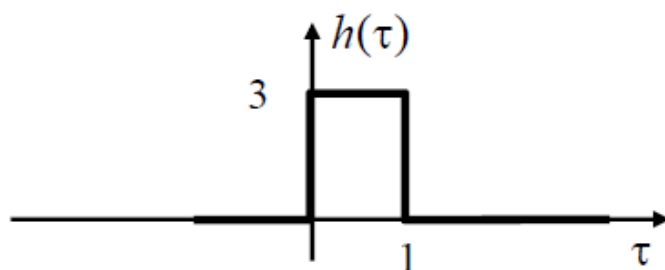
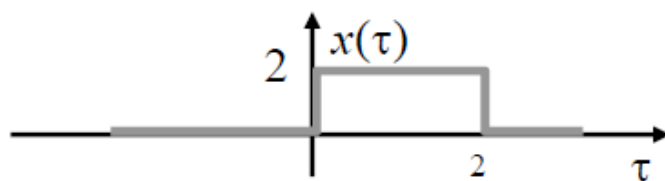
By “Properties of Convolution”...
these two forms are
Equal

This is why we can
flip either signal

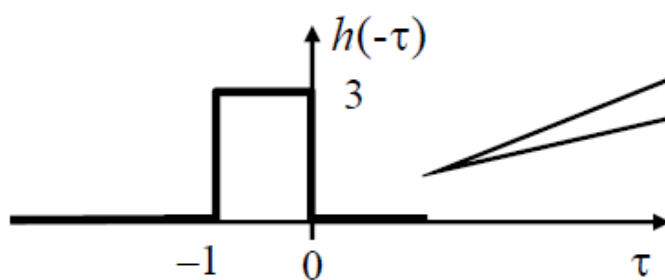
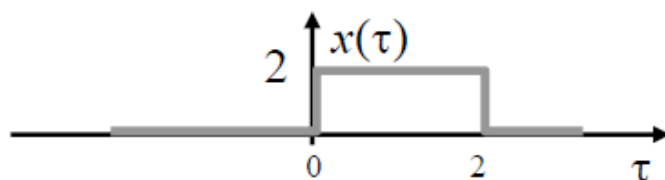
Convolve these two signals:



Step #1: Write as Function of τ

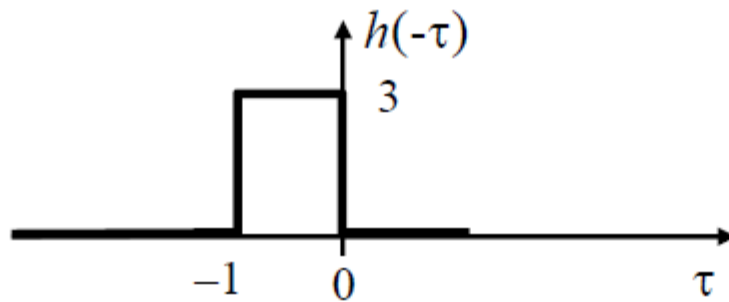
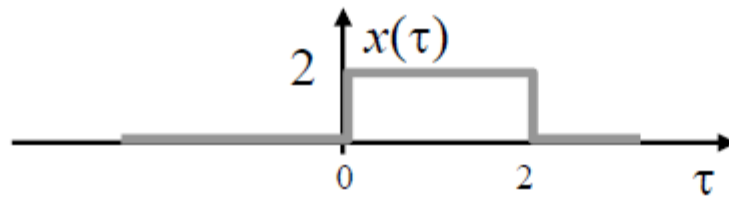


Step #2: Flip $h(\tau)$ to get $h(-\tau)$



**Usually Easier
to Flip the
Shorter Signal**

Step #3: Find Edges of Flipped Signal



$\tau_{L,0} = -1$

$\tau_{R,0} = 0$

Motivating Step #4: Shift by t to get $h(t-\tau)$ & Its Edges

Just looking at 2 “arbitrary” t values

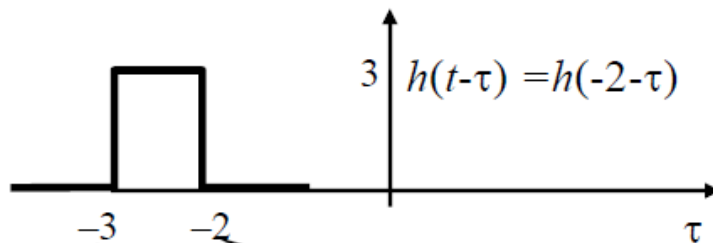
In Each Case We Get

$$\tau_{L,t} = t + \tau_{L,0}$$

$$\tau_{R,t} = t + \tau_{R,0}$$

For $t = -2$

For $t = 2$



$$\tau_{L,t} = t + \tau_{L,0}$$

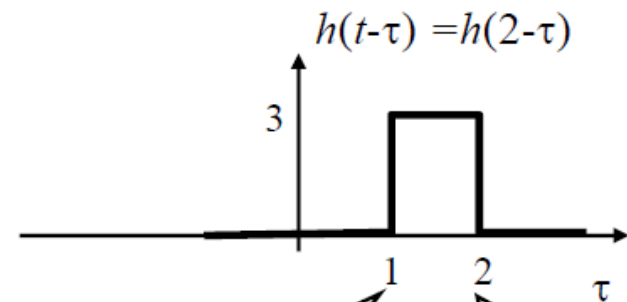
$$\tau_{L,t} = t - 1$$

$$\tau_{L,-2} = -2 - 1$$

$$\tau_{R,t} = t + \tau_{R,0}$$

$$\tau_{R,t} = t + 0$$

$$\tau_{R,-2} = -2 + 0$$



$$\tau_{L,t} = t + \tau_{L,0}$$

$$\tau_{L,t} = t - 1$$

$$\tau_{L,2} = 2 - 1$$

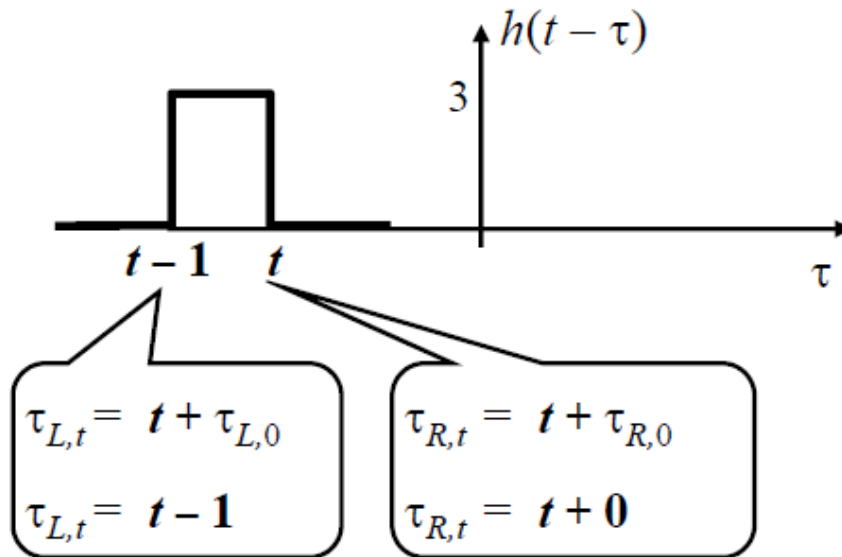
$$\tau_{R,t} = t + \tau_{R,0}$$

$$\tau_{R,t} = t + 0$$

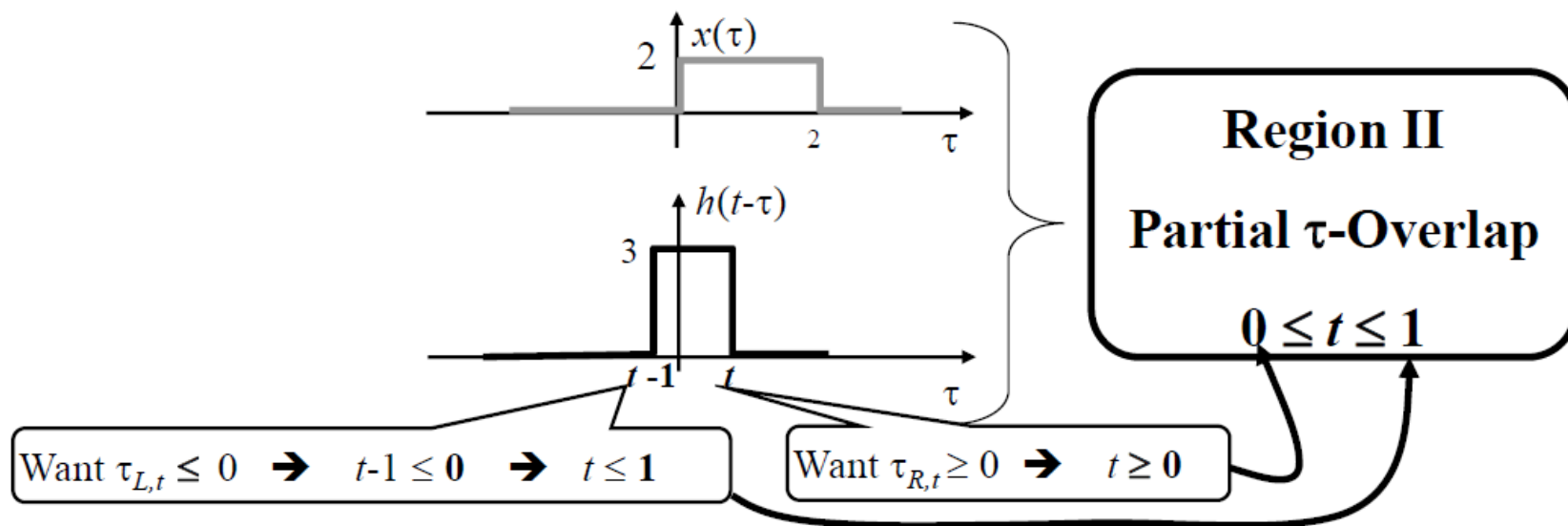
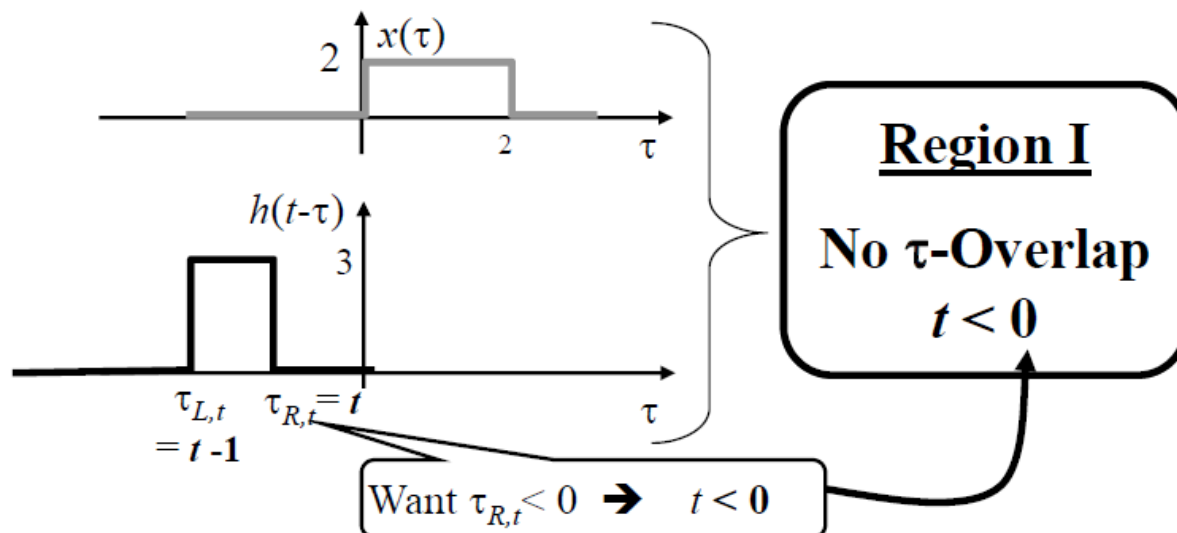
$$\tau_{R,2} = 2 + 0$$

Doing Step #4: Shift by t to get $h(t-\tau)$ & Its Edges

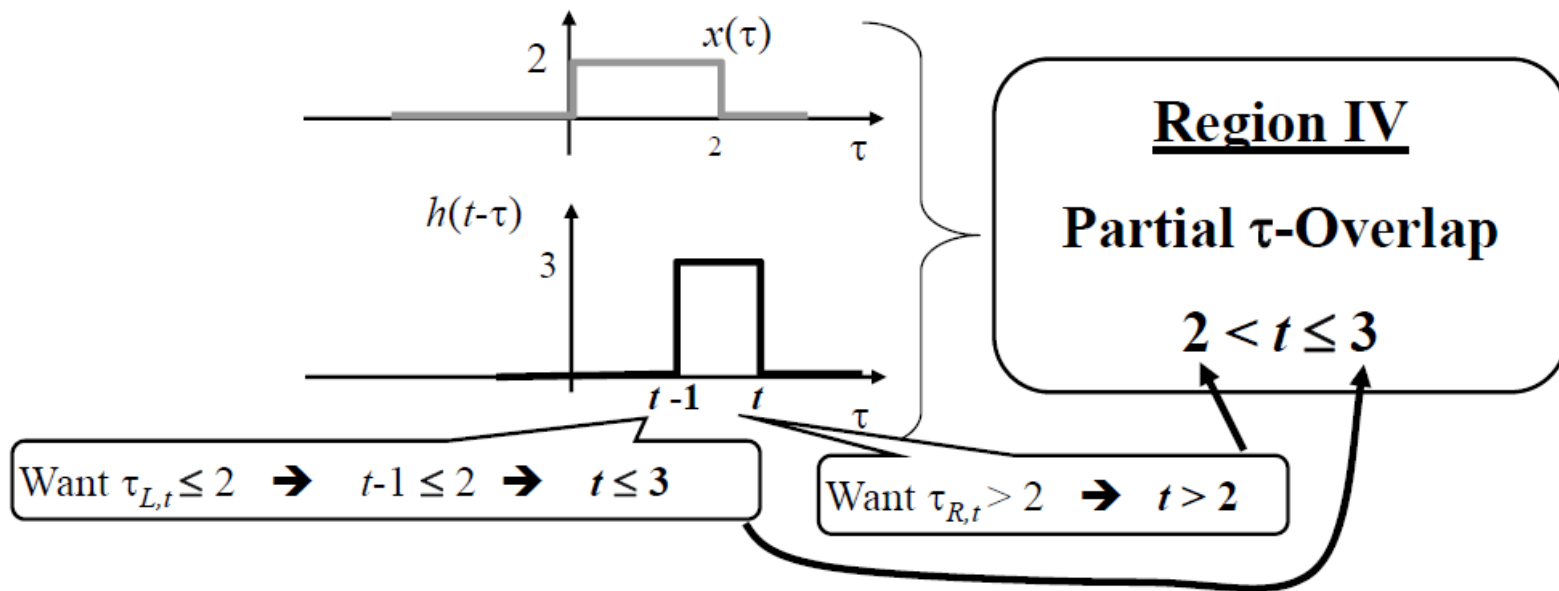
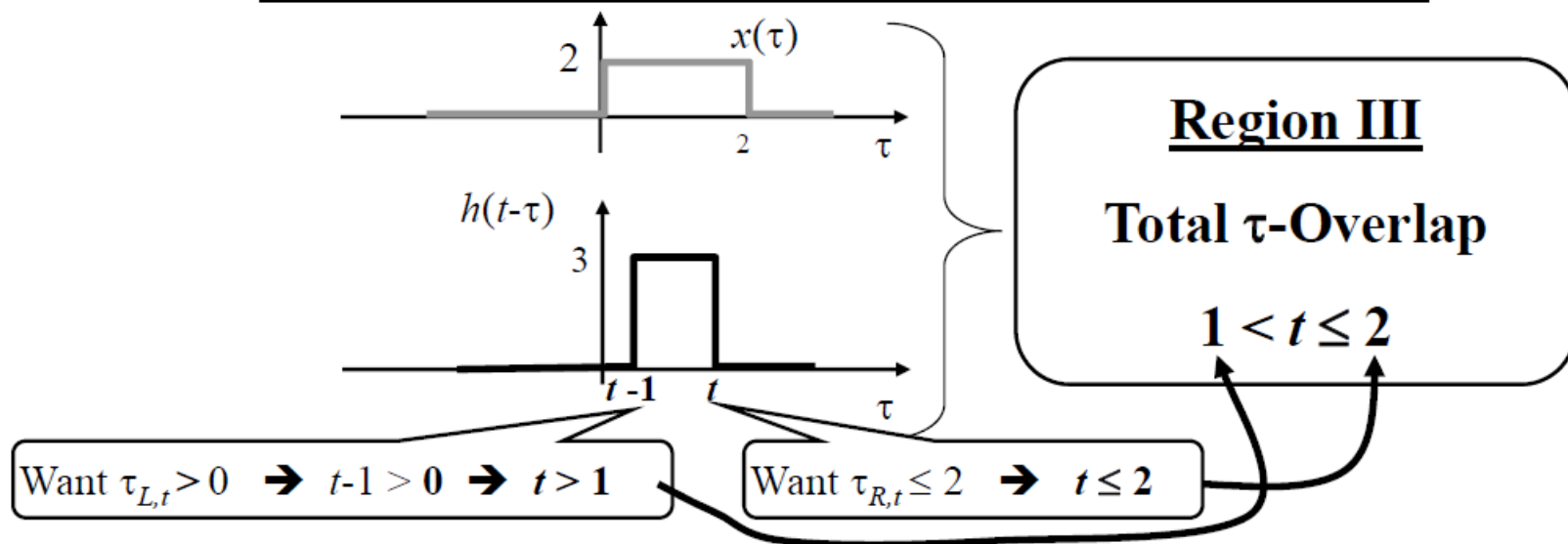
For Arbitrary Shift by t



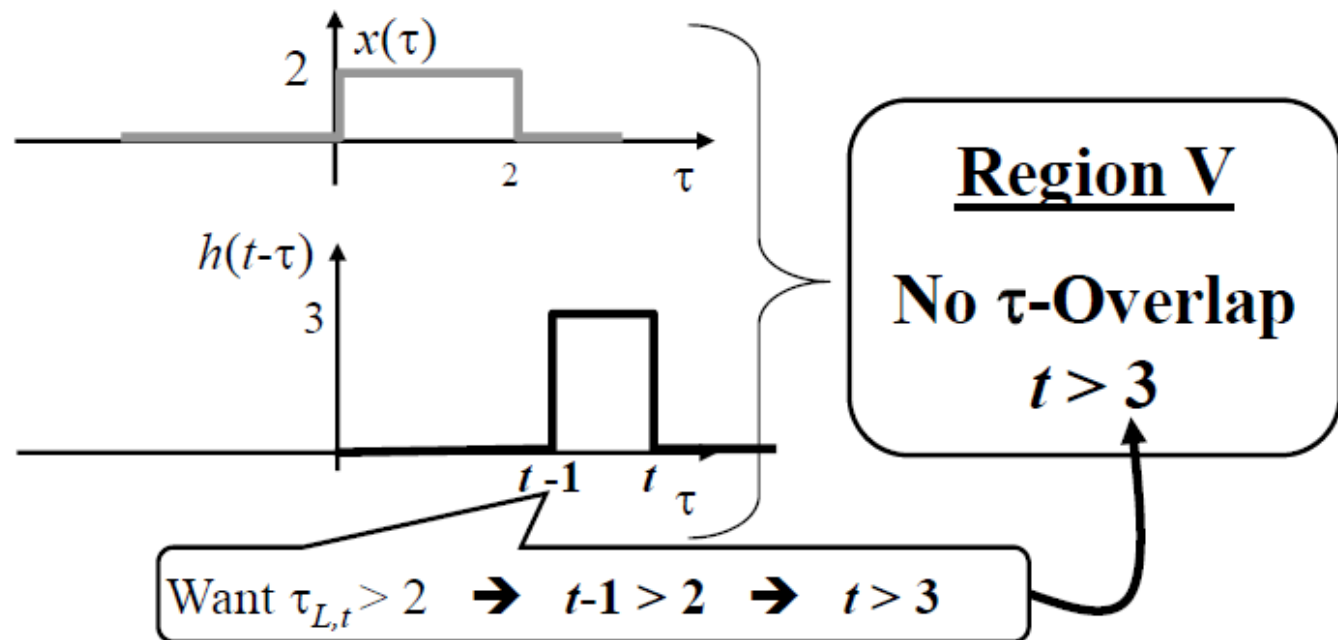
Step #5: Find Regions of τ -Overlap



Step #5 (Continued): Find Regions of τ -Overlap

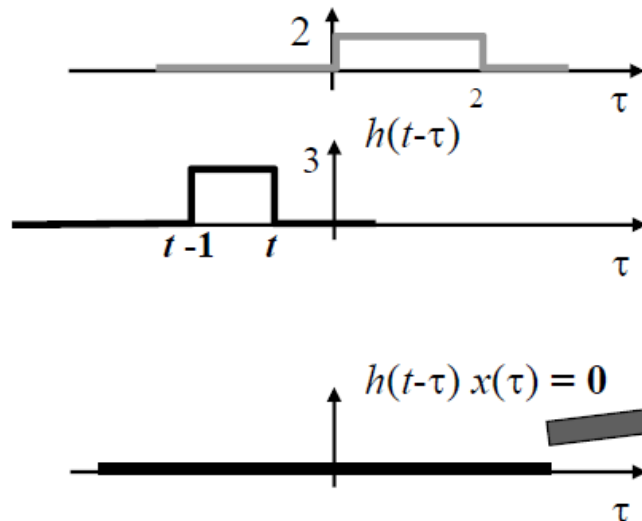


Step #5 (Continued): Find Regions of τ -Overlap



Step #6: Form Product & Integrate For Each Region

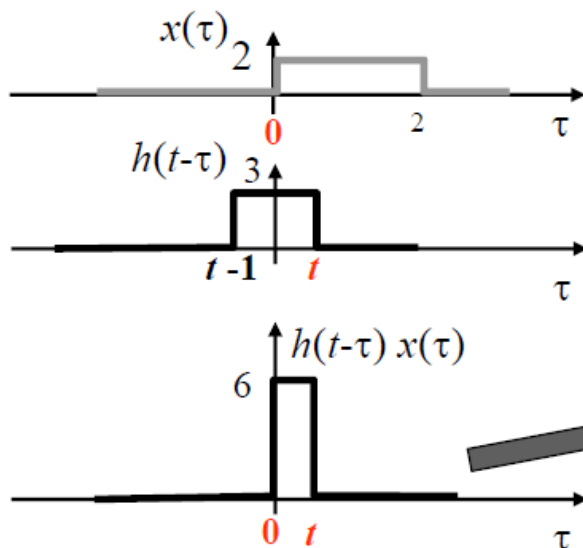
Region I: $t < 0$



$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
 &= \int_{-\infty}^{\infty} 0d\tau = 0 \\
 y(t) &= 0 \quad \text{for all } t < 0
 \end{aligned}$$

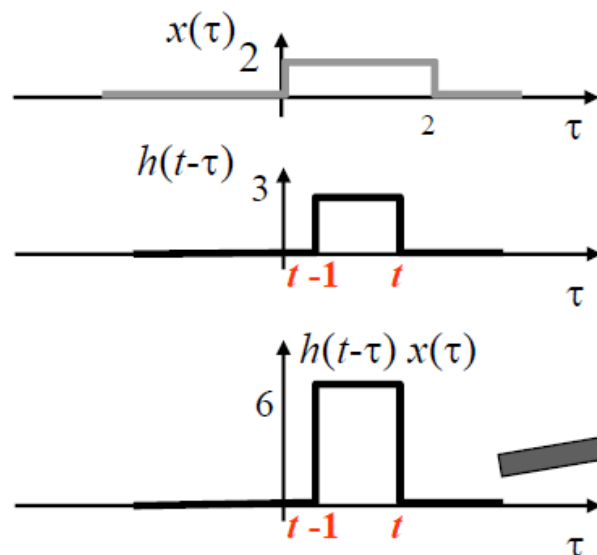
With 0 integrand
the limits don't
matter!!!

Region II: $0 \leq t \leq 1$



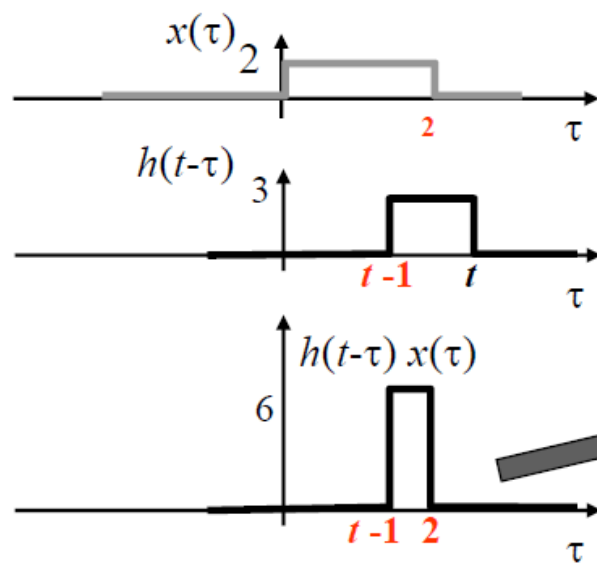
$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
 &= \int_0^t 6d\tau = [6\tau]_0^t = 6t - 6 \times 0 = 6t \\
 y(t) &= 6t \quad \text{for } 0 \leq t \leq 1
 \end{aligned}$$

Step #6 (Continued): Form Product & Integrate For Each Region



Region III: $1 < t \leq 2$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{t-1}^t 6d\tau = [6\tau]_{t-1}^t = 6t - 6(t-1) = 6 \\ y(t) &= 6 \quad \text{for all } t \text{ such that: } 1 < t \leq 2 \end{aligned}$$

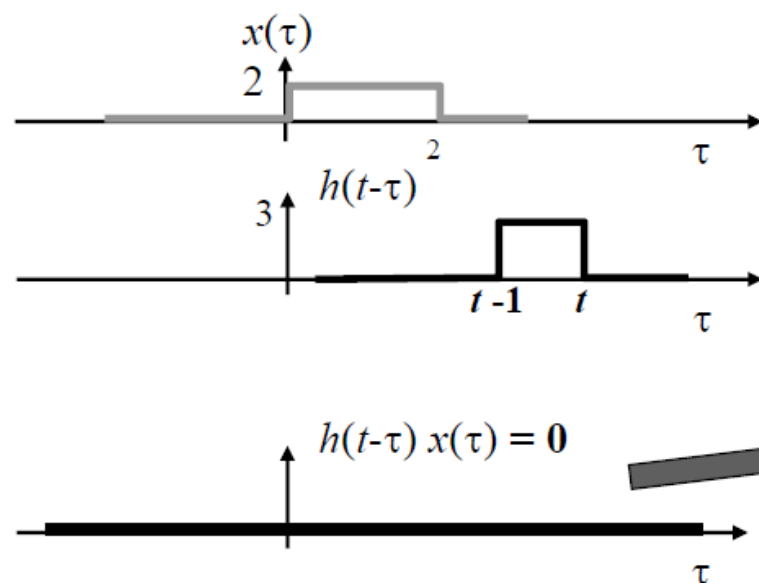


Region IV: $2 < t \leq 3$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{t-1}^2 6d\tau = [6\tau]_{t-1}^2 = 6 \times 2 - 6(t-1) = -6t + 18 \\ y(t) &= -6t + 18 \quad \text{for } 2 < t \leq 3 \end{aligned}$$

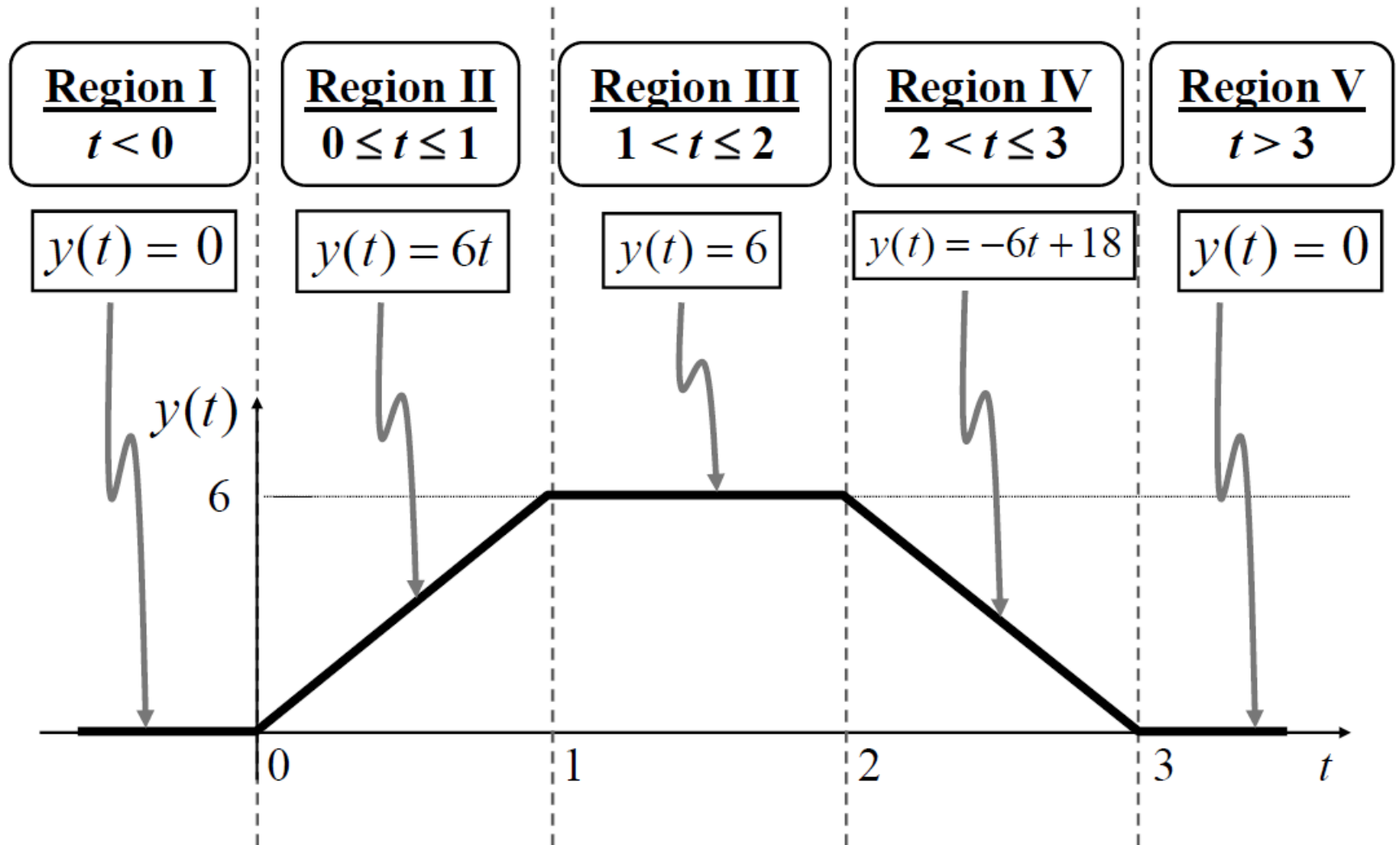
Step #6 (Continued): Form Product & Integrate For Each Region

Region V: $t > 3$



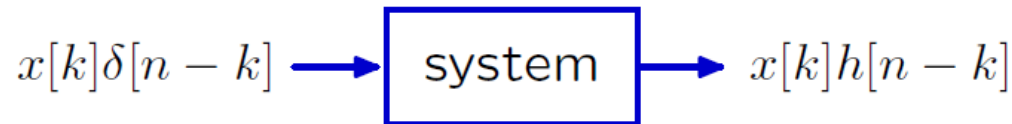
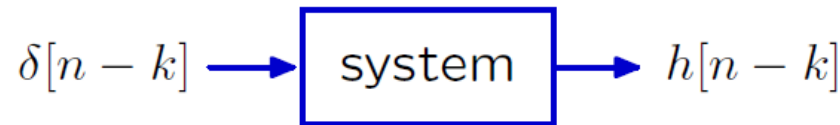
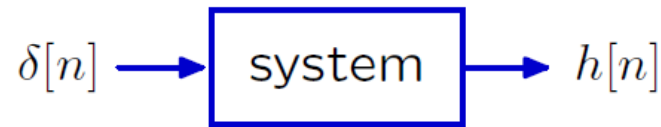
$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} 0d\tau = 0 \\ y(t) &= 0 \quad \text{for all } t > 3 \end{aligned}$$

Step #7: “Assemble” Output Signal



Discrete Convolution

If a system is linear and time-invariant (LTI) then its output is the sum of weighted and shifted unit-sample responses.



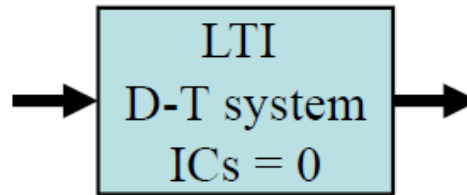
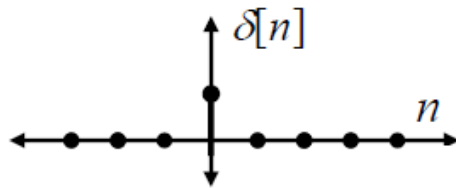
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - k] \longrightarrow \text{system} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k]$$

Discrete Convolution

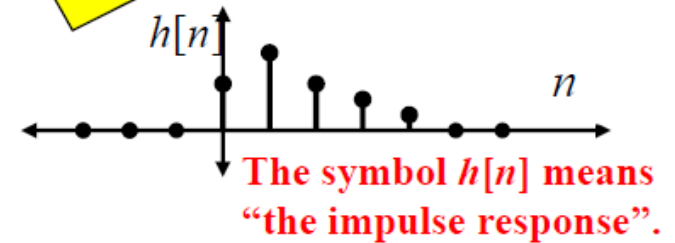
Recall: Impulse Response

Earlier we introduced the concept of impulse response...

...what comes out of a system when the input is an impulse (delta sequence)



Note: If system is causal,
then $h[n] = 0$ for $n < 0$



Noting that the ZT of $\delta[n] = 1$ and using the properties of the transfer function and the Z transform we said that

$$h[n] = Z^{-1} \{ H(z) Z \{ \delta[n] \} \}$$

$$h[n] = Z^{-1} \{ H(z) \}$$

$$h[n] = IDTFT \{ H(\Omega) \}$$

So...once we have either $H(z)$ or $H(\Omega)$ we can get the impulse response $h[n]$

Convolution Property and System Output

Let $x[n]$ be a signal with DTFT $X(\Omega)$ and ZT of $X(z)$

$$x[n] \leftrightarrow X(\Omega)$$

$$x[n] \leftrightarrow X(z)$$

Consider a system w/ freq resp $H(\Omega)$ & trans func $H(z)$

$$h[n] \leftrightarrow H(\Omega)$$

$$h[n] \leftrightarrow H(z)$$

We've spent much time using these tools to analyze system outputs this way:

$$Y(\Omega) = H(\Omega)X(\Omega) \leftrightarrow y[n] = DTFT^{-1}\{H(\Omega)X(\Omega)\}$$

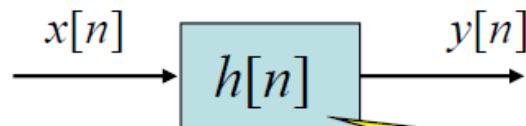
$$Y(z) = H(z)X(z) \leftrightarrow y[n] = Z^{-1}\{H(z)X(z)\}$$

The convolution property of the DTFT and ZT gives an alternate way to find $y[n]$:

$$DTFT^{-1}\{X(\Omega)H(\Omega)\} = x[n] * h[n]$$

$$Z^{-1}\{X(z)H(z)\} = x[n] * h[n]$$

$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$



$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

LTI System with impulse response $h[n]$

**“Convoluting”
input $x[n]$ with the
impulse response
 $h[n]$ gives the
output $y[n]$!**

Convolution for *Causal* System & with *Causal* Input

An arbitrary LTI system's output can be found using the general convolution form:

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

General LTI System

If the system is causal then $h[n] = 0$ for $n < 0 \dots$ Thus $h[n-m] = 0$ for $m > n \dots$ so:

$$y[n] = \sum_{m=-\infty}^n x[m]h[n-m]$$

Causal LTI System

If the input is causal then $x[n] = 0$ for $n < 0 \dots$ so:

$$y[n] = \sum_{m=0}^{\infty} x[m]h[n-m]$$

Causal Input & General LTI System

If the system & signal are both causal then

$$y[n] = \sum_{m=0}^n x[m]h[n-m]$$

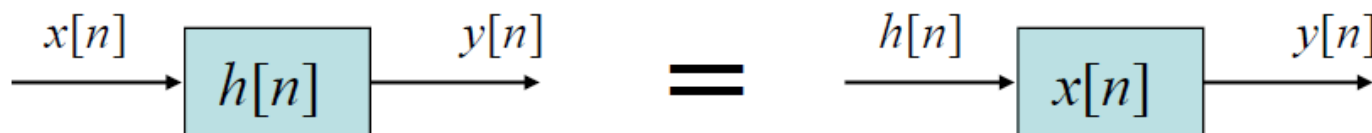
Causal Input & Causal LTI System

Convolution Properties (can sometimes exploit to make things easier)

1. Commutativity

$$x[n] * h[n] = h[n] * x[n]$$

$$\sum_{m=-\infty}^{\infty} x[m]h[n-m] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$



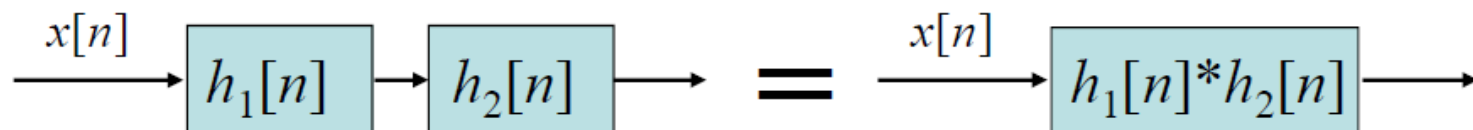
This is obvious from the frequency domain (or z domain) viewpoint:

$$x[n] * h[n] = h[n] * x[n] \quad \Rightarrow \quad X(\Omega)H(\Omega) = H(\Omega)X(\Omega)$$

2. Associativity

$$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$$

\Rightarrow Can combine cascade into single equivalent system

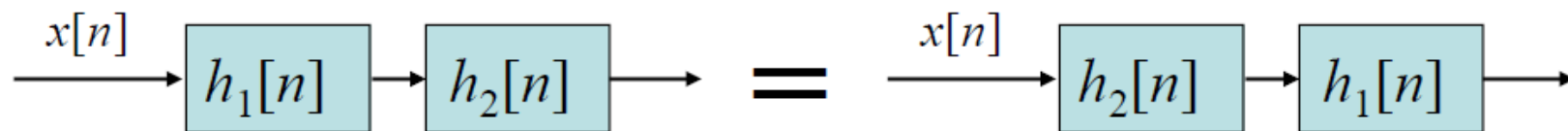


This is obvious from the frequency domain (or z domain) viewpoint:

$$[X(\Omega)H_1(\Omega)]H_2(\Omega) = X(\Omega)[H_1(\Omega)H_2(\Omega)]$$

Tells us what the Freq
Resp is for a cascade

Associativity together with commutativity says we **can interchange the order of two cascaded systems**:

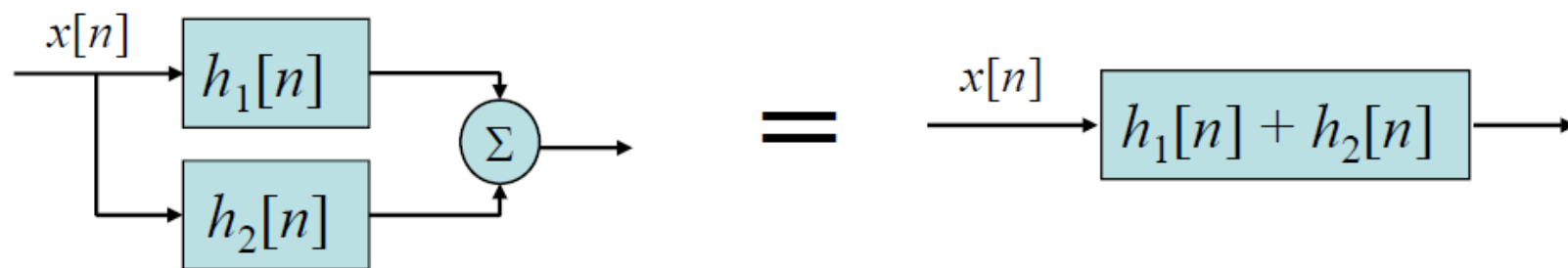


Warning: This holds in theory but in practice there may be physical issues that prevent this!!!

3. **Distributivity**

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

\Rightarrow can combine sum of two outputs into a single system (or vice versa)



With commutativity this says we can split a complicated input into sum of simple ones... which is nothing more than “linearity”!!

Graphical Convolution – To Visualize & Test Real Systems

Can do convolution this way when signals are known numerically or by equation

- Convolution involves the sum of a product of two signals: $x[i]h[n - i]$
- At each output index n , the product changes

“Commutativity” says we can flip either $x[i]$ or $h[i]$ and get the same answer

Step 1: Write both as functions of i : $x[i]$ & $h[i]$

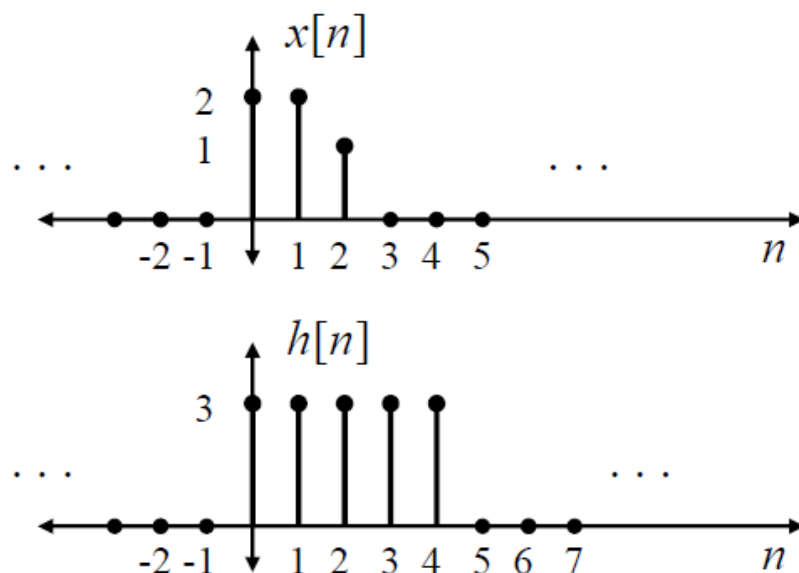
Step 2: Flip $h[i]$ to get $h[-i]$ (The book calls this “fold”)

Repeat for each n {

Step 3: For each output index n value of interest, shift by n to get $h[n - i]$
(Note: positive n gives right shift!!!!)

Step 4: Form product $x[i]h[n - i]$ and sum its elements to get the number $y[n]$

Example of Graphical Convolution



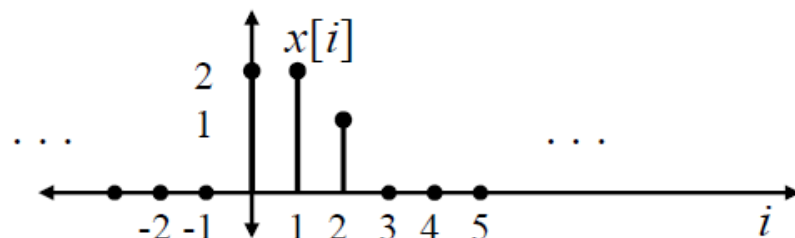
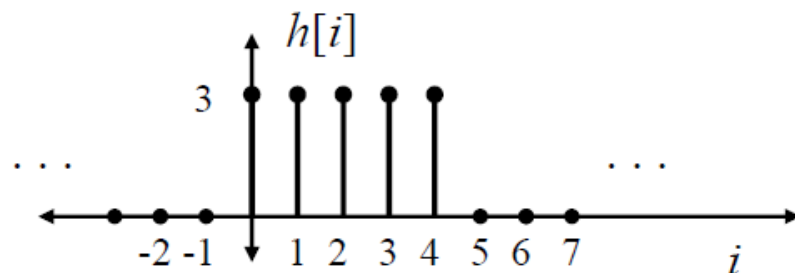
Find $y[n]=x[n]*h[n]$
for all
integer values of n

Solution

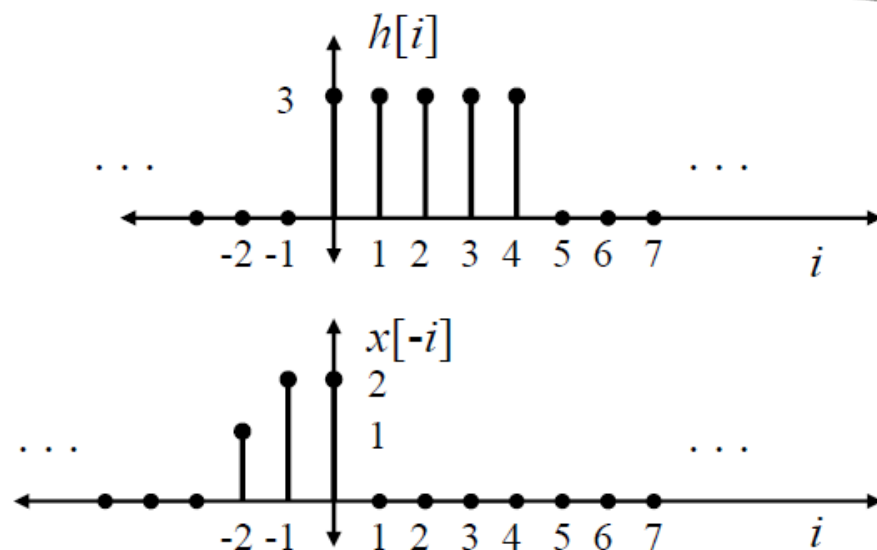
For this problem I choose to flip $x[n]$

My personal preference is to flip the shorter signal although I sometimes don't follow that "rule"... only through lots of practice can you learn how to best choose which one to flip.

Step 1: Write both as functions of i : $x[i]$ & $h[i]$



Step 2: Flip $x[i]$ to get $x[-i]$



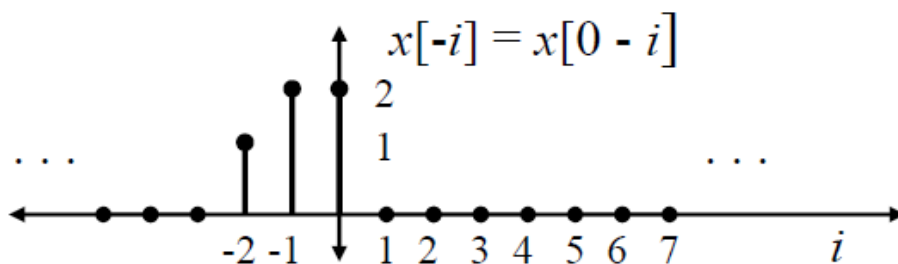
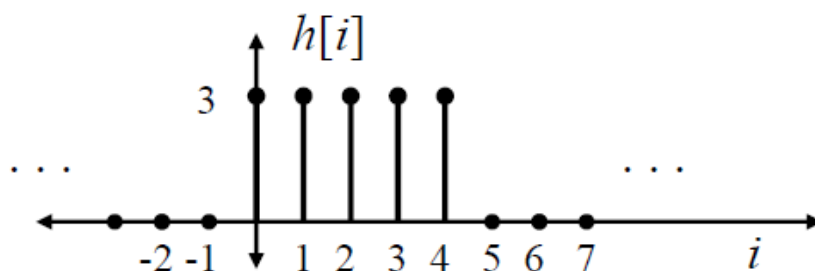
“Commutativity” says we can flip either $x[i]$ or $h[i]$ and get the same answer...
Here I flipped $x[i]$

We want a solution for $n = \dots -2, -1, 0, 1, 2, \dots$ so must do Steps 3&4 for all n .

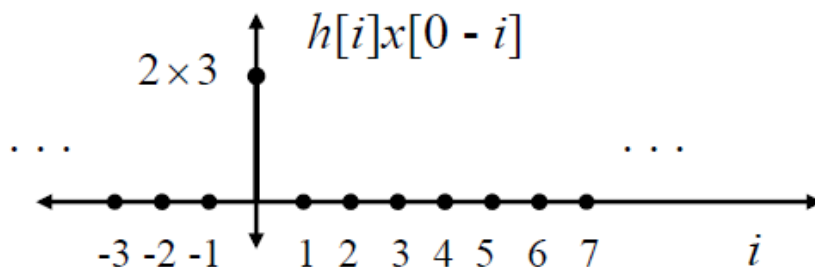
But... let's first do: **Steps 3&4 for $n = 0$** and then proceed from there.

Step 3: For $n = 0$, shift by n to get $x[n - i]$

For $n = 0$ case there is no shift!
 $x[0 - i] = x[-i]$



Step 4: For $n = 0$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$



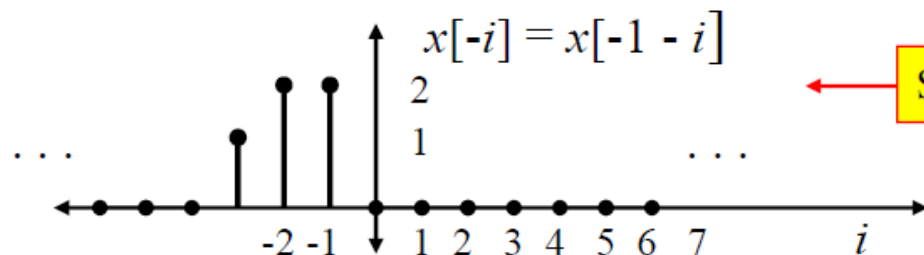
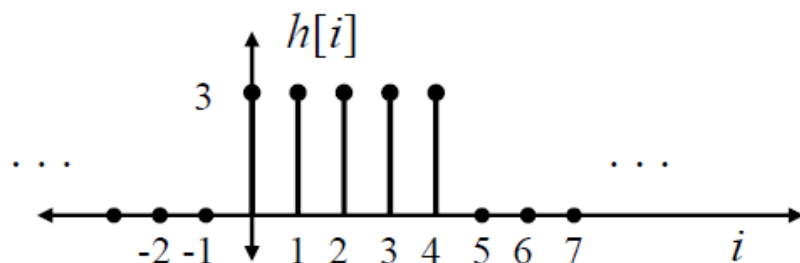
Sum over $i \Rightarrow$

$$y[0] = 6$$

Steps 3&4 for all $n < 0$

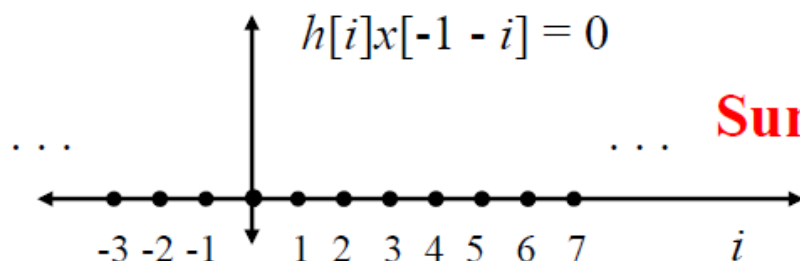
Step 3: For $n < 0$, shift by n to get $x[n - i]$

Negative n gives a left-shift



Shown here for $n = -1$

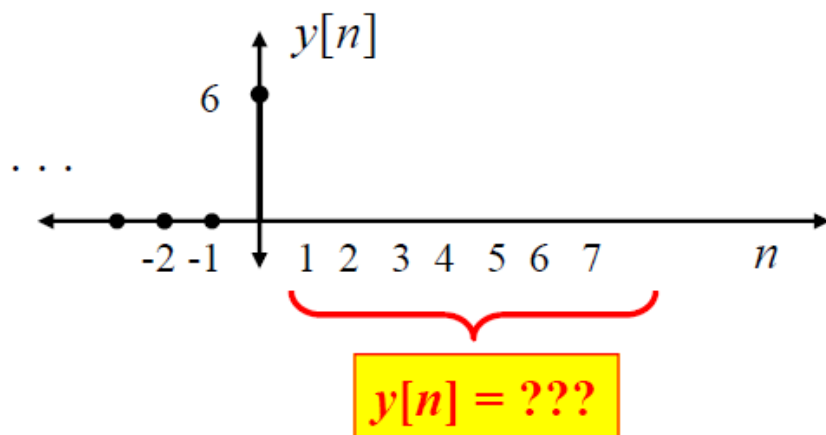
Step 4: For $n < 0$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$



Sum over $i \Rightarrow y[n] = 0 \quad \forall n < 0$

So... what we know so far is that:

$$y[n] = \begin{cases} 0, & \forall n < 0 \\ 6, & n = 0 \end{cases}$$

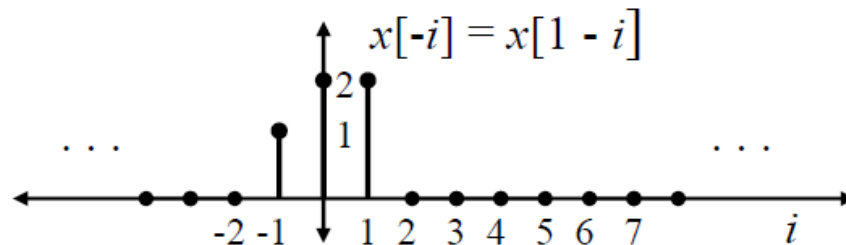
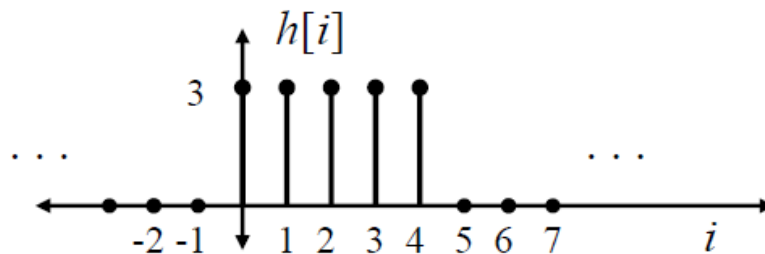


So now we have to do Steps 3&4 for $n > 0$...

Steps 3&4 for $n = 1$

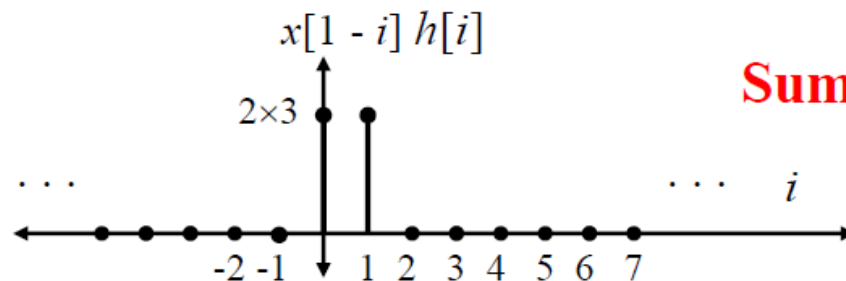
Step 3: For $n = 1$, shift by n to get $x[n - i]$

Positive n gives a Right-shift



shifted to the right by one

Step 4: For $n = 1$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

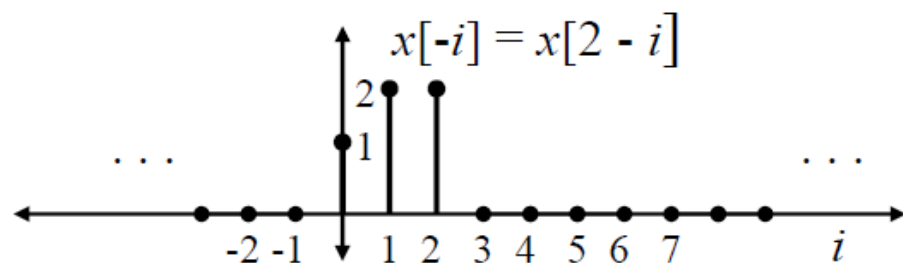
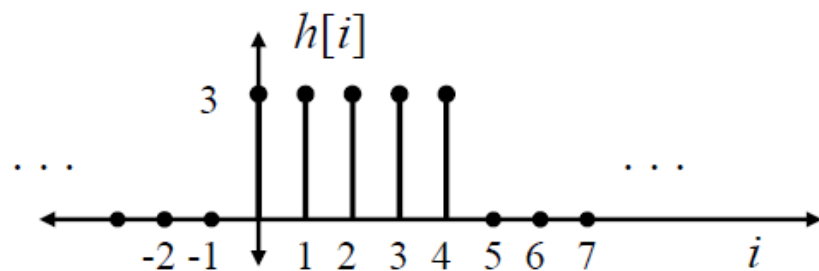


Sum over $i \Rightarrow y[1] = 6 + 6 = 12$

Steps 3&4 for $n = 2$

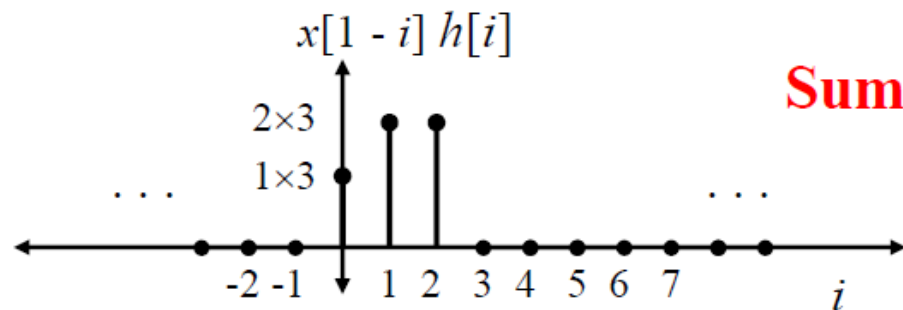
Step 3: For $n = 2$, shift by n to get $x[n - i]$

Positive n gives a Right-shift



shifted to the
right by two

Step 4: For $n = 2$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

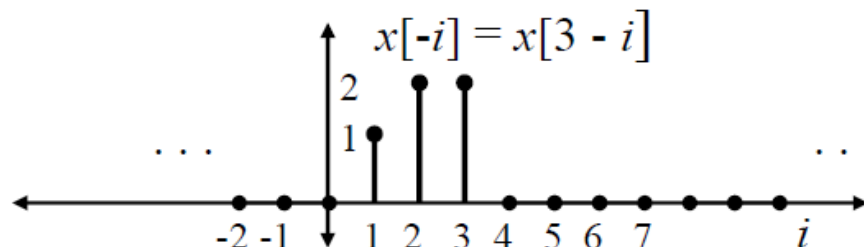
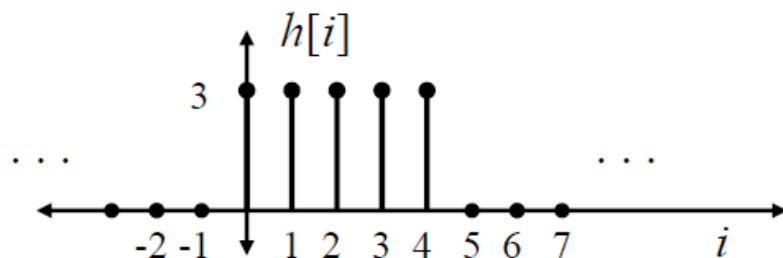


Sum over $i \Rightarrow y[2] = 3 + 6 + 6 = 15$

Steps 3&4 for $n = 3$

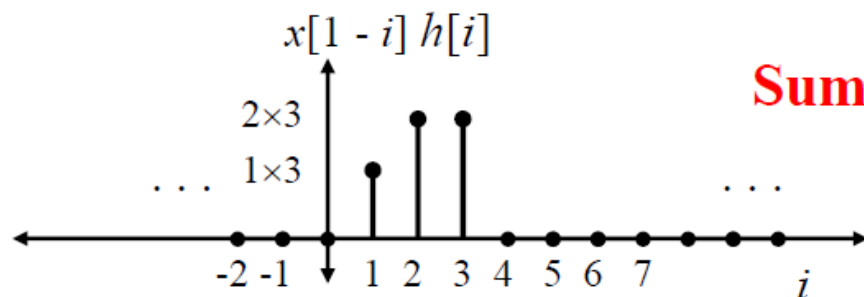
Step 3: For $n = 3$, shift by n to get $x[n - i]$

Positive n gives a Right-shift



shifted to the
right by three

Step 4: For $n = 3$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

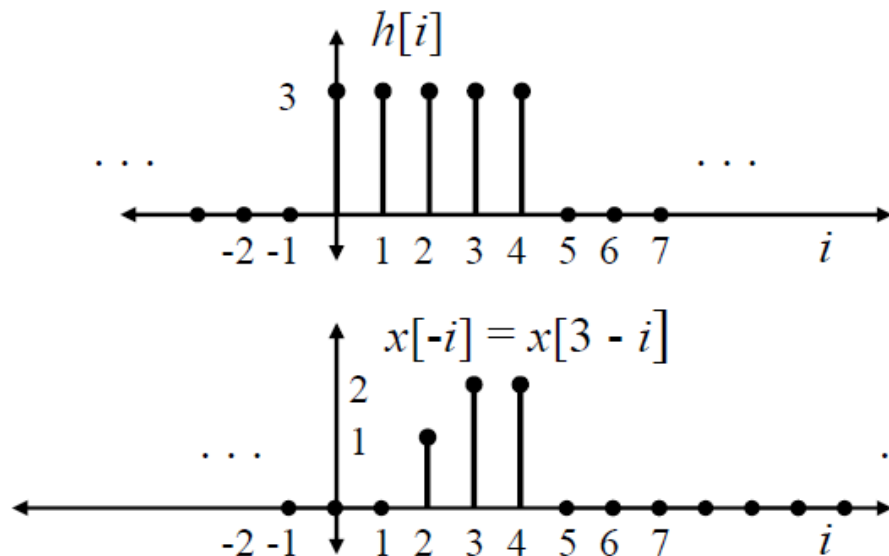


Sum over $i \Rightarrow y[3] = 3 + 6 + 6 = 15$

Steps 3&4 for $n = 4$

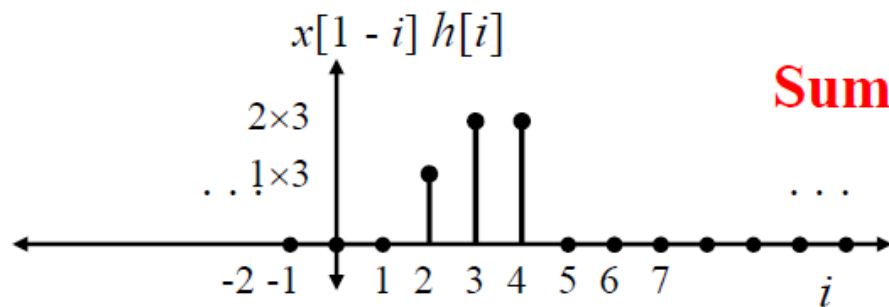
Step 3: For $n = 4$, shift by n to get $x[n - i]$

Positive n gives a Right-shift



shifted to the right by four

Step 4: For $n = 4$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

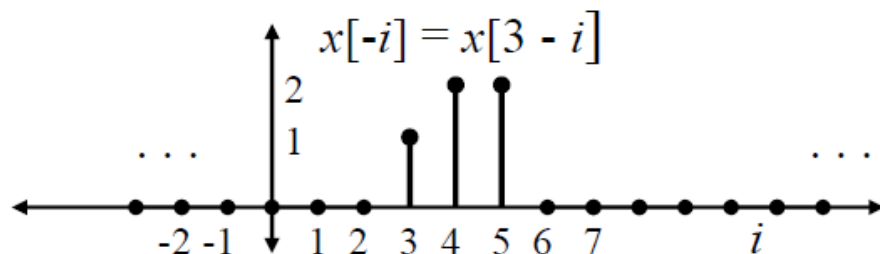
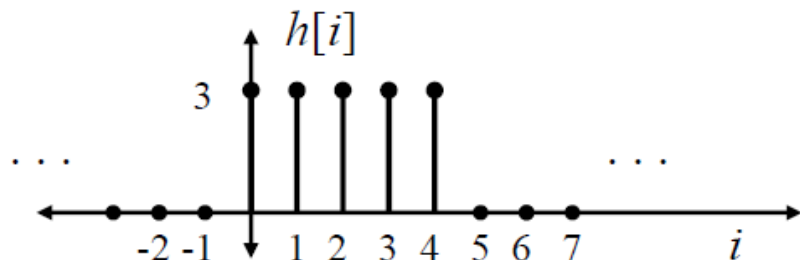


Sum over $i \Rightarrow y[4] = 3 + 6 + 6 = 15$

Steps 3&4 for $n = 5$

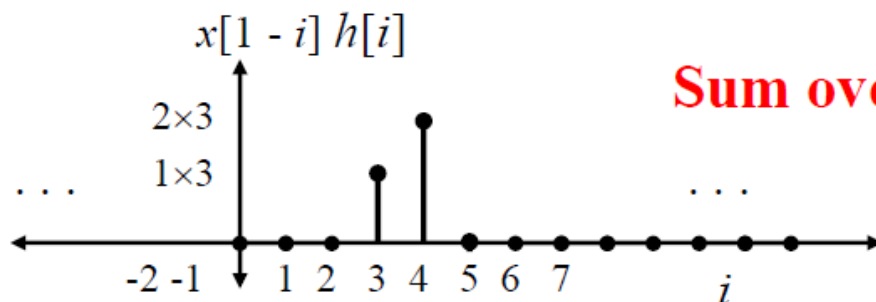
Step 3: For $n = 5$, shift by n to get $x[n - i]$

Positive n gives a Right-shift



shifted to the
right by five

Step 4: For $n = 5$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

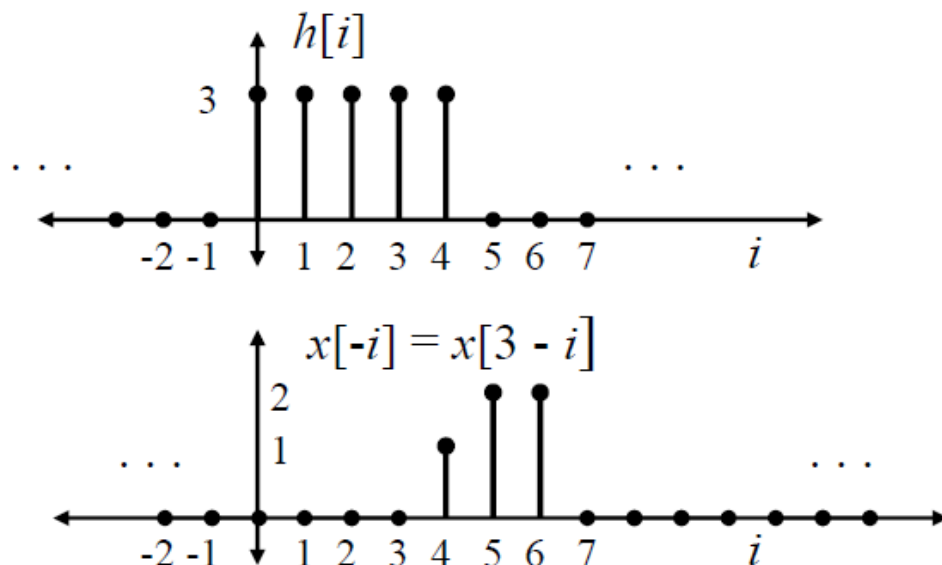


Sum over $i \Rightarrow y[5] = 3 + 6 = 9$

Steps 3&4 for $n = 6$

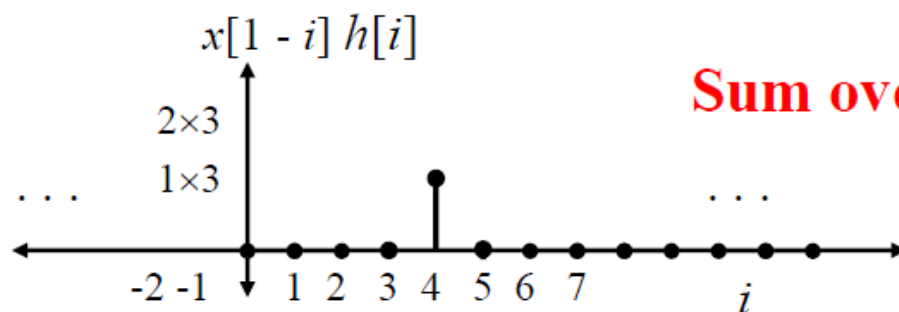
Step 3: For $n = 6$, shift by n to get $x[n - i]$

Positive n gives a Right-shift



shifted to the
right by six

Step 4: For $n = 6$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$

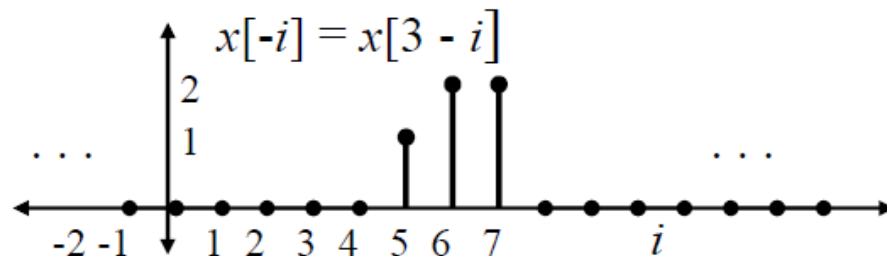
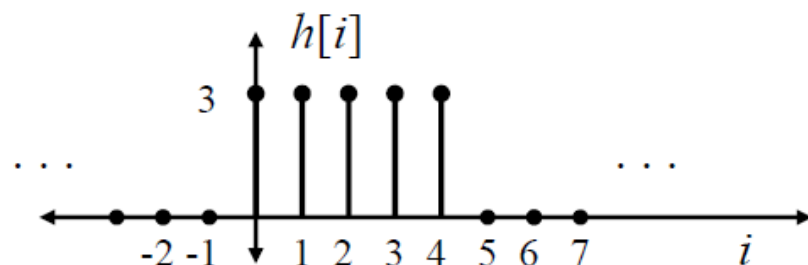


Sum over $i \Rightarrow y[6] = 3$

Steps 3&4 for all $n > 6$

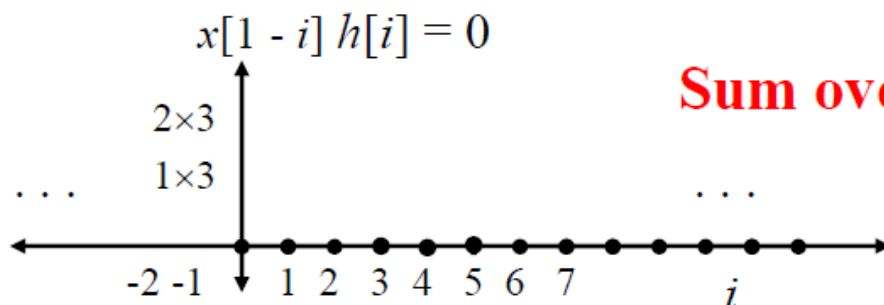
Step 3: For $n > 6$, shift by n to get $x[n - i]$

Positive n gives a Right-shift



shifted to the
right by seven

Step 4: For $n > 6$, Form the product $x[i]h[n - i]$ and sum its elements to give $y[n]$



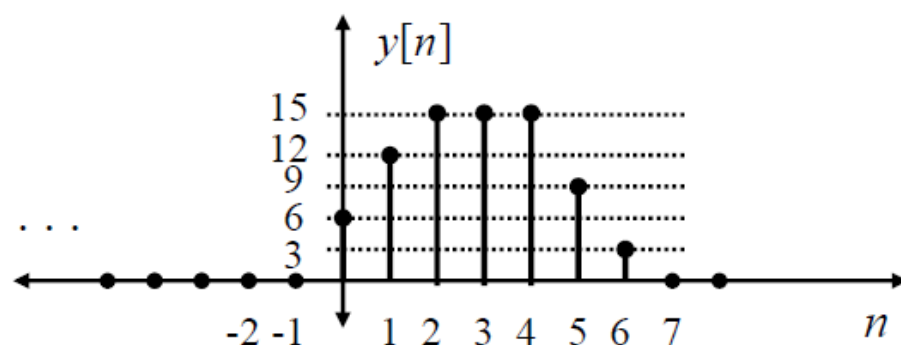
Sum over $i \Rightarrow$

$$y[n] = 0 \quad \forall n > 6$$

So... now we know the values of $y[n]$ for all values of n

We just need to put it all together as a function...

Here it is easiest to just plot it... you could also list it as a table.



Note that convolving these kinds of signals gives a “ramp-up” at the beginning and a “ramp-down” at the end.

Various kinds of “transients” at the beginning and end of a convolution are common.

Image Convolutions

