Optimization Techniques Section 4

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Convergence

- We have tried to find some x^k that converge to the desired point x*(most often a local minimizer). The fundamental question is how fast the convergence is.
- Let's define errors at each iteration step as $|e_k| = |x^k - x^*|$ and $|e_{k+1}| = |x^{k+1} - x^*|$

Convergence

- Linear Convergance :

There exist a constant $0<c_1<1$

where $|e_{k+1}| \leq c_1 * |e_k|$

- If c_1 is very close to 0 then superlinear convergence.
- Quadratic Convergance :

There exist a constant $0<c_2<1$

where $|e_{k+1}| \leq c_2 * |e_k|^2$

Convergence Example

- Two methods are given; one of them showing linear convergence and the other one showing quadratic convergence.
- After certain number of steps errors reach 3 digit precision for both of the methods ($|e_k| <$ 0,001).
- How many more steps (iterations) will be necessary if we require 12 digits of precision ?
- $c_1=c_2=1/2$

Convergence Example

- In case of Linear Convergence : c1=0.5
- $|e_{k+1}| \leq 0.5^*|e_k|$
- $|e_{k+2}| \leq 0.5^*|e_{k+1}|$
- $|e_{k+2}| \leq 0.5^*0.5^*|e_k|$
- …
- $|e_{k+n}| \leq 0.5^{n*}|e_k|$

- $|e_k| \leq 0.001$ (It is given.)
- $|e_{k+n}| \leq 10^{-12}$ (It is required.)
- $|e_{k+n}| \leq 0.5^{n*}10^{-3}$
- $0.5^n \approx 10^{-9}$
- $\log_{10}2^{-n} = \log_{10}10^{-9}$
- $-n * \log_{10}2 = -9$ ($\log_{10}2 \approx 0.301$)
- $n \approx 30$

Convergence Example

- In case of quadratic Convergence : c2=0.5
- $|e_{k+1}| \leq 0.5^*|e_k|^2$
- $|e_{k+2}| \leq 0.5^*|e_{k+1}|^2$
- $|e_{k+2}| \leq 0.5^*(0.5^*|e_k|^2)^2$
- $|e_{k+3}| \leq 0.5^*(0.5^*(0.5^*|e_k|^2)^2)^2$

- $|e_k| \leq 10^{-3}$ (It is given.)
- $|e_{k+n}| \leq 10^{-12}$ (It is required.)
- $|e_k|^2=(10^{-3})^2=10^{-6}$
- $(|e_k|^2)^2 = 10^{-12}$
- n = 2

with N dimensions

- $f(x)=(1/2) * x^T * q * x + b^T * x + c$
- $f: R^n \text{-->} R^1$
- q--> n*n
- b--> n*1
- c--> 1*1
- x--> n*1
-
- df=q*x+b
- ddf=q

- opt_Ndim_v2.m

To avoid local minimums

- Random restart
- Non Derivative Techniques

References

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