Yıldız Technical University
Computer Engineering Department
2023-2024 Spring
BLM3620 Digital Signal Processing
Homework 4, Form: $\boxed{\text{A}}$

Name: ______________________

Surname: ______________________

Student I.D.: ______________________

Signature: ______________________

Signal	FT
$x(t)$	$X(w)$
$y(t)$	$Y(w)$
$\frac{1}{2\pi}\int\limits_{-\infty}^{\infty} X(w)e^{jwt}dw$	$\int\limits_{-\infty}^{\infty} x(t)e^{-jwt}dt$
$\delta(t)$	1
$\Pi(t) = \begin{cases} 0, & \lvert t\rvert > \frac{1}{2} \\ 1, & \lvert t\rvert \leq \frac{1}{2} \end{cases}$	$sinc\left(\frac{w}{2\pi}\right)$
$\Lambda(t) = \begin{cases} 0, & \lvert t\rvert > 1 \\ 1-\lvert t\rvert, & \lvert t\rvert \leq 1 \end{cases}$	$sinc^2\left(\frac{w}{2\pi}\right)$
1	$2\pi\delta(w)$
e^{-jw_0t}	$2\pi\delta(w-w_0)$
$e^{-\lvert a\rvert t}u(t)$	$\frac{1}{\lvert a\rvert+jw}$
$e^{\lvert a\rvert t}u(-t)$	$\frac{1}{\lvert a\rvert-jw}$
$e^{-\lvert at\rvert}$	$\frac{2\lvert a\rvert}{\lvert a\rvert^2+w^2}$
$e^{-\pi t^2}$	$e^{-\frac{w^2}{4\pi}}$
$u(t)$	$\pi\delta(w)+\frac{1}{jw}$
$\cos(w_0t)$	$\pi\delta(w+w_0)+\pi\delta(w-w_0)$
$\sin(w_0t)$	$j\pi\delta(w+w_0)-j\pi\delta(w-w_0)$
$\sum\limits_{n=-\infty}^{\infty}\delta(t-n)$	$\sum\limits_{n=-\infty}^{\infty}\delta(w-n)$
$t^nx(t)$	$j^n\frac{\mathrm{d}^nX(w)}{\mathrm{d}w^n}$
$\lvert t\rvert$	$-\frac{2}{w^2}$
$\frac{1}{1+t^2}$	$\pi e^{-\lvert w\rvert}$
$ax(t)+by(t)$	$aX(w)+bY(w)$
$x(t-a)$	$X(w)e^{-jwa}$
$x(\frac{t}{a})$	$\lvert a\rvert X(aw)$
$x(t)*y(t)$	$X(w)Y(w)$
$x(t)y(t)$	$\frac{1}{2\pi}X(w)*Y(w)$
$x(t)e^{jw_0t}$	$X(w-w_0)$
$x^*(t)$	$X^*(-w)$
$x(-t)$	$X(-w)$
$\frac{\mathrm{d}x(t)}{\mathrm{d}t}$	$jwX(w)$
$\int\limits_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{jw}X(w)$
$x(t)$ is real	$X(w)=X^*(-w)$
$x(t)$ is real and even	$X(w)$ is real and even
$x(t)$ is real and odd	$X(w)$ is imaginary and odd
$\int\limits_{-\infty}^{\infty}\lvert x(t)\rvert^2dt$	$\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\lvert X(w)\rvert^2dw$

Sequence	DTFT
$x[n]$	$X(e^{jw})$
$y[n]$	$Y(e^{jw})$
$\frac{1}{2\pi}\int\limits_{-\pi}^{\pi} X(e^{jw})e^{jwn}dw$	$\sum\limits_{n=-\infty}^{\infty} x[n]e^{-jwn}$
$\delta[n]$	1
$\delta[n-a]$	e^{-jwa}
$\sum\limits_{m=-\infty}^{\infty} \delta[n-m]$	$2\pi \sum\limits_{k=-\infty}^{\infty} \delta(w-2\pi k)$
e^{-jan}	$2\pi \sum\limits_{k=-\infty}^{\infty} \delta(w+a-2\pi k)$
$u[n]$	$\frac{1}{1-e^{-jw}} + \pi \sum\limits_{k=-\infty}^{\infty} \delta(w-2\pi k)$
$a^n u[n]$, $\|a\| < 1$	$\frac{1}{1-ae^{-jw}}$
$-a^n u[-n-1]$, $\|a\| > 1$	$\frac{1}{1-ae^{-jw}}$
$(n+1)a^n u[n]$, $\|a\| < 1$	$\frac{1}{\left(1-ae^{-jw}\right)^2}$
$\cos(na)$	$\pi \sum\limits_{k=-\infty}^{\infty} \delta(w-a-2\pi k) + \delta(w+a-2\pi k)$
$\sin(na)$	$\frac{\pi}{j} \sum\limits_{k=-\infty}^{\infty} \delta(w-a-2\pi k) - \delta(w+a-2\pi k)$
$x[n] = \begin{cases} 0, & \|n\| > N \\ 1, & \|n\| \leq N \end{cases}$	$\frac{\sin\left(w\left(N+\frac{1}{2}\right)\right)}{\sin\left(\frac{w}{2}\right)}$
$\frac{\sin(An)}{\pi n}$	$X(e^{jw}) = \begin{cases} 1, & \|w\| \leq A \\ 0, & A < \|w\| \leq \pi \end{cases}$, $X(e^{jw})$ periodic by 2π
$ax[n] + by[n]$	$aX(e^{jw}) + bY(e^{jw})$
$x[n-n_0]$	$X(e^{jw})e^{-jwn_0}$
$x[n]e^{jwn_0}$	$X(e^{j(w-w_0)})$
$x^*[n]$	$X^*(e^{-jw})$
$x[-n]$	$X(e^{-jw})$
$x[n] * y[n]$	$X(e^{jw})Y(e^{jw})$
$x[n]y[n]$	$\frac{1}{2\pi}\int\limits_{2\pi} X(e^{j\theta})Y(e^{j(w-\theta)})d\theta$
$x[n] - x[n-1]$	$\left(1-e^{-jw}\right) X(e^{jw})$
$\sum\limits_{k=-\infty}^{\infty} x[k]$	$\frac{1}{\left(1-e^{-jw}\right)} + \pi X(1) \sum\limits_{k=-\infty}^{\infty} \delta(w-2\pi k)$
$nx[n]$	$j\frac{\mathrm{d}X(e^{jw})}{\mathrm{d}w}$
$x[n]$ is real	$X(e^{jw}) = X^*(e^{-jw})$
$x[n]$ is real and even	$X(e^{jw})$ is real and even
$x[n]$ is real and odd	$X(e^{jw})$ is imaginary and odd
$\sum\limits_{n=-\infty}^{\infty} \left\|x[n]\right\|^2$	$\frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \left\|X(e^{jw})\right\|^2 dw$

Sequence	DFT
$x[n]$	$X[k]$
$y[n]$	$Y[k]$
$\frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi\frac{kn}{N}}$	$\sum_{n=0}^{N-1} x[n]e^{-j2\pi\frac{kn}{N}}$
$\sum_{k=-\infty}^{\infty} \delta[n+Nk]$	1 (period N)
1 (period N)	$N\sum_{m=-\infty}^{\infty} \delta[k+Nm]$
$e^{j2\pi k_0 n}$	$N\delta[((k-k_0))_N]$
$\cos\left(2\pi\frac{k_0 n}{N}\right)$	$\frac{N}{2}\left(\delta[((k-k_0))_N]+\delta[((k+k_0))_N]\right)$
$ax[n]+by[n]$	$aX[k]+bY[k]$
$x[((n-m))_N]$	$X[k]e^{-j2\pi\frac{km}{N}}$
$X[n]$	$NX[((-k))_N]$
$x[n]y[n]$	$\frac{1}{N}X[k]\circledast Y[k]$
$x[n]\circledast y[n]$	$X[k]Y[k]$
$x^*[n]$	$X^*[((-k))_N]$
$x[((-n))_N]$	$X^*[k]$
$\mathrm{Re}\left\{x[n]\right\}$	$\frac{1}{2}\left(X[((k))_N]+X^*[((-k))_N]\right)$
$j\,\mathrm{Im}\left\{x[n]\right\}$	$\frac{1}{2}\left(X[((k))_N]-X^*[((-k))_N]\right)$
$\frac{1}{2}\left(x[((n))_N]+x^*[((-n))_N]\right)$	$\mathrm{Re}\left\{X[k]\right\}$
$\frac{1}{2}\left(x[((n))_N]-x^*[((-n))_N]\right)$	$j\,\mathrm{Im}\left\{X[k]\right\}$
$\sum_{n=0}^{N-1} \left\|x[n]\right\|^2$	$\frac{1}{N}\sum_{k=0}^{N-1} \left\|X[k]\right\|^2$

Sequence	ZT	Region of Convergence
$x[n]$	$X(z)$	
$y[n]$	$Y(z)$	
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\|>\|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\|<\|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|>\|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|<\|a\|$
$\delta[n]$	1	All z
$\delta[n-n_0]$	z^{-n_0}	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\|>1$
$\cos(\omega_0 n)u[n]$	$\frac{1-z^{-1}\cos(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$	$\|z\|>1$
$\sin(\omega_0 n)u[n]$	$\frac{z^{-1}\sin(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$	$\|z\|>1$
$a^n\cos(\omega_0 n)u[n]$	$\frac{1-az^{-1}\cos(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}$	$\|z\|>\|a\|$
$a^n\sin(\omega_0 n)u[n]$	$\frac{az^{-1}\sin(\omega_0)}{1-a2z^{-1}\cos(\omega_0)+a^2z^{-2}}$	$\|z\|>\|a\|$
$Ax[n]+By[n]$	$AX(z)+BY(z)$	
$x[n-n_0]$	$X(z)z^{-n_0}$	
$a^n x[n]$	$X(a^{-1}z)$	
$x^*[n]$	$X^*(z^*)$	
$x[-n]$	$X(z^{-1})$	
$x[n]*y[n]$	$X(z)Y(z)$	
$nx[n]$	$-z\frac{dX(z)}{dz}$	
$x[n]$ is causal	$x(0)=\lim_{z\to\infty} X(z)$	
$x[n]$ is causal	$x(\infty)=\lim_{z\to 1}[z-1]X(z)$	

1. Determine the fundamental period of $x[n]$.
$$x[n] = \cos(\frac{n\pi}{10}) + \sin(\frac{n\pi}{15})$$

2. Determine whether or not the signal $x[n]$ is periodic.
$$x[n] = \sin(\sqrt{2} + 0.2n)$$

3. Given that real valued signal $x_1[n]$ is even by definition $x_1[n] = x_1[-n]$, and real valued signal $x_2[n]$ is odd by definition $x_2[n] = -x_2[-n]$, determine symmetricality (even/odd) of $y[n]$.
$$y[n] = x_1[n] \cdot x_2[n]$$

4. Given that the power of real valued signal $x[n]$ is defined as $P = \sum_{n=-\infty}^{\infty} x^2[n]$, compute the power in $y[n]$.
$$y[n] = 2^n \cdot u[-n]$$

5. Given that $x[n]$ is the system input and $y[n]$ is the system output, determine whether or not the following systems is time (shift)-invariant.
$$y[n] = x[n] \cdot u[n]$$

6. Given that $x[n]$ is the system input and $y[n]$ is the system output, determine whether or not the following systems is linear.
$$y[n] = \text{Im}\left(x[n]\right)$$

7. Given that $x[n]$ is the system input and $y[n]$ is the system output, determine whether or not the following systems is casual.
$$y[n] = x[|n|]$$

8. Given that $x[n]$ is the system input and $y[n]$ is the system output, determine unit sample response $h[n]$ of the system.
$$y[n] = 0.5y[n-1] + 4x[n-2]$$

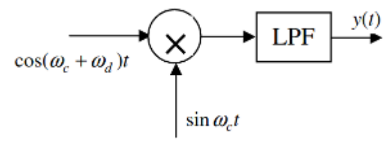
9. The responses of a linear time (shift)-invariant system to specified inputs are defined as follows:

input		reponse
name	symbol	
unit sample	$\delta[n]$	$h[n]$
unit step	$u[n]$	$s[n]$

calculate $h[n]$ for the system, given that $s[n] = u[n] - u[n-5]$.

10. Find the Fourier transform of $x(t)$.
$$x(t) = \begin{cases} \frac{1}{2}, & -T < t < T \\ 0, & other \end{cases}$$

11. Given that the cut-off frequency of the low pass filter (LPF) is w_c evaluate the output $y(t)$. Note: LPF allows frequency values between $-w_c < w < w_c$.



$$x[n] = 2\delta[n+2] - \delta[n+1] + 3\delta[n] - \delta[n-1] + 2\delta[n-2]$$

12. Given that $x[n] \overset{DTFT}{\Longleftrightarrow} X(e^{jw})$ is a DTFT pair, evaluate $X(e^{jw})|_{w=\pi}$ without explicitly finding $X(e^{jw})$.
$$x[n] = 2\delta[n+2] - \delta[n+1] + 3\delta[n] - \delta[n-1] + 2\delta[n-2]$$

13. Given that $x[n] \overset{DTFT}{\Longleftrightarrow} X(e^{jw})$ and $y[n] \overset{DTFT}{\Longleftrightarrow} Y(e^{jw})$ are DTFT pairs, prove the convolution theorem.
$$x[n] * y[n] \overset{DTFT}{\Longleftrightarrow} X(e^{jw})Y(e^{jw})$$

14. Find the inverse DTFT of $X(e^{jw})$.
$$X(e^{jw}) = \cos^2(w)$$

15. Given the 6-point sequence $x[n] = [4, -1, 4, -1, 4, -1]$, determine its 6-point DFT sequence $X[k]$.

16. If the 4-point DFT an unknown length-4 sequence $v[n]$ is $V[k] = [1, 4+j, -1, 4-j]$, determine $v[n]$.

17. Find z-transforms of $x[n]$.
$$x[n] = 6\delta[n] - 7\delta[n-3] - 2\delta[n] - 9\delta[n-5]$$

18. If the region of convergence (ROC) for any $x[n] \overset{ZT}{\Longleftrightarrow} X(z)$ z-transform pair includes the unit cirle in the complex plane then, $x[n] \overset{DTFT}{\Longleftrightarrow} X(e^{jw})$ DTFT pair can also be calculated (converges). Given that the following $X(z)$ includes the unit circle in its region of convergence, evaluate DTFT of $x[n]$ at $w = \pi$.
$$X(z) = \frac{z + 2z^{-2} + z^{-3}}{1 - 3z^{-4} + z^{-5}}$$

19. Evaluate $h[n] * x[n]$ using the convolution property of z-transform.

$$h[n] = (0.5)^n u[n]$$
$$x[n] = 3^n u[-n]$$

20. Given that $x[n] \overset{ZT}{\Longleftrightarrow} X(z)$ is a z-transform pair find $x[n]$.

$$X(z) = 2 + 5(z^2 + z^{-2})$$

21. Given that $x[n] \overset{ZT}{\Longleftrightarrow} X(z)$ is a z-transform pair find $x[n]$ for $|z| > 2$.

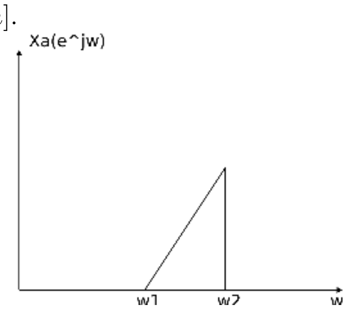
$$X(z) = \frac{1}{1 + 3z^{-1} + 2z^{-2}}$$

22. A continuous-time sinusoid $a_1(t) = \cos(w_1 t + 0.1\pi)$ is sampled at $f_{s_1} = 40Hz$ to give $a_1[n]$, and a second continuous-time sinusoid $a_2(t) = \cos(w_2 t + 0.1\pi)$ is sampled at $f_{s_2} = 50\ Hz$ to give $a_2[n]$. If $w_2 = 30\pi\ rad/s$, determine w_1 so that $a_1[n] = a_2[n]$. Assume there is no aliasing when sampling $a_1(t)$.

23. A complex bandpass filter is constructed by frequency shifting a running sum filter. evaluate and plot the magnitude frequency response of the complex bandpass filter $\left|H_B(e^{jw})\right|$.

$$h[n] = \sum_{k=0}^{4} \delta[n-k]$$
$$h_B[n] = h[n] e^{jw_0 n}$$
$$h_B[n] \overset{DTFT}{\Longleftrightarrow} H_B(e^{jw})$$

24. A complex bandpass analog signal $x_a(t)$ has Fourier transform that is non-zero over the range of $[w_1, w_2]$. The signal is sampled to produce the sequence $x[n] = x_a(nT_s)$. What is the smallest sampling frequency that can be used so that $x_a(t)$ may be recovered from its samples $x[n]$.



25. Plot STFT (Short Time Fourier Transform) representation of a 1D chirp signal with different window sizes. Compare the results with FT of the same signal. Comment on what might be an optimal window size.