SINAV SORULARI ÇÖZÜMLERİ

1) $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin x - 1}{\cot^2 x} = \dfrac{0}{0}$ bl.

$$= \lim_{x\to\frac{\pi}{2}} \frac{\sin x - 1}{\dfrac{\cos^2 x}{\sin^2 x}} = \lim_{x\to\frac{\pi}{2}} \frac{(\sin x - 1)\sin^2 x}{\cos^2 x} \cdot \frac{(\sin x + 1)}{(\sin x + 1)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{(\sin^2 x - 1)\sin^2 x}{\cos^2 x(\sin x + 1)} = \lim_{x\to\frac{\pi}{2}} \frac{-\cancel{\cos^2 x}\,.\sin^2 x}{\cancel{\cos^2 x}\,(\sin x + 1)} = -\frac{1}{2} \;//$$

2) $\lim\limits_{x\to\pi} \dfrac{\sin(2x)}{x^2 - \pi x} = \dfrac{0}{0}$ bl.

$$= \lim_{x\to\pi} \frac{2.\sin x.\cos x}{x\,(x-\pi)} \quad , \quad x - \pi = t \Rightarrow x = t + \pi, \quad x \to \pi \Rightarrow t = 0$$

$$= \lim_{t\to 0} \frac{2.\sin(\pi+t).\cos(\pi+t)}{(\pi+t).t} \qquad \sin(\pi+t) = -\sin t, \quad \cos(\pi+t) = -\cos t$$

$$= \lim_{t\to 0} \frac{2.(-\sin t).(-\cos t)}{t.(\pi+t)} = \lim_{t\to 0} \underbrace{\frac{\sin t}{t}}_{1} \cdot \underbrace{\frac{2\cos t}{(\pi+t)}}_{\frac{2}{\pi}} = \frac{2}{\pi} \;//$$

3) $\lim\limits_{x\to\infty} \dfrac{x - \sqrt{1+x^2}}{x - \sqrt{x}}$ $\left(\dfrac{\infty}{\infty}\text{ bl}\right)$

$$= \lim_{x\to\infty} \frac{x - \sqrt{x^2\left(\frac{1}{x^2}+1\right)}}{x\left(1 - \frac{1}{\sqrt{x}}\right)} = \lim_{x\to\infty} \frac{x - \overset{=x}{|x|}\sqrt{\frac{1}{x^2}+1}}{x\left(1 - \frac{1}{\sqrt{x}}\right)}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}\left(1 - \sqrt{\cancel{\tfrac{1}{x^2}}^{\to 0}+1}\right)}{\cancel{x}\left(1 - \underset{\to 0}{\cancel{\tfrac{1}{\sqrt{x}}}}\right)} = \frac{1-1}{1} = 0 \;//$$

4) $$\lim_{x\to-\infty}\left(2x+\sqrt{4x^2+3x}\right)=(\infty-\infty \text{ bl.})$$

$$=\lim_{x\to-\infty}\frac{\left(2x+\sqrt{4x^2+3x}\right)\left(2x-\sqrt{4x^2+3x}\right)}{\left(2x-\sqrt{4x^2+3x}\right)}$$

$$=\lim_{x\to-\infty}\frac{\cancel{4x^2}-\cancel{4x^2}-3x}{2x-\sqrt{4x^2+3x}}=\lim_{x\to-\infty}\frac{-3x}{2x-\underbrace{|x|}_{-x}\sqrt{4+\frac{3}{x}}}=\lim_{x\to-\infty}\frac{-3x}{2x+x\sqrt{4+\frac{3}{x^2}}}$$

$$=\lim_{x\to-\infty}\frac{-3\cancel{x}}{\cancel{x}\left(2+\sqrt{4+\underbrace{\frac{3}{x^2}}_{0}}\right)}=-\frac{3}{4}$$ //

5) $$\lim_{x\to0}\frac{1-\cos(\sin x)}{2x^2}\quad\left(\frac{0}{0}\text{ bl.}\right)$$

$$=\lim_{x\to0}\frac{\left(1-\cos(\sin x)\right)}{2x^2}\cdot\frac{\left(1+\cos(\sin x)\right)}{\left(1+\cos(\sin x)\right)}$$

$$=\lim_{x\to0}\frac{1-\cos^2(\sin x)}{2x^2\left(1+\cos(\sin x)\right)}=\lim_{x\to0}\frac{\sin^2(\sin x)}{2x^2\left(1+\cos(\sin x)\right)}$$

$$=\lim_{x\to0}\underbrace{\frac{\sin^2(\sin x)}{\sin^2x}}_{1}\cdot\underbrace{\frac{\sin^2x}{x^2}}_{1}\cdot\frac{1}{2\left(1+\cos(\sin x)\right)}=\frac{1}{4}$$ //

6) $$\lim_{x\to2}\frac{|x^2-4|}{\sqrt[4]{x-2}}$$

$$\lim_{x\to2^+}\frac{|x^2-4|}{\sqrt[4]{x-2}}=\lim_{x\to2^+}\frac{(x-2)(x+2)}{(x-2)^{1/4}}=\lim_{x\to2^+}(x-2)^{3/4}(x+2)=0$$

$$\lim_{x\to2^-}\frac{|x^2-4|}{\sqrt[4]{x-2}}=\lim_{x\to2^-}\frac{-(x-2)(x+2)}{(x-2)^{1/4}}=\lim_{x\to2^-}-(x-2)^{3/4}(x+2)=0$$

$\Big] =$

$$\lim_{x\to2}\frac{|x^2-4|}{\sqrt{x-2}}=0$$ //

CS2

7) $f(x) = \ln(\ln(\ln x)) + \sqrt{9-x^2}$

$\underbrace{\ln(\ln(\ln x))}_{} > 0$

$\Downarrow$

$\ln(\ln x) > 0$

$\Downarrow$

$\ln x > 1$

$\Downarrow$

$\boxed{x > e}$

$\underline{x > 0}$

$\ln x > 0$

$\underline{x > 1}$

$9 - x^2 \geq 0$

x		-3		3	
$9-x^2$	$-$	0	$+$	0	$-$

$[-3, 3]$

T.K. : $(e, 3]$

8)

$f(x) = \dfrac{1}{\sqrt{|x| - x}} + \ln\left(\dfrac{9-x^2}{x^2+x}\right)$

$|x| - x > 0$

$|x| > x$

$\boxed{x < 0}$

$(-\infty, 0)$

$\dfrac{9-x^2}{x^2+x} > 0$

$\dfrac{(3-x)(3+x)}{x(x+1)} > 0$

x		-3		-1		0		3	
$9-x^2$	$-$	0	$+$		$+$		$+$	0	$-$
x	$-$		$-$		$-$	0	$+$		$+$
$x+1$	$-$		$-$		$+$		$+$		$+$
$\dfrac{9-x^2}{x(x+1)}$	$-$		$+$		$-$		$+$		$-$

$(-3, -1) \cup (0, 3)$

$(-\infty, 0)$ ve $(-3, -1) \cup (0, 3)$

$\boxed{\text{T.K} : (-3, -1)}$

//

9) $f(x)=\dfrac{3|x-2|}{x^2(4-x^2)}$

$x=0$, $x=\pm 2$ süreksizlik noktaları

$\underline{x=0}$

$\lim\limits_{x\to 0}\dfrac{3|x-2|}{x^2(2-x)(2+x)}=\lim\limits_{x\to 0}\dfrac{3\cancel{(2-x)}}{x^2\cancel{(2-x)}(2+x)}=\infty$ sonsuz (esas) süreksizdir.

$\underline{x=2}$

$\lim\limits_{x\to 2^+}\dfrac{3|x-2|}{x^2(2-x)(2+x)}=\lim\limits_{x\to 2^+}\dfrac{3\cancel{(\overline{x-2})}}{x^2\cancel{(2-x)}(2+x)}=\dfrac{-3}{16}$

$\lim\limits_{x\to 2^-}\dfrac{3|x-2|}{x^2(2-x)(2+x)}=\lim\limits_{x\to 2^-}\dfrac{3\cancel{(2-x)}}{x^2\cancel{(2-x)}(2+x)}=\dfrac{3}{16}$

$\neq$ sıçramalı süreksizdir.

$\underline{x=-2}$

$\lim\limits_{x\to -2^+}\dfrac{3|x-2|}{x^2(2-x)(2+x)}=\lim\limits_{x\to -2}\dfrac{3\cancel{(2-x)}}{x^2\cancel{(2-x)}(2+x)}=\infty$ sonsuz (esas) süreksiz

12) $f(x)=\dfrac{2|x-1|}{x^2-x^3}$

$x^2(1-x)=0 \Rightarrow x=0$, $x=1$ süreksizlik noktaları

$\underline{x=0}$

$\lim\limits_{x\to 0}\dfrac{2|x-1|}{x^2(1-x)}=\lim\limits_{x\to 0}\dfrac{2\cancel{(1-x)}}{x^2\cancel{(1-x)}}=\infty$ sonsuz (esas) süreksizlik

$\underline{x=1}$

$\lim\limits_{x\to 1^+}\dfrac{2|x-1|}{x^2(1-x)}=\lim\limits_{x\to 1^+}\dfrac{2\cancel{(\overline{x-1})}}{x^2\cancel{(1-x)}}=-\dfrac{2}{1}=-2$

$\lim\limits_{x\to 1^-}\dfrac{2|x-1|}{x^2(1-x)}=\lim\limits_{x\to 1^-}\dfrac{2\cancel{(1-x)}}{x^2\cancel{(1-x)}}=\dfrac{2}{1}=2$

$\neq$ sıçramalı süreksizlik

10) $f(x) = \begin{cases} \frac{\pi}{2} & , x=0 \\ \sin\left(\frac{x}{3}\right) & , 0<x<3 \\ 2^{\frac{1}{x-4}} & , 3\leq x<4 \text{ ve } 4<x\leq 5 \\ \frac{\pi}{2} & , x=4 \end{cases}$

<u>$x=0$</u>

$$\left.\begin{array}{l} \lim\limits_{x\to 0^+} \sin\left(\frac{x}{3}\right) = 0 \\ f(0) = \frac{\pi}{2} \end{array}\right] \neq$$

$x=0$ da kaldırılabilir süreksizdir.

<u>$x=3$</u>

$$\left.\begin{array}{l} \lim\limits_{x\to 3^+} 2^{\frac{1}{x-4}} = \frac{1}{2} = f(3) \\ \lim\limits_{x\to 3^-} \sin\left(\frac{x}{3}\right) = \sin 1 \end{array}\right] \neq$$

$x=3$ de sıçramalı süreksizdir.

<u>$x=4$</u>

$$\left.\begin{array}{l} \lim\limits_{x\to 4^+} 2^{\frac{1}{x-4}} = 2^{\infty} = \infty \\ \lim\limits_{x\to 4^-} 2^{\frac{1}{x-4}} = 2^{-\infty} = 0 \end{array}\right]$$

$x=4$ de sonsuz (esas) süreksizdir.

11)
$$f(x) = \begin{cases} x & , \; x \leq -2 \\ 1/(x+2) & , \; -2 < x \leq 1 \\ \dfrac{\sin(1-\sqrt{x})}{x-1} & , \; x > 1 \end{cases}$$

i) $\lim\limits_{x \to -2} f(x)$, $\lim\limits_{x \to 1} f(x)$

$$\left.\begin{array}{l} \lim\limits_{x \to -2^-} x = -2 \\[2ex] \lim\limits_{x \to -2^+} \dfrac{1}{x+2} = \infty \end{array}\right] \quad \lim\limits_{x \to -2} f(x) \text{ mevcut değildir.}$$

$$\lim_{x \to 1^-} \frac{1}{x+2} = \frac{1}{3}$$

$$\lim_{x \to 1^+} \frac{\sin(1-\sqrt{x})}{x-1} = \lim_{x \to 1^+} \frac{\sin(1-\sqrt{x})}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x \to 1^+} \underbrace{\frac{\sin(1-\sqrt{x})}{(1-\sqrt{x})}}_{1} \cdot \frac{-1}{(\sqrt{x}+1)} = -\frac{1}{2}$$

$\neq$ olduğundan $\lim\limits_{x \to 1} f(x)$ mevcut değildir.

ii) $\lim\limits_{x \to -2^+} \dfrac{1}{x+2} = \infty$ olduğundan $x=-2$ de sonsuz (esas) süreksiz

$$\left.\begin{array}{l} \lim\limits_{x \to 1^-} \dfrac{1}{x+2} = \dfrac{1}{3} = f(1) \\[2ex] \lim\limits_{x \to 1^+} \dfrac{\sin(1-\sqrt{x})}{x-1} = -\dfrac{1}{2} \end{array}\right] \neq \text{ olduğundan}$$

$x=1$ de $f(x)$ sıçramalı süreksiz

13) $f(x)=\dfrac{x^2}{1-\cos x}$ $\qquad x=0$ da fonksiyon tanımsız.

$$\lim_{x\to 0}\frac{x^2}{1-\cos x}=\lim_{x\to 0}\frac{x^2(1+\cos x)}{1-\cos^2 x}=\lim_{x\to 0}\underbrace{\frac{x^2}{\sin^2 x}}_{1}\cdot(1+\cos x)=2.$$

$x=0$ da $f(x)$ kaldırılabilir süreksizliğe sahiptir.

$$F(x)=\begin{cases}\dfrac{x^2}{1-\cos x}, & x\neq 0\\ 2, & x=0\end{cases}$$

14) $f(x)=\dfrac{|x^2-9|}{x^2-4x+3}$

$x^2-4x+3=0 \Rightarrow (x-3)(x-1)=0 \Rightarrow x=1\ , x=3$ süreksizlik noktaları

$\underline{x=1}$

$$\lim_{x\to 1^+}\frac{|x^2-9|}{(x-1)(x-3)}=\lim_{x\to 1^+}\frac{-(x-\cancel{3})(x+3)}{(x-1)(x-\cancel{3})}=-\infty$$ sonsuz (esas) süreksizlik

$\underline{x=3}$

$$\lim_{x\to 3^+}\frac{|x^2-9|}{(x-1)(x-3)}=\lim_{x\to 3^+}\frac{(x-\cancel{3})(x+3)}{(x-1)(x-\cancel{3})}=\frac{6}{2}=3$$

$$\lim_{x\to 3^-}\frac{|x^2-9|}{(x-1)(x-3)}=\lim_{x\to 3^-}\frac{-(x-\cancel{3})(x+3)}{(x-1)(x-\cancel{3})}=-3$$

$\neq$ sıçramalı süreksizlik

15) $$f(x)=\begin{cases} x^2 \sin\frac{1}{x^2} & , x<0 \\ a-\arcsin\left(\frac{x+1}{2}\right) & , 0\leq x<1 \\ \frac{a}{b}+\arctan\sqrt{3}\,x & , x\geqslant 1 \end{cases}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} x^2.\sin\frac{1}{x^2}=0 \\ \lim\limits_{x\to 0^+} a-\arcsin\left(\frac{x+1}{2}\right)=a-\frac{\pi}{6}=f(0) \end{array}\right] \Rightarrow a-\frac{\pi}{6}=0 \Rightarrow \boxed{a=\frac{\pi}{6}} \,//$$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^-} \left(a-\arcsin\left(\frac{x+1}{2}\right)\right)= a-\frac{\pi}{2}=\frac{\pi}{6}-\frac{\pi}{2}=\frac{-\pi}{3} \\ \lim\limits_{x\to 1^+} \left(\frac{a}{b}+\arctan\sqrt{3}\,x\right)=\frac{a}{b}+\frac{\pi}{3}=f(1) \end{array}\right] \quad \frac{\pi/6}{b}+\frac{\pi}{3}=-\frac{\pi}{3} \quad \boxed{b=-\frac{1}{4}} \,//$$

16) Ara Değer Teoremi: Eğer f $[a,b]$ aralığında sürekli bir fonksiyon ve y_0 da $f(a)$ ve $f(b)$ arasında herhangi bir değer ise bu durumda $y_0=f(c)$ olacak şekilde (a,b) aralığında bazı c' ler vardır.

$f(x)=x-\cos x$ olsun. f fonksiyonu $[0,\frac{\pi}{2}]$ aralığında sürekli, $f(0)=-1$ ve $f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$ olduğundan, Ara Değer Teoremine göre,

$$f(0)=-1 < f(c)=0 < f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$$

$f(c)=0=c-\cos c$ olacak şekilde bir $c\in[0,\frac{\pi}{2}]$ vardır.

$$\cos c = c \,//$$

17)

$x>0 \quad , f(x)=\frac{1}{\sqrt{x}}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}=\lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h.\sqrt{x}\sqrt{x+h}}\cdot\frac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}$$

$$=\lim_{h\to 0}\frac{x-(x+h)}{h\sqrt{x}.\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}=\lim_{h\to 0}\frac{-\cancel{h}}{\cancel{h}\sqrt{x}\sqrt{x+h}}\cdot\frac{1}{(\sqrt{x}+\sqrt{x+h})}$$

$$=-\frac{1}{2x\sqrt{x}}=\frac{-1}{2\sqrt{x^3}} \quad //$$

18)

$$f(x)=\begin{cases}\sqrt[3]{x}\,(1-\cos x) & , x\geq 0\\ \sin x & , x<0\end{cases}$$

$$\left.\begin{array}{l}\lim\limits_{x\to 0^+}\sqrt[3]{x}\,(1-\cos x)=0=f(0)\\ \lim\limits_{x\to 0^-}\sin x=0\end{array}\right]\Rightarrow f(x),\ x=0 \text{ da süreklidir.}$$

$$\left.\begin{array}{l}f'_+(0)=\lim\limits_{h\to 0^+}\frac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0^+}\sqrt[3]{h}\,\underbrace{\frac{(1-\cos h)}{h}}_{0}=0\\ f'_-(0)=\lim\limits_{h\to 0^-}\frac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0^-}\frac{\sin h}{h}=1\end{array}\right]\neq$$

$f'_+(0)\neq f'_-(0)$ olduğundan $f(x)$, $x=0$ da türevlenemezdir.

19) $f(x)=\begin{cases} x^2+x & , x<0 \\ x+1 & , x\geqslant 0 \end{cases}$

$x=0$ da sürekli mi?

$\lim\limits_{x\to 0^+} x+1=1=f(0)$

$\lim\limits_{x\to 0^-} x^2+x=0$

$\neq$ olduğundan $f(x)$, $x=0$ da sürekli değildir.

Dolayısıyla $x=0$ da türevi mevcut değildir.

20) $f(x)=\begin{cases} \tan(\sin x) & , x\leq 0 \\ \frac{1}{x}\sin x^2 & , x>0 \end{cases}$

$\lim\limits_{x\to 0^+} \frac{1}{x}\sin x^2 = \lim\limits_{x\to 0^+} \underbrace{\frac{\sin x^2}{x^2}}_{1}\cdot x = 0$

$\lim\limits_{x\to 0^-} \tan(\sin x) = 0 = f(0)$

$\lim\limits_{x\to 0} f(x)=f(0)$ olduğundan $x=0$ da süreklidir.

$f'_+(0)=\lim\limits_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h\to 0^+} \frac{\frac{1}{h}\sin h^2}{h} = \lim\limits_{h\to 0^+} \frac{\sin h^2}{h^2}=1$

$f'_-(0)=\lim\limits_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h\to 0^-} \frac{\tan(\sin h)}{h} = \lim\limits_{h\to 0^-} \underbrace{\frac{\tan(\sin h)}{\sin h}}_{1}\cdot \underbrace{\frac{\sin h}{h}}_{1}$

$=1$

$f'_+(0)=f'_-(0)=1$ olduğundan $f(x)$, $x=1$ de türevlenebilirdir.

$f'(0)=1$ //

21) $f(x)=\begin{cases} x+1 & , x<0 \\ \cos^2 x & , x\geq 0 \end{cases}$

$\left.\begin{array}{l} \lim\limits_{x\to 0^+} \cos^2 x = 1 = f(0) \\ \lim\limits_{x\to 0^-} x+1 = 1 \end{array}\right] =$ oldugundan $x=0$ da süreklidir.

$$f'_+(0)=\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0^+}\frac{\cos^2 h-1}{h}=\lim_{h\to 0^+}\frac{-\sin^2 h}{\frac{h^2}{1}}.h=0$$

$$f'_-(0)=\lim_{h\to 0^-}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0^-}\frac{h+1-1}{h}=1$$

$f'_+(0)\neq f'_-(0)$ oldugundan $x=0$ da türevi mevcut degildir.

22) $g(0)=8$, $f(x)=x.g(x)$

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h\,g(h)-0.\,g(0)}{h}$$

$$f'(0)=\lim_{h\to 0}\frac{\cancel{h}\,g(h)}{\cancel{h}}=g(0)=8$$

23) $0 < x < 1$, $f(x) = \arcsin x - \arccos\sqrt{1-x^2}$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{\frac{2x}{2\sqrt{1-x^2}}}{\sqrt{1-(1-x^2)}} = \frac{1}{\sqrt{1-x^2}} - \frac{\cancel{x}}{\cancel{x}\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$$

$f'(x) = 0 \Rightarrow f(x) = c$ seklinde sabit bir fonksiyondur.

24) $f(g(x)) = x$ $\qquad f'(x) = 1 + (f(x))^2$

$$(f(g(x)))' = 1$$

$$g'(x) \cdot f'(g(x)) = 1$$

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{1+(f(g(x)))^2} = \frac{1}{1+x^2} \quad //$$

25) $x \geq 0 \quad , f(x) = \frac{\pi}{4} + \arctan\sqrt{e^{2x}-1} - \arccos e^{-x}$

$$f'(x) = \frac{\frac{2e^{2x}}{2\sqrt{e^{2x}-1}}}{1+e^{2x}-1} - \frac{e^{-x}}{\sqrt{1-e^{-2x}}} = \frac{1}{\sqrt{e^{2x}-1}} - \frac{e^{2x}}{e^{2x}\sqrt{e^{2x}-1}} = 0$$

$f'(x) = 0 \Rightarrow f(x)$ sabit bir fonksiyondur.

$x = 0 \Rightarrow f(x) = \frac{\pi}{4}$ //

26) $f\left(\frac{\pi}{2}\right) = 6 \quad , f'\left(\frac{\pi}{2}\right) = 3 \quad , g(x) = [f(x)]^{\sin x}$

$$g(x) = [f(x)]^{\sin x} \qquad g\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right)^{\sin\frac{\pi}{2}} = 6$$

$$\ln g(x) = \sin x \cdot \ln(f(x))$$

$$\frac{g'(x)}{g(x)} = \cos x \cdot \ln(f(x)) + \sin x \cdot \frac{f'(x)}{f(x)}$$

$$\frac{g'\left(\frac{\pi}{2}\right)}{g\left(\frac{\pi}{2}\right)} = \underbrace{\cos\frac{\pi}{2}}_{0} \cdot \ln f\left(\frac{\pi}{2}\right) + \underbrace{\sin\frac{\pi}{2}}_{1} \cdot \frac{f'\left(\frac{\pi}{2}\right)}{f\left(\frac{\pi}{2}\right)}$$

$$g'\left(\frac{\pi}{2}\right) = 6 \cdot \frac{3}{6} \Rightarrow g'\left(\frac{\pi}{2}\right) = 3$$ //

27) $x^2y^2 + \tan(x+y) - 1 = 0 \quad , P\left(\frac{\pi}{4}, 0\right)$

$$\underbrace{2xy^2}_{0} + \underbrace{2x^2yy'}_{0} + (1+y')\left(1+\tan^2(x+y)\right) = 0$$

$$\left(1+y'_{1p}\right)\underbrace{\left(1+\tan^2\frac{\pi}{4}\right)}_{2} = 0$$

$$1+y'_{1p} = 0 \Rightarrow \underline{y'_{1p} = -1}$$

T.D.D. : $y - y_0 = f'(x_0)(x - x_0)$

$$y - 0 = -1\left(x - \frac{\pi}{4}\right)$$

$$y = -x + \frac{\pi}{4} \quad //$$

28) $y\sin\left(\frac{1}{y}\right) + x\cos\left(\frac{1}{y}\right) = -2x \quad , \quad P\left(0, \frac{1}{\pi}\right)$

$$\underbrace{y'\cdot\sin\left(\frac{1}{y}\right)}_{0} - y'\cdot\frac{1}{y}\cos\left(\frac{1}{y}\right) + \cos\left(\frac{1}{y}\right) + x\frac{y'}{y^2}\underbrace{\sin\left(\frac{1}{y}\right)}_{0} + 2 = 0$$

$$y'_{1p}\pi - 1 + 2 = 0$$

$$y'_{1p}\pi = -1 \Rightarrow \underline{y'_{1p} = -\frac{1}{\pi}}$$

T.D.D. : $y - \frac{1}{\pi} = -\frac{1}{\pi}(x - 0)$

$$y = -\frac{1}{\pi}x + \frac{1}{\pi} \quad //$$

29) $2^{xy} + \ln\left[e + \arcsin\frac{y}{x}\right] = 1 + x \quad , \quad P(1,0)$

$$(\underset{0}{y} + \underset{1}{x}y')\cdot 2^{xy}\cdot\ln 2 + \frac{\dfrac{\frac{y'x-y}{x^2}}{\sqrt{1-\frac{y^2}{x^2}}}}{e + \arcsin\frac{y}{x}} = 1$$

$$y'_{1p}\cdot\ln 2 + \frac{y'_{1p}}{e} = 1$$

$$y'_{1p}\left(\ln 2 + \frac{1}{e}\right) = 1$$

$$y'_{1p} = \frac{e}{e\ln 2 + 1}$$

T.D.D. : $y - 0 = \frac{e}{e\ln 2 + 1}(x - 1)$

$$y = \frac{e}{e\ln 2 + 1}x - \frac{e}{e\ln 2 + 1} \quad //$$

30) $\cos(x-y) = x e^x \quad , P\left(0, \frac{\pi}{2}\right)$

$\cos(x-y) - xe^x = 0$

$-(1-y').\sin(x-y) - e^x - \underbrace{xe^x}_{0} = 0$

$(1-y'_{|p}).\sin\frac{\pi}{2} - 1 = 0$

$1 - y'_p = 1$

$\underline{y'_{|p} = 0}$ T.D.D: $y - \frac{\pi}{2} = 0\,(x-0)$

$\underline{y = \frac{\pi}{2}}$ //

31) $\sin(xy) = 1 - x^2 - y^2 + x^2y^3$

$\sin 0 = 1 - x^2 - 0$ $P_1\,(1,0)$, $P_2(-1,0)$

$1 - x^2 = 0 \Rightarrow \underline{x = \pm 1}$

$(y + xy')\cos(xy) = -2x - \underbrace{2yy'}_{0} + \underbrace{2xy^3}_{0} + \underbrace{3x^2y^2y'}_{0}$

$\underline{P_1(1,0)}$ $m_1 = y'_{|P_1} = -2$

$\underline{P_2(-1,0)}$ $m_2 = y'_{|P_2} = -2$ $m_1 = m_2$ old. dan teğetler birbirine paraleldir.

32) $y - 2\cos(\pi y - x) = 2x + 3 \quad , \quad P(0,1)$

$y' + 2(\pi y' - 1)\sin(\pi y - x) = 2$

$y'_{|p} + 2(\pi y'_{|p} - 1)\sin(\pi - 0) = 2$

$y'_{|p} = 2$

T.D.D : $y - 1 = 2\,(x-0)$

$y = 2x + 1$ //

33) $g(1) = h'(1) = g'(1) = h(1) = 2 \quad , f(x) = \left[g(x^2) \right]^{h(x)}$

$$f(x) = \left[g(x^2)\right]^{h(x)} \quad , \quad f(1) = (g(1))^{h(1)} = 2^2 = 4$$

$$\ln f(x) = h(x) \cdot \ln g(x^2)$$

$$\frac{f'(x)}{f(x)} = h'(x) \cdot \ln g(x^2) + h(x) \cdot \frac{2x\, g'(x^2)}{g(x^2)}$$

$$\frac{f'(1)}{f(1)} = h'(1) \cdot \ln g(1) + h(1) \; \frac{2\, g'(1)}{g(1)}$$

$$f'(1) = 4 \cdot \left(2 \cdot \ln 2 + \cancel{2} \cdot \frac{2 \cdot 2}{\cancel{2}}\right)$$

$$f'(1) = 8 \ln 2 + 16 \quad //$$

35) $f(x) = \dfrac{1+x}{\sqrt{1+x^2}} \quad , \quad f : (-\infty, 1] \longrightarrow \mathbb{R}$

$$f'(x) = \frac{\sqrt{1+x^2} - (1+x) \cdot \frac{1}{\cancel{2}} \cdot \cancel{2}x \, (1+x^2)^{-1/2}}{1+x^2}$$

$$f'(x) = \frac{1 + x^2 - x \cdot (1+x)}{(1+x^2)^{3/2}} = \frac{1 + \cancel{x^2} - x - \cancel{x^2}}{(1+x^2)^{3/2}} = \frac{1-x}{(1+x^2)^{3/2}} > 0$$

$f'(x) > 0 \Rightarrow f$, 1-1 dir. Dolayısıyla tersi mevcuttur.

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))}$$

$$\frac{1+x}{\sqrt{1+x^2}} = 0 \Rightarrow x = -1 \qquad f'(-1) = \frac{2}{2^{3/2}} = \frac{1}{\sqrt{2}}$$

$$(f^{-1})'(0) = \frac{1}{f'(-1)} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} \quad //$$

36) $f(x) = e^{\arctan x}$

$$f'(x) = \frac{1}{1+x^2} e^{\arctan x} > 0 \qquad e^{\arctan x} = e^{\pi/3}$$

$$\arctan x = \frac{\pi}{3} \Rightarrow x = \sqrt{3}$$

f, 1-1 dir ve tersi mevcuttur.

$$(f^{-1})'(e^{\pi/3}) = \frac{1}{f'(f^{-1}(e^{\pi/3}))} = \frac{1}{f'(\sqrt{3})}$$

$$= \frac{1}{1+3} e^{\arctan\sqrt{3}} = \frac{1}{4} e^{\pi/3} \quad //$$

37) $f(x) = 1 + x + \ln(1+x^2)$

$$f'(x) = 1 + \frac{2x}{1+x^2} = \frac{1+x^2+2x}{1+x^2} = \frac{(1+x)^2}{1+x^2} > 0 \Rightarrow 1\text{-}1 \Rightarrow \text{tersi var.}$$

$$1 + x + \ln(1+x^2) = 1$$

$$x + \ln(1+x^2) = 0$$

$$\Rightarrow x = 0$$

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}$$

$$f'(0) = \frac{1}{1} = 1$$

$$(f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{1} = 1 \quad //$$

38) $y = f(x)$, $x = 1$, $2x + y - 1 = 0$

$$2x + y - 1 = 0 \Rightarrow y = -2x + 1 \Rightarrow m_N = -2$$

$$m_N \cdot m_T = -1 \Rightarrow \boxed{m_T = \frac{1}{2} = f'(1)}$$

$$x = 1 \Rightarrow 2.1 + y - 1 = 0$$

$$y = -1$$

$$f(1) = -1$$

$$(f^{-1})'(-1) = \frac{1}{f'(1)} = \frac{1}{\frac{1}{2}} = 2 \quad //$$

39) $f(x) = 2 + \arctan x + e^{2x}$

$f'(x) = \frac{1}{1+x^2} + 2e^{2x} > 0 \Rightarrow 1\text{-}1 \Rightarrow$ tersi mevcut

$2 + \arctan x + e^{2x} = 3 \Rightarrow \underbrace{x=0}$, $f'(0) = 1+2 = 3$

$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(0)} = \frac{1}{3}$ //

40) $g(t) = t^3 + 7t + 21$, $g(-2) = -1$

$g'(t) = 3t^2 + 7$ $\quad g^{-1}(-1) = -2$

$g^{-1}(t) \approx L(t) = g^{-1}(-1) + (g^{-1})'(-1)\,(x+1)$

$(g^{-1})'(-1) = \frac{1}{g'(g^{-1}(-1))} = \frac{1}{g'(-2)} = \frac{1}{12+7} = \frac{1}{19}$

$g^{-1}(t) \approx L(t) = -2 + \frac{1}{19}(x+1)$ //

34) $f(x) = (\cos x^4)^{\arctan x^2}$, $f'(0)$

$\ln f(x) = \arctan x^2 \ln(\cos x^4)$, $f(0) = 1$

$\frac{f'(x)}{f(x)} = \frac{2x}{1+x^4} \cdot \ln(\cos x^4) + \arctan x^2 \cdot \frac{-4x^3 \cdot \sin x^4}{\cos x^4}$

$\frac{f'(0)}{f(0)} = 0 \Rightarrow f'(0) = 0$ //

43) $(1,001)^5 - 3(1,001)^{7/3} + 2$

$f(x) = x^5 - 3x^{7/3} + 2$, $a = 1$,

$f'(x) = 5x^4 - 7x^{\frac{4}{3}}$, $f(1) = 0$, $f'(1) = -2$

$dy \approx \Delta y \Rightarrow f'(a)\,dx \approx f(a+\Delta x) - f(a)$

$dx = \Delta x = 1,001 - 1 = 0,001$

$f'(1).(0,001) \approx f(1,001) - f(1)$

$f(1,001) \approx -0,002$ //

42) $g(1) = g'(1) = 4$ $f(x) = \dfrac{g(x^2)}{1+x^2}$, $f(1,25)$

$a = 1$, $f(1) = \dfrac{g(1)}{2} = 2$, $f'(x) = \dfrac{2x\,g'(x^2)(1+x^2) - 2x\,g(x^2)}{(1+x^2)^2}$

$f'(1) = \dfrac{4g'(1) - 2g(1)}{4}$

$f'(1) = \dfrac{16-8}{4} = 2$

$f(1,25) \approx L(1,25) = f(1) + f'(1)(1,25-1)$

$= 2 + 2(0,25) = 2,5$

$f(1,25) \approx 2,5$ //

41) $f(x) = (x^3+x-1)^7 + \arctan(x^4-1)$, $x=1$, $f(1,02)$

$f'(x) = 7(x^3+x-1)^6(3x^2+1) + \dfrac{4x^3}{1+(x^4-1)^2}$

$f(1) = 1$, $f'(1) = 28 + 4 = 32$

$f(1,02) \approx L(1,02) = f(1) + f'(1)(1,02-1)$

$= 1 + 32 . 0,02 = 1,64$

$f(1,02) \approx 1,64$ //

46) $\sqrt[3]{28}$

$f(x) = \sqrt[3]{x}$, $a = 27$, $f(x) \approx L(x) = f(a) + f'(a)(x-a)$

$f'(x) = \frac{1}{3} x^{-2/3}$ $f(27) = 3$, $f'(27) = \frac{1}{27}$

$\sqrt[3]{28} = f(28) \approx L(28) = f(27) + f'(27)(28-27)$

$= 3 + \frac{1}{27} = 3{,}037$

$\sqrt[3]{28} \approx 3{,}037$ //

45) $g(2) = -4$, $g'(x) = \sqrt{x^2+5}$, $g(2.05)$

$g(x) \approx L(x) = g(a) + g'(a)(x-a)$ $a = 2$, $g'(2) = \sqrt{9} = 3$

$g(2.05) \approx L(2.05) = g(2) + g'(2)(2.05-2)$

$= -4 + 3(0{,}05)$

$= -4 + 0{,}15 = -3{,}85$

$g(2.05) \approx -3{,}85$ //

44) $\sqrt[3]{(63)^2}$

$f(x) = \sqrt[3]{x^2}$, $a = 64$

$f'(x) = \frac{2}{3} x^{-1/3}$ $f(64) = 16$, $f'(64) = \frac{1}{6}$

$f(63) \approx L(63) = f(64) + f'(64)(63-64)$

$= 16 + \frac{1}{6}(-1) = 16 - 0{,}16 = 15{,}84$

$\sqrt[3]{(63)^2} \approx 15{,}84$ //

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47)

$$\lim_{x\to 0^+} x^{\frac{1}{\ln(e^x-1)}} = 0^\infty \text{ bl.}$$

$$y = x^{\frac{1}{\ln(e^x-1)}}$$

$$\ln y = \frac{1}{\ln(e^x-1)} \ln x \Rightarrow \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln x}{\ln(e^x-1)} \quad \left(\frac{\infty}{\infty} \text{ bl.}\right)$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{e^x}{e^x-1}} = \lim_{x\to 0^+} \frac{e^x-1}{xe^x} = \frac{0}{0} \text{ bl.}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{e^x}{e^x+xe^x} = 1$$

$$\lim_{x\to 0^+} \ln y = \ln \lim_{x\to 0^+} y = 1$$

$$\Rightarrow \lim_{x\to 0^+} x^{\frac{1}{\ln(e^x-1)}} = e^1 = e \;//$$

48)

$$\lim_{x\to\infty} \left[1+2^x+3^x\right]^{1/x} = \infty^0 \text{ bl.}$$

$$\lim_{x\to\infty} \left[1+2^x+3^x\right]^{1/x} = \lim_{x\to\infty} \left[3^x\left(\frac{1}{3^x}+\left(\frac{2}{3}\right)^x+1\right)\right]^{1/x}$$

$$= \lim_{x\to\infty} 3.\left[\underbrace{\frac{1}{3^x}}_{0}+\underbrace{\left(\frac{2}{3}\right)^x}_{0}+1\right]^{1/x}$$

$$= 3.1 = 3 \;//$$

49)

$$\lim_{x\to 4}\left[\frac{4}{\pi}\arctan\left(\frac{x}{4}\right)\right]^{\tan\left(\frac{\pi x}{8}\right)} = 1^{\infty} \text{ bl.}$$

$$y=\left[\frac{4}{\pi}\arctan\left(\frac{x}{4}\right)\right]^{\tan\left(\frac{\pi x}{8}\right)}$$

$$\ln y = \tan\left(\frac{\pi x}{8}\right)\cdot \ln\left(\frac{4}{\pi}\arctan\frac{x}{4}\right)$$

$$\lim_{x\to 4}\ln y = \lim_{x\to 4}\frac{\ln\left(\frac{4}{\pi}\cdot\arctan\frac{x}{4}\right)}{\cot\left(\frac{\pi x}{8}\right)} = \frac{0}{0} \text{ bl}$$

$$\overset{L'H}{=} \lim_{x\to 4}\frac{\cancel{\frac{4}{\pi}}\cdot\frac{1/4}{1+\frac{x^2}{16}}}{\cancel{\frac{4}{\pi}}\cdot\arctan\frac{x}{4}}\cdot\frac{1}{-\frac{\pi}{8}\cdot\underbrace{\operatorname{cosec}^2\left(\frac{\pi x}{8}\right)}_{\frac{1}{\sin^2\left(\frac{\pi x}{8}\right)}}}$$

$$= \lim_{x\to 4}\frac{-8\sin^2\left(\frac{\pi x}{8}\right)}{4\pi\left(1+\frac{x^2}{16}\right)\cdot\arctan\frac{x}{4}} = \frac{-\cancel{8}}{\cancel{4}\pi\cdot\cancel{2}\cdot\frac{\pi}{\cancel{4}}} = -\frac{4}{\pi^2}$$

$$\lim_{x\to 4}\ln y = \ln\lim_{x\to 4} y = -\frac{4}{\pi^2}$$

$$\lim_{x\to 4} y = e^{-\frac{4}{\pi^2}} \,//$$

50)

$$\lim_{x\to0^+}\left(2-e^{\sqrt{x}}\right)^{\frac{1}{x}} \quad (1^\infty \text{ bl.})$$

$$y=\left(2-e^{\sqrt{x}}\right)^{\frac{1}{x}}$$

$$\ln y=\frac{1}{x}\ln\left(2-e^{\sqrt{x}}\right)$$

$$\lim_{x\to0^+}\ln y=\lim_{x\to0^+}\frac{\ln\left(2-e^{\sqrt{x}}\right)}{x}=\left(\frac{0}{0}\text{ bl.}\right)$$

$$\overset{L'H}{=}\lim_{x\to0^+}\frac{-\frac{1}{2\sqrt{x}}\cdot e^{\sqrt{x}}}{2-e^{\sqrt{x}}}=-\infty$$

$$\lim_{x\to0^+}\ln y=\ln\left(\lim_{x\to0^+}y\right)=-\infty$$

$$\Rightarrow \lim_{x\to0^+}\left(2-e^{\sqrt{x}}\right)^{\frac{1}{x}}=e^{-\infty}=0 \,//$$

51)

$$\lim_{x\to0^+}(\cot x)^{1/\ln x} \quad (\infty^0 \text{ bl})$$

$$y=(\cot x)^{1/\ln x}$$

$$\ln y=\frac{1}{\ln x}\cdot\ln\cot x$$

$$\lim_{x\to0^+}\ln y=\lim_{x\to0^+}\frac{\ln\cot x}{\ln x}\overset{\frac{\infty}{\infty}\text{ bl}}{\underset{L'H}{=}}\lim_{x\to0^+}\frac{\dfrac{-1/\sin^2x}{\frac{\cos x}{\sin x}}}{\frac{1}{x}}=\lim_{x\to0^+}\frac{-x\cdot\cancel{\sin x}}{\sin^{\cancel{2}}x\cdot\cos x}$$

$$=\lim_{x\to0^+}\frac{-x}{\sin x\cdot\cos x}=\lim_{x\to0^+}\underbrace{\frac{x}{\sin x}}_{1}\cdot\frac{-1}{\cos x}=-1$$

$$\lim_{x\to0^+}\ln y=\ln\left(\lim_{x\to0^+}y\right)=-1$$

$$\Rightarrow \lim_{x\to0^+}(\cot x)^{1/\ln x}=e^{-1}=\frac{1}{e}\,//$$

52)

$$\lim_{x\to\infty} \frac{e^{\arctan x}-x}{\ln(1+x^2)+x} \left(\frac{\infty}{\infty} \text{ bl.}\right) \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+x^2}\cdot e^{\arctan x}-1}{\frac{2x}{1+x^2}+1}$$

$$= \frac{0.\, e^{\pi/2}-1}{0+1} = -1 \quad //$$

53)

$$\lim_{x\to 0^+} \left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}} \quad (1^\infty \text{ bl.})$$

$$y = \left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}}$$

$$\ln y = \frac{2}{x} \ln\left(\frac{a^x+b^x}{2}\right)$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln\left(\frac{a^x+b^x}{2}\right)}{\frac{x}{2}} \left(\frac{0}{0} \text{ bl.}\right) \overset{L'H}{=} \lim_{x\to 0^+} \frac{2\cdot \frac{a^x \ln a + b^x \ln b}{2(a^x+b^x)}}{\frac{1}{2}}$$

$$= \lim_{x\to 0^+} \frac{2(a^x \ln a + b^x \ln b)}{a^x+b^x} = \ln a + \ln b$$

$$\lim_{x\to 0^+} \ln y = \ln\left(\lim_{x\to 0^+} y\right) = \ln(a.b)$$

$$\Rightarrow \lim_{x\to 0^+} \left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}} = a.b \quad //$$

54)

$$\lim_{x\to 1^+} \left[\frac{4}{\pi} \arctan x\right]^{\frac{3}{x^2+2x-3}} \qquad (1^\infty \text{ bl.})$$

$$y = \left[\frac{4}{\pi} \arctan x\right]^{\frac{3}{x^2+2x-3}}$$

$$\ln y = \frac{3}{(x^2+2x-3)} \ln\left(\frac{4}{\pi} \arctan x\right)$$

$$\lim_{x\to 1^+} \ln y = \lim_{x\to 1^+} \frac{3 . \ln\left(\frac{4}{\pi} \arctan x\right)}{(x^2+2x-3)} \qquad \left(\frac{0}{0} \text{ bl.}\right)$$

$$\overset{L'H}{=} \lim_{x\to 1^+} \frac{3 . \dfrac{\cancel{\frac{4}{\pi}} \cdot \frac{1}{1+x^2}}{\cancel{\frac{4}{\pi}} \arctan x}}{2x+2} = \lim_{x\to 1^+} \frac{3}{(2x+2)(1+x^2) \arctan x}$$

$$\lim_{x\to 1^+} \ln y = \frac{3}{\cancel{4} . 2 . \frac{\pi}{\cancel{4}}} = \frac{3}{2\pi}$$

$$\ln\left(\lim_{x\to 1^+} y\right) = \frac{3}{2\pi} \Rightarrow \lim_{x\to 1^+} \left[\frac{4}{\pi} \arctan x\right]^{\frac{3}{x^2+2x-3}} = e^{\frac{3}{2\pi}} \;//$$

55)

$$\lim_{x\to 0^+} (1+\sin 2x)^{\frac{1}{x}} \qquad (1^\infty \text{ bl.})$$

$$y = (1+\sin 2x)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \ln(1+2x)$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(1+2x)}{x} \qquad \left(\frac{0}{0} \text{ bl.}\right)$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{2}{1+2x}}{1} = 2$$

$$\lim_{x\to 0^+} \ln y = 2$$

$$\ln\left(\lim_{x\to 0^+} y\right) = 2 \Rightarrow \lim_{x\to 0^+} (1+\sin 2x)^{\frac{1}{x}} = e^2 \;//$$