Olasılıksal Robotik

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Eşanlı Konum Belirleme ve Haritalama - SLAM

- Full SLAM

$$p\left(x_{1:t}, m \middle| z_{1:t}, u_{1:t}\right)$$

- Online SLAM

$$p\left(x_t, m \middle| z_{1:t}, u_{1:t}\right)$$

Eşanlı Konum Belirleme ve Haritalama - SLAM

- **Full SLAM with correspondence variable**

$$p\left(x_{1:t}, m, c_t \,\middle|\, z_{1:t}, u_{1:t}\right)$$

- **Online SLAM with correspondence variable**

$$p\left(x_t, m, c_t \,\middle|\, z_{1:t}, u_{1:t}\right)$$

EKF SLAM

- **EKF SLAM =**
  - EKF tabanlı hareket modeli
  - EKF tabanlı sensör modeli
  - Online SLAM
  - Ençok Olabilirlik veri ilişkilendirme(Maximum Likelihood data association)

EKF SLAM

- **EKF SLAM =**
  - Özellik tabanlı haritalar (Feature based map)
  - Noktasal mihenktaşları (point landmarks)
  - Toplam landmark sayısı < 1000
  - Hareket ve sensör modeli için Gauss gürültüsü varsayımı

EKF SLAM – Bilinen ölçüm mihenktaşı ilişkisi
EKF – Known Correspondence

- EKF konum belirleme ile büyük benzerlik taşır
- Durum uzayına (y_t), robot konumuna (x_t) ek olarak landmarkların konum bilgisi (m) de eklenir

$$y_t = \begin{pmatrix} x_t \\ m \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

$$\begin{pmatrix} m_1 \\ m_i \\ \cdots \\ m_N \end{pmatrix}$$

$$\begin{pmatrix} m_{i,x} \\ m_{i,y} \\ m_{i,s} \end{pmatrix}$$

$$y_t \Big|_{3N+3\times 1}$$

EKF SLAM - İlklendirme

$$\mu_0 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{(3N+3)\times 1}$$

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \infty & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \infty \end{pmatrix}_{(3N+3)\times(3N+3)}$$

EKF SLAM – Hareket Modeli

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin(\theta) + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta_t) \\ \frac{v_t}{\omega_t}\cos(\theta) - \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta_t) \\ \omega_t \Delta_t + \gamma \Delta_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

EKF SLAM – Hareket Modeli

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \end{pmatrix}_{(3)\times(3N+3)}$$

$$y_t = y_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t}\sin(\theta) + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta_t) \\ \frac{v_t}{\omega_t}\cos(\theta) - \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta_t) \\ \omega_t \Delta_t + \gamma \Delta_t \end{pmatrix}$$

$$y_t = \underbrace{y_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t}\sin(\theta) + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta_t) \\ \frac{v_t}{\omega_t}\cos(\theta) - \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta_t) \\ \omega_t \Delta_t + \gamma \Delta_t \end{pmatrix}}_{g(u_t, y_{t-1})} + \mathcal{N}\left(0, F_x^T R_t F_x\right)$$

EKF SLAM – Hareket Modeli

$$g\left(u_t, y_{t-1}\right) \approx g\left(\mu_{u_t}, \mu_{t-1}\right) + G_t\left(y_{t-1}, \mu_{t-1}\right)$$

$$G_t = \left.\frac{\partial g\left(u_t, y_{t-1}\right)}{\partial y_{t-1}}\right|_{u_t, y_{t-1}=\mu_{t-1}}$$

$$G_t = g'\left(u_t, \mu_{t-1}\right)$$

$$G_t = I + F_x^T g_t F_x$$

$$g_t = \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t}\cos\left(\mu_{t-1,\theta}\right) + \frac{v_t}{\omega_t}\cos\left(\mu_{t-1,\theta} + \omega_t \Delta_t\right) \\ 0 & 0 & -\frac{v_t}{\omega_t}\sin\left(\mu_{t-1,\theta}\right) + \frac{v_t}{\omega_t}\sin\left(\mu_{t-1,\theta} + \omega_t \Delta_t\right) \\ 0 & 0 & 0 \end{pmatrix}$$

EKF SLAM – Hareket Modeli

$$\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\left(\mu_{t-1,\theta}\right) + \frac{v_t}{\omega_t}\sin\left(\mu_{t-1,\theta} + \omega_t\Delta_t\right) \\ \frac{v_t}{\omega_t}\cos\left(\mu_{t-1,\theta}\right) - \frac{v_t}{\omega_t}\cos\left(\mu_{t-1,\theta} + \omega_t\Delta_t\right) \\ \omega_t\Delta_t \end{pmatrix}$$

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$$

EKF SLAM – Hareket Modeli

Algorithm EKF_SLAM_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$)**:**

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$$

$$\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

$$G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$$

$$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + F_x^T \, R_t \, F_x$$

EKF SLAM – Sensör Modeli

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

for all observed features $z_t^i = (r_t^i \;\; \phi_t^i \;\; s_t^i)^T$ *do*

$$j = c_t^i$$

if landmark j *never seen before*

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \; \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \; \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$

endif

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

$$q = \delta^T \delta$$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \mathrm{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$$

EKF SLAM – Sensör Modeli

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0\cdots0 & 0 & 0 & 0 & 0\cdots0 \\ 0 & 1 & 0 & 0\cdots0 & 0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 1 & 0\cdots0 & 0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 0 & 0\cdots0 & 1 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 0 & 0\cdots0 & 0 & 1 & 0 & 0\cdots0 \\ 0 & 0 & 0 & \underbrace{0\cdots0}_{3j-3} & 0 & 0 & 1 & \underbrace{0\cdots0}_{3N-3j} \end{pmatrix}$$

$$H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$$

$$K_t^i = \bar{\Sigma}_t \; H_t^{iT} (H_t^i \; \bar{\Sigma}_t \; H_t^{iT} + Q_t)^{-1}$$

$$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$$

$$\bar{\Sigma}_t = (I - K_t^i \; H_t^i) \; \bar{\Sigma}_t$$

endfor

$\mu_t = \bar{\mu}_t$

$\Sigma_t = \bar{\Sigma}_t$

return μ_t, Σ_t