

ROB599_HW5

Analyzing the dynamics of knee actuator

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Clean Up

```
close all
clear
clc
```

Problem 1A: Determine the system parameters

```
% Define the system variables
syms s
syms I b
syms G_s

plant_tf = 1/(I*s + b);

% Evaluate at DC Gain
G_s_0 = 10^(7.6/20);
plant = subs(plant_tf, s, 0);
equation = abs(plant) == G_s_0;

b_eval = eval(solve(equation, b))

% Evaluate at Cutoff Frequency
G_s_omega = 10^(4.6/20);
wc = 6;

equation = sqrt(1/((I*wc)^2 + b_eval^2)) == G_s_omega;
I_eval = abs(eval(solve(equation, I)));
I_eval = I_eval(1) % two solutions, pick the positive one
```

```
b_eval =

    0.4169
```

```
I_eval =

    0.0693
```

Problem 1B: Bode Plot of the Actuator

```
% Defining the Transfer Function of the Actuator
TF = tf([1], [I_eval b_eval]);

% Plot the Bode plot
figure('Position', [100, 100, 1200, 800]);

% Create Bode plot
opts = bodeoptions;
opts.Grid = 'on';
opts.FreqUnits = 'rad/s';
opts.PhaseWrapping = 'off';
opts.Title.String = ''; % Remove "Bode Diagram" text
opts.XLabel.String = 'Frequency (rad/s)'; % Set custom x-label
bodeplot(TF, opts)

% Get the axes handles
h = gcf;
axesObjs = findobj(h, 'Type', 'axes');

% Set the properties for the Bode plot lines
bodePlotLines = findobj(h, 'Type', 'line');
set(bodePlotLines, 'LineWidth', 2, 'Color', 'blue');
```

```

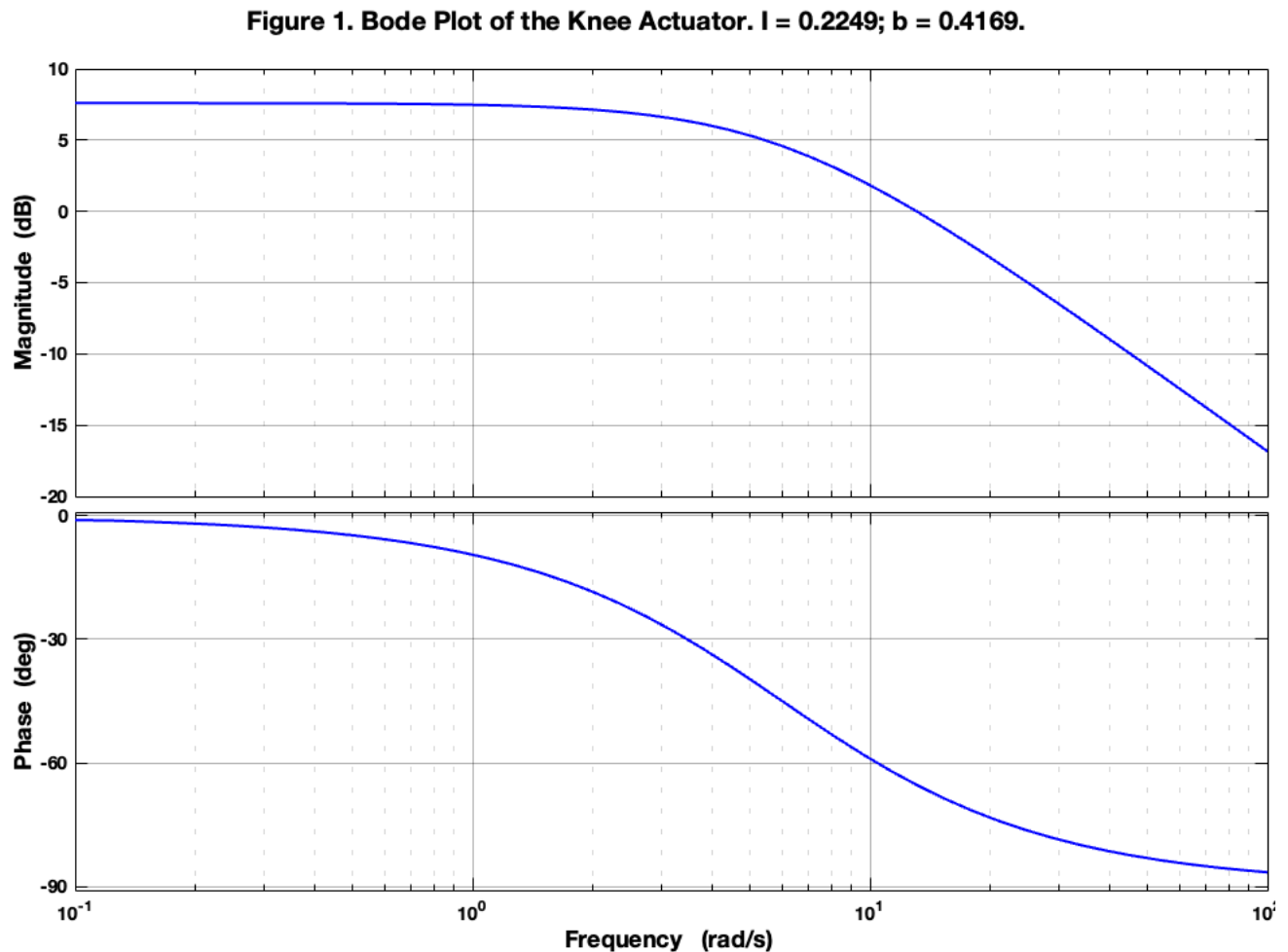
% Set the properties for the axes
for i = 1:length(axes0bjs)
    set(axes0bjs(i), 'FontSize', 14, 'FontWeight', 'bold');
    grid(axes0bjs(i), 'on');
    set(axes0bjs(i), 'GridAlpha', 0.3);
    set(axes0bjs(i), 'MinorGridAlpha', 0.15);
    set(axes0bjs(i), 'XScale', 'log');
end

% Hide all default titles and text
delete(findobj(h, 'Type', 'text'));

% Set the title and labels
sgtitle('Figure 1. Bode Plot of the Knee Actuator. I = 0.2249; b = 0.4169.', 'FontSize', 20, 'FontWeight', 'bold');
xlabel(axes0bjs(2), 'Frequency ', 'FontSize', 18, 'FontWeight', 'bold'); % Only lower plot
set(get(axes0bjs(1), 'XLabel'), 'String', ''); % Remove x-label from upper plot
ylabel(axes0bjs(2), 'Magnitude ', 'FontSize', 18, 'FontWeight', 'bold');
ylabel(axes0bjs(1), 'Phase ', 'FontSize', 18, 'FontWeight', 'bold');

print(gcf, 'Figure 1. Bode Plot of the Knee Actuator.png', '-dpng', '-r300');

```



Problem 2A: Determine the Transfer Function of the Plant from Torque to Angle

```
plant = tf([1], [I_eval b_eval 0])
```

```
plant =
```

$$\frac{1}{0.06931 \text{ s}^2 + 0.4169 \text{ s}}$$

Continuous-time transfer function.

Problem 2B: Plot the Root Locus & Step Response

```

% kp/kd = 10
controller = tf([1 10], [1])
L = controller * plant

% Plot the Root Locus
figure('Color', 'white', 'Position', [100, 100, 1200, 800]);
rlocus(L)
axis([-25 5 -9, 9]);
set(gca, 'FontSize', 14);
set(findall(gca, 'Type', 'Line'), 'LineWidth', 3);
sgrid([0.2, 0.4, 0.6, 0.8], [2, 4, 6, 8, 10, 15, 20]);
title('Figure 2. Knee Actuator. Root Locus, Kd Range. Kp/Kd = 10.', 'FontWeight', 'bold', 'FontSize', 24);
xlabel('Real Axis', 'FontWeight', 'bold', 'FontSize', 18);
ylabel('Imaginary Axis', 'FontWeight', 'bold', 'FontSize', 18);
grid on;
box on;

print(gcf, 'Figure 2. Root Locus of Knee Actuator Controller.png', '-dpng', '-r300');

% Analyze the Step Response
% Define the closed loop system
kd = 0.69; % from the root locus
kp = 10 * kd;
controller = tf([kd kp], [1]);
CL = feedback(controller * plant, 1)

% Plot the Step Response
figure('Color', 'white', 'Position', [100, 100, 1200, 800]);
step(CL)
set(gca, 'FontSize', 14);
set(findall(gca, 'Type', 'Line'), 'LineWidth', 3);
title('Figure 3. Knee Actuator. Step Response, Kd = 0.69, Kp = 6.9.', 'FontWeight', 'bold', 'FontSize', 24);
xlabel('Time', 'FontWeight', 'bold', 'FontSize', 18);
ylabel('Angle (rad)', 'FontWeight', 'bold', 'FontSize', 18);
grid on;
box on;

print(gcf, 'Figure 3. Step Response of Knee Actuator Controller.png', '-dpng', '-r300');

```

controller =

$$s + 10$$

Continuous-time transfer function.

L =

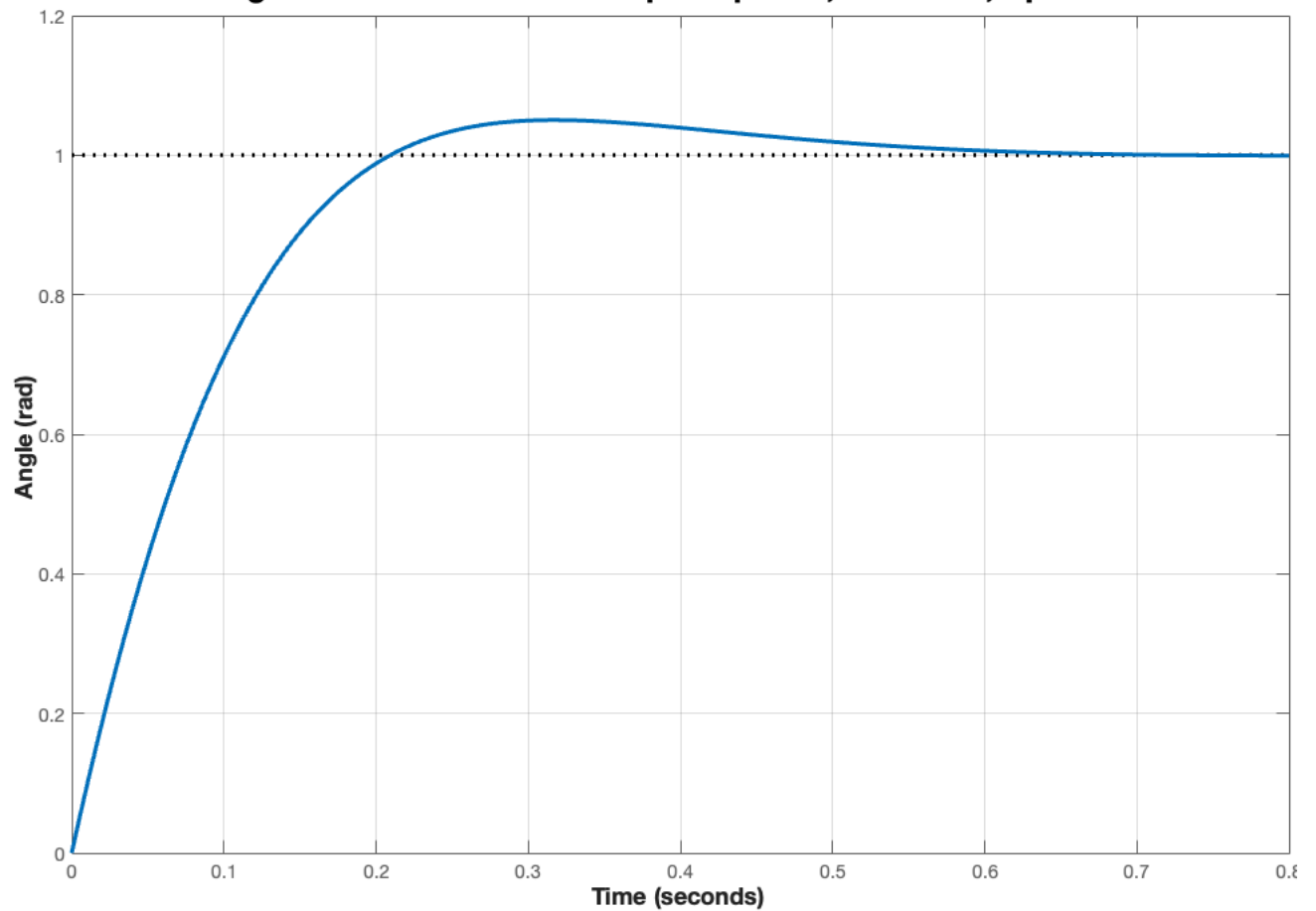
$$\frac{s + 10}{0.06931 s^2 + 0.4169 s}$$

Continuous-time transfer function.

CL =

$$\frac{0.69 s + 6.9}{0.06931 s^2 + 1.107 s + 6.9}$$

Continuous-time transfer function.

Figure 3. Knee Actuator. Step Response, $K_d = 0.69$, $K_p = 6.9$.**Problem 2C: Design Lead Compensator**

```
% Define the desired characteristics
PM_d = 45;
e_SS = 0.05;

% Evaluated Quantities from the Bode Plot of the Plant with Gain K
K = 9; % K>8.337 ensures e_ss < 0.05

% Plot the Bode plot of the plant with gain K
figure('Position', [100, 100, 1200, 800]);
% Create Bode plot
opts = bodeoptions;
opts.Grid = 'on';
opts.FreqUnits = 'rad/s';
opts.PhaseWrapping = 'off';
opts.Title.String = ''; % Remove "Bode Diagram" text
opts.XLabel.String = 'Frequency (rad/s)'; % Set custom x-label
margin(K * plant, opts)

% Get the axes handles
h = gcf;
axesObjs = findobj(h, 'Type', 'axes');

% Set the properties for the Bode plot lines
bodePlotLines = findobj(h, 'Type', 'line');
set(bodePlotLines, 'LineWidth', 2, 'Color', 'blue');

% Set the properties for the axes
for i = 1:length(axesObjs)
    set(axesObjs(i), 'FontSize', 14, 'FontWeight', 'bold');
    grid(axesObjs(i), 'on');
    set(axesObjs(i), 'GridAlpha', 0.3);
    set(axesObjs(i), 'MinorGridAlpha', 0.15);
    set(axesObjs(i), 'XScale', 'log');
end

% Hide all default titles and text
```

```

delete(findobj(h, 'Type', 'text'));

% Set the title and labels
sgtitle('Figure 4. Bode Plot of the Knee Actuator with Gain K = 9.', 'FontSize', 20, 'FontWeight', 'bold');
xlabel(axesObjs(2), 'Frequency ', 'FontSize', 18, 'FontWeight', 'bold'); % Only lower plot
set(get(axesObjs(1), 'XLabel'), 'String', ''); % Remove x-label from upper plot
ylabel(axesObjs(2), 'Magnitude ', 'FontSize', 18, 'FontWeight', 'bold');
ylabel(axesObjs(1), 'Phase ', 'FontSize', 18, 'FontWeight', 'bold');

print(gcf, 'Figure 4. Bode Plot of the Knee Actuator with Gain K = 9.png', '-dpng', '-r300');

% Noting from Bode Plot
PM_c = 29.5;
wm = 13.0;

% Computing corresponding zc and pc
phi_m = PM_d - PM_c + 5;
a = (1 - sind(phi_m)) / (1 + sind(phi_m));
CrossGain = 20*log10(sqrt(a));

zc = - wm * sqrt(a);
pc = zc / a;

C_s = tf(K*pc/zc*[1 -zc], [1 -pc])

% Plot the Bode Plot of the System with Lead Compensator

% Create Bode plot
figure('Position', [100, 100, 1200, 800]);
opts = bodeoptions;
opts.Grid = 'on';
opts.FreqUnits = 'rad/s';
opts.PhaseWrapping = 'off';
opts.Title.String = ''; % Remove "Bode Diagram" text
opts.XLabel.String = 'Frequency (rad/s)'; % Set custom x-label
margin(C_s * plant, opts)

% Get the axes handles
h = gcf;
axesObjs = findobj(h, 'Type', 'axes');

% Set the properties for the Bode plot lines
bodePlotLines = findobj(h, 'Type', 'line');
set(bodePlotLines, 'LineWidth', 2, 'Color', 'blue');

% Set the properties for the axes
for i = 1:length(axesObjs)
    set(axesObjs(i), 'FontSize', 14, 'FontWeight', 'bold');
    grid(axesObjs(i), 'on');
    set(axesObjs(i), 'GridAlpha', 0.3);
    set(axesObjs(i), 'MinorGridAlpha', 0.15);
    set(axesObjs(i), 'XScale', 'log');
end

% Hide all default titles and text
delete(findobj(h, 'Type', 'text'));

% Set the title and labels
controllerTF = "$$C(s) = " + num2str(K*pc/zc) + "\frac{s + " + num2str(-zc) + "}{s + " + num2str(-pc) + "}$";
sgtitle('Figure 5. Bode Plot of the Knee Actuator with Lead Compensator ' + controllerTF, 'FontSize', 20, 'FontWeight', 'bold', 'Interpreter', 'latex');
xlabel(axesObjs(2), 'Frequency ', 'FontSize', 18, 'FontWeight', 'bold');
set(get(axesObjs(1), 'XLabel'), 'String', '');
ylabel(axesObjs(2), 'Magnitude ', 'FontSize', 18, 'FontWeight', 'bold');
ylabel(axesObjs(1), 'Phase ', 'FontSize', 18, 'FontWeight', 'bold');

print(gcf, 'Figure 5. Bode Plot of the Knee Actuator with Lead Compensator.png', '-dpng', '-r300');

```

C_s =

$$\frac{18.7 s + 168.7}{s + 18.74}$$

Continuous-time transfer function.

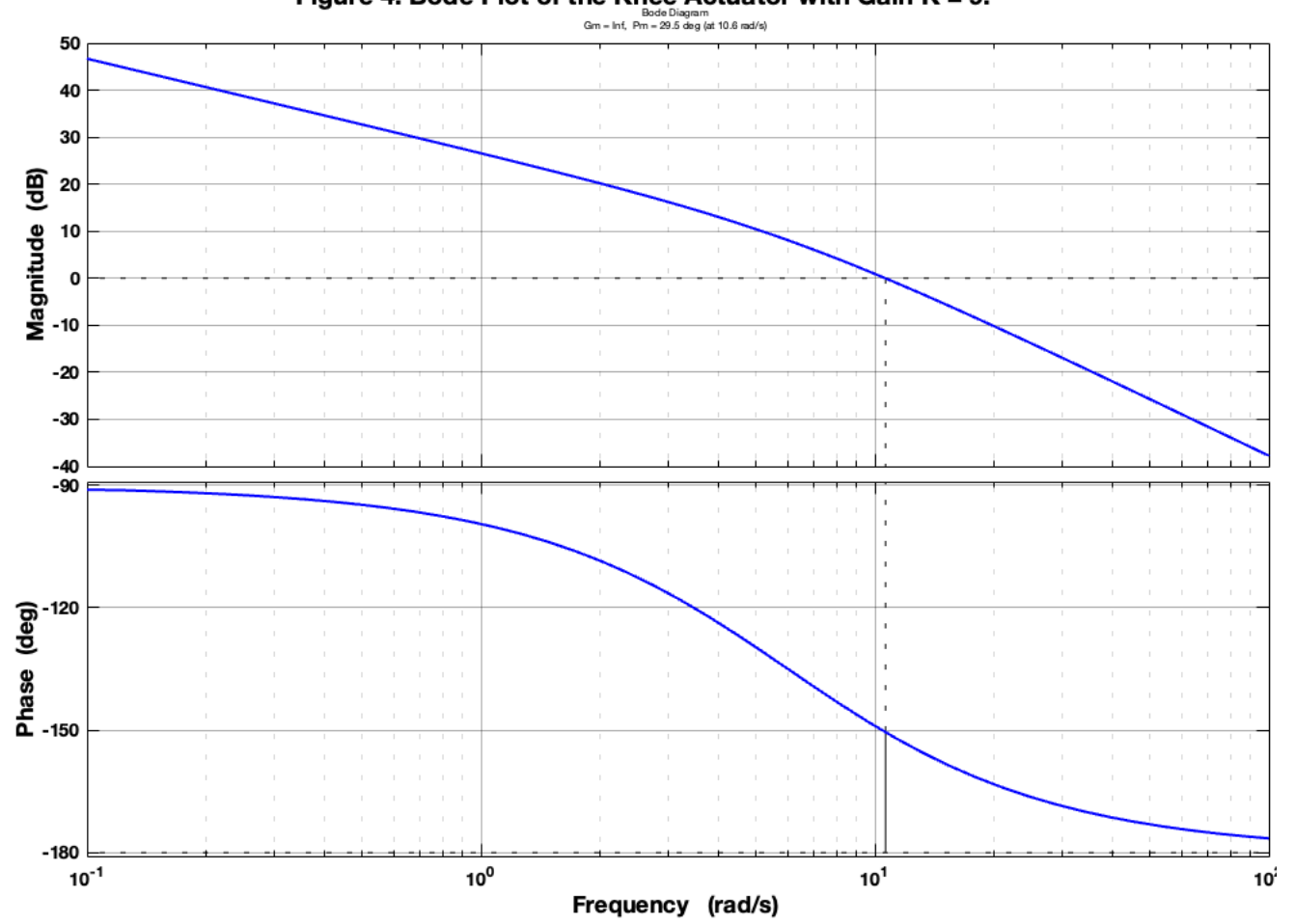
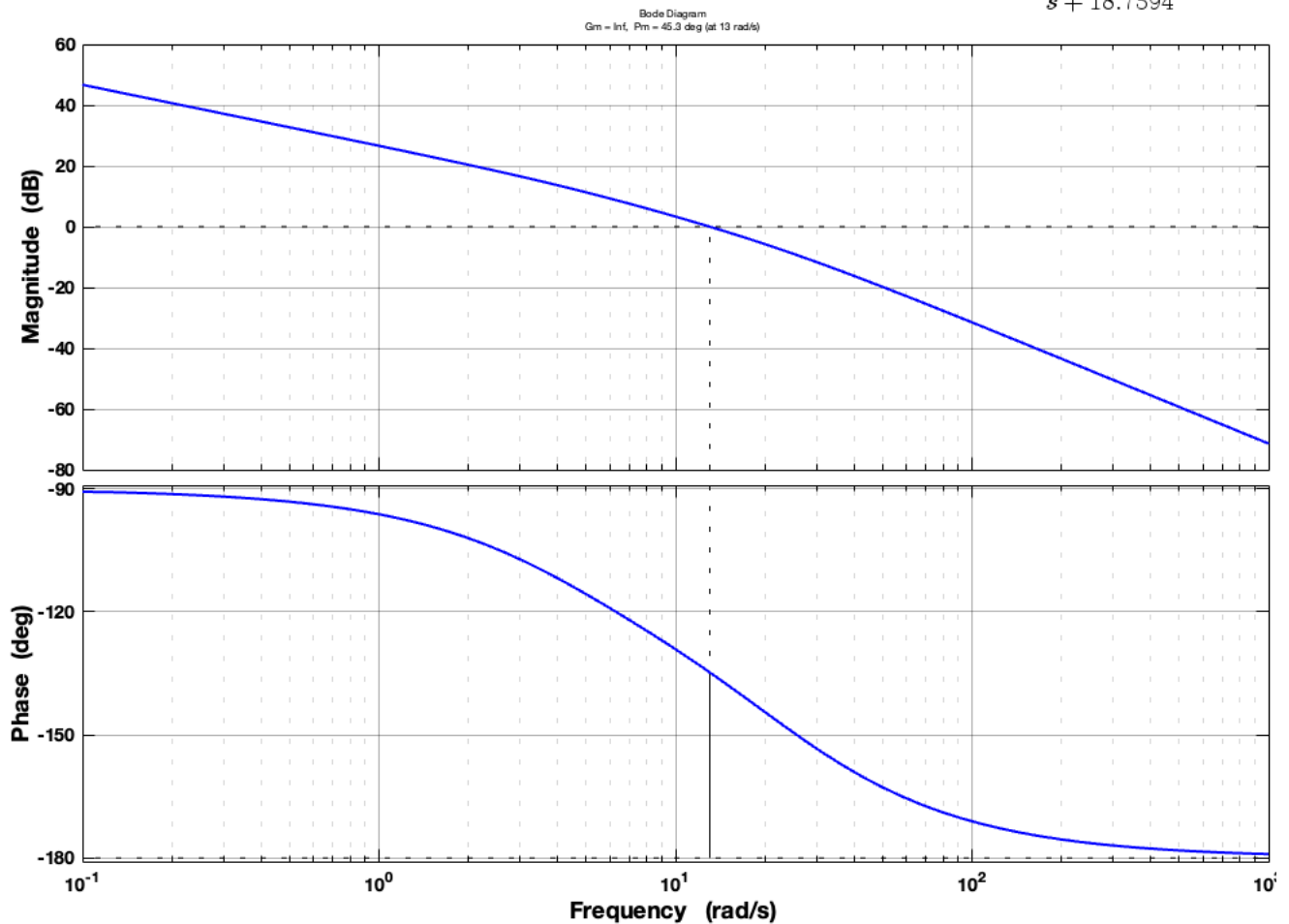
Figure 4. Bode Plot of the Knee Actuator with Gain K = 9.

Figure 5. Bode Plot of the Knee Actuator with Lead Compensator $C(s) = 18.7011 \frac{s + 9.0184}{s + 18.7394}$



Problem 2D: Create a Simulink Model and Simulate Ramp Input Signal

```
% Recall the transfer function of the system
C_s = tf(K*pc/zc*[1 -zc], [1 -pc]);
P_s = plant;

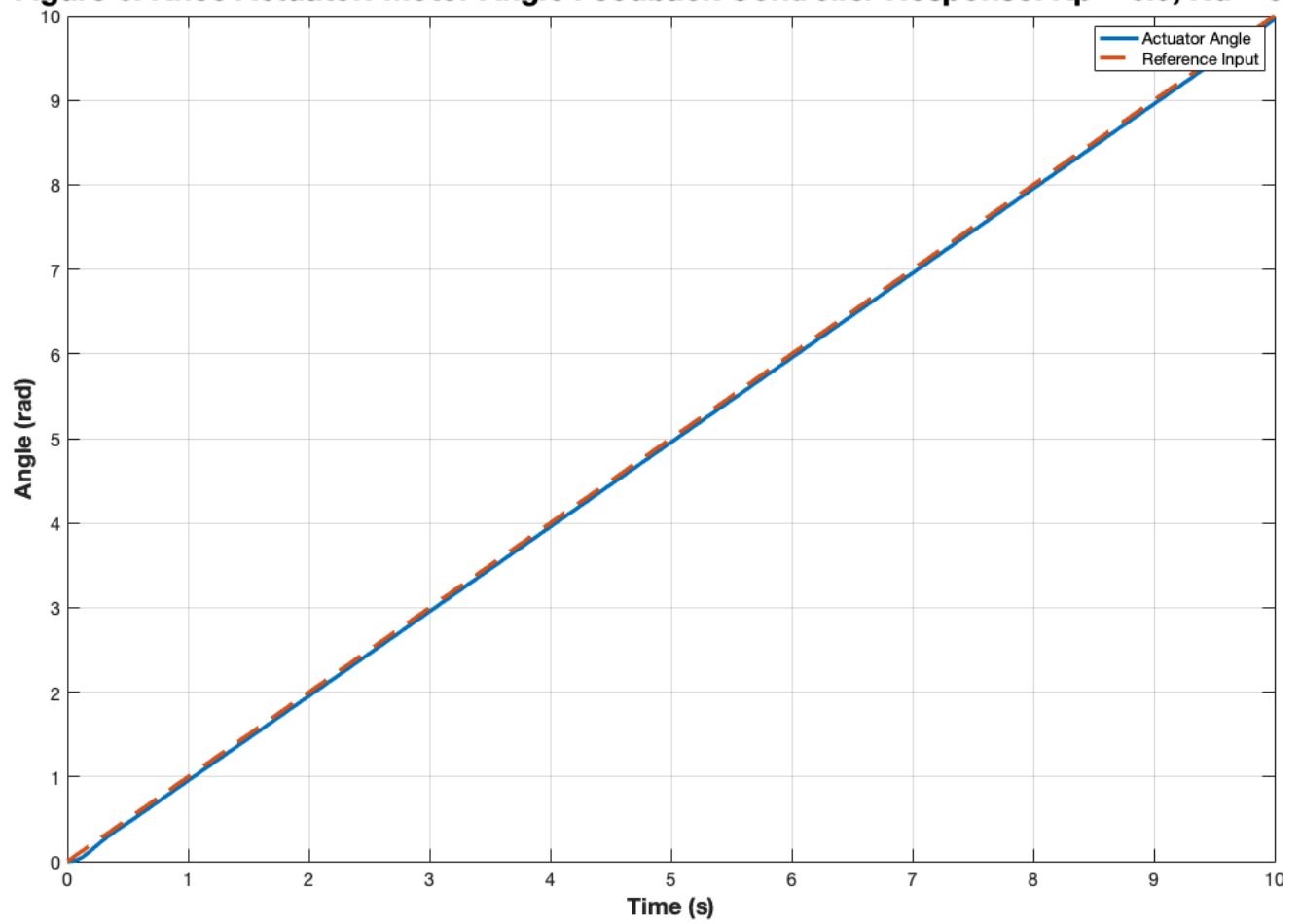
% Run the Simulink Model
simOut = sim('ROB599_HW5_P2D', 'SimulationMode', 'normal', 'StopTime', '10');

% Get the Step Response Data (Motor Angle Feedback)
refSignalTime = simOut.ref_signal.Time;
refInput = simOut.ref_signal.Data;

% Get the Step Response (Load Angle Feedback)
simResponseTime = simOut.theta.Time;
simResponse = simOut.theta.Data;

% Create Figure for Motor Angle Feedback Step Response
figure('Color', 'white', 'Position', [100, 100, 1200, 800]);
plot(simResponseTime, simResponse, 'LineWidth', 3);
hold on;
set(gca, 'FontSize', 14);
plot(refSignalTime, refInput, '--', 'LineWidth', 3);
title('Figure 6. Knee Actuator. Motor Angle Feedback Controller Response. Kp = 6.9, Kd = 0.69', 'FontWeight', 'bold', 'FontSize', 24);
xlabel('Time (s)', 'FontWeight', 'bold', 'FontSize', 18);
ylabel('Angle (rad)', 'FontWeight', 'bold', 'FontSize', 18);
legend('Actuator Angle', 'Reference Input');
grid on;
box on;

% Save motor angle feedback plot
print(gcf, 'Figure 6. Knee Actuator. Motor Angle Feedback Controller Response.png', '-dpng', '-r300');
```


Figure 6. Knee Actuator. Motor Angle Feedback Controller Response. $K_p = 6.9$, $K_d = 0$ 

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