

Additional Results: Empirical copula displacement interpolation: a direct optimization approach with applications

Oliver Grothe

Karlsruher Institut für Technologie, Institut für Operations Research

and

Anika Kaplan ^{*}

Karlsruher Institut für Technologie, Institut für Operations Research

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1 Results: Location Polynomials and test statistic

The exhaustive results for ECDI for parameterized copulas of the chosen types of parametric copulas with fixed parameters are displayed in Figure 1 up to Figure 12. For each type, the ECDI-procedure was conducted 50 times. On the left of each figure the location polynomials with degree $K = 2, 3, 4$ for the first of fifty runs is displayed. On the right the test statistics for both margins of all 50 repetitions are summarized in a boxplot. Next, the behavior of Kendall's Tau in the interpolation process is displayed. On the left there is the actual Kendall's Tau values, which are estimated based on the constructed samples. On the right the corresponding changes in Kendall's Tau behavior, i.e. an estimate for the derivatives of the left sides functions. As a comparison the Kendall's Tau of the interpolation between the parametric copulas under the assumption that the copula parameter is the external parameter is also displayed. The largest deviations from equal-speed movement are necessary for Gaussian copulas interpolating between parameters -0.4 and 0.4 . While the theoretical values for Kendall's τ are the same for cases 2 (Gaussian), 4 (Clayton) and 5 (Gumbel), in case 2 the movement is constant-speeded and both other cases differ widely from constant-speed movement. Even for case 3 (Gaussian) and case 6 (Gumbel), where the differences of Kendall's τ in the starting samples is small, constant-speed movement does not suffice in the Gumbel case.

The differences between the different degrees of the location polynomials can best be seen in the figures

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on Kendall's τ for case 4 (Clayton) and case 6 (Gumbel, small differences in correlation). In Figure ?? of the main section, the differences between the location polynomials are quantified.

2 Further Results: Weather Application

In Figures 13 and 14 the results for ECDI on the weather data along the slope of the Zugspitze are found for location polynomials of degree $K = 3$ and $K = 4$, respectively. Visually they appear similar, since there are only small differences in the resulting location polynomials.

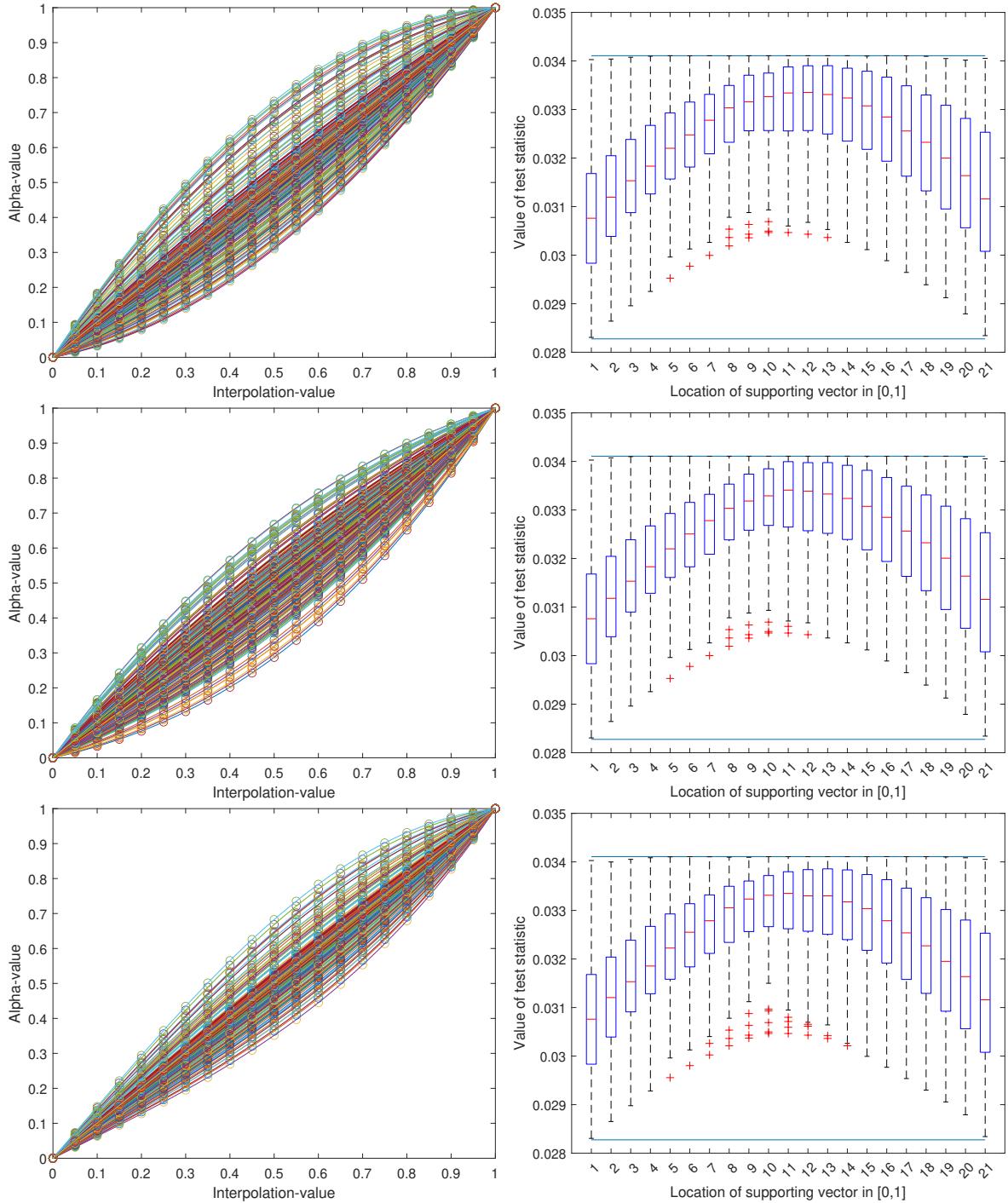


Figure 1: For samples from Gaussian copulas with parameters -0.4 and 0.4 and different degrees ($K = 2, 3, 4$) of the location polynomials, the location polynomials and corresponding test statistics for 50 iterations are presented.

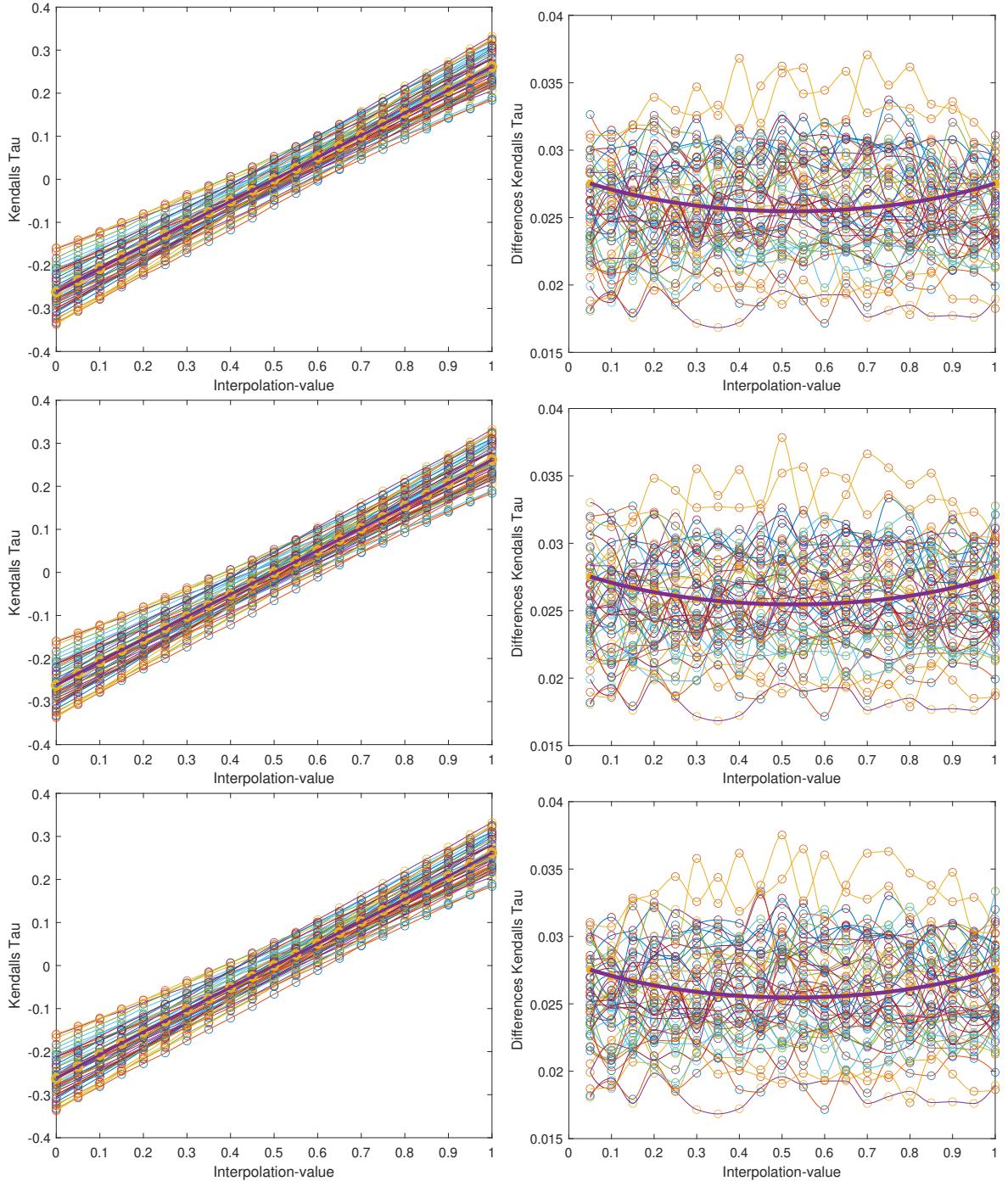


Figure 2: For samples from Gaussian copulas with parameters -0.4 and 0.4 and different degrees ($K = 2, 3, 4$) of the location polynomials, the values of Kendall's Tau in the interpolation process and the changes of Kendall's Tau are displayed.

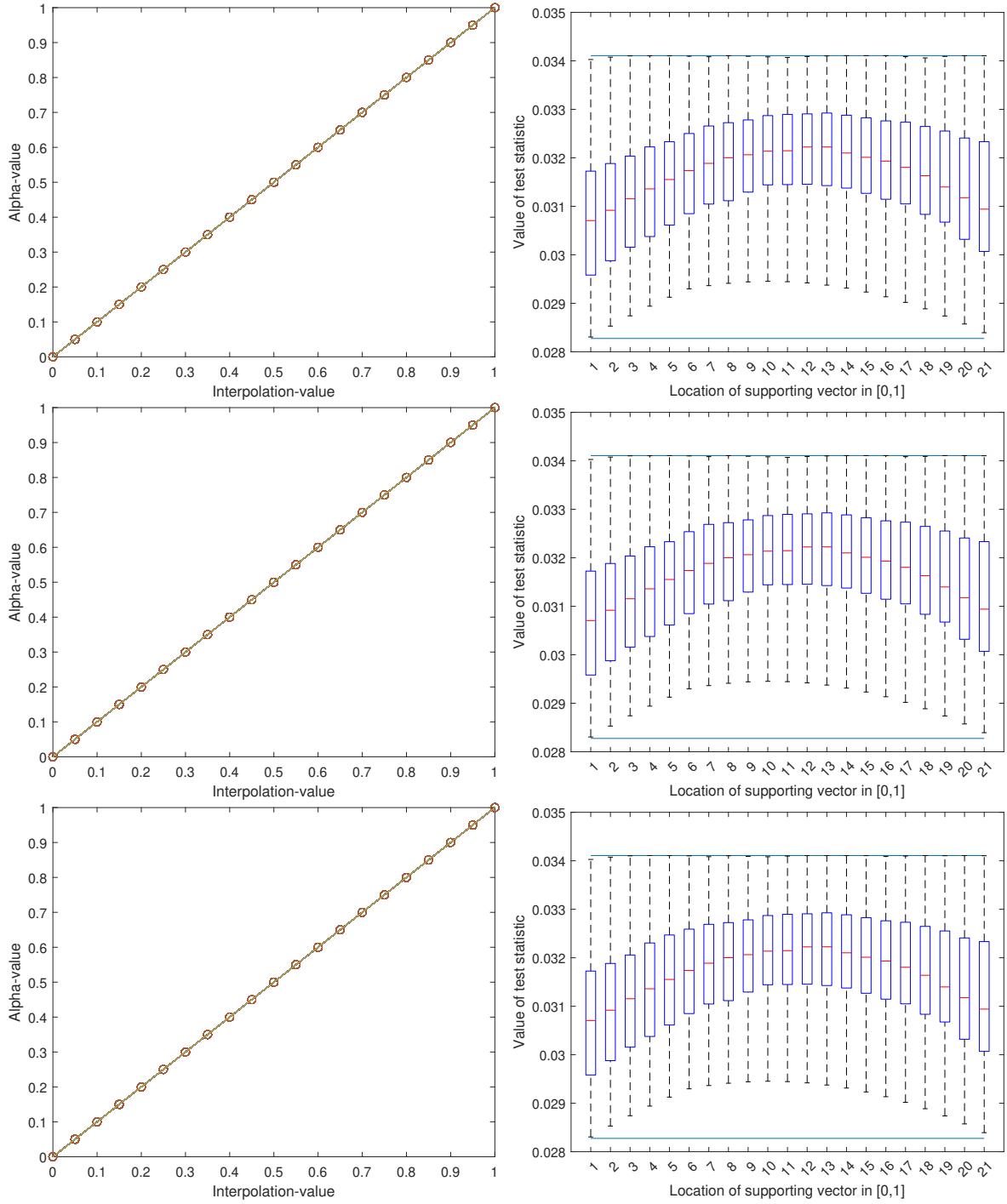


Figure 3: For samples from Gaussian copulas with parameters 0.2 and 0.7 and different degrees ($K = 2, 3, 4$) of the location polynomials, the location polynomials and corresponding test statistics for 50 iterations are presented.

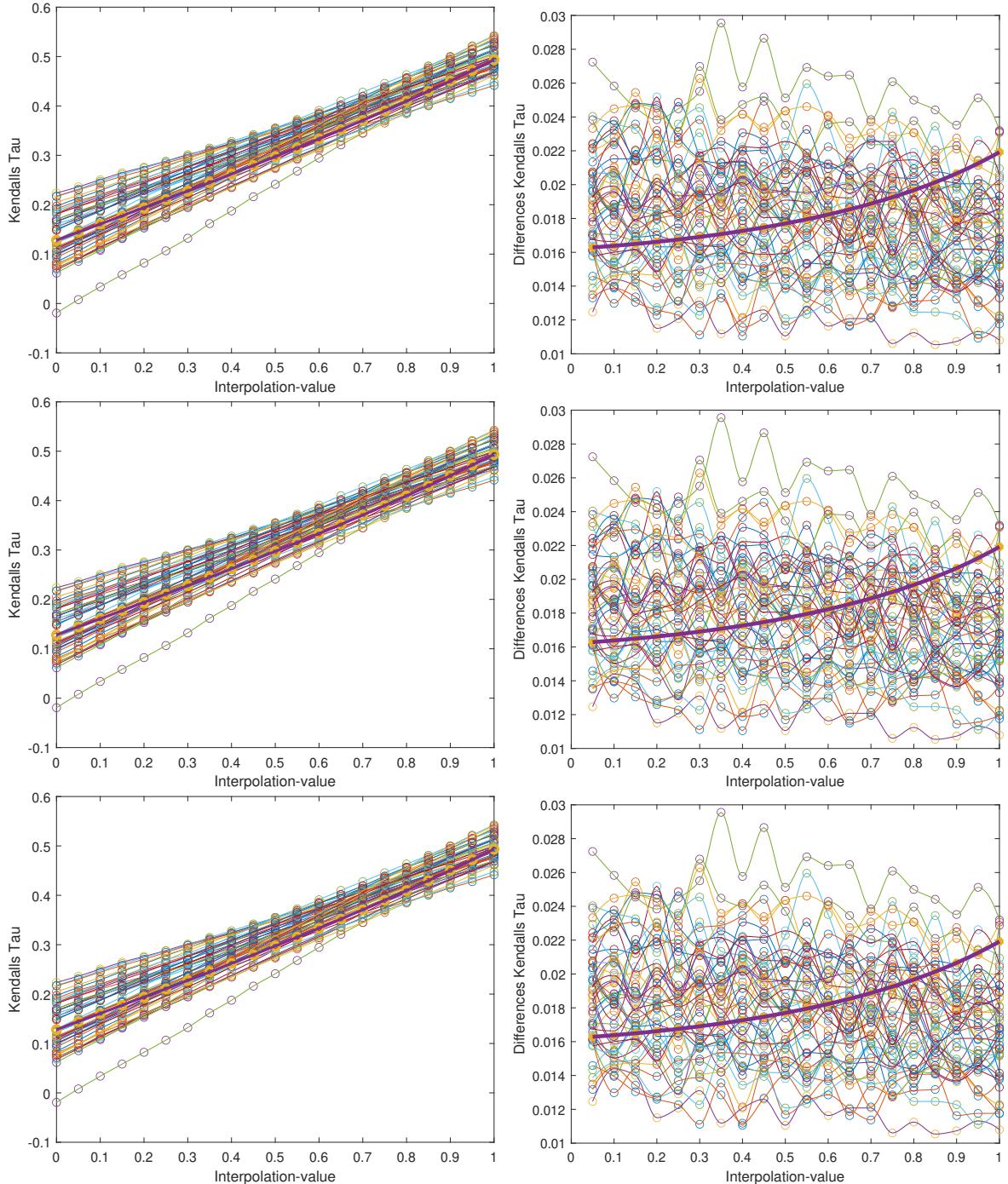


Figure 4: For samples from Gaussian copulas with parameters 0.2 and 0.7 and different degrees ($K = 2, 3, 4$) of the location polynomials, the values of Kendall's Tau in the interpolation process and the changes of Kendall's Tau are displayed.

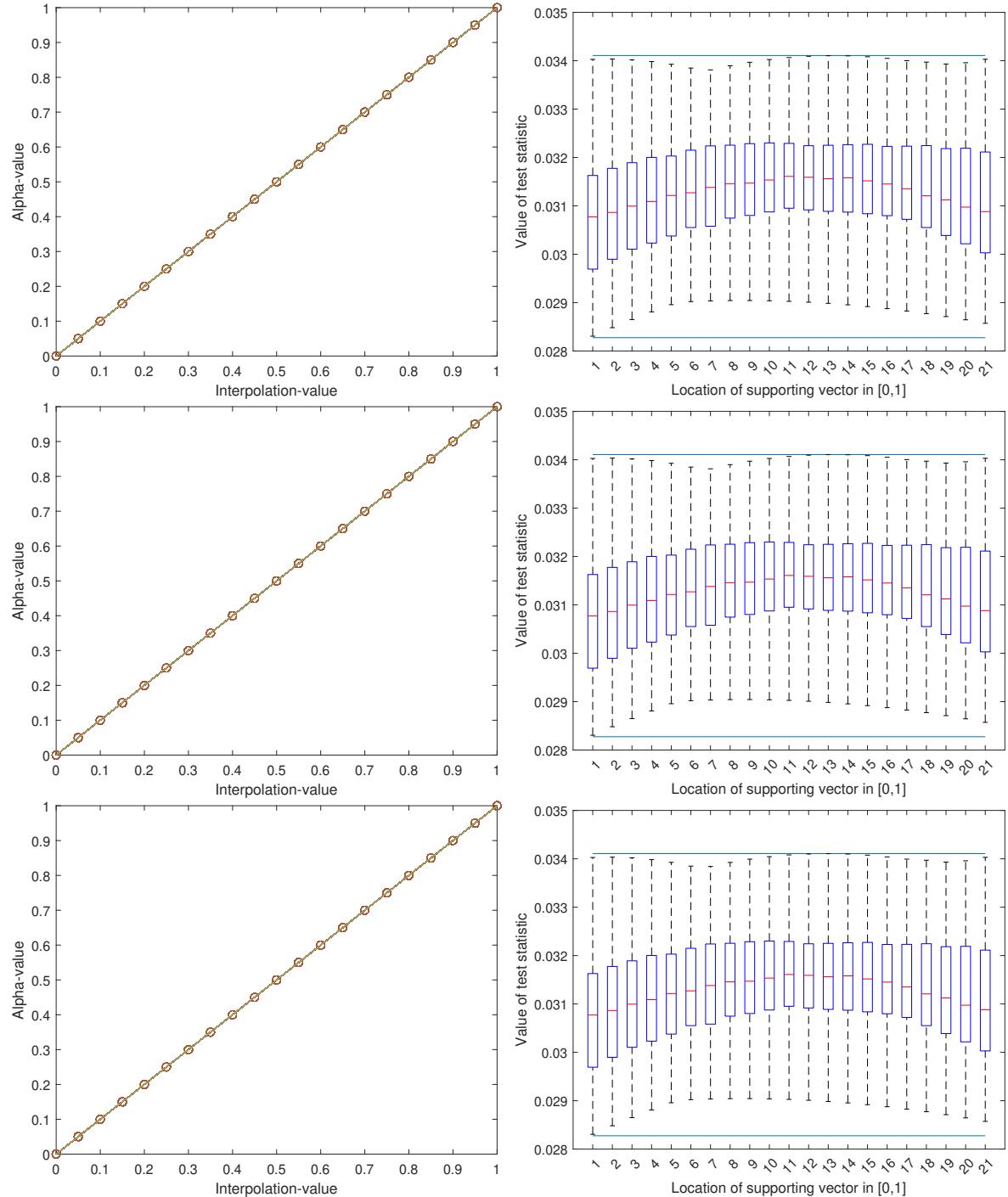


Figure 5: For samples from Gaussian copulas with parameters 0.5 and 0.8 and different degrees ($K = 2, 3, 4$) of the location polynomials, the location polynomials and corresponding test statistics for 50 iterations are presented.

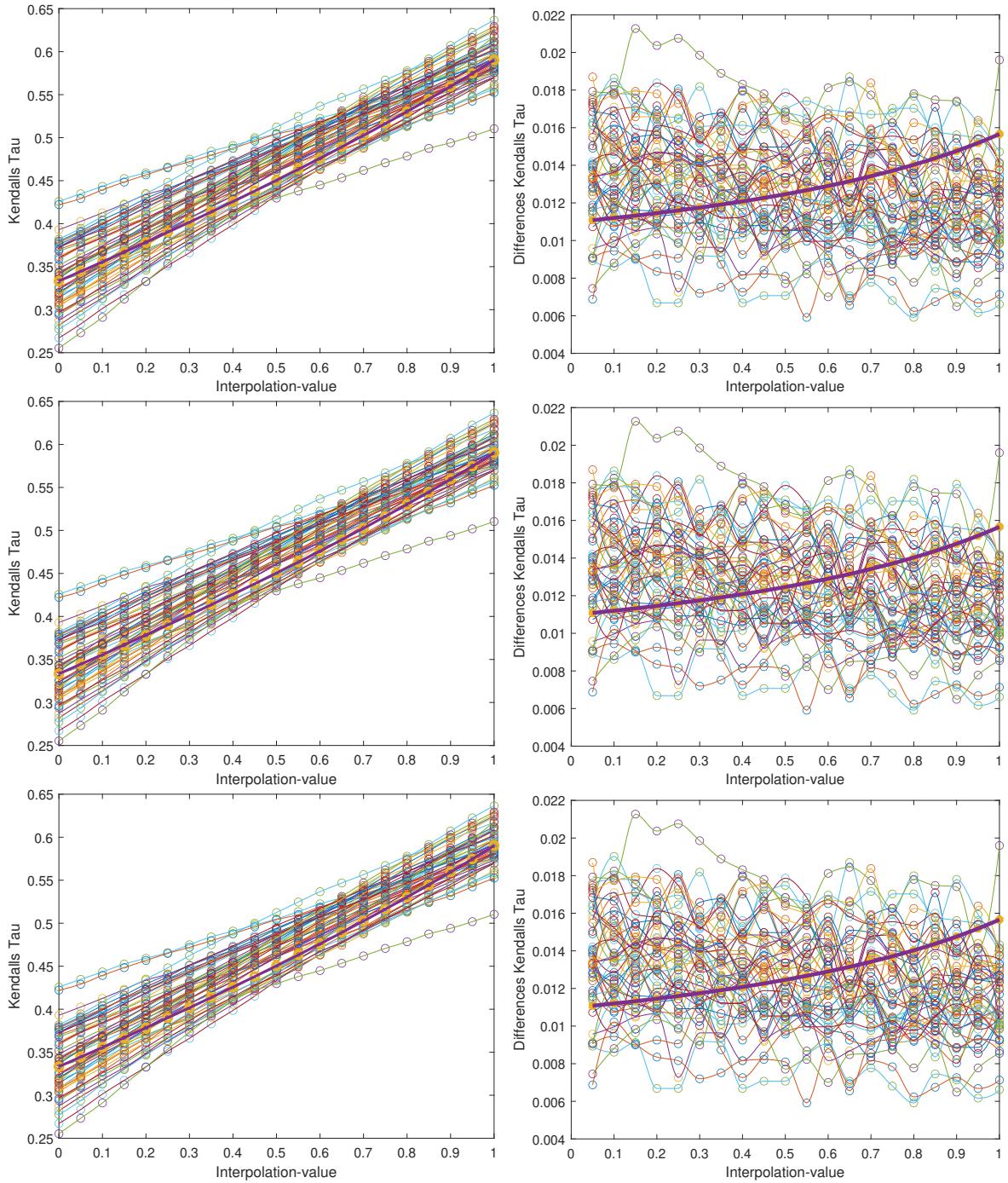


Figure 6: For samples from Gaussian copulas with parameters 0.5 and 0.8 and different degrees ($K = 2, 3, 4$) of the location polynomials, the values of Kendall's Tau in the interpolation process and the changes of Kendall's Tau are displayed.

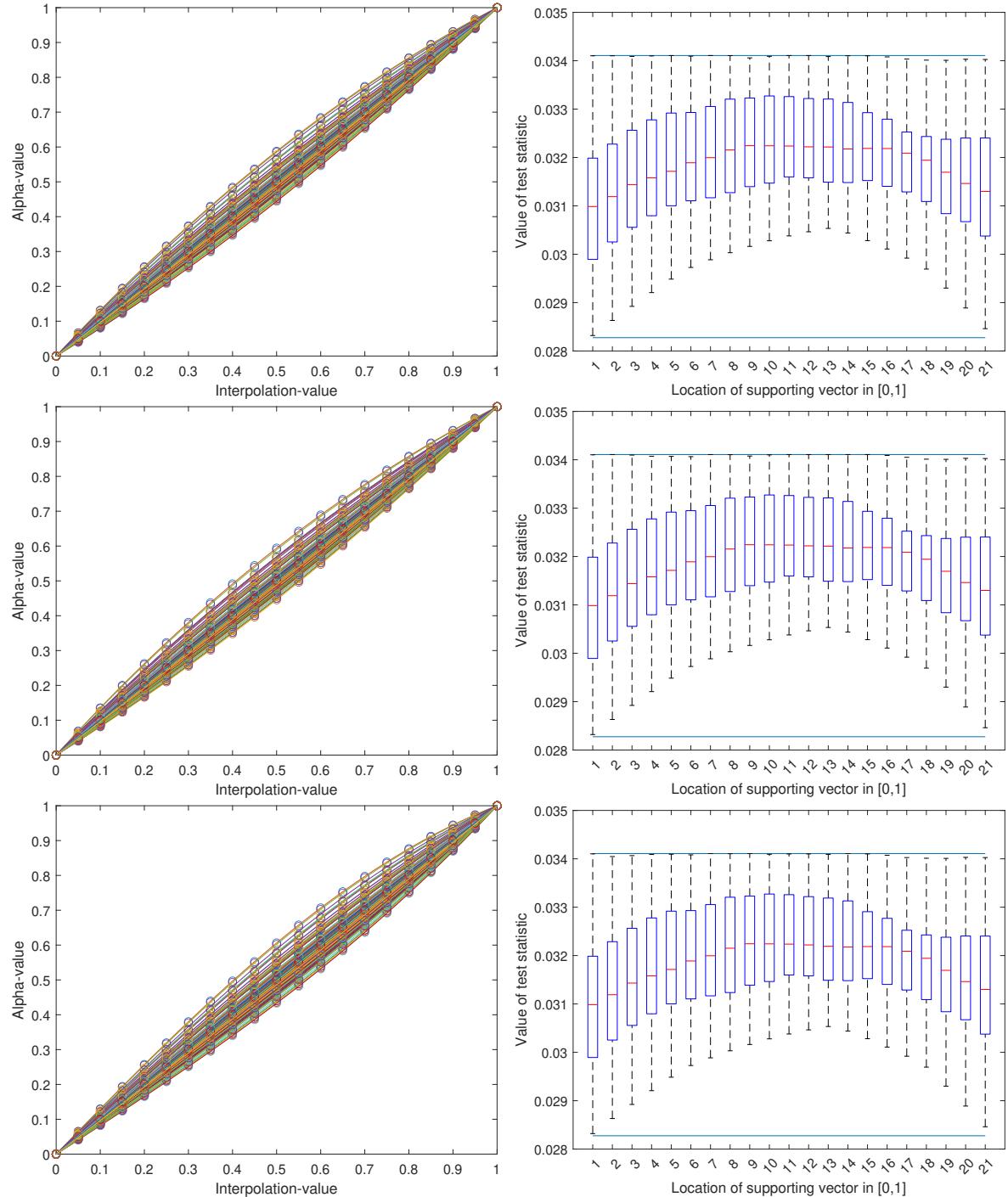


Figure 7: For samples from Clayton copulas with parameters 0.2941 and 1.9497 and different degrees ($K = 2, 3, 4$) of the location polynomials, the location polynomials and corresponding test statistics for 50 iterations are presented.

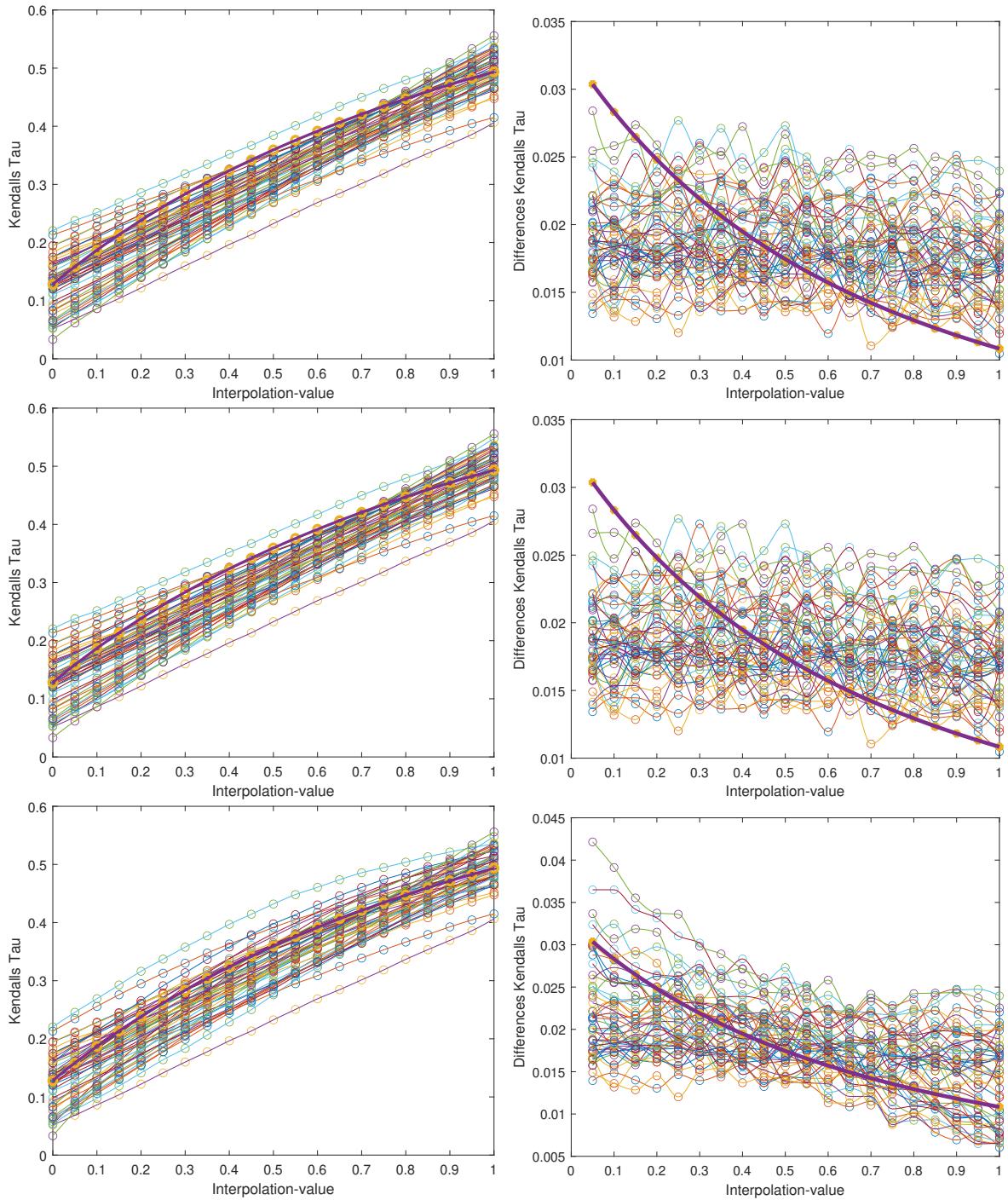


Figure 8: For samples from Clayton copulas with parameters 0.2941 and 1.9497 and different degrees ($K = 2, 3, 4$) of the location polynomials, the values of Kendall's Tau in the interpolation process and the changes of Kendall's Tau are displayed.

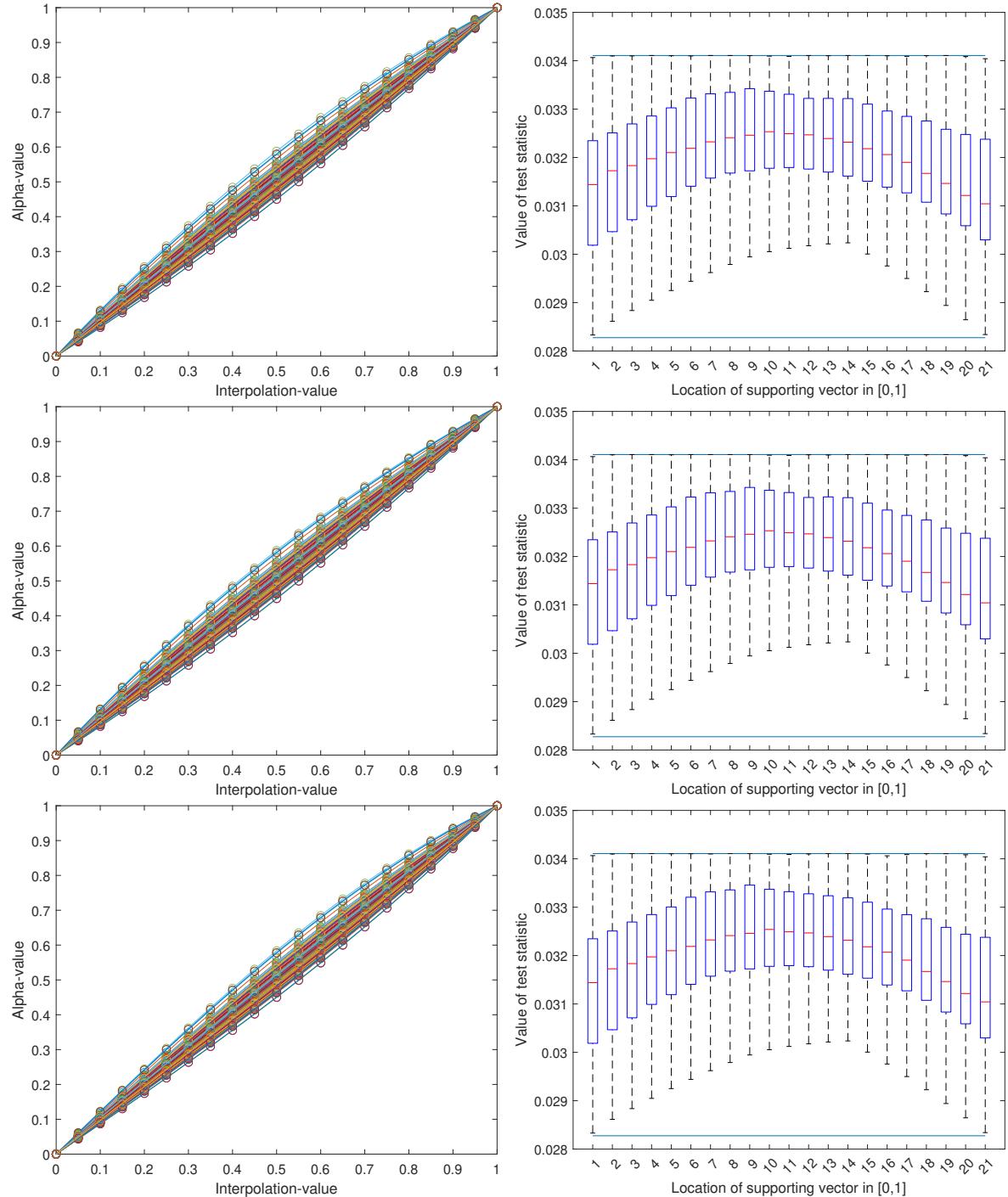


Figure 9: For samples from Gumbel copulas with parameters 1.1470 and 1.9749 and different degrees ($K = 2, 3, 4$) of the location polynomials, the location polynomials and corresponding test statistics for 50 iterations are presented.

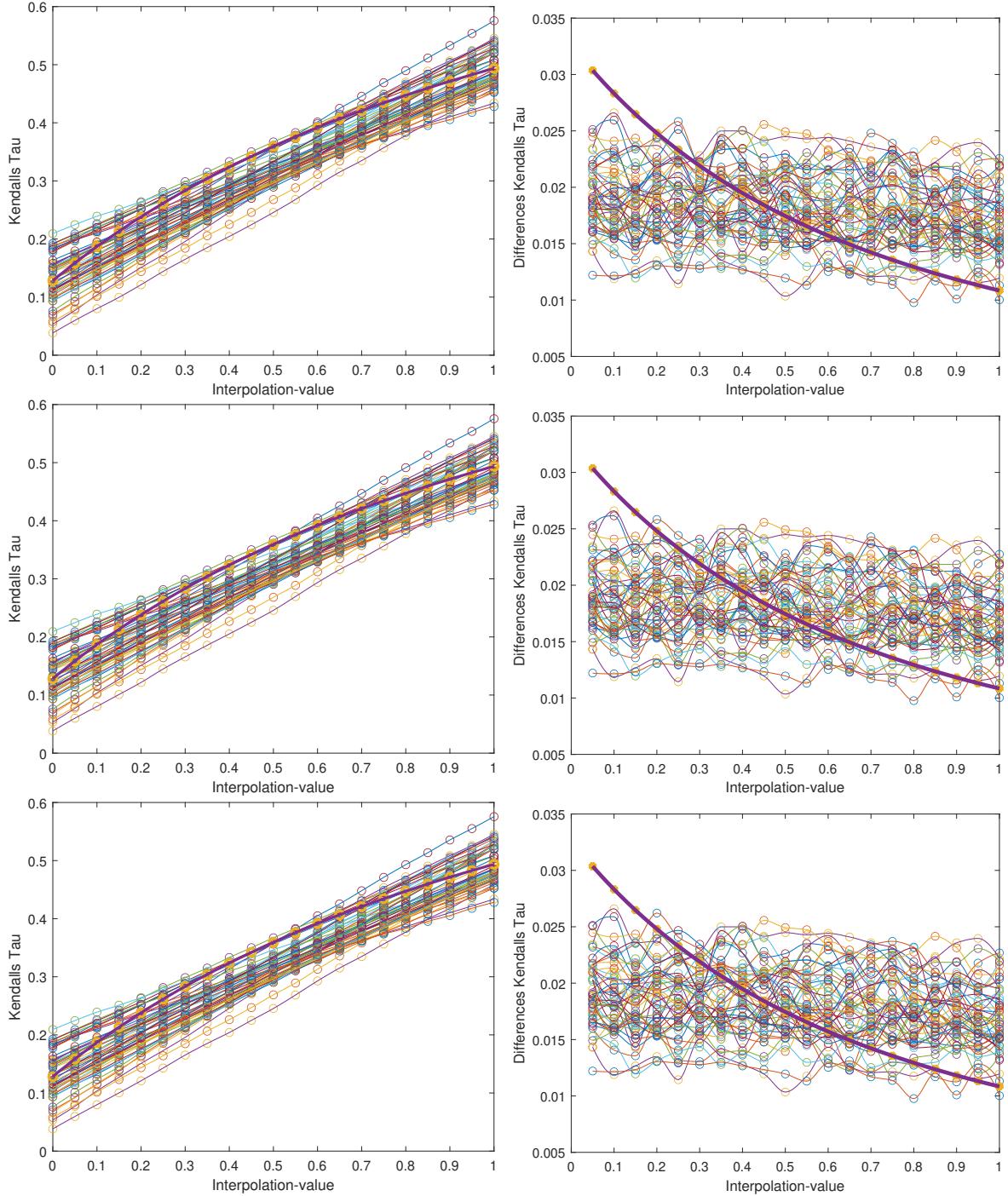


Figure 10: For samples from Gumbel copulas with parameters 1.1470 and 1.9749 and different degrees ($K = 2, 3, 4$) of the location polynomials, the values of Kendall's Tau in the interpolation process and the changes of Kendall's Tau are displayed.

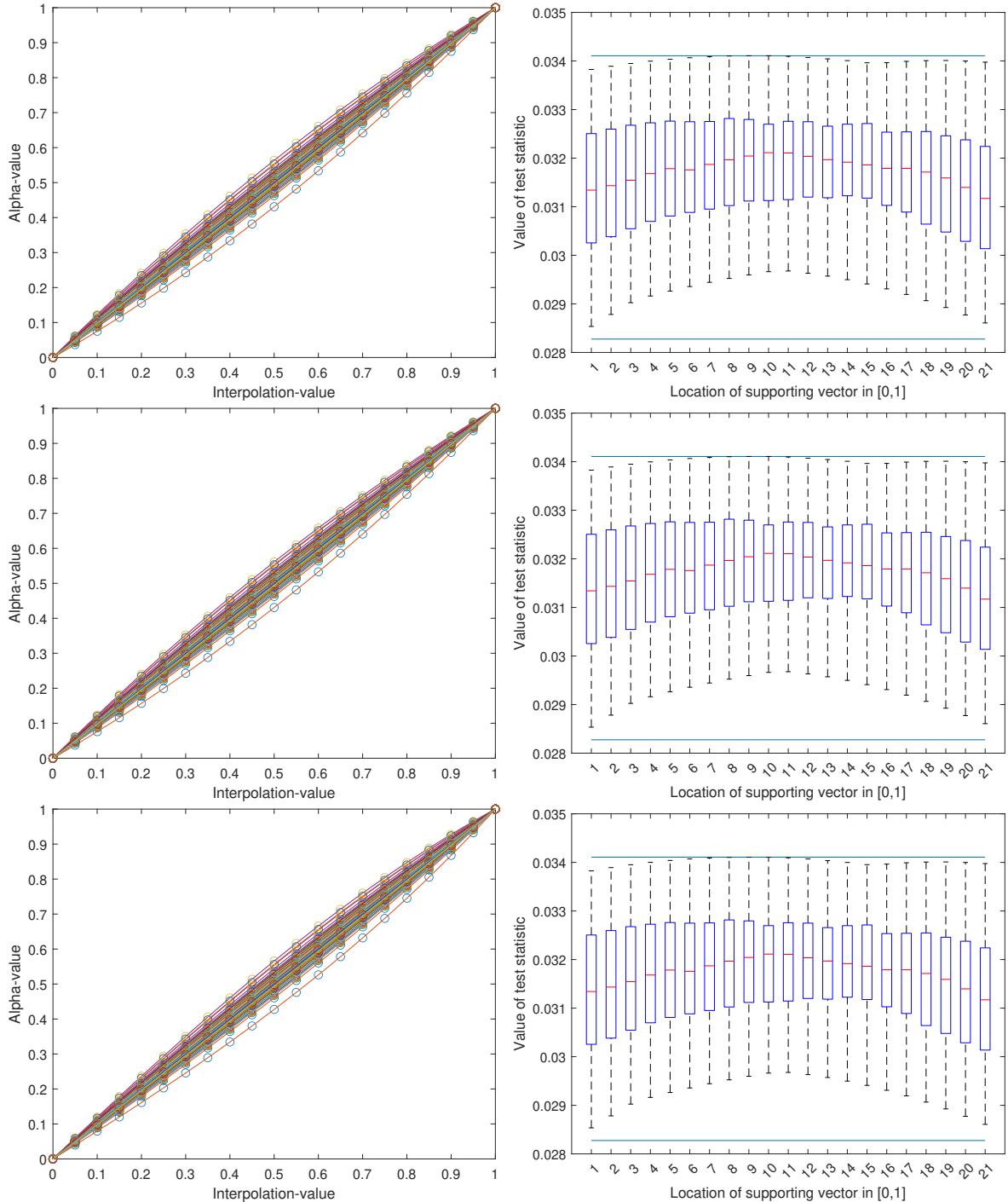


Figure 11: For samples from Gumbel copulas with parameters 1.5 and 2.4410 and different degrees ($K = 2, 3, 4$) of the location polynomials, the location polynomials and corresponding test statistics for 50 iterations are presented.

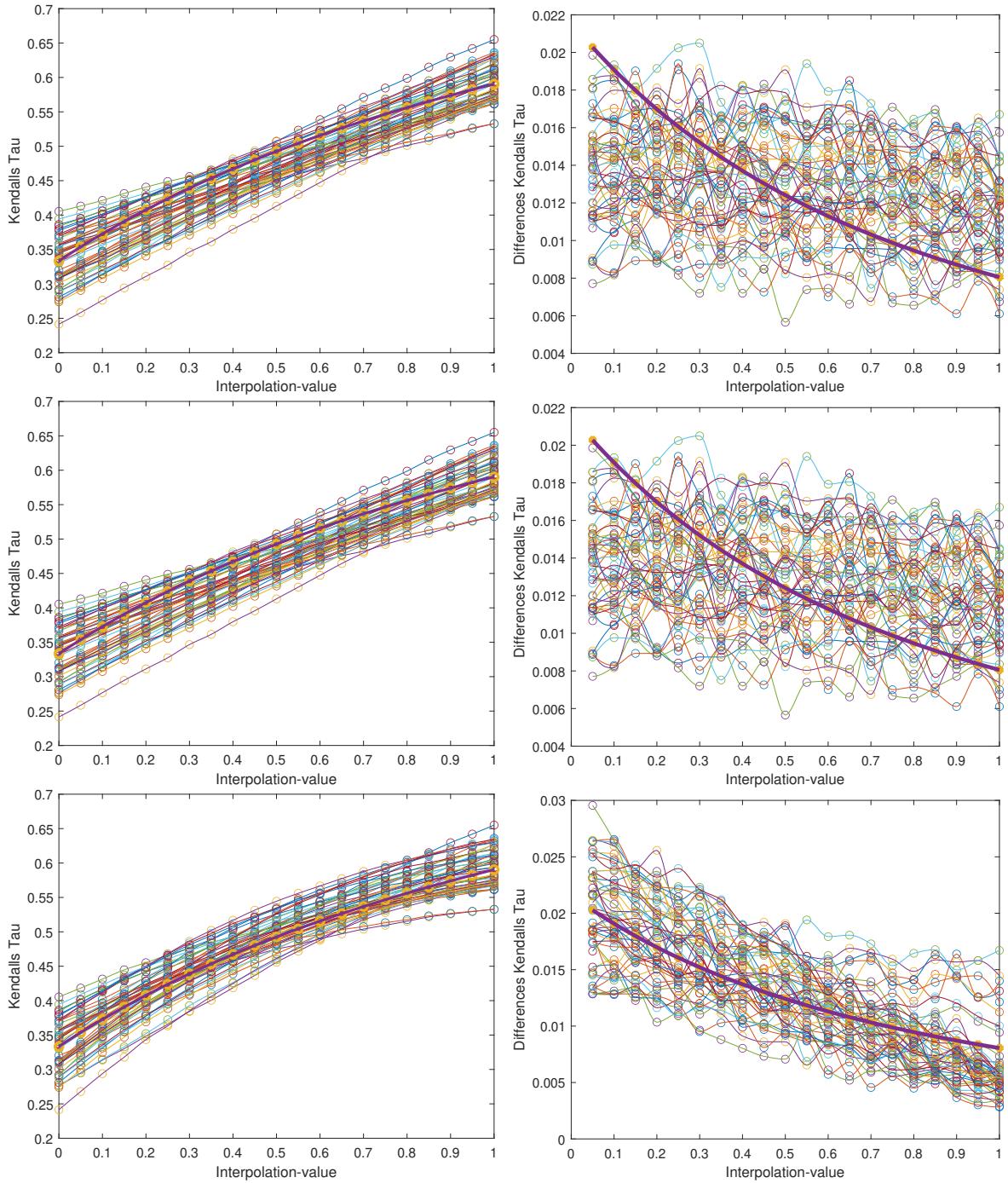


Figure 12: For samples from Gumbel copulas with parameters 1.5 and 2.4410 and different degrees ($K = 2, 3, 4$) of the location polynomials, the values of Kendall's Tau in the interpolation process and the changes of Kendall's Tau are displayed.

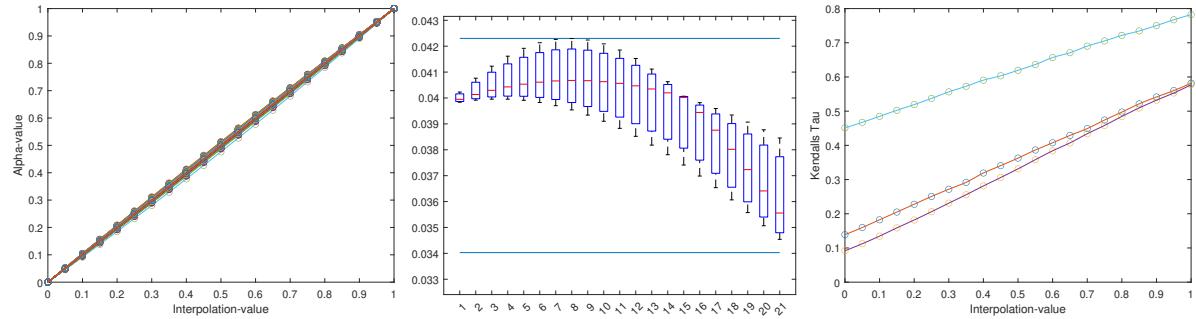


Figure 13: The results (location polynomials, values of test statistic, Kendall's tau) of ECDI for weather data at shifting altitudes with location polynomials of degree three.

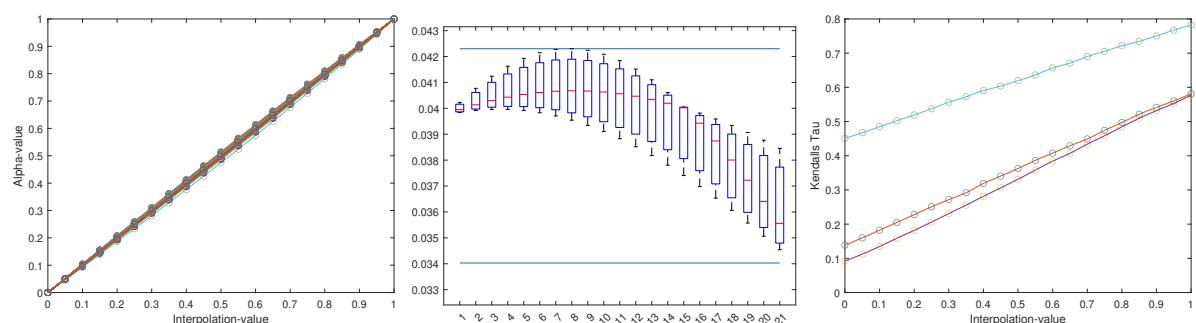


Figure 14: The results (location polynomials, values of test statistic, Kendall's tau) of ECDI for weather data at shifting altitudes with location polynomials of degree four.