Exercise5

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1 Statistical Parameter Estimation 2024

1.1 Exercise 5

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[14]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

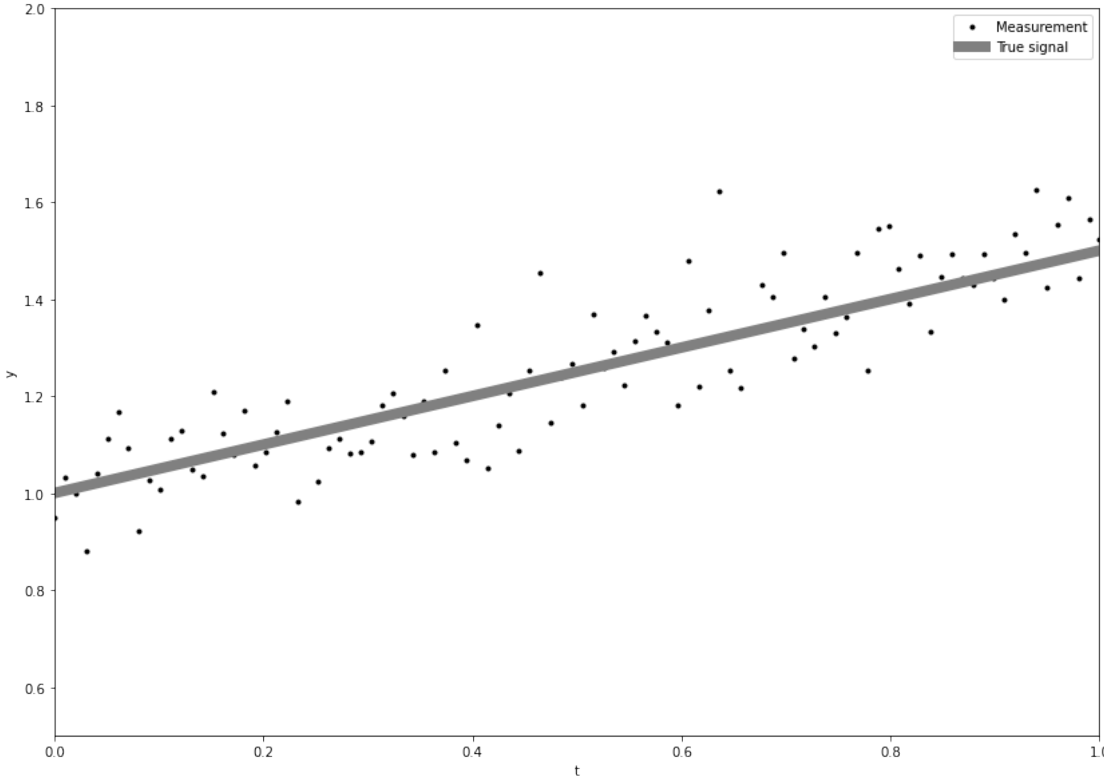
1.1.1 Task 1.

Consider the linear regression model $y_k = \theta_1 + \theta_2 t_k + \epsilon_k$,

where $\epsilon_k \sim N(0, \sigma^2)$ and prior of $(\theta) = (\theta_1, \theta_2)^T$ is Gaussian with known mean and covariance $\sim N(m_0, P_0)$ (fig 3.1)

[21]:
```
# Create the measurement data

tk = np.linspace(0,1,100)

# True signal (y = 1/2*t+1)

y_true = 1/2*tk+1

# Noise N(0,sigma^2)
sigma = 0.3
eps_k = np.random.normal(loc=0, scale=sigma**2, size=tk.shape)

# Measurements

yk = y_true + eps_k

# Visualize the measurement against the true signal

plt.figure(figsize=(14,10))
plt.plot(tk, yk, 'k.', label="Measurement")
plt.plot(tk, y_true, lw=8, c="gray", label="True signal")
plt.xlim([0,1])
```

```
plt.ylim((0.5, 2))
plt.xlabel("t")
plt.ylabel("y")
plt.legend()
plt.show()
```



1.1.2 Task 2.

Batch linear regression (fig 3.2)

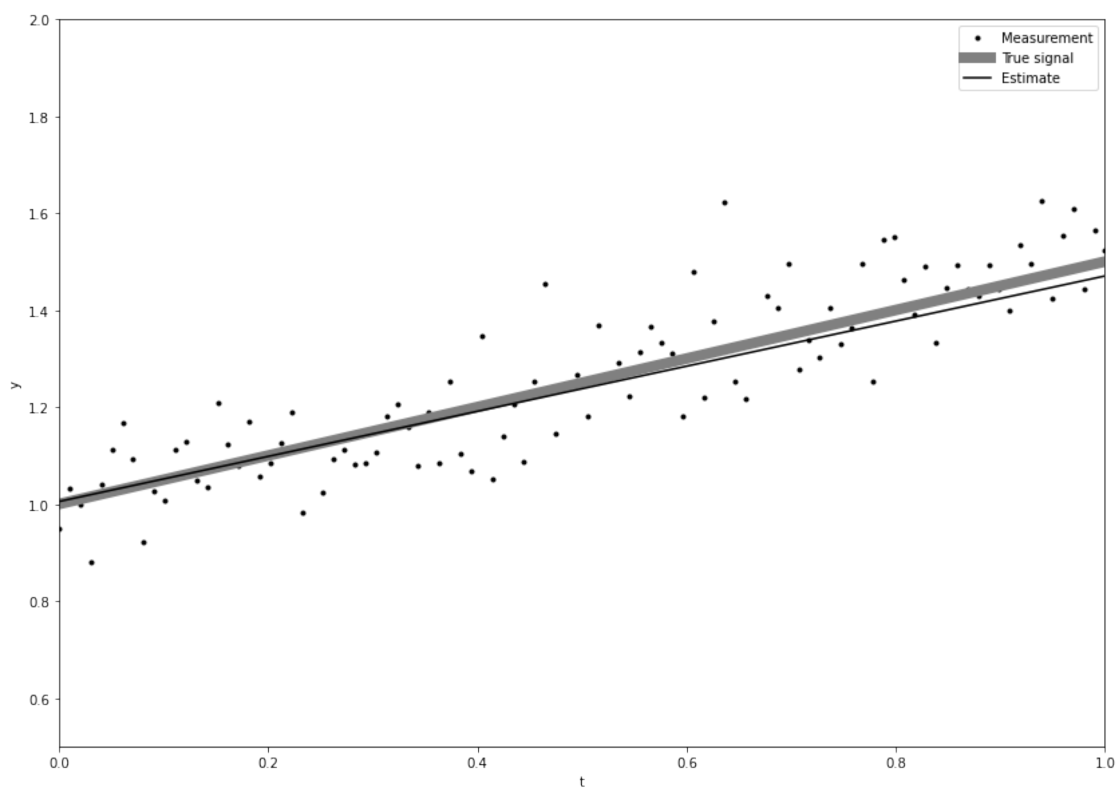
Mean and covariance:

$$
\begin{aligned}
\mathbf{m}_T &= \mathbf{P}_T^{-1} + \frac{1}{\sigma^2}\mathbf{H}^T\mathbf{H}\frac{1}{\sigma^2}\mathbf{H}^T\mathbf{y} + \mathbf{P}_T^{-1}\mathbf{m}_{T-1} \\
\mathbf{P}_T &= \mathbf{P}_{T-1}^{-1} + \frac{1}{\sigma^2}\mathbf{H}^T\mathbf{H}
\end{aligned},
$$

where $\mathbf{t}_T = (t_1, t_2, t_3, ..., t_{k=T})^T$, $\mathbf{H} = (\mathbf{1}\ \mathbf{t}_T)$ and $\mathbf{y} = (y_1, y_2, y_3, ..., y_{k=T})$

[38]:
```
# Initial mean and covariance (guess)
mT = np.zeros(2)
PT = np.eye(len(mT))
BATCH_SIZE = 5
```

```
for T in range(0,len(tk), BATCH_SIZE):
    tT = tk[T : T + BATCH_SIZE]
    y = yk[T:T+BATCH_SIZE]
    H = np.vstack((np.ones_like(tT), tT)).T

    mT = (np.linalg.inv(np.linalg.inv(PT) + 1 / sigma**2 * H.T @ H) @
                    (1 / sigma**2 * H.T @ y + np.linalg.inv(PT)@mT))
    PT = np.linalg.inv(np.linalg.inv(PT) + 1 / sigma**2 * H.T @ H)


theta_est = np.random.multivariate_normal(mean=mT, cov=PT)
y_pred = theta_est[0]+theta_est[1]*tk
# Visualize the regression model

plt.figure(figsize=(14, 10))
plt.plot(tk, yk, "k.", label="Measurement")
plt.plot(tk, y_true, lw=8, c="gray", label="True signal")
plt.plot(tk, y_pred, 'k', label="Estimate")
plt.xlim([0, 1])
plt.ylim((0.5, 2))
plt.xlabel("t")
plt.ylabel("y")
plt.legend()
plt.show()
```



1.1.3 Task 3.

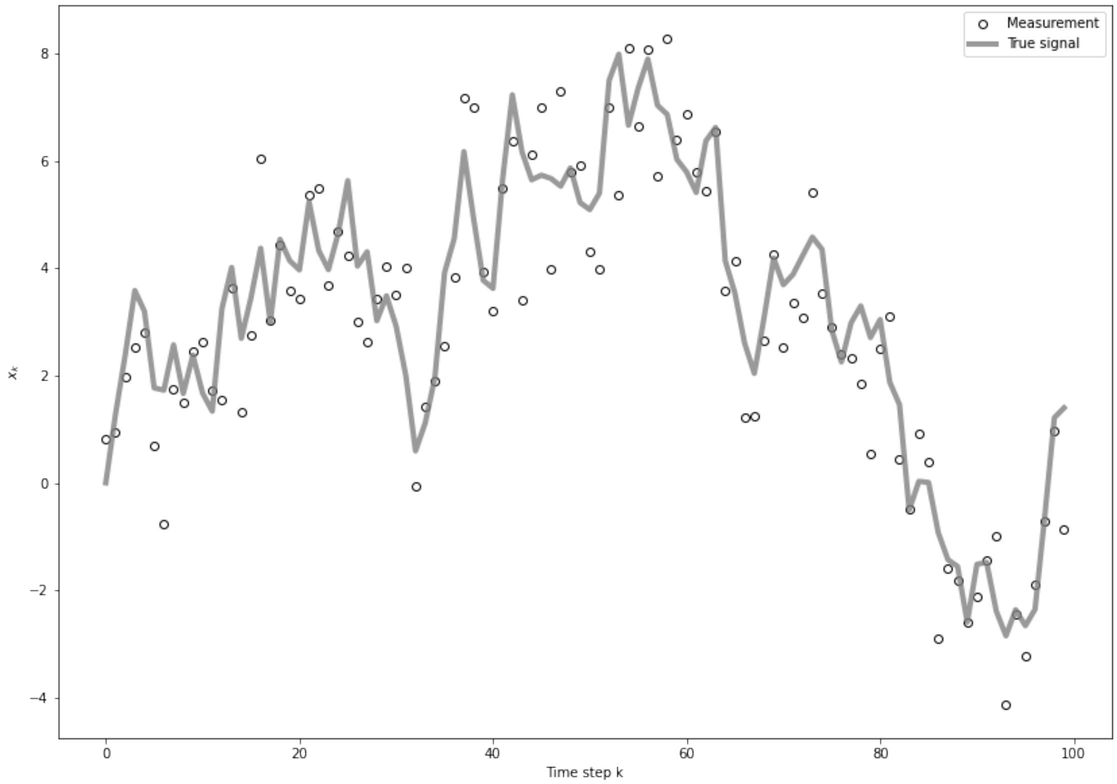
Gaussian random walk (fig 6.1)

Formula:

$$
\begin{aligned}
x_k &= x_{k-1} + q \\
y_k &= x_k + r,
\end{aligned}
$$

where $q \sim N(0, Q)$ and $r \sim N(0, R)$

[40]:
```
Q = R = 1


xk = np.empty((100))

yk = np.empty_like(xk)
xk[0] = 0

yk[0] = xk[0] + np.random.normal(0,R)

for k in range(1,len(xk)):

    xk[k] = xk[k-1] + np.random.normal(0,Q)

    yk[k] = xk[k] + np.random.normal(0, R)


plt.figure(figsize=(14, 10))

plt.plot(yk, "ko", mfc="white", label="Measurement")

plt.plot(xk, lw=4, alpha=0.8, c="gray", label="True signal")

plt.xlabel("Time step k")

plt.ylabel("$x_k$")

plt.legend()

plt.show()
```


Measurement
True signal
x_k
Time step k
8
6
4
2
0
−2
−4
0
20
40
60
80
100