Intelligent Agent for Open Face Chinese Poker Web-Application

8th May 2015

Submitted May 2015 in partial fulfilment of the conditions of the award of the degree BSc (Honours) Computer Science

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I hereby declare that this dissertation is all my own work, except as indicated in the text:

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Abstract

Artificial Intelligence (AI) is a crucial and high-interest area of research in the field of Computer Science, which has gained increased traction in recent decades due largely to the requirements for increasingly sophisticated AI in games, and the implications that development of *algorithms* and *heuristics* in this area can have beyond the domain of game-playing in other fields of study such as *Automated Planning and Scheduling*. Poker games provide a challenge to *Intelligent Agents* because of many factors including *combinatorial explosions* of *game trees*, elements of *hidden information* such as the cards other players hold, as well as stochastic elements due to not knowing which cards are yet to be dealt. This differentiates Poker games from more traditional games such as Checkers, which is a deterministic *perfect information* game meaning that each player has the same complete knowledge of the game state at any stage and that there is a finite set of moves for each player. Therefore at each stage there is an *optimal move* leading to a winning strategy, and in this manner these games can effectively be *solved*, and so when playing against a competent Agent for such a game it is impossible to win, only to draw. Poker games are different because of the aforementioned stochastic elements, imperfect information and complex search trees, necessitating the use of more sophisticated algorithms in order for an Agent to perform competently versus intelligent opponents. While there has been lots of research into Intelligent Agents for traditional board games and variants of Poker such as *Texas Hold'Em*, lesser known variants such as the relatively new *Open Face Chinese Poker* have not been explored to the same degree. Agents for Open Face Chinese Poker often suffer from poor performance due to reliance on simple algorithms and methods such as *Rule-Based Systems*, which can lead to predictable or sub-optimal play that is additionally largely domain specific. This dissertation considers the merits and limitations of various AI techniques, and implements an Intelligent Agent for a bespoke Open Face Chinese Poker Web-Application, with discussions of the range of technologies used and methodologies employed in creating a functional final product. It is advisable for readers unfamiliar with Poker, Computer Science or any of the italicised terms found throughout to familiarise themselves with the definitions found in the glossary section.

Contents

1 Introduction

1.1 Outline of Introduction

Section 1.2 covers an introduction to Open Face Chinese Poker, explaining the basic rules and giving illustrated examples of the game board and the scoring system. Readers unfamiliar with the game will benefit largely from reading this section in conjunction with reference to the glossary of terms.

Section 1.3 outlines the problem description, considering the limitations of existing Intelligent Agents and the reasons for these limitations.

Section 1.4 outlines specific aims and objectives for the project based on functional, non-functional and usability requirements for function, performance and meeting user demands.

Section 1.5 discusses the potential for ethical issues in this project and its area of research.

1.2 Introduction to Open Face Chinese Poker

Open Face Chinese Poker (commonly abbreviated to *OFCP*) is a perfect or imperfect information (depending on variant)[1] card game, and is a variant of *Chinese Poker*. In OFCP, players take turns placing cards face-up into three distinct *rows* (*bottom*, *middle* and *top*), creating the best possible *poker hands* they can in order to score points from each other. Stronger hands score bonus points called *royalties* and royalty-scoring hands in higher rows score more points than equivalent hands in lower rows. For example, a *Royal House* gives +25 bonus points in bottom row, or +50 in the middle row. Players score +1 point for each row they win in addition to any royalties. In the case that a player wins all 3 rows they score a *scoop* bonus which grants an additional point for each row, for a total of +6 base points before royalties are calculated. OFCP is a zero-sum game, meaning any gains by one player are balanced with losses by another player; players win points directly from their opponents, so if a player's score is +16 points then in a 1v1 game it is therefore implied that their opponent's score would be -16.

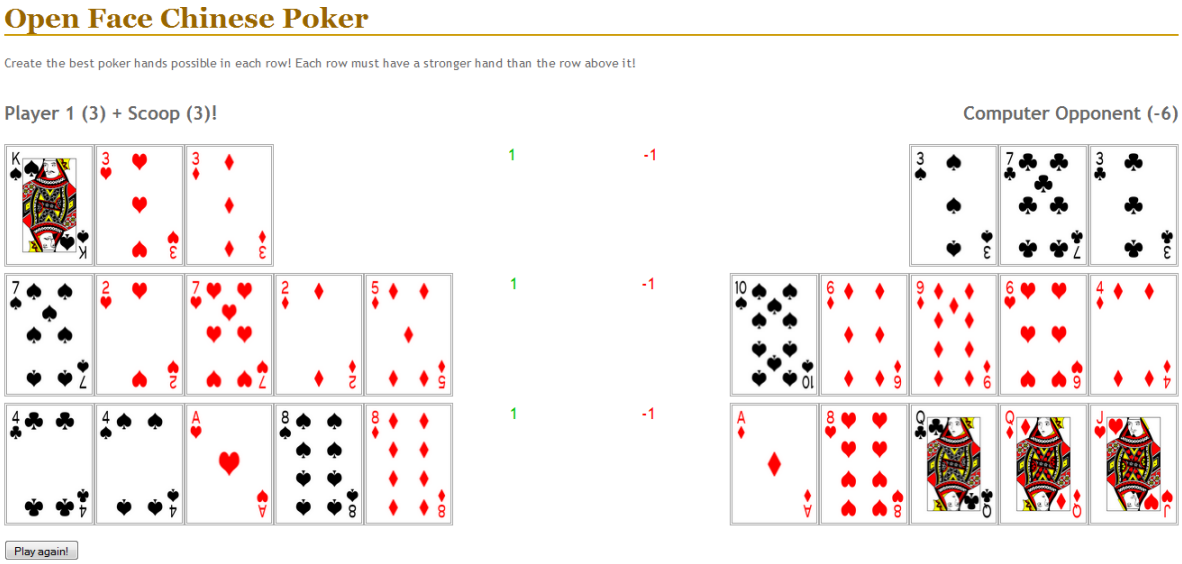


Fig 1.2.1 – layout of the board after a round of Open Face Chinese Poker (screenshot from an early prototype of the application)

Figure 1.2.1 shows the results and layout of the board after a game of OFCP. Player 1 wins bottom with a *Two Pair* 8s and 4s versus a *Pair* of Qs for +1 point. Player 1 also wins middle row with Two Pair 7s and

[1] Standard OFCP is a perfect information game, but other variants such as *Pineapple OFC* also feature elements of hidden information

2s versus Pair of 6s for +1 point. In top row Player 1 and the computer opponent both have a Pair of 3s, and so the third card is taken into account as the *kicker*, with Player 1 clinching the win for +1 point with a King kicker versus a 7. Further to the individual row scores, because Player 1 won every row they score an additional scoop bonus of +3.

One important caveat of the game is that hands in lower rows must be stronger than those on the rows above; if a player creates a top-heavy board then that player's hand is invalid, which is known as *fouling*. When a player fouls their opponents automatically scoop them for +6 each (+1 for each row and +3 scoop bonus) in addition to any royalties, and any of the fouled player's royalties are disqualified. In the case that all players foul, no points are awarded.

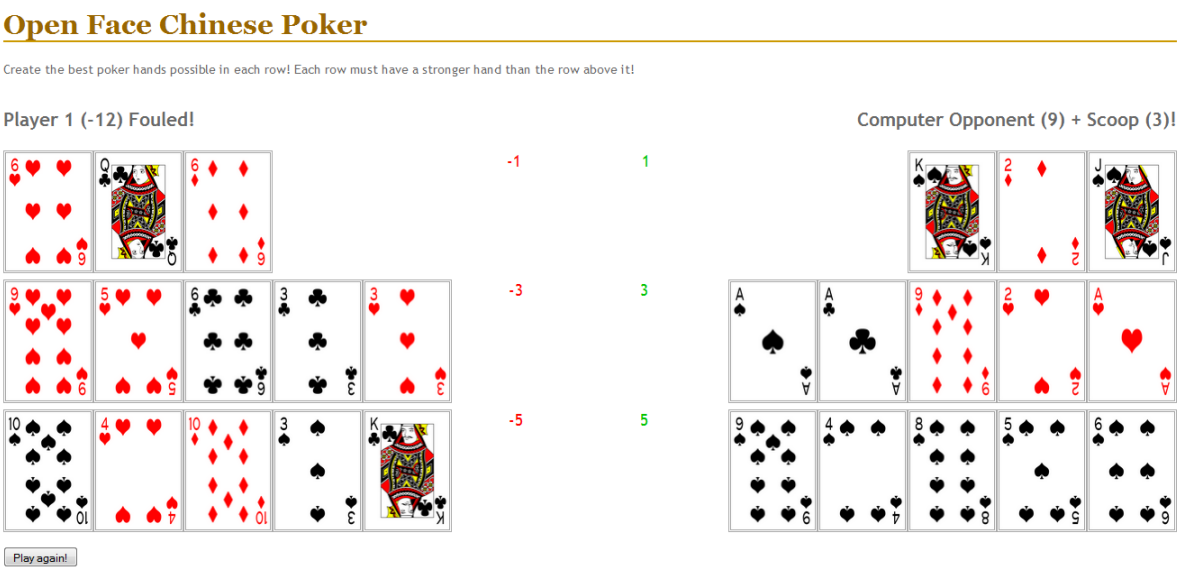


Fig 1.2.2 – Player 1 fouls and so their opponent wins an automatic scoop bonus plus royalties

In the game shown in Fig 1.2.2 Player 1 has Pair of 10s in bottom, Pair of 3s in middle and Pair of 6s in top. Because the top row contains a stronger poker hand than the row below it, the hand is invalid and the player fouls. On top of the +3 scoop bonus and points for individual row wins, the computer opponent wins further points for royalties because of its *Flush* in bottom and *Three of A Kind* in middle.

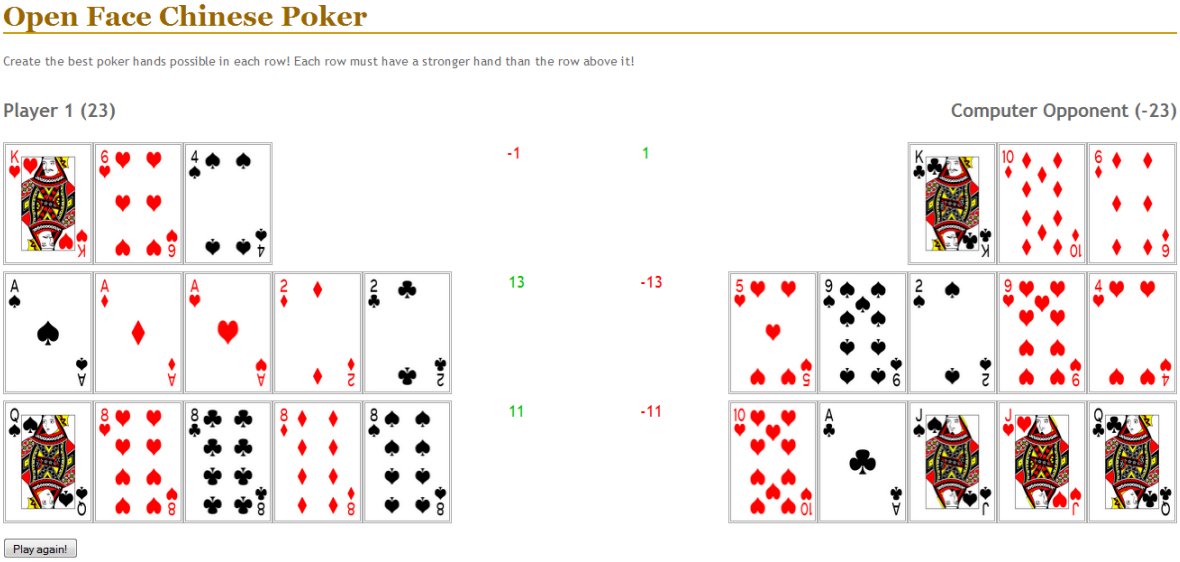


Fig 1.2.3 – Player 1 wins a lot of bonus points from royalties for strong hands

Fig 1.2.3 indicates how many points can be won from strong hands. Player 1 wins bottom row with *Four of a Kind* 8s versus the computer opponent's Pair of Jacks. Player 1 receives +1 point for winning the row, with an additional +10 points royalty for Four of a Kind on bottom. Player 1 also wins middle row with a *Full House*, Aces full of Deuces versus the computer opponent's Pair of 9s, scoring +1 point for winning the row and an additional +12 royalty. Player 1 and the computer opponent both have *High Card* King in the top row, but the computer opponent wins the row because their kicker of 10 beats Player 1's 6 kicker. The computer opponent wins +1 point for winning this row but the hand is not strong enough to score any additional royalties. The points each player wins are taken from their opponent, so the final score for Player 1 is $-1 + 13 + 11 = 23$, and the computer opponent's score is the inverse of this, -23.

1.3 Problem Description

Creating sophisticated Intelligent Agents for games with large branching factors poses a significant challenge, as for the Agent to perform well it must be able to search many moves ahead of the current state in order to find the optimal move. With every additional level of depth searched in a game tree the complexity of the search increases exponentially due to combinatorial explosion; at a depth of d with a branching factor B there would be approximately B^d potential nodes to explore. A brute-force Depth First Search would therefore have a time complexity of $O(B^d)$ which is prohibitively complex for any reasonably exhaustive search of a game with a high branching factor. Consider for example Chess, which has an estimated branching factor of 35 (Mandziuk, 2010, p4). In his influential paper *Programming a Computer for Playing Chess*, Claude Shannon estimated a lower-bound for the game-tree complexity of Chess of 10^{120}, asserting "A machine operating at the rate of one variation per second would require over 10^{90} years to calculate the first move" (Shannon, 1950, p4). While in theory exhaustively analysing a game of chess from start to finish is possible, it remains implausible with any conceivable modern computer despite huge advances in processing speed and power since 1950.

Search Depth	**Approximate Nodes**	**Approximate Time to Search (10^6 nodes/sec)**
1	35	3.5×10^{-5} seconds
2	35^2	1.2×10^{-3} seconds
3	35^3	4.3×10^{-2} seconds
4	35^4	1.5 seconds
5	35^5	52.5 seconds
6	35^6	30.6 minutes
7	35^7	17.9 hours
8	35^8	26 days
9	35^9	2.5 years
10	35^{10}	87.5 years
11	35^{11}	3061 years
12	35^{12}	$\sim 100,000$ years
20	35^{20}	$\sim 2.4 \times 10^{17}$ years
30	35^{30}	$\sim 6.7 \times 10^{32}$ years
40	35^{40}	$\sim 1.8 \times 10^{48}$ years

Table 1.3.1 showing the intractability of a brute force search of a Chess game tree

Open Face Chinese Poker does not have such an astronomical space state or game-tree complexity as Chess, although unlike Chess it incorporates stochastic elements because of its nature as a card game, and the subsequent challenge this poses for an Intelligent Agent makes it an interesting area of research. Considering there are $52! \approx 8 \times 10^{67}$ permutations of a standard deck of cards, representation of each possible state in a game tree invokes a combinatorial explosion of branching factor. An upper-bound[2] for possible final

[2]This estimate is generous as it includes duplicate states e.g. rows with the exact same cards but in different orders

game states is 4.96×10^{14} with an upper-bound estimate for the game tree size of 5.77×10^{32} which poses a similarly daunting complexity. The game tree size was calculated by assuming that both player's first 5 cards are known and then taking into account from there each player's sequential turns for the duration of the game, using the following formula:

$$\left(\binom{3}{1}\cdot 42\right) \times \left(\binom{3}{1}\cdot 41\right) \times \cdots \times \left(\binom{3}{1}\cdot 30\right) \times \left(\binom{2}{1}\cdot 29\right) \times \left(\binom{2}{1}\cdot 28\right) \times \left(\binom{1}{1}\cdot 27\right) \times \left(\binom{1}{1}\cdot 26\right)$$

Search Depth	**Approximate Nodes**	**Approximate Time to Search (10^6 nodes/sec)**
1	1.55×10^4	1.5×10^{-2} seconds
2	2.18×10^8	3.6 minutes
3	2.75×10^{12}	31.8 days
4	3.12×10^{16}	9.9×10^2 years
5	3.15×10^{20}	9.99×10^6 years
6	2.81×10^{24}	8.9×10^{10} years
7	1.47×10^{28}	4.65×10^{14} years
8	2.22×10^{31}	0.7×10^{18} years

Table 1.3.2 showing the possible states to consider in an exhaustive search of an Open Face Chinese Poker game tree

The rapid expansion of the search space makes it clear that more processing power is not in itself a solution to the problem, as combinatorial explosion quickly makes deep searches of these games impractical for any realistic purposes; even a machine that could search 1 Trillion (10^{12}) nodes per second would take 7×10^{11} years to perform such an exhaustive search of an entire OFCP game tree.

However, despite the fact that modern computers are still not powerful enough to perform such complex and exhaustive searches, competent Intelligent Agents for games such as Chess have emerged regardless. The reason this is possible is because of different approaches to the problem, or through pruning algorithms which reduce the complexity of the search by removing branches of the tree that do not provide beneficial information. A variety of algorithms and heuristics have been formalised over the decades, with the creation of a multitude of Intelligent Agents of varying competence, with wide coverage of more traditional games such as Chess and Checkers, and even for more complex games such as Go. In terms of artificial intelligence applications for Poker, more well-known variants such as Texas Hold'Em have seen lots of interest while more obscure variants such as OFCP have remained somewhat untouched.

Intelligent Agents do exist for OFCP, but publicly available implementations largely play predictably or sub-optimally, often favouring conservative play over taking risks, even in situations where fouling would not be heavily penalised (such as when the human opponent is in a fouling position themselves). There has been little formal research into Intelligent Agents for OFCP, and as such limited source material is available. Most existing bots for the game are designed for Poker sites which do not have any source code made available, or made by development teams which license the software out to Poker players for money such as in the case of Warren[3], which uses neural networks to learn and improve over time as it plays against human opponents. Even with such advanced techniques the game has not been solved, as evidenced by the modest claim that "(Warren) plays almost perfect Open Face Chinese Poker after the 6th card". Other implementations of Agents for OFCP by hobbyist programmers for personal projects often make use of simple algorithms which leads to sub-optimal, exploitable play because of predictable or naïve behaviour. This project aims to create an Intelligent Agent that performs well versus competent human players, and to do so will need to be able to do everything a human can in terms of making judgement calls, exploiting the player's game position in order to inform its decision as to whether it should take a risk or play conservatively to avoid fouling itself.

[3]Warren is an OFCP bot which uses neural networks to train and improve, licensed out for use at a cost of between $7-79/month depending on chosen package https://www.playwarren.com/ (Accessed 2015)

1.4 Aims and Objectives

The project aim is to create a functional, competent Intelligent Agent for a bespoke Open Face Chinese Poker Web-Application, which can be broken down simply into two main objectives.

1. Create a game environment for Open Face Chinese Poker (Web-Application)
2. Create an Intelligent Agent which interfaces with the Application and plays the game

Beyond these simple functional requirements the Intelligent Agent must meet expectations for competent performance, making strong plays and avoiding the pitfalls of more simplistic Agents. To create a credible degree of perceived intelligence, the Agent must play tactically, avoiding fouling on one hand but not playing in such a risk averse and conservative manner as to limit potential for scoring and exploitation of a weak player position. In addition to this, the Agent must be able to make its decisions quickly in order to minimise delay in line with user demands for responsiveness.

With regards to the Web-Application itself there are multiple demands to be met in terms of reliability, usability and functionality. The interface must be intuitive and aesthetically appealing, and must feel responsive and efficient. As it will be a Web-Application accessible via the internet efforts must be made to optimise performance and decrease loads times through means such as compression of assets and redirection minimisation, also enabling larger support for multiple concurrent users and increasing scalability.

1.5 Ethics

Ethical issues can be a concern in Poker games when playing for money; consider for example the implications of a human player unknowingly playing versus a Poker Bot. This is certainly a major concern for variants of Poker such as Texas Hold'Em which has a large online following with numerous websites and applications where virtual Poker games are played for real money. Open Face Chinese Poker on the other hand is generally only played for money in home games, with friends or at casinos as a side-game during or after a Texas Hold'Em tournament. While some websites do exist for playing OFCP for money, these do not have the same level of following as more popular variants of Poker, and the Intelligent Agent produced in this project will not provide any kind of support for integration with these websites.

In the context of the proposed application, no money will be involved and participants will be playing purely for entertainment value and research interest in Artificial Intelligence. This mitigates potential ethical issues as the project focuses not on aspects of gambling but rather on the technologies and methodologies used in creating the Web-Application and system architecture, as well as the pros and cons of various algorithms for producing a sophisticated Intelligent Agent.

2 Background

2.1 Game Theory

In game theory, a zero-sum perfect information game such as Chess or Checkers can be formalised as a search problem, representing possible game states in a game tree (Özcan, 2004, p282). Such a tree can be explored and analysed in order to determine the best move from a given position, and to do so it is necessary to implement some form of search algorithm.

2.1.1 Minimax

While game theory has been discussed at least as far back as 1713, it wasn't until John von Neumman's research into the field that game theory was established as a mathematical discipline. Neumann's early efforts focused on proving the Minimax Theorem, which was published in his 1928 paper *Zur Theorie der Gesellschaftsspiele* (The Theory of Board Games). Proving the Minimax Theorem was considered by Neumman to be of utmost importance, saying "I thought there was nothing worth publishing until the Minimax Theorem was proved" (Casti, 1996, p19). The Minimax Theorem states that in any finite zero-sum two player game

there exists strategies for each player that minimise their maximum losses. These strategies must consider their adversary's possible responses, and the strategy which minimises a player's maximum losses is known as the optimal strategy. Expanding on his 1928 paper, Neumman co-authored *Theory of Games and Economic Behavior* in 1944, which presented a different proof of the Minimax Theorem, extending the theorem to include imperfect information games and games with more than two players. This 1944 publication was the first formal, coherent book in the field of game theory and is the basis for modern game theory.

Minimax searches a game tree, looking for moves that maximise a player's chance of winning. A minimax approach uses an evaluation function to score possible game states, choosing the moves that lead to the highest expected value for a given Agent. Implementation of a minimax search assumes that the opponent will play optimally and thus choose the next state with the lowest score from the Agent's perspective. A naïve minimax approach will produce optimal results assuming a perfect evaluation function, but explores nodes unnecessarily and takes a very long time because it will exhaustively expand the game tree, and due to combinatorial explosion is prohibitively slow for all but the most simple of games, or when exploration of the tree is limited to a certain depth. Tic-Tac-Toe for example has $9! = 362,880$ possible outcomes, which is feasible for a computer to compute, but this huge amount of nodes to consider for such a simple game demonstrates the impracticality of this naïve approach. For these reasons its use is limited in games with high branching factors unless optimisations are made.

2.1.2 Alpha-Beta Pruning

Alpha-Beta Pruning is an optimisation of the minimax algorithm, reducing the number of nodes that must be visited in a search. It was discovered independently a number of times by various researchers throughout the mid 20th century, and Arthur Samuel implemented an early version for Checkers in 1959 (Samuel, 1959, p210-229). This optimisation of the minimax algorithm is achieved through keeping track of and updating two scores; Alpha, which keeps a record of the best score possible by any means; and Beta, the worst-case scenario for the opponent. Any move with a value lower than this Alpha score can safely be pruned since a better move has already been found, and with the Beta value it can be assumed that any discovered move with a value higher than this will not be used by the opponent. With Alpha-Beta Pruning, a branch of the game tree is pruned when at least one possibility is found that is worse than a previously examined move. Pruning away branches in this way does not affect the quality of the result as these branches could not possibly influence the final decision, and therefore Alpha-Beta Pruning returns the exact same move as a standard minimax approach would, but does so faster and more efficiently. In the worst case Alpha-Beta Pruning must examine $O(b^d)$ leaf nodes, although assuming that best moves are always searched first, the number of leaf nodes evaluated will be as low as $O(\sqrt{b^d})$, reducing the branching factor to its square root, effectively meaning a search can be made to twice the depth with the same amount of computation (Russell, 2003).

2.2 Solving Games

Even with optimised approaches to game tree searches, managing and mitigating the complexity of game trees remains difficult due to high average branching factors. Simple games such as Tic-Tac-Toe can be trivially solved, and even more complex games like Checkers can be weakly solved with traditional algorithms. However, solving more complex games such as Chess and Go still remains a challenge. Chess can be partially solved with retrograde algorithms, solved for only some positions or when using smaller non-standard boards. Solving Chess is generally considered impossible with modern technology, and it is debatable as to whether future improvements in computer processing speeds will one day allow for brute-force solving of the game. Intelligent Agents for Go using traditional approaches generally can only perform well in a limited capacity, such as when playing on a much smaller than usual board. It is not expected for full-size 19x19 board Go to be solved anywhere in the near future, although modern approaches to creating Intelligent Agents for Go produce Agents that can play to a good standard.

Game	State-Space Complexity	Solved	Method
Tic-Tac-Toe	10^3	Strongly	Traditional Algorithms
Checkers	10^{20}	Weakly	Traditional Algorithms
Chess	10^{47}	Partially	Retrograde Analysis
Go 19x19	10^{171}	-	-

Table 2.2.1 - Showing the current state of some popular board games

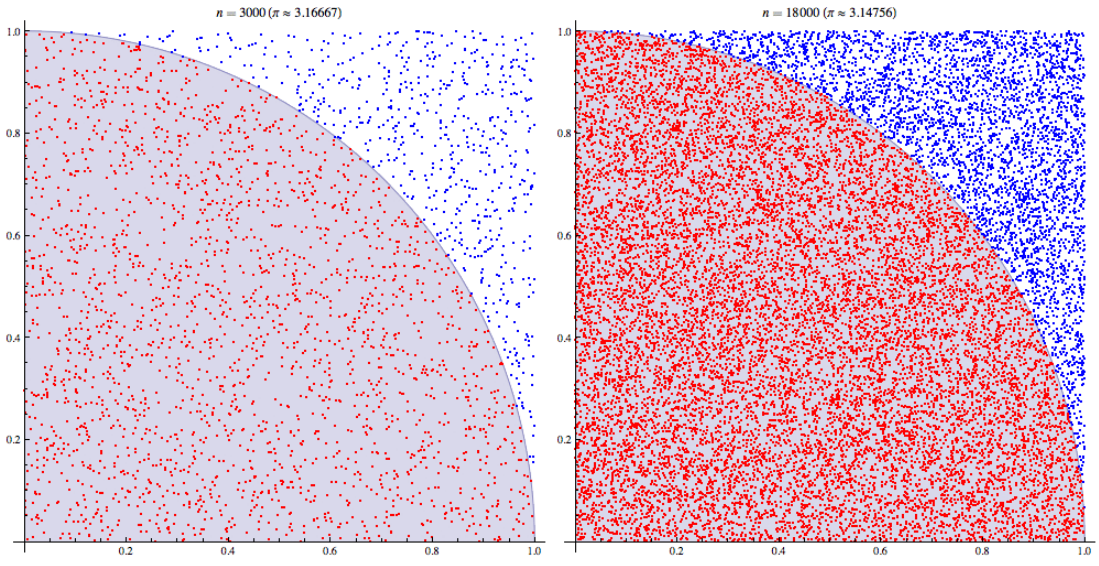
2.3 Games of Chance and Imperfect Information

The aforementioned board games have all been zero-sum perfect information deterministic games, which makes them well suited for the application of traditional search methods such as minimax and its derivatives. However, games such as Poker are stochastic in nature because of random cards, and in addition to any elements of imperfect information, this presents a challenge for implementation of an Intelligent Agent. Imperfect Information game trees can be made, using chance and decision nodes that are grouped into information sets, however since these nodes are not independent it means that algorithms such as Alpha-Beta Pruning cannot be used (Dizman, n.d, p3).

2.4 Monte Carlo Methods

Monte Carlo methods are computational algorithms which use random sampling over multiple trial runs in order to obtain usable, numerical results. Modern Monte Carlo methods have origins in the 1940s during work on nuclear weapons projects, and were used in the simulations required for the Manhattan Project, although were limited by the lack of technology at the time (Eckhardt, 1987, p131).

Implementation of Monte Carlo methods generally involves defining possible inputs and then randomly generating inputs within this range, computing the value or result for each simulation and finally aggregating results. An example of this process is illustrated below in the context of calculating π by random distribution of points within a given area. As the number of trials increases the fraction of points distributed within the circle's area diverges on $\frac{\pi}{4}$, which can then be multiplied by 4 to give a reasonably accurate estimation of π.



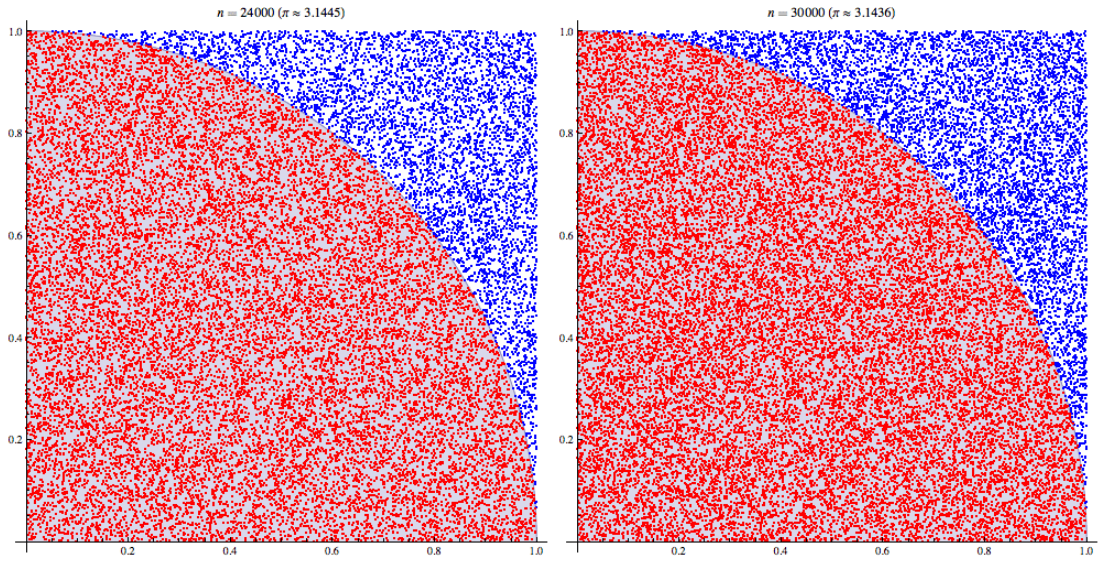


Fig 2.4.1 - Monte Carlo used in estimating the value of pi

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Monte Carlo Methods have applications in a wide range of fields from physics and mathematics to business and finance, and can be particularly useful in applications where domain specific knowledge is limited or difficult to implement.

2.5 Monte Carlo Tree Search

Monte Carlo methods can be used with good results for applications in games, and since 2006 have been developed into a technique called *Monte Carlo Tree Search (MCTS)*, a term coined by Rémi Coulom during his application of the method to play Go through expansion of the search tree based on random sampling (Coulom, 2007, p72-83). In MCTS, simulated games or *playouts* are played out to the end through random move selection, and the final result of each simulation is used to weight the nodes used, meaning that better nodes are more likely to be chosen in future playouts. MCTS in its most basic form follows a fairly simply process, generating and scoring a search tree node by node according to the results of the simulated playouts. The diagram below illustrates this.

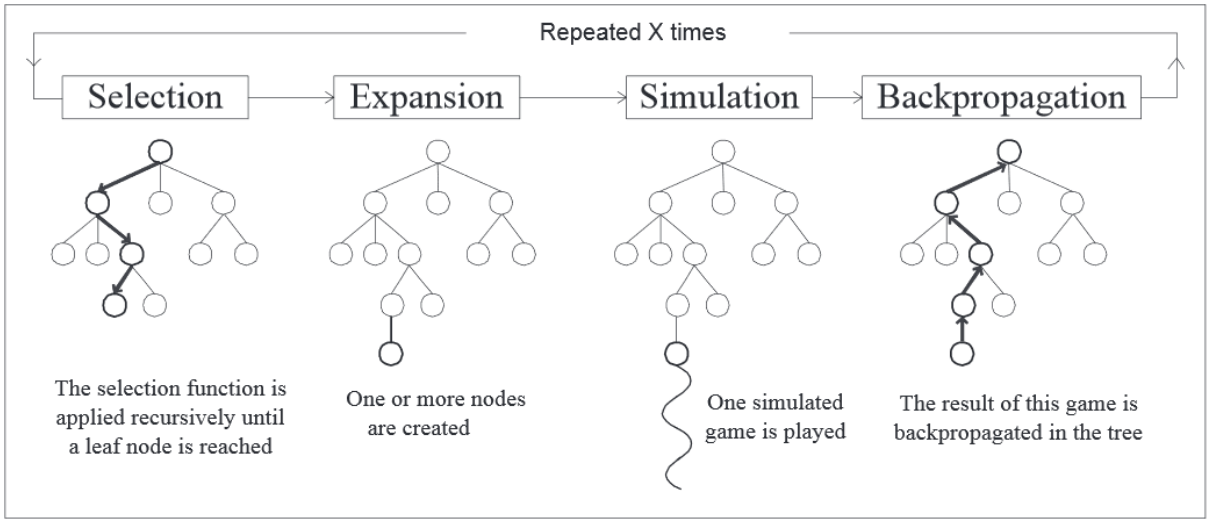


Fig 2.5.1 - Methodology of a Monte Carlo Tree Search (Figure from Chaslot et al., 2008)

MCTS has quickly gained traction as a strong general purpose algorithm for AI in games due to its effective results with potentially low space and time complexity. The game tree in MCTS grows asymmetrically, with more promising branches being favoured for exploration. Because of this MCTS can be more efficient than traditional algorithms and produce better results as it is better suited to dealing with the problems posed by high branching factors. One of the most enticing benefits of MCTS is that it requires no strategic or tactical knowledge about a problem domain other than end conditions and legal moves, making MCTS implementations flexible and applicable to a variety of problem domains with little modification (Browne et al., 2012, p9).

2.5.1 Upper Confidence Bounds Enhancement

While the basic MCTS algorithm is not perfect, there have been many proposed enhancements. Perhaps the most well known of these uses an *Upper Confidence Bounds* formula formalised by Kocsis and Szepesvári (2006, p1-12), as shown below.

$$v_i + C \times \sqrt{\frac{\ln N}{n_i}}$$

v_i is the estimated value of a node, C is a tunable exploration parameter, n_i is the number of times the nodes has been visited and N is the total number of times the node's parent has been visited. This enhancement provides a strong balance between exploitation of high scoring nodes and exploration of nodes that have not been explored thoroughly. As with any method based on random sampling, initial estimates will not be reliable but over time and with more iterations the results will diverge on finding the optimal move. Use of UCB with MCTS is known as the *Upper Confidence Bounds for Trees* (UCT) method.

Because of the asymmetric tree growth MCTS is suitable for deep and broad explorations of larger search spaces that would be infeasible for a typical search algorithm, and because of the adaptability in regards to exploitation and exploration, optimal moves will be found and focused on. MCTS therefore has revolutionised Artificial Intelligence approaches, most notably in Go which has seen various successful applications of MCTS and MCTS with UCT to create sophisticated Intelligent Agents which present a credible challenge to even skilled human opponents[4].

2.5.2 Information Set Monte Carlo Tree Search

Information Set Monte Carlo Tree Search (ISMCTS) follows on from application of MCTS to games involving hidden information, aiming to overcome the shortcoming of previous approaches to dealing with hidden information such as determinisation techniques. ISMCTS uses randomisations of the current game state to make guesses of hidden information, with each determinisation conceivably representing what could actually be the current game state based on the Agent's observations so far (Whitehouse et al, 2013, p101).

ISMTCS is useful for games like traditional Texas Hold'Em poker where elements of hidden information mean an Agent lacks all necessary information to make a well-informed decision. ISMTCS models various possible game state variations of what other players could have, attempting to guess other players cards based on previous information. An advantage of ISMCTS over approaches such as determinised UCT is the use of better strategies which more closely resemble the decision making processes that a human player would make (Cowling et al, 2012, p121).

This approach is not needed for standard OFCP because it is a perfect information game. However, ISMCTS could come into play in an implementation of a custom variant of OFCP such as Pineapple OFC, which does have elements of hidden information due to players secretly discarding one of their dealt cards each hand. ISMCTS could be used to model which cards are unlikely to have appeared based on how the opponent plays; for example, if a player has a King on bottom row but does not pair it in the next hand then it is almost certain that the discarded card is not a King. Information can be built in this way to influence determined probabilities of certain cards appearing and the Intelligent Agent can act appropriately, tweaking expected probabilities for outs and changing strategy to match this information.

[4]Human-Computer Go Challenges Records http://www.computer-go.info/h-c/index.html (Accessed 2015)

2.6 Hand Evaluation Algorithms

For any Poker game - and indeed for any Poker AI - hand evaluator functions are necessary to rank, score and compare hands. A naïve approach to the problem could involve step-by-step comparison of a given hand against each entry in a comprehensive list of hands and matching ranks. Such a technique would generally prove efficient in terms of time-complexity and could prove to be significantly faster than performing scoring calculations on demand. One problem with this approach however is that it would necessarily involve pre-computation and scoring of every single possible hand combination, which would undoubtedly be a lengthy process. Another issue is potential memory and space constraints, dependent on the system and application in question. There are $\binom{52}{5} = 2,598,960$ possible Poker Hands, and representation of each would amount to a sizeable lookup table[5]. This could prove problematic for example in the case of a pre-packaged application; it would be incredibly undesirable to include such a large file.

In situations where such a technique is infeasible or undesirable, for example due to memory constraints, other approaches are possible. On-demand hand evaluation can be achieved in a variety of ways, for example through a simple histogram approach which reads in a hand and records the frequency of each card rank. Using this information the evaluator would be able to work out the hands rank based on highest rank frequencies. For example, a hand consisting of "2 of diamonds, 2 of hearts, 5 of clubs, 7 of spades, King of diamonds" would be represented in a histogram as seen below.

Card Rank	2	3	4	5	6	7	8	9	10	J	Q	K	A
Rank Frequency	2	0	0	1	0	1	0	0	0	0	0	1	0

Table 2.6.1 - Demonstrating how a histogram approach would store information about a hand

In this example the evaluator would then analyse the histogram, recognising that the highest frequency rank was 2 (indicating a pair), and that no other frequency was higher than 1 (indicating there is no second pair), and that the highest ranked card with a frequency above 0 was a King. A potential format of this information could be along the lines of $[1, 2, 13]$, where the first number indicates the overall hand rank (1 signifying a Pair), the second number indicating the rank of the card with this frequency (2), and the third number indicating the kicker (13 signifying a King). This could then be used directly elsewhere for comparison, or interpreted into a more readable format such as "Pair of 2s, King kicker".

The advantage of this approach comes in its simplicity as it is trivial to implement and easy to understand, and provides a strong compromise between space and time complexity. One limitation to note is that simply mapping frequencies of hands is not enough to produce complete results; additional consideration must be made to check for straights and flushes. However, this does not pose a significant challenge, as simple checks can be made if the highest frequency found is 1 to see whether all the suits are the same, or if the 5 cards are sequential.

These kind of approaches to hand evaluation have reasonable efficiency, with the potential to evaluate many hundreds of thousands of poker hands per second, and considering the relative ease of implementation are perfect for most purposes. However, more efficient algorithms require less processing power enabling higher throughput and faster execution times, and as such can be hugely beneficial for large scale applications. Other approaches to hand evaluators generally are not so simple, using shrewd techniques and taking full advantage of the efficiency of low level languages such as C. One such evaluator is Cactus Kev's[6], the principle component of which is the realisation that while there are 2,598,960 possible combinations of Poker Hands, these can be sorted into 7462 distinct categories. Through the use of various methods including efficient card representations, bitwise operations, lookup tables and binary searches, an incredibly fast hand evaluator can be constructed. The main disadvantage of using such an approach is the complexity to implement, and the ease of deployment is entirely dependent on the specific environment due to the nature of compiled C code. In addition, the feasibility of using such an evaluator is dependent on the compatibility of any other technologies and languages used.

[5] For example, this Poker hand lookup table is 100MB https://github.com/chenosaurus/poker-evaluator/blob/master/data/HandRanks.dat (Accessed 2015)

[6] For more information on how this algorithm works see here http://www.suffecool.net/poker/evaluator.html (Accessed 2015)

3 Design and Approach

3.1 Requirements Specification

The requirements for this project were driven by the need to meet user demands and expectations for the Application, as well as ensuring the quality of the Intelligent Agent produced. The functional requirements below describe what the system will do.

1. Create a game environment for Open Face Chinese Poker (Web-Application)
   (a) The Application must implement appropriate rules for the game - fouling, scoring system
   (b) Enable alternate round support - player acts first in Round 1, Intelligent Agent acts first in Round 2 etc.
   (c) Use appropriate networking technology to support concurrent connections so multiple users can play at once
2. Create an Intelligent Agent to play Open Face Chinese Poker
   (a) The Intelligent Agent must interface with the Web-Application in order to make its moves
   (b) The Intelligent Agent must adhere to all rules of the game
   (c) The Intelligent Agent must not use any information a human player in its position would not have (i.e. it will not cheat)

Building on these are non-functional requirements specifying how the system will work, which fall into 4 categories: Reliability, Usability, Supportability and Performance.

- **Reliability**
  1. The System must be reliable with minimal errors and failures: Long Mean-Time-To-Failure
  2. Uptime of the Application should be maximised, meaning minimal downtime for upgrades and avoidance of fatal crashes
  3. Implement support for performance monitoring through logs of errors, server actions and statistics (e.g. time taken for requests)
- **Usability**
  1. The Application should be responsive and quick to load
     (a) The website should load in an acceptable time ($<$1s is optimal)
     (b) The Application should feel responsive e.g. when dragging cards
  2. Interface elements should be intuitive and easy to understand (clearly labelled buttons, instructions)
  3. The aesthetics of the Application should be pleasing - good layout, colour scheme, graphical elements
- **Supportability**
  1. Maximise potential for scalability through maximised throughput and appropriate optimisations to reduce Application's footprint
  2. Reconfiguring the Application should not require extensive or problematic changes.
     (a) Tweaking values should be easy e.g. time limit for Intelligent Agent's decisions
     (b) Adding features shouldn't break the system - achieved through good design structure
  3. The Application should be compatible with as many devices as possible

4. Installation of the Application should be easy, requiring minimal reconfiguration

- **Performance**

1. Users should not have to wait more than 5 seconds after their input for the Intelligent Agent to make its move
   (a) Employ heuristics and efficient algorithms to optimise performance
2. Stress Requirements: The Application must be able to support at least 100 concurrent users
3. Throughput: At least 95% of the time, the Application should take no more than 5 seconds for any request
4. The Intelligent Agent must play competently and provide a significant level of challenge to human opponent

3.2 Use Case and Data Flow Diagrams

Most of the complexity of the Application will be hidden from the user, with heavy processing of data and algorithms handled behind the scenes. In terms of interaction with the application there will be a frontend interface which will allow the user to interact with and play against the Intelligent Agent. At its highest level of abstraction from the user's perspective, a use case can simply boil down to the diagram below.

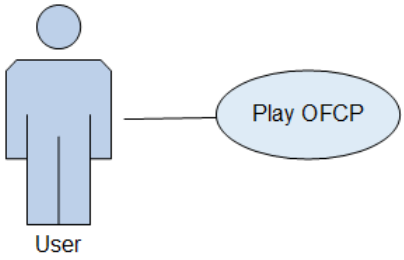


Fig 3.2.1 - Highly abstract Use Case Diagram for the Application from an end user's perspective

A more useful analysis of how a user will interact with the Application considers individual actions a user will need to be able to carry out, which is presented below.

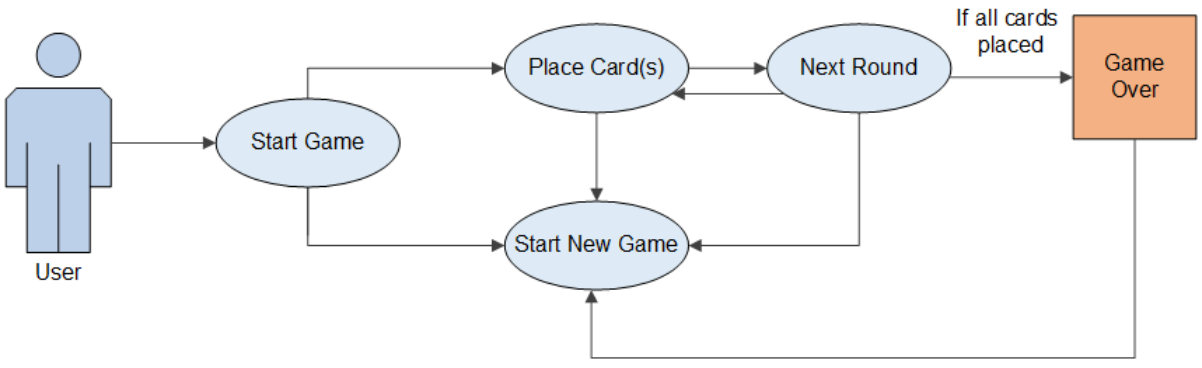


Fig 3.2.2 - Individual actions and events necessary for the user to interact with the Application

For the Application to function there must be a sequential process of user inputs followed by Application outputs, demanding further inputs and so on. For example, after a user places their cards for one round, the Application must output a response with the Intelligent Agent's card placements as well as the next card for the player to place. In order for this to work there must be at least 2 tiers for the application: a frontend and a backend. The frontend will be responsible for displaying information to the user and handling the

interface, while the backend will be responsible for the main processing tasks such as processing the game state, determining the Intelligent Agent's moves and handling the game logic. This will require a 2 way flow of information from the frontend of the Application to the backend, with the frontend sending data describing the user's inputs to the backend, which will process this and return a formatted response which the frontend can interpret and display to the user.

In practice, the Application will make use of a standard three tier architecture, with a frontend for client interacton; an Application layer for the processing tasks and communication between the higher and lower tiers; and a database for storage and management of game state information. A data flow diagram has been constructed to visualise this process.

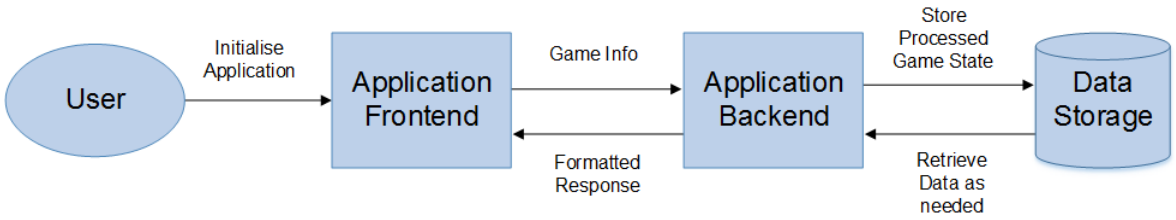


Fig 3.2.3 - Data Flow Diagram modelling the 2 way flow of information in the Application

3.3 Design Overview

Most client requests will be handled by a Web Server serving static content such as images, CSS and JavaScript files. Dynamic requests, however, will be passed by the Web Server to an Application server, which will then process the request, undertaking any necessary interactions with other pieces of software before formatting and returning a response. This response will then be passed up from the Application Server to the Web Server and from there back to the client's browser. The diagram below illustrates this process.

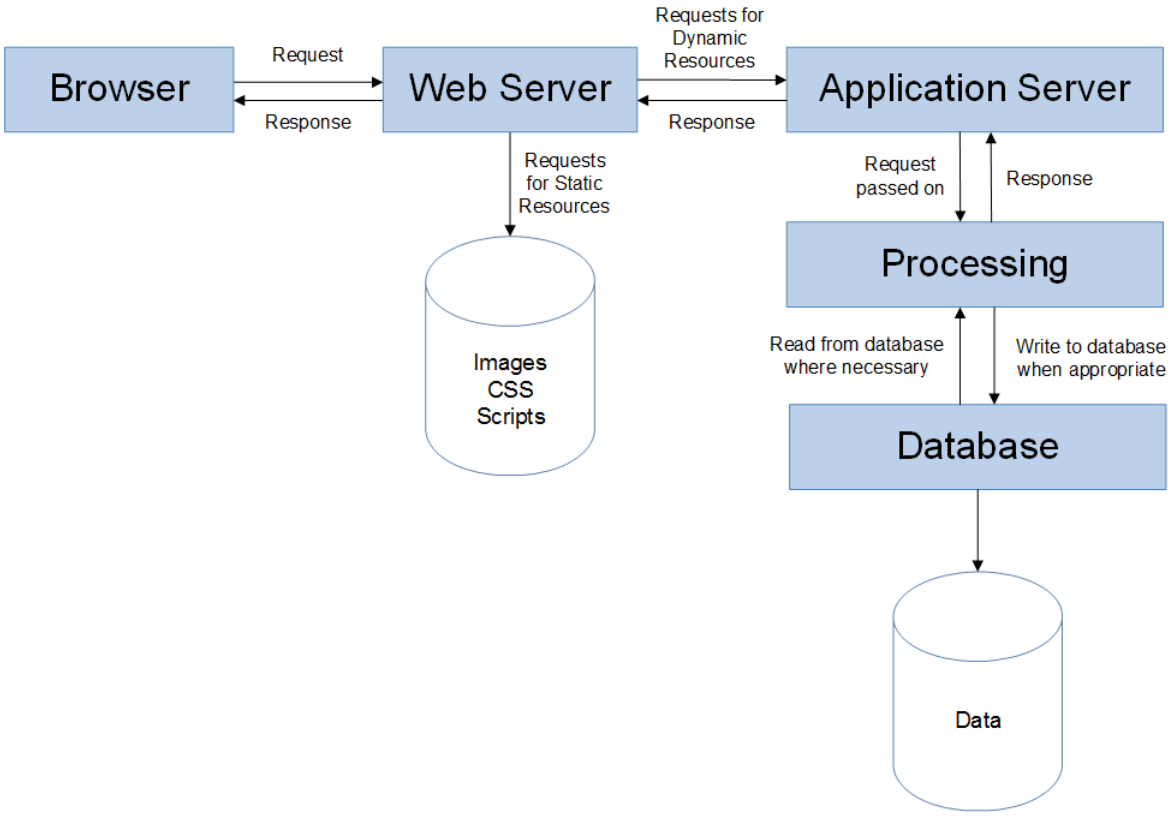


Fig 3.3.1 - System Architecture Diagram

A lower level of abstraction from this System Architecture considers specific technologies which will enable the system to function as a whole. The Application will be hosted using a Virtual Private Server running Ubuntu 14.04, aided by various technologies. Use of HTML, CSS and JavaScript will be largely responsible for the frontend of the application and will be served using an Nginx reverse proxy server. The Jinja2 template engine will be utilised in order to produce dynamic web pages, which will be served through Nginx by an Application Server implemented with CherryPy, a Pythonic Web Application Framework. User actions which require a response will be handled using JavaScript which will send HTTP POST requests containing the game state information to the CherryPy server. Here, the request will be parsed and sent on to the backend for processing, which will be handled with Python scripts. The backend will validate the sent game state and call helper functions and scripts to handle processes such as hand evaluation and scoring, handling the Intelligent Agent's turn and dealing cards. If everything is in order and once all necessary processes have been completed, the game state will be stored in a MongoDB document-oriented database, and an appropriate response containing information for the frontend will be produced in JSON format and sent back to the client. The JavaScript will parse and handle this response, reflecting the updates to the game state on the web page by dynamically creating and updating HTML DOM Element Objects.

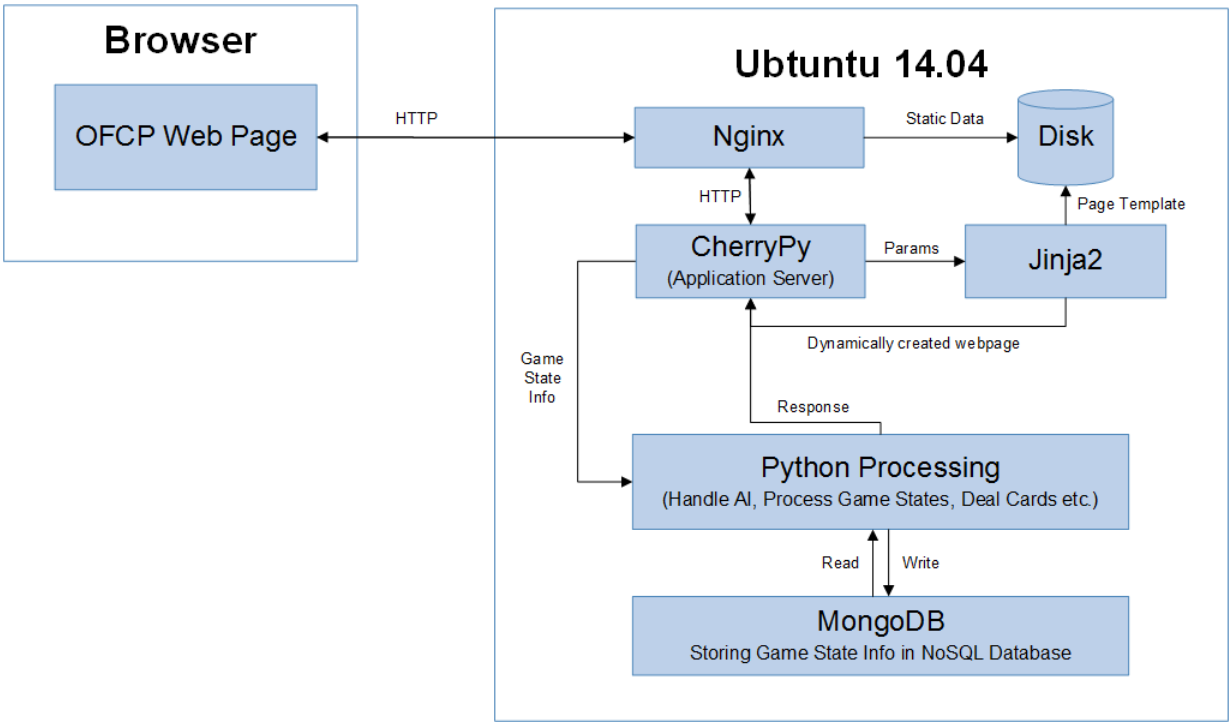


Fig 3.3.2 - Software Architecture Diagram

3.4 Wireframe

The Application's frontend will consist of one dynamic webpage, using Jinja2 templates to enable persistent information across multiple rounds and page refreshes, with content dynamically updated and created using JavaScript as required. Following Jakob Nielsen's general principles for interaction design (Nielsen, 1994, p115), the interface for the site will be made as intuitive as possible with a clear, well-structured layout. The simpler the interface, the easier to understand and use, and following this concept the wireframe mockup presented below was produced.

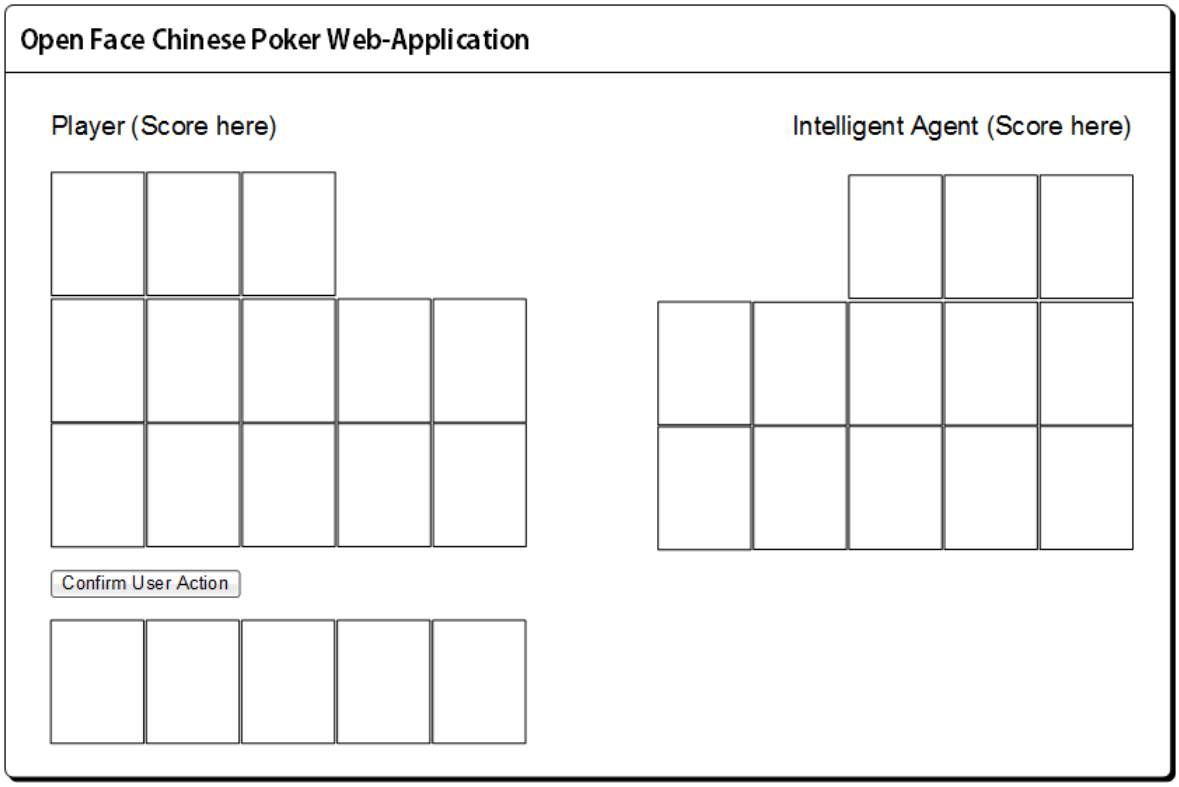


Fig 3.4.1 - Wireframe of the Interface demonstrating a clear, simple layout in adherence to principles for interaction design

The player and Intelligent Agent have distinct, separate play areas on either side of the screen where they will place their respective cards. Use of the interface is as simple as possible, with cards to be placed appearing at the bottom of the screen to be dragged and dropped into free positions on the board. The process of starting a game, finalising card placements and moving onto a new round will all be controlled through one dynamic button, making the game process as simple and intuitive as possible for the user; there are no surprises and behaviour remains consistent throughout the game. This is in strict adherence to Nielsen's aforementioned usability principles, specifically regarding consistency.

3.5 Design Principles

The design principle to adhere to throughout the process of creating the application will be Rapid Application Development using Evolutionary Prototyping, creating a functional prototype at each stage which can be refined and updated to implement new features and meet changing requirements per user feedback. This approach is ideal for the needs of the project as it allows for a flexible approach and means that emphasis can be put on development, creating a functional or semi-functional application at each stage, implementing some of the planned features, meeting some of the requirements and being ready to build upon and develop further into a new version which improves upon itself. This flexible style is naturally advantageous over a more traditional approach such as the Waterfall model which involves rigorously defining specifications from the start, which means making changes down the line becomes increasingly difficult and costly; such a style of development was appropriated from other industries before more suitable methodologies of software development were formalised. Rapid Application Development allows for a versatile development-driven approach which naturally incorporates user feedback, design and testing as part of the cyclic process, which hopefully will result in a better, well-rounded final product.

4 Implementation

4.1 Technologies and Architecture

4.1.1 Hosting with Virtual Private Server

The Web-Application was deployed on a Virtual Private Server hosted by DigitalOcean[7]. It was set up with a custom stack along the lines of a LEMP stack, although with some minor changes, using Ubuntu 14.04, Nginx, MongoDB and Python. This allowed for a powerful, customisable and robust architecture that met the needs of the project well. Originally a simple LAMP stack was used, but this was reconfigured and modified over time meeting changing requirements and implementing changes to increase performance; the lightweight nature of nginx is naturally advantageous over the bulkier Apache and better suited the needs of this project[8].

4.1.2 Frontend (OFCP Game Webpage)

The page for the Web-Application was created with HTML, CSS and Javascript, with Jinja2 templates used to create persistent, consistent content across page refreshes and round changes. This was important so that the scores were retained across multiple rounds, and was useful for storing information pertaining to each specific page such as the game id and and a boolean used in ascertaining whether the player or Intelligent Agent acts first. This also allowed for dynamic creation of page elements, namely, the cards placed, which meant that these could be displayed across page refreshes without being lost.

```
<!-- middle row for player 1 -->
{% for i in range(1,6) %}
    <div class="cardl" id="p1_middle{{ i }}" ondrop="drop(event)" ondragover="allowDrop(event)">
        {% if game_state['properties1']['cards']['items']['position' ~ (i+5)] is not none %}
            <img ondragstart="return false;" width="109" height="150" src="../static/cards/{{game_state['properties1']['cards']['items']['position' ~
(i+5)]}}.png" name="{{game_state['properties1']['cards']['items']['position' ~ (i+5)]}}" style="...">
        {% endif %}
    </div>
{% endfor %}
```

Fig 4.1.2.1 - Use of the Jinja2 Templating Engine in creating a dynamic webpage

Most of the aesthetics were controlled through external style sheets, allowing for easy customisation of layouts. That said, certain styling information was controlled inline or through modification with JavaScript. One example of this is the styling of the board positions for the player; when a user drags a card their board positions are highlighted in green for increased usability and visual appeal, which is controlled through JavaScript in the drag event handlers.

Early prototypes of the application had a very simple, plain interface which was clear to use, although not very visually appealing. In response to user feedback the interface had a visual rework, using a better colour scheme with updated graphics and more visually appealing elements.

[7] See https://www.digitalocean.com/ for more information on the services they provide (Accessed 2015)

[8] For more information on the differences between Apache and Nginx see here https://www.digitalocean.com/community/tutorials/apache-vs-nginx-practical-considerations (Accessed 2015)

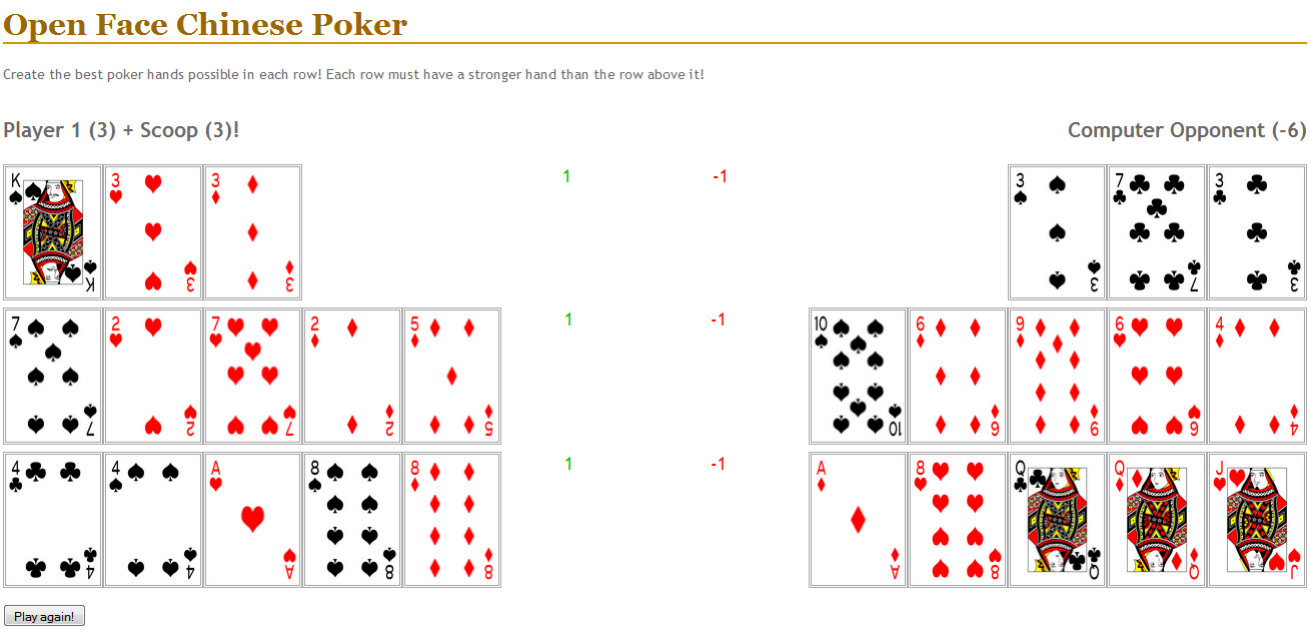


Fig 4.1.2.2 - Early prototypes of the interface were functional but basic

Fig 4.1.2.3 - In response to user feedback a more visually appealing interface was produced

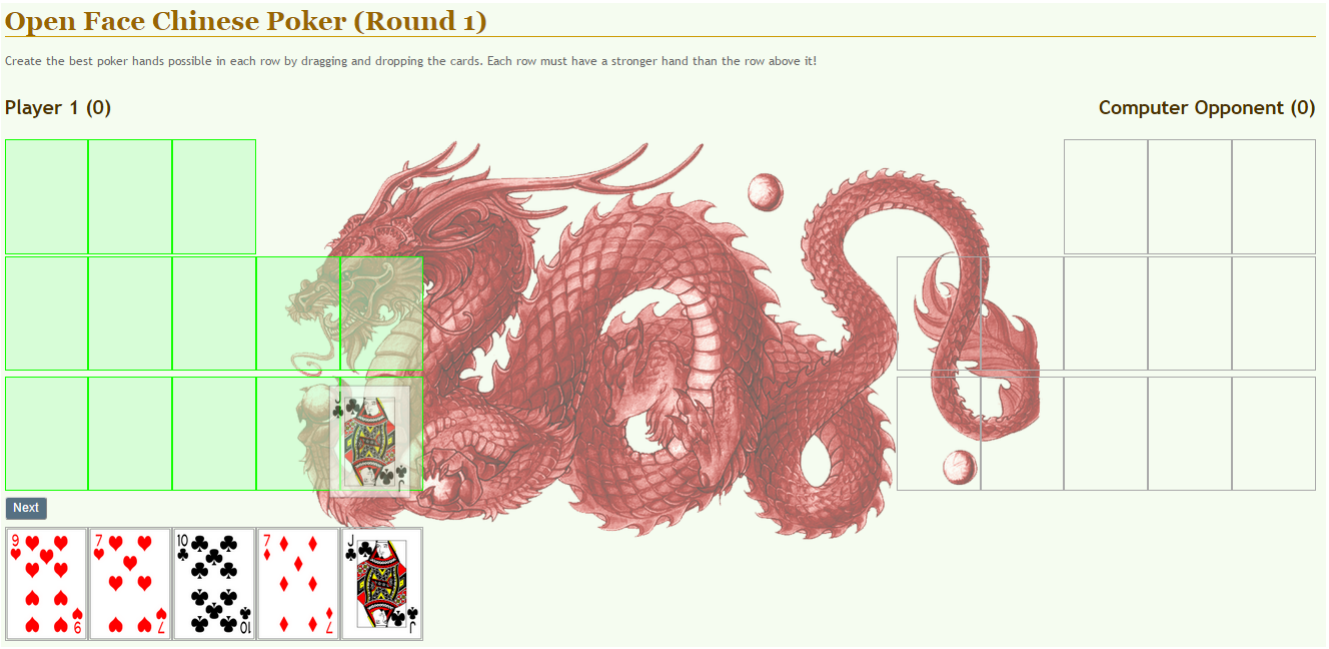


Fig 4.1.2.4 - dragging a card highlights the positions on the board for aesthetic appeal and usability

4.1.3 Middle Tier (Web Server and Application Server)

The application makes use of pure python networking with CherryPy, which is an efficient and lightweight Web Framework. Using CherryPy for the application server worked well as it can handle up to 1000 concurrent requests, which is more than enough for the requirements of this project. This part of the application is responsible for handling requests for dynamic content, acting as a middle man between the frontend and the backend of the system. The CherryPy server handles parameters received and processes these, returning a dynamically created template as appropriate. It is also responsible for listening for HTTP POST requests from the frontend, which contain game state information that is decoded and sent to the backend for processing, before returning the formatted response to the client.

CherryPy was set up to operate behind an nginx reverse-proxy[9], reducing stress on the Application server as nginx was configured to handle client requests for static resources. CherryPy could have handled these requests as well but configuring the application in this manner means that scalability in the future is more feasible, and played to the strengths of nginx which can handle large amounts of concurrent users and excels at serving static content.

4.1.4 Backend (Processing and Database)

The core functionality of the backend is handled entirely through Python scripts, handling the various necessary processes such as validation of game states, dealing of cards, handling the Intelligent Agent's card placements, hand evaluation and scoring and communication with the database. These processes were subdivided into multiple scripts to form a more coherent structure, with independent scripts called as needed from a main game_logic handler. The diagram below shows a breakdown of the structure of the backend.

[9]This tutorial was incredibly helpful for this purpose: https://www.digitalocean.com/community/tutorials/how-to-deploy-python-wsgi-applications-using-a-cherrypy-web-server-behind-nginx (Accessed 2015)

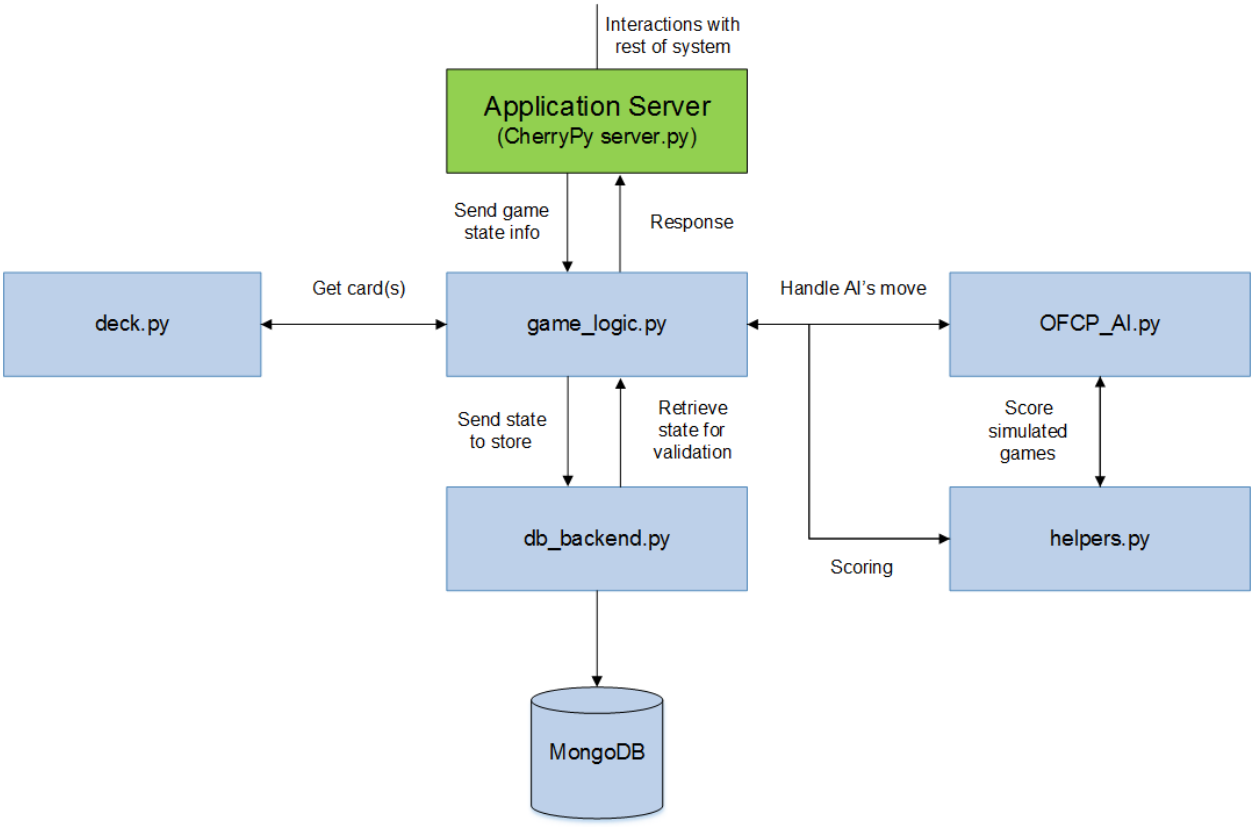


Fig 4.1.4.1 - Structure of the system's backend

Game_logic.py processes the game state, working out what stage the game is in and handling the necessary logic and game flow. This main script is responsible for validating the game state passed from the frontend against the recorded game state in the MongoDB database. Cards are retrieved from a deck object (initialised at the start of a round) which are passed back to the player or through to the AI script as appropriate. In OFCP_AI.py the Agent's moves are decided through running Monte Carlo simulations of possible placements of the given card(s) in the current game state. Use of supplementary functions in helpers.py allows the AI to score simulated game boards in order to calculate and aggregate the expected value for various moves. Once the Agent has determined its move this is passed back to the main game_logic.py script, which formats this information in the game state to be returned to the frontend along with the player's next card, and updates the stored state in the database. At the end of a round game_logic.py calls a scoring function in helpers.py and then updates the database and returns this scoring information to the frontend for display.

MongoDB is a document-oriented database (as opposed to a traditional relational database e.g. using SQL), which decreased development time and reduced complexity as there was no need to constantly transform the data when reading from the database into the python backend. MongoDB offers a scalable and high performance solution to data storage, and allows for flexible data structures, for example with optional values being handled trivially. The database gets type information from the data itself meaning it can map easily into program objects, which is specifically advantageous in this application because of the use of dictionaries to store the game state. The flexibility and ease of deployment of such a style of database makes its use well suited for web-applications such as this one. For ease of deployability and redployability, MongoDB was set up to run in a virtual docker environment.

4.1.5 Distributed Version Control System

Use of version control is paramount as it allows for undesirable changes to be rolled back easily, and if something goes wrong there are always working versions available to roll back to. In this project I opted for the use of GitHub due to its high usability and support. Using multiple branches (master and experimental) meant

that a stable version could be maintained while new features were implemented safely on the experimental branch without affecting or potentially breaking the master version. GitHub also allows for releases of different versions of the software, which was useful for maintaining historical releases of stable, working applications, for example a legacy client-side only prototype of the application.

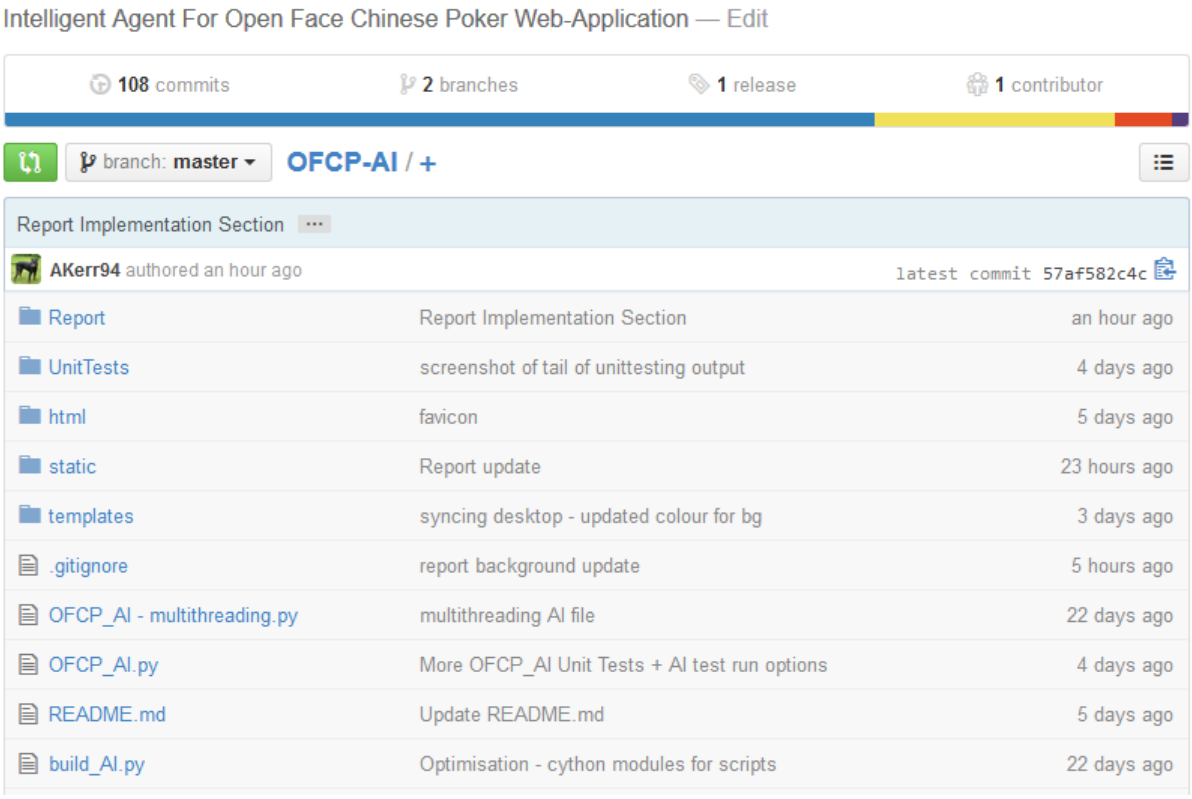


Fig 4.1.5.1 - Throughout the development of the application, project files were backed up in a private GitHub repository OFCP-AI

The advantage of a Distributed Version Control System such as GitHub in comparison to Local or Centralised Version Control is that there is not one single point of failure; without proper alternative backups there is a credible risk of losing everything in the case of a failure. With a DVCS the entire repository is mirrored by clients meaning there are multiple backups, any of which can be used to restore the repository.

4.2 Intelligent Agent

4.2.1 General Approach

The general method used to implement the Intelligent Agent was a simple Monte Carlo approach. Although there are 13 separate positions to decide between for the placement of a card, because the order of cards in each row does not matter the effective choice is between either top, middle or bottom row. The Agent carries out Monte Carlo simulations of its given card in each of the rows many times, populating the rest of the board with random cards and scoring the game state, recording this result. After all simulations have been carried out, results are aggregated and the row which produced the best results is chosen for the card placement. The random nature of this methodology means initial results are naturally unreliable, but as the iterations increase the results produced become more and more reliable.

4.2.2 First 5 Card Placements

For the Intelligent Agent to determine the optimal placements for the first 5 cards, analysis is first performed to find all permutations of possible card placements. This results in an initial total of $\binom{13}{5} = 1287$ states, but

this can be pruned down further by removing duplicate states with the same cards in each row but ordered differently, reducing the maximum possible states to consider to around 240 which is a more manageable amount although still isn't ideal.

Here, heuristics and domain specific knowledge are implemented to further prune the number of states considered. The hand is evaluated, mapping the frequencies of each occurring rank in a histogram. In the case that the hand contains a Pair, Three of a Kind, or Four of a Kind then any states which do not place these cards in the same row can be pruned as they would almost inevitably be sub-optimal. The better the initial starting hand, the fewer possible states the Agent must consider.

In the worst case scenario with 5 unique ranks and no flush or straight potential, some last resort pruning is implemented, removing certain state combinations that can be considered unlikely to produce optimal results, such as placing all cards in middle, or having three cards dumped in the top row. This reduces the number of states to consider, although will still likely mean the Agent must consider ~ 200 different states. Considering the requirement for the Agent to return its move in 5 seconds, this will likely result in suboptimal placements as there will only be $\sim \frac{5}{200}$ seconds worth of computation time spent simulating each state. Such a limitation is unfortunate, and is discussed later in this report in the Evaluation section.

The results of each game playout are stored and after all simulations have been completed the results are aggregated and the most promising scoring state's card placements are chosen.

4.2.3 1 Card Placements

Subsequent card placements are a lot easier and do not require such bespoke analysis, simply carrying out simulations of the given card's placement in each row for as many seconds as possible. The allowed simulation time is hardcoded in the AI script, and was tweaked in response to user feedback and demands for the Agent's performance. A 5 second response meets the requirements specified, although users generally found that 5 seconds seemed too slow, and a sweet spot for trade-off between responsiveness and usable results was between 2 and 4 seconds. With 1 card placement and only 3 choices to make (place card in top, middle or bottom), thousands of simulations for each placement can be achieved even on low-spec hardware, which generally produced close to optimal results.

4.3 Optimisations

The Web-Application functions well, and the System Architecture is robust. Performance issues arise in the backend however due to heavy processing demands from the Intelligent Agent. As aforementioned, use of heuristics to prune the potential game states for the initial 5 placements were implemented in order to reduce the complexity of the search space, and this had a dramatic improvement on the Agent's performance.

Further optimisation could be had through implementation of a more sophisticated algorithm, as well as by using a more efficient language. The hand evaluator used produces complete results and has reasonable efficiency, although again in this regard a more efficient evaluator would have increased overall performance. Time constraints on the project were the main reason why fundamental improvements to these areas of the system could not be made, although attempts were made to mitigate performance issues through further heuristics and techniques.

For example, using Cython[10] noticeably increased the performance of the system, although involved compiling the modules after every change, which in many ways subverted the usefulness of developing in Python in the first place, and produced very bulky compiled scripts. Fortunately however, a more suitable solution was found through the use of PyPy[11], a *Just In Time* compiler capable of converting Python to machine code at run time. The speed increase with PyPy was comparable to that of Cython and preferable in the sense that it simply worked - there was no need to compile modules every time a change was made - which allowed for faster deployment of updates and produced just as good results.

4.4 Live Implementation

As of the time of writing (May, 2015), the application is available to play at the following URI:
http://www.alastairkerr.co.uk/ofc

[10] Cython is an optimising compiler for Python http://www.cython.org (Accessed 2015)
[11] http://www.pypy.org (Accessed 2015)

5 Testing

5.1 Validation of Inputs

Tests were run throughout the development process to ensure proper validation of inputs, as well as making sure that appropriate exception handling was implemented. This was achieved through various means including type checks, value checks, try catch blocks and assertions. Below is an example block of code in which some of these methods can be seen.

```
if (type(hand) is not str):
    print "Invalid hand (required type = string), " + str(hand) + " is " + str(type(hand)) + "\n"
    return None

if (len(hand) != 15):
    print "Invalid Hand. Required format e.g: c09c10c11c12c13 (clubs straight flush 9->King).\n"
    return None

try:
    print ('Reading in hand: ' + str(hand) + '. Reformatting now...\n' )
    cards_list = []
    formatted_hand = ""

    rank_dic = {'10':'T', '11':'J', '12':'Q', '13':'K', '14':'A'}

    for i in xrange(0,15,3):            # decode string to get each card name. index 0 -> 14 step 3
        suit = hand[i]
        rank_p1 = hand[i+1]
        rank_p2 = hand[i+2]

        suit = suit.upper()             # evaluator needs suit as uppercase char

        if suit not in ('H','D','S','C'):
            print "Invalid suit! Expected H, D, S or C. Actual:", suit
            return None

        rank = int(rank_p1 + rank_p2)   # get numerical value for rank
        if ( rank < 1 or rank > 14):
            print "Invalid rank. Accepted range 1-14.\n"
            return None
```

Fig 5.1.1 - Sample code from function 'reformat_hand_xyy_yx' in helpers.py script: use of input validation and try except blocks to catch errors

Individual tests were run with test data comprising of a mixture of valid and invalid inputs, testing that the validation could catch invalid data and invalid types, and throw proper exceptions or print out appropriate usage messages.

```
test_items = ( 'c05c06c07c08c09','s05c05h09s08d13','h13c01s03d05c07','invalid',100,'fakestring',('i','am','invalid'),'123456789112345' )
for item in test_items:
    format_resp = helpers.reformat_hand_xyy_yx(item)
    if format_resp != None:
        print 'Formatted ' + str(item) + ' -> ' + str(format_resp) + '\n'
```

Fig 5.1.2 - Test inputs to ensure function works as intended

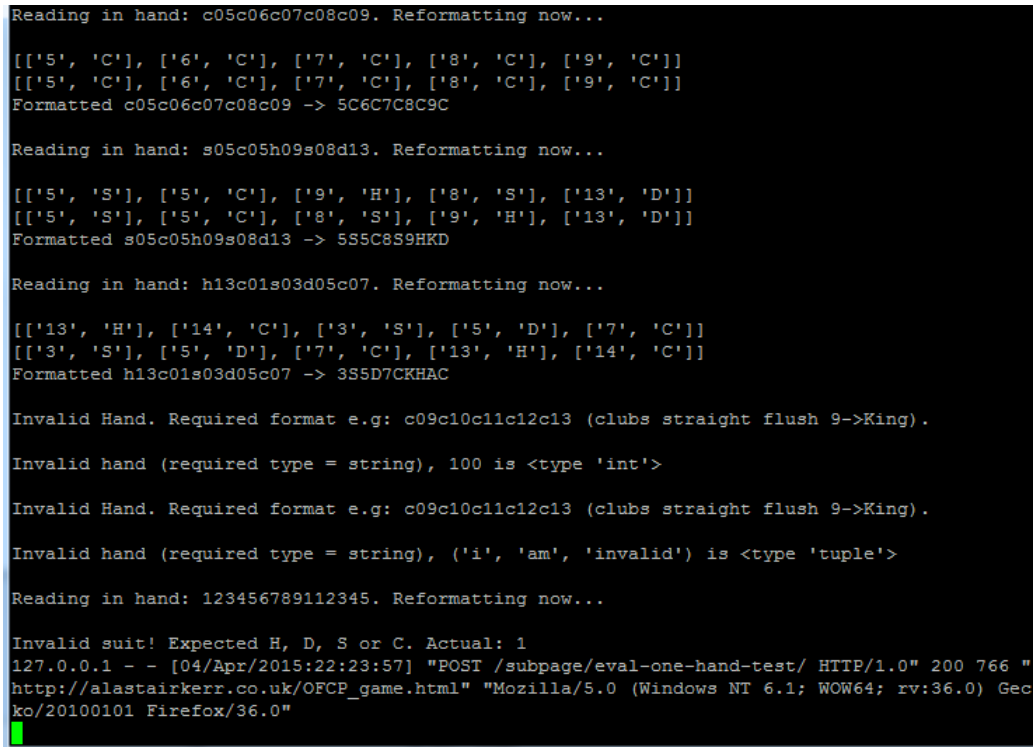
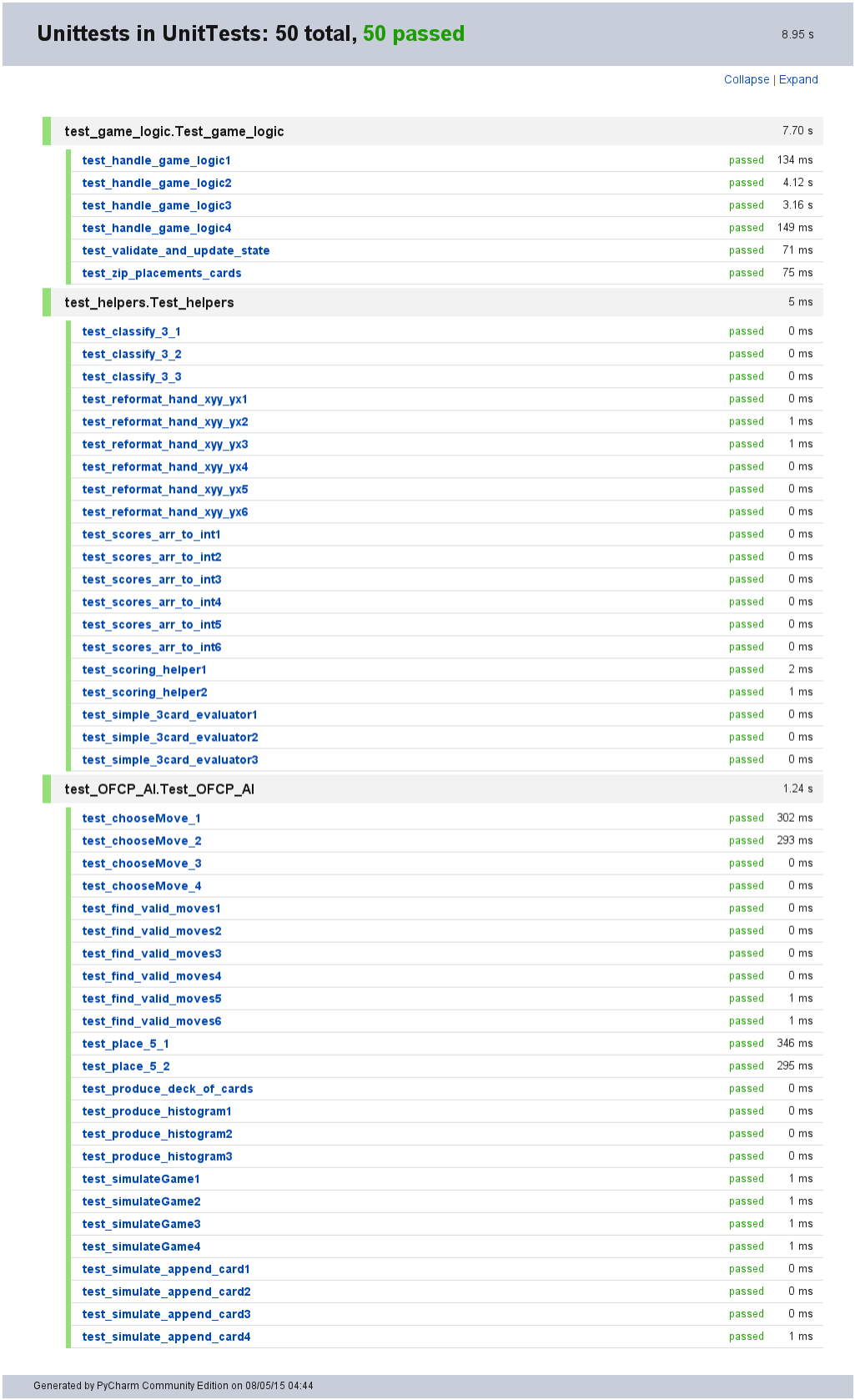


Fig 5.1.3 - Output – invalid hands are handled properly, throw exceptions/ print usage messages rather than throwing errors

5.2 Unit and Integration Testing

Modular testing of individual functions was carried out using the Python unittest framework, in addition to integration testing of the interaction between different parts of the system. The aim of these tests was to ensure that outputs were as expected, and to test validation measures in functions by passing in erroneous data and ensuring exceptions were properly handled.



Unittests in UnitTests: 50 total, 50 passed 8.95 s

Collapse | Expand

test_game_logic.Test_game_logic		7.70 s
test_handle_game_logic1	passed	134 ms
test_handle_game_logic2	passed	4.12 s
test_handle_game_logic3	passed	3.16 s
test_handle_game_logic4	passed	149 ms
test_validate_and_update_state	passed	71 ms
test_zip_placements_cards	passed	75 ms
test_helpers.Test_helpers		5 ms
test_classify_3_1	passed	0 ms
test_classify_3_2	passed	0 ms
test_classify_3_3	passed	0 ms
test_reformat_hand_xyy_yx1	passed	0 ms
test_reformat_hand_xyy_yx2	passed	1 ms
test_reformat_hand_xyy_yx3	passed	1 ms
test_reformat_hand_xyy_yx4	passed	0 ms
test_reformat_hand_xyy_yx5	passed	0 ms
test_reformat_hand_xyy_yx6	passed	0 ms
test_scores_arr_to_int1	passed	0 ms
test_scores_arr_to_int2	passed	0 ms
test_scores_arr_to_int3	passed	0 ms
test_scores_arr_to_int4	passed	0 ms
test_scores_arr_to_int5	passed	0 ms
test_scores_arr_to_int6	passed	0 ms
test_scoring_helper1	passed	2 ms
test_scoring_helper2	passed	1 ms
test_simple_3card_evaluator1	passed	0 ms
test_simple_3card_evaluator2	passed	0 ms
test_simple_3card_evaluator3	passed	0 ms
test_OFCP_AI.Test_OFCP_AI		1.24 s
test_chooseMove_1	passed	302 ms
test_chooseMove_2	passed	293 ms
test_chooseMove_3	passed	0 ms
test_chooseMove_4	passed	0 ms
test_find_valid_moves1	passed	0 ms
test_find_valid_moves2	passed	0 ms
test_find_valid_moves3	passed	0 ms
test_find_valid_moves4	passed	0 ms
test_find_valid_moves5	passed	1 ms
test_find_valid_moves6	passed	1 ms
test_place_5_1	passed	346 ms
test_place_5_2	passed	295 ms
test_produce_deck_of_cards	passed	0 ms
test_produce_histogram1	passed	0 ms
test_produce_histogram2	passed	0 ms
test_produce_histogram3	passed	0 ms
test_simulateGame1	passed	1 ms
test_simulateGame2	passed	1 ms
test_simulateGame3	passed	1 ms
test_simulateGame4	passed	1 ms
test_simulate_append_card1	passed	0 ms
test_simulate_append_card2	passed	0 ms
test_simulate_append_card3	passed	0 ms
test_simulate_append_card4	passed	1 ms

Generated by PyCharm Community Edition on 08/05/15 04:44

Fig 5.2.1 - Unit Testing Output (All tests passed)

The tests ran are included in the Unittest subdirectory of the code submitted, along with evidence of output. All tests ran passed, or in the case of failure, appropriate fixes were implemented, and then the tests were run again to ensure these changes had the desired effect. This only occurred in a few cases where proper input validation was lacking, and in one case, a minor bug where the wrong variable was being returned by the OFCP_AI chooseMove function.

5.3 User and Usability Tests

User testing was an ongoing process throughout the development life cycle of the project, in line with the demands of a Rapid Application Development approach. User feedback was crucial at every stage impacting decisions regarding tweaking the Intelligent Agent's behaviour, making UI changes and updates and general feedback about potential improvements.

In addition to getting user feedback after major code changes, usability tests were conducted to make sure that users were able to use the application properly, asking users to play through several rounds in the application and seeing how well they were able to undertake this task.

User	Test	Expected Result	Actual Result
1	Start the game	User presses Play button	User pressed play button
1	Place cards	User drags cards onto board	User read instructions then dragged cards
1	Confirm Input	Users confirms action	User paused and then pressed button
1	Place next card	User waits for AI, places next card	User placed card after a brief hesitation
1	Continue playing game	User places cards until game end	User continued play seamlessly
1	Move onto next round	User presses Play Next Round	User read scores, then pressed button
1	Play 3 Rounds	Users plays game for 3 rounds	User continued play

Table 5.3.1 - Usability test with User #1.

The results from this first usability test were very promising; it is clear that the design choices for a simple, clear layout paid off. The user did hesitate momentarily at times, but quickly managed to work out how to proceed without any prompting. When asked for feedback the user said they found the interface visually appealing, although did mention that it was initially distracting. The reason for this was because of the dragon background, which at the time stood out very vibrantly, as seen below.

Fig 5.3.1 - Original Redesign of the interface which featured a very vibrant, bold dragon background image

In response to this feedback I made a subtle change to the interface in order to mitigate the potentially distracting nature of this background; the ease of use and intuitive nature of the interface is more important than simple visual appeal. The redesigned interface simply used a slightly transparent version of the same image, as shown below.

Fig 5.3.2 - Rework of the interface to make background less distracting, making the UI more intuitive

After making these changes I conducted further tests with two new participants, one of whom was familiar with the game Open Face Chinese Poker, and one who was not. The results of these usability tests are presented below.

User	Test	Expected Result	Actual Result
2	Start the game	User presses Play button	User scanned interface, pressed play button
2	Place cards	User drags cards onto board	User dragged cards after a few seconds
2	Confirm Input	Users confirms action	User pressed button
2	Place next card	User waits for AI, places next card	User reacted faster, placing card quickly
2	Continue playing game	User places cards until game end	User continued play seamlessly
2	Move onto next round	User presses Play Next Round	User read scores, then pressed button
2	Play 3 Rounds	Users plays game for 3 rounds	User continued play at a steady progress
3	Start the game	User presses Play button	User quickly pressed play button
3	Place cards	User drags cards onto board	User immediately began dragging cards
3	Confirm Input	Users confirms action	User pressed button after a brief wait
3	Place next card	User waits for AI, places next card	User placed card almost immediately
3	Continue playing game	User places cards until game end	User continued play seamlessly
3	Move onto next round	User presses Play Next Round	User analysed scores, then pressed button
3	Play 3 Rounds	Users plays game for 3 rounds	User continued play without issue

Table 5.3.2 - 2nd round of usability tests

Overall I was pleased with the results garnered from these usability tests. I believe the steps I took to increase the usability of the interface such as highlighting where to place cards when dragging cards, as well as sticking to a simple layout paid off, as evidenced by the fact that the users quickly picked up on how to play the game and use the UI effectively. The change to the background image seemed well received when shown to User #1 from the first usability test, and based off of how quickly User #2 and #3 picked up on the interface it would seem that the interface has good usability.

6 Evaluation

6.1 Aims and Objectives

Requirement	Met?	Notes
Create a game environment for OFCP	Yes	Frontend Web-Application
Implement appropriate rules for the game	Yes	Proper Royalties are calculated
Enable alternate round support	Yes	Using persistent variable
Support concurrent connections	Yes	Robust networking setup
Create an Intelligent Agent to play OFCP	Yes	Using Monte Carlo Methods
Agent must interface with the application	Yes	2 way communication between front and backend
Agent must adhere to all rules of game	Yes	-
The Agent must not cheat	Yes	AI does not use any information it shouldn't
Long Mean-Time-To-Failure	Yes	Servers are resistant to fatal crashes
Maximise application uptime	Yes	Application uptime at least 99% of the time
Support performance monitoring	Yes	Error logs, database records, server output logs
Application must be responsive and quick to load	Yes	Good hosting provider, responsive interface
Website should load in <1s	Yes	Assuming client has a good internet connection
Application should feel responsive	Yes	Dragging cards works well
UI should be intuitive and easy to understand	Yes	Clear layout, buttons etc
Pleasing aesthetic	Yes	Updated colour scheme and graphics
Maximise scalability	Partially	Reasonable scalability, room for improvement
Reconfigurability	Yes	No need for extensive work to make changes
Able to tweak values e.g. AI time limit	Yes	Simply edit values in source code
Adding features should not break the system	Yes	Overall good design structure enables this
Application should maximise compatibility	Partially	Minimal, basic support. Room for improvement
Installation of the application should be easy	Yes	Snapshots for droplet - 60 second redeployment
<5s wait for AI's move	Yes	Hardcoded 5s time limit
Optimise Performance	Partially	PyPy, heuristics. Room for improvement
Support 100 concurrent users	Yes	CherryPy can handle 1000 concurrent users
Take no longer than 5s for any request	Yes	Application is responsive. 3-5s limits of AIs turns
Agent must play competently	Partially	Plays adequately but outplayed by skilled humans

Table 6.1.1 - Measuring how well the final application met the specified requirements

To confirm the performance of the site, a website speed testing service was used, which produced the following result.

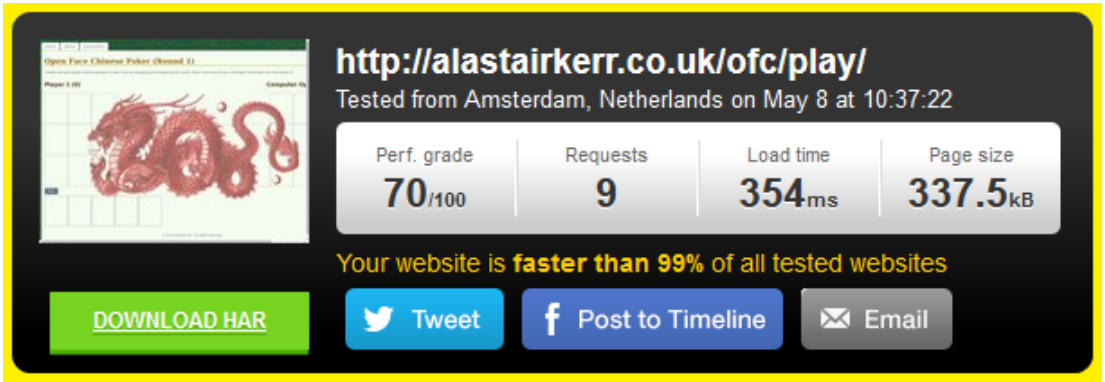


Fig 6.1.1 - Performance of the website as rated by http://www.pingdom.com

6.2 Critical Appraisal

6.2.1 Frontend

The frontend for the application is functional and aesthetically pleasing, but would have benefited from being designed better from the start. For example, originally hard-coding player's card image objects rather than dynamically creating them with JavaScript lead to problems down the line which required hack-like workarounds until later refactoring improved the design. One important issue to note with the frontend is cross-platform compatibility. Browsers such as older versions of Internet Explorer are not supported, and the application does not display well on smaller screens. Some basic efforts were made to support as many platforms as possible, with measures taken to ensure that the board's structure would not break when viewed on a low resolution device or when zoomed in at the browser level. In addition to this, basic support for mobile devices was implemented with an iOS HTML5 drag and drop shim, necessitated because of my choice to implement the application with pure JavaScript. In hindsight, use of JQuery may have been more appropriate as drag and drop event handlers would automatically have cross-platform support, but development in this manner required me to really get to grips with understanding and becoming competent with JavaScript and so in my mind was a fundamentally important and useful process.

6.2.2 Backend

The backend works well overall and I am very pleased with the system structure and design. Performance wise if different technologies and languages were used there is a huge potential for increased efficiency. For example writing the heavy-lifting backend scripts in C would have been hugely beneficial, and using a more efficient hand evaluator algorithm that could evaluate millions of hands per second rather than hundreds of thousands could shave off seconds of processing time. This hypothetical performance increase could either lead to increased responsiveness or allow for more games to be simulated by the Intelligent Agent making it more likely to find optimal solutions for hand placements. In addition the overall footprint of the application would be reduced, allowing for increased scalability.

Throughout the course of the development of the application, many games were played versus the various iterations of the Intelligent Agent, from its origins with entirely naïve placements through to its current iteration, using a Monte Carlo approach aided by domain specific knowledge. The improvements over time were therefore incredibly noticeable, with early prototypes struggling to even produce a legitimate, non-fouling board, and later iterations able to make optimal plays which implicitly took into account the opposing player's moves. Despite the increasing ability of the Agent over time, however, the simple approach limited its potential and it is unfortunate that time constraints prevented me from producing a more sophisticated implementation, and it is something I intend to work on improving in the future.

6.2.3 Software Methodology

The choice of a flexible software methodology worked well overall because of the evolving requirements and design choices, as well as the individual nature of the project. In comparison, in a large team of developers issues with this approach could arise from lacking a clear design focus and having limited control - a necessary trade-off that is an inevitable consequence of the increased flexibility this methodology enables. One important pitfall to avoid with Rapid Application Development is focusing too much on individual components without getting a clear view of the system's design, making minor changes without considering possibilities for an improved design structure. While there is generally a clear design phase before entering the initial implementation stage, there can be a tendency to omit a renewed design phase in subsequent implementation cycles leading to a lack of documentation which can have large consequences down the line; as Gerber et al. (2007) state in the analysis of one case study "... due to the fact that the design was not formally documented and reviewed, the discrepancy was only discovered after the implementation phase. This situation caused conflict between developers and analysts and in the end necessitated a redesign effort which put unnecessary pressure due to time constraints and limited resources on the whole development team". Design choices in early prototypes had a carry-on effect which meant that later down the line code refactoring was necessary in order to create a more coherent system structure, which potentially could have been avoided or reduced with a stricter design philosophy.

An apt example of this is seen in legacy prototypes of the application, which were client-side only. This was a choice that was made in order to quickly create a functional prototype, using JavaScript to simulate processes that would be handled elsewhere in the final application's architecture (such as dealing cards). This was useful because it resulted in a functional application which implemented some of the planned features, leading to a clearer understanding of the needs of the project, but had to be adapted later in order to create a more logical system structure which could meet the requirements, such as backend processing to handle the game states and calculate the Intelligent Agent's moves.

6.3 Improvements

In line with user feedback, there is a demand for multiplayer support for players vs players as well as players vs AI(s). Implementing this would allow for a more well-rounded, feature-full application and is certainly a consideration for the future.

Other feedback from users indicated a desire for support of other variants such as Pineapple OFC. This is indeed something I am interested in doing, as Pineapple includes elements of hidden information due to players placing 2 of 3 cards at once and choosing 1 to discard, the identity of which is known only to them. This adds another layer of depth to the game, and increases the complexity of game which would require further modifications and improvements to the Intelligent Agent. Performance issues faced in the Agent for standard OFC would become even more of an obstacle which would almost certainly mean that use of more sophisticated algorithms would be necessary.

The most important area of improvement in my mind is the performance of the Intelligent Agent. While the general ability of the Intelligent Agent is adequate, it is clear that there is room for improvement. Despite generally choosing optimal placements it is important that the limitations of the method used are acknowledged, as Whitehouse et al assert "with any method based on random simulation, it is inevitable that poor quality moves will be chosen with nonzero probability, due to a particularly lucky run of simulations making the move appear better than it is" (Whitehouse et al, 2013, p104). Due to the need for a compromise between finding the optimal solution and finishing the request in a reasonable amount of time the number of simulated games is limited and therefore it is possible that sub-optimal plays will be over-valued due to the element of randomness. This potential for inaccurate evaluation of a move's strength has an inverse relationship with the number of iterations of simulated games - as the number of iterations increase the result diverges to the optimal solution, meaning that with an infinite amount of simulations the probability of finding the best move is 1. The performance of the Intelligent Agent therefore could be improved with more processing power and/or a longer allocated time to simulate games, although it is important to note that there are diminishing returns with this strategy; doubling the iterations does not mean that the results will be twice as good.

This ties in with the choices made for the implementation and configuration of the Agent, specifically in regards to compromises between responsiveness and finding the best move - in the specified requirements the Agent was intended to take no more than 5 seconds to calculate its turn, and the application satisfies this requirement. However, increasing the iterations would make satisfying this criteria infeasible without increased processing power or through further optimisation. This could be achieved in various ways, for example by rewriting the application in a more efficient language such as C, or by implementing more advanced heuristics to reduce the complexity of the calculations, or using a different algorithm such as a more advanced implementation of the Monte Carlo method like Monte Carlo Tree Search with UCT, as discussed in Section 2 of this dissertation. Overall the implementation of the Intelligent Agent satisfies the specified requirements and performs adequately for its intended purpose in the scope of this application, but for larger scale implementations would likely need to make use of one or more of these changes in order to achieve increased scalability, for example for use in a commercial application with thousands of concurrent users.

6.4 Reflection

This project was a daunting task, but the experience was in all a thoroughly enjoyable one. It enabled me to build on a wider skill-set, and required me to immerse myself in several different aspects of Computer Science and Software Engineering. Artificial Intelligence was an area I had only very little prior experience

with, and while conceptually at times difficult to follow, I feel that I have learnt a lot and it is has definitely sparked an interest in this field of study. Creation of the final product required a culmination of various technologies and disciplines, giving me broad exposure to many new concepts along the way. Sharpening up my programming abilities in Python and JavaScript was certainly a beneficial process, and I also learned a lot about web design and web technologies in the process. Furthermore, hosting the application on a dedicated website using a Virtual Private Server required me to take on a role of System Administration, brushing up on UNIX commands, learning more about security measures and gaining a wider understanding of networking and system architecture.

7 Conclusion

In conclusion the Intelligent Agent produced performed adequately well and largely ticked the boxes in terms of requirements specified, demonstrating the versatility of Monte Carlo methods; with little domain specific knowledge, usable results were obtainable. The Web-Application itself worked well for its intended purpose and shows promising scalability, with a robust and sensible system architecture, and the final interface produced was user-friendly and visually appealing. The technologies and methodologies employed enabled very fast turnover of prototypes and new software iterations, which turned out to be critically useful in making improvements to the system in response to user feedback.

Room for improvement can be had in many aspects however. Using Python allowed for quick development but the application could have benefited from the use of a more efficient language such as C, although these performance concerns were partially mitigated through the use of PyPy. The Intelligent Agent's simplistic approach could be expanded upon and improved, and fundamentally there are limitations with the approach used. The simple Monte Carlo methods used produce good results but are CPU intensive and necessarily require a tradeoff at some level between finding an optimal solution and getting fast results. It would be interesting to note the effect and improvements that implementation of more sophisticated Monte Carlo methods such as MCTS with UCT could achieve, and I look forward to further development of the Intelligent Agent in this respect.

8 Glossary of Terms

8.1 Poker Variants

- **Texas Hold'Em** is a popular variant of Poker where each player receives 2 cards for use individually in combination with 5 community cards shared between all players, with players combining any of their available cards in order to create the strongest standard poker hand possible
- **Chinese Poker** is a variant of poker where players are dealt 13 cards which they must arrange into three rows, placed face down. Players announce in clockwise order whether or not they wish to play their hand, and then all players announce their royalties and show their cards
- **Open Face Chinese Poker** is a variant of Chinese Poker where players act in clockwise order, receiving first 5 cards which are placed face up and then one card at a time until all players have placed 13 cards. Players must create valid hands consisting of stronger poker hands in lower rows, and score points from their opponents for winning corresponding rows. Additional points known as royalties can be won for particularly strong hands
- **Pineapple OFC** is a sub-variant of Open Face Chinese Poker, following the same basic rules as the standard variant with the distinction that in subsequent rounds after the initial 5 card placements, players receive 3 cards and choose to place 2 and discard 1. This introduces higher action play as well as elements of hidden information as other players are unaware of which cards their opponents have discarded, although there is potential to infer this information based off of how the player acts

8.2 Poker Hands Guide (Weakest to Strongest)

- **High Card** is the lowest ranking poker hand and is the default when no other hands have been made. An example High Card is the Jack of Spades, which would beat any hand comprised of a High Card ranked 10 or less, but would lose to a High Card Queen, King, Ace, or any stronger poker hand.
- **Pair** is the second weakest poker hand. A player has a pair when they have two cards of the same rank, for example the 7 of Hearts and 7 and Clubs would form a Pair of 7s.
- **Two Pair** is the next strongest hand rank, consisting of two different pairs. For example having the 5 of Spades, 5 of Diamonds, 9 of Hearts and 9 of Clubs would form Two Pair 9s and 5s. When comparing a Two Pair to another Two Pair the highest ranked pair takes precedence.
- **Three of a Kind** or a **Set** beats Two Pairs, Pairs and High Cards and consists of three same-ranked cards, such as the Ace of Spades, Ace of Diamonds and Ace of Hearts, which would form Three of a Kind Aces
- **Straight** is a hand where a player has 5 sequential cards, such as 4 of Diamonds, 5 of Hearts, 6 of Hearts, 7 of Clubs, 8 of Spades, which would form a Straight 8 High. The lowest ranked straight spans Ace to 5 and the highest ranked straight spans 10 to Ace. It is important to note that straights do not wrap around; you cannot form a straight such as Queen, King, Ace, Deuce, 3.
- **Flush** is one of the stronger poker hands, consisting of cards which are all the same suit. For example the 6 of Hearts, 9 of Hearts, Jack of Hearts, Queen of Hearts and King of Hearts would form a King High Flush.
- **Full House**, sometimes known as a **Boat**, is one of the strongest poker hands available, and comprises both a Three of a Kind and an additional Pair. For example Three of a Kind Tens with Pair of Jacks would combine to form a Full House, Tens full of Jacks.
- **Four of a Kind** is a hand obtained when a player has every instance of a particular card rank, such as King of Hearts, King of Diamonds, King of Clubs and King of Spades which would form Four of a Kind Kings.
- **Straight Flush** is effectively the strongest poker hand possible, consisting of 5 sequential same suited cards. For example the 4 of Spades, 5 of Spades, 6 of Spades, 7 of Spades and 8 of Spades would form a Straight Flush 8 High.
- **Royal Flush** is a special instance of a Straight Flush where a player has the 10, Jack, Queen, King and Ace of a particular suit. Royal Flushes are particularly rare; in Texas Hold'Em for example the probability of getting a Royal Flush is approximately 0.000154%.

8.3 Open Face Chinese Poker Terminology

- A **row** is a set structure for placing cards. At the end of a game when scoring occurs poker hands in each player's rows are compared to the opposing player's hand in their corresponding row. There are three different rows as described below
- **Bottom Row** or **Back Hand** is the foundation row, and consists of 5 cards. Out of all three rows this must have the strongest poker hand or the player fouls.
- **Middle Row** or **Middle Hand** also consists of 5 cards. It must have a weaker hand than Bottom Row in order for the player's hand to be valid.
- **Top Row** or **Front Hand** consists of 3 cards, meaning the best possible hand here is a Three of a Kind (in most variants 3 card straights and flushes do not count). Top Row must have a weaker hand than both middle and bottom row.

- **Scoop** is a bonus awarded to a player when they win all 3 rows against an opponent. On top of the standard +1 point per row won, the player is granted an additional bonus of +3 points which is also taken from their opponent.
- **Fouling** occurs when a player plays an invalid hand, for example by putting a stronger hand in their middle row than their bottom row. When a player fouls any of their royalties are null, and their opponent is automatically awarded a scoop bonus so long as their hand is valid.
- **Kicker** is the term used to describe the next card taken into account when comparing two otherwise equal hands. For example, if both Player 1 and Player 2 have a Pair of 8s then the rest of their cards would be considered, and whichever player has the highest rank wins. If both players kicker is equivalent then the next highest kicker will come into play and so on.
- **Royalties** are bonus points awarded to player's for particularly strong hands. Just like any other points a player wins, they are taken directly from opposing players. Hands in higher rows score higher royalties than equivalent hands in lower rows. For example, a Full House in bottom row is worth 6 points in bottom or 12 points in middle. Another example is that a Three of a Kind in bottom row scores no royalty, but gets 2 bonus points in middle and between 10 and 22 points in top depending on the rank (Three of a Kind Deuces scores 10 points up to Three of a Kind Aces with 22 points). See here for a full list of royalties: http://www.wsop.com/2013/Open_Face_Chinese_Structure_Sheet.pdf

8.4 Computer Science and Mathematical Terminology

- **Algorithms** are precisely defined step-by-step instructions describing a set of operations to be performed
- **Artificial Intelligence** is a field of study in Computer Science which focuses on simulating intelligent behaviour in computers.
- **Automated Planning and Scheduling** is a branch of artificial intelligence which focuses on strategies and actions to be performed by Intelligent Agents or autonomous robots. Research into optimisations and improved heuristics is a key area of interest due to the potentially huge complexity of problems and solutions.
- The **Branching Factor** is the average amount of branches from a node in a tree, indicating the tree's complexity. At a depth of d with a branching factor of B there would be approximately B^d nodes.
- **Combinatorial Explosion** is the phenomenon of exponential growth encountered in search problems. Deeper searches become increasingly complex multiplying by the branching factor at each level
- A **Game Tree** represents possible states in a game, with each node representing a possible position and each edge representing a possible move
- **Game Theory** is a branch of mathematics focused on determining optimal strategies and decisions in competitive situations
- **Heuristics** are techniques used when classical approaches would fail or be prohibitively slow to complete. Heuristics use intelligent guesses to reduce the complexity of problems. For example in the context of a Chess game rather than trying to explore every single possible move and subsequent tree branch, a heuristic based approach would ignore branches starting with clearly bad moves
- **Hidden Information** is relevant information available to one or more agents but not others. For example in a Poker game, the cards that players individually possess or have discarded are known to them but are hidden from other players.
- An **Intelligent Agent** is an autonomous software entity that perceives and acts upon its environment, performing reasoning in order to solve problems and determine actions, exhibiting goal-oriented behaviour.

- The **Optimal Move** or **Optimal Strategy** is the move or strategy that will lead to the most favourable outcome for an Agent
- **Perfect Information Games** are games where at any stage all relevant information is available to an agent in order to inform its decision. In comparison in an **Imperfect Information Game** certain information about the game state or prior actions are unknown.
- **Rule Based Systems** store expert knowledge for a particular domain as a set of rules which can be used in various artificial intelligence applications. Rule Based Systems are used extensively for a wide range of purposes such as game playing, credit card authorisation and fraud detection.
- **Solved Games** are games where the outcome (Win/Loss/Draw) can be predicted at any stage assuming optimal play by all players
- A **Zero-Sum Game** is a game in which a player's gains are losses are balanced by another player's gains or losses. Open Face Chinese Poker is an example of a Zero-Sum Game as a player's gains are directly taken from their opponent.

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