PHYS 688: Numerical Methods for AstroPhysics
Homework #3: Linear Algebra and FFTs

Marina von Steinkirch

March 27, 2013

$\boxed{\text{Q.1}}$ $\boxed{\text{Q.2}}$ **See outputs from source codes.**

$\boxed{\text{Q.3}}$ **(Time Series) The equation of motion for a damped driven pendulum is:**

$$ml\frac{d\theta^2}{dt^2} = F_g + F_d + F_r,$$

where θ is the angle of the pendulum from vertical, $F_r = f_0 \cos(\omega_d)$ is the driving force with amplitude f_0 and driving frequency ω_d, $F_d = \kappa\nu$ is the resistive force and $F_g = mg\sin(\theta)$ is the gravitational force on the pendulum. We can write this as a system of the form:

$$\frac{\theta}{dt} = \omega,$$

$$\frac{\omega}{dt} = -q\omega - \sin(\theta) + b\cos(\omega_d),$$

where q is a *scaled damping coefficient* and b is a *scaled forcing amplitude*. We also took $l = g$ to simplify things.

1. We integrate this system using *4th-order Runge-Kutta* and a *uniform timestep* (equally spaced points to do Fourier analysis). We chose the parameters $q = 0.5$, $b = 0.9$, and $\omega_d = 2/3$. We integrate for tmax = 100 periods and we pick a timestep dt = 0.05 that gives a converged solution. We plot $\omega - \theta$ (left top plots in the below figures) and we can see the period motion for $0.5 \leq b < 10.0$.

2. We perform the *discrete Fourier transform* (DFT) of $\theta(t)$ and plot the power spectrum for values $0.5 \leq b \leq 13.5$ (figures above), where for $b \geq 10.0$ the behaviour is chaotic.

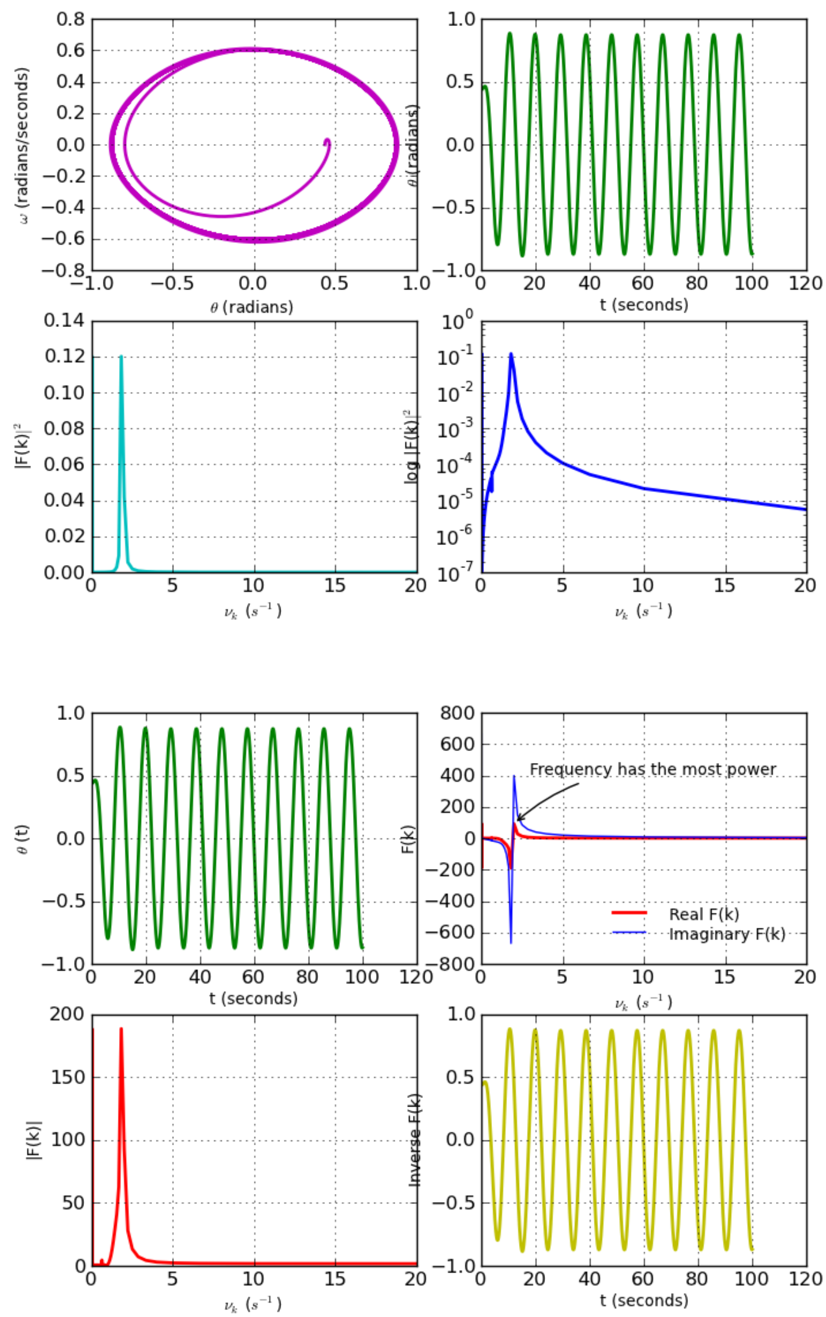


Figure 1: **Periodic Regime, b = 0.5**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

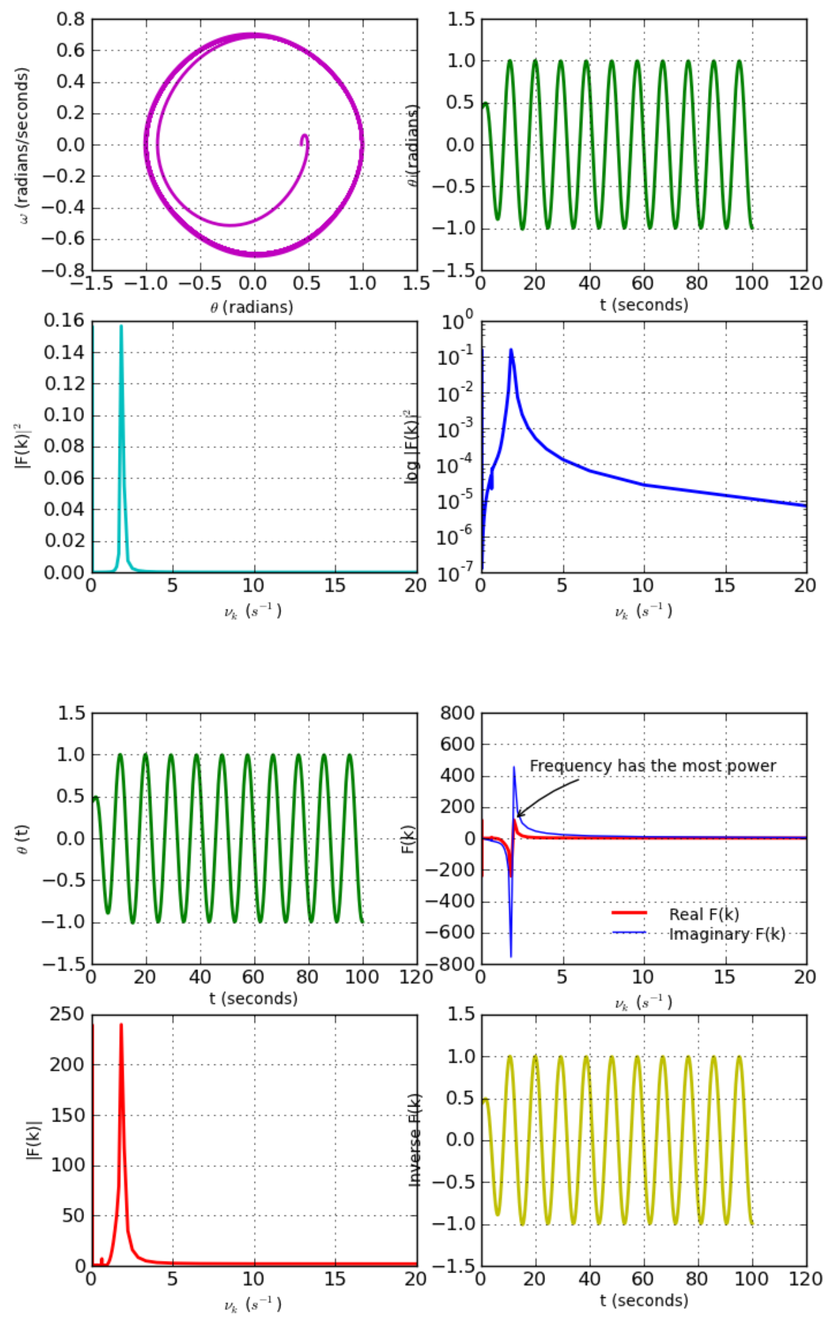


Figure 2: **Periodic Regime, b = 0.55**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

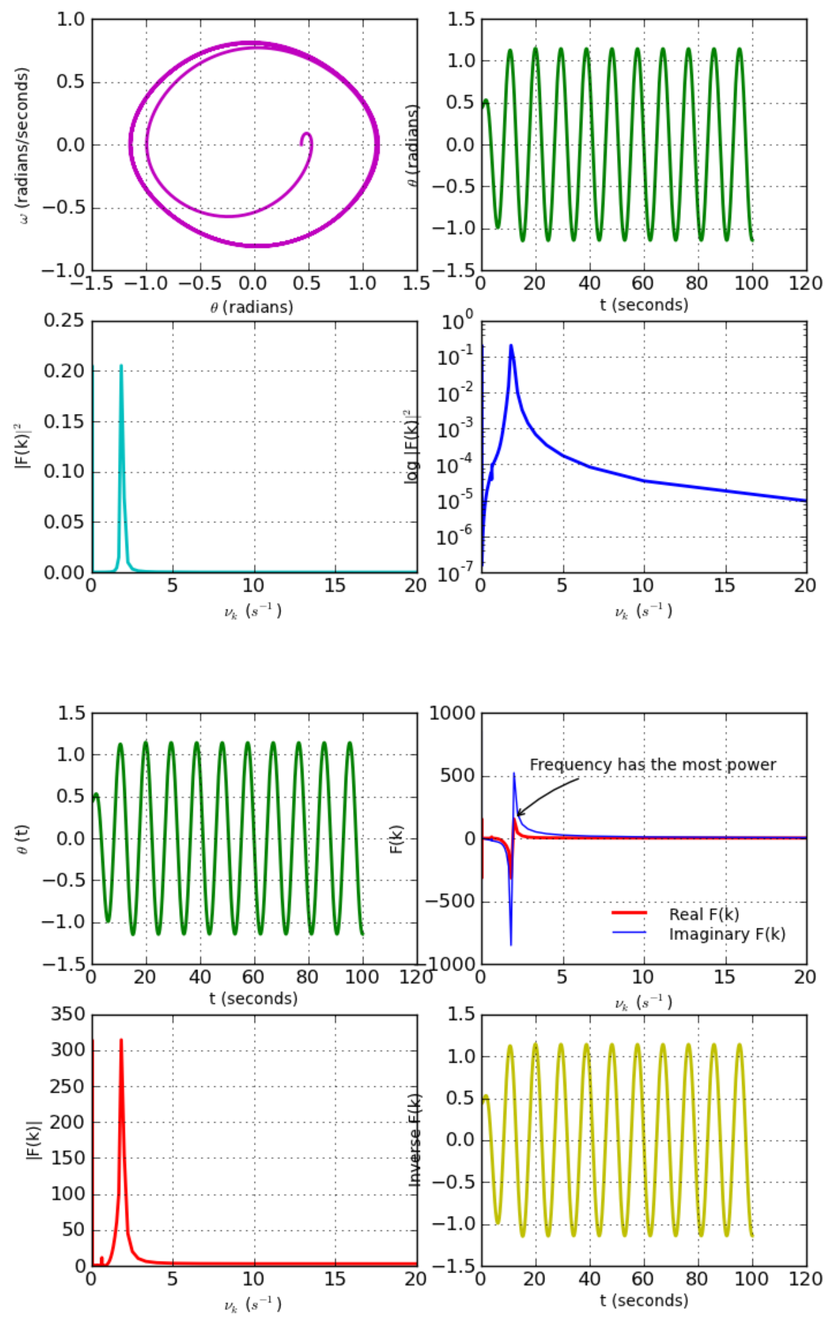


Figure 3: **Periodic Regime, b = 0.6**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

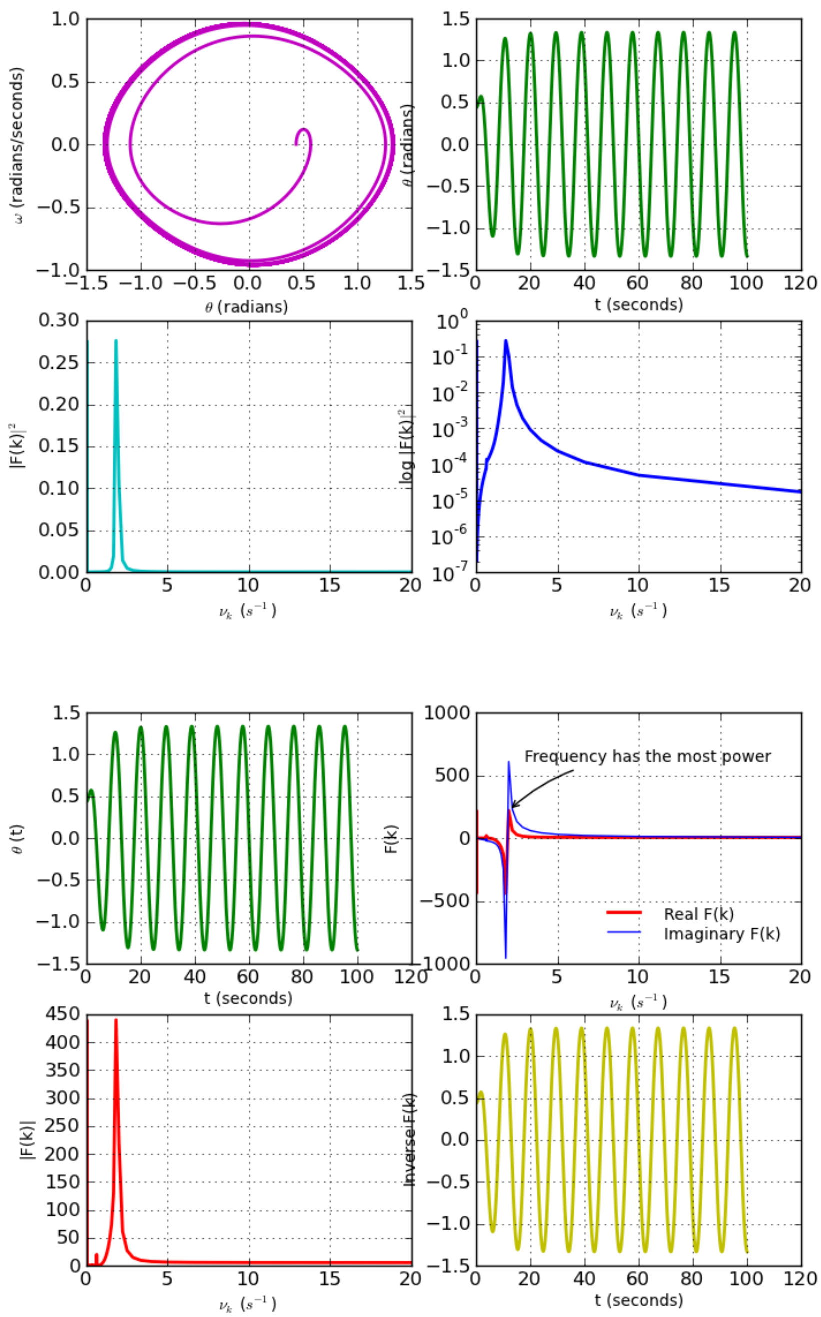


Figure 4: **Periodic Regime, b = 0.65**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

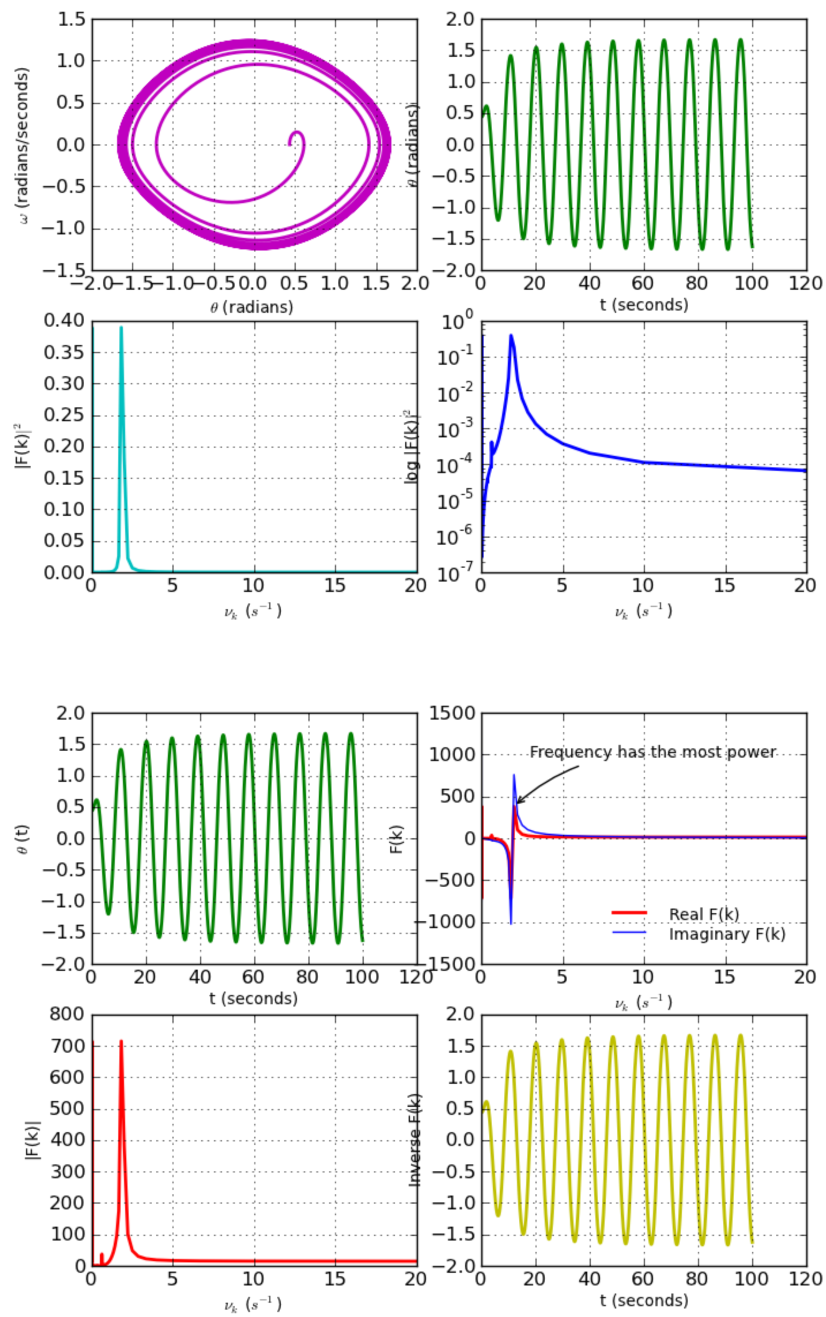


Figure 5: **Periodic Regime, b = 0.07**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

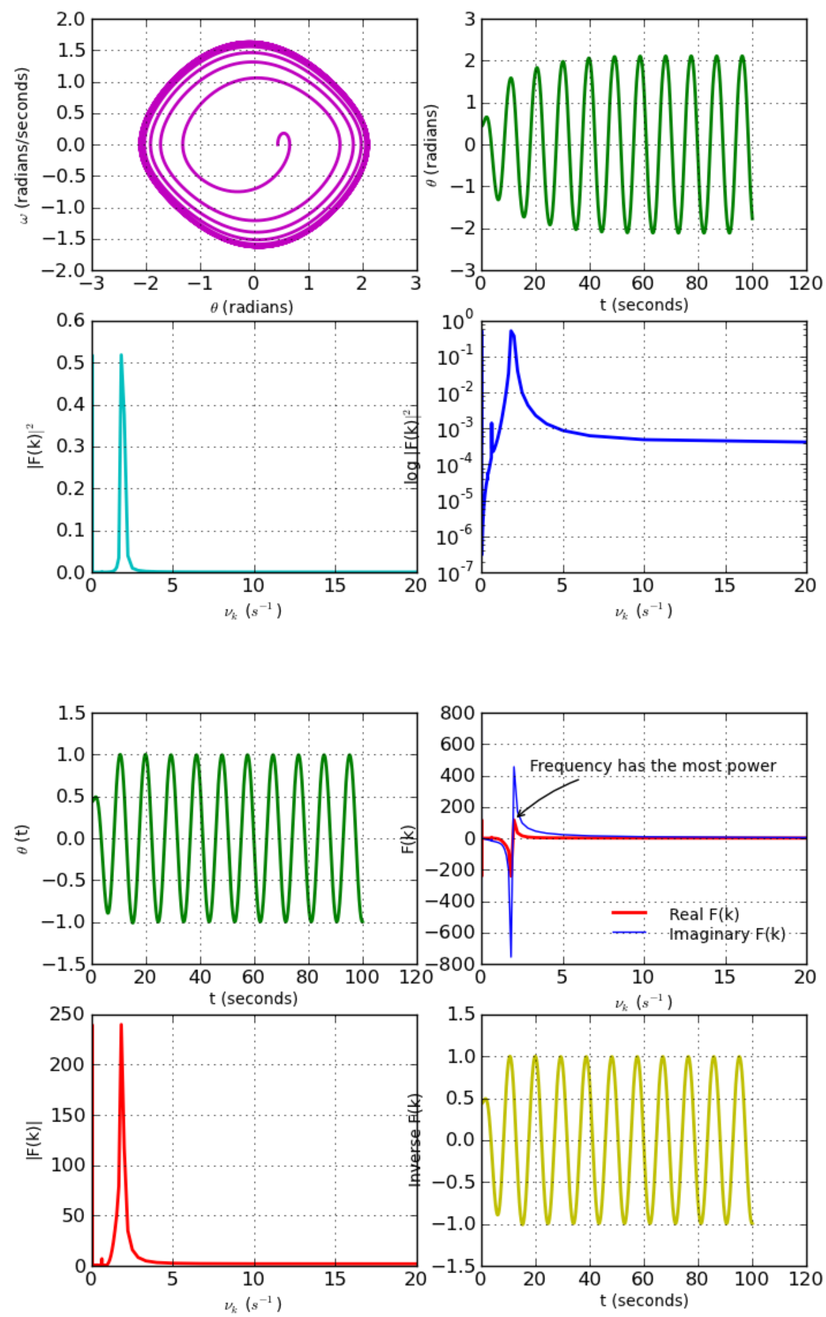


Figure 6: **Periodic Regime, b = 0.75**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

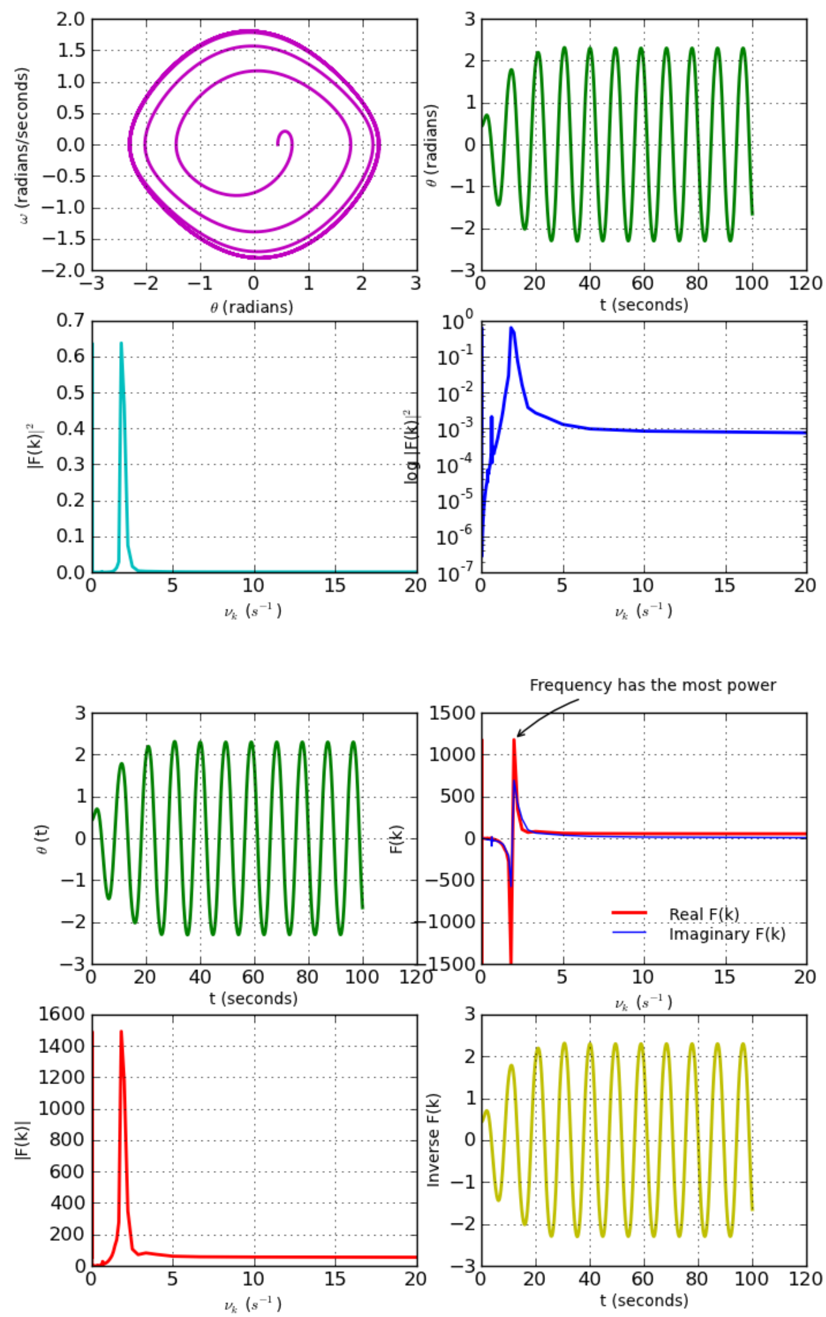


Figure 7: **Periodic Regime, b = 0.8**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

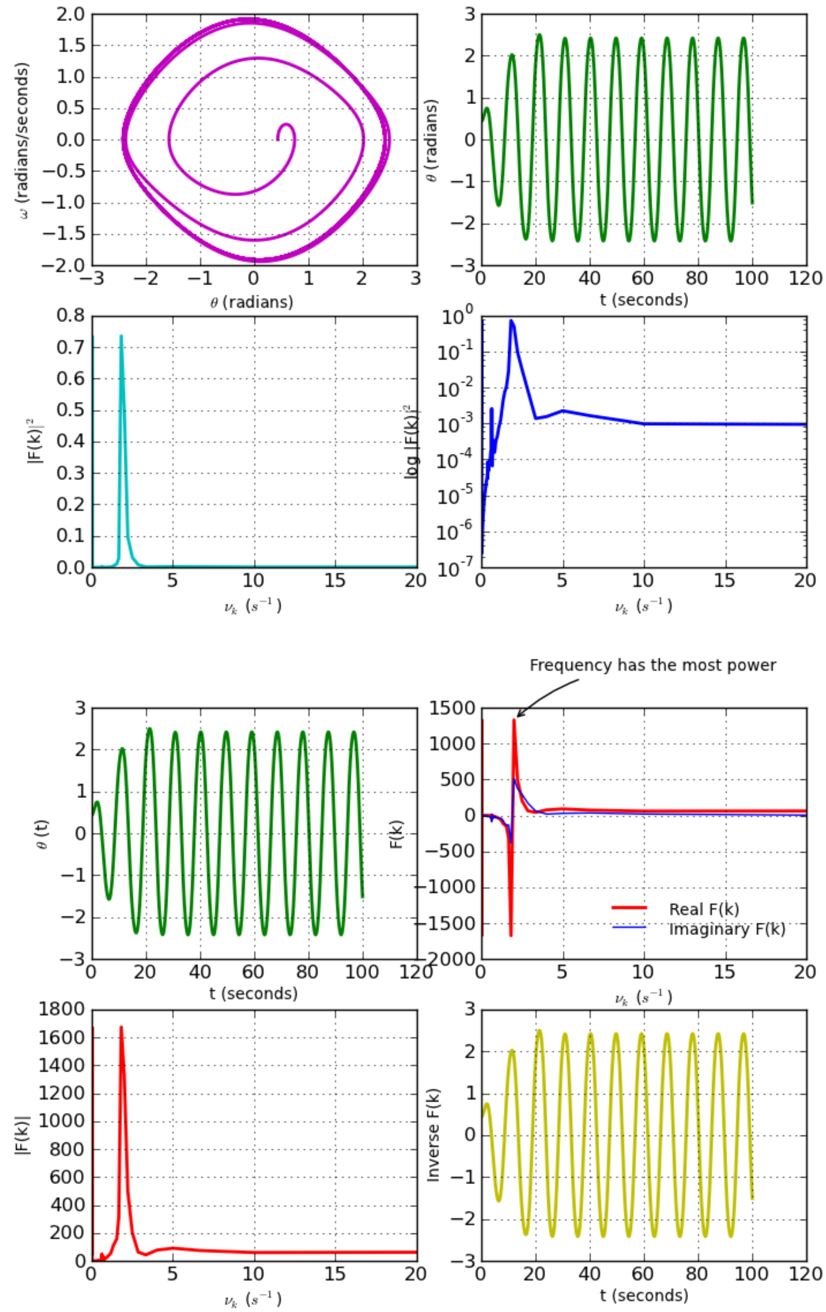


Figure 8: **Periodic Regime, b = 0.85**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

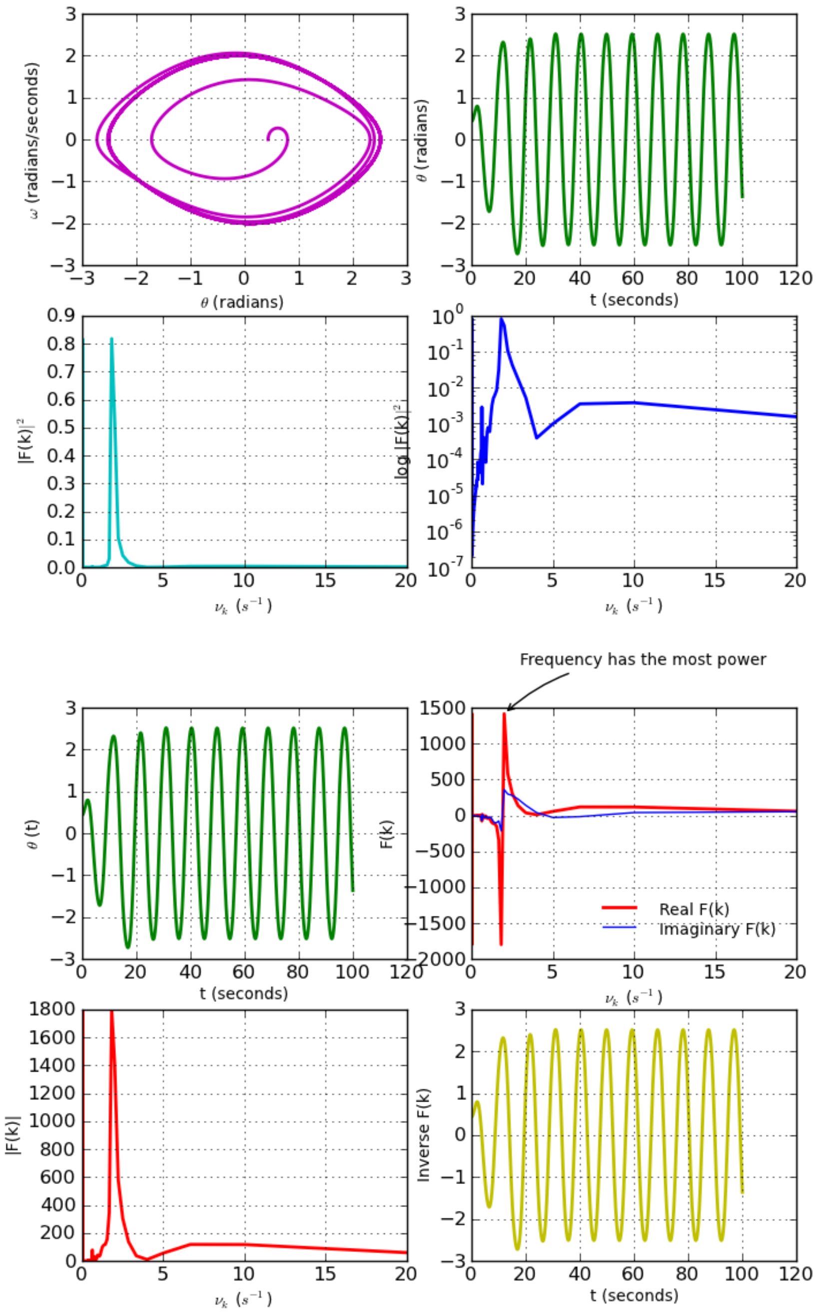


Figure 9: **Periodic Regime, b = 0.9**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

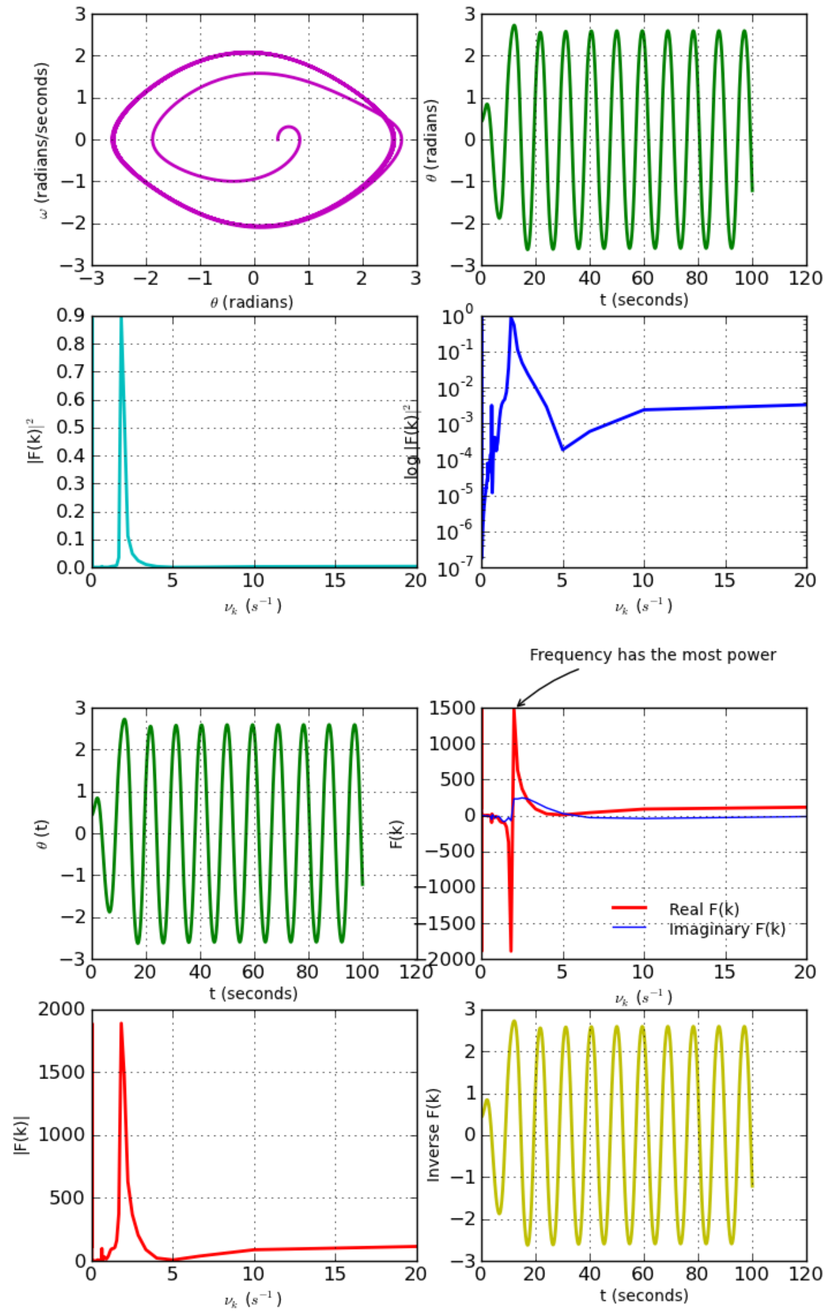


Figure 10: **Periodic Regime, b = 0.95**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

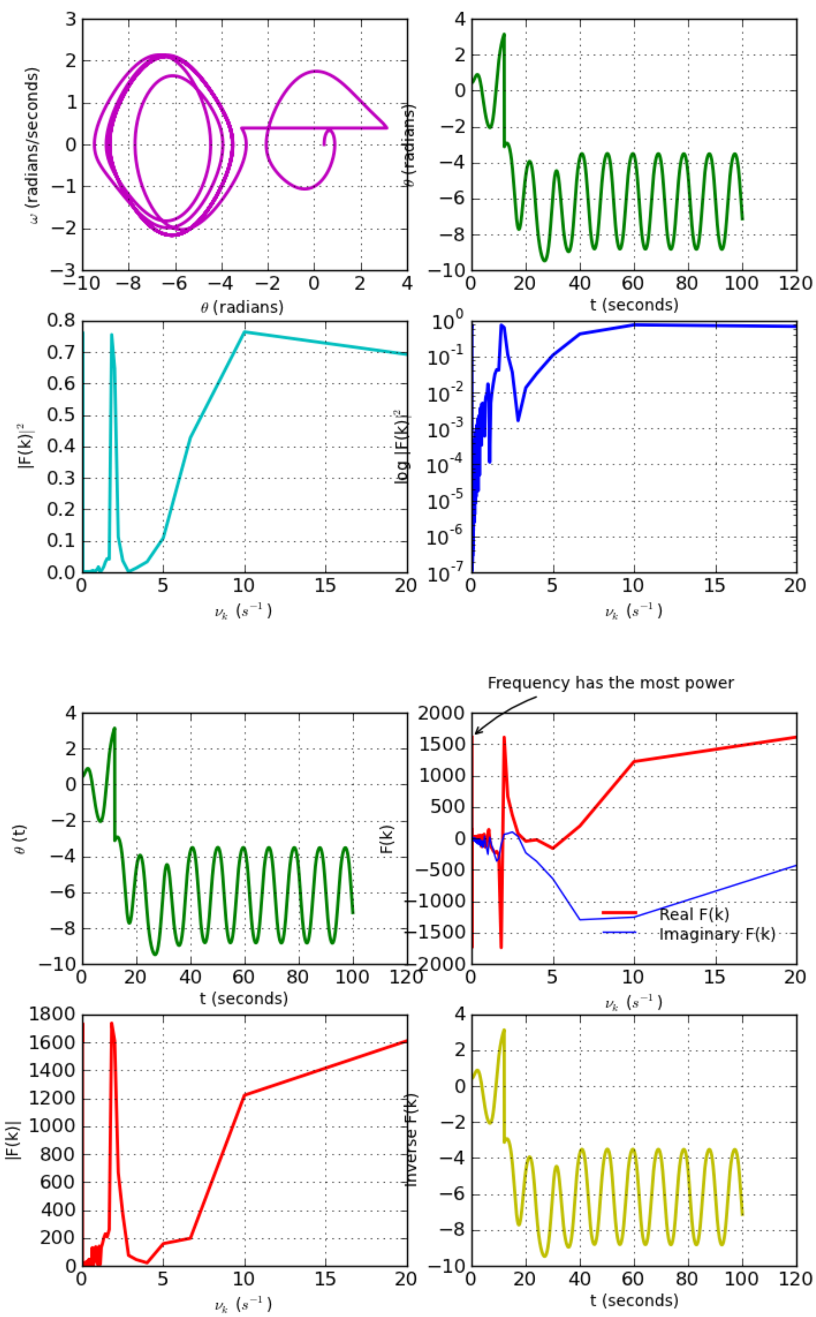


Figure 11: **Chaotic Regime, b = 1.0**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

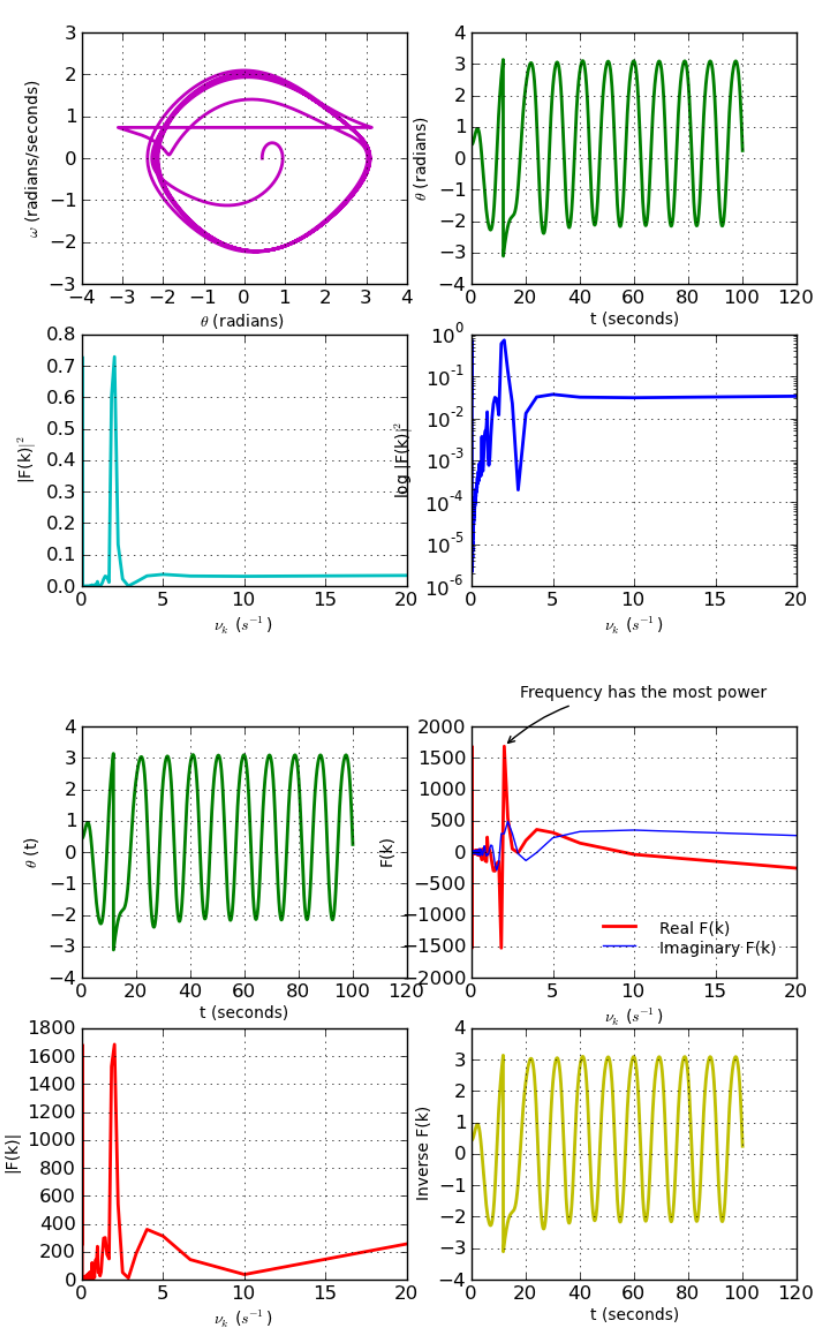


Figure 12: **Chaotic Regime, b = 10.5**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

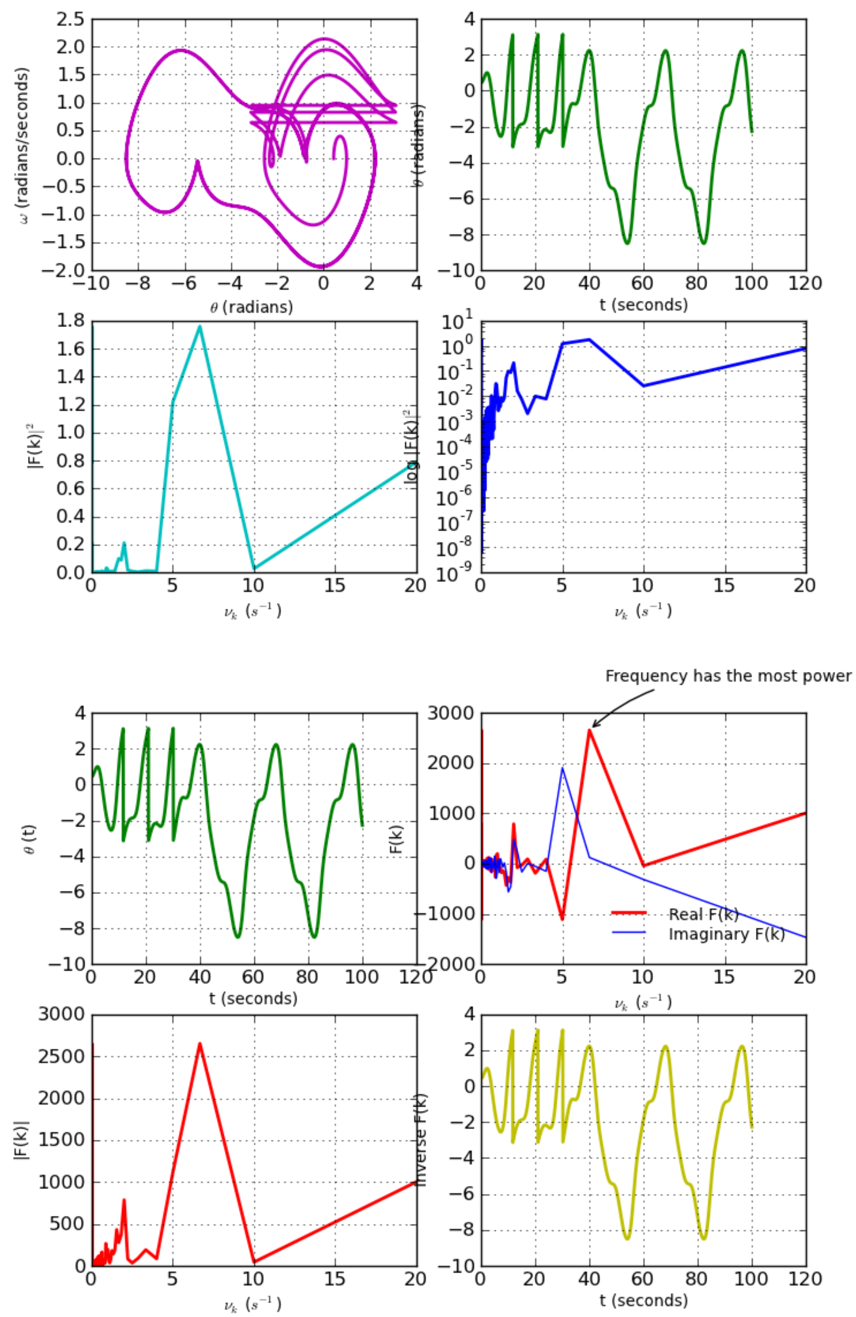


Figure 13: **Chaotic Regime, b = 11.1**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

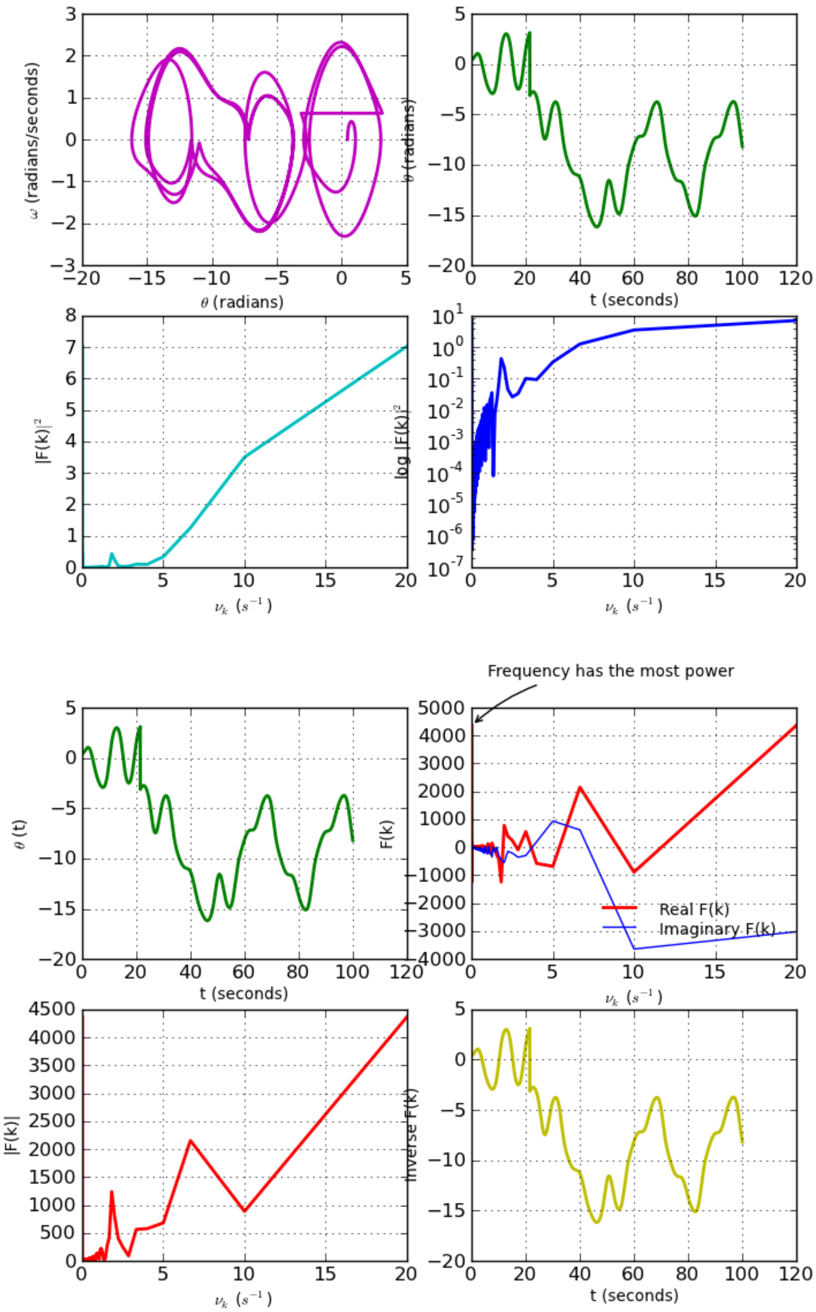


Figure 14: **Chaotic Regime, b = 11.5**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

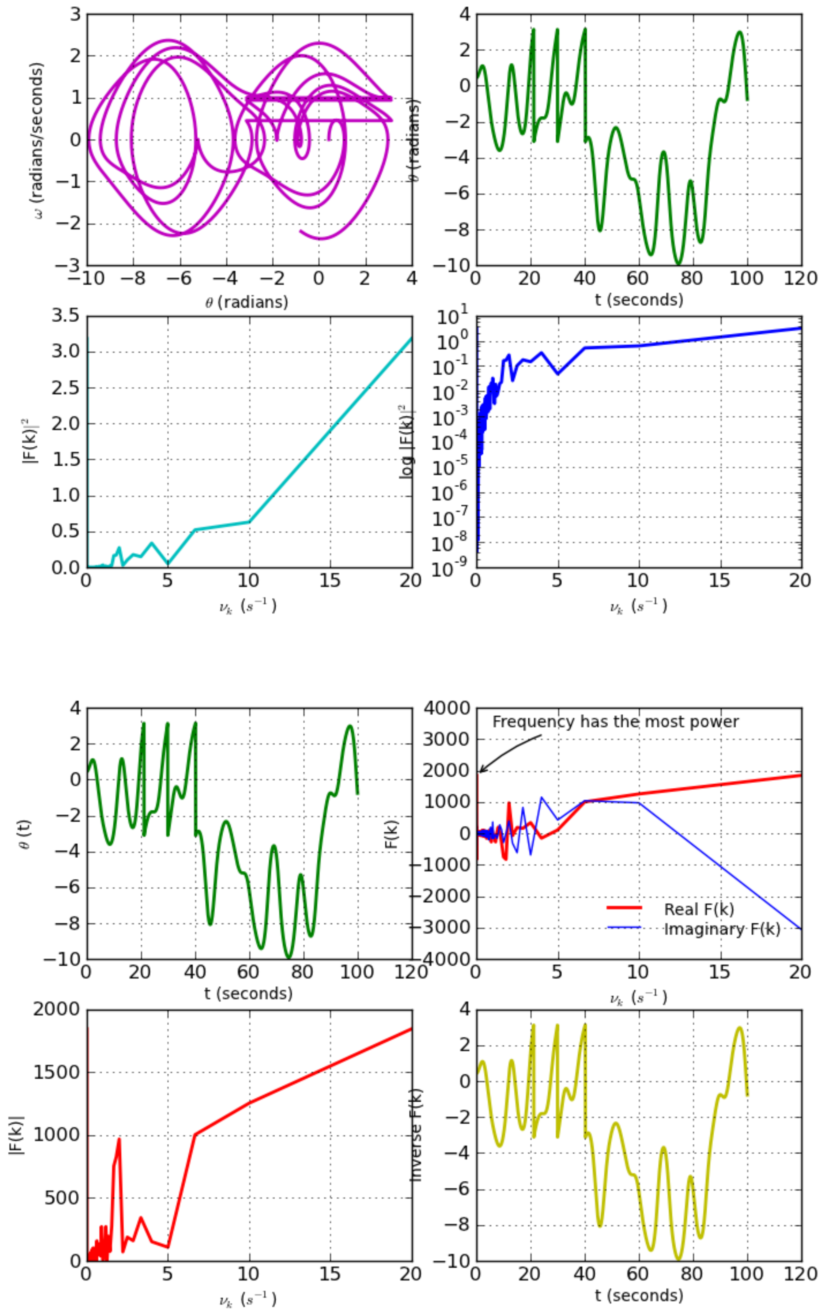


Figure 15: **Chaotic Regime, b = 12**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

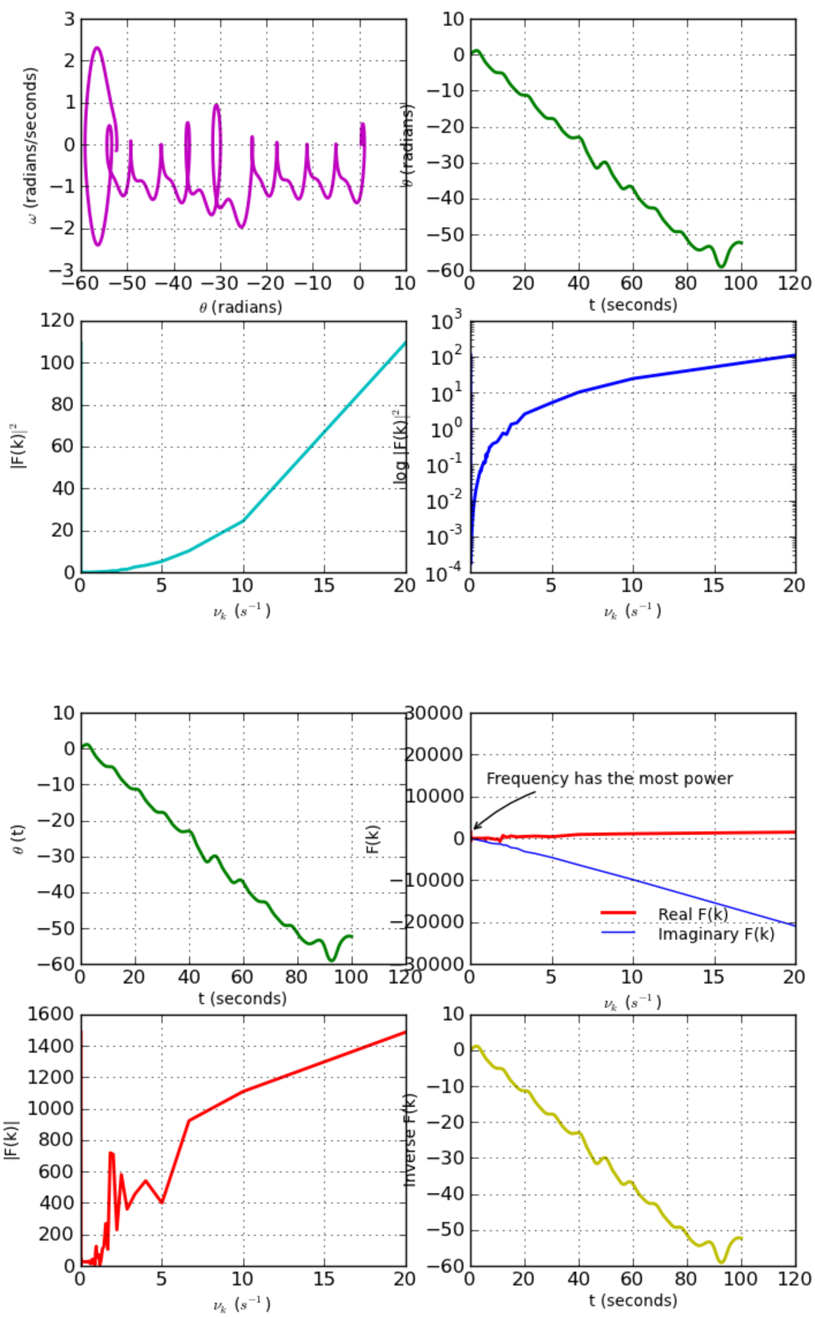
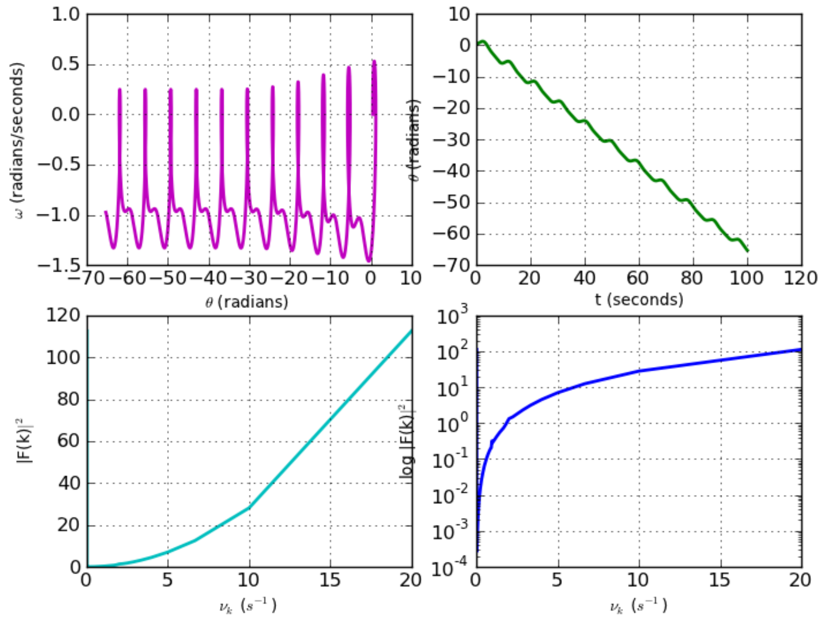


Figure 16: **Chaotic Regime, b = 12.5**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).



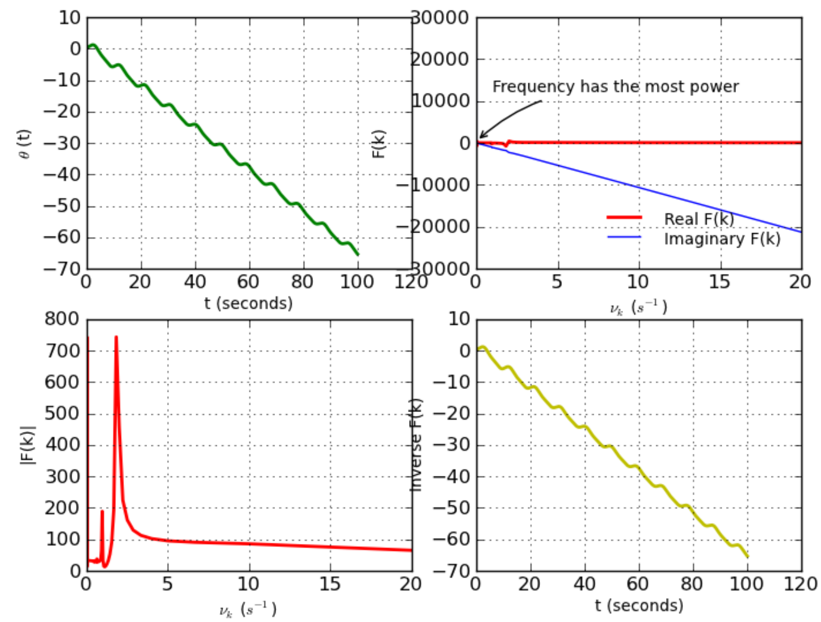


Figure 17: **Chaotic Regime, b = 13**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).

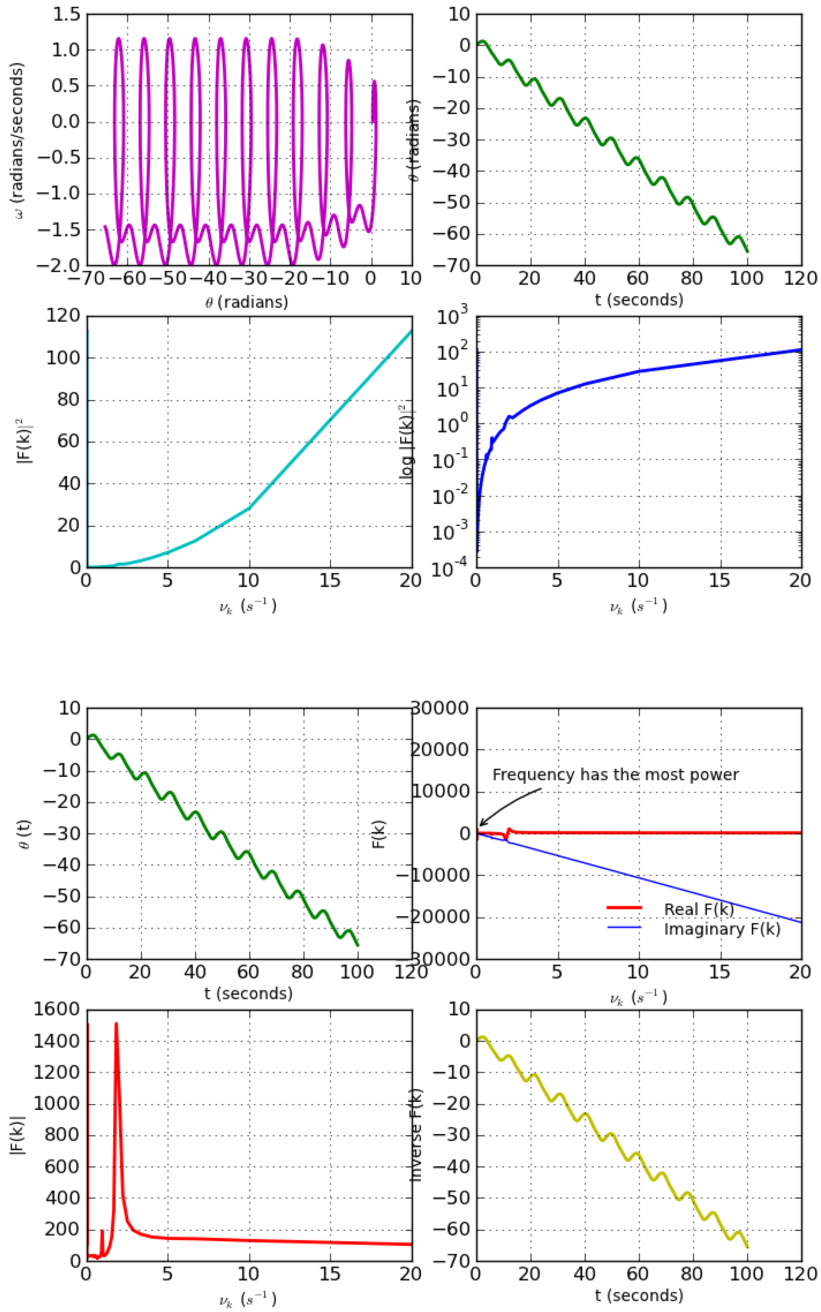


Figure 18: **Chaotic Regime, b = 13.5**: (left) *Phase space* and *power spectrum* for the damped driven pendulum. (right) The *discrete Fourier transform* (DFT) and its inverse, returning to the original function).