PHYS 688: Numerical Methods for AstroPhysics
Homework #6: 2D Rayleigh-Taylor Instability

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Implement a model of *Rayleigh-Taylor instability* in the `pyro code` [?].

1. The *initial conditions* for density defines a heavy fluid over a light fluid, separated by an interface:

$$\rho = \begin{cases} \rho_{\text{down}} & \text{if } y < 0.5 \cdot y_{\text{center}} \\ \rho_{\text{up}} & \text{if } y \geq 0.5 \cdot y_{\text{center}}, \end{cases}$$

   where $\rho_{\text{up}} > \rho_{\text{down}}$ are **arbitrary inputs** and $y_{\text{center}} = y_{\text{min}} + y_{\text{max}}$.

2. The *evolution* for pressure is given by hydrostatic equilibrium for the gravity constant, g,

$$\frac{\partial p}{\partial z} + \rho g = 0,$$

   which translates as

$$p = \begin{cases} p_0 + \rho_{\text{down}} \cdot g \cdot y & \text{if } y < 0.5 \cdot y_{\text{center}} \\ p_0 + \rho_{\text{down}} \cdot g \cdot y_{\text{center}} + \rho_{\text{up}} \cdot g \cdot (y - y_{\text{center}}) & \text{if } y \geq 0.5 \cdot y_{\text{center}}, \end{cases}$$

   where p_0 is an **arbitrary input**.

3. The *perturbation* is applied to the y-velocity,

$$v = A \sin(2\pi x / L_x) \cdot e^{-(y - y_{\text{center}})^2/\sigma^2},$$

   and should be small, where A is the amplitude and σ the vertical extent of the perturbation, both **arbitrary inputs**.

4. The *boundary conditions* are *periodic* on the sides (x) and *reflecting* at the top and bottom (y).

5. The *domain* is made *tall*, with $y \in [0, 3]$ and *narrow*, with $x \in [0, 1]$.

Using the following input parameters:

- $\rho_{\rm down} = 1.0$,
- $\rho_{\rm up} = 0.2$,
- $p_0 = 1.0$,
- $g = 1.0$.

We test many possibilities for the perturbation parameters, A and σ. First, if A is too large, *e.g.,* $A = 0.8$, it quickly creates shocks, which interferes with the process, as we can see in the Fig. **??**.

If we set σ to be too small, the fluid only propagates, as we can see in the Fig. **??**.

The tuning is important in this problem. Reducing A and increasing σ to find a good development of the perturbation in the plane dividing the two liquids, avoiding shocks. For example, we make $A = 0.1, \sigma = 0.1$, as in Fig. **??**, the RT-instability still did not have enough time to form before the shock.

A set of parameters that showed the RT-instability without being interfered by shock to the boundaries were $\sigma = 0.4$ and $A = 0.1$, as we can see in the Fig. **??**.

To compare resolution, we run the same parameters above but with a smaller grid, [100, 300]. Altought the RT-instabilities seems to appears fast (smaller grids), they present very low resolution, as we see in the Fig. **??**.

References

[1] *Mike Zingale's Class*, *http://bender.astro.sunysb.edu/hydro_by_example/compressible/*

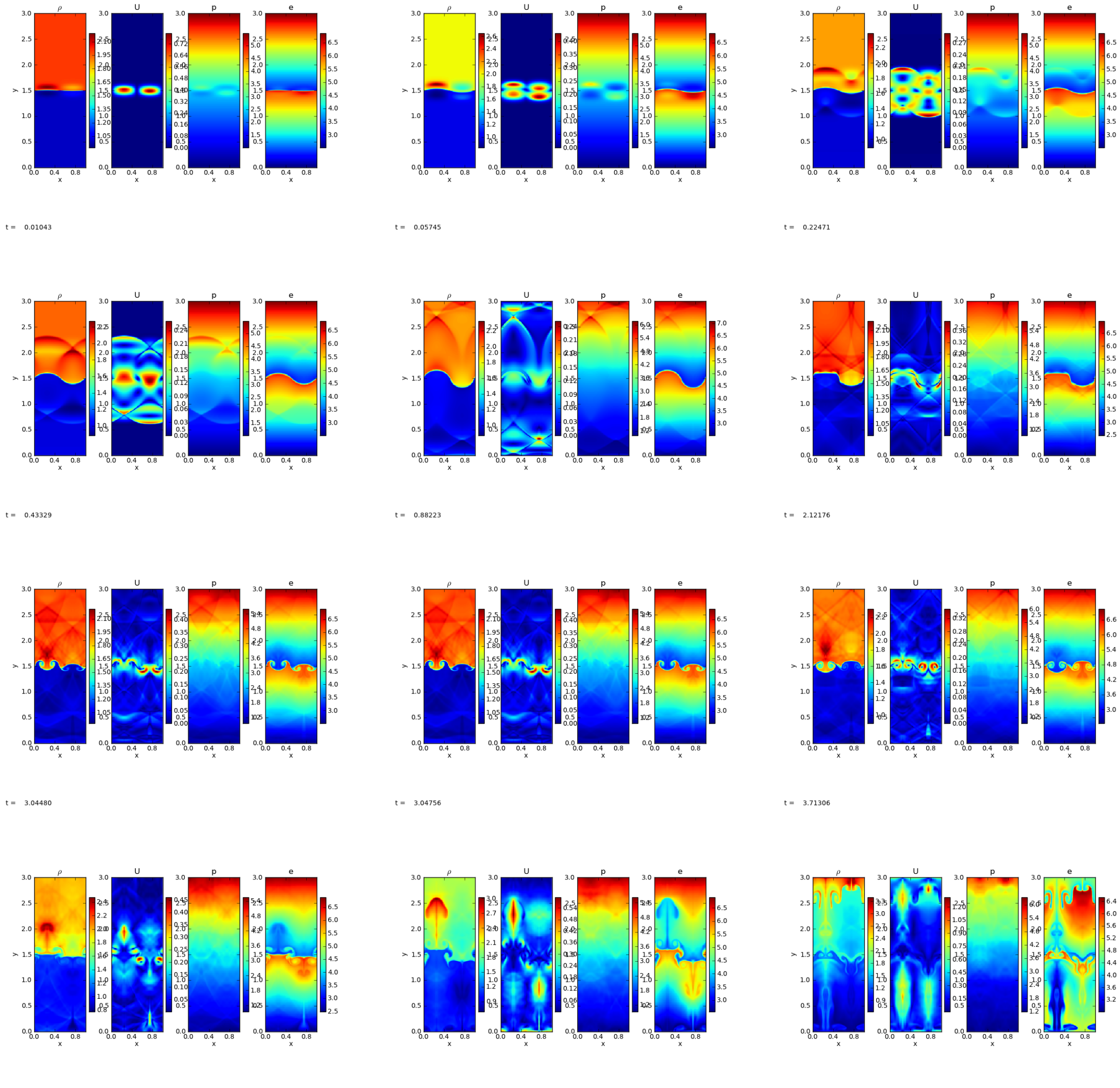


Figure 1: Rayleigh-Taylor instability with shocks: here the amplitude of the perturbation in the velocity is too large. $A = 0.8, \sigma = 0.08$, grids $[200, 600]$

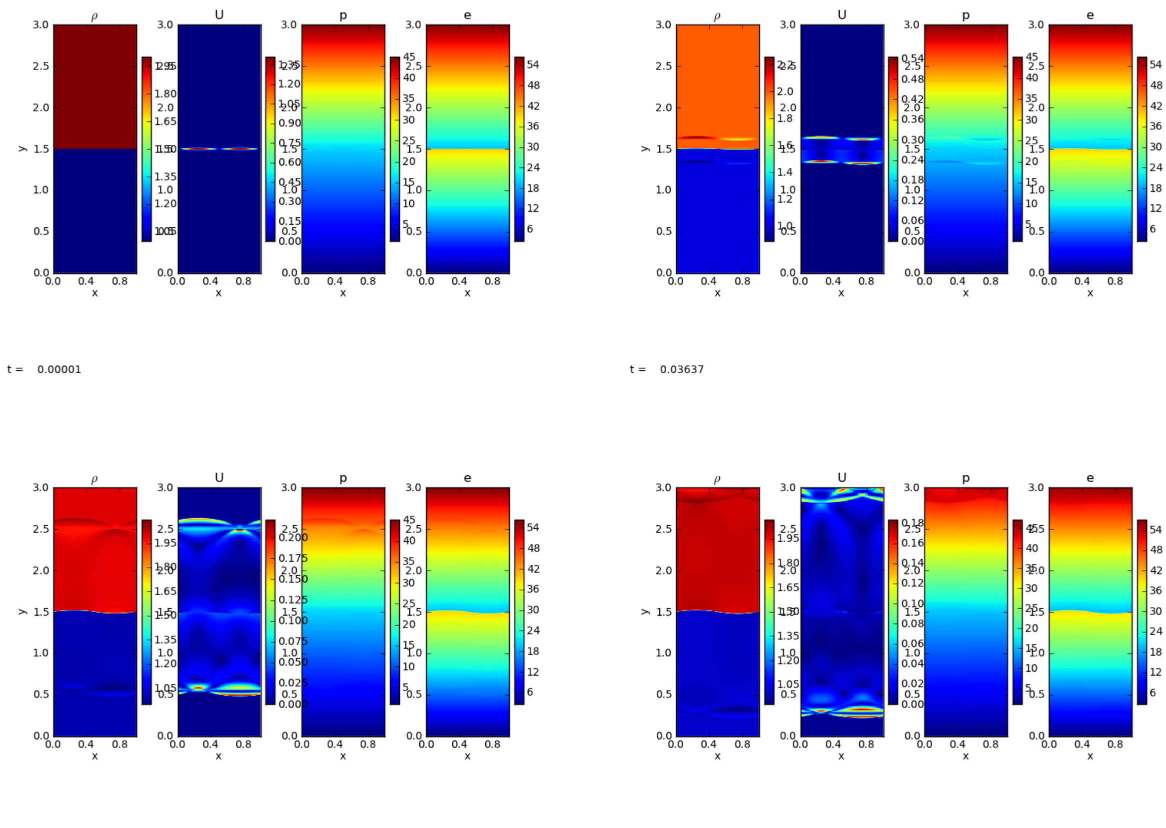


Figure 2: Rayleigh-Taylor instability not seen and interfered by shocks: here σ is too small. $A = 1.0$ and $\sigma = 0.001$, grid $[200, 600]$

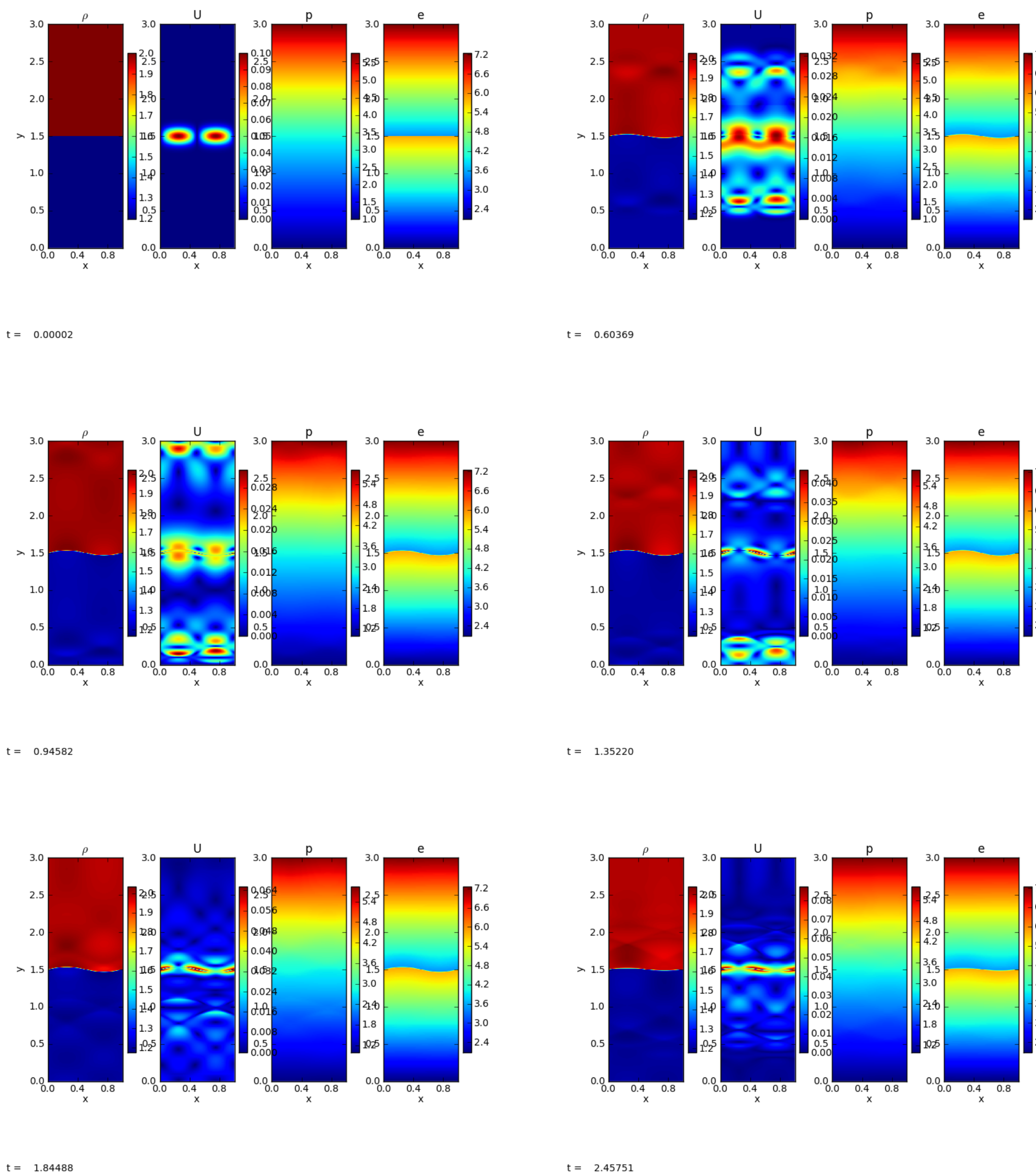


Figure 3: Rayleigh-Taylor instability not clear: increasing $\sigma = 0.1$ and decreasing $A = 0.1$, still is not enough. Meshes: $[200, 600]$

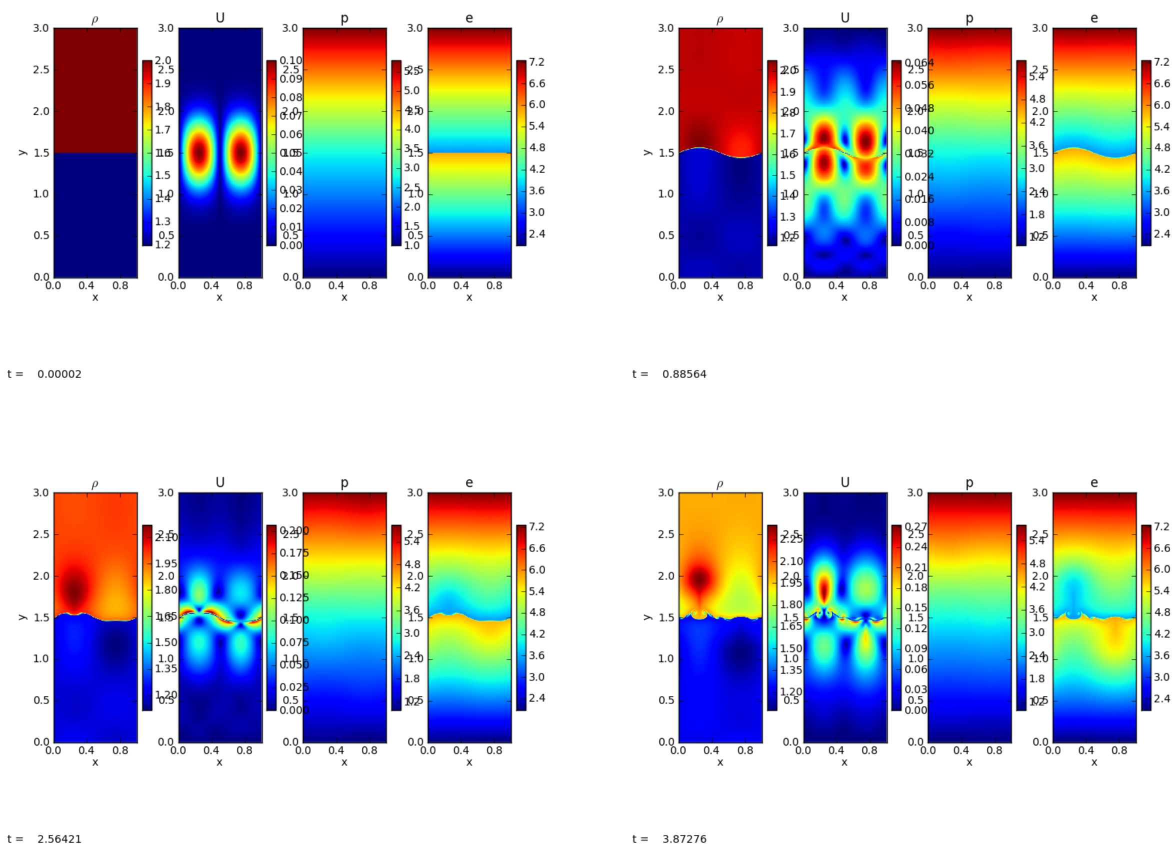
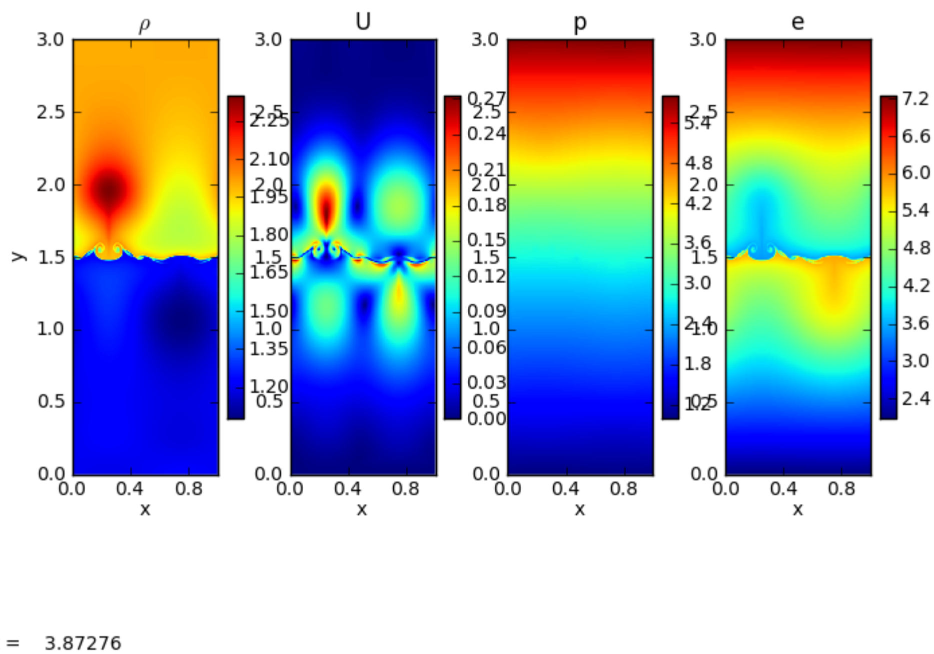


Figure 4: Rayleigh-Taylor instability formed: with $\sigma = 0.4$ and $A = 0.01$, and grids $[256, 768]$.



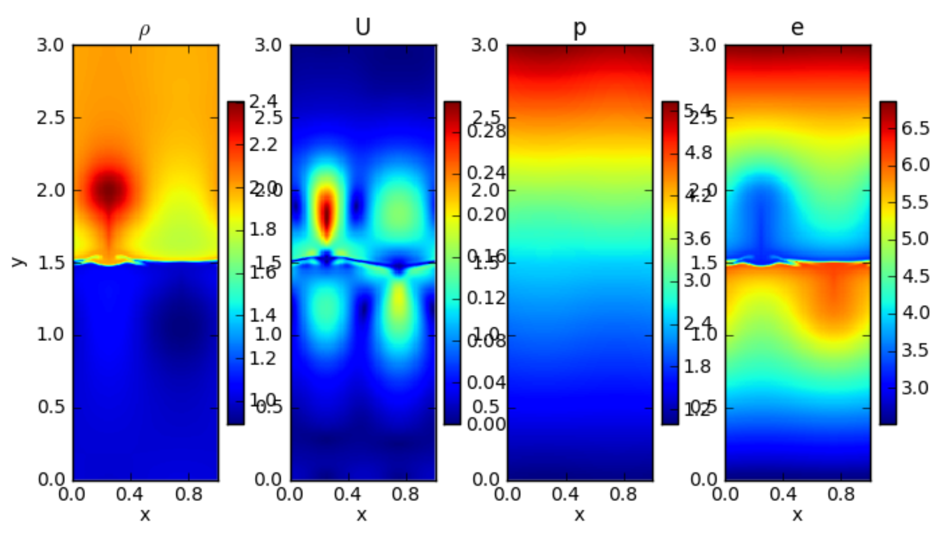


Figure 5: Rayleigh-Taylor instabilities for $\sigma = 0.4$ and $A = 0.01$, and two sizes of grids: (top) $[256, 768]$ and (bottom) $[100, 300]$.