Recall that from eqs(3.1) and (3.2), we have

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) = \mathbf{F}(\mathbf{x}^* + \delta\mathbf{x}) = \mathbf{F}(\mathbf{x}^*) + \mathbf{F}'(\mathbf{x}^*)\delta\mathbf{x} + \cdots,$$

where $\delta\mathbf{x} = \mathbf{x} - \mathbf{x}^*$. Also, note that at a fixed point, we have $\mathbf{F}(\mathbf{x}^*) = 0$, so we can simplify the linearised dynamical equations to

$$\dot{\mathbf{x}} \approx \mathbf{F}'(\mathbf{x}^*)\delta\mathbf{x}.$$

Note, however, that $\dot{\mathbf{x}} = \delta\dot{\mathbf{x}}$, because

$$\delta\dot{\mathbf{x}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}^*,$$

and $\dot{\mathbf{x}}^* = 0$ because it's a constant. So we can write the dynamics as

$$\delta\dot{\mathbf{x}} \approx \mathbf{F}'(\mathbf{x}^*)\delta\mathbf{x}.$$

Let's assume, for simplicity, that $\delta\mathbf{x}$ lies along an eigenvector of $\mathbf{F}'(\mathbf{x}^*)$. Then we have

$$\delta\dot{\mathbf{x}} \approx \lambda\,\delta\mathbf{x},$$

where λ is the eigenvalue. Note, also that we can integrate this expression,

$$\begin{aligned}
\delta\dot{\mathbf{x}} = \frac{\partial\delta\mathbf{x}}{\partial t} &\approx \lambda\,\delta\mathbf{x} \\
\int \frac{d\delta\mathbf{x}}{\delta\mathbf{x}} &= \int \lambda dt \\
\log(\delta\mathbf{x}) &= \lambda t + const \\
\delta\mathbf{x} &= \mathbf{A}\cdot e^{\lambda t},
\end{aligned}$$

where the constant of integration in the 2nd last line has become the constant $\mathbf{A}$ in the final line after exponentiation. Let's look at the behaviour for different types of λ.

If $Re(\lambda) < 0$, the exponentiation implies that $\delta\mathbf{x} \to 0$ as $t \to \infty$, and hence $\mathbf{x} \to \mathbf{x}^*$. We have an attracting mode.

If $Re(\lambda) > 0$, the exponentiation implies that $\delta\mathbf{x} \to \infty$ as $t \to \infty$. We have a repelling mode.

Suppose $Im(\lambda) \neq 0$. Let's call this imaginary component ω then the exponentiation includes a component of the form $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$. The imaginary component induces an oscillation. Note that the magnitude of this oscillation is still given by $\exp[Re(\lambda)t]$.

Note that an arbitrary perturbation can usually be decomposed into a linear combination of perturbations along the eigenmodes, and the behaviour of the perturbation is then given by its behaviour along the eigenmodes.