AEW Worksheet 9

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MATH 1920

Name: ____________________

Collaborators: ____________________

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1

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{\text{T}}\boxed{\text{F}}$ $\lim_{m,n\to\infty}\left(\left(\frac{1}{n}\right)^{\alpha}+\left(\frac{2}{n}\right)^{\alpha}+\cdots+\left(\frac{n}{n}\right)^{\alpha}\right)\frac{1}{n}\cdot\left(\left(\frac{1}{m}\right)^{\beta+1}+\left(\frac{2}{m}\right)^{\beta+1}+\cdots+\left(\frac{m}{m}\right)^{\beta+1}\right)\frac{1}{m}=\int_0^1\int_0^1 x^{\alpha}y^{\beta+1}\,dx\,dy$

(b) $\boxed{\text{T}}\boxed{\text{F}}$ $\int_{y=1}^{4}\int_{x=0}^{1}\left(x^2+\sqrt{y}\right)\sin\left(x^2y^2\right)dx\,dy\le 9$

(c) $\boxed{\text{T}}\boxed{\text{F}}$ $\int_{r=-1}^{1}\int_{\theta=0}^{1}e^{r^2+\theta^2}\,d\theta\,dr=\left[\int_{r=-1}^{1}e^{r^2}\,dr\right]\left[\int_{\theta=0}^{1}e^{\theta^2}\,d\theta\right]$

(d) $\boxed{\text{T}}\boxed{\text{F}}$ The integral $\int_{\phi=0}^{\pi/2}\int_{\theta=0}^{\pi/2}\int_{\rho=0}^{1}\rho^2\sin\theta\,d\rho\,d\theta\,d\phi$ gives the volume of 1/4 of a sphere.

2

(a) Evaluate

$$\iint_D \frac{1}{(x^2+y^2)^{n/2}}\,dA,$$

where n is an integer and D is the region bounded by the circles with center the origin and radii r and R, $0<r<R$.

(b) For what values of n does the integral have a limit as $r\to 0^+$?

(c) Find

$$\iiint_E \frac{1}{(x^2+y^2+z^2)^{n/2}}\,dV,$$

where E is the region bounded by the spheres with center the origin and radii r and R, $O<r<R$

(d) For what values of n does the integral in part (b) have a limit as $r\to 0^+$?

3

Given the triple integrals below:

$$\pi\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{\ln 3}\frac{\pi e^{2x}\sin\left(\pi y^2\right)}{y^2}\,dx\,dy\,dz=2a+b$$

$$\int_b^a\int_0^{\pi/4}\int_0^{\sec\phi}(\rho\cos\phi)\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=\frac{\pi}{4}$$

Find the values of a and b.

4

Radium 223 decays with a half-life of 11.43 days; Radium 224, with a half life of 3.632 days As a result, the probability that an atom of Radium 223 will decay at a time x days has a density function $p(x)=me^{-mx}$, where $m=0.06064$ and the probability that an atom of Radium 224 will decay at a time y days has a density function $q(y)=ne^{-ny}$, where $n=0.1908$

(a) Assuming that the decay times of the two atoms is independent, find the probability that an atom of Radium 223 will decay before an atom of Radium 224.