AEW Worksheet 7

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MATH 1920

Name: ____________________

Collaborators: ____________________

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1

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{T}\boxed{F}$ If (x, y) is a local minimum of a function f then f is differentiable at (x, y) and $\nabla f(x, y) = 0$.

(b) $\boxed{T}\boxed{F}$ If x is a minimum of f given the constraints $g(x) = h(x) = 0$ then $\nabla f(x) = \lambda \nabla g(x)$ and $\nabla f(x) = \mu \nabla h(x)$ for some scalars λ and μ

(c) $\boxed{T}\boxed{F}$ For any unit vector $\mathbf{u}$ and any point $\mathbf{a}$, $Df_{-\mathbf{u}}(\mathbf{a}) = -Df_{\mathbf{u}}(\mathbf{a})$.

2

(a) Find and evaluate critical points (local maximum or minimum) of

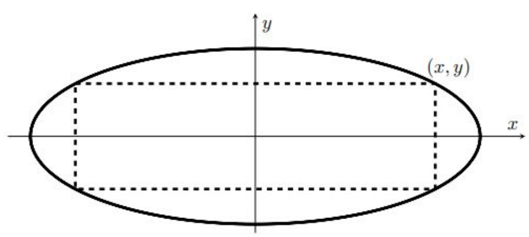
$$f(x, y, z) = -x\log x - 2y\log y - 3z\log z\,,\ \text{subject to the constraint } g(x, y, z) = x + 2y + 3z = 1$$

(b) Find the critical points of

$$f(x, y, z) = \left(5x^2\right) + \left(5y^2\right) + \left(5z^2\right)\,,\ \text{subject to the constraint } g(x, y, z) = xyz = 6$$

3

In this problem we will consider rectangles that are inscribed in the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The sides of the inscribed rectangles are parallel to the coordinate axes, as in the figure below. **Note:** a, b are positive constants.



(a) Using the method of Lagrange multipliers, find the rectangle of largest area that can be inscribed in the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. What is its area?

(b) For the inscribed rectangle of maximum area, what are the coordinates of its vertex that lies in the first quadrant?

4 (Challenge+)

The equation below, where n is a positive integer, a and b are constants, describes a unique surface. Define the extreme points to be the points on the surface with a maximum distance from the origin. For this problem you must use method of Lagrange Multipliers.

$$\frac{x^{2n}}{a^{2n}} + \frac{y^{2n}}{b^{2n}} = 1$$

(a) Find all the extreme points on the surface with $n = 2$. What is the distance between the extreme points and the origin, in terms of a and b?

(b) Find all the extreme points on the surface for integers $n > 2$. What is the distance between the extreme points and the origin, in terms of a, b, and n? (assume the object has symmetry about the origin for x and y, and $n > 2$)

(c) Suppose $a = b$, find the distance between these extreme points and the origin in part (a) and (b).