AEW Worksheet 10

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MATH 1920

Name: ______________________________

Collaborators: ______________________________

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1

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) T F If a particle travels in a closed loop then the total work done on the particle over the loop is zero

(b) T F If there exists a closed curve C in D such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ then $\mathbf{F}$ is conservative on D.

(c) T F $\int_{-C} f ds = -\int_C f ds$

2

Evaluate

$$I = \int_C x^2 y \, dx + (x - 2y) \, dy$$

over the parts of the parabola $y = x^2$ from (0,0) to (1,1).

3

Sketch the gradient vector field for $f(x, y) = x^2 + y^2$ as well as several contours for this function.

4

(a) Find a constant a such that the vector field

$$F(x, y) = \langle ax^2 y - y^3, 3x^2 - 3xy^2 \rangle$$

is conservative or else show that there is no such constant a. If conservative, find a potential function.

(b) Find constants a and b such that the vector field

$$\vec{F} = \langle ay^2, 2xy + 2yz, by^2 + z^2 \rangle$$

is conservative or else show that there is no such constants. If conservative, find a potential function.

(c) Using part b, find the integral $\int_C \vec{F} \cdot d\vec{r}$, where the curve C is parametrized by $x = e^t - te^t, y = 2t^2, z = 3t, 0 \leq t \leq 1$

(d) Using part (b) and (c), give an equation of the surface S that contains all points P so that $\int_O^P \vec{F} \cdot d\vec{r} = 1$, where $O = (0, 0, 0)$ is the origin.

5

(a) Prove that the rotation field

$$\mathbf{F} = \frac{\langle -y, x \rangle}{|\mathbf{r}|^p},$$

where $\mathbf{r} = \langle x, y \rangle$ is not conservative for $p \neq 2$

(b) For $p = 2$, show that F is conservative on any region not containing the origin.

(c) Find a potential function for $\mathbf{F}$ when $p = 2$