AEW Worksheet 11

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MATH 1920

Name: ______________________

Collaborators: ______________________

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# 1

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{T}\boxed{F}$ If S is the unit sphere $x^2 + y^2 + z^2 = 1$ and $a, b, c$ are real numbers, then $\iint_S |ax + by + cz|\, dS \geq 0$

(b) $\boxed{T}\boxed{F}$ If $S = \{(x, y, z) : f(x, y, z) = k\}$ is a level surface of a smooth function f with no critical points on S, then S must be orientable.

(c) $\boxed{T}\boxed{F}$ If $F_Z(z) = P(X + Y \leq z) = \int_{y=-\infty}^{+\infty} \int_{x=-\infty}^{z-y} f_{XY}(x, y)dxdy$, then $f_Z(z) = \int_{-\infty}^{+\infty} \left(\frac{\partial}{\partial z} \int_{-\infty}^{z-y} f_{XY}(x, y)dx\right) dy$ assume $Z = X + Y$ and the region $D_z : x + y \leq z$ is shaded.

# 2

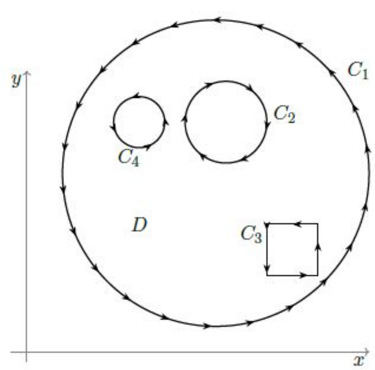
Suppose R is a positive real number. Let S be the cone given by the equation $z = \sqrt{x^2 + y^2}$ with $0 \leq z \leq R$, oriented downward. Compute the flux of $G = \langle xz, yz, xy\rangle$ across S.

# 3

Suppose that D is the bounded region in the plane that has boundary given by the oriented simple closed piecewise smooth curves $C_1, C_2, C_3$, and $C_4$ as in the picture. Suppose $\mathbf{F} = \langle P, Q, 0\rangle : \mathbb{R}^3 \to \mathbb{R}^3$ is a vector field and P and Q have continuous partial derivatives on $\mathbb{R}^2$. If the following is true,

$$\oint_{C_k} \mathbf{F} \cdot d\mathbf{r} = 2^k$$

find $\iint_D (Q_x - P_y)\, dA = \iint_D (\nabla \times \mathbf{F}) \cdot \hat{k} dA$




# 4

Define $\mathbf{G} = \left\langle 2zxe^{x^2-y^2}, -2zye^{x^2-y^2}, e^{x^2-y^2} + 2z\right\rangle$, $\mathbf{H} = \langle 0, x, -y\rangle$ and $\mathbf{F} = \mathbf{G} + \mathbf{H}$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the line segment from (1,2,4) to (-1,1,1).

## 5 (Challenge)

Prove that for the vector field,

$$\mathbf{F} = \langle my + g(x, y), nx + h(x, y) \rangle$$

and the positively oriented curve C around any isosceles right triangle (that is, right triangle having legs of equal length), the following must be true,

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = a^2$$

where $a$ is the length of the legs of the triangle (i.e., show that there is a condition for which the statement above holds true. Note that you should not pick a specific triangle or value of $a$ when completing this problem. Likewise, do not pick specific values of $g(x, y)$ and $h(x, y)$ (neither $m$ nor $n$)).

## 6 (Challenge)

Evaluate the following line integral,

$$\int_{\mathcal{C}} (e^{\sinh(\ln(\ln(-x^{2020} \cdot e^{-x^{2021}} + 2022)))} + y)dx + (3x + y)dy$$

on the non-closed path C connecting $M(0, 0)$ to $N(2, 2)$, then to $P(2, 4)$, and then to $Q(0, 6)$.