



1

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) ☐ T ☐ F $\lim_{\rho \rightarrow 0} \frac{\rho \sin(\phi) \cos(\theta) \rho \sin(\phi) \sin(\theta) \rho \cos(\phi)}{\rho^2}$ in spherical coordinates does not exist.

(b) ☐ T ☐ F If $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$, then $\lim_{x \rightarrow 0} f(x,0) = 0$.

(c) ☐ T ☐ F If $\lim_{x \rightarrow 0} f(x,0) = 0$, and $\lim_{y \rightarrow 0} f(0,y) = 0$, then $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$.

2

Sketch in the xy-plane the domain of

$$f(x,y) = \frac{\sqrt{4-y^2}}{\ln(y-x^2)}$$

3

(a) Find the limit, if it exists, or show that the limit does not exist

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{(x+y+z)^4}{x^4 + y^4 + z^4}$$

(b) Is there a real number α such that f is continuous at $(0,0)$?

$$\text{Let } f(x,y) = \begin{cases} \frac{x^2 \sin^2(y)}{3x^2 + 2y^2} & \text{if } (x,y) \neq (0,0) \\ \alpha & \text{if } (x,y) = (0,0) \end{cases}$$

4

A particle travels along the parametric curve $\langle e^{-t} \cos t, e^{-t} \sin t \rangle$ starts at $(1,0)$ at time $t = 0$ and then spirals into the origin $(0,0)$ as $t \rightarrow \infty$. How far will the particle have traveled when it reaches the origin?

5 (Challenge)

Find the length of the curve $\mathbf{r}(t) = \langle t^m, t^m, t^{3m/2} \rangle$, for $0 \leq a \leq t \leq b$, where m is a real number. Express the result in terms of m , a , and b .

6 (Challenge)

(a) Show that the limit below does not exist when a , b , and c are nonzero real numbers and m and n are positive integers.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{ax^m y^n}{bx^{m+n} + cy^{m+n}}$$

(b) Show that the limit below does not exist when a , b , and c are nonzero real numbers and n and p are positive integers with $p \geq n$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{ax^{2(p-n)} y^n}{bx^{2p} + cy^p}$$