AEW Worksheet 6

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MATH 1920

Name: ______________________

Collaborators: ______________________

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1

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{\text{T}}$ $\boxed{\text{F}}$ If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ where $a, b \neq 0$, then $x = a + r\cos\theta$ and $y = b + r\sin\theta$

(b) $\boxed{\text{T}}$ $\boxed{\text{F}}$ There exists a function f with continuous second-order partial derivatives such that $f_x(x,y) = kx + y^2$ and $f_y(x,y) = x - y^2$ for constant k.

(c) $\boxed{\text{T}}$ $\boxed{\text{F}}$ Suppose their exist an angle of inclination ψ and $z = f(x,y)$, then $\psi = \tan^{-1}\left(\left\|\nabla f_{(a,b)}\right\| \sin\theta\right)$

2

Consider the discriminant,

$$D(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} -12\pi x^2 + 2\pi a & b \\ b & -12\pi y^2 + 2\pi a \end{vmatrix}$$

where $D(x,y)$ is in the determinant form.

(a) Compute the determinant in only terms of x, y, a and b.

(b) By considering the second derivative test, describe the classification of critical points, if $b = 0$, only terms of x, y, and a.

(c) Find a function $f(x,y)$ that could represent this discriminant, if $a = 2$ and $b = 0$. Assume that the function behaves such that $f(0,0) = 0$, $f(1,0) = \pi$, and $f(0,1) = \pi$.

(d) Find critical points of f and determine whether the points are local minima, maximum or saddle points.

3

The N corporation (maker of the finest Y) has recently merged with the J (maker of the finest Z). Currently Y sell for three dollars each and Z sell for nine dollars each. By combining their production the new company enjoys economy of scope and is now able to produce y Y and z Z at a cost of $10 + \frac{1}{2}y^2 + \frac{1}{3}z^3 - yz$ dollars. Determine how many Y and Z respectively should be made in order to maximize profit. Also, verify that your answer is a maximum by using the second derivative test. **Note:** Profit = Revenue − Cost

4

Suppose that $f(x,y)$ is differentiable and $f\left(t^3 - t + 1, 2 - t^2\right) = t^4 - 4t^3 + 4t + 6$

$$\text{Find } \frac{\partial f}{\partial x}(1,1) \text{ and } \frac{\partial f}{\partial y}(1,1)$$

5

Suppose we know the following:

$$\frac{d}{dt}f(\mathbf{c}(t)) = 2 \quad \text{if} \quad \vec{c}(t) = \langle t, t \rangle$$

$$\frac{d}{dt}f(\mathbf{c}(t)) = 3 \quad \text{if} \quad \vec{c}(t) = \langle t, -t \rangle$$

Find $\nabla f(0, 0)$.