AEW Worksheet 5

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MATH 1920

Name: ______________________________

Collaborators: ______________________________

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1

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{T}$ $\boxed{F}$ $f_y(a,b) = \lim_{y\to b} \frac{f(a,y)-f(a,b)}{y-b}$

(b) $\boxed{T}$ $\boxed{F}$ If $f(x,y) = \sin x + \sin y$ then $-\sqrt{2} \leq D_u f(x,y) \leq \sqrt{2}$ for all unit vectors u.

(c) $\boxed{T}$ $\boxed{F}$ If $f_x(a,b)$ and $f_y(a,b)$ both exist then f is locally linear at (a,b)

2

The kinetic energy of a body with mass m and velocity v is $K = \frac{1}{2}mv^2$. Find a product of two partial derivatives with respect to m and v such that kinetic energy is constant.

3

Find the directional derivative of $f(x,y,z) = xz^2 - 3xy + 2xyz - 3x + 5y - 17$ from the point (2,-6,3) in the direction of the origin.

4

Find (x_0, y_0) so that the plane tangent to the surface $z = f(x,y) = x^2 + 3xy - y^2$ at $(x_0, y_0, f(x_0, y_0))$ is parallel to the plane $16x - 2y - 2z = 23$.

5

Let

$$f(x,y) = e^{x^2+2y^2}$$

Find the unit vector $\mathbf{u} = \langle a, b\rangle$ which minimizes the directional derivative $D_{\mathbf{u}} f$ at the point $(x,y) = (2,3)$.

6 (Challenge)

Find $\frac{\partial f}{\partial x}(0,1)$ and $\frac{\partial f}{\partial y}(0,1)$ for:

$$f(x,y) = \sin x + y^2 \cos x + y^4 \arctan\left(x\left(y^2 - 1\right)\right) + \ln\left(2e^{\sin x} - 1\right) \sec(xy) \tan(y-1)$$

Hint: there is an *extremely easy* way and there is a hard way.