AEW T/F

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1 Miscellaneous

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) T F $\lim_{\rho \to 0} \frac{\rho \sin(\phi)\cos(\theta)\rho \sin(\phi)\sin(\theta)\rho \cos(\phi)}{\rho^2}$ in spherical coordinates does not exist.

(b) T F If $\lim_{(x,y)\to(0,0)} f(x,y) = 0$, then $\lim_{x\to 0} f(x,0) = 0$.

(c) T F If $\lim_{x\to 0} f(x,0) = 0$, and $\lim_{y\to 0} f(0,y) = 0$, then $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

(d) T F If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ where $a, b \neq 0$, then $x = a + r\cos\theta$ and $y = b + r\sin\theta$

2 Chapter 13 VECTOR GEOMETRY

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) T F Suppose a vector v is defined as $v = \langle a_2 - a_1, b_2 - b_1 \rangle$, then the slope is given by $\frac{b_2 - b_1}{a_2 - a_1}$ where a and b are non-zero constants.

(b) T F For any vectors u and v in $\mathbb{R}^n$, $|u + v| = |u| + |v|$.

(c) T F For any vectors u and v in $\mathbb{R}^n$, $|u + v| \leq |u| + |v|$.

(d) T F For any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$.

(e) T F For any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$

3 Chapter 14 CALCULUS OF VECTOR-VALUED FUNCTIONS

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) T F If there is no solution t to the equation $u_0 + t\mathbf{u} = v_0 + t\mathbf{v}$ then the lines given by $\{u_0 + t\mathbf{u} : t \in \mathbb{R}\}$ and $\{v_0 + t\mathbf{v} : t \in \mathbb{R}\}$ do not intersect.

(b) T F For any line in $\mathbb{R}^3$ and a point not on that line, there is exactly one plane that is normal to the line and contains the point.

(c) T F If $|\mathbf{r}(t)| = 1$ for all t, then $|\mathbf{r}'(t)|$ is constant.

4 Chapter 15 DIFFERENTIATION IN SEVERAL VARIABLES

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) T F Suppose their exist an angle of inclination ψ and $z = f(x,y)$, then $\psi = \tan^{-1}\left(\left\|\nabla f_{(a,b)}\right\| \sin\theta\right)$

(b) T F If (x,y) is a local minimum of a function f then f is differentiable at (x,y) and $\nabla f(x,y) = 0$.

(c) T F If x is a minimum of f given the constraints $g(x) = h(x) = 0$ then $\nabla f(x) = \lambda \nabla g(x)$ and $\nabla f(x) = \mu \nabla h(x)$ for some scalars λ and μ

(d) $\boxed{\text{T}}\boxed{\text{F}}$ $f_y(a,b) = \lim_{y\to b} \frac{f(a,y)-f(a,b)}{y-b}$

(e) $\boxed{\text{T}}\boxed{\text{F}}$ If $f(x,y) = \sin x + \sin y$ then $-\sqrt{2} \le D_u f(x,y) \le \sqrt{2}$ for all unit vectors u.

(f) $\boxed{\text{T}}\boxed{\text{F}}$ If $f_x(a,b)$ and $f_y(a,b)$ both exist then f is locally linear at (a,b)

(g) $\boxed{\text{T}}\boxed{\text{F}}$ For any unit vector $\mathbf{u}$ and any point $\mathbf{a}$, $Df_{-\mathbf{u}}(\mathbf{a}) = -Df_{\mathbf{u}}(\mathbf{a})$.

(h) $\boxed{\text{T}}\boxed{\text{F}}$ There exists a function f with continuous second-order partial derivatives such that $f_x(x,y) = kx + y^2$ and $f_y(x,y) = x - y^2$ for constant k.

5 Chapter 16 MULTIPLE INTEGRATION

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{\text{T}}\boxed{\text{F}}$ $\int_0^1 \int_0^x \sqrt{x+y^2}\,dy\,dx = \int_0^x \int_0^1 \sqrt{x+y^2}\,dx\,dy$

(b) $\boxed{\text{T}}\boxed{\text{F}}$ $\int_0^b \int_{x^2}^{a-x} f(x,y)\,dy\,dx = \int_0^{a-b} \int_0^{\sqrt{y}} f(x,y)\,dx\,dy + \int_{a-b}^{a} \int_0^{a-y} f(x,y)\,dx\,dy$ (assume $a > b$)

(c) $\boxed{\text{T}}\boxed{\text{F}}$ If $f(x,y) = g(x)h(y)$, then $\iint_D f(x,y)\,dA = \left(\iint_D g(x)\,dA\right)\left(\iint_D h(y)\,dA\right)$

(d) $\boxed{\text{T}}\boxed{\text{F}}$ $\lim_{m,n\to\infty} \left(\left(\frac{1}{n}\right)^\alpha + \left(\frac{2}{n}\right)^\alpha + \cdots + \left(\frac{n}{n}\right)^\alpha\right) \frac{1}{n} \cdot \left(\left(\frac{1}{m}\right)^{\beta+1} + \left(\frac{2}{m}\right)^{\beta+1} + \cdots + \left(\frac{m}{m}\right)^{\beta+1}\right) \frac{1}{m} = \int_0^1 \int_0^1 x^\alpha y^{\beta+1}\,dx\,dy$

(e) $\boxed{\text{T}}\boxed{\text{F}}$ $\int_{y=1}^{4} \int_{x=0}^{1} \left(x^2 + \sqrt{y}\right) \sin\left(x^2 y^2\right)\,dx\,dy \le 9$

(f) $\boxed{\text{T}}\boxed{\text{F}}$ $\int_{r=-1}^{1} \int_{\theta=0}^{1} e^{r^2+\theta^2}\,d\theta\,dr = \left[\int_{r=-1}^{1} e^{r^2}\,dr\right]\left[\int_{\theta=0}^{1} e^{\theta^2}\,d\theta\right]$

(g) $\boxed{\text{T}}\boxed{\text{F}}$ The integral $\int_{\phi=0}^{\pi/2} \int_{\theta=0}^{\pi/2} \int_{\rho=0}^{1} \rho^2 \sin\theta\,d\rho\,d\theta\,d\phi$ gives the volume of 1/4 of a sphere.

6 Chapter 17 LINE AND SURFACE INTEGRALS

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{\text{T}}\boxed{\text{F}}$ A surface integral is always a positive quantity.

(b) $\boxed{\text{T}}\boxed{\text{F}}$ If a particle travels in a closed loop then the total work done on the particle over the loop is zero

(c) $\boxed{\text{T}}\boxed{\text{F}}$ If there exists a closed curve C in D such that $\int_C \mathbf{F}\cdot d\mathbf{r} = 0$ then $\mathbf{F}$ is conservative on D.

(d) $\boxed{\text{T}}\boxed{\text{F}}$ $\int_{-C} f\,ds = -\int_C f\,ds$

(e) $\boxed{\text{T}}\boxed{\text{F}}$ If S is the unit sphere $x^2+y^2+z^2 = 1$ and a, b, c are real numbers, then $\iint_S |ax+by+cz|\,dS \ge 0$

(f) $\boxed{\text{T}}\boxed{\text{F}}$ If $F_Z(z) = P(X+Y \le z) = \int_{y=-\infty}^{+\infty} \int_{x=-\infty}^{z-y} f_{XY}(x,y)\,dx\,dy$, then $f_Z(z) = \int_{-\infty}^{+\infty} \left(\frac{\partial}{\partial z} \int_{-\infty}^{z-y} f_{XY}(x,y)\,dx\right) dy$ assume $Z = X+Y$ and the region $D_z : x+y \le z$ is shaded.

(g) $\boxed{\text{T}}\boxed{\text{F}}$ If $S = \{(x,y,z) : f(x,y,z) = k\}$ is a level surface of a smooth function f with no critical points on S, then S must be orientable.

7 Chapter 18 FUNDAMENTAL THEOREMS OF VECTOR ANALYSIS

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{\text{T}}\boxed{\text{F}}$ The flux of $F = \langle x, 0, 0\rangle$ across a sphere of radius 1 at the origin is strictly less than its flux across a sphere of radius 2 at the origin, where both are outwardly oriented.

(b) $\boxed{\text{T}}\boxed{\text{F}}$ If S is be the unit sphere centered at the origin, oriented outwards with normal vector **n** and the integral $I = \iint_S D_{\mathbf{n}} f dS$ where $D_{\mathbf{n}}$ is the directional derivative along **n**, then $I = \iiint_E div(\nabla f) dV$ where E is a solid sphere (assume f is a continuous function).

(c) $\boxed{\text{T}}\boxed{\text{F}}$ If $\vec{F} = \left(x - \frac{2}{3}x^3, \frac{-4}{3}y^3, \frac{-8}{3}z^3\right)$ and $\mathcal{J} = \iint_S \vec{F} \cdot \vec{n} dS$, then $\mathcal{J}$ is maximized with surface S described as $1 = 2x^2 + 4y^2 + 8z^2$