AEW Worksheet 8

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MATH 1920

Name: ______________________________

Collaborators: ______________________________

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1

Determine if the following statements are true(T) or false(F). Mark the correct answer. No justification needed.

(a) $\boxed{\text{T}}\ \boxed{\text{F}}$ $\int_0^1 \int_0^x \sqrt{x+y^2}\,dy\,dx = \int_0^x \int_0^1 \sqrt{x+y^2}\,dx\,dy$

(b) $\boxed{\text{T}}\ \boxed{\text{F}}$ $\int_0^b \int_{x^2}^{a-x} f(x,y)\,dy\,dx = \int_0^{a-b} \int_0^{\sqrt{y}} f(x,y)\,dx\,dy + \int_{a-b}^{a} \int_0^{a-y} f(x,y)\,dx\,dy$ (assume $a > b$)

(c) $\boxed{\text{T}}\ \boxed{\text{F}}$ If $f(x,y) = g(x)h(y)$, then $\iint_D f(x,y)\,dA = \left(\iint_D g(x)\,dA\right)\left(\iint_D h(y)\,dA\right)$

2 (Challenge)

Given that the volume of a region $\mathcal{W}$ is

$$V = \iiint_{\mathcal{W}} 1\,dV = \frac{4}{15}$$

where $\mathcal{W}$ is bounded by the surfaces given by $x = 0$, $x = 1$, $z = a$, $y = 0$, $z + y + x^2 = 4$, $y + x^2 - 4 = -a$. Find all possible values for a (assume that $a \in (0, \infty)$).

3

Set up, but do **NOT** evaluate, the following triple integrals to find the volume. Draw the associated 2D region in a coordinate plane.

(a) $y = x^2 + 4z^2$ and $y = 2x + 3$

(b) $z = \sqrt{x^2 + 4y^2}$ and $z = 1$

(c) A shape with vertices located at $(0,0,0)$, $(a,0,0)$, $(0,b,0)$, and $(0,0,c)$

(d) The solid common to the cylinders $z = \sin x$ and $z = \sin y$ over the square $R = \{(x,y) : 0 \le x \le \pi, 0 \le y \le \pi\}$

(e) The wedge of the square column $|x| + |y| = 1$ created by the planes $z = 0$ and $x + y + z = 1$.

4

Set up, but do **NOT** evaluate, the following double integrals. Draw the associated 2D region in a coordinate plane.

(a) $\int\int_D e^{\frac{x}{y}}\,dA$, $D = \{(x,y) | 1 \le y \le 2, y \le x \le y^3\}$

(b) $\int\int_D 4xy - y^3\,dA$, is the region bounded by $y = \sqrt{x}$ and $y = x^3$.

(c) $\int\int_D 6x^2 - 40y\,dA$, D is the triangle with vertices (0,3), (1,1), (5,3).

(d) The volume of the solid that lies below the surface given by $z = 16xy + 200$ and lies in the region in the xy-plane bounded by $y = x^2$ and $y = 8 - x^2$.