Домашняя работа №3

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№1

$$S=\begin{pmatrix}\cos\theta & \sin\theta e^{-i\varphi}\\ \sin\theta e^{i\varphi} & -\cos\theta\end{pmatrix}$$

1. $\det(S-\lambda I)=0$

$$\begin{vmatrix}\cos\theta-\lambda & \sin\theta e^{-i\varphi}\\ \sin\theta e^{i\varphi} & -\cos\theta-\lambda\end{vmatrix} = -(\cos\theta-\lambda)(\cos\theta+\lambda)-\sin^2\theta=$$

$$=-(\cos^2\theta-\lambda^2)-\sin^2\theta=-\cos^2\theta-\sin^2\theta+\lambda^2=\lambda^2-1=0$$

$\lambda_{1,2}=\pm1$ - собственные значения

При $\lambda_1=1$

$$\begin{pmatrix}\cos\theta-1 & \sin\theta e^{-i\varphi}\\ \sin\theta e^{i\varphi} & -\cos\theta-1\end{pmatrix}\begin{pmatrix}\psi_1^1\\ \psi_2^1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{cases}\psi_1^1(\cos\theta-1)+\psi_2^1\sin\theta e^{-i\varphi}=0\\ \psi_1^1\sin\theta e^{i\varphi}-\psi_2^1(\cos\theta+1)=0\end{cases} \rightarrow \psi_1^1=\frac{\psi_2^1(\cos\theta+1)}{\sin\theta e^{i\varphi}}=\operatorname{ctg}\frac{\theta}{2}e^{-i\varphi}\psi_2^1$$

$$\overline{\psi}_1=\begin{pmatrix}\psi_1^1\\ \psi_2^1\end{pmatrix}=\psi_2^1\begin{pmatrix}\operatorname{ctg}\frac{\theta}{2}e^{-i\varphi}\\ 1\end{pmatrix}=C\begin{pmatrix}\operatorname{ctg}\frac{\theta}{2}e^{-i\varphi}\\ 1\end{pmatrix};$$

$\overline{\psi}_1=\begin{pmatrix}\operatorname{ctg}\frac{\theta}{2}e^{-i\varphi}\\ 1\end{pmatrix}$ - собственный вектор

$$\|\overline{\psi}_1\|=\sqrt{\langle\overline{\psi}_1|\overline{\psi}_1\rangle}=\sqrt{\psi_1^{1*}\psi_1^1+\psi_2^{1*}\cdot\psi_2^1}=$$

$$=\sqrt{\operatorname{ctg}\frac{\theta}{2}e^{-i\varphi}\cdot\operatorname{ctg}\frac{\theta}{2}e^{i\varphi}+1}=\sqrt{\operatorname{ctg}^2\frac{\theta}{2}+1}=\sqrt{\frac{1}{\sin^2\frac{\theta}{2}}}=\frac{1}{\sin\frac{\theta}{2}}>0$$

$$\overline{\psi}_{1\,\text{норм}}=\begin{pmatrix}\cos\frac{\theta}{2}e^{-i\varphi}\\ \sin\frac{\theta}{2}\end{pmatrix}$$

При $\lambda_2 = -1$

$$\begin{pmatrix} \cos\theta+1 & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta+1 \end{pmatrix} \begin{pmatrix} \psi_1^2 \\ \psi_2^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} \psi_1^2(\cos\theta+1) + \psi_2^2 \sin\theta e^{-i\varphi} = 0 \\ \psi_1^2 \sin\theta e^{i\varphi} + \psi_2^2(1-\cos\theta) = 0 \end{cases} \rightarrow \psi_1^2 = \frac{\psi_2^2(\cos\theta-1)}{\sin\theta e^{i\varphi}}$$

$$\overline{\Psi}_2 = \begin{pmatrix} \psi_1^2 \\ \psi_2^2 \end{pmatrix} = \psi_2^2 \begin{pmatrix} \frac{\cos\theta-1}{\sin\theta e^{i\varphi}} \\ 1 \end{pmatrix} = C \begin{pmatrix} \frac{\cos\theta-1}{\sin\theta e^{i\varphi}} \\ 1 \end{pmatrix};$$

$\overline{\Psi}_2 = \begin{pmatrix} \frac{\cos\theta-1}{\sin\theta e^{i\varphi}} \\ 1 \end{pmatrix}$ – собственный вектор

$$\|\overline{\Psi}_2\| = \sqrt{\langle \overline{\Psi}_2 | \overline{\Psi}_2 \rangle} = \sqrt{\psi_1^{2*}\psi_1^2 + \psi_2^{2*}\psi_2^2} =$$

$$= \sqrt{\frac{\cos\theta-1}{\sin\theta e^{-i\varphi}} \cdot \frac{\cos\theta-1}{\sin\theta e^{i\varphi}} + 1} = \sqrt{tg^2\frac{\theta}{2} + 1} = \sqrt{\frac{1}{\cos^2\frac{\theta}{2}}} = \frac{1}{\cos\frac{\theta}{2}} > 0$$

$$\overline{\Psi}_{2\,норм} = \begin{pmatrix} -tg\frac{\theta}{2} e^{-i\varphi} \cdot \cos\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} -\sin\frac{\theta}{2} e^{-i\varphi} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

Проверим ортогональность собственных векторов

$$\langle \overline{\Psi}_1 | \overline{\Psi}_2 \rangle = \psi_1^{1*}\psi_1^2 + \psi_2^{1*}\psi_2^2 = ctg\frac{\theta}{2} e^{i\varphi} \cdot (-tg\frac{\theta}{2} e^{-i\varphi}) + 1 =$$

$= -1 + 1 = 0 \Rightarrow$ векторы ортогональны