Deep Bayes. Theoretical Tasks

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Problem 1

R.v. $\xi \sim Pois(\lambda)$, if $\xi = k$ we perform k Bernoulli trials with the probability of success p.
R.v. η — number of successful outcomes of Bernoulli trials.
Prove that $\eta \sim Pois(\lambda p)$.

Proof

$$\mathbb{P}(\eta = k) = \mathbb{P}(\xi = k) \cdot p^k + \mathbb{P}(\xi = k+1) \cdot p^k(1-p) \cdot (k+1) + \mathbb{P}(\xi = k+2) \cdot p^k(1-p)^2 \cdot C_{k+1}^2 + \cdots =$$
$$= \sum_{i=0}^{\infty} \mathbb{P}(\xi = k+i) p^k (1-p)^i C_{k+i}^i = \sum_{i=0}^{\infty} \left(\frac{\lambda^{k+i} e^{-\lambda}}{(k+i)!} p^k (1-p)^i \frac{(k+i)!}{i!k!} \right) = \frac{e^{-\lambda}\lambda^k p^k}{k!} \sum_{i=0}^{\infty} \frac{\lambda^i (1-p)^i}{i!}$$

The sum in the last term is exactly Taylor expansion of the exponent. Therefore:

$$\mathbb{P}(\eta = k) = \frac{e^{-\lambda}\lambda^k p^k}{k!} e^{\lambda(1-p)} = \frac{(\lambda p)^k e^{-\lambda p}}{k!}$$

Which means, that $\eta \sim Pois(\lambda p)$

Problem 2

Strict reviewer: $t_1 \sim \mathcal{N}(30, 100)$
Kind reviewer: $t_2 \sim \mathcal{N}(20, 25)$
Reviewer is chosen with prob 0.5. Find $\mathbb{P}(kind|t = 10)$.

Solution

$$\mathbb{P}(kind|t = 10) = \frac{\mathbb{P}(kind \cap t = 10)}{\mathbb{P}(t = 10)} = \frac{0.5\mathbb{P}(t_2 = 10)}{0.5\mathbb{P}(t_2 = 10) + 0.5\mathbb{P}(t_1 = 10)}$$

Since, density of the normal distribution is known, we can easily calculate this probability:

$$\mathbb{P}(kind|t = 10) = \frac{0.5\frac{1}{5}\exp(-\frac{(20-10)^2}{2\cdot 25})}{0.5\frac{1}{5}\exp(-\frac{(20-10)^2}{2\cdot 25}) + 0.5\frac{1}{10}\exp(-\frac{(30-10)^2}{2\cdot 100})} = \frac{0.1\exp(-2)}{0.1\exp(-2) + 0.05\exp(-2)} = \frac{10}{15} = \frac{2}{3}$$