Demonstrating the Equivalence and Efficiency of the Nested Model Compared to a Single GE Model

Introduction

This document demonstrates that:

1. The nested model is an equivalent and efficient alternative to a single n-period GE model with embedded climate policy
2. Solving the entire problem within a single GE model is computationally infeasible

Overview of the Two Approaches

1. Nested Model (Current Approach)

- **Outer Loop:** Policy maker uses MCTS to optimize climate policy over n periods
- **Inner Loop:** Two-period GE model solves for equilibrium at each node
- **State Transitions:** States update based on GE outcomes, damages, and learning

2. Single GE Model with Embedded Climate Policy

- **Objective Function:** Incorporates policy maker's objective directly over n periods
- **Agents and Firms:** Optimize over entire n-period horizon with endogenous policy
- **Equilibrium Concept:** DSGE model with embedded policy maker optimization

Theoretical Equivalence

Dynamic Programming Framework

Both approaches solve:

$$\max_{\{a_t\}} \mathbb{E}\left[\sum_{t=0}^{n} \beta^t \left(u(S_t) - D(E_t, \theta_t)Y_t\right)\right]$$

subject to:

$$S_{t+1} = f(S_t, a_t, \epsilon_t)$$

Both models satisfy the Bellman equation, ensuring theoretical equivalence.

Computational Efficiency Analysis

Dimensionality Comparison

Nested Model

- **State Space:** Managed efficiently by MCTS
- **Action Space:** Explored selectively through MCTS
- **Computation:** Solves smaller problems sequentially

Single GE Model

- **State Space:** All variables over n periods
- **Action Space:** Combined space of all controls
- **Computation:** Must solve everything simultaneously

Mathematical Complexity

Let:

- m: Economic variables per period
- k: Number of agents
- a: Actions per agent per period

Single GE model complexity:

$$\text{Total Variables} = n \times (k \times m + \text{Policy Variables})$$

$$\text{Computational Complexity} = O(a^{kn})$$

Demonstrating Unsolvability of Single GE Model

Theoretical Barriers

1. **Fixed-Point Problems**
   - High-dimensional space proportional to n
   - Nonlinear dynamics make finding contractions difficult
2. **Expectations Formation**
   - Agents must form expectations over entire horizon
   - Interdependent optimization problems due to endogenous policy
3. **Computational Limits**
   - Bellman's curse of dimensionality
   - Nonlinear equation systems grow exponentially

Single GE Model Formulation

Attempts to solve:

$$\max_{\{C_t, L_t, I_t, K_t^i, \theta_t^i, \tau_t\}_{t=0}^n} \mathbb{E}\left[\sum_{t=0}^{n} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\nu}}{1+\nu} - D(E_t, \theta_t) Y_t\right)\right]$$

Subject to:

- Economic constraints
- Policy constraints
- Expectations formation
- Market clearing conditions

Key Infeasibility Factors

1. **High Dimensionality**
   - Variables and constraints grow exponentially with n
   - Non-convexities from technology choices and adjustment costs
2. **Interdependencies**
   - Agents' expectations depend on future policy
   - Policy depends on agents' behavior
3. **Computational Resources**
   - Memory and processing requirements exceed practical limits
   - No known algorithms for efficient solution

Advantages of Nested Model

1. **Problem Decomposition**
    - Separates policy optimization from equilibrium computation
    - Maintains theoretical optimality through Bellman equation
2. **Computational Efficiency**
    - MCTS scales logarithmically with simulations
    - Parallel processing possible
    - Results can be cached and reused

Conclusion

The nested model:

- Is theoretically equivalent to the single GE model
- Provides the only computationally feasible solution approach
- Maintains optimality while managing complexity

The single GE model is demonstrably unsolvable due to:

- Computational complexity
- High-dimensional state space
- Interdependent expectations