DOMAIN OPTIMIZATION PROBLEMS WITH A BOUNDARY VALUE PROBLEM AS A CONSTRAINT

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Abstract. In this paper, an overview of domain optimization problems is given. The optimization problem in which an objective function depending on a domain through the solution of a boundary value problem defined on the domain should be minimized (or maximized) with respect to the domain is called a domain optimization problem. Some examples of the optimization problems are given. For a simple domain optimization problem, the first order and the second order necessary conditions are derived. The existence of, and the numerical methods for the optimal domain are briefly discussed.

Keywords. Domain optimization; distributed parameter sytem; boundary-value problem; first & second variation; existence of the solution.

INTRODUCTION

The purpose of this paper is to give an overview on "domain optimization problems" from the author's viewpoint. The domain optimization problems have recently drawn interest on some investigators who wishes to design the shape of solid substances optimal in some sense. The objective function in these problems depends on the domain through not only the integration but also the solution of a boundary value problem defined on the domain; the optimization problems are, then, thought to be a kind of shape optimization problems.

Examples of Domain Optimization Problems

We shall give some typical examples of the domain optimization problems. Cea (1981) enumerates many such problems; the reader who is interested can consult his paper.

Torsional rigidity, principal frequency. Polya (1948) completely solved the following optimization problems:

Problem 1 (maximum torsional rigidity), minimize

$$\frac{\int_\Omega |\nabla u|^2 dx}{\{\int_\Omega u\ dx\}^2} \quad \text{with respect to } \Omega \in R^2,$$

where Ω has the prescribed area and u is the solution of the boundary value problem

$$\Delta u(x) = -2, \qquad x \in \Omega, \tag{1}$$

$$u(x) = 0, \qquad x \in \partial\Omega; \tag{2}$$

problem 2 (minimum principal frequency), minimize

$$\frac{\int_\Omega |\nabla u|^2 dx}{\int_\Omega u^2 dx} \quad \text{with respect to } \Omega \in R^2,$$

where Ω has the prescribed area and u is the solution of

$$\Delta u(x) + \lambda u(x) = 0, \quad x \in \Omega, \tag{3}$$

$$u(x) = 0, \quad X \in \partial\Omega, \tag{4}$$

λ being determined together with the minimum. The first problem is to find the shape of the cross section of an elastic bar so that the torsional rigidity of the bar is maximized. The second one is to determine, for example, the shape of an elastic membrane which has the lowest principal tone. Polya gave the final answers to these problems using the Steiner's symmetrization in isoperimetric problems; the optimal shapes are circles. Banichuk (1975) investigated the similar problems.

Minimum-drag flow. Pironneau (1973, 1974) investigated the shape design problem of minimum-drag flow. His problem is, for example, as follows: Let Ω be a domain in R^3 bounded by two closed sufaces, S and Γ, S being contained in Γ and Γ being fixed; minimize

$$\int_\Omega \frac{1}{2} \sum_{i,j=1}^{3} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 dx$$

with respect to S, where Ω has a prescribed area and $\vec{u} = (u_1, u_2, u_3)$ is the solution of

$$\Delta\vec{u}(x) - \nabla p(x) = 0,\ \nabla\cdot\vec{u} = 0, \quad x \in \Omega, \tag{5}$$

$$\vec{u} = 0, \quad x \in S, \tag{6}$$

$$\vec{u} = \vec{a}, \quad x \in \Gamma, \tag{7}$$

the Stokes equations with the pressure field p, $\vec{a}$ being a given velocity field at Γ. Pironneau gave a first order necessary condition to this problem. Koda (1984) considered the similar problem of non-stationary flow through his sensitivity analysis.

Miscellaneous problems. From the shape design problem for plasma columns in toroidal devices for thermal nuclear fusion, the present author (1986a) was led to the following problem: Maximize

$$\int_\Omega g(x,u(x))\ dx \quad \text{with respect to } \Omega,$$

where g(x,u) is a given function, Ω must satisfy for a given function h(x)

$$\int_\Omega h(x)\ dx = \text{const.}, \tag{8}$$

and u is the solution of

$$\Delta u(x) - \vec{v}\cdot\nabla u = f(x,u), \quad x \in \Omega, \tag{9}$$

$$u = \kappa\ (\text{const.}) \quad x \in \partial\Omega, \tag{10}$$

$\underset{\sim}{f}(x,u)$ being an appropriate nonlinear function and v being a given vector field. He defined anumbiguously the first variation of the solution, corresponding to domain variations, to the boundary value problem (9), (10). Then he derived the first order necessary condition in two ways of representation.

He and his students (Fujii, Ichikawa and Kozai, 1985) showed an intuitive method for deriving the first order necessary condition as well as the first variation of the solution in the case of objective functions that depend on the gradient of the solution, ∇u.

Chenais (1975) transformed a domain identification problem with a boundary value problem of second kind into a domain optimization problem such as those described here and showed the existence of the optimal domain introducing a special class of domains.

Zolesio (1981b) investigated a kind of free boundary problems tranforming each of them into a domain optimization problem.

Problem Statement

Main features of the above problems lie on the fact that the objective function must be optimized with respect to the domain on which a boundary value problem is defined; in this regard, our problems are different from other shape optimization problems in which the domain is fixed (see, for example, Sakawa and Ukai, 1980). In order to simply obtain an outline of the theory, we shall focus on the following simple (or prototype) problem:

Optimization problem. Given a smooth function $g(x,u)$ of $x = (x_1, x_2) \in R^2$ and $u \in R$, minimize $J(\Omega;u)$ defined by

$$J(\Omega;u) \equiv \int_\Omega g(x,u(x))\, dx \tag{11}$$

with respect to Ω in R^2, subject to the subsidiary constraint

$$I(\Omega) \equiv \int_\Omega h(x)\, dx = c(\text{ const.}), \tag{12}$$

where $u(x)$ is the solution of the boundary value problem

$$\Delta u(x) - k(x)u(x) = f(x), \quad x \in \Omega, \tag{13}$$

$$u(x) = \kappa(\text{const.}), \quad x \in \partial\Omega. \tag{14}$$

Throughout the paper, the given functions $h(x)$, $k(x) \geq 0$ and $f(x)$ are assumed to be defined on R^2 and to be sufficiently smooth for the sake of simplicity. Note that (12) is a generalization of the condition that Ω has a prescribed area.

FIRST ORDER NECESSARY CONDITION

Definition of Boundary Variation

In order to investigate the above Optimization problem, it is natural to consider variations of the preassigned domain Ω. These domain vriations are incarnated as variations of the boundary of Ω, $\Gamma \equiv \partial\Omega$. Let $\rho(s)$ be a sufficiently smooth function of the arclength s introduced on Γ. We erect at each point on Γ the normal and plot on it the segment $\varepsilon\rho(s)$ so that positive $\varepsilon\rho(s)$ lies on the outward normal $n = (n_1, n_2)$, where ε is a positive number. If ε is small enough, the end-points of the segments form a closed curve which enclose a new domain; hereafter, the curve will be denoted by Γ_ε and the new domain, Ω_ε. We shall call $\varepsilon\rho(s)$ the boundary variation and say that $\Omega_\varepsilon(\Gamma_\varepsilon)$ is obtained from $\Omega(\Gamma)$ by this variation.

First variation of the solution

If we try to calculate the first variation of the objective function defined by (11) in accordance with the description of the conventional calculus of variation, we are inclined to know the first variation of the solution to (13), (14). Let u_ε be the solution of

$$\Delta u_\varepsilon(x) - k(x)u_\varepsilon(x) = f(x), \quad x \in \Omega_\varepsilon, \tag{15}$$

$$u_\varepsilon(x) = \kappa, \quad x \in \Gamma_\varepsilon, \tag{16}$$

where Γ_ε is $\partial\Omega_\varepsilon$. The solution u_ε is different from u even in the overlapped part of the domains, $\Omega\cap\Omega_\varepsilon$. Thus, the following question arises: Does there exist a function ϕ so that, roughly speaking,

$$u_\varepsilon(x) - u(x) = \varepsilon\phi(x) + o(\varepsilon), \quad x \in \Omega\cap\Omega_\varepsilon \tag{17}$$

holds? Here $o(\varepsilon)$ is a quantity such that $\varepsilon^{-1}o(\varepsilon) \to 0$ as $\varepsilon \to 0$. We call ϕ the first variation of the solution. This notion will play a crucial role in calculation of the first variation of objective function $J(\Omega;u)$.

For the boundary value problem (9), (10), the present author (1986a) showed the existence of the first variation ϕ of the solution. As for the boundary value problem (13), (14), he has shown the following theorem in the coexisting paper (Fujii, 1986b).

Theorem 1. If we define $\phi_\varepsilon(x)$ by

$$\phi_\varepsilon(x) \equiv \varepsilon^{-1}(u_\varepsilon(x) - u(x)),$$

then there exists a function $\phi \in C^1(\bar{\Omega})\cap C^2(\Omega)$ such that

$$\phi_\varepsilon \to \phi, \quad \frac{\partial\phi_\varepsilon}{\partial x_i} \to \frac{\partial\phi}{\partial x_i}, \quad \frac{\partial^2\phi_\varepsilon}{\partial x_i\partial x_j} \to \frac{\partial^2\phi}{\partial x_i\partial x_j}, \quad i,j = 1,2,$$

uniformly on every compact subdomain of Ω; in other words,

$$u_\varepsilon(x) - u(x) = \varepsilon\phi(x) + o(\varepsilon), \quad x \in \Omega_\varepsilon\cap\Omega \tag{18}$$

Function $\phi(x)$ is given as the solution of the boundary value problem:

$$\Delta\phi(x) - k(x)\phi(x) = 0, \quad x \in \Omega,$$

$$\phi(x) = -\frac{\partial u}{\partial n}\rho(x), \quad x \in \Gamma.$$

This theorem follows from the well-known maximum principle (Courant, 1962) and Schauder's interior estimates (Courant, 1962; Gilbarg and Trudinger, 1983).

First Order Necessary Condition

Along the description of the conventional calculus of variation, we calculate the first variation of the objective function. Observe that $J(\Omega_\varepsilon;u_\varepsilon)$ is given by

$$J(\Omega_\varepsilon;u_\varepsilon) = \int_{\Omega_\varepsilon} g(x,u_\varepsilon(x))\, dx,$$

where, of course, Ω_ε is obtained from Ω by boundary variation $\varepsilon\rho(s)$ and u_ε is the solution of (15), (16). If we define the first variation $\delta^{(1)}J$ by

$$J(\Omega_\varepsilon;u_\varepsilon) - J(\Omega;u) = \varepsilon\delta^{(1)}J + o(\varepsilon), \tag{19}$$

it is calculated to be

$$\delta^{(1)}J = \int_\Gamma g(x,u)\rho(x)\, d\Gamma + \int_\Omega \frac{\partial g}{\partial u}(x,u)\Phi(x)\, dx, \quad (20)$$

where Theorem 1 is used. In order to eliminate $\Phi(x)$ from (20), we introduce a function $p(x)$ through

$$\Delta p(x) - k(x)p(x) = \frac{\partial g}{\partial u}(x,u(x)), \quad x \in \Omega, \quad (21)$$

$$p(x) = 0, \quad x \in \Gamma, \quad (22)$$

obtaining

$$\int_\Omega \frac{\partial g}{\partial u}(x,u)\Phi(x)\, dx = -\int_\Gamma \frac{\partial u}{\partial n}\frac{\partial p}{\partial n}\rho(x)\, d\Gamma.$$

Substitution of this relation into (20) yields

$$\delta^{(1)}J = \int_\Gamma \{g(x,u) - \frac{\partial p}{\partial n}\frac{\partial u}{\partial n}\}\rho(x)\, d\Gamma. \quad (23)$$

On the other hand, Ω_ε must satisfy (12). Hence, we have

$$\delta^{(1)}I = \int_\Gamma h(x)\rho(x)\, d\Gamma = 0, \quad (24)$$

where $\delta^{(1)}I$ is defined in the same manner as $\delta^{(1)}J$.

If Ω is the optimal domain of Optimization problem, $\delta^{(1)}J$ must vanish for an arbitrary $\rho(s)$ which satisfies (24). Thus, we arrived at the following theorem (Fujii, 1986b, see also 1986a).

Theorem 2. Let Ω be an optimal domain and $u(x)$ be the corresponding solution. Then, there exists a constant λ such that

$$g(x,u) - \frac{\partial p}{\partial n}\frac{\partial u}{\partial n} - \lambda h(x) = 0, \quad x \in \Gamma \quad (25)$$

holds, where $p(x)$ is the solution of the boundary value problem (21), (22).

The similar results are obtained by the present author and his students (Fujii, Ichikawa and Kozai, 1985) for other types of objective functions and subsidiary constraints.

Alternative Approaches

We shall briefly survey the alternative methods for deriving the first order necessary condition for optimality.

Zolesio (1981a) introduced a time varying vector field $V(t,x)$ in R^n to define a mapping $T_t(V)$ in R^n by $T_t(V)$: $x \to X(t,x)$, where $X(t,x)$ is the solution of

$$\frac{d}{dt} X(t,x) = V(t,X(t,x)),$$

$$X(0,x) = x.$$

Thus, $T_t(V)$ represents a motion of the preassigned domain Ω in such a way that each point x in Ω is moved to a new point $X(t,x)$. He calculated the derivatives of objective functions with respect to this motion assuming the existence of the first derivatives (these derivatives correspond to our first variation of the solutuion) of the solution to boundary value problems of various types. He called his method the material derivative method. This method is general and sophisticated. It seems, however, to be circumlocutory for our purpose since it is of no use to introduce motions of the interior points of Ω.

Rousselet (1982, 1983) introduced a vector field $F(x)$ to define a mapping $x \to \Phi(x) = x + F(x)$ in a neighbourhood of Ω. He calculated the sensitivity of an objective function with respect to this mapping and derived the first order necessary condition for optimality for vibrating membranes.

Koda (1982, 1984) made use of the one parameter family of transformations found in the well-known book by Gelfand and Fomin (Gelfand and Fomin, 1963) to calculate the first variation of an objective function introducing adjoint variables; he assumed tacitly the existence of the first variation of the solution however. He derived the first order necessary condition for the minimal drag shape in non-stationary flow of fluid.

In order to derive the first order necessary condition for maximal torsional rigidity of elastic bars, Banichuk (1976) used specialty that the Euler-Lagrange equation for the objective function turns out to be the partial differential equation of the boundary value problem in the constraints.

The present author (Fujii, 1985) used conformal mappings to calculate the first variation of an objective function in the case of Neumann problems; this method applies, of course, also to the case of Dirichlet problems.

SECOND ORDER NECESSARY CONDITIONS

In order to decrease the number of candidates for, and to resarch features of the optimal domain, it is favorable to derive the second order necessary conditions. To this end, we shall first examine the second variation of the solution to the boundary value problem (13), (14). As far as the author knows, there is no paper except one by the present author (Fujii, 1986b) which deals with the second order necessary conditions together with the second variation of the solution for the domain optimization problems.

Let Ω_ε be the domain obtained from Ω by boundary variation $\varepsilon\rho(s)$ as in the previous section, and u_ε be the coresponding solution of (15), (16). We shall call $\psi(x)$ in

$$u_\varepsilon(x) - u(x) = \varepsilon\phi(x) + \varepsilon^2\psi(x) + o(\varepsilon), \quad x \in \Omega_\varepsilon \cap \Omega \quad (26)$$

the second variation of the solution, where $\phi(x)$ is the first variation of the solution given in Theorem 1. Let us define $\psi_\varepsilon(x)$ by

$$\psi_\varepsilon(x) \equiv \varepsilon^{-1}(u_\varepsilon(x) - u(x) - \varepsilon\phi(x)), \quad x \in \Omega \cap \Omega_\varepsilon.$$

Then, we have

Theorem 3. For the sequence $\{\psi_\varepsilon\}$, there exists a function $\psi \in C^1(\bar{\Omega}) \cap C^2(\Omega)$ such that

$$\psi_\varepsilon \to \psi, \quad \frac{\partial \psi_\varepsilon}{\partial x_i} \to \frac{\partial \psi}{\partial x_i}, \quad \frac{\partial^2 \psi_\varepsilon}{\partial x_i \partial x_j} \to \frac{\partial^2 \psi}{\partial x_i \partial x_j}, \quad i = 1,2$$

uniformly on every compact subdomain of Ω. In other words, we have

$$u_\varepsilon(x) - u(x) = \varepsilon\phi(x) + \varepsilon^2\psi(x) + o(\varepsilon^2), \quad x \in \Omega \cap \Omega_\varepsilon,$$

where $o(\varepsilon^2)$ is a uniform estimate on every compact subdomain of $\Omega \cap \Omega_\varepsilon$. The function $\psi(x)$ is given as the solution of

$$\Delta\psi(x) - k(x)\psi(x) = 0, \quad x \in \Omega, \quad (27)$$

$$\psi(x) = -\frac{\partial u}{\partial n}\sigma - \frac{\partial \phi}{\partial n}\rho - \frac{1}{2}\frac{\partial^2 u}{\partial n^2}\rho^2, \quad x \in \Gamma, \quad (28)$$

where u is the solution of (13) and (14), and ϕ is given by Theorem 1.

Thus, the existence of the second variation $\psi(x)$ of the solution is shown and the characterization of

$\Psi(x)$ is given.

It is possible, from Theorem 3, to calculate the second variation $\delta^{(2)}J$ of the objective function defined by

$$J(\Omega_\varepsilon;u_\varepsilon) - J(\Omega;u) = \varepsilon\delta^{(1)}J + \varepsilon^2\delta^{(2)}J + o(\varepsilon^2). \tag{29}$$

The result reads

$$\delta^{(2)}J = \frac{1}{2}\int_\Gamma \{\frac{1}{R}\, g(x,u) + \frac{\partial g}{\partial x}(x,u)\cdot\vec{n} - \frac{\partial g}{\partial u}(x,u)\frac{\partial u}{\partial n} - \frac{\partial^2 u}{\partial n^2}\frac{\partial p}{\partial n}\}\,\rho^2\, d\Gamma - \int_\Gamma \frac{\partial p}{\partial n}\frac{\partial \phi}{\partial n}\,\rho\, d\Gamma + \frac{1}{2}\int_\Omega \frac{\partial^2 g}{\partial u^2}(x,u)\phi^2\, dx, \tag{30}$$

where R is the radius of the curvature at $d\Gamma$ and is defined to be positive when the curve is concave to the domain. On the other hand, Ω must satisfy (12). It follows from this that $\rho(s)$ should satisfy

$$\delta^{(2)}I = -\int_\Gamma (\frac{1}{R}\, h + \frac{\partial h}{\partial n})\,\rho^2(x)\, d\Gamma = 0, \tag{31}$$

where $\delta^{(2)}I$ is defined in the same manner as $\delta^{(2)}J$.

If Ω is the optimal domain, $\delta^{(2)}J$ given by (30) must be nonnegative for every $\rho(s)$ that satisfies (24) and (31). Hence we have the following second order necessary conditions of Kuhn-Tucker type for optimality.

Theorem 4. Let Ω be a domain bounded by a smooth boundary. Necessary conditions that Ω attain a minimum to the optimization problem are that there exists a constant λ (Lagrange multiplier) such that

$$g(x,u) - \frac{\partial u}{\partial n}\frac{\partial p}{\partial n} - \lambda\, h(x) = 0, \quad x \in \Gamma \tag{32}$$

holds, and that, for every $\rho(s)$ which satisfies (24),

$$\delta^{(2)}J - \lambda\, \delta^{(2)}I \geq 0 \tag{33}$$

holds, where $u(x)$ is the solution of (13),(14) and $p(x)$, the solution of (21), (22), and $\delta^{(2)}J$, $\delta^{(2)}I$ are given by (30), (31), respectively.

Once we assume the existence of the second variation of the solution, it is now easy to derive the second order necessary conditions for other types of boundary value problems, objective functions and subsidiary constraints. For more detials, the reader should refer the coexisting paper (Fujii, 1986b).

EXISTENCE OF THE OPTIMAL DOMAIN

It is hopeless to show the existence of an optimal domain itself without imposing any condition on a class of domains in which we seek the optimal domain. Chenais (1975) introduced an important class of domains to prove the existence of an optimal domain in a certain domain optimization problem. The present author (Fujii, 1984) proved some lower-semicontinuity theorems for this class of domains to show the existence of the optimal domain for two kinds of boundary value problems. We shall briefly review these results.

Chenais (1975) considered the following domain optimization problem:
Let G and D be two bounded domains in R^n, Euclidean n-space, and let a given function f belong to $L^2(D)$ and u_d be given in $L^2(G)$; find Ω $(G \subset \Omega \subset D)$ which minimizes $J(\Omega)$ given by

$$J(\Omega) \equiv \int_G (u|_G - u_d)^2\, dx, \tag{34}$$

where $u|_G$ is the restriction of u to G, and u is the solution of the boundary value problem

$$\Delta u(x) - u(x) = f(x), \qquad x \in \Omega, \tag{35}$$

$$\frac{\partial u}{\partial n} = 0, \qquad x \in \partial\Omega, \tag{36}$$

$f(x)$ being a given function.

He sought the optimal domain in the class of domains mentioned above deriving many important and useful properties of this class and proved the existence of the optimal domain; his objective function does not, however, depend on the gradient of the solution, ∇u. For the same class of domains, the present author (Fujii, 1984) derived some lower-semicontinuity theorems for objective functions that depend on the gradient of the solution, ∇u. He showed, for example, the following result.

Theorem 5. Let a nonnegative function $g(q)$ be continuous in $q \in R^2$ and convex. Then, there exists an optimal domain Ω^* in the class of domains introduced by Chenais of the optimization problem:

Minimize $\int_\Omega g(\nabla u(x))\, dx$ with respect to Ω in the class, where u is the generalized solution of

$$\Delta u(x) - k(x)u(x) = f(x), \quad x \in \Omega,$$

$$\frac{\partial u}{\partial n} = 0, \qquad x \in \partial\Omega$$

and Ω must satisfy

$$\int_\Omega h(x)\, dx = c., \tag{14}$$

$f(x)$ being a given smooth function of $x \in R^2$. If Ω^* is smooth enough, the corresponding solution u^* is the classical one; hence, the optimal domain exists in the classical sense.

To his regret, however, his result for Dirichlet problem is not complete in the classical sense. In other words, the existence of the optimal domain for Optimization problem is not completely shown. This point should be overcome in the near future.

NUMERICAL METHODS

We can hardly expect analytic solutions of the domain optimization problems except the special problems such as Polya's problems stated in the first section. Thus, it is desired, from the practical point of view, to develop numerical methods for solutions.

Almost all the existing numerical methods (Cea, 1981b; Pironneau, 1973; Koda, 1982,1984; Marrocco and Pironneau, 1978) are on the basis of the first variation of the objective function. Since these methods usually require many iterations of solving the boundary value problems, they are not thought to be much efficient. If we shall succede in making use of the second variation, stated above, of the objective function to derive the numerical methods like the Newtonian methods in the conventional mathematical programming, we are able to expect shortening of the computational times and the costs.

In this case, however, if the problem enjoys high symmetry such as symmetry of spatial rotation and translation, we must add the, so to speak, "localization factor", for example

$$\alpha \int_{\Omega} (x_1{}^2 + x_2{}^2)\, dx$$

to the objective function in order to confirm the convergence of the iterations required, α being a weight.

CONCLUDING REMARKS

We have given an outline of the theory for the domain optimization problems that have a boundary value problem as a main constraint. For a linear boundary value problem with a Dirichlet condition as an example, the existence of the first and the second variation, corresponding to a boundary variation, of the solution are established; they are shown to be given as solutions of boundary value problems of the same type. From these results, we were able to formulate the first and the second variations of the objective function. The first order necessary condition derived in this paper is obtained also by other methods, for example, the material derivative method, the sensitivity analysis, conformal mapping and so on. The second order necessary conditions of Kuhn-Tucker type has recently been derived by the present author.

Many problems are left unsolved. We can enumerate them from the theoretical point of view: For example,

1 complete existence theory for the optimal domain in the case of Dirichlet Problem;
2 sufficient conditions for the optimal domain;
3 properties, foreseen from the necessary conditions, of the optimal diomain;
4 analytic solutions of the optimal domain, if possible.

From the practical point of view, the efficient numerical methods are desired first of all.

For other boundary value problems such as Neumann problems, we can mention that it is easy to derive the first and the second order necessary conditions if we assume the existence of the variations of the solution and if the objective function does not depend on the gradient of the solution.

If the investigation, including to settle those problems above, will be brought to succesive degrees of completion, the domain optimization method will find many application fields such as design problems in aeronautics.

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