Übungsblatt 4

Verteilungsfkt. ist rechtsseitig stetig

Merke: $P(X < a) = \lim_{x \to a-} F(x)$, $P(X \leq a) = F(a) = \lim_{x \to a+} F(x)$

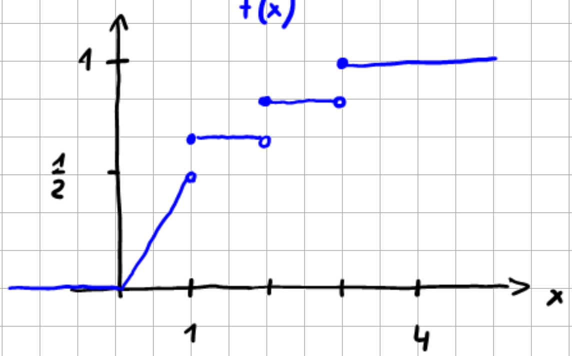
$$P(a \leq X \leq b) = P(X \leq b) - P(X < a) = F(b) - \lim_{x \to a-} F(x)$$

$$P(a < X < b) = P(X < b) - P(X \leq a) = \lim_{x \to b-} F(x) - F(a)$$

$$P(a \leq X < b) = P(X < b) - P(X < a) = \lim_{x \to b-} F(x) - \lim_{x \to a-} F(x)$$

4.1

a)



$P(X < 1) = \lim_{x \to 1-} F(x) = \frac{1}{2}$

$P(X \leq 1) = F(1) = \frac{2}{3}$

b) $P(X = 1) = P(X \leq 1) - P(X < 1) = F(1) - \lim_{x \to 1-} F(x) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

Sprunghöhe bei 1

$P(X > \frac{1}{2}) = \frac{3}{4}$

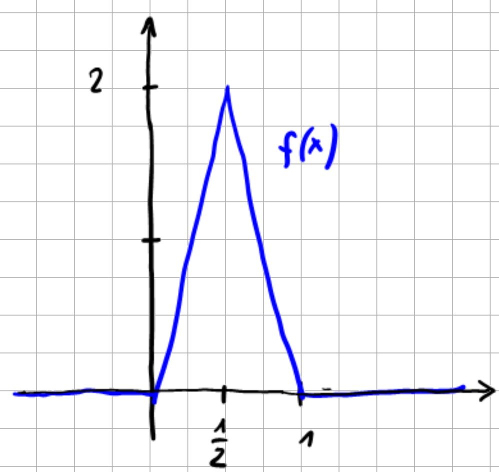
$P(2 < X \leq 4) = \frac{1}{6}$

$P(X < 3) = \lim_{x \to 3-} F(x) = \frac{5}{6}$

4.2

a)

$$f(x) = \begin{cases} 0 , & x < 0 \\ 4x , & 0 \leq x < \frac{1}{2} \\ -4x + 4 , & \frac{1}{2} \leq x < 1 \\ 0 , & x \geq 1 \end{cases}$$



b) $P(\frac{1}{4} < X < \frac{3}{4}) = \frac{3}{4}$

c) $E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx = \int_0^{\frac{1}{2}} x \cdot 4x\, dx + \int_{\frac{1}{2}}^{1} \underbrace{x(-4x+4)}_{-4x^2+4x}\, dx$

$$= \left[\frac{4}{3}x^3\right]_0^{\frac{1}{2}} + 4\left[-\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_{\frac{1}{2}}^{1} = 4\left(\frac{1}{24} - \frac{1}{3} + \frac{1}{2} + \frac{1}{24} - \frac{1}{8}\right)$$

$$= \frac{1}{2}$$

4.3

a) $1 = \int_{-\infty}^{\infty} f(x)\, dx = a\int_0^2 (4x - 2x^2)\, dx = \frac{8}{3}a \implies a = \frac{3}{8}$

b)
$$F(x) = \begin{cases} 0 & , \; x < 0 \\ \frac{3}{4}\int_0^x (2t - t^2)\, dt = \frac{1}{4}(3x^2 - x^3) & , \; 0 \leq x < 2 \\ 1 & , \; x \geq 2 \end{cases}$$

c) $P(X > 1) = 1 - F(1) = 1 - \frac{1}{2} = \frac{1}{2}$