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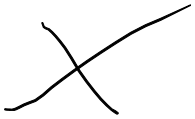
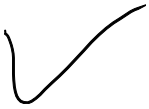
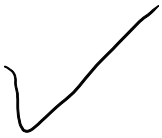
Mathematik – Übung

Hochschule **Rosenheim**
University of Applied Sciences

Algebraische Strukturen

Fragen?

*** (kommutative) (Halb-)Gruppe?** Welche der folgenden Mengen besitzt welche algebraische Struktur?

	Halbgruppe?	Gruppe?	Kommutativ?
a) $(\mathbb{N} = \{1, 2, 3, ...\}, +)$			
b) $(\mathbb{Z}, +)$			
c) $(\mathbb{Z}\backslash\{0\}, \cdot)$			
d) $(\mathbb{Q}\backslash\{0\}, \cdot)$			

Eigener Lösungsversuch.

	Halbgruppe?	Gruppe?	Kommutativ?
a) $(\mathbb{N} = \{1, 2, 3, ...\}, +)$			
b) $(\mathbb{Z}, +)$			
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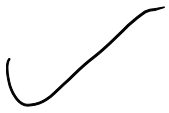
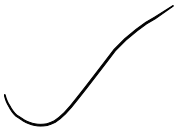
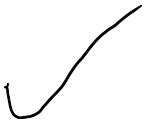
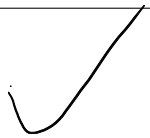
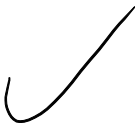
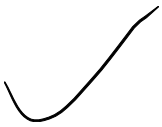
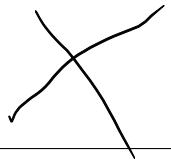
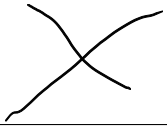
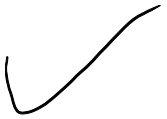
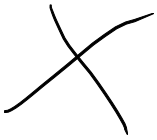
d) $(\mathbb{Q}\backslash\{0\}, \cdot)$			

*** Algebraische Struktur von $\mathbb{Z}_n$.** Welche der folgenden Mengen besitzt welche algebraische Struktur?

	Halbgruppe?	Gruppe?	Kommutativ?
a) $(\mathbb{Z}_3, +)$ $\overline{-a} = \overline{n-a}$			
b) $(\mathbb{Z}_3 \backslash \{0\}, \cdot)$			
c) $(\mathbb{Z}_4, +)$			
d) $(\mathbb{Z}_4 \backslash \{0\}, \cdot)$			
e) $(\mathbb{Z}_4, \cdot)$			

Eigener Lösungsversuch.

	Halbgruppe?	Gruppe?	Kommutativ?
a) $(\mathbb{Z}_3, +)$			
b) $(\mathbb{Z}_3 \backslash \{0\}, \cdot)$			
c) $(\mathbb{Z}_4, +)$			
d) $(\mathbb{Z}_4 \backslash \{0\}, \cdot)$			
e) $(\mathbb{Z}_4, \cdot)$			

Zusammenfassung: Algebraische Struktur von $\mathbb{Z}_n$.

$$\forall n \in \mathbb{N}: \boxed{(\mathbb{Z}_n, +) \text{ ist eine abelsche Gruppe}}$$

$$\boxed{(\mathbb{Z}_n \backslash \{0\}, \cdot) \text{ ist abelsche Gruppe} \iff \text{n ist prim}}$$

Inverse berechnen.

1. Berechnen Sie (falls möglich) $\overline{5}^{-1}$ in $\mathbb{Z}_{1024}$.
2. Berechnen Sie (falls möglich) $\overline{2}^{-1}$ in $\mathbb{Z}_{1024}$.
3. Wann ist $\overline{a} \in \mathbb{Z}_n$ bzgl. $\cdot$ invertierbar?

Lösung.

1.) $\overline{5}x = 1 + q \cdot 1024 \iff 5x + 1024 \cdot (-q) = 1$ → diophant Gleich.

2.) $2x = 1 \iff 2x = 1 + q \cdot 1024 \iff 2x - 1024(-q) = 1$

$ggT(2; 1024) = 2 \neq 1$; d.h. nicht lösbar

3.) $\overline{a} \cdot \overline{x} = \overline{1} \iff ax = 1 + q \cdot n \iff ax + n \cdot (-q) = 1$

$\overline{a}$ invertierbar in $(\mathbb{Z}_n \backslash \{0\}; \cdot) \iff ggT(a; n) = 1$ diophant. Gl.

Lösung.

Eigener Lösungsversuch.

Invertierbarkeitskriterium. Sind 537 und 8491 in $\mathbb{Z}_{63481}$ invertierbar?

Lösung.

Eigener Lösungsversuch.