Lösung – Blatt 12

12.1

a) Wahr

b) Falsch

c) Wahr

d) Falsch

12.2

$$\int_0^1 \underbrace{\frac{1}{1+x^2}}_{f(x)} \, dx \approx \frac{1}{6}\left(f(0) + 4\cdot f\left(\tfrac{1}{2}\right) + f(1)\right) = \frac{1}{6}\left(1 + 4\cdot\frac{4}{5} + \frac{1}{2}\right) = \frac{47}{60}$$

absoluter Fehler : $\frac{\pi}{4} - \frac{47}{60} \approx 2.065 \cdot 10^{-3}$

12.3

$n=2$: $$\int_0^4 \underbrace{\frac{x}{1+x}}_{f(x)} \, dx \approx \underbrace{\frac{4}{2}}_{\frac{b-a}{n}}\left(\frac{f(0)}{2} + f(2) + \frac{f(4)}{2}\right) = 2\cdot\left(\frac{2}{3} + \frac{4}{10}\right) = \frac{32}{15}$$

$n=4$: $$\int_0^4 \frac{x}{1+x} \, dx \approx \frac{f(0)}{2} + f(1) + f(2) + f(3) + \frac{f(4)}{2} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{10} = \frac{139}{60}$$

Exakter Wert: $$\int_0^4 \frac{x+1-1}{1+x} \, dx = \int_0^4 \left(1 - \frac{1}{1+x}\right) dx = \Big[x - \ln(1+x)\Big]_0^4 = 4 - \ln 5$$

Absolute Fehler:

$n=2$: $\quad 4 - \ln 5 - \frac{32}{15} \approx 0.257$

$n=4$: $\quad 4 - \ln 5 - \frac{139}{60} \approx 0.074$

12.4

(a) $\int_{-1}^{1} \cos x \, dx = \left[\sin x\right]_{-1}^{1} \approx 1.682942$

Trapezregel: $\int_{-1}^{1} \cos x \, dx \approx 2\left(\frac{\cos(-1)}{2} + \frac{\cos(1)}{2}\right) \approx 1.080605$

$\Longrightarrow$ absoluter Fehler: 0.6

Summenformel für Trapezregel mit $n=4$:

$$\int_{-1}^{1} \cos x \, dx \approx \frac{2}{4}\left(\frac{\cos(-1)}{2} + \cos(-0.5) + \cos(0) + \cos(0.5) + \frac{\cos(1)}{2}\right)$$

$$\approx 1.647734$$

$\Longrightarrow$ absoluter Fehler: 0.0352

Simpson-Regel: $\int_{-1}^{1} \cos x \, dx \approx \frac{2}{6}\left(\cos(-1) + 4 \cdot \cos(0) + \cos(1)\right)$

$$\approx 1.693535$$

$\Longrightarrow$ absoluter Fehler: 0.0106

b) $\int_{-1}^{1} \overbrace{(3x^2 - 2e^x - 5)}^{f(x)} \, dx = \left[x^3 - 2e^x - 5x\right]_{-1}^{1} \approx -12.7008$

Trapezregel: $\int_{-1}^{1} f(x) \, dx \approx 2\left(\frac{f(-1)}{2} + \frac{f(1)}{2}\right) \approx -10.1723$

$\Longrightarrow$ absoluter Fehler ≈ 2.5285

Summenformel für Trapezregel mit $n=4$:

$$\int_{-1}^{1} f(x) \, dx \approx \frac{2}{4}\left(\frac{f(-1)}{2} + f(-0.5) + f(0) + f(0.5) + \frac{f(1)}{2}\right)$$

$$\approx -12.5483$$

$\Longrightarrow$ absoluter Fehler ≈ 0.1525

Simpson-Regel: $\int_{-1}^{1} f(x) \, dx \approx \frac{2}{6}\left(f(-1) + 4 \cdot f(0) + f(1)\right)$

$$\approx -12.7241$$

$\Longrightarrow$ absoluter Fehler ≈ 0.0233

12.5 $\int_0^1 f(t)\,dt \approx \frac{1}{4}\left(3f\left(\frac{1}{3}\right)+f(1)\right)$

Hat mind. Ord. 2, d.h. exakt für konstante Funktionen und Geraden.

$\frac{1}{3} = \int_0^1 t^2\,dt = \frac{1}{4}\left(3\cdot\left(\frac{1}{3}\right)^2+1\right) = \frac{1}{4}\cdot\frac{4}{3} = \frac{1}{3}$ ✓

$\frac{1}{4} = \int_0^1 t^3\,dt \neq \frac{1}{4}\left(3\cdot\left(\frac{1}{3}\right)^3+1\right) = \frac{1}{4}\cdot\frac{10}{9} = \frac{5}{18}$

Die Quadraturformel hat Ordnung 3.