$q = 11$, Nachrichtenlänge $m = 4$, $n = 6$ Stellen ausgewertet

a) Codierung von $(1, 2, 0, 1)$

Polynom $= 1 + 2x + x^3$

Auswertung an 6 Stellen $1, 2, \ldots, 6$

$p(1) = 1 + 2 + 1 = 4$

$p(2) = 1 + 4 + 8 = 13 = 2$

$p(3) = 1 + 6 + 27 = 34 = 1$

$p(4) = 1 + 8 + 64 = 73 = 7$

$p(5) = 1 + 10 + 125 = 136 = 4$

$p(6) = 1 + 12 + 216 = 229 = 9$

Codewort:

$(4, 2, 1, 7, 4, 9)$

b) 1 Ausfall: $(4, 2, 1, \varepsilon, 4, 9)$

z. B.: Erste 4 Stellen $4, 2, 1, 4 \xrightarrow[\text{Auswertung}]{} 1, 2, 3, 5$

$g_1(x) = (x-2)(x-3)(x-5)$

$g_2(x) = (x-1)(x-3)(x-5)$

$g_3(x) = (x-1)(x-2)(x-5)$

$g_5(x) = (x-1)(x-2)(x-3)$

$g_1(1) = (-1)(-2)(-4) = -8 = 3$

$g_2(2) = (+1)(-1)(-3) = +3 = 8$

$g_3(3) = (+2)(+1)(-2) = -4 = 7$

$g_5(5) = (+4)(+3)(+2) = 24 = 2$

$$g_1(x) = (x^2 - 5x + 6)\,(x-5)$$
$$= x^3 - 5x^2 + 6x - 5x^2 + 25x - 30$$
$$= x^3 - 10x^2 + 31x - 30$$
$$= x^3 + x^2 + 9x + 3$$

$$g_2(x) = (x^2 - 4x + 3)(x-5)$$
$$= x^3 - 4x^2 + 3x - 5x^2 + 20x - 15$$
$$= x^3 - 9x^2 + 23x - 15$$
$$= x^3 + 2x^2 + x + 7$$

$$g_3(x) = (x^2 - 3x + 2)(x-5)$$
$$= x^3 - 3x^2 + 2x - 5x^2 + 15x - 10$$
$$= x^3 - 8x^2 + 17x - 10$$
$$= x^3 + 3x^2 + 6x + 1$$

$$g_5(x) = (x^2 - 3x + 2)\,(x-3)$$
$$= x^3 - 3x^2 + 2x - 3x^2 + 9x - 6$$
$$= x^3 - 6x^2 + 11x - 6$$
$$= x^3 + 5x^2 + 5$$

<u>Multiplikative Inverse</u>

$g_1(1) = 3$ $\quad g_1^{-1}(1) = 4$ $\quad 7 \cdot 4 = 12 = 1$

$g_2(2) = 3$ $\quad g_2^{-1}(2) = 4$

$g_3(3) = 7$ $\quad g_3^{-1}(3) = 8$ $\quad 7 \cdot 8 = 56 = 1$

$g_5(5) = 2$ $\quad g_5^{-1}(5) = 6$ $\quad 2 \cdot 6 = 12 = 1$

Produkte $P(u_i) \cdot g_i^{-1}(u_i)$:

$P(1) \cdot g_1^{-1}(1) = 4 \cdot 4 = 16 = 5$

$P(2) \cdot g_2^{-1}(2) = 2 \cdot 4 = 8 = 8$

$P(3) \cdot g_3^{-1}(3) = 1 \cdot 8 = 8 = 8$

$P(5) \cdot g_5^{-1}(5) = 4 \cdot 6 = 24 = 2$

Ergebnis:

$$
\begin{aligned}
P(x) &= 5g_1(x) + 8g_2(x) + 8g_3(x) + 2g_5(x) \\
&= 5x^3 + 5x^2 + 45x + 15 \\
&\quad 8x^3 + 16x^2 + 8x + 56 + \\
&\quad 8x^3 + 24x^2 + 48x + 8 + \\
&\quad 2x^3 + 10x^2 + 10 \\
&= 23x^3 + 55x^2 + 101x + 89 \\
&= x^3 \qquad + 2x \qquad + 1
\end{aligned}
$$

$\longrightarrow$ ursprüngliche Nachricht: $(1, 2, 0, 1)$

$\quad 1 \quad 2x \quad 0x^2 \quad x^3$

c) angekommen: $(4, 2, 1, 7, 4, 0)$ $\frac{n-m}{2} = \frac{6-4}{2} = 1$ Fehler korrigierbar

Polynome: $f(x)$ Grad $\lceil \frac{n-m}{2} \rceil = 1$ $f(x) = f_0 + f_1 x$

$g(x)$ Grad $\lceil \frac{n-m}{2} \rceil + m - 1 = 4$

$g(x) = g_0 + g_1 x + g_2 x^2 + g_3 x^3 + g_4 x^4$

$\longrightarrow$

$$p(x,y) = y f(x) + g(x) =$$
$$= y f_0 + f_1 xy + g_0 + g_1 x + g_2 x^2 + g_3 x^3 + g_4 x^4$$

Paare $(u_i, p(u_i)) = (x,y)$ für $p(x,y) = 0$:

$(1,4), (2,2), (3,1), (4,7), (5,4), (6,0)$ $\longrightarrow$ einsetzen

$$4f_0 + 4f_1 + g_0 + g_1 + g_2 + g_3 + g_4 = 0$$
$$2f_0 + 4f_1 + g_0 + 2g_1 + 4g_2 + 8g_3 + 16g_4 = 0$$
$$f_0 + 3f_1 + g_0 + 3g_1 + 9g_2 + 27g_3 + 81g_4 = 0$$
$$7f_0 + 28f_1 + g_0 + 4g_1 + 16g_2 + 64g_3 + 256g_4 = 0$$
$$4f_0 + 20f_1 + g_0 + 5g_1 + 25g_2 + 125g_3 + 625g_4 = 0$$
$$g_0 + 6g_1 + 36g_2 + 216g_3 + 1296g_4 = 0$$

Inverse:

2 – 6
3 – 4
4 – 3
5 – 9
6 – 2
7 – 8
8 – 7
9 – 5
10 – 10

④

Gauß-Elimination :

	f_0	f_1	g_0	g_1	g_2	g_3	g_4	
x	4	4	1	1	1	1	1	
	2	4	1	2	4	8	5	$1\cdot\left(-\frac{1}{2}\right) = (-6) = 5$
	1	3	1	3	9	5	4	$1\cdot\left(-\frac{1}{4}\right) = (-3) = 8$
	7	6	1	4	5	9	3	$1\cdot\left(-\frac{7}{4}\right) = (-7\cdot 3) = -21 = 1$
	4	9	1	5	3	4	9	$1\cdot(-1) = 10$
	0	0	1	6	3	7	9	

	4	4	1	1	1	1	1	
x	0	2	6	7	9	2	10	
	0	2	9	0	6	2	1	$1\cdot(-1) = 10$
	0	10	2	5	6	10	4	$1\cdot\left(-\frac{10}{2}\right) = (-5) = 6$
	0	5	0	4	2	3	8	$1\cdot\left(-\frac{5}{2}\right) = (-5\cdot 6) = -30 = 3$
	0	0	1	6	3	7	9	

	4	4	1	1	1	1	1	
	0	2	6	7	9	2	10	
x	0	0	3	4	8	0	2	
	0	0	5	3	5	0	9	$1\cdot\left(-\frac{5}{3}\right) = (-5\cdot 4) = -20 = 2$
	0	0	7	3	7	9	5	$1\cdot\left(-\frac{7}{3}\right) = (-7\cdot 4) = -28 = 5$
	0	0	1	6	3	7	9	$1\cdot\left(-\frac{1}{3}\right) = -4 = 7$

0	0	0	0	10	0	2	
0	0	0	1	3	9	~~10~~ 4	
0	0	0	1	4	7	~~8~~ 1	

0	0	0	1	3	9	~~10~~ 4	
0	0	0	0	10	0	2	$\cdot(-1) = 10$
0	0	0	1	4	7	~~8~~ 1	$1\cdot$ ~~$\left(-\frac{1}{2}\right) = -6 = 5$~~

0	0	0	0	10	0	2
0	0	0	0	1	9	8
0	0	0	0	0	9	10

$1 \cdot (-\frac{1}{10}) = -10 = 1$

gesamt:

	f_0	f_1	g_0	g_1	g_2	g_3	g_4
I	4	4	1	1	1	1	1
II	0	2	6	7	9	2	10
III	0	0	3	4	8	0	2
IV	0	0	0	1	3	9	4
V	0	0	0	0	10	0	2
VI	0	0	0	0	0	9	10

VI g_4 beliebig, z.B. $g_4 = 1$: $9g_3 = -10$, $g_3 = 5 \cdot 1 = \underline{\underline{5}}$

V $10g_2 = -2g_4 = -2$: $g_2 = -10 \cdot 2 = -20 = \underline{\underline{2}}$

IV $g_1 + 3 \cdot 2 + 9 \cdot 5 + 4 = 0$

$g_1 = -55 = \underline{\underline{0}}$

III $3g_0 + 8 \cdot 2 + 2 = 0$

$3g_0 = -7 \rightarrow g_0 = -4 \cdot 7 = -28 = \underline{\underline{5}}$

II $2f_1 + 6 \cdot 5 + 7 \cdot 0 + 9 \cdot 2 + 2 \cdot 5 + 10 = 0$

$2f_1 + 8 + 7 + 10 + 10 = 0$

$2f_1 = -35 = 9 \rightarrow f_1 = 6 \cdot 9 = 54 = \underline{\underline{10}}$

I $4f_0 + 4 \cdot 10 + 5 + 2 + 5 + 1 = 0$

$4f_0 = -53 = 2 \rightarrow f_0 = 3 \cdot 2 = 6$

$f(x) = 6 + 10x$

$g(x) = 5 + 2x^2 + 5x^3 + x^4$

Polynomdivision $\quad g(x) : f(x)$

$$(x^4 + 5x^3 + 2x^2 + 5) : (10x + 6) = 10x^3 + 9x + 10$$

$$\begin{array}{l}
-(x^4 + 60x^3) \\
\hline
\quad - \quad - \quad 2x^2 + 5 \\
\qquad -(90x^2 + 54x) \\
\hline
\qquad\qquad x + 5 \\
\qquad\qquad -(100x + 60) \\
\hline
\qquad\qquad - \quad -
\end{array}$$

gesendete Nachricht: $\quad -\dfrac{g(x)}{f(x)} = -(10x^3 + 9x + 10)$

$$= x^3 + 2x + 1$$

(von hinten lesen) $\longrightarrow \quad (1, 2, 0, 1)$