Übungsblatt 7 – Lösung

7.1 $E[X_i] = 75$, $\mathrm{Var}[X_i] = 25 \overset{\text{ZGWS}}{\Longrightarrow} \bar{X} \sim N\left(\overset{\mu}{75}, \frac{\overset{\sigma^2}{25}}{n}\right)$

Ges.: n, so dass $P(-5 < \bar{X} - 75 < 5) \geq 0.9$ für $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

$$P(-5 < \bar{X} - 75 < 5) = P\left(\frac{-5}{5/\sqrt{n}} < \underbrace{\frac{\bar{X} - 75}{5/\sqrt{n}}}_{= Z \sim N(0,1)} < \frac{5}{5/\sqrt{n}}\right) = P(-\sqrt{n} < Z < \sqrt{n}) =$$

$$\overset{6.3}{=} 2P(Z < \sqrt{n}) - 1 \geq 0.9$$

$$\Longleftrightarrow P(Z < \sqrt{n}) \geq 0.95 \Longleftrightarrow \sqrt{n} = \mathrm{qnorm}(0.95, 0, 1) \approx 1.645$$

$$\Longrightarrow n \geq 3$$

7.2

$$\left.\begin{array}{l} E[X_i] = 50,\ \mathrm{Var}[X_i] = 100 \overset{\text{ZGWS}}{\Longrightarrow} \sum_{i=1}^{20} X_i \sim N\left(\overbrace{20 \cdot 50}^{n\mu}, \overset{n\cdot\sigma^2}{2000}\right) \\ E[Y_i] = 52,\ \mathrm{Var}[Y_i] = 225 \overset{\text{ZGWS}}{\Longrightarrow} \sum_{i=1}^{20} Y_i \sim N(1040, 4500) \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow \sum_{i=1}^{20} X_i - \sum_{i=1}^{20} Y_i \sim N(-40, 6500)$$

$\Longleftarrow$ $\begin{cases} E[X \pm Y] = E[X] \pm E[Y] \\ \mathrm{Var}[X \pm Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] \end{cases}$ (X, Y unabh.)

$$P\left(\sum_{i=1}^{20} X_i < \sum_{i=1}^{20} Y_i\right) = P\left(\sum_{i=1}^{20} X_i - \sum_{i=1}^{20} Y_i < 0\right) = \mathrm{pnorm}(0, -40, \sqrt{65} \cdot 10)$$

$$\approx 69.0\%$$

7.3

$$E[X_i] = 100,\ \mathrm{Var}[X_i] = 900 \overset{\text{ZGWS}}{\Longrightarrow} \sum_{i=1}^{n} X_i \sim N(100n, 900n)$$

Ges.: n mit $P\left(\sum_{i=1}^{n} X_i \geq 2000\right) \geq 0.95$

$$\Longleftrightarrow 1 - P\left(\underbrace{\frac{\sum_{i=1}^{n} X_i - 100n}{30\sqrt{n}}}_{Z \sim N(0,1)} < \frac{2000 - 100n}{30\sqrt{n}}\right) \geq 0.95$$

$$\Longleftrightarrow \quad P\left(Z < \frac{2000 - 100n}{30\sqrt{n}}\right) \leq 0.05$$

$$\Longleftrightarrow \quad \frac{2000 - 100n}{30\sqrt{n}} = qnorm(0.05, 0, 1) \approx -1.645$$

$$\Longleftrightarrow \quad 100n - 1.645 \cdot 30\sqrt{n} - 2000 = 0$$

$$\Longleftrightarrow \quad \underbrace{n}_{\sqrt{n}^2} - 1.645 \cdot 0.3\sqrt{n} - 20 = 0$$

$$\Longleftrightarrow \quad \sqrt{n} = \frac{1.645 \cdot 0.3 \pm \sqrt{1.645^2 \cdot 0.09 + 80}}{2} \quad \Longleftrightarrow \quad \sqrt{n} \approx 4.726$$

$$\Longrightarrow \quad n \geq 23$$