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CA 2 – Data representation

The lecture is based on the work and the documents of Prof. Dr. Theodor Tempelmeier

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Goal

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Goal

CA::Data representation

- Important basics
- ASCII
- Unicode and UTF
- Data types: Numbers

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Important basics

Which numeral systems do you know?

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Important basics

Numeral systems

- DEC: 0, 1, ..., 9; e.g.: 291
- BIN: 0, 1; e.g.: 100100011
- HEX: 0, 1, ..., 9, A, B, ..., F; e.g.: 0x123

Conversion between:

- HEX <-> DEC
- **BIN <-> HEX**
- DEC <-> BIN

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Important basics - hints

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Important basics - short exercise 1/2

Convert `HEX:0xC0FE` to `BIN`.

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Important basics - short exercise 2/2

Convert `BIN:1100000011011110` to `HEX`.

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Binary system

Why is the binary (dual) system used in computer science?

Binary system for digits and characters

- Technically easy to realise (0/1)
- Well understood theoretical basis
  - Boolean algebra
  - Formal logic

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Subtraction is reduced to addition

Idea: Complementation and addition of the complement

Example: $11 - 6$ **in binary system**

```
11: -> 01011
 6: -> 00110
complement of 6:  11001
                +    1
                ------
                 11010
addition of 11 + (-6):
              11: 01011
              -6: 11010
              ---------
                 X00101 => 5
```

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Codes

Which codes for characters do you know?

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ASCII (American Standard Code for Information Interchange)

Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL (null)	32	20	040		Space	64	40	100	@	@	96	60	140	`	`
1	1	001	SOH (start of heading)	33	21	041	!	!	65	41	101	A	A	97	61	141	a	a
2	2	002	STX (start of text)	34	22	042	"	"	66	42	102	B	B	98	62	142	b	b
3	3	003	ETX (end of text)	35	23	043	#	#	67	43	103	C	C	99	63	143	c	c
4	4	004	EOT (end of transmission)	36	24	044	$	$	68	44	104	D	D	100	64	144	d	d
5	5	005	ENQ (enquiry)	37	25	045	%	%	69	45	105	E	E	101	65	145	e	e
6	6	006	ACK (acknowledge)	38	26	046	&	&	70	46	106	F	F	102	66	146	f	f
7	7	007	BEL (bell)	39	27	047	'	'	71	47	107	G	G	103	67	147	g	g
8	8	010	BS (backspace)	40	28	050	(	(	72	48	110	H	H	104	68	150	h	h
9	9	011	TAB (horizontal tab)	41	29	051	)	)	73	49	111	I	I	105	69	151	i	i
10	A	012	LF (NL line feed, new line)	42	2A	052	*	*	74	4A	112	J	J	106	6A	152	j	j
11	B	013	VT (vertical tab)	43	2B	053	+	+	75	4B	113	K	K	107	6B	153	k	k
12	C	014	FF (NP form feed, new page)	44	2C	054	,	,	76	4C	114	L	L	108	6C	154	l	l
13	D	015	CR (carriage return)	45	2D	055	-	-	77	4D	115	M	M	109	6D	155	m	m
14	E	016	SO (shift out)	46	2E	056	.	.	78	4E	116	N	N	110	6E	156	n	n
15	F	017	SI (shift in)	47	2F	057	/	/	79	4F	117	O	O	111	6F	157	o	o
16	10	020	DLE (data link escape)	48	30	060	0	0	80	50	120	P	P	112	70	160	p	p
17	11	021	DC1 (device control 1)	49	31	061	1	1	81	51	121	Q	Q	113	71	161	q	q
18	12	022	DC2 (device control 2)	50	32	062	2	2	82	52	122	R	R	114	72	162	r	r
19	13	023	DC3 (device control 3)	51	33	063	3	3	83	53	123	S	S	115	73	163	s	s
20	14	024	DC4 (device control 4)	52	34	064	4	4	84	54	124	T	T	116	74	164	t	t
21	15	025	NAK (negative acknowledge)	53	35	065	5	5	85	55	125	U	U	117	75	165	u	u
22	16	026	SYN (synchronous idle)	54	36	066	6	6	86	56	126	V	V	118	76	166	v	v
23	17	027	ETB (end of trans. block)	55	37	067	7	7	87	57	127	W	W	119	77	167	w	w
24	18	030	CAN (cancel)	56	38	070	8	8	88	58	130	X	X	120	78	170	x	x
25	19	031	EM (end of medium)	57	39	071	9	9	89	59	131	Y	Y	121	79	171	y	y
26	1A	032	SUB (substitute)	58	3A	072	:	:	90	5A	132	Z	Z	122	7A	172	z	z
27	1B	033	ESC (escape)	59	3B	073	;	;	91	5B	133	[	[	123	7B	173	{	{
28	1C	034	FS (file separator)	60	3C	074	<	<	92	5C	134	\	\	124	7C	174			|
29	1D	035	GS (group separator)	61	3D	075	=	=	93	5D	135	]	]	125	7D	175	}	}
30	1E	036	RS (record separator)	62	3E	076	>	>	94	5E	136	^	^	126	7E	176	~	~
31	1F	037	US (unit separator)	63	3F	077	?	?	95	5F	137	_	_	127	7F	177	&#127;	DEL

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Extended ASCII codes

128	Ç	144	É	160	á	176	▒	192	└	208	╨	224	α	240	≡
129	ü	145	æ	161	í	177	▒	193	┴	209	╤	225	ß	241	±
130	é	146	Æ	162	ó	178	▓	194	┬	210	╥	226	Γ	242	≥
131	â	147	ô	163	ú	179	│	195	├	211	╙	227	π	243	≤
132	ä	148	ö	164	ñ	180	┤	196	─	212	╘	228	Σ	244	⌠
133	à	149	ò	165	Ñ	181	╡	197	┼	213	╒	229	σ	245	⌡
134	å	150	û	166	ª	182	╢	198	╞	214	╓	230	µ	246	÷
135	ç	151	ù	167	º	183	╖	199	╟	215	╫	231	τ	247	≈
136	ê	152	ÿ	168	¿	184	╕	200	╚	216	╪	232	Φ	248	°
137	ë	153	Ö	169	⌐	185	╣	201	╔	217	┘	233	Θ	249	∙
138	è	154	Ü	170	¬	186	║	202	╩	218	┌	234	Ω	250	·
139	ï	155	¢	171	½	187	╗	203	╦	219	█	235	δ	251	√
140	î	156	£	172	¼	188	╝	204	╠	220	▄	236	∞	252	n
141	ì	157	¥	173	¡	189	╜	205	═	221	▌	237	φ	253	²
142	Ä	158	₧	174	«	190	╛	206	╬	222	▐	238	ε	254	■
143	Å	159	ƒ	175	»	191	┐	207	╧	223	▀	239	∩	255	

Source: www.LookupTables.com

[source: asciitable.com]

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ASCII

ASCII - American Standard Code for Information Interchange

Any problems with ASCII?

Unicode

- International standard (ISO 10646)
- For every character one code
- In the long term: A digital code is defined for each meaningful character or text element of all known cultures, countries/languages, and character systems.
- Is constantly extended
- `http://www.unicode.org`

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Unicode

Character range:

first code U+00 0000

last code U+10 FFFF

Character sets

Name	Unit	Calculation	#chars	first	last
UCS-2	16 Bit	2^{16}	65536	U+0000	U+FFFF
UCS-4	17 Planes	$17 * 2^{16}$	1114112	U+00 0000	U+10 FFFF

Examples:

Unicode	Full number	Character
U+0041	00 0041	A
U+1F600	01 F600	😀

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Unicode 14.0 - Planes

Plane 0 00 0000-00 FFFF BMP Basic Multilingual Plane	Plane 1 01 0000-01 FFFF SMP Supplementary Multilingual Plane	Plane 2 02 0000-02 FFFF SIP Supplementary Ideographic Plane	Plane 3 03 0000-03 FFFF unassigned	Plane 4 04 0000-04 FFFF unassigned
Plane 5 05 0000-05 FFFF unassigned	Plane 6 06 0000-06 FFFF unassigned	Plane 7 07 0000-07 FFFF unassigned	Plane 8 08 0000-08 FFFF unassigned	Plane 9 09 0000-09 FFFF unassigned
Plane 10 0A 0000-0A FFFF unassigned	Plane 11 0B 0000-0B FFFF unassigned	Plane 12 0C 0000-0C FFFF unassigned	Plane 13 0D 0000-0D FFFF unassigned	Plane 14 0E 0000-0E FFFF SSP Supplementary Special-purpose Plane
Plane 15 0F 0000-0F FFFF SPUA-A Supplementary Private Use Area planes	Plane 16 10 0000-10 FFFF SPUA-A Supplementary Private Use Area planes			

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Unicode

Enter unicode characters

OS	Program	Keyboard shortcut
Linux	Terminal, xed, LibreOffice	`CTRL+SHIFT+U + HEX Number`
Windows	Microsoft Word, Excel, WordPad	`HEX Number + ALT+C`
macOS*	Console, Text	`ALT + HEX Number`

More shortcuts: wikipedia.org

*must be enabled as input source

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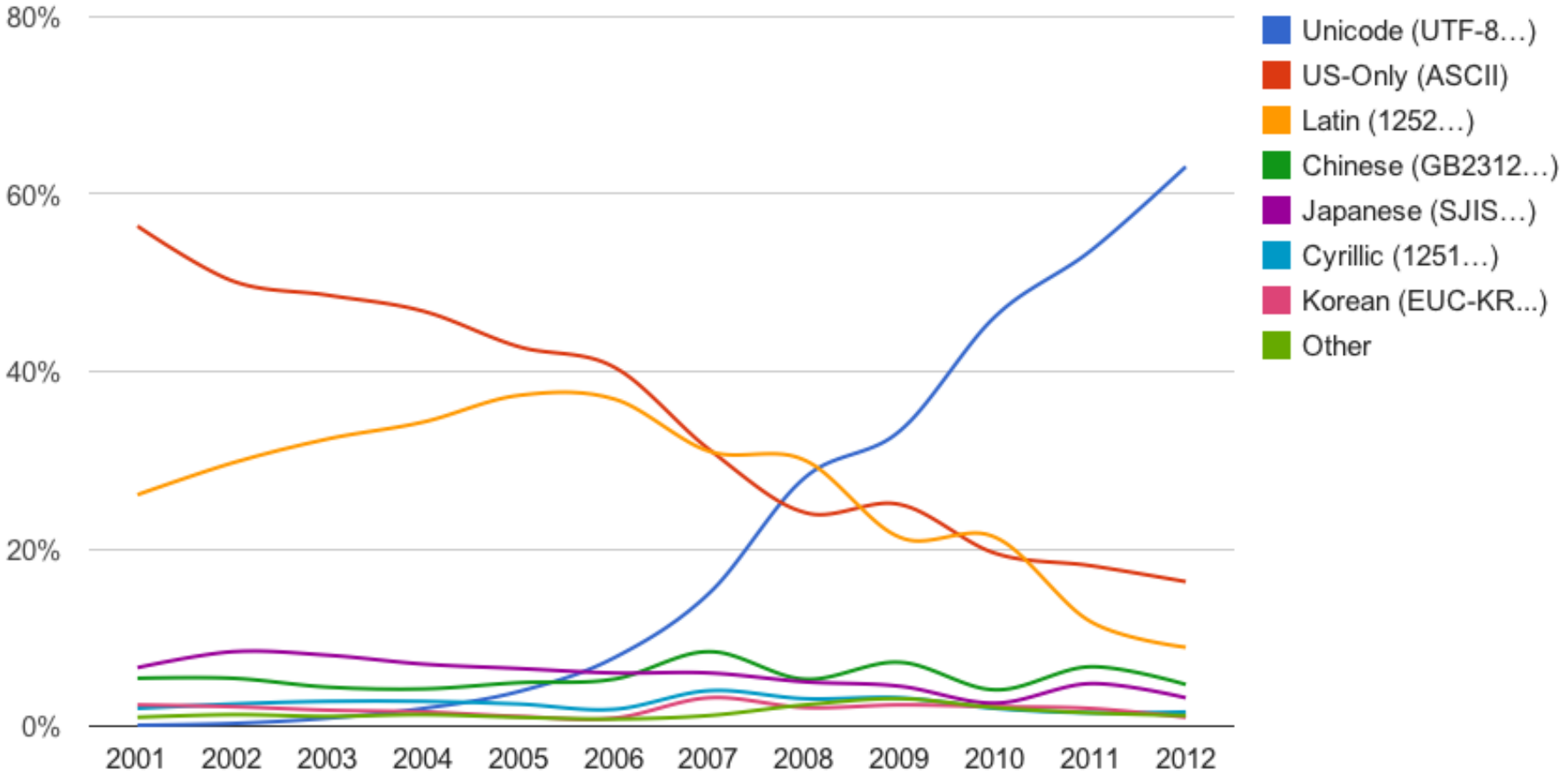
Unicode usage



[source: googleblog.blogspot.com], Link to current statistics: w3techs.com

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Unicode

Character set vs. character encoding?

Unicode vs **UTF**

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UTF - Unicode Transformation Format

UTF maps all unicode code points to a unique sequence of bytes.

Used for

- Store information into files, databases, ...
- Transfer data (websites, e-mail, ...)

Choice depends on

- Storage space
- Source code compatibility
- Interoperability with other systems
- Runtime for encoding/decoding

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UTF - Unicode Transformation Format

Overview of UTF encodings

Encoding	Bits	Length	Common use
UTF-8	8-bit	Variable length: 1 to 4 bytes	Internet, Linux
UTF-16	16-bit	Variable length: 2 or 4 bytes	Qt, Java, Tcl
UTF-32	32-bit	Fixed length: 4 bytes	

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UTF-8

UTF-8 length

Number of bytes	Bits for code point	Unicode range	Comment
1	7	0 - 00 007F	Compatible with ASCII
2	11	80 - 00 07FF	
3	16	800 - 00 FFFF	
4	21	1 0000 - 10 FFFF	

UTF-8 encoding details

Unicode range	Byte 1	Byte 2	Byte 3	Byte 4
0 - 00 007F	0xxxxxxx			
80 - 00 07FF	110xxxxx	10xxxxxx		
800 - 00 FFFF	1110xxxx	10xxxxxx	10xxxxxx	
1 0000 - 10 FFFF	11110xxx	10xxxxxx	10xxxxxx	10xxxxxx

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UTF-8 - example

Encode the „ü“ into UTF-8!

[ü: https://en.wikipedia.org/wiki/Latin-1_Supplement_(Unicode_block)]

```
1  ü -> 252 -> 0xFC
2
3  ü in Unicode:
4  U+00 00FC (8 bits -> 2 bytes required)
5  F    C
6  1111 1100
7
8  ü in UTF-8:
9  11000011 10111100
10 C   3    B   C     -> 0xC3BC
```

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UTF-16

UTF-16 length

Number of bytes	Bits for code point	Unicode range	Comment
2	16	0 - 00 FFFF	
4	20	01 0000 - 10 FFFF	subtraction required: U+XXXXXX - 0x10000

UTF-16 encoding details

Unicode range	Byte 1	Byte 2	Byte 3	Byte 4
0 - 00 FFFF	xxxxxxxx	xxxxxxxx		
	High surrogate		Low surrogate	
01 0000 - 10 FFFF	110110xx	xxxxxxxx	110111xx	xxxxxxxx

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UTF-16 - example

Encode the „😀“ (U+1F600) into UTF-16!

```
4 byte variant and therefore correction required:
0x1F600 - 0x10000 = 0xF600

F    6    0    0
1111 0110 0000 0000

In UTF-16:
 High surrogate      Low surrogate
11011000 00111101  11011110 00000000
D   8    3   D     D   E    0   0     -> 0xD83DDE00
```

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UTF-32

UTF-32 length

Number of bytes	Bits for code point	Unicode range	Comment
4	21	00 0000 - 10 FFFF	directly representable

UTF-32 encoding details

Unicode range	Byte 1	Byte 2	Byte 3	Byte 4
0 - 10 FFFF	00000000	000xxxxx	xxxxxxxx	xxxxxxxx

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UTF-32 - example

Encode the „😀“ (U+1F600) into UTF-32!

```
1 Only the 4 byte variant exists
2 0x1F600
3
4 1    F    6    0    0
5 0001 1111 0110 0000 0000
6
7 In UTF-32:
8 00000000 00000001 11110110 00000000
9 0    0    0    1    F    6    0    0     -> 0x0001F600
```

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Numbers

Type	Common data type	Realisation
Integer	unsigned int, int, ...	Hardware: ALU
Floating point – binary	float, double, ...	Hardware: FPU
Floating point – decimal	decimal32, decimal64, ...	Mostly in software
Fixed point – binary	Often not well integrated	Mostly in software
Fixed point – decimal	Often not well integrated	Mostly in software

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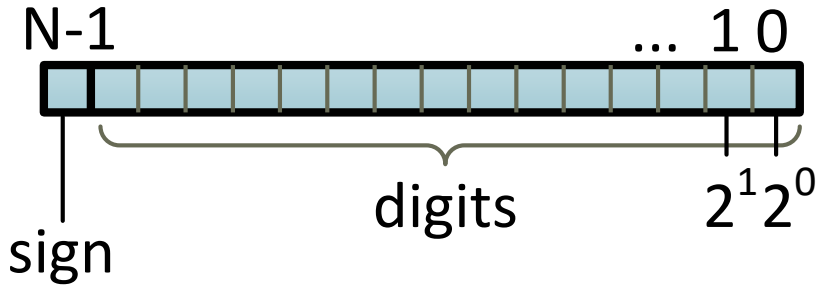
Integer (signed)

Example: `short int`



Positive number: The weight for position i is 2^i
Negative number: The sign is interpreted as -2^N
Example `short int`: Minimum: -32768; Maximum: 32767

limits: `http://www.cplusplus.com/reference/climits`

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Integers

Any problems with integers?

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Fixed width integer types (since C99)

Available types:

Bits	signed	unsigned
8	int8_t	uint8_t
16	int16_t	uint16_t
32	int32_t	uint32_t
64	int64_t	uint64_t

Example:

```
1 #include <stdint.h>
2 ...
3 int16_t val_16 = 5;
```

More details: https://en.cppreference.com/w/cpp/types/integer

Floating point – binary

Usually scientific numbers with mantissa and exponent.
Requires hardware support (FPU - floating point unit).

Format: $x = m \cdot B^e$ (m = mantissa, B = basis, and e = exponent)

Examples:

- C: `float x;`
- Ada: `x: float`

Floating point – binary

Floating point binary formats are defined in the **IEEE Standard for Floating-Point Arithmetic (IEEE 754)**.

Name	Common name	Number of bits	Characteristic	Mantissa
binary16	Half precision	16	5 bits; $c = e + 15$	10 bits
binary32	Single precision	32	8 bits; $c = e + 127$	23 bits
binary64	Double precision	64	11 bits; $c = e + 1023$	52 bits
binary128	Quadruple precision	128	15 bits; $c = e + 16383$	112 bits
binary256	Octuple precision	256	19 bits; $c = e + 262143$	236 bits

IEEE 754 on Wikipedia: `https://en.wikipedia.org/wiki/IEEE_754`

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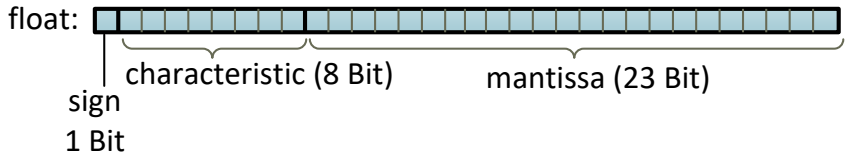
Floating point – binary

Example: `float (single precision)`



Exponent	$-126, \ldots, +127$	Exponent is represented via the characteristic
Characteristic	$c = e + 127$	
Mantissa	$1 \leq m < B$	Is normalised in the binary system: `1.MMM...M` Advantage: `1` doesn't have to be saved!

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Floating point – binary

Convert the decimal number 1.75 into the binary32 (float) representation.

```
1.75 -> binary:
01.11000...0   -> it has already the required form
                  of 1.MMM...M (=> e=0)

c = e + 127 = 0 + 127 = 127

S|C       |M
0|01111111|11000000000000000000000|

Hex representation:
0x3fe00000
```

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Floating point – binary

Let's do some (binary) floating point number crunching.

Nr.	Code	different	equal
1	`36.2 != 36.2`		
2	`0.362 * 100.0 != 36.2`		
3	`0.362 * (100.0 / 100.0) != 0.362`		
4	`(0.362 * 100.0) / 100.0 != 0.362`		

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Floating point – decimal

Floating point decimal formats are defined in the **IEEE Standard for Floating-Point Arithmetic (IEEE 754)**.

Format: $x = (-1)^{\text{signbit}} \times 10^{\text{exponentbits}_2 - 101_{10}} \times \text{truesignificand}_{10}$

Name	Number of decimal digits	Exponent min.	Exponent max.
decimal32	7	-95	+96
decimal64	16	-383	+384
decimal128	34	-6143	+6144

IEEE 754 on Wikipedia: `https://en.wikipedia.org/wiki/IEEE_754`

- Possible in gnu C with `_Decimal32`, `_Decimal64`, and `_Decimal128`
- Example C: `_Decimal32 x = 0.1df;`
- Possible in gnu C++ with `decimal32`, `decimal64`, and `decimal128`
- Example C++: `std::decimal::decimal32 x(0.1);`

More details on the format (on Wikipedia): `https://en.wikipedia.org/wiki/Decimal32_floating-point_format`

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Floating point – decimal

Let's do some (decimal) floating point number crunching.

Nr.	Code	different	equal
1	`36.2 != 36.2`		
2	`0.362 * 100.0 != 36.2`		
3	`0.362 * (100.0 / 100.0) != 0.362`		
4	`(0.362 * 100.0) / 100.0 != 0.362`		

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Fixed point

Fixed point numbers have a fixed imaginary point that is not moved.

Usage:

- Areas where rounding errors must be avoided (e.g. commercial applications)
- If no floating point hardware (FPU) is available (e.g. in embedded systems)
- Devices use the numbers in this format anyway (e.g. analog/digital converter)

Two variants:

Type	Usage
Binary fixed point	technical
Decimal fixed point	economical

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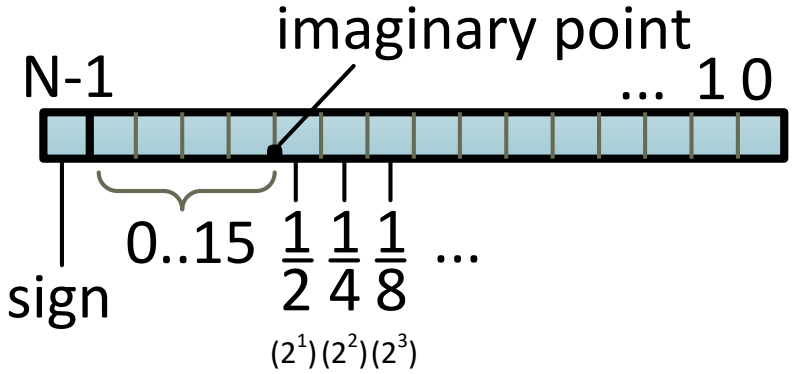
Fixed point – binary

Uses integers with an imaginary binary point.

Often without specialised hardware: *Poor man's floating point*.

Ada: `type analog_input is delta 0.125 range -16.0..15.0;`



[C++ library example: Compositional Numeric Library]

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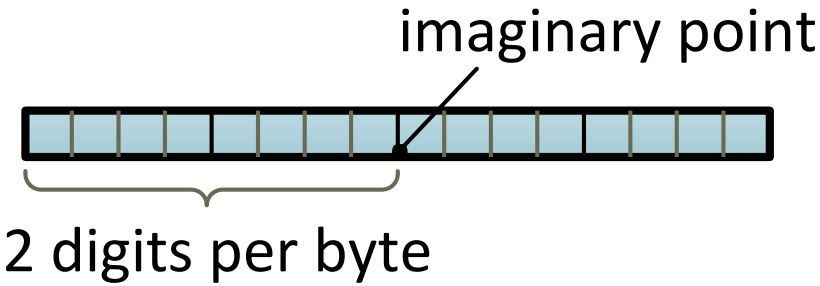
Fixed point – decimal

Uses the binary coded decimal (BCD) system with an imaginary decimal point and BCD arithmetic.

Used in IBM main frame. Sometimes there exists specialised hardware.

BCD: Every digit $(0-9)$ is represented by 4 bits

Ada: `type money is delta 0.01 digits 8;`



[C++ library example: Decimal data type for C++]

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When to use what?
A first try for a property overview

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Summary and outlook

Summary

- Important basics
- ASCII
- Unicode and UTF
- Data types: Numbers

Outlook

- Logical hardware