H.W. №2

Problem 1.

$S = 5 + 9 + 13 + \ldots + 89$

$a_1 = 5; \quad d = 4;$

$a_n = a_1 + (n-1) \cdot d;$

$\cancel{5+}(n-1) \cdot 4 = 89 - 5$

$\cancel{5+} 4n - 4 = 84$

$n = 22$

$S = \sum_{k=1}^{22} [5 + (k-1) \cdot 4]$

Problem 2.

$\sum_{k=3}^{15} (2k+1) = \sum_{j=1}^{13} [2(j+2)+1] = \sum_{j=1}^{13} (2j+5);$

Problem 3

$a_1 = 12; \quad a_{10} = 57$

$a_n = a_{n-1} + d$

$a_{10} = a_9 + d = a_1 + (n-1)d = 12 + \cancel{d}\, 9d = 57$

$9d = 45$

$d = 5$

$a_{25} = 12 + (25-1) \cdot 5 = 132$

Problem 4

$a_1 = 7 \cdot 15 = 105$

$a_n = 7 \cdot 142 = 994$

$n = \frac{a_n - a_1}{d} + 1 = \frac{994 - 105}{7} + 1 = 128$

$S = \frac{n}{2}(a_1 + a_n) = \frac{128}{2}(105 + 994) = 64 \cdot 1099 = 70336;$

Problem 5

$S = \sum_{k=1}^{n} (3k+2); \quad S = 2650; \quad S = \frac{n}{2}[a_1 + a_n] \Rightarrow 3n^2 + 7n - 5300 = 0$

$a_1 = 3(1) + 2 = 5 \quad S = \frac{n}{2}[5 + 3n + 2]; \quad n_{1,2} = \frac{-7 \pm \sqrt{49 - (4 \cdot 3 \cdot 5300)}}{6};$

$a_n = 3n + 2 \quad S = \frac{n}{2}[3n + 7]; \quad n = \frac{245}{6} \approx 40{,}88$

[illegible]

Problem 6

$a_5 = 20$

$a_{15} = 60$

$a_{10} = \frac{a_5 + a_{15}}{2} = 40;$

Problem 7

$a_1 = 5$

$d = 0,5$

$n = 20$

$S_{20} = \frac{20}{2}[2 \cdot 5 + (20-1) \cdot 0,5];$

$S_{20} = 10(10 + \frac{19}{2});$

$S_{20} = 100 + 95;$

$S_{20} = 195;$

Problem 8

$a_1 = 11$

$d = 3$

$\frac{n}{2}[22 + (3n - 3)] > 1000$

$22n + 3n^2 - 3n > 2000$

$3n^2 + 19n - 2000 > 0$

$n_1 = \frac{-19 \pm \sqrt{19^2 - 4 \cdot 3 \cdot (-2000)}}{6};$

$n = 156,03$

$n = \frac{-19 + 156,03}{6} \approx 22,84$

$S_{23} = \frac{23(3 \cdot 23 + 19)}{2} = 1012$

Problem 9

$\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k \quad k = 0;$

$j = k - 3; \ j = 0;$

$\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k = \sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3} = \sum_{j=0}^{9} 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)^j = \sum_{j=0}^{9}\left(\frac{1}{2}\right)^{j+1}$

Problem 10

$a_2 = -6; \ a_5 = 48; \quad -6 = a_1 R \Rightarrow \frac{48}{-6} = \frac{\cancel{a_1} R^{\cancel{4}\,3}}{\cancel{a_1} \cancel{R}} \Rightarrow R = -2; \quad a_1 = 3;$

$a_n = a_1 R^{n-1} \qquad 48 = a_1 R^4$

$a_{10} = 3 \cdot (-2)^9 = -1536$

Problem 11

$a_4 = 54;$

$a_7 = 1458;$

$R - ?$

$$\frac{\cancel{1458}^{27}}{\cancel{54}} = \frac{\cancel{a_1} R^{\cancel{6}\,3}}{\cancel{a_1} \cancel{R^4}} \Rightarrow R = \sqrt[3]{27};\ R = 3;$$

Problem 12

$S_{15} - ?$ $\qquad S_n = a_1 \frac{1 - R^n}{1 - R}$

$a_1 = 8$ $\qquad S_{15} = 8\,\frac{\left(1 - \left(\frac{3}{4}\right)^{15}\right)}{\frac{1}{4}} \Rightarrow 32\left(1 - \left(\frac{3}{4}\right)^{15}\right) \Rightarrow \approx 32$

$R = \frac{3}{4}$

Problem 13

No Solution

Problem 14

$(2x^4 - 3x^3 + x - 5) + (x^3 - 2x^2 + 4x + 7) = 2x^4 - 3x^3 + x - 5 + x^3 - 2x^2 + 4x + 7 =$

$= 2x^4 - 2x^3 - 2x^2 + 5x + 2$

Problem 15

$(x^2 - x + 2)(x^2 + x + 1) = x^4 + \cancel{x^3} + \cancel{x^2} - \cancel{x^3} - \cancel{x^2} - \cancel{x} + 2x^2 + \cancel{2x} + 2 = x^4 + 2x^2 + x + 2$

Problem 16

$24x^3y^2z^5 \qquad 36x^5y^3z^2$

GCD:

$12,\ x^3, y^2, z^2$

LCM:

$72,\ x^5 y^3, z^5$

Problem 17

$x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4) = (x - 3)(x + 3)(x - 2)(x + 2)$

Problem 19

$6x^3 + 11x^2 - 31x + 15 \;|\; 3x - 2$

$(6x^3 - 4x^2) \qquad 2x^2 + 5x - 7$ (Remainder 1)

$15x^2 - 31x$

$-(15x^2 - 10x)$

$-21x + 15$

$-(-21x + 14)$

Problem 18

$(2x+3y)^5$

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

$k=0 \quad \binom{5}{0}(2x)^5(3y)^0 = 1 \cdot 32x^5 \cdot 1 = 32x^5$

$k=1 \quad \binom{5}{1}(2x)^4(3y)^1 = 5 \cdot 16x^4 \cdot 3y = 240x^4y$

$k=2 \quad \binom{5}{2}(2x)^3(3y)^2 = 10 \cdot 8x^3 \cdot 9y^2 = 720x^3y^2$

$k=3 \quad \binom{5}{3}(2x)^2(3y)^3 = 10 \cdot 4x^2 \cdot 27y^3 = 1080x^2y^3$

$k=4 \quad \binom{5}{4}(2x)^1(3y)^4 = 5 \cdot 2x \cdot 81y^4 = 810xy^4$

$k=5 \quad \binom{5}{5}(2x)^0(3y)^5 = 1 \cdot 1 \cdot 243y^5 = 243y^5$

Problem 20

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$