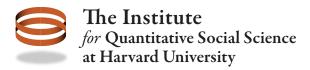
Regression Models in R



Outline

- Introduction
- 2 Linear regression
- 3 Interactions and factors
- 4 Regression with binary outcomes
- Multilevel Modeling
- 6 Exercise solutions

prototype

Wrap-up

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Workshop description

- This is an intermediate/advanced R course
- Appropriate for those with basic knowledge of R
- This is not a statistics course!
- Learning objectives:
 - Learn the R formula interface
 - Specify factor contrasts to test specific hypotheses
 - Perform model comparisons
 - Run and interpret variety of regression models in R

Materials and Setup

Lab computer users: Log in using the user name and password on the board to your left.

Everyone should have R installed -if not:

- Open a web browser and go to http://cran.r-project.org and download and install it
- Also helpful to install RStudo (download from http://rstudio.com)

Materials for this workshop include slides, example data sets, and example code.

- Download materials from http://tutorials.iq.harvard.edu/R/Rstatistics.zip
- Extract materials from RStatistics.zip (on lab machines *right-click -> WinZip -> Extract to here*) and move to your desktop.

Launch RStudio

- Open the RStudio program from the Windows start menu
- Open up today's R script
 - In RStudio, Go to File => Open Script
 - Locate and open the Rstatistics.R script in the Rstatistics folder on your desktop
- Go to Tools => Set working directory => To source file location (more on the working directory later)
- I encourage you to add your own notes to this file!

Set working directory

It is often helpful to start your R session by setting your working directory so you don't have to type the full path names to your data and other files

```
> # set the working directory
> # setwd("~/Desktop/Rstatistics")
> # setwd("C:/Users/dataclass/Desktop/Rstatistics")
>
```

You might also start by listing the files in your working directory

Load the states data

The states.dta data comes from http:

//anawida.de/teach/SS14/anawida/4.linReg/data/states.dta.txt and appears to have originally appeared in *Statistics with Stata* by Lawrence C. Hamilton.

```
> # read the states data
> states.data <- readRDS("dataSets/states.rds")</pre>
> #get labels
> states.info <- data.frame(attributes(states.data)[c("names", "var.labels")])
> #look at last few labels
> tail(states.info, 8)
                              var.labels
    names
14 csat Mean composite SAT score
                    Mean verbal SAT score
15 vsat.
16 msat
                      Mean math SAT score
17 percent
                % HS graduates taking SAT
18 expense Per pupil expenditures prim&sec
19
  income Median household income, $1,000
                      % adults HS diploma
    high
21 college
                  % adults college degree
```

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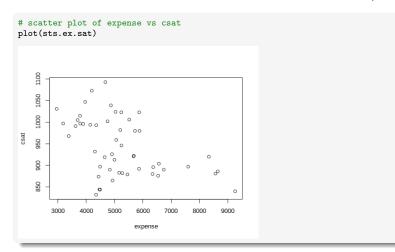
Examine the data before fitting models

Start by examining the data to check for problems.

```
> # summary of expense and csat columns, all rows
> sts.ex.sat <- subset(states.data, select = c("expense", "csat"))</pre>
> summary(sts.ex.sat)
   expense csat
Min. :2960 Min. :832
1st Qu.:4352 1st Qu.: 888
Median: 5000 Median: 926
Mean :5236 Mean : 944
3rd Qu.:5794 3rd Qu.: 997
Max. :9259 Max. :1093
> # correlation between expense and csat
> cor(sts.ex.sat)
     expense csat
expense 1.000 -0.466
csat -0.466 1.000
```

Plot the data before fitting models

Plot the data to look for multivariate outliers, non-linear relationships etc.



Linear regression example

- Linear regression models can be fit with the lm() function
- For example, we can use 1m to predict SAT scores based on per-pupal expenditures:

```
> # Fit our regression model
> sat.mod <- lm(csat ~ expense, # regression formula
               data=states.data) # data set
> # Summarize and print the results
> summary(sat.mod) # show regression coefficients table
Call:
lm(formula = csat ~ expense, data = states.data)
Residuals:
   Min 1Q Median 3Q Max
-131.81 -38.08 5.61 37.85 136.50
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 1060.73244 32.70090 32.44 < 2e-16
expense -0.02228 0.00604 -3.69 0.00056
```

Residual standard error: 59.8 on 49 degrees of freedom

Why is the association between expense and SAT scores negative?

Many people find it surprising that the per-capita expenditure on students is negatively related to SAT scores. The beauty of multiple regression is that we can try to pull these apart. What would the association between expense and SAT scores be if there were no difference among the states in the percentage of students taking the SAT?

```
> summary(lm(csat ~ expense + percent, data = states.data))
Call:
lm(formula = csat ~ expense + percent, data = states.data)
Residuals:
  Min 10 Median 30 Max
-62.92 -24.32 1.74 15.50 75.62
Coefficients:
          Estimate Std. Error t value Pr(>|t|)
(Intercept) 989.8074 18.3958 53.81 < 2e-16
expense 0.0086 0.0042 2.05 0.046
percent -2.5377 0.2249 -11.28 4.2e-15
```

The Im class and methods

OK, we fit our model. Now what?

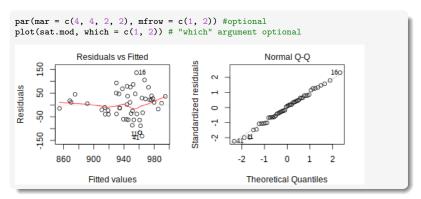
• Examine the model object:

```
> class(sat.mod)
[1] "lm"
> names(sat.mod)
[1] "coefficients" "residuals" "effects" "rank"
[5] "fitted.values" "assign" "qr" "df.residual"
[9] "xlevels" "call" "terms" "model"
> methods(class = class(sat.mod))[1:9]
[1] "add1.lm" "alias.lm" "anova.lm"
[4] "case.names.lm" "confint.lm" "cooks.distance.lm"
[7] "deviance.lm" "dfbeta.lm" "dfbetas.lm"
```

• Use function methods to get more information about the fit

Linear Regression Assumptions

- Ordinary least squares regression relies on several assumptions, including that the residuals are normally distributed and homoscedastic, the errors are independent and the relationships are linear.
- Investigate these assumptions visually by plotting your model:



Comparing models

Do congressional voting patterns predict SAT scores over and above expense? Fit two models and compare them:

```
> # fit another model, adding house and senate as predictors
> sat.voting.mod <- lm(csat ~ expense + house + senate,
                      data = na.omit(states.data))
> sat.mod <- update(sat.mod, data=na.omit(states.data))</pre>
> # compare using the anova() function
> anova(sat.mod, sat.voting.mod)
Analysis of Variance Table
Model 1: csat ~ expense
Model 2: csat ~ expense + house + senate
           RSS Df Sum of Sq F Pr(>F)
     46 169050
     44 149284 2 19766 2.91 0.065
> coef(summary(sat.voting.mod))
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1082.9344 38.63381 28.03 1.07e-29
expense -0.0187 0.00969 -1.93 6.00e-02
house -1.4424 0.60048 -2.40 2.06e-02
       0.4982 0.51356 0.97 3.37e-01
senate
```

Exercise 0: least squares regression

Use the *states.rds* data set. Fit a model predicting energy consumed per capita (energy) from the percentage of residents living in metropolitan areas (metro). Be sure to

- Examine/plot the data before fitting the model
- 2 Print and interpret the model summary
- plot the model to look for deviations from modeling assumptions

Select one or more additional predictors to add to your model and repeat steps 1-3. Is this model significantly better than the model with *metro* as the only predictor?

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Modeling interactions

Interactions allow us assess the extent to which the association between one predictor and the outcome depends on a second predictor. For example: Does the association between expense and SAT scores depend on the median income in the state?

```
> #Add the interaction to the model

> sat.expense.by.percent <- lm(csat ~ expense*income,

+ data=states.data)

> #Show the results

> coef(summary(sat.expense.by.percent)) # show regression coefficients table

Estimate Std. Error t value Pr(>|t|)

(Intercept) 1380.36423 172.086252 8.02 0.000000000237

expense -0.06384 0.032701 -1.95 0.056878369245

income -10.49785 4.991463 -2.10 0.040832525071

expense:income 0.00138 0.000864 1.60 0.115539488253
```

Regression with categorical predictors

Let's try to predict SAT scores from region, a categorical variable. Note that you must make sure R does not think your categorical variable is numeric.

```
> # make sure R knows region is categorical
> str(states.data$region)
Factor w/ 4 levels "West", "N. East", ...: 3 1 1 3 1 1 2 3 NA 3 ...
> states.data$region <- factor(states.data$region)</pre>
> #Add region to the model
> sat.region <- lm(csat ~ region,
                 data=states.data)
> #Show the results
> coef(summary(sat.region)) # show regression coefficients table
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 946.3 14.8 63.958 1.35e-46
regionN. East -56.8 23.1 -2.453 1.80e-02
regionSouth -16.3 19.9 -0.819 4.17e-01
regionMidwest 63.8 21.4 2.986 4.51e-03
> anova(sat.region) # show ANOVA table
Analysis of Variance Table
Response: csat
         Df Sum Sq Mean Sq F value Pr(>F)
region 3 82049 27350 9.61 0.000049
Residuals 46 130912 2846
```

Setting factor reference groups and contrasts

In the previous example we use the default contrasts for region. The default in R is treatment contrasts, with the first level as the reference. We can change the reference group or use another coding scheme using the C function.

```
> # print default contrasts
> contrasts(states.data$region)
       N. East South Midwest
West
N. East 1 0
South
Midwest 0 0
> # change the reference group
> coef(summary(lm(csat ~ C(region, base=4),
               data=states.data)))
                  Estimate Std. Error t value Pr(>|t|)
(Intercept) 1010.1 15.4 65.59 4.30e-47
C(region, base = 4)1 -63.8 21.4 -2.99 4.51e-03
C(region, base = 4)2 -120.5 23.5 -5.12 5.80e-06
C(region, base = 4)3 -80.1 20.4 -3.93 2.83e-04
> # change the coding scheme
> coef(summary(lm(csat ~ C(region, contr.helmert),
               data=states.data)))
                       Estimate Std. Error t value Pr(>|t|)
```

Exercise 1: interactions and factors

Use the states data set.

- Add on to the regression equation that you created in exercise 1 by generating an interaction term and testing the interaction.
- Try adding region to the model. Are there significant differences across the four regions?

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Logistic regression

This far we have used the lm function to fit our regression models. lm is great, but limited—in particular it only fits models for continuous dependent variables. For categorical dependent variables we can use the glm() function. For these models we will use a different dataset, drawn from the National Health Interview Survey. From the CDC website:

The National Health Interview Survey (NHIS) has monitored the health of the nation since 1957. NHIS data on a broad range of health topics are collected through personal household interviews. For over 50 years, the U.S. Census Bureau has been the data collection agent for the National Health Interview Survey. Survey results have been instrumental in providing data to track health status, health care access, and progress toward achieving national health objectives.

Load the National Health Interview Survey data:

```
> NH11 <- readRDS("dataSets/NatHealth2011.rds")
> labs <- attributes(NH11)$labels
>
```

←□ → ←□ → ← □ → ← □ →

Logistic regression example

Let's predict the probability of being diagnosed with hypertension based on age, sex, sleep, and bmi

```
> str(NH11$hypev) # check stucture of hypev
Factor w/ 5 levels "1 Yes", "2 No", ...: 2 2 1 2 2 1 2 2 1 2 ...
> levels(NH11$hypev) # check levels of hypev
[1] "1 Yes" "2 No"
                                          "7 Refused"
[4] "8 Not ascertained" "9 Don't know"
> # collapse all missing values to NA
> NH11$hypev <- factor(NH11$hypev, levels=c("2 No", "1 Yes"))
> # run our regression model
> hyp.out <- glm(hypev~age_p+sex+sleep+bmi,</pre>
               data=NH11, family="binomial")
> coef(summary(hyp.out))
           Estimate Std. Error z value Pr(>|z|)
(Intercept) -4.26947 0.056495 -75.57 0.00e+00
age_p 0.06070 0.000823 73.78 0.00e+00
sex2 Female -0.14403 0.026798 -5.37 7.68e-08
sleep -0.00704 0.001640 -4.29 1.78e-05
bmi 0.01857 0.000951 19.53 6.49e-85
```

Logistic regression coefficients

Generalized linear models use link functions, so raw coefficients are difficult to interpret. For example, the age coefficient of .06 in the previous model tells us that for every one unit increase in age, the log odds of hypertension diagnosis increases by 0.06. Since most of us are not used to thinking in log odds this is not too helpful!

One solution is to transform the coefficients to make them easier to interpret

```
> hyp.out.tab <- coef(summary(hyp.out))
> hyp.out.tab[, "Estimate"] <- exp(coef(hyp.out))
> hyp.out.tab

Estimate Std. Error z value Pr(>|z|)
(Intercept) 0.014 0.056495 -75.57 0.00e+00
age_p 1.063 0.000823 73.78 0.00e+00
sex2 Female 0.866 0.026798 -5.37 7.68e-08
sleep 0.993 0.001640 -4.29 1.78e-05
bmi 1.019 0.000951 19.53 6.49e-85
```

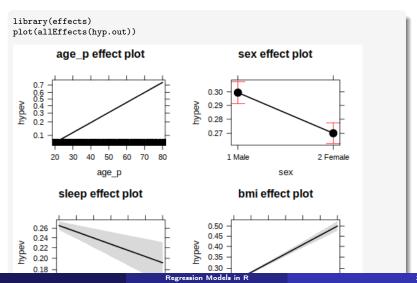
Generating predicted values

In addition to transforming the log-odds produced by glm to odds, we can use the predict() function to make direct statements about the predictors in our model. For example, we can ask "How much more likely is a 63 year old female to have hypertension compared to a 33 year old female?".

This tells us that a 33 year old female has a 13% probability of having been diagnosed with hypertension, while and 63 year old female has a 48% probability of having been diagnosed.

Packages for computing and graphing predicted values

Instead of doing all this ourselves, we can use the effects package to compute quantities of interest for us (cf. the Zelig package).



Exercise 2: logistic regression

Use the NH11 data set that we loaded earlier.

- Use glm to conduct a logistic regression to predict ever worked (everwrk) using age (age_p) and marital status (r_{maritl}).
- 2 Predict the probability of working for each level of marital status.

Note that the data is not perfectly clean and ready to be modeled. You will need to clean up at least some of the variables before fitting the model.

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Multilevel modeling overview

- Multi-level (AKA hierarchical) models are a type of mixed-effects models
- Used to model variation due to group membership where the goal is to generalize to a population of groups
- Can model different intercepts and/or slopes for each group
- Mixed-effecs models include two types of predictors: fixed-effects and random effects

```
Fixed-effects observed levels are of direct interest (.e.g, sex, political party...)
```

Random-effects observed levels not of direct interest: goal is to make inferences to a population represented by observed levels

- In R the Ime4 package is the most popular for mixed effects models
 - Use the lmer function for liner mixed models, glmer for generalized mixed models

```
> library(lme4)
>
```

The Exam data

The Exam data set contans exam scores of 4,059 students from 65 schools in Inner London. The variable names are as follows:

```
school School ID - a factor.
normexam Normalized exam score.
  schgend School gender - a factor. Levels are 'mixed', 'boys', and 'girls'.
   schavg School average of intake score.
        vr Student level Verbal Reasoning (VR) score band at intake - a
           factor. Levels are 'bottom 25%', 'mid 50%', and 'top 25%'.
    intake Band of student's intake score - a factor. Levels are 'bottom
           25%', 'mid 50%' and 'top 25%'./
standLRT Standardised LR test score.
      sex Sex of the student - levels are 'F' and 'M'.
     type School type - levels are 'Mxd' and 'Sngl'.
  student Student id (within school) - a factor
```

```
> Exam <- readRDS("dataSets/Exam.rds")
>
```

The null model and ICC

As a preliminary step it is often useful to partition the variance in the dependent variable into the various levels. This can be accomplished by running a null model (i.e., a model with a random effects grouping structure, but no fixed-effects predictors).

```
> # null model, grouping by school but not fixed effects.
> Norm1 <-lmer(normexam ~ 1 + (1|school),</pre>
               data=Exam, REML = FALSE)
> summary(Norm1)
Linear mixed model fit by maximum likelihood ['lmerMod']
Formula: normexam ~ 1 + (1 | school)
  Data: Exam
    ATC
          BIC logLik deviance df.resid
  10826 10844 -5410 10820
                                     3984
Scaled residuals:
  Min 1Q Median 3Q Max
-3.902 -0.646 0.003 0.698 3.636
Random effects:
Groups Name
                  Variance Std.Dev.
school (Intercept) 0.169 0.412
```

Adding fixed-effects predictors

Predict exam scores from student's standardized tests scores

```
> Norm2 <-lmer(normexam~standLRT + (1|school),
             data=Exam.
             REML = FALSE)
> summary(Norm2)
Linear mixed model fit by maximum likelihood ['lmerMod']
Formula: normexam ~ standLRT + (1 | school)
  Data: Exam
    ATC
        BIC logLik deviance df.resid
   9143 9168 -4568 9135 3954
Scaled residuals:
  Min 10 Median 30 Max
-3.700 -0.625 0.024 0.678 3.262
Random effects:
Groups Name
              Variance Std.Dev.
school (Intercept) 0.0919 0.303
Residual 0.5670 0.753
Number of obs: 3958, groups: school, 65
Fixed effects:
```

Multiple degree of freedom comparisons

As with 1m and g1m models, you can compare the two 1mer models using the anova function.

```
> anova(Norm1, Norm2)
Data: Exam
Models:
Norm1: normexam ~ 1 + (1 | school)
Norm2: normexam ~ standLRT + (1 | school)

Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
Norm1 3 10826 10844 -5410 10820
Norm2 4 9143 9169 -4568 9135 1684 1 <2e-16
```

Random slopes

Add a random effect of students' standardized test scores as well. Now in addition to estimating the distribution of intercepts across schools, we also estimate the distribution of the slope of exam on standardized test.

```
> Norm3 <- lmer(normexam~standLRT + (standLRT|school), data=Exam,</pre>
               REML = FALSE)
> summary(Norm3)
Linear mixed model fit by maximum likelihood ['lmerMod']
Formula: normexam ~ standLRT + (standLRT | school)
  Data: Exam
    AIC
         BIC logLik deviance df.resid
   9108 9146 -4548 9096
Scaled residuals:
  Min 1Q Median 3Q Max
-3.813 -0.634 0.033 0.673 3.452
Random effects:
Groups Name
                  Variance Std.Dev. Corr
school (Intercept) 0.0899 0.300
        standLRT 0.0141 0.119 0.51
Residual
                0.5552 0.745
Number of obs: 3958, groups: school
```

Test the significance of the random slope

To test the significance of a random slope just compare models with and without the random slope term

```
> anova(Norm2, Norm3)
Data: Exam
Models:
Norm2: normexam ~ standLRT + (1 | school)
Norm3: normexam ~ standLRT + (standLRT | school)
Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
Norm2 4 9143 9169 -4568 9135
Norm3 6 9108 9146 -4548 9096 39 2 0.0000000035
```

Exercise 3: multilevel modeling

Use the dataset, bh1996: data(bh1996, package="multilevel") From the data documentation:

Variables are Cohesion (COHES), Leadership Climate (LEAD), Well-Being (WBEING) and Work Hours (HRS). Each of these variables has two variants - a group mean version that replicates each group mean for every individual, and a within-group version where the group mean is subtracted from each individual response. The group mean version is designated with a G. (e.g., G.HRS), and the within-group version is designated with a W. (e.g., W.HRS).

- Create a null model predicting wellbeing ("WBEING")
- Calculate the ICC for your null model
- Run a second multi-level model that adds two individual-level predictors, average number of hours worked ("HRS") and leadership skills ("LEAD") to the model and interpret your output.
- Now, add a random effect of average number of hours worked ("HRS") to the model and interpret your output. Test the significance of this random term.

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Exercise 0 prototype

Use the *states.rds* data set. states <- readRDS("dataSets/states.rds")

Fit a model predicting energy consumed per capita (energy) from the percentage of residents living in metropolitan areas (metro). Be sure to

Examine/plot the data before fitting the model

```
states.en.met <- subset(states, select = c("metro", "energy"))
summary(states.en.met)
plot(states.en.met)
cor(states.en.met, use="pairwise")</pre>
```

Print and interpret the model summary
mod.en.met <- lm(energy ~ metro, data = states)
summary(mod.en.met)</pre>

• plot the model to look for deviations from modeling assumptions plot(mod.en.met)

Select one or more additional predictors to add to your model and repeat steps 1-3. Is this model significantly better than the model with *metro* as the only predictor?

```
states.en.met.pop.wst <- subset(states, select = c("energy", "metro", "pop", "waste"))
summary(states.en.met.pop.wst)
plot(states.en.met.pop.wst)
cor(states.en.met.pop.wst, use = "pairwise")
mod.en.met.pop.waste <- lm(energy ~ metro + pop + waste, data = states)
summary(mod.en.met.pop.waste)</pre>
```

Exercise 1: prototype

Use the states data set.

• Add on to the regression equation that you created in exercise 1 by generating an interaction term and testing the interaction.

```
mod.en.metro.by.waste <- lm(energy ~ metro * waste, data = states)</pre>
```

Try adding a region to the model. Are there significant differences across the four regions?

```
mod.en.region <- lm(energy ~ metro * waste + region, data = states)
anova(mod.en.region)</pre>
```

Exercise 2 prototype

Use the NH11 data set that we loaded earlier. Note that the data is not perfectly clean and ready to be modeled. You will need to clean up at least some of the variables before fitting the model.

Use glm to conduct a logistic regression to predict ever worked (everwrk) using age (age_p) and marital status (r_{maritl}).

Predict the probability of working for each level of marital status.

```
library(effects)
data.frame(Effect("r_maritl", mod.wk.age.mar))
```

Exercise 3 prototype

```
Use the dataset, bh1996: data(bh1996, package="multilevel")
```

From the data documentation:

Variables are Cohesion (COHES), Leadership Climate (LEAD), Well-Being (WBEING) and Work Hours (HRS). Each of these variables has two variants - a group mean version that replicates each group mean for every individual, and a within-group version where the group mean is subtracted from each individual response. The group mean version is designated with a G. (e.g., G.HRS), and the within-group version is designated with a W. (e.g., W.HRS).

Note that the group identifier is named "GRP".

```
Create a null model predicting wellbeing ("WBEING") library(lme4) mod.grp0 <- lmer(WBEING ^{\circ} 1 + (1 | GRP), data = bh1996) summary(mod.grp0) => library(lme4) > mod.grp0 <- lmer(WBEING ^{\circ} 1 + (1 | GRP), data = bh1996) > summary(mod.grp0) Linear mixed model fit by REML ['ImerMod'] Formula: WBEING ^{\circ} 1 + (1 | GRP) Data: bh1996 REML criterion at convergence: 19347
```

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Wrap-up

Help us make this workshop better!

• Please take a moment to fill out a very short

feedback form

- These workshops exist for you tell us what you need!
- http://tinyurl.com/RstatisticsFeedback

Additional resources

- IQSS workshops: http://projects.iq.harvard.edu/rtc/filter_by/workshops
- IQSS statistical consulting: http://rtc.iq.harvard.edu
- Zelig
 - Website: http://gking.harvard.edu/zelig
 - Documentation: http://r.iq.harvard.edu/docs/zelig.pdf
- Ameila
 - Website: http://gking.harvard.edu/Amelia/
 - Documetation: http://r.iq.harvard.edu/docs/amelia/amelia.pdf