Regression Models in R

The Institute
for Quantitative Social Science
at Harvard University

Outline

Topic

Workshop description

- This is an intermediate/advanced R course
- Appropriate for those with basic knowledge of R
- This is not a statistics course!
- Learning objectives:
    - Learn the R formula interface
    - Specify factor contrasts to test specific hypotheses
    - Perform model comparisons
    - Run and interpret variety of regression models in R
    - Create and use imputed data sets in regression models

Materials and Setup

- Lab computer users:

  USERNAME dataclass
  PASSWORD dataclass

- Download materials from `http:j.mp/r-stats`
- Extract materials from RStatistics.zip (on lab machines *right-click -> WinZip -> Extract to here*) and move to your desktop.

Launch RStudio

- Open the RStudio program from the Windows start menu
- Open up today's R script
  - In RStudio, Go to File => Open Script
  - Locate and open the `Rstatistics.R` script in the Rstatistics folder on your desktop
- Go to Tools => Set working directory => To source file location (more on the working directory later)
- I encourage you to add your own notes to this file!

Set working directory

It is often helpful to start your R session by setting your working directory so you don't have to type the full path names to your data and other files

```
> # set the working directory
> # setwd("~/Desktop/Rstatistics")
> # setwd("C:/Users/dataclass/Desktop/Rstatistics")
>
```

You might also start by listing the files in your working directory

```
> getwd() # where am I?
[1] "/home/izahn/Documents/Work/IQSS/Classes/IQSS_Stats_Workshops/R/Rstatistics
> list.files("dataSets") # files in the dataSets folder
[1] "Exam.rds"           "NatHealth2008MI"    "NatHealth2011.rds"
[4] "states.dta"         "states.rds"
```

Load the states data

The *states.dta* data comes from `http://anawida.de/teach/SS14/anawida/4.linReg/data/states.dta.txt` and appears to have originally appeared in *Statistics with Stata* by Lawrence C. Hamilton.

```
> # read the states data
> states.data <- readRDS("dataSets/states.rds")
> #get labels
> states.info <- data.frame(attributes(states.data)[c("names", "var.labels")])
> #look at last few labels
> tail(states.info, 8)
     names                      var.labels
14    csat        Mean composite SAT score
15    vsat            Mean verbal SAT score
16    msat              Mean math SAT score
17 percent       % HS graduates taking SAT
18 expense Per pupil expenditures prim&sec
19  income Median household income, $1,000
20    high              % adults HS diploma
21 college         % adults college degree
```

Topic

Examine the data before fitting models

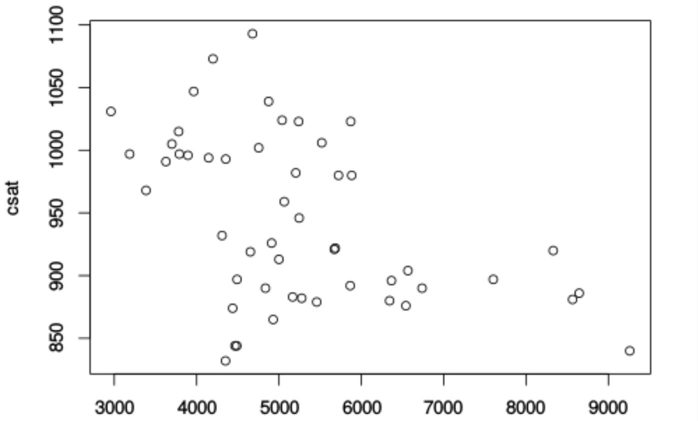
Start by examining the data to check for problems.

```
> # summary of expense and csat columns, all rows
> summary(states.data[ , c("expense", "csat")])
    expense          csat     
 Min.   :2960   Min.   : 832  
 1st Qu.:4352   1st Qu.: 888  
 Median :5000   Median : 926  
 Mean   :5236   Mean   : 944  
 3rd Qu.:5794   3rd Qu.: 997  
 Max.   :9259   Max.   :1093  
> # correlation between expense and csat
> cor(states.data[ , c("expense", "csat")])[1, 2]
[1] -0.466
```

Plot the data before fitting models

Plot the data to look for multivariate outliers, non-linear relationships etc.

```
# scatter plot of expense vs csat
plot(states.data[ , c("expense", "csat")])
```



Linear regression example

- Linear regression models can be fit with the `lm()` function
- For example, we can use `lm` to predict SAT scores based on per-pupal expenditures:

```
> # Fit our regression model
> sat.mod <- lm(csat ~ expense, # regression formula
+               data=states.data) # data set
> # Summarize and print the results
> summary(sat.mod) # show regression coefficients table

Call:
lm(formula = csat ~ expense, data = states.data)

Residuals:
    Min      1Q  Median      3Q     Max
-131.81  -38.08    5.61   37.85  136.50

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 1060.73244   32.70090   32.44  < 2e-16
expense       -0.02228    0.00604   -3.69  0.00056

Residual standard error: 59.8 on 49 degrees of freedom
Multiple R-squared:  0.217,  Adjusted R-squared:  0.201
```

Why is the association between expense and SAT scores *negative*?

Many people find it surprising that the per-capita expenditure on students is negatively related to SAT scores. The beauty of multiple regression is that we can try to pull these apart. What would the association between expense and SAT scores be if there were no difference among the states in the percentage of students taking the SAT?

```
> summary(lm(csat ~ expense + percent, data = states.data))

Call:
lm(formula = csat ~ expense + percent, data = states.data)

Residuals:
   Min     1Q Median     3Q    Max
-62.92 -24.32   1.74  15.50  75.62

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 989.8074    18.3958   53.81  < 2e-16
expense       0.0086     0.0042    2.05    0.046
percent      -2.5377     0.2249  -11.28  4.2e-15
```

The lm class and methods

OK, we fit our model. Now what?

- Examine the model object:

```
> class(sat.mod)
[1] "lm"
> names(sat.mod)
 [1] "coefficients"  "residuals"     "effects"       "rank"
 [5] "fitted.values" "assign"        "qr"            "df.residual"
 [9] "xlevels"       "call"          "terms"         "model"
> methods(class = class(sat.mod))[1:9]
[1] "add1.lm"          "alias.lm"           "anova.lm"
[4] "case.names.lm"    "confint.lm"         "cooks.distance.lm"
[7] "deviance.lm"      "dfbeta.lm"          "dfbetas.lm"
```

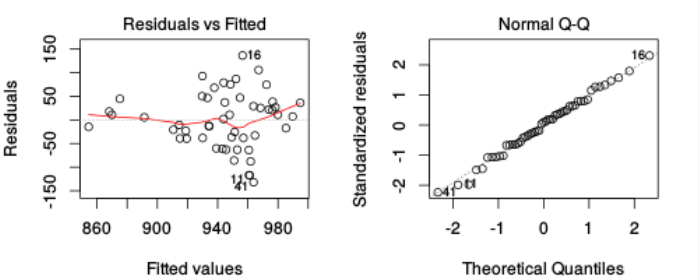
- Use function methods to get more information about the fit

```
> confint(sat.mod)
                2.5 %    97.5 %
(Intercept) 995.0175 1126.4474
expense      -0.0344   -0.0101
> # hist(residuals(sat.mod))
>
```

Linear Regression Assumptions

- Ordinary least squares regression relies on several assumptions, including that the residuals are normally distributed and homoscedastic, the errors are independent and the relationships are linear.
- Investigate these assumptions visually by plotting your model:

```
par(mar = c(4, 4, 2, 2), mfrow = c(1, 2)) #optional
plot(sat.mod, which = c(1, 2)) # "which" argument optional
```



Comparing models

Do congressional voting patterns predict SAT scores over and above expense? Fit two models and compare them:

```
> # fit another model, adding house and senate as predictors
> sat.voting.mod <-  lm(csat ~ expense + house + senate,
+                       data = na.omit(states.data))
> sat.mod <- update(sat.mod, data=na.omit(states.data))
> # compare using the anova() function
> anova(sat.mod, sat.voting.mod)
Analysis of Variance Table

Model 1: csat ~ expense
Model 2: csat ~ expense + house + senate
  Res.Df    RSS Df Sum of Sq    F Pr(>F)
1     46 169050
2     44 149284  2     19766 2.91  0.065
> coef(summary(sat.voting.mod))
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 1082.9344   38.63381   28.03 1.07e-29
expense       -0.0187    0.00969   -1.93 6.00e-02
house         -1.4424    0.60048   -2.40 2.06e-02
senate         0.4982    0.51356    0.97 3.37e-01
```

Exercise 0: least squares regression

Use the *states.rds* data set. Fit a model predicting energy consumed per capita (energy) from the percentage of residents living in metropolitan areas (metro). Be sure to

1. Examine/plot the data before fitting the model
2. Print and interpret the model `summary`
3. `plot` the model to look for deviations from modeling assumptions

Select one or more additional predictors to add to your model and repeat steps 1-3. Is this model significantly better than the model with *metro* as the only predictor?

Topic

Modeling interactions

Interactions allow us assess the extent to which the association between one predictor and the outcome depends on a second predictor. For example: Does the association between expense and SAT scores depend on the median income in the state?

```
>   #Add the interaction to the model
> sat.expense.by.percent <- lm(csat ~ expense*income,
+                              data=states.data)
> #Show the results
>   coef(summary(sat.expense.by.percent)) # show regression coefficients table
                 Estimate Std. Error t value        Pr(>|t|)
(Intercept)    1380.36423 172.086252    8.02 0.000000000237
expense          -0.06384   0.032701   -1.95 0.056878369245
income          -10.49785   4.991463   -2.10 0.040832525071
expense:income    0.00138   0.000864    1.60 0.115539488253
```

Regression with categorical predictors

Let's try to predict SAT scores from region, a categorical variable. Note that you must make sure R does not think your categorical variable is numeric.

```
> # make sure R knows region is categorical
> str(states.data$region)
 Factor w/ 4 levels "West","N. East",..: 3 1 1 3 1 1 2 3 NA 3 ...
> states.data$region <- factor(states.data$region)
> #Add region to the model
> sat.region <- lm(csat ~ region,
+                  data=states.data)
> #Show the results
> coef(summary(sat.region)) # show regression coefficients table
             Estimate Std. Error t value Pr(>|t|)
(Intercept)     946.3       14.8  63.958 1.35e-46
regionN. East   -56.8       23.1  -2.453 1.80e-02
regionSouth     -16.3       19.9  -0.819 4.17e-01
regionMidwest    63.8       21.4   2.986 4.51e-03
> anova(sat.region) # show ANOVA table
Analysis of Variance Table

Response: csat
          Df Sum Sq Mean Sq F value   Pr(>F)
region     3  82049   27350    9.61 0.000049
Residuals 46 130912    2846
```

Setting factor reference groups and contrasts

In the previous example we use the default contrasts for region. The default in R is treatment contrasts, with the first level as the reference. We can change the reference group or use another coding scheme using the `C` function.

```
> # print default contrasts
> contrasts(states.data$region)
        N. East South Midwest
West          0     0       0
N. East       1     0       0
South         0     1       0
Midwest       0     0       1
> # change the reference group
> coef(summary(lm(csat ~ C(region, base=4),
+                 data=states.data)))
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)           1010.1       15.4   65.59 4.30e-47
C(region, base = 4)1   -63.8       21.4   -2.99 4.51e-03
C(region, base = 4)2  -120.5       23.5   -5.12 5.80e-06
C(region, base = 4)3   -80.1       20.4   -3.93 2.83e-04
> # change the coding scheme
> coef(summary(lm(csat ~ C(region, contr.helmert),
+                 data=states.data)))
                         Estimate Std. Error t value Pr(>|t|)
```

Exercise 1: interactions and factors

Use the states data set.

1. Add on to the regression equation that you created in exercise 1 by generating an interaction term and testing the interaction.
2. Try adding a categorical variable to your regression (remember, it will need to be dummy coded). You could use region or, or generate a new categorical variable from one of the continuous variables in the dataset.
3. Are there significant differences across the four regions?

Topic

Logistic regression

This far we have used the `lm` function to fit our regression models. `lm` is great, but limited–in particular it only fits models for continuous dependent variables. For categorical dependent variables we can use the `glm()` function. For these models we will use a different dataset, drawn from the National Health Interview Survey. From the CDC website:

> *The National Health Interview Survey (NHIS) has monitored the health of the nation since 1957. NHIS data on a broad range of health topics are collected through personal household interviews. For over 50 years, the U.S. Census Bureau has been the data collection agent for the National Health Interview Survey. Survey results have been instrumental in providing data to track health status, health care access, and progress toward achieving national health objectives.*

Load the National Health Interview Survey data:

```
> NH11 <- readRDS("dataSets/NatHealth2011.rds")
> labs <- attributes(NH11)$labels
>
```

Logistic regression example

Let's predict the probability of being diagnosed with hypertension based on age, sex, sleep, and bmi

```
> str(NH11$hypev) # check stucture of hypev
 Factor w/ 5 levels "1 Yes","2 No",..: 2 2 1 2 2 1 2 2 1 2 ...
> levels(NH11$hypev) # check levels of hypev
[1] "1 Yes"              "2 No"               "7 Refused"
[4] "8 Not ascertained" "9 Don't know"
> # collapse all missing values to NA
> NH11$hypev <- factor(NH11$hypev, levels=c("2 No", "1 Yes"))
> # run our regression model
> hyp.out <- glm(hypev~age_p+sex+sleep+bmi,
+               data=NH11, family="binomial")
> coef(summary(hyp.out))
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -4.26947   0.056495  -75.57 0.00e+00
age_p        0.06070   0.000823   73.78 0.00e+00
sex2 Female -0.14403   0.026798   -5.37 7.68e-08
sleep       -0.00704   0.001640   -4.29 1.78e-05
bmi          0.01857   0.000951   19.53 6.49e-85
```

Logistic regression coefficients

Generalized linear models use link functions, so raw coefficients are difficult to interpret. For example, the age coefficient of .06 in the previous model tells us that for every one unit increase in age, the log odds of hypertension diagnosis increases by 0.06. Since most of us are not used to thinking in log odds this is not too helpful!

One solution is to transform the coefficients to make them easier to interpret

```
> hyp.out.tab <- coef(summary(hyp.out))
> hyp.out.tab[, "Estimate"] <- exp(coef(hyp.out))
> hyp.out.tab
            Estimate Std. Error z value Pr(>|z|)
(Intercept)    0.014   0.056495  -75.57 0.00e+00
age_p          1.063   0.000823   73.78 0.00e+00
sex2 Female    0.866   0.026798   -5.37 7.68e-08
sleep          0.993   0.001640   -4.29 1.78e-05
bmi            1.019   0.000951   19.53 6.49e-85
```

Generating predicted values

In addition to transforming the log-odds produced by `glm` to odds, we can use the `predict()` function to make direct statements about the predictors in our model. For example, we can ask "How much more likely is a 63 year old female to have hypertension compared to a 33 year old female?".

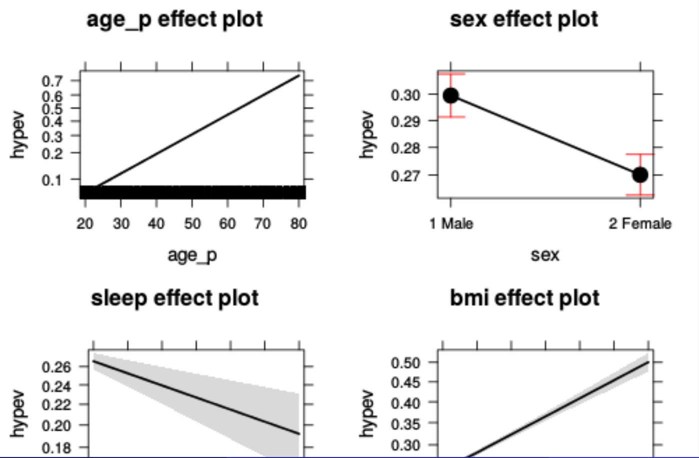
```
> # Create a dataset with predictors set at desired levels
> predDat <- with(NH11,
+                 expand.grid(age_p = c(33, 63),
+                             sex = "2 Female",
+                             bmi = mean(bmi, na.rm = TRUE),
+                             sleep = mean(sleep, na.rm = TRUE)))
> # predict hypertension at those levels
> cbind(predDat, predict(hyp.out, type = "response",
+                        se.fit = TRUE, interval="confidence",
+                        newdata = predDat))
  age_p      sex  bmi sleep   fit  se.fit residual.scale
1    33 2 Female 29.9  7.86 0.129 0.00285              1
2    63 2 Female 29.9  7.86 0.478 0.00482              1
```

This tells us that a 33 year old female has a 13% probability of having been diagnosed with hypertension, while and 63 year old female has a 48% probability of having been diagnosed.

Packages for computing and graphing predicted values

Instead of doing all this ourselves, we can use the effects package to compute quantities of interest for us (cf. the Zelig package).

```
library(effects)
plot(allEffects(hyp.out))
```



Exercise 2: logistic regression

Use the NH11 data set.

1. Use glm to conduct a logistic regression to predict ever worked (everwrk) using age (age_{p}) and marital status (r_{maritl}).
2. Predict the probability of working for each level of marital status.

Note that the data is not perfectly clean and ready to be modeled. You will need to clean up at leas some of the variables before fitting the model.

Topic

Multilevel modeling overview

- Multi-level (AKA hierarchical) models are a type of mixed-effects models
- Used to model variation due to group membership where the goal is to generalize to a population of groups
- Can model different intercepts and/or slopes for each group
- Mixed-effecs models include two types of predictors: fixed-effects and random effects

  Fixed-effects observed levels are of direct interest (.e.g, sex, political party. . .)

  Random-effects observed levels not of direct interest: goal is to make inferences to a population represented by observed levels
- In R the lme4 package is the most popular for mixed effects models
  - Use the `lmer` function for liner mixed models, `glmer` for generalized mixed models

```
> library(lme4)
>
```

The Exam data

The Exam data set contans exam scores of 4,059 students from 65 schools in Inner London. The variable names are as follows:

- **school** School ID - a factor.
- **normexam** Normalized exam score.
- **schgend** School gender - a factor. Levels are 'mixed', 'boys', and 'girls'.
- **schavg** School average of intake score.
- **vr** Student level Verbal Reasoning (VR) score band at intake - a factor. Levels are 'bottom 25%', 'mid 50%', and 'top 25%'.
- **intake** Band of student's intake score - a factor. Levels are 'bottom 25%', 'mid 50%' and 'top 25%'./
- **standLRT** Standardised LR test score.
- **sex** Sex of the student - levels are 'F' and 'M'.
- **type** School type - levels are 'Mxd' and 'Sngl'.
- **student** Student id (within school) - a factor

```
> Exam <- readRDS("dataSets/Exam.rds")
>
```

The null model and ICC

As a preliminary step it is often useful to partition the variance in the dependent variable into the various levels. This can be accomplished by running a null model (i.e., a model with a random effects grouping structure, but no fixed-effects predictors).

```
> # null model, grouping by school but not fixed effects.
> Norm1 <-lmer(normexam ~ 1 + (1|school),
+                 data=Exam, REML = FALSE)
> summary(Norm1)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: normexam ~ 1 + (1 | school)
   Data: Exam

     AIC      BIC   logLik deviance df.resid
   10826    10844    -5410    10820     3984

Scaled residuals:
   Min      1Q  Median      3Q     Max
-3.902 -0.646  0.003  0.698  3.636

Random effects:
 Groups   Name        Variance Std.Dev.
 school   (Intercept) 0.169    0.412
 Residual             0.848    0.921
```

Adding fixed-effects predictors

Predict exam scores from student's standardized tests scores

```
> Norm2 <-lmer(normexam~standLRT + (1|school),
+              data=Exam,
+              REML = FALSE)
> summary(Norm2)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: normexam ~ standLRT + (1 | school)
   Data: Exam

     AIC      BIC   logLik deviance df.resid
    9143     9168    -4568     9135     3954

Scaled residuals:
   Min      1Q Median     3Q    Max
-3.700 -0.625  0.024  0.678  3.262

Random effects:
 Groups   Name        Variance Std.Dev.
 school   (Intercept) 0.0919   0.303
 Residual             0.5670   0.753
Number of obs: 3958, groups: school, 65

Fixed effects:
            Estimate Std. Error t value
```

Multiple degree of freedom comparisons

As with `lm` and `glm` models, you can compare the two `lmer` models using the `anova` function.

```
> anova(Norm1, Norm2)
Data: Exam
Models:
Norm1: normexam ~ 1 + (1 | school)
Norm2: normexam ~ standLRT + (1 | school)
      Df   AIC   BIC logLik deviance Chisq Chi Df Pr(>Chisq)
Norm1  3 10826 10844  -5410    10820
Norm2  4  9143  9169  -4568     9135  1684      1     <2e-16
```

Random slopes

Add a random effect of students' standardized test scores as well. Now in addition to estimating the distribution of intercepts across schools, we also estimate the distribution of the slope of exam on standardized test.

```
> Norm3 <- lmer(normexam~standLRT + (standLRT|school), data=Exam,
+               REML = FALSE)
> summary(Norm3)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: normexam ~ standLRT + (standLRT | school)
   Data: Exam

     AIC      BIC   logLik deviance df.resid
    9108     9146    -4548     9096     3952

Scaled residuals:
   Min     1Q Median     3Q    Max
-3.813 -0.634  0.033  0.673  3.452

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 school   (Intercept) 0.0899   0.300
          standLRT    0.0141   0.119    0.51
 Residual             0.5552   0.745
Number of obs: 3958, groups: school, 65
```

Test the significance of the random slope

To test the significance of a random slope just compare models with and without the random slope term

```
> anova(Norm2, Norm3)
Data: Exam
Models:
Norm2: normexam ~ standLRT + (1 | school)
Norm3: normexam ~ standLRT + (standLRT | school)
      Df  AIC  BIC logLik deviance Chisq Chi Df   Pr(>Chisq)
Norm2  4 9143 9169  -4568     9135
Norm3  6 9108 9146  -4548     9096    39      2 0.0000000035
```

Exercise 3: multilevel modeling

Use the dataset, bh1996:

```
data(bh1996, package="multilevel")
```

From the data documentation:

> *Variables are Cohesion (COHES), Leadership Climate (LEAD), Well-Being (WBEING) and Work Hours (HRS). Each of these variables has two variants - a group mean version that replicates each group mean for every individual, and a within-group version where the group mean is subtracted from each individual response. The group mean version is designated with a G. (e.g., G.HRS), and the within-group version is designated with a W. (e.g., W.HRS).*

1. Create a null model predicting wellbeing ("WBEING")
2. Calculate the ICC for your null model
3. Run a second multi-level model that adds two individual-level predictors, average number of hours worked ("HRS") and leadership skills ("LEAD") to the model and interpret your output.
4. Now, add a random effect of average number of hours worked ("HRS") to the model and interpret your output. Test the significance of this random term.
5. Finally, add a group-level term, workplace cohesion ("G.COHES") to the

Topic

Multiple imputation

- Majority of datasets contain missing data
- Produces a variety of problems and limitations to data analysis
- Multiple imputation (MI) generates multiple, complete datasets that contain estimations of missing data points

Multiple imputation

Earlier we wanted to compare a model predicting bmi from demographic variables to a model including demographics and substantive predictors. We omitted missing data so that we could fit both models to the same data. That is a common practice, but it has many problems (which we unfortunately don't have time to discuss in detail). A popular solution is to use multiple imputation to fill in the missing values with reasonable placeholders.

MI is typically thought of as involving three steps:

- Selection of imputation model
- Generation of imputed datasets
- Combining results across imputed datasets

There are a number of packages for doing this in R: we will use the Amelia package because it is powerful, fast, and easy to use. You can refer to the Amelia documentation for more information about its imputation procedures: `http:r.iq.harvard.edu/docs/amelia/amelia.pdf`

Creating imputed data sets

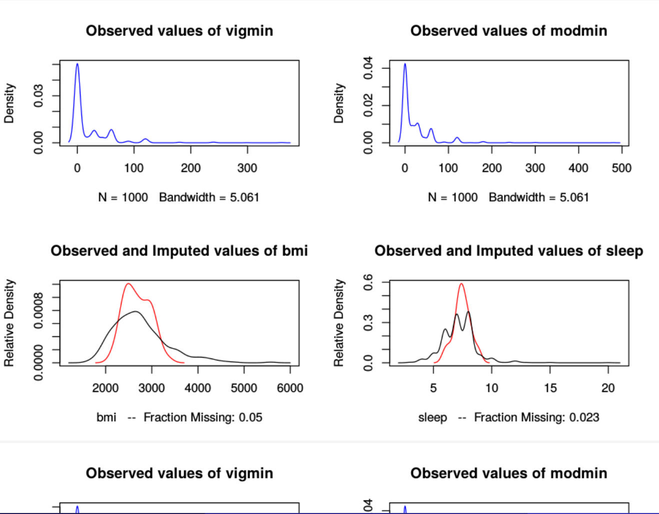
We're going to create several datasets to look at a model predicting the number of days of work missed/year (wkdayr)

```
> # load the Amelia package
> library(Amelia)
> # help(package="Amelia")
> # load a smaller version of NH
> NH08.mi <- readRDS("dataSets/NatHealth2008MI")
> # generate five imputed data sets
> amelia.log <- capture.output( # suppress amelia's chattiness
+   NatHealth.MI <- amelia(NH08.mi,
+                          m=5,
+                          idvars=c("id")))
>
```

Checking imputed values

Compare imputed values to observed values

```
plot(NatHealth.MI, which.vars=9:12)
```

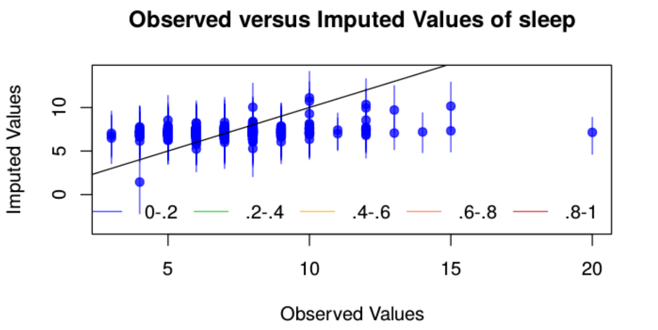


Checking imputed values: overimputation

Overimputation strategy:

- Treat every observed value as if it was missing
- Impute many values for that observed value
- Examine the correspondence between imputed and observed values

```
overimpute(NatHealth.MI, var="sleep")
```



Using imputed data sets in regression models

Zelig makes it very easy to use imputed data sets – just point to the list of imputed data sets in the `data` argument

```
> library(Zelig)
> nhImp.out <- zelig(wkdayr ~ cigsday + modmin + sleep, model = "ls",
+                    data = NatHealth.MI$imputations, cite = FALSE)
>
> coef(summary(nhImp.out))
              Value Std. Error t-stat p-value
(Intercept) -6.0745     5.6803  -1.07  0.2963
cigsday      0.0252     0.1326   0.19  0.8496
modmin      -0.0318     0.0227  -1.40  0.1610
sleep        1.7233     0.8624   2.00  0.0669
```

For separate results, use print(summary(x), subset = i:j).

Exercise 2: multiple imputation

1. Using Amelia, generate 5 imputed versions of the Exam dataset. Make sure you tell Amelia which variables are nominal, and that school is the id variable.
2. Create plots that compare imputed values to observed values
3. Overimpute the variable "schavg"

Topic

Help us make this workshop better!

- Please take a moment to fill out a very short

feedback form

- These workshops exist for you – tell us what you need!
- `http:tinyurl.com/RstatisticsFeedback`

Additional resources

- IQSS workshops: `http:projects.iq.harvard.edu/rtc/filter_by/workshops`
- IQSS statistical consulting: `http:rtc.iq.harvard.edu`
- Zelig
  - Website: `http:gking.harvard.edu/zelig`
  - Documentation: `http:r.iq.harvard.edu/docs/zelig.pdf`
- Ameila
  - Website: `http:gking.harvard.edu/Amelia/`
  - Documetation: `http:r.iq.harvard.edu/docs/amelia/amelia.pdf`