EE2111A Activity Sheet - Week 4 Studio 1

Start	Duration	Activity
0:00	30 mins	Briefing
0:30	60 mins	Steady state solution of AC circuit
		Activity #1(a): Analytical method
		Activity #1(b): CircuitLab simulation
1:30	30 mins	Activity #2: Frequency dependence of response
2:00	30 mins	Activity #3: RLC circuit

Learning Objectives

To be able to

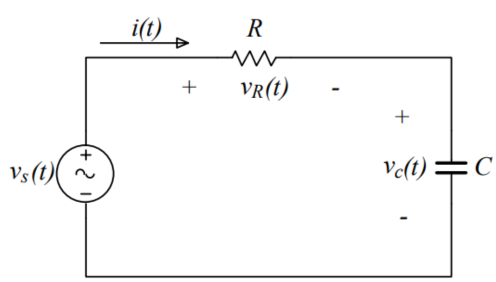
- write time-domain KVL or KCL equation, and
- find the steady-state solution

of an AC circuit driven by sinusoidal input.

Activity #1(a) - RC circuit: Steady state solution (analytical method)

Group size: This is an individual activity.

I. KVL in time domain



- Write the KVL equation around the loop with capacitor voltage v_C as the unknown variable. It will be a non-homogeneous first-order ordinary differential equation (ODE).

II. Steady state solution

You will not be asked in this module to solve the ODE analytically. However, to give you an idea of how it can be done, the steady state solution (particular solution of ODE) is given. You are required use the solution given to complete this part of the activity.

If

$$v_S(t) = V_m \sin(\omega t),$$

then the KVL equation is:

$$\tau \frac{dv_c}{dt} + v_c = V_m \sin(\omega t), \tag{1}$$

Check whether you derived the same equation in Step I.

where, $\tau = R \times C$.

In the steady state, the capacitor voltage is also a sine function of the same frequency ω, but with unknown amplitude A and phase ϕ. So, one can express the steady state capacitor voltage as

$$v_c(t) = A \sin(\omega t + \phi)$$

and, therefore,

$$\frac{dv_c}{dt} = \omega A \cos(\omega t + \phi).$$

Substituting these in equation 1,

$$\omega \tau A \cos(\omega t + \phi) + A \sin(\omega t + \phi) = V_m \sin(\omega t).$$

Expanding $\cos(\omega t + \phi)$ and $\sin(\omega t + \phi)$ using trigonometric identity and, then, rearranging the terms,

$$(\omega \tau A \cos\phi + A \sin\phi) \cos(\omega t) + (A \cos\phi - \omega \tau A \sin\phi) \sin(\omega t) - V_m \sin(\omega t) = 0.$$

This is true if and only if the following two conditions hold:

$$\begin{aligned} A \sin\phi + \omega \tau A \cos\phi &= 0, \\ A \cos\phi - \omega \tau A \sin\phi &= V_m. \end{aligned}$$

We get the value of ϕ from the first of these two conditions:

$$\phi = -\tan^{-1}(\omega \tau). \tag{2}$$

With the value of ϕ is known, we can find A using the second condition:

$$A = \frac{1}{(\cos\phi - \omega \tau \sin\phi)} V_m. \tag{3}$$

To do:

1. Use the formulae given in equations 2 and 3 to determine the steady state capacitor voltage if
   - R: 100 Ω,
   - C: 10 μF,
   - Source voltage: 10 V sine wave of frequency 1000 Hz.

Can you find, using the same approach, the steady state solution of a series RL or series RLC driven by sinusoidal voltage?

Activity #1(b) - CircuitLab simulation

Group size: This is an individual activity.

Use CircuitLab simulator for time-domain simulation of the circuit in Activity #1(a). Determine from the simulation plots

(i) Amplitude of the steady state capacitor voltage $v_C(t)$

(ii) Phase shift of $v_C(t)$ with respect to $v_S(t)$.

Compare the values obtained from simulation with those obtained analytically in Activity #1(a).

Activity #2 - Frequency dependence of response

Group size: This is an individual activity.

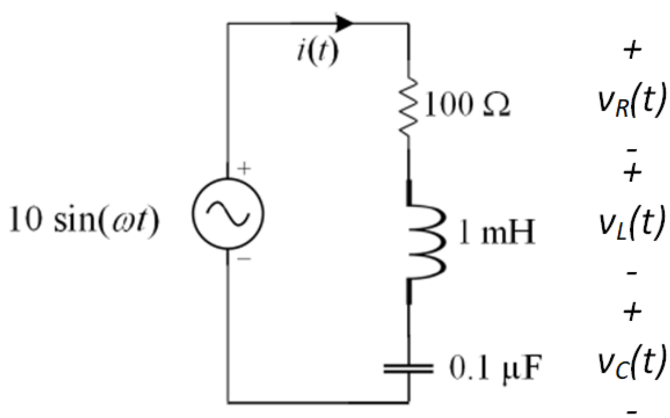
You might have noticed in Activity #1(A) that the expressions for A and ϕ include ω. The response of ac circuit containing energy-storing component(s) varies if the frequency is varied. In this activity, you will verify that through CircuitLab simulation.

1. Use the same RC circuit used in Activity #1.
2. You have already found the values of A and ϕ with 1 kHz source voltage. Simulate with a few different frequencies, *e.g.*, 500 Hz, and 2 kHz. For each case, determine the values of A and ϕ.
3. Plot amplitude versus frequency and phase shift versus frequency.

(OPTIONAL) Repeat this experiment with 0.1 H inductor in place of the 10 μF capacitor.

Activity #3 - RLC circuit

Group size: This is an individual activity.



1. Write the KVL equation of the circuit above in terms of the capacitor voltage v_c. It will be a non-homogeneous second-order ODE in term of v_c.
2. (**OPTIONAL**) Using CircuitLab, determine the amplitude, and the phase shift (with respect to the source voltage) of
   (a) $v_R(t)$,
   (b) $v_L(t)$,
   (c) $v_C(t)$, and
   (d) $i(t)$.

   if $\omega = 10^5$ rad/s.

eLogbook:

A. Activity #1(a):

- Part I: derivation of the non-homogeneous ODE
- Part II: Values of the amplitude and phase shift obtained, and the time-domain expression describing $v_c(t)$.

B. Activity #1(b):

- Graphs of $v_S(t)$, $v_c(t)$ and $I(t)$ from the simulation
- Values of the amplitude and phase obtained from the simulation
- Comparison between results obtained from the simulation and results obtained analytically.

C. Activity #2:

- The plots shown amplitude versus frequency and phase versus frequency of the RC circuit.
- If RL circuit is simulated, amplitude versus frequency plot and phase versus frequency plot for the RL circuit.

D. Activity #3:

- Graphs of $v_S(t)$, $v_R(t)$, $v_L(t)$, $v_C(t)$, and $i(t)$ obtained from the simulation.
- (**OPTIONAL**) Amplitude and phase shift with respect to the source voltage of
  (a) $v_R(t)$,
  (b) $v_L(t)$,
  (c) $v_C(t)$, and
  (d) $i(t)$.