EE2111A Week 6 Studio 1
Ni Qingqing

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Review: Wk5S2
Review: Activity 1
Review: Activity 2
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Intro
I. Potential Divider
Gain
II. RC Low-pass Filter
More on Gain
Frequency Response
Cutoff Frequency
III. RC High-pass Filter
IV. RC Bandpass Filter
Practice Qn. Tut

EE2111A Week 6 Studio 1
Filters & Tutorials

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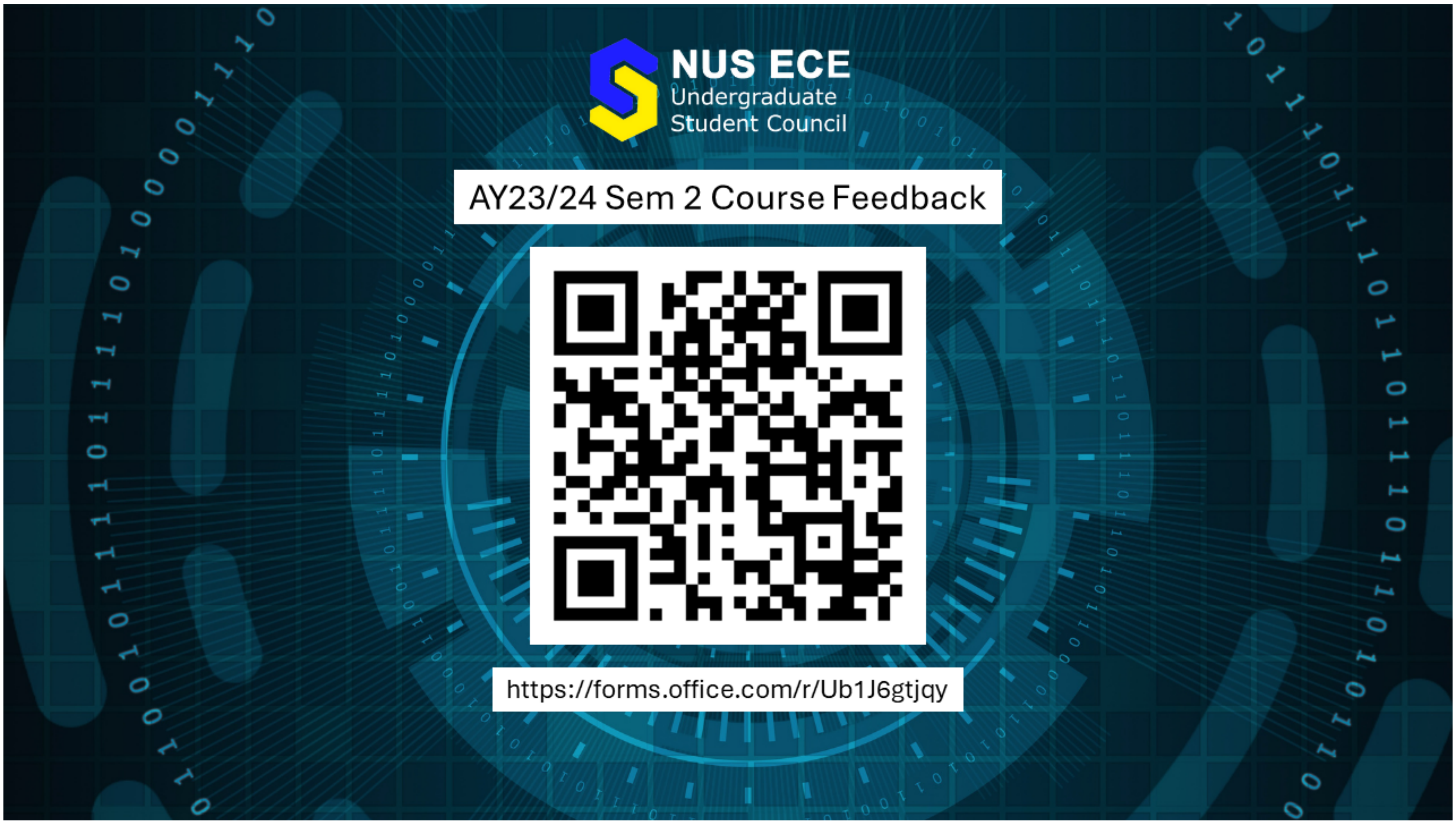
Year 1 EE Module Feedback Survey



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Learning Objectives

We will first revise the following:

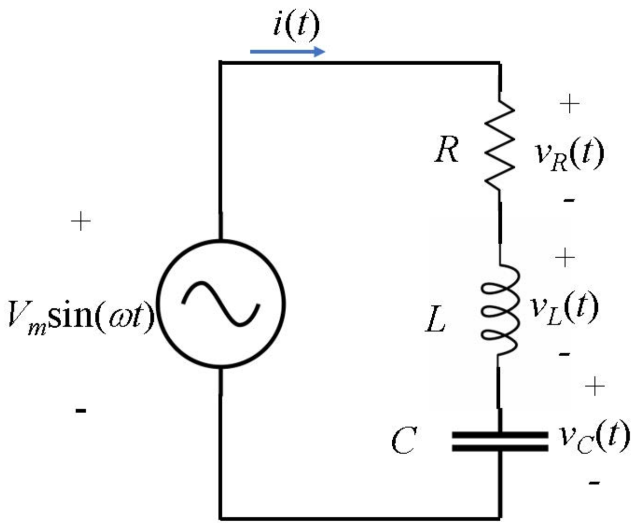
- KVL /KCL in AC Circuit.

At the end of this session, you should be able to:

- appreciate how RC and RL filters work.
- design RC/RL filters for given specifications.

Activity 1: KVL in a series RLC Circuit

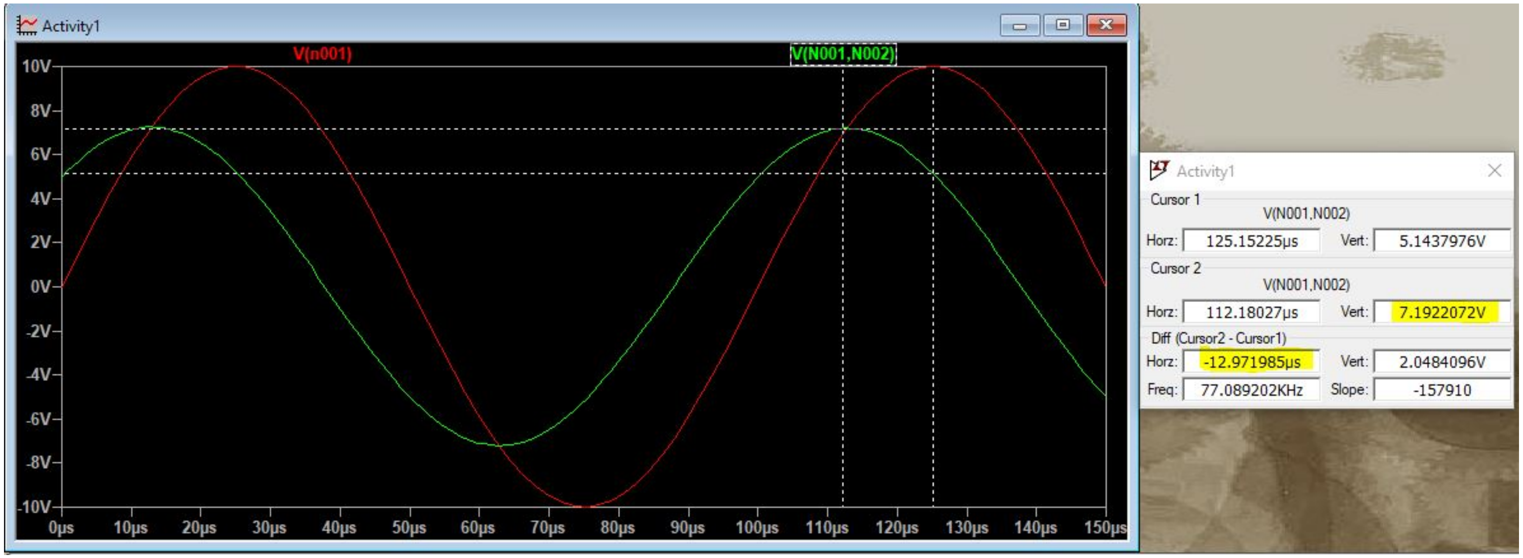
Revisit the steps to derive phasor expression of voltages:



- Measure the amplitude of $v_C(t)$, $v_L(t)$, $v_R(t)$, and their time shift w.r.t. $v_S(t)$
- Calculate phase shift w.r.t. $v_S(t)$ from time shift
- (Optional) Write down time domain expression $v_C(t)$, $v_L(t)$, $v_R(t)$, and $v_S(t)$
- Derive $\bar{V}_C$, $\bar{V}_L$, $\bar{V}_R$, and $\bar{V}_S$
- Verify: $\bar{V}_S = \bar{V}_C + \bar{V}_L + \bar{V}_R$

Activity 1: Derive $\bar{V}_R$

Result simulated with LTSpice, Red: Source Voltage, $V_S(t)$, Green: Voltage Cross component R/L/C.



- Measurement: $V_{R,Amp} = 7.19V$, $\Delta t = -12.97\mu s$
- Calculate:
  $\Delta\phi = \omega\Delta t = 2\pi f\Delta t = 2\pi \times 10kHz \times (-12.97)\mu s = -0.81$ rads
- Since V_S is the reference signal, it has a phase angle of 0. Comparing to V_S, V_R has a phase shift of $\Delta\phi = -0.81$ rads. Hence the time domain expression is:

$$v_R(t) = Asin(\omega t + \phi) = Asin(\omega t + (0 + \Delta\phi)) = 7.19sin(\omega t - 0.81)$$

- Derive: $\bar{V}_R = 7.19\angle -0.81$,

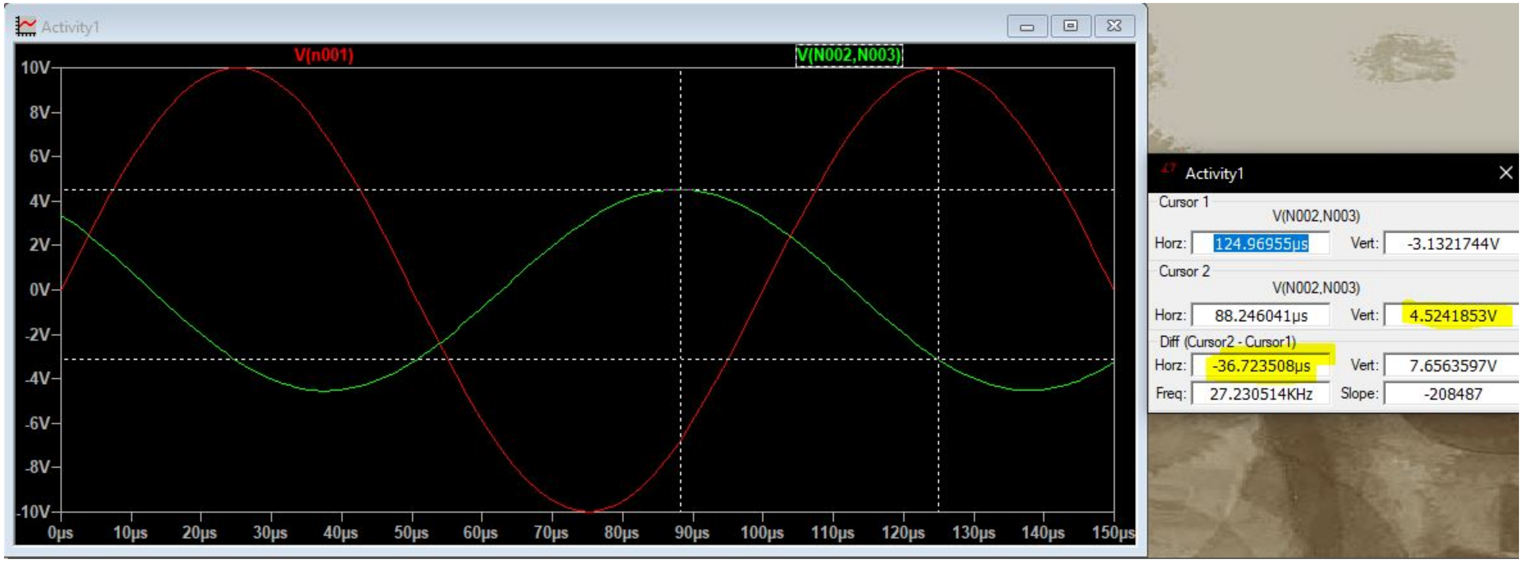
Activity 1: Derive $\bar{V}_C$

Result simulated with LTSpice, Red: Source Voltage, $V_s(t)$, Green: Voltage Cross component R/L/C.

- Measurement: $V_{C,Amp} = 11.45V$, $\Delta t = 12.97\mu s$
- Calculate: $\Delta\phi = \omega\Delta t = 2\pi f \Delta t = 2\pi \times 10kHz \times 12.97\mu s = 0.81$ rads
- Derive: $\bar{V}_C = 11.45\angle 0.81$,

Activity 1: Derive $\bar{V}_L$

Result simulated with LTSpice, Red: Source Voltage, $V_S(t)$, Green: Voltage Cross component R/L/C.



- Measurement: $V_{L,Amp} = 4.52V$, $\Delta t = -36.72\mu s$
- Calculate:
  $\Delta\phi = \omega\Delta t = 2\pi f \Delta t = 2\pi \times 10kHz \times (-36.72)\mu s = -2.31$ rads
- Derive: $\bar{V}_L = 4.52\angle - 2.31$,

Activity 1: Verify KVL

To verify KVL, we want to show that $\bar{V}_S = \bar{V}_R + \bar{V}_C + \bar{V}_L$.

- $\bar{V}_S = 10\angle 0$ (Why 0?)
- $\bar{V}_R = 7.19\angle -0.81$
- $\bar{V}_C = 11.45\angle 0.81$
- $\bar{V}_L = 4.52\angle -2.31$
- Can we directly sum them up? You can do so with a calculator!
  - Method 1: convert polar form ($A\angle\phi$) into rectangular form ($a + jb$)

$$A\angle\phi = Acos\phi + jAsin\phi$$

  - Method 2: Set complex display as $A\angle\phi$ and sum up polar form directly with a calculator!
- $\bar{V}_R + \bar{V}_C + \bar{V}_L = 9.81\angle -0.024 \approx 10\angle 0$

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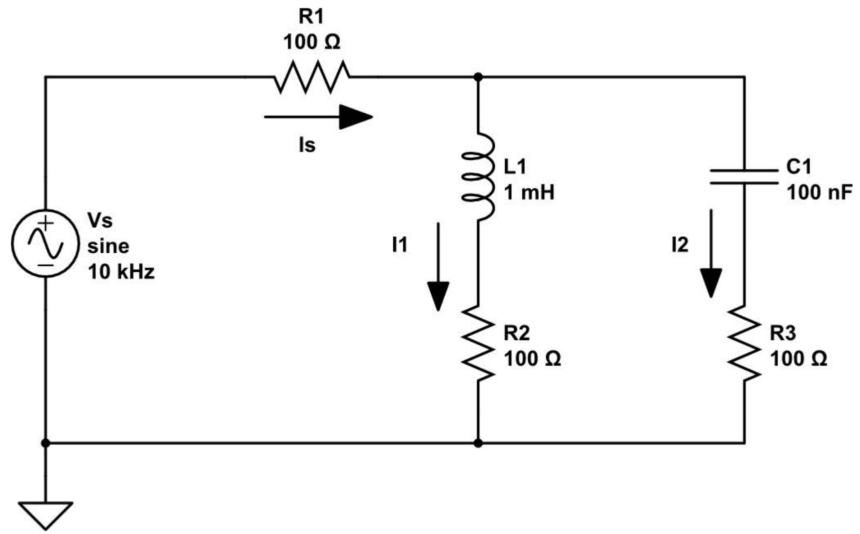
Activity 2

Revisit the steps to derive phasor expression of currents:



- Measure the amplitude of $v_{R1}(t)$, $v_{R2}(t)$, $v_{R3}(t)$, and their time shift w.r.t. $v_S(t)$
- Calculate phase shift w.r.t. $v_S(t)$ from time shift
- Calculate the amplitude of $i_S(t)$, $i_1(t)$, $i_2(t)$ with Ohm's Law
- Derive $\bar{I}_S$, $\bar{I}_1$, and $\bar{I}_2$
- Verify $\bar{I}_S = \bar{I}_1 + \bar{I}_2$

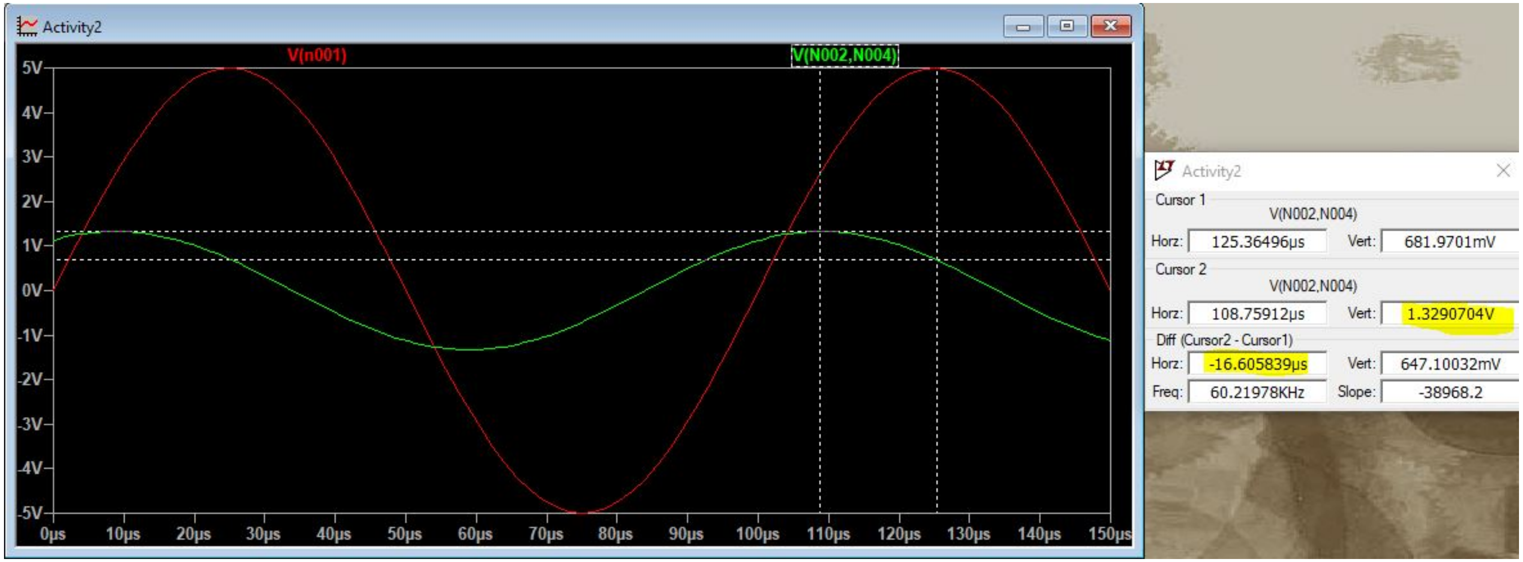
Activity 2: Derive $\bar{I}_1$

Result simulated with LTSpice, Red: Source Voltage, $V_S(t)$, Green: Voltage Cross component R/L/C.

- Measurement: $V_{R2,Amp} = 2.11V$, $\Delta t = 9.12\mu s$
- Calculate: $\Delta\phi = \omega\Delta t = 2\pi f \Delta t = 2\pi \times 10kHz \times 9.12\mu s = 0.57$ rads
- Calculate: $I_{1,Amp} = \frac{V_{R2,Amp}}{R2} = \frac{2.11}{100} = 0.0211$ A
- For a resistor, the voltage across it and the current through it is always in phase. Hence, compared to the source voltage, I_S has a phase shift of 0.57 rads!
- Derive: $\bar{I}_1 = 0.0211\angle 0.57$

Activity 2: Derive $\bar{I}_2$

Result simulated with LTSpice, Red: Source Voltage, $V_S(t)$, Green: Voltage Cross component R/L/C.



- Measurement: $V_{R3,Amp} = 1.33V$, $\Delta t = -16.61\mu s$
- Calculate:
  $\Delta\phi = \omega\Delta t = 2\pi f \Delta t = 2\pi \times 10kHz \times (-16.61)\mu s = -1.04$ rads
- Calculate: $I_{2,Amp} = \frac{V_{R3,Amp}}{R3} = \frac{1.33}{100} = 0.0133$ A
- For a resistor, the voltage across it and the current through it is always in phase. Hence, compared to the source voltage, I_S has a phase shift of -1.04 rads!
- Derive: $\bar{I}_2 = 0.0133\angle - 1.04$

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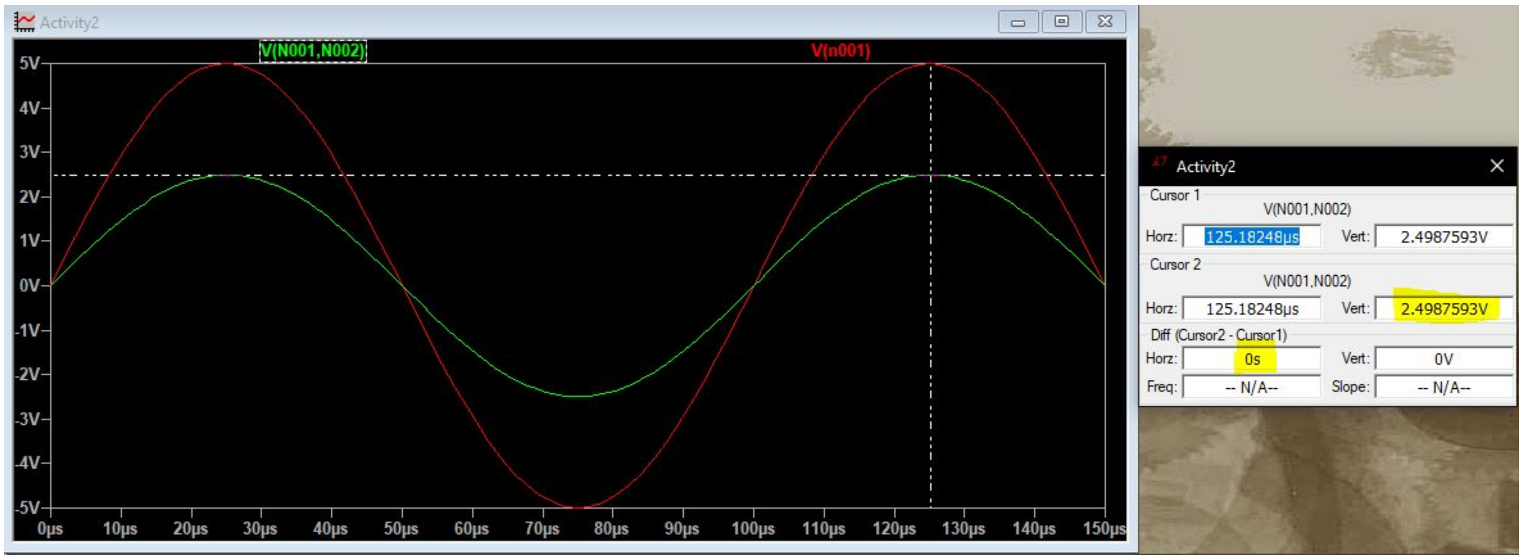
Activity 2: Derive $\bar{I_S}$

Result simulated with LTSpice, Red: Source Voltage, $V_S(t)$, Green: Voltage Cross component R/L/C.



- Measurement: $V_{R1,Amp} = 2.5V$, $\Delta t = 0\mu s$ (?)
- Calculate: $\Delta\phi = \omega\Delta t = 2\pi f\Delta t = 2\pi \times 10kHz \times 0\mu s = 0$ rads
- Calculate: $I_{S,Amp} = \frac{V_{R1,Amp}}{R1} = \frac{2.5}{100} = 0.025$ A
- For a resistor, the voltage across it and the current through it is always in phase. Hence, compared to the source voltage, I_S has a phase shift of 0 rads!
- Derive: $\bar{I_S} = 0.025\angle 0$,

Activity 2: Verify KCL

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To verify KCL, we want to show that $\bar{I}_S = \bar{I}_1 + \bar{I}_2$.

- $\bar{I}_S = 0.025\angle 0$
- $\bar{I}_1 = 0.0211\angle 0.57$
- $\bar{I}_2 = 0.0133\angle -1.04$
- $\bar{I}_1 + \bar{I}_2 = 0.024\angle -0.0034 \approx 0.025\angle 0$

Can you tell that this circuit is resonating?

- When resonating, the complex circuit is equivalent to a pure resistive circuit.
- In a pure resistive circuit, the Voltage source and Voltage across a resistor are always in-phase.
- In a mixed circuit with R, L, and C, the Voltage source and Voltage across a resistor are USUALLY out-of-phase. The exception happens when the circuit is at **resonance**.
- What's the equivalent R of the Activity 2 circuit?

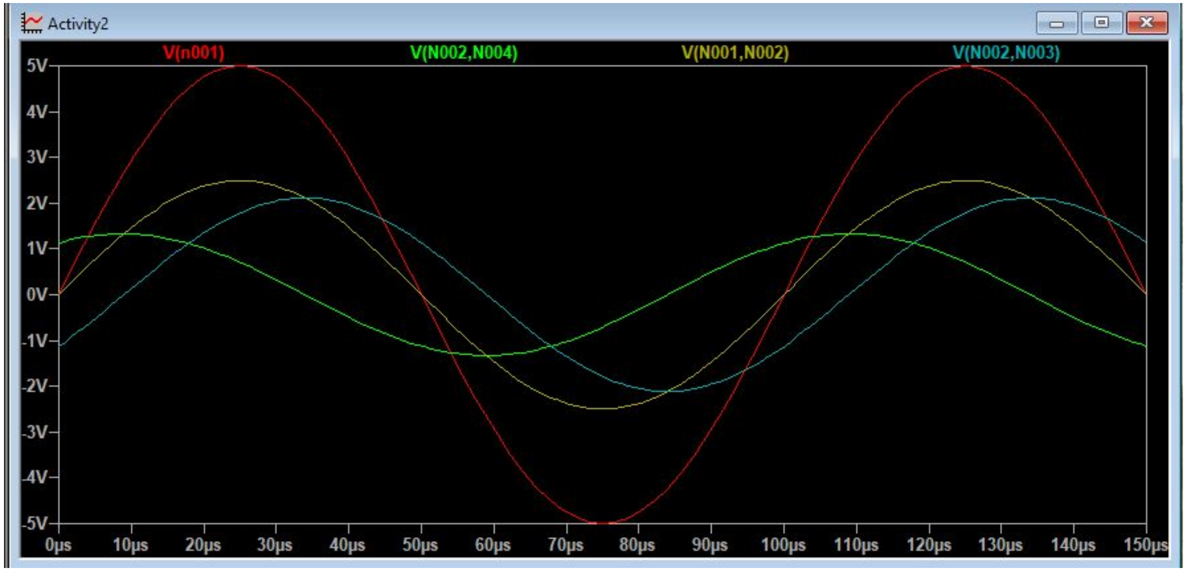

Figure 1: Red: V_S, Green: V_{R2}, Yellow: V_{R1}, Blue: V_{R3}

What is a filter

A filter is a **2-port** device that **takes in** a signal $V_{in}(t)$ at the input port, **modifies** it in some way, and **sends out** the modified signal $V_{out}(t)$ on the output port. Signals are often simply time-varying voltages.

Today, we will go through four basic filters:

- Potential Divider as a Filter
- RC Low-pass Filter
- RC High-pass Filter
- Bandpass Filter

And characterize filters with:

- Gain
- Cutoff Frequency
- Frequency response (Bode Plot)

Potential Divider as a Filter

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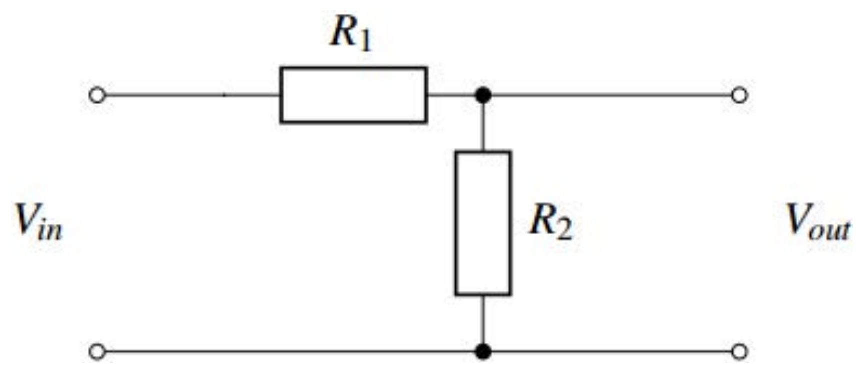
For this potential divider circuit:



$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$$

So the output signal is simply a **scaled** version of the input signal.

If the input signal is time-varying, the output signal will also vary with time:

$$V_{out}(t) = \frac{R_2}{R_1 + R_2} V_{in}(t)$$

Filter Gain

In the expression:

$$V_{out}(t) = \frac{R_2}{R_1 + R_2} V_{in}(t)$$

The scaling factor G is what we call the Gain of the filter.

$$V_{out}(t) = GV_{in}(t)$$

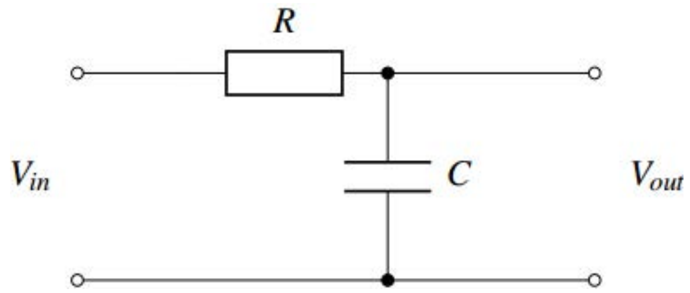
$$G = \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

Notice that:

- The gain quantifies the ratio between the output signal and input signal, it takes value from the range of $(0, +\infty]$
  - When $G < 1$, the output signal has a smaller amplitude compared to the input signal
  - When $G > 1$, the output signal has a greater amplitude compared to the input signal
  - WHen $G = 1$, the input and output signals have the same amplitude.
- The gain G here is **independent** of the input signal. This is **not** generally the case for other circuits.

RC Low-pass Filter I

For this RC circuit:



- If V_{in} is not time-varying (DC signal), C serves as an open circuit:
  - C will initially charge up to V_{in} through R.
  - Once C is fully charged, it acts like an open circuit, no current flows through C.
  - At this point, there is no current flow through the R as well, and there is no voltage drop across R
  - In the steady-state:

$$V_{out} = V_{in}$$

RC Low-pass Filter II

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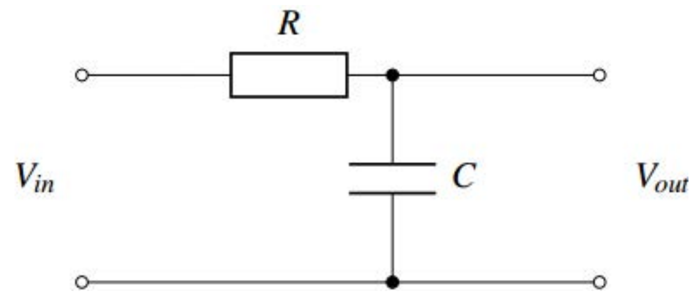


- If V_{in} is time-varying (AC signal), we can analyze this AC circuits:
  - Let the angular frequency, $\omega = 2\pi f$
  - Let the impedance of R and C be Z_R and Z_C respectively.
  - Let $\bar{V_{out}}$ and $\bar{V_{in}}$ be the phasor of output and input signal
  - The complex gain $\bar{G}$:

$$\bar{G} = \frac{\bar{V_{out}}}{\bar{V_{in}}} = \frac{Z_C}{Z_R + Z_C} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

Complex Gain? Gain?

We've derived the expression of the **Complex Gain** of the RC Low-pass filter earlier.

However, when referring to **Gain**, we usually mean the **magnitude of Complex Gain**, $|G|$. It can be found by taking the modulus of $\bar{G}$.

For a complex number $z = a + jb$, the modulus is given by $|z| = \sqrt{a^2 + b^2}$.

$$
\begin{aligned}
|G| &= \left| \frac{1}{1 + j\omega RC} \right| = \frac{|1|}{|1 + j\omega RC|} \\
&= \frac{1}{\sqrt{1^2 + (\omega RC)^2}} \\
&= \frac{1}{\sqrt{1 + (\omega RC)^2}}
\end{aligned}
$$

You can use the calculator "Abs" or "$|\square|$" function to calculate $|G|$.

Gain and Gain in dB

We have now derived $|G|$ as a ratio between V_{out} and V_{in}.

But usually, we would like to express Gain in the unit of decibels (dB).
To convert $|G|$ into G_{dB}:

$$G_{dB} = 20log_{10}(|G|)$$

Representing gain as a ratio:

- When $|G| < 1$, amplitude of output signal smaller than input signal
- When $|G| > 1$, amplitude of output signal greater than input signal
- When $|G| = 1$, amplitude of output signal equals to input signal

Representing gain in dB:

- When $G_{dB} < 0$, amplitude of output signal smaller than input signal
- When $G_{dB} > 0$, amplitude of output signal greater than input signal
- When $G_{dB} = 0$, amplitude of output signal equals to input signal

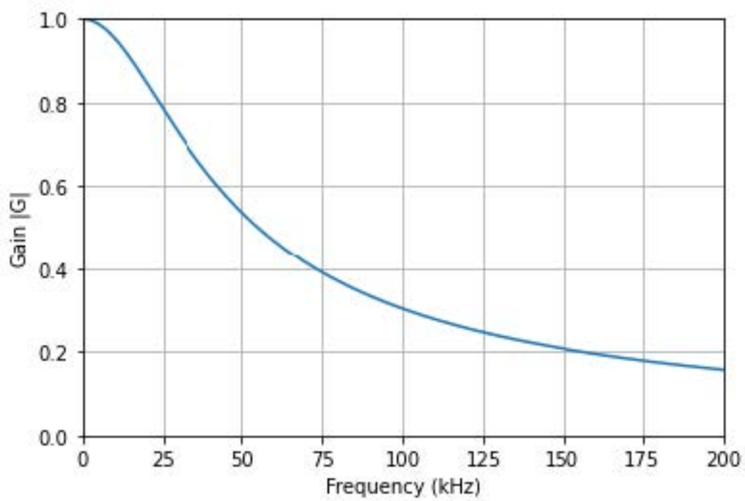
With G_{dB}, we are using 0 to gauge amplification and suppression, we can avoid values like 0.0001 or 10000. Which also helps with better Frequency Response plotting.

Frequency Response of filter in $|G|$ and G_{dB} I

You should notice that the gain of RC low-pass filter depends on the frequency of the input signal, as ω is part of the term.

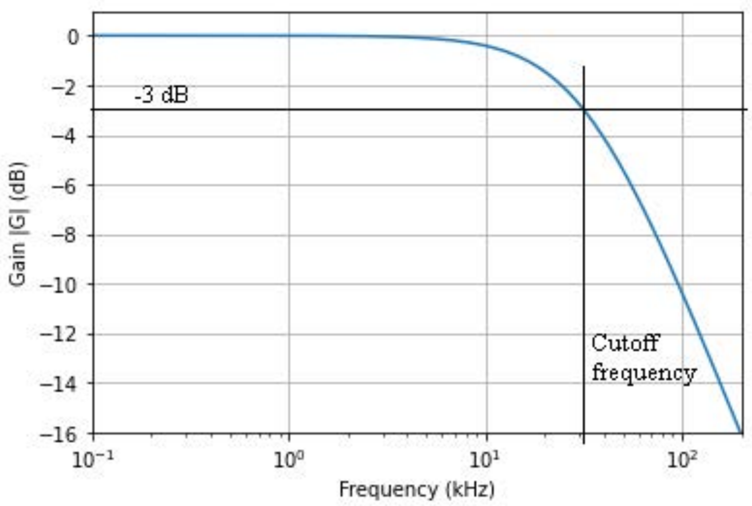
Let's plot $|G|$ and G_{dB} against the input signal frequency respectively:

$|G|$ vs. frequency



f is in linear scale

G_{dB} vs. frequency



f is in logarithmic scale

As you can see, the filter allows low frequencies to pass through and attenuates the high frequencies.

R = 1k Ω, C = 5 nF

Frequency Response of filter in $|G|$ and G_{dB} II

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In both plots, we plotted the exact same thing, but which one is preferred?

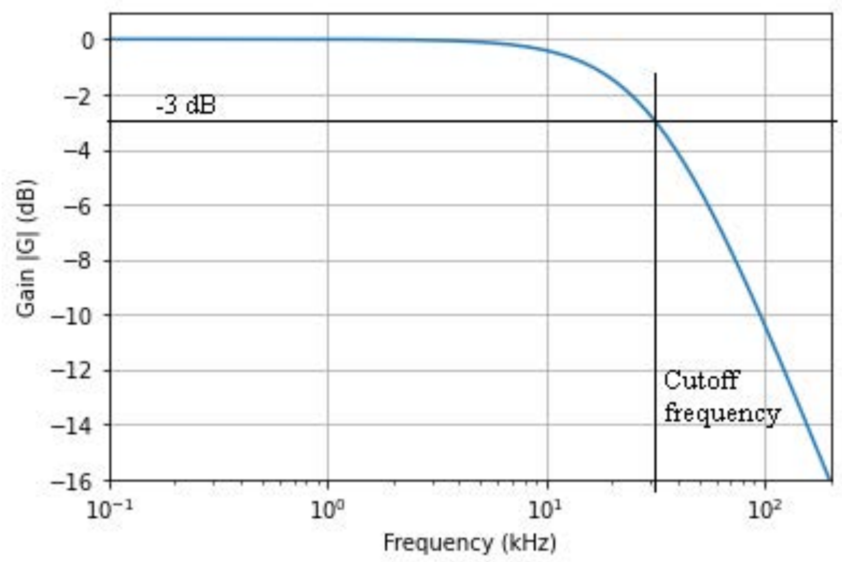
To an electrical engineer, the Bode Plot (the one on the right) is preferred!

Bode Plot

- In a logarithmic scale, multiples have the same distance. (e.g.: doubling: the distance 30 to 60 matches the distance 40 to 80 and 100 to 200).
- logarithmic scales are useful where arithmetic ratios are more significant than arithmetic differences.
- Also, in logarithmic scales, we have the lower frequency region expanded, and higher frequency compressed, this would give us a better view over a large range of frequencies while still being able to observe the frequency response trend properly.

Cutoff Frequency: -3dB

In the frequency response plot below, one particular point is marked:



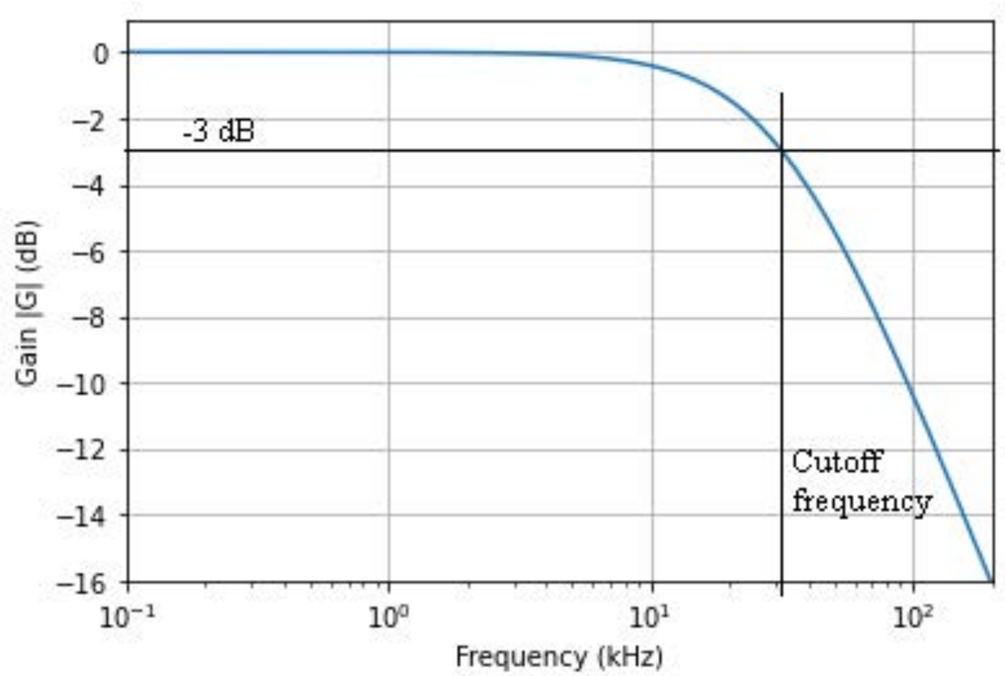
At a particular input signal frequency, the response drops to -3dB.
In linear scale, it means that the output signal amplitude is equal to the input signal amplitude scaled by a factor of 0.707.

This particular input frequency is known as the **cutoff frequency**.

$$f_{cutoff} = \frac{1}{2\pi RC}$$

Frequency Response: Bode Plot

In the frequency response plot below, you may find all the information needed for a filter:



- Gain at each input frequency
- Cutoff Frequency (-3dB point)
- Attenuation effectiveness (roll-off gradient) (Not examinable)

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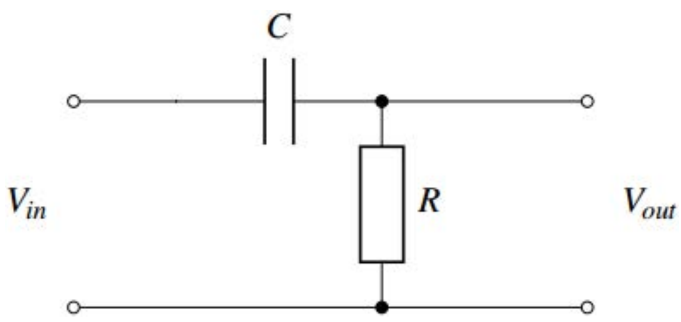
RC High-pass Filter I

For this RC circuit:



- If V_{in} is not time-varying (DC signal), C serves as an open circuit:
  - C will initially charge up to V_{in} through R.
  - Once C is fully charged, it acts like an open circuit, no current flows through C.
  - At this point, there is no current flow through the R as well, and there is no voltage drop across R
  - In the steady-state:

$$V_{out} = 0V$$

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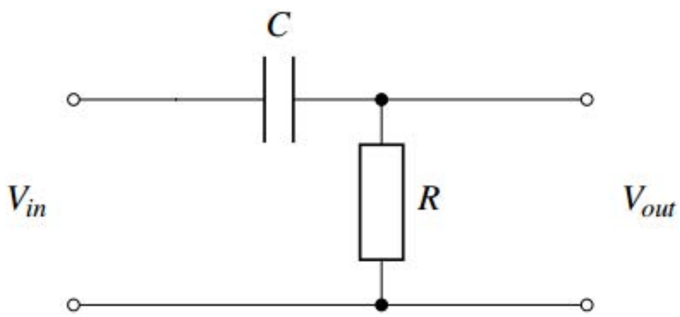


- If V_{in} is time-varying (AC signal), we can analyze this AC circuits:
  - Let the angular frequency, $\omega = 2\pi f$
  - Let the impedance of R and C be Z_R and Z_C respectively.
  - Let $\bar{V_{out}}$ and $\bar{V_{in}}$ be the phasor of output and input signal
  - The complex gain $\bar{G}$:

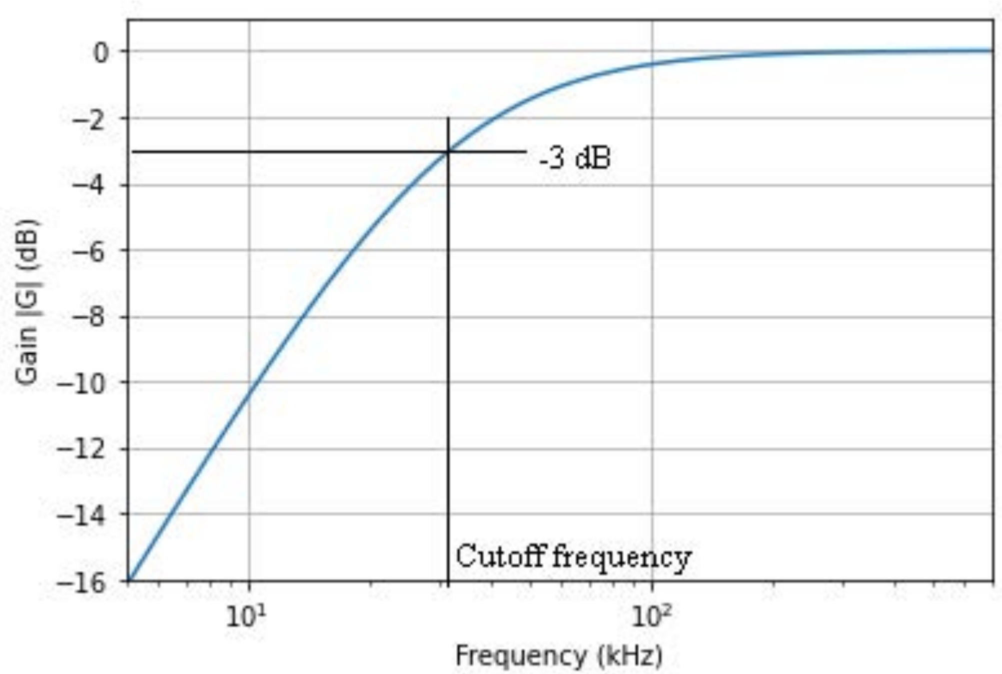
$$\bar{G} = \frac{\bar{V_{out}}}{\bar{V_{in}}} = \frac{Z_R}{Z_R + Z_C} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

  - The gain $|G|$:

$$|G| = \frac{j\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

High-pass filter Frequency Response

Let's plot the frequency response of this high-pass filter:



As you can see, the filter allows high frequencies to pass through and attenuates the low frequencies.

The cutoff frequency remains the same:

$$f_{cutoff} = \frac{1}{2\pi RC}$$

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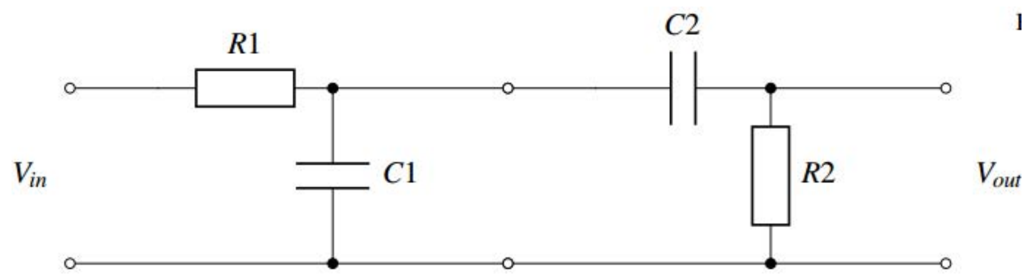
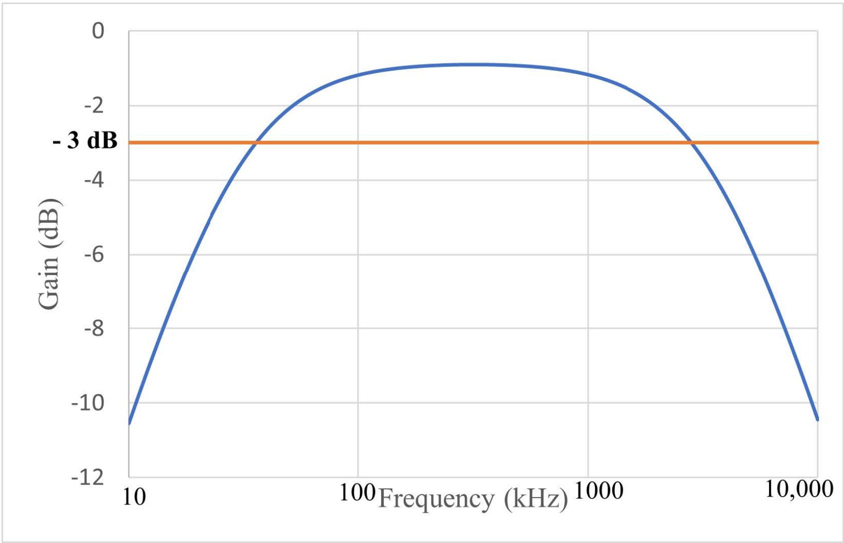
RC Bandpass Filter

If we want a filter that only allows frequencies between f_1 and f_2 to pass through, we can combine a RC high-pass filter with a RC low-pass filter.

Such filters are called **band-pass** filters.



Which has a frequency response like shown below:



The difference $f_2 - f_1$ is known as the **bandwidth** of the filter.
Such filters have two cutoff frequencies.

Practice Question Tutorial

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We will go through some practice questions and the quiz questions.
Organized notes will be uploaded after the class within 2 days.

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III. RC High-pass Filter

IV. RC Bandpass Filter

Practice Qn. Tut

Thank You

Any questions?