DC coupling (most cases)
↓
measuring voltage.

AC coupling → remove average value

trigger level → most of times is in between
if custom level → set above the starting level.

AC: phase shifts. (0 → 2π)
frequency; amplitude · phase.

An AC circuit is only linear component.
↳ All component has the same frequency and sinusoidal wave.

Ex 7.

$v_c(t) = 100\cos(250t)$ $\omega = 200$.

C: 100 μF

$Z_c = \frac{1}{\omega C} \angle -90^\circ = \frac{1}{200\times 100\times 10^{-6}} = 50\angle -90^\circ = 0 - j50$

$I_c = \frac{V_c}{Z_c} = \frac{100\angle 0}{50\angle -90^\circ} = 2\angle 90^\circ = 2 + j0$

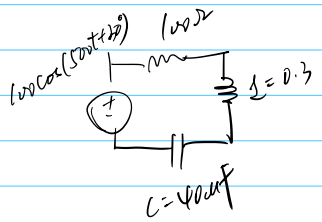
$V_c = 100\angle 0 = 100$.

$v_R(t) = 100\cos(200t)$ 50 Ω.

$Z_R = R = 50$.

$Z_R = \frac{V_R}{Z_R} = \frac{100}{50} = 2\angle 0$.

$V_R = 100\angle 0$.



$Z_L = \omega L \angle 90^\circ = 150\angle 90^\circ = 0 + j150$

$Z_C = \frac{1}{\omega C} \angle -90^\circ = 50\angle -90^\circ = 0 - j50$

$Z_R = 100\angle 0 = 100 + j0$

$Z_{eq} = 100 + j100 = 141\angle 45^\circ$

$V_S = 100\angle 30^\circ$.

$V_L = Z_L I$

$= 150\angle 90^\circ \cdot 0.33\angle 30^\circ$

$= 50\angle 120^\circ$

$I_S = \frac{V_S}{Z_{eq}} = \frac{100\angle 30}{141\angle 45^\circ} = 0.707\angle -15^\circ$.

$I_S = 0.707\cos(250t - 15^\circ)$

$V_L = Z_L I$

$= 50 \angle 90^\circ \times 0.33 \angle 30^\circ$

$= 16.67 \angle -60^\circ$

$V_R = RI = 100 \times 0.707 \angle -45^\circ = 70.7 \angle -45^\circ$

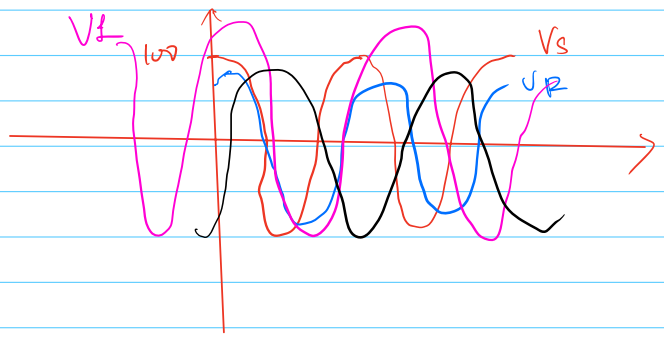
$V_L = Z_L I = 150 \angle 90^\circ \times 0.707 \angle -45^\circ = 106 \angle 75^\circ$

$V_C = Z_C I = 50 \angle -90^\circ \times 0.707 \angle -45^\circ = 35.4 \angle -105^\circ$

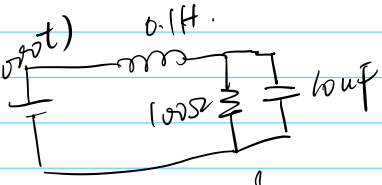
$V_R = 70.7 \cos(500t - 15^\circ)$

$V_L = 106 \cos(500t + 75^\circ)$

$V_C = 35.4 \cos(500t - 105^\circ)$



$V_c(t) = 10 \sin(1000t)$



$I = \frac{V}{Z_{eq}} = \frac{10 \angle 90}{70.71 \angle 45^\circ} = 0.141 \angle 135^\circ$

$V_L = Z_L I = 100 \angle 90^\circ \times 0.141 \angle 45^\circ$

$= 14.1 \angle 225^\circ$

$V_C = Z_{RC} I = 70.71 \angle -45^\circ \times 0.141 \angle -45^\circ$

$= 10 \angle -180^\circ$

$V_s = 10 \angle 90^\circ$

$I_R = \frac{V_C}{R} = \frac{10 \angle -180^\circ}{100 \angle 0} = 0.1 \angle -180^\circ$

$I_C = \frac{V_R}{Z_C} = \frac{10 \angle -180^\circ}{100 \angle -90^\circ} = 0.1 \angle -90^\circ$

$V_s = 10 \angle 90^\circ$

$Z_L = j\omega L = j100$

$Z_C = j\omega C = -j100$

$R = 100 + j0$

$Z_{eq} = Z_L + Z_{RC}$

$Z_{RC} = \frac{Z_R \times Z_C}{Z_R + Z_C}$

$= \frac{100 \angle 0 \times 100 \angle -90^\circ}{141.4 \angle -45^\circ}$

$= \frac{10000 \angle -90^\circ}{200 \angle -45^\circ}$

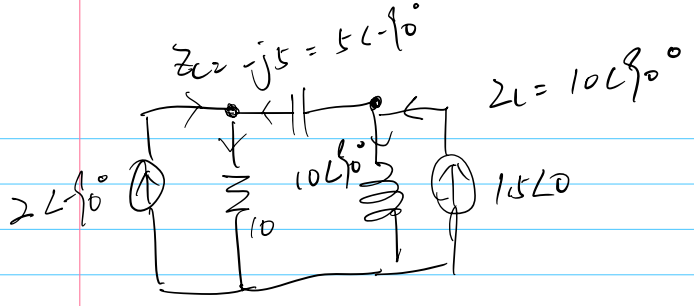
$= 70.71 \angle -45^\circ$

$Z_{eq} = Z_L + Z_{RC}$

$= 0 + j100 + 50 - j50$

$= 50 + j50$

$= 70.71 \angle 45^\circ$



$$\frac{\bar{V}_1}{10} = 2\angle 0^\circ + \frac{\bar{V}_2 - \bar{V}_1}{Z_C}$$

$$\frac{\bar{V}_1}{10} + \frac{\bar{V}_1 - \bar{V}_2}{Z_C} = 2\angle 0^\circ$$

$$10(5\angle -90^\circ)V_1 = 2\angle 0^\circ (5\angle -90^\circ) + \bar{V}_2 - \bar{V}_1$$

$$50\angle -90^\circ V_1 = 10\angle -180^\circ + \bar{V}_2 - \bar{V}_1$$

$$(1 - j50)V_1 = -10 + \bar{V}_2 - \bar{V}_1$$

$$(1 - j50)V_1 - V_2 = -10$$

$$\frac{V_2 - V_1}{Z_C} + 1.5\angle 0 = \frac{\bar{V}_2}{Z_L}$$

$$\frac{\bar{V}_2}{Z_L} + \frac{V_2 - V_1}{Z_C} = 1.5\angle 0$$

$$Z_C\bar{V}_2 + Z_L(V_2 - V_1) = 1.5\angle 0 (Z_L \times Z_C)$$

$$-j5\bar{V}_2 + 10(\bar{V}_2 - \bar{V}_1) = 1.5\angle 0 (10\angle 90^\circ \times 5\angle -90^\circ)$$

$$-j5\bar{V}_2 + 10\bar{V}_2 - \bar{V}_1 = 1.5\angle 0 (50\angle 0)$$

$$(10 - j5)\bar{V}_2 - \bar{V}_1 = 75$$

$$\frac{\bar{V}_1}{10} + \frac{V_1}{Z_C} - \frac{V_2}{Z_C} = 2\angle 0^\circ$$

$$0.1\bar{V}_1 + 0.2j\bar{V}_1 - 0.2j\bar{V}_2 = -j2$$

$$(0.1 + 0.2j)\bar{V}_1 - 0.2j\bar{V}_2 = -j2$$

$$-0.1j\bar{V}_2 + 0.2j\bar{V}_2 - 0.2jV_1 = 1.5$$

$$0.1j\bar{V}_2 - 0.2jV_1 = 1.5$$

$$0.2j\bar{V}_2 - 0.4jV_1 = 3$$

$$0.1 - j0.2\,V_1 = 3 - j2$$

$$V_1 = \frac{3 - j2}{0.1 - j0.2}$$

$$= \frac{\sqrt{13}\angle -33.69}{\sqrt{0.05}\angle -63.43}$$

$$= \frac{\sqrt{13}}{\sqrt{0.05}}\angle 29.7^\circ$$

$$v_1 = 16.1\cos(100t + 29.7^\circ)$$

$= 16.1 \angle 2\pi \cdot j^{-}$

$V_L = L \frac{di}{dt}$

A.

$10i + V_L = V_S$

$10i + L\frac{di}{dt} = V_S$

$\frac{di}{dt} + 100i = 100\sin(100t)$ $\quad i = C\frac{dV_C}{dt}$

$10i + V_C = V_S$

$10C\frac{dV_C}{dt} + V_C = 10\sin(10^4 t)$

$10\times 100\times 10^{-6}\frac{dV_C}{dt} + V_C = 10\sin(10^4 t)$

$\frac{dV_C}{dt} + 1000V_C = 100000\sin(10^4 t)$

B-1) $\bar{V}_1 = 20\angle\frac{\pi}{5}$

B-2) $\bar{V}_2 = 150\sin(\omega t + \pi) = 50\angle\pi$

B-3) $\bar{V}_3 = 200\angle\frac{\pi}{2}$.

B-4) $\bar{I}_1 = 100\angle -\frac{\pi}{3}$.

B-5) $\bar{I}_2 = 5\cos(\omega t - \frac{\pi}{4} + \pi) = 5\cos(t + \frac{3}{4}\pi) = 5\cos(t + \frac{5}{4}\pi)$

$= 5\angle\frac{5\pi}{4}$.

B-6) $\bar{I}_3 = 50\angle\frac{\pi}{4}$.

C-1) $v_1 = 100\sin(\omega t + \frac{\pi}{4})$

C-2) $v_2(t) = 200\sin(\omega t)$

C-3) $v_3(t) = 10\sin(\omega t - \frac{\pi}{4})$

C-4) $i_1(t) = 0.1\sin(\omega t - \frac{\pi}{6})$

C-5) $i_2(t) = 1.5\sin(\omega t - \frac{\pi}{6})$

C-6) $i_3(t) = 0.42\sin(\omega t + \frac{\pi}{3})$

D-1 i) $\bar{Z} = R + jX$

$= 100 + j\omega L$

$= 100 + j\,0.002\omega$

D ii) $\bar{Z} = R + jX$

$= 100 + j\omega L + j\frac{-1}{\omega C}$

$= 100 + j\,0.1\omega + j\,(-10000\omega^{-1})$

$= 100 + j\,(0.1\omega - 10000\omega^{-1})$

D iii) $Z = R + jX$

$= 100 + j\left(-\frac{1}{\omega C}\right)$

$= 100 + j\left(-\frac{1}{100\pi \times 10^{-6}}\right)$

$= 100 - j\,318.3\ \Omega$

D iv) $Z = R + jX$

$= 0 + j\,0.002\pi - j\,1000\ \Omega^{-1}$

D-2

i) $\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{Z_L}$

$= \frac{1}{100} + \frac{1}{j100}$

$= \frac{j100 + 100}{100 \times j100}$

$= \frac{100 + j100}{10000\angle 90}$

$= \frac{\sqrt{20000}\angle 45^\circ}{10000\angle 90}$

$= \frac{10000\angle 90^\circ}{\sqrt{20000}\angle 45^\circ}$

$= 70.7\angle\frac{\pi}{4}$

ii) $\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}$

$= \frac{1}{100} + \frac{1}{j\omega L} - j100\,(100\times10^{-6})$

$= \frac{\sqrt{20000}}{10000}\angle 45^\circ - j\,0.01$

$= 0.01 + j\,0.01 - j\,0.01$

$= 0.01$

$= 100\ \Omega$

iii) $\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{Z_C}$

$= \frac{1}{100} - j\omega C$

$= \frac{1}{100} - j100\,(100\times10^{-6})$

$= 0.01 - j\,0.01$

$= 0.0141\angle 45^\circ$

$= 70.71\angle -45^\circ$

iv) $\frac{1}{Z_{eq}} = \frac{1}{Z_L} + \frac{1}{Z_C}$

$= \frac{1}{j100} - j\,100 \times 100 \times 10^{-6}$

$= \frac{1}{j100} - j\,0.01$

$= 0.01\angle 90^\circ - 0.01\angle 90^\circ$

$\frac{1}{Z_{eq}} = 0$

$Z_{eq} = \frac{1}{0} \to \infty$

D-3

$Z_{AB} = \frac{1}{10+j10} + \frac{1}{10}$

$= \frac{1\angle 0}{\sqrt{200}\angle 45^\circ} + 0.1$

$= \frac{1}{\sqrt{200}}\angle -45^\circ + 0.1\angle 0$

$= 0.05 - j0.05 + 0.1$

$= 0.15 - j0.05$

$= 0.158\angle -18.43$

$= 6.32\angle 0.32$ rad Ω.

ii) $\frac{1}{Z_{AB}} = \frac{1}{10+j10} - \frac{1}{j10}$

$= \frac{1\angle 0}{\sqrt{200}\angle 45^\circ} - \frac{1\angle 0}{10\angle 90^\circ}$

$= \frac{1}{\sqrt{200}}\angle 45^\circ - 0.1\angle -90^\circ$

$= 0.05 - j0.05 - (-j0.1)$

$= 0.05 + j0.05$

$= 0.0707\angle 45^\circ$

$= 14.1\angle -45^\circ = 14.1\angle -\frac{\pi}{4}$.

iii) $Z_{eq} = R + Z_L$

$= 10 + j\omega L$

$= 10 + j100(100 \times 10^{-3})$

$= 10 + j10$

$= 14.14\angle 45^\circ$

$= 10\sqrt{2}\angle \frac{\pi}{4}$.

iv) $f = 50$ Hz $\quad \omega = 2\pi f = 314.2$ rad/s

$Z_{AB} = R + Z_L + Z_C$

$= 30 + j314 \times 0.1 - j\frac{1}{314(100\times10^{-6})}$

$= 30 + j31.4 - j31.8$

$= 30 + j - 0.445$

$= \sqrt{909.17}\ \angle -0.79$

$= 30 \angle -0.01$ rad Ω.

v) $f = 50$ Hz $\omega = 2\pi f = 314.2$

$\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}$

$= 0.02 + \frac{1}{j314.2(0.001)} - j\frac{1}{314.2 \times 10 \times 10^{-3}}$

$= 0.02 + \frac{1 \angle 0}{0.314 \angle 90^\circ} - j\,3.142$

$= 0.02 + 3.18 \angle -90^\circ - j\,3.142$

$= 0.02 - j\,3.18 - j\,3.142$

$= 0.02 - j\,6.32$

$= 6.32 \angle 89.81$

$= 0.158 \angle -1.56$ rad Ω.

$= \frac{1}{50} + \frac{1}{j0.314}$

$= \frac{50 + j0.314}{50 \angle 0 \times 0.314 \angle 90^\circ}$

$= \frac{50 + j0.314}{15.7 \angle 90^\circ}$

$= \frac{50 \angle 0.359}{15.7 \angle 90^\circ}$

$= 3.18 \angle -89.64 - j\,3.14$

$= 0.01999 - j\,3.18 - j\,3.14$

$= 0.01999 - j\,6.32$

$= 6.32 \angle -89.81$

$= 0.158 \angle 89.81$

$= 0.158 \angle 1.56$.

E. a) $V_s = 200\sin(2\pi 50 t)$

$Z_{eq} = R + Z_L$

$= 10 + j\omega L$

$= 10 + j2\pi 50(0.1)$

$= 10 + j10\pi$.

$\bar{I}_s = \frac{\bar{V}_s}{Z_{eq}} = \frac{200 \angle 0}{10 + j10\pi}$

$I_s = 1.840 - j5.178$

$= 6.07 \angle -1.26$

$i_s(t) = 6.1 \sin(2\pi 50 t - 1.26)$

$V_1 = R\bar{I}_s$

$= 10 I_s$

$= 60.7 \angle -1.26$

$= 61 \sin(2\pi 50 t - 1.26)$

$V_2 = j\omega L \cdot I_s$

$= j2\pi 50(0.1) \times 6.07 \angle -1.26$

$= 181.06 + j57.805$

$= 190 \sin(2\pi 50 t - 0.308)$

b) $\bar{V} = 71 \angle 0 \qquad \omega = 100\pi$

$Z_{eq} = \dfrac{1}{\frac{1}{10} + j\omega C}$

$= 9.101 - j2.85$

$= 9.54 \angle -0.304$

$I_s = 7.1 + j2.23$

$= 7.404 \angle 0.304$

$i_1 = \dfrac{Z_c}{R + Z_c} I_s$

$= 7.1 - j3.303$

$i_2 = \dfrac{\bar{V}_c}{Z_c}$

$= \dfrac{71}{\frac{1}{j\omega C}}$

$= 2223A$

c) $\bar{V}_s = 50$

$Z_{eq} = \dfrac{1}{\frac{1}{R+Z_L} + j\omega C}$

$= \dfrac{1}{\frac{1}{10+j30} + j0.03}$

$= 100$

$i_s(t) = \dfrac{V_s}{Z_{eq}} = \dfrac{50}{100} = 0.5A$

E-2.

$Z = 8 + j6\,\Omega$

$i_s(t) = 0.4 - 0.3i$

$= 0.5 \angle -0.635$

Amplitude is 0.5

$\phi' = -0.64.$

$i_s = 5\sin(1000\pi t - \frac{\pi}{6})$

$\omega = 1000\pi.$

E-3. $\bar{I}_s = \frac{\bar{V}_s}{Z_{eq}}$

$Z_{eq} = \frac{\bar{V}_s}{\bar{I}_s}$

$= \frac{230\angle 0}{5\angle -\frac{\pi}{6}.}$

$= \frac{230 + j0}{4.33 - j2.5}$

$= 39.8 + j23$

$= 46\angle 0.524 \left/ \frac{\pi}{6}.\right.$

b) $Z_{eq} = R + j\omega L$

$= 39.8 + j23.$

$L = 0.073$

$= 73\,mH$

E-4. impedance is smallest at resonance.

$\omega_n = \frac{1}{\sqrt{LC}} = 3162.7$

$f = 503\,Hz.$