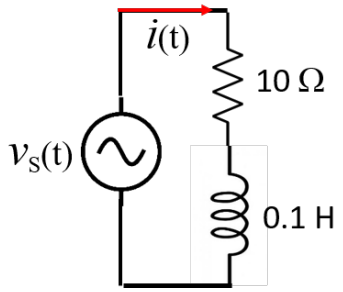
AC Circuit Practice Problems

A. KVL equation in time-domain

i. Write the time-domain KVL equation (ODE) for the following circuit if

$$v_S(t) = 10\sin(100t).$$



Answer: $\frac{di}{dt} + 100i = 100\sin(100t)$

ii. Write the time-domain KVL equation (ODE) of a series RC circuit keeping capacitor voltage as the unknown variable given that

$$v_S(t) = 10\sin(10^4 t),\ C = 100\ \mu F,\ R = 10\ \Omega.$$

Answer: $\frac{dv_C}{dt} + 10^3 v_C = 10^4\sin(10^4 t)$

B. Write the phasor representation of a sinusoidal voltage/ current using the convention

$$A\sin(\omega t + \phi) \Longleftrightarrow Ae^{j\phi} = A\angle\phi.$$

B-1

$$v_1(t) = 20\sin(\omega t + \frac{\pi}{5})\ \ volt,$$

B-2

$$v_2(t) = -50\sin(\omega t)\ \ volt,$$

B-3

$$v_3(t) = 200\cos(\omega t)\ \ volt,$$

B-4

$$i_1(t) = 100\sin(10^4 t - \frac{\pi}{3})\ \ mA,$$

B-5

$$i_2(t) = -5\cos(\omega t - \frac{\pi}{4})\ \ A,$$

B-6

$$i_3(t) = 50\sin(100t + \frac{\pi}{4})\ \ mA.$$

Answer:

$$\bar{V}_1 = 20\angle\frac{\pi}{5}\ V,\ \bar{V}_2 = 50\angle\pi\ V,\ \bar{V}_3 = 200\angle\frac{\pi}{2}\ V,$$

$$\bar{I}_1 = 100\angle -\frac{\pi}{3}\ mA,\ \bar{I}_2 = 5\angle\frac{5\pi}{4}\ A\ or\ 5\angle -\frac{3\pi}{4},\ \bar{I}_3 = 50\angle\frac{\pi}{4}\ mA.$$

C. Convert phasor into time-domain function using the convention

$$A\sin(\omega t + \phi) \Leftrightarrow Ae^{j\phi} = A\angle\phi.$$

Use ω as the radian frequency.

C-1

$$\bar{V}_1 = 100\angle\frac{\pi}{4}\ \ mV,$$

C-2

$$\bar{V}_2 = 200\angle 0\ \ V,$$

C-3

$$\bar{V}_3 = 10\angle -\frac{\pi}{4}\ \ V,$$

C-4

$$\bar{I}_1 = 100\angle -\frac{\pi}{6}\ \ mA,$$

C-5

$$\bar{I}_2 = 1.5\angle -\frac{\pi}{6}\ \ A,$$

C-6

$$\bar{I}_3 = 420\angle\frac{\pi}{3}\ \ mA.$$

Answer:

$$v_1 = 100\sin(\omega t + \frac{\pi}{4})\ mV,\ v_2 = 200\sin(\omega t)\ V,\ v_3 = 10\sin(\omega t - \frac{\pi}{4})\ V,$$

$$i_1 = 100\sin(\omega t - \frac{\pi}{6})\ mA,\ i_2 = 1.5\sin(\omega t - \frac{\pi}{6})\ A,\ i_3 = 420\sin(\omega t + \frac{\pi}{3})\ mA$$

D. Impedance

D-1 Find the equivalent complex impedance for each of the following questions assuming that the components are connected in series. Express your answers in terms of the radian frequency ω:

i.

$$R = 100\Omega, L = 2\ mH$$

ii.

$$R = 100\Omega, L = 0.1\ H, C = 1000\ \mu F$$

iii.

$$R = 100\Omega, C = 100\ \mu F$$

iv.

$$L = 2\ mH, C = 1000\ \mu F$$

Answer:

$$i.\ 100 + j2 \times 10^{-3}\omega\ \ \Omega$$

$$ii.\ 100 + j(0.1\omega - \frac{10^3}{\omega})\ \ \Omega$$

$$iii.\ 100 - j\frac{10^4}{\omega}\ \ \Omega$$

$$iv.\ \ + j(\frac{\omega}{500} - \frac{1000}{\omega})\ \ \Omega$$

D-2 Find the equivalent complex impedance for the following questions assuming that the components are connect in parallel. Assume ω = 100 rad/s:

i.

$$R = 100\Omega, L = 1\ H$$

ii.

$$R = 100\Omega, L = 1\ H, C = 100\ \mu F$$

iii.

$$R = 100\Omega, C = 100\ \mu F$$

iv.

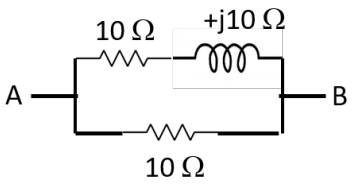
$$L = 1\ H, C = 100\ \mu F$$

Answer:

$$i.\ 70.7\angle\frac{\pi}{4}\ \ \Omega,\quad ii.\ 100\ \ \Omega,\quad iii.70.7\angle -\frac{\pi}{4},\ \ \Omega,\quad iv.\ \infty$$

D-3 Find the equivalent impedance Z_{AB} between the points A and B for each of the following:

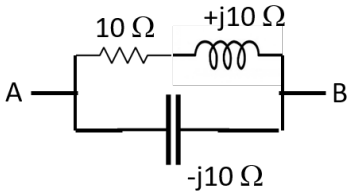
i. Z_{AB} = ?



Answer:

$$Z_{AB} = 6 + j2\ \ \Omega = 6.32\angle 0.32\ rad\ \ \Omega.$$

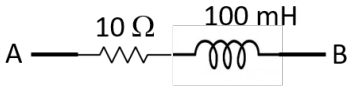
ii. Z_{AB} = ?



Answer:

$$Z_{AB} = 10 - j10 \ \Omega = 10\sqrt{2}\angle -\frac{\pi}{4} \ rad \ \Omega.$$

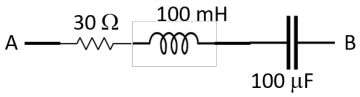
iii. What is Z_{AB} if $\omega = 100$ rad/s?



Answer:

$$Z_{AB} = 10 + j10 \ \Omega = 10\sqrt{2}\angle +\frac{\pi}{4} \ rad \ \Omega.$$

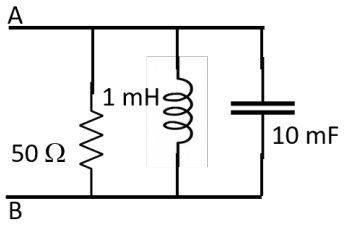
iv. What is Z_{AB} if $f = 50$ Hz?



Answer:

$$Z_{AB} = 30 - j0.4 \ \Omega \approx 30\angle -0.01 \ rad \ \Omega.$$

v. What is Z_{AB} if $f = 50$ Hz?



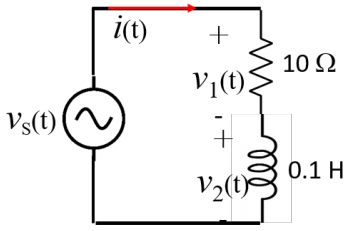
Answer:

$$Z_{AB} \approx 10 + j20 \ \Omega = 22.36\angle 1.1 \ rad \ \Omega.$$

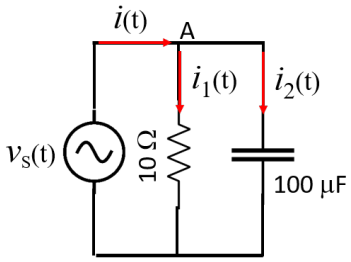
E. Circuit analysis:

E-1. Use phasor and impedance to determine the unknown currents/ voltages as indicated in each circuit.

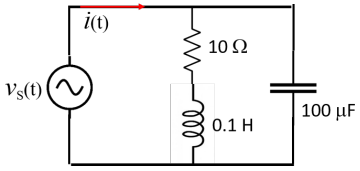
a. If $v_S = 200\sin(2\pi 50t)$, find i, v_1 and v_2. Express your answers as functions of time.



b. In the circuit below, the voltage source is sinusoidal with RMS voltage 71 V and frequency 50 Hz. Determine the RMS values of i, i_1, and i_2.



c. In the circuit below source is 50 V (rms) and $\omega = 300$ rad/s. Determine the RMS value of the source current.



Answer:

$$a.\ i = 6.1\sin(\omega t - 1.26)\ amp,\ \ v_1 = 61\sin(\omega t - 1.26),$$
$$v_2 = 191\sin(\omega t + 0.31),\ \ \omega = 2\pi \times 50\ rad/s$$
$$b.\ I_1 = 7.1\ A,\ \ I_2 = 2.23\ A,\ \ I = 7.44\ A.$$
$$c.\ I = 0.5\ A$$

E-2 A circuit with impedance $Z = 8 + j6\ \Omega$ is connected to a sinusoidal voltage source with amplitude of 5V.

a. What is the amplitude of the resulting current?

b. What is the phase of current waveform with respect to the source voltage?

Answer:

$$a.\ 0.5\ A,\ \ b.\ -0.64\ rad.$$

E-3. An equipment connected to the utility supply 230 V, 50 Hz draws RMS current of 5 A, and the current lags the voltage by $\frac{\pi}{6}$ rad.

a. Find the impedance $Z\angle\theta$ of the equipment.

b. If the impedance is modelled as series connection of two components, determine the values of the individual components.

Answer:

$$a.\ Z = 46\angle\frac{\pi}{6}\ \Omega,\ \ b.\ R = 39.8\ \Omega,\ L = 73\ mH.$$

E-4. Determine the frequency (Hz) at which the impedance of a series R-L-C is the smallest if $R = 100\ \Omega$, $L = 0.1\ mH$, and $C = 1000\ \mu F$?

Answer: 503 Hz.