How do systems Sense their environments?

All engineering systems have sensors. Without sensors, a system would not have any knowledge about its environment, and therefore would not be very useful.

Take an air-conditioning unit as an example. It requires a temperature sensor to know the temperature in the room. If it is much warmer than the desired temperature (also known as a *set-point*), the air-conditioner has to blow cold air to cool the room. If the temperature is cooler than the set-point, the air-conditioner should stop blowing cold air, and simply monitor the temperature. In addition to a temperature sensor, most air-conditioners will have other sensors. For example, they may have buttons or touch sensors to allow the user to change the set-point. Since buttons allow the system to sense when the user is pressing them, they are sensors too.

In this chapter, we will learn about various kinds of sensors, how they work, and how they are typically connected to an engineering system.

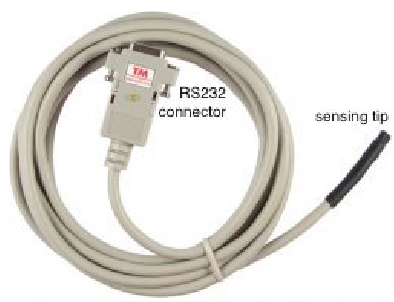


Figure 1: TM temperature sensor from papouch.com.

How sensors work?

Sensors come in various shapes and forms, ranging from simple circuit elements to integrated circuits (ICs) to sophisticated modules that can be directly interfaced using standard ports (e.g. USB port, serial port).

A digital temperature sensor

Let us begin by exploring an off-the-shelf digital temperature sensor (TM) from papouch.com that can be connected to the RS232 serial port of a computer.

If you have a desktop computer or an older laptop, it'll probably have one or more RS232 serial ports (often called COM ports) that looks something like the one shown in Figure 2.

Figure 2: RS232 serial port on desktop PC.

Newer laptops often only come with USB ports, but one can get a USB-to-RS232 dongle that will provide a RS232 serial port to connect to. In either case, once we have a RS232 serial port on a computer, we simply connect the sensor to that port and run a serial terminal software (e.g. termite) to obtain the data from the sensor:

```
+025.3C
+025.4C
+025.3C
```

This sensor automatically sends out a temperature measurement every 10 seconds, once it is connected to a RS232 serial port.

In practice, we may want to connect our sensor to an embedded computer (most embedded computers have serial ports), and read the temperature measurements from our own embedded software. While the details of the software would depend on the details of the embedded computer, it is not hard to write software that reads the sensor measurements. For example, if the embedded computer runs our software in Python on Linux, we may write something like this:

```
import serial
with serial.Serial('/dev/ttyS1', 9600) as port:  # open serial port @ 9600 bps
    s = port.readline()                          # read sensor output as string
    s = s[:-1]                                   # remove the trailing 'C'
    temperature = float(s)                       # convert string to float
```

Okay, so that wasn't so hard!

Understanding sensor specifications

If we search for "temperature sensor" on the web, we will find many! So how do we select one? Are all of them the same, and we simply pick one? The answer is no! To choose one, we have to carefully look at the *technical specifications* of the sensor (typically in the *data sheet* for that sensor), and see which ones meet our application needs. Then other factors such as price, size, power consumption, etc may help narrow down the choice further.

Technical parameters

Measurable range	–55 to +125 °C
Accuracy	±0.5 °C within range from –10 °C to +85 °C and ±2 °C outside of this range
Resolution	0,1°C
Operating temperature of electronic	–40 to +85 °C
Communication	ASCII, described below
Measurement speed	the first measurement within 1 sec, subsequently once per 10 sec ±2 %
Communication line	RS232 (simplified)
Communication parameters	9600 Bd, 8 bits, 1 stop-bit, parity – none

Figure 3: Temperature sensor specifications from the data sheet.

- What is the difference between accuracy and resolution?
- How can it be that the operating temperature range of the electronics is narrower than the measurable range of the sensor? Wouldn't the electronics get damaged if we tried to measure a temperature of, say, 100°C?

Let us take a look at the technical specifications for the temperature sensor that we explored in the previous section (see Figure 3). The specifications tell us what is the temperature range that the sensor can measure, to what precision (resolution), to what accuracy, what interface the sensor reports its measurements over, and how often.

How does a digital sensor work?

We used a digital temperature sensor that gave us temperature data over a serial port. But how does it really measure temperature? And how does the measurement get converted to the ASCII text that is sent over the serial port?

Let us first assume that we have an *analog sensor* that produces a voltage that changes with the quantity we wish to measure. In the digital temperature sensor module above, we will perhaps find an analog temperature sensor that produces a voltage output that is dependent on the temperature:

$$V_o = f(T) \tag{1}$$

where V_o is the voltage output in Volts, T is the temperature in °C, and $f(\cdot)$ is some monotonic function.

- Why do we require that $f(\cdot)$ be monotonic? What would happen if it was not?

The output of this analog temperature sensor will be a voltage. If we have a way to measure this voltage, then we have a working sensor. Microcontrollers typically have analog-to-digital converters (ADCs) that can measure voltage, and also have many standard ports. So our temperature sensor module would need a microcontroller with an ADC and a RS232 serial port:

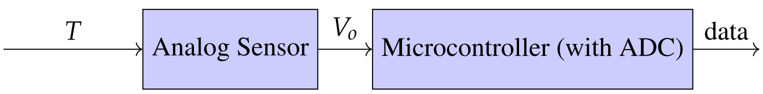


Figure 4: Functional breakdown of a typical digital sensor measuring T.

The microcontroller would run a simple program to convert the voltage into temperature:

$$T = g(V_o) \tag{2}$$

where $g \approx f^{-1}$. This value can then be sent as ASCII text over the RS232 serial port.

How does an analog sensor work?

In the previous section, we assumed that we had an analog sensor that produces a voltage depending on the parameter that we measure. In some cases, it is easy to find such a device or material. For example:

- *Photovoltaic materials* (used in solar cells) directly convert light into electricity at the atomic level. The atoms absorb photons of light and release electrons that can flow in an electric circuit.

- *Piezoelectric materials* naturally produces a voltage depending on the force applied on the material.

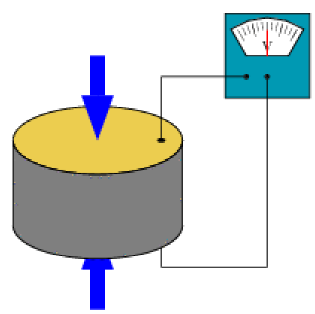


Figure 5: A piezoelectric material produces a voltage depending on the force applied on it.

However, quite often it is not possible to find such materials or devices directly. It is more common to find circuit elements whose electrical properties (e.g. resistance, capacitance, inductance) change with the quantity to be measured. We will call such circuit elements *sensing elements*.

- Can you find a device whose output voltage changes in response to a magnetic field? What is it called?

For example, a *thermistor* is a type of resistor whose resistance R_s is strongly dependent[1] on temperature T:

$$\log R_s = \frac{\beta}{T} + \alpha \tag{3}$$

[1] Standard resistors exhibit a weak temperature dependence.

for some constants α, β. We can incorporate the thermistor (or other sensing elements) into an electrical circuit that generates a voltage that can be measured:

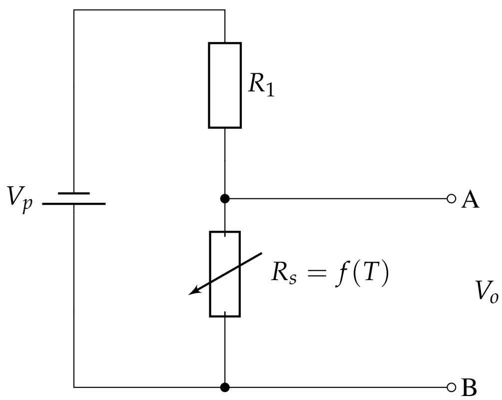


Figure 6: Simple electrical circuit to convert temperature T to voltage V_o that can be measured.

In this circuit, V_p is a voltage source of our choice (perhaps a battery), R_1 is a known resistance and R_s is our sensing element (thermistor). By analyzing the circuit, we easily see that:

$$V_o = \frac{R_s}{R_s + R_1} V_p. \tag{4}$$

Since V_p and R_1 are known, we can calculate R_s once we measure V_o:

$$R_s = \frac{V_o}{V_p - V_o} R_1. \tag{5}$$

The temperature T is easily obtained once R_s is known:

$$T = \frac{\beta}{\log R_s - \alpha}. \tag{6}$$

- Can you find a device whose resistance depends on the intensity of light incident on it? What is it called?
- In the circuit shown in Figure 6, how would you choose the values of V_p and R_1?

If we find other sensing elements whose electrical properties depend on other quantities of interest, we can use the same technique to make other sensors.

How do you measure the output voltage?

Once we know how to build an analog sensor, we can ignore the detailed circuit and simply think of it as a voltage source V_o with an output resistance R_o:

For the circuit in Figure 6, we can compute R_o using Thevenin's theorem:

$$R_o = \frac{R_1 R_s}{R_1 + R_s}. \tag{7}$$

- In modeling an analog sensor as a simple voltage source with an output resistance, are we making an approximation?

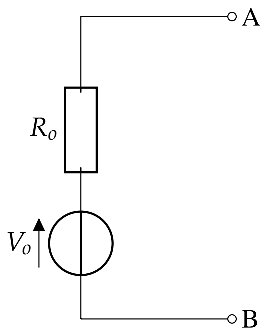


Figure 7: Equivalent circuit of an analog sensor.

We can measure the output voltage between nodes A and B using a voltmeter or an oscilloscope, or in the case of the digital sensor, using an ADC. When we connect the leads of our measuring instrument to the nodes A and B, a tiny amount of current has to flow through the measuring instrument for it to sense the voltage. So the measuring instrument can be thought of as having an input resistance (aka *input impedance*) R_i. The instrument measures voltage V_{AB} between nodes A and B:

$$V_{AB} = \frac{R_i}{R_i + R_o} V_o. \tag{8}$$

Clearly this is not the same as V_o, unless $R_o = 0$ or $R_i = \infty$. In order for our measurement to be as close to V_o as possible, we require that $R_o \ll R_i$.

- Should we choose R_1 to be small or large for $R_o \ll R_i$?
- What is the effect of this choice of R_1 on the power consumed by the sensor?

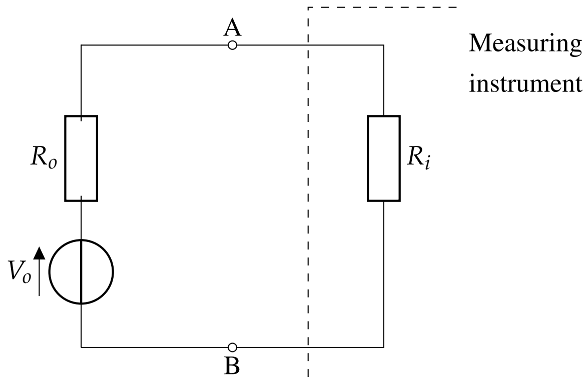


Figure 8: Equivalent circuit of an analog sensor and measuring instrument.

How do active sensors work?

So far, the sensors we have studied are *passive sensors*. They sense their environment without transmitting any energy. In some cases, sensors actively transmit energy in order to sense their environment. For example, a flash on a camera actively emits light. The light reflects off the subject and is received by the camera's CCD sensor to generate an image. The camera + flash system is therefore considered to be an *active sensor*. Typically, this means that active sensors require an energy source to operate, but passive sensors may operate with or without one.

Many active sensors are commonly found in engineering systems:

- Ultrasound ranging sensors, commonly used in reverse sensors in cars, emit a high frequency sound that echoes off obstacles. A measurement of the time taken for the sound to reach the obstacle and return back provides an estimate of the range to the obstacle.
- Drones commonly use ultrasound rangefinders to measure their altitude above the ground. An underwater version of ultrasound rangefinders are known as depth sounders, and are very commonly found on boats and ships.
- Laser rangefinders work on a similar principle, but use laser light instead of ultrasound to find range. Handheld laser rangefinders are commonly used in the construction and renovation industry, and by military.
- Active infrared proximity sensors are commonly used as touch-less switches for taps and towel dispensers in public washrooms.

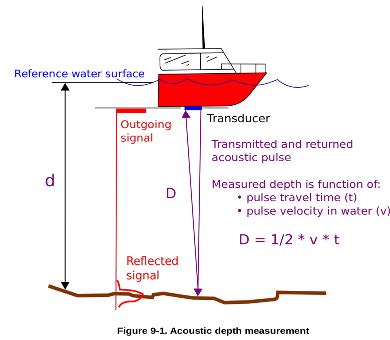


Figure 9: Principle of operation of a depth sounder.

Supplementary Reading

- How do photovoltaics work?
- Piezoelectric sensors
- Hall effect sensor
- Photoresistor

Introduction to DC motor

Electricity used in different ways. One of them is to do mechanical work, *e.g*, moving an object, turning an object, and exerting force on a stationary object. Such applications require mechanical power. Converting electrical power to mechanical power is possible using different kinds of devices; electrical motor is one such device. We find motors in practical applications ranging from small appliances to large industrial drives.

Although the term *motor* is generally associated with machines capable of producing rotary motion, there are motors that produce linear motion which are known as *linear motors*.

Input power and output power of motor

- Input power (electrical): $Power = Voltage \times Current$
- Output power (mechanical)
  1. Rotary: $Power = Torque \times Angular\ velocity$
  2. Linear: $Power = Force \times Velocity$

Types of motor

Depending on the nature of the electrical source, motors are classified into two categories:

1. **DC motors** (powered by DC source) and
2. **AC motors** (powered by an AC source).

Each is further divided into different groups depending on how the motor works or how it is built.

Only DC motors are covered in this course. The principle of operation and required calculations are explained with one particular type of DC motors known as **permanent magnet DC (PMDC) motor**. Few other DC motors are briefly mentioned at the end of this chapter.

Unit of rotational speed

Revolutions per minute (**RPM**) is the most common unit used for specifying the speed of a rotating motor. However, the SI unit **radians per second**, represented using the Greek letter ω, is also used which is related to RPM by the following:

In one revolution, any point on the circumference of the motor shaft goes through an angular change of 2π radians.

$$\begin{aligned} \omega &= 2\pi \times (revolution\ per\ second), \\ &= 2\pi \times (\frac{RPM}{60}). \end{aligned} \tag{9}$$

Conversion efficiency

As in any other energy conversion process, there are losses in motors and other electro-mechanical systems. Some factors contributing to these losses are mentioned in this section.

The efficiency of a motor producing rotary motion is

$$\eta = \frac{P_{out}}{P_{in}} = \frac{torque \times speed}{voltage \times current}.$$

In motors, the interaction between the electrical system and the mechanical system takes place through a magnetic field coupling which is governed by the following laws of physics.

- *Lorentz force:* A magnetic field exerts force on a current-carrying conductor.

- *Faraday's law of induction:* Any change in a magnetic field gives rise to a potential difference across a conductor placed in that field.

In the process of the conversion, energy is lost at different stages. Some of these losses are listed here.

- **Copper loss:** I^2R loss in the electrical conductors

- **Iron loss:** Loss due to hysteresis and eddy currents in the magnetic core

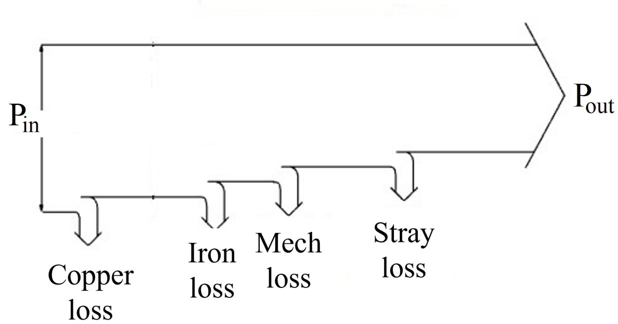
- **Mechanical loss:** due to bearing friction and air friction

- **Stray loss:** small amount of loss due to factors not listed here.

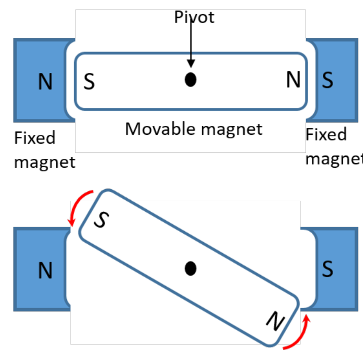
Conversion efficiency

$$\begin{aligned}\eta &= \frac{P_{out}}{P_{in}} \\ &= \frac{P_{out}}{P_{out} + P_{loss}}.\end{aligned}$$

where P_{loss} is the sum of losses. Power flow in a dc motor is illustrated in the following **Sankey diagram**.



How does a DC motor work?



Let us consider a simple arrangement of magnets shown in the margin where the inner magnet is pivoted at the center and the outer magnetic poles are fixed in position. If no external force is applied, the inner magnet naturally aligns itself with the outer poles. If the inner magnet is moved away from this position and then released, magnetic forces will bring it back to the equilibrium.

This simple mechanism illustrates how magnetic forces can be exploited to create rotary motion. The inner magnet in this illustration cannot rotate continuously. However, if at least one of the magnets, either the inner or the outer, is an electromagnet, then continuous motion can be realized by changing the polarity of the electromagnet at appropriate instants.

Stator and Rotor

All motors have

1. a stationary part called the **stator**, and
2. a rotating part called the **rotor**.

In most DC motors, the rotor is an electromagnet and the stator is either permanent magnet or another electromagnet. Depending on the construction of the stator, the DC motors can be classified as

1. **Wound field DC motors** (both rotor and stator are electromagnets) or
2. **Permanent magnet DC (PMDC)** (stator is made of permanent magnets).

Characteristic properties of DC motors are explained in this note using a PMDC motor. But, some examples of the wound filed DC motors are given at the end of the chapter.

Wound field motors are further classified into different categories according to the ways the stator coils are energized.

Structure of a DC motor

The cross-section of a wound-field DC motor, with two stator poles, is shown in Figure 10. The central magnetic core with winding of current-carrying conductors makes the rotor - also known as **armature**.

Electromagnetic torque or Motor torque

A current-carrying conductor placed in a magnetic field experiences a force exerted on it (*Lorentz force*). If the conductor is at right angles with the direction of the magnetic field, the magnitude of the force (unit: Newton) is

$$F = BIL,$$

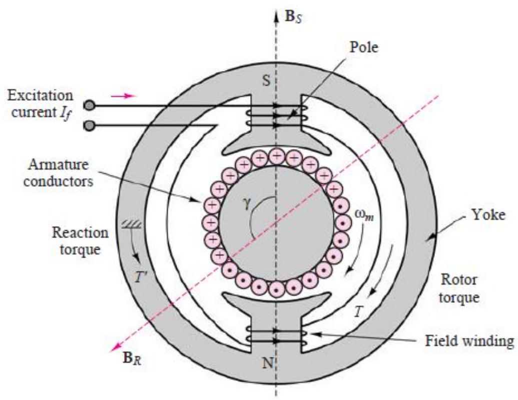
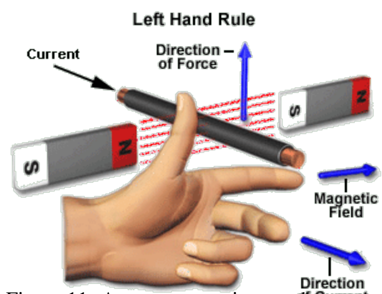


Figure 10: Cross-section of a DC motor: The grey circle in the middle is the rotor core. Small circles around the circumference of the rotor core are conductors that form loops. Half of these conductors are indicated with a $\times$ sign meaning current through them flows into the paper, while current flows out of the paper through the remaining conductors which are marked with a dot (.). Torque is produced when current flows through the **armature** conductors. In this drawing, the second magnet (poles indicated using N and S in the yoke) is also an electromagnet. So this is a wound field motor.

where, B is magnetic flux density (unit: Tesla), I is current (unit: ampere) and L is length (unit: meter) of the conductor. The direction of the exerted force is explained using **Fleming's left hand rule** (see the illustration in the margin). Force is perpendicular to both current direction and field direction.



Let us consider a rectangular conductor loop carrying current I (Amp) placed in a magnetic field of flux density B (Tesla) (Figure 11).

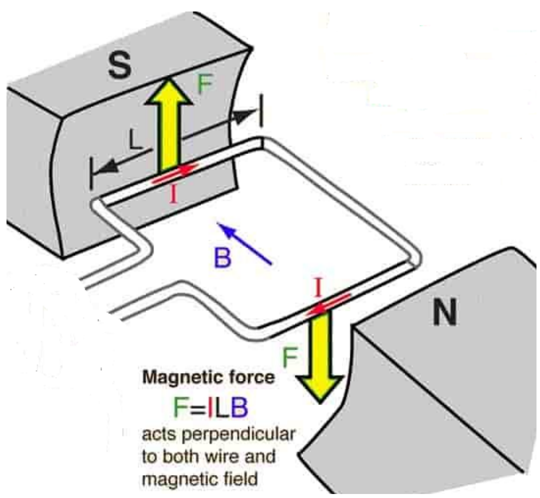


Figure 11: A current-carrying conductor loop placed in a magnetic field experiences a torque.

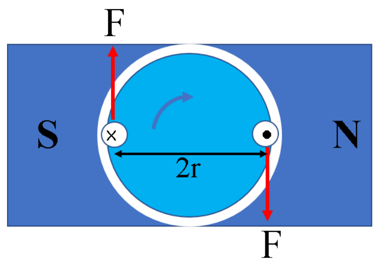
The sections of the loop that runs parallel to the direction of B field do not experience any force (Fleming's left hand rule). Force (yellow arrows in the figure) is exerted on two sections of the loop where current is perpendicular to the B field. Assuming that the lengths of these two sections are equal, and the B field is uniform, the forces on both sides are equal to

$$F = BIL$$

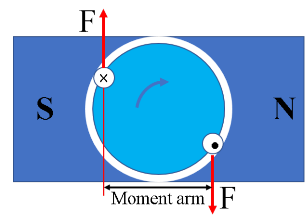
but opposite in direction. These forces forms a moment that tries to rotate the loop.

In a DC motor, many such loops are placed around the rotor core (Figure 10) and the sum of moments of individual loops is the torque produced.

For ease of illustration, a two-dimensional drawing is used with cross ($\times$) and dot ($\cdot$) representing currents into the page and out of the page, respectively.



If the radius of the rotor is r m, then the resulting moment produced by one conductor loop is



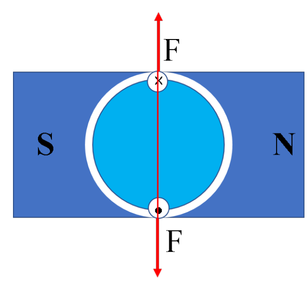
$$\begin{aligned} T_{loop1} &= (F)(2r), \\ &= 2BILr. \end{aligned} \tag{10}$$

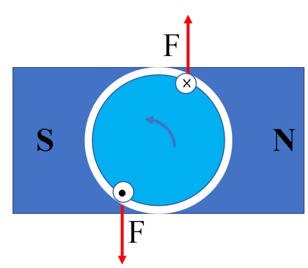
With this moment (clockwise in this illustration) acting on the loop, the rotor is set in motion. As it rotates, the perpendicular distance between the two forces is decreased (figure in the margin) resulting in decreasing magnitude of the moment. The variation of the moment can be expressed as

$$T_{loop1} = 2BILr\sin\theta, \tag{11}$$

where, θ is the angle the loop makes with an axis perpendicular to the B field.

The moment will be to zero when $\theta = 0$, *i.e.*, the loop is perpendicular to the B field (the middle figure in the margin). But, due to inertia, the loop continues to rotate. As soon as it moves beyond the zero-moment position, the direction of moment is reversed (the bottom figure in the margin).





If nothing is done, the loop will oscillate back and forth around the zero-moment position and will eventually get stuck after loosing energy due to friction. To make the loop rotate continuously, the rotor current must be reversed as soon as the loop goes past the zero-moment position. This process of changing the direction of rotor current is called **commutation**. How commutation works is explained later under the section titled**Commutator**.

Figure 12 shows the variation of torques as functions of θ for three different loops which are placed at 120° (mechanical angle) offset from one another. The torques shown in the figure are always positive implying that commutation is in place. The resultant torque, *i.e.*, the sum of the three torques is also shown. The resultant torque is not constant rather shows ripples.

For a given motor, the length L, the radius r, and the number of loops in the rotor are fixed. Also, the flux density B is constant in a PMDC motor. Under these conditions, the developed electromagnetic torque (T_e) is proportional to the rotor current or the armature current (I_a),

The stator field (B) in some wound-field DC motors varies with operating condition. You will learn more about them in the section *Different types of DC motors*.

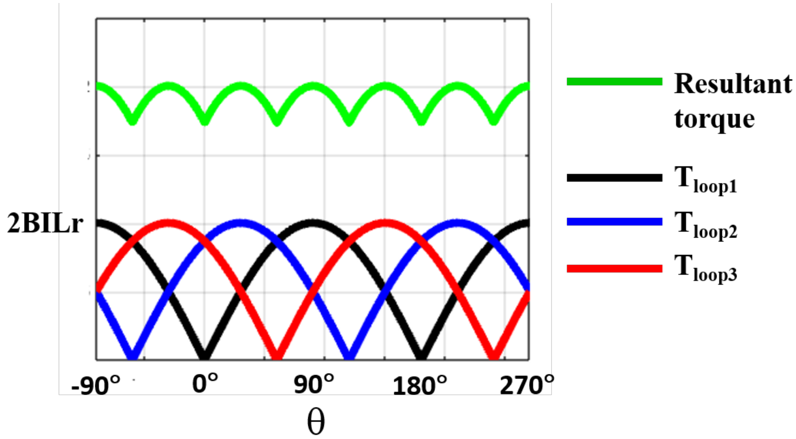


Figure 12: Torque produced in three rotor loops placed 120° apart and the resultant torque are shown. The resultant torque has ripples. Placing more loops around the rotor core (as shown in Figure 10) will make the ripple smaller. Many loops spread around the rotor core will produce a torque that can be considered constant.

$$T_e = K_t I_a. \tag{12}$$

The **torque constant** K_t is a characteristic parameter of a DC motor. Its unit is $\frac{Nm}{A}$.

Back EMF

The current-carrying conductor loops of a rotating rotor experience change in magnetic flux and, hence, a voltage is induced between the ends of the rotor coil (Faraday's law). This induced voltage, known as the **back EMF** (electromotive force) or **counter EMF**, opposes the source voltage that tries to push current through the rotor coil. The back emf (V_e) is proportional to the speed of rotation:

$$V_e = K_e \omega. \tag{13}$$

The **back emf constant** K_e is another key parameter for characterizing a motor. Its unit is $\frac{V}{rad/s}$.

If SI units are used for all quantities (current, voltage, torque and speed) then for a PMDC motor

$$K_t = K_e.$$

It holds if the unit of speed is rad/s but not when the unit is RPM.

Model of a PMDC motor

Let us develop a mathematical model that will relate the input (electrical quantities) to the output (mechanical quantities).

Electrical Characteristics

The equivalent electrical circuit model of a PMDC motor is shown in Figure 13. Though the stator is not shown in this illustration, it is understood that the rotor coil is immersed in a constant B field produced by the permanent motor stator.
Applying KVL:

$$V_a - I_a R_a - L_a \frac{dI_a}{dt} - V_e = 0.$$

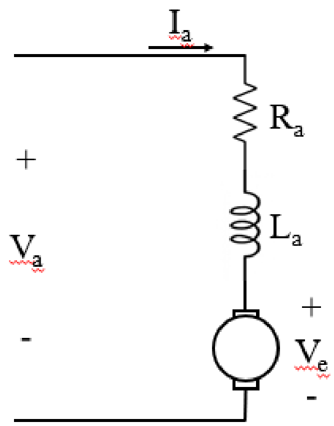


Figure 13: Electrical circuit model of a PMDC motor. V_a: voltage applied to the rotor (armature), R_a: resistance of the rotor coil, L_a: inductance of the rotor coil, V_e: back emf

Reorganizing the terms,

$$\frac{L_a}{R_a}\frac{dI_a}{dt} + I_a = \frac{V_a - V_e}{R_a},$$

which represents a first-order system with time constant

$$\tau_e = \frac{L_a}{R_a}$$

known as the **electrical time constant** of the motor.

The armature current does not reach a constant value immediately after a PMDC motor is connected to the power source.

Once the initial transient has decayed, the motor current reaches the steady state value, and $\frac{dI_a}{dt} = 0$. Then,

$$I_a = \frac{V_a - V_e}{R_a}. \tag{14}$$

From equations 12 and 13, we know that the torque produced is proportional to armature current I_a and the back emf V_e is proportional to the speed, respectively:

$$T_e = K_t I_a, \quad V_e = K_e \omega.$$

Then, the equation 14 can be used to derive the relationship between the torque and the speed (mechanical characteristics).

Speed-torque graph

Substituting equations 12 and 13 in equation 14,

$$\frac{T_e}{K_t} = \frac{V_a}{R_a} - \frac{K_e}{R_a}\omega.$$

After rearranging,

$$\omega = \frac{V_a}{K_e} - \frac{R_a}{K_t K_e} T_e. \tag{15}$$

The speed-torque graphs of a PMDC motor for different applied voltages are shown in Figure 14. The graph is a straight line with slope of $-\frac{R_a}{K_t K_e}$. At the intersections on the two axes:

$$\omega_{nl} = \frac{V_a}{K_e}, \quad when \;\; T_e = 0, \tag{16}$$

and

$$T_{stall} = K_t \frac{V_a}{R_a}, \quad when \;\; \omega = 0. \tag{17}$$

- The first of these two is the **no-load speed** ω_{nl} which is the **maximum rotor speed** that can be achieved for a given V_a. It happens when the EM torque produced is zero.

- The second one is the **stall torque** T_{stall} which is the **maximum EM torque** that the motor can produce for a given V_a, and the corresponding speed is zero.

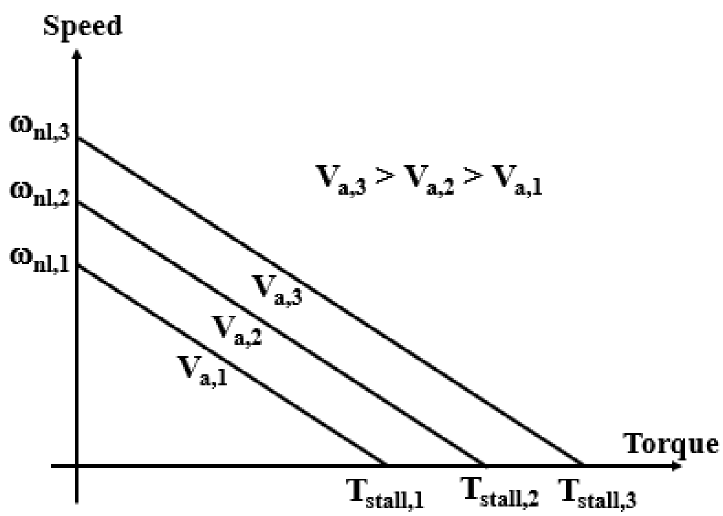


Figure 14: Speed versus torque graph of a PMDC motor for three different applied voltages $V_{a,3} > V_{a,2} > V_{a,1}$. If the applied voltage V_a is changed, the slope of the speed-torque graph (equation 15) does not change but the graph is shifted upward or downward.

- Do you see the similarity between the speed-torque graph of PMDC motor and the load-line of a practical voltage source or a linear circuit?

For a given V_a, the motor will produce certain amount of torque and will spin at a specific speed such that the operating point (speed and torque) lies on the speed-torque graph for that particular V_a. And the torque produced must satisfy the condition for mechanical equilibrium, *i.e.*, the net torque is zero. Mechanical equilibrium is explained in the subsection *Mechanical characteristics*.

It can be concluded from the speed-torque graph that

- **for a given V_a, if higher torque is required then speed is lowered, and *vice versa*.**

- **if the torque (T_e) is to remain constant, the speed can be increased (or decreased) by increasing (or decreasing) the voltage (V_a).**

Mechanical Characteristics

Besides the electromagnetic torque T_e produced by the motor, several other torques act on the system. They are

- $J\frac{d\omega}{dt}$: torque for the rotational acceleration/ deceleration of the rotor (speed is not constant). J in this expression is the moment of inertia of the rotating mass. Accelerating/ decelerating torque is required during speeding up or slowing down.

- $T_\omega = k_f\omega$: torque that is proportional to the speed of rotation. The friction at the bearing and the air friction contribute to this. k_f in this expression is the friction coefficient. In reality, the bearing friction is a non-linear function of speed, but a linear model is used here for simplification.

- T_L: torque due to mechanical load. Different applications have different types of load torque. One example of a pulley drive used to lift an object is shown later in this section.

These torques oppose the electromagnetic torque produced by the motor and, therefore, for mechanical equilibrium:

$$T_e - J\frac{d\omega}{dt} - k_f\omega - T_L = 0. \tag{18}$$

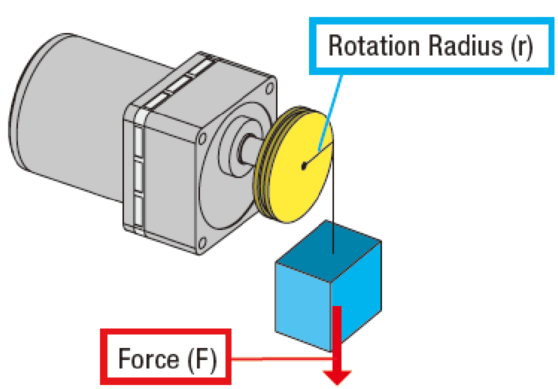
This represents a first-order dynamic system with time constant

$$\tau_m = \frac{J}{k_f}$$

known as the **mechanical time constant** of the motor.

Example of constant load torque

Load torque calculation is different for different applications. The example shown below is for a pulley drive used to lift an object.



For this case, when the object is lifted upward at a constant speed, motor must provide a force to counteract the force of gravity. This force acts at a point away from the center of the pulley. Hence, the load torque is

$$T_L = F \times r = (mg) \times r,$$

where, $F = mg$ is the force of gravity and r is the radius of the pulley. The load torque in this case is independent of the speed of hoisting. This is an example of constant torque load. Note that, at the start of lifting, an additional *acceleration torque* ($J\frac{d\omega}{dt}$) is required, and before stopping the torque produced should be less than $F \times r$ to cause deceleration.

There are applications, where the load torque varies with speed. The speed-torque relationship is nonlinear for fan load or pump load.

Operating point

The speed-torque graphs of four different types of load are shown in Figure 15. The figure also shows the speed-torque graph of a PMDC motor. When the load is attached to the motor, the equilibrium is at the point of intersection between the motor's speed-torque graph and the load's speed-torque graph. The operating torque and the operating speed are determined by that point of intersection.

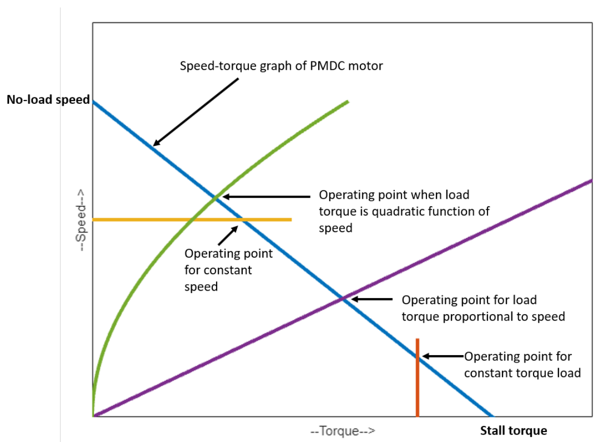


Figure 15: Speed-torque graph of PMDC motor and four different types of load.

Self-regulating nature of DC motor

- If the torque demand is increased, the motor must produce larger torque (T_e) to maintain the equilibrium and, hence, armature current (I_a) is increased. As a consequence, the back emf

  $$V_e = V_a - I_a R_a$$

  is decreased resulting in decreased speed

  $$\omega = \frac{V_e}{K_e}.$$

- A decrease in torque demand leads to decrease in armature current (I_a). As a consequence, the back EMF is increased and the speed goes up.

Electromagnetic (developed) torque and Shaft torque

The power developed by the motor is

$$P_{dev} = T_e \times \omega,$$

$$P_{dev} = K_t I_a \times \frac{V_e}{K_e}.$$

Taking $K_t = K_e$,

$$P_{dev} = V_e I_a.$$

Part of this developed power is used to overcome the bearing friction, and hence wasted as heat, which is known as the **rotational loss** at the bearings (P_{rot}). This is a mechanical power loss. The mechanical power available for driving the load is

$$\begin{aligned} P_{sh} &= P_{dev} - P_{rot}, \\ T_{sh}\omega &= T_e\omega - T_{fb}\omega. \end{aligned}$$

The torque

$$T_{sh} = T_e - T_{fb}$$

is called the **shaft torque**.

Commutator

It was shown in section "*How does a DC motor work?*" that when a rotor conductor crosses the zero-moment position, the direction of current must be reversed to keep the moment in the same direction.

A simple commutator device, known as *split-ring commutator*, is shown in Figure 16. It is an annular metal ring split by insulator at regular intervals, and is wrapped around the motor shaft so that it rotates along with the shaft. The drawing shown in Figure 16 has two split segments.

How does it work?

Rotor coils are connected to the conducting segments of the commutator ring. The power connectors supplying current to the rotor are connected to carbon brushes or metal brushes which are pressed against the split ring using spring-loaded housings. As the shaft rotates, conducting segments of the split-ring comes in contact alternately with the positive and the negative terminals of the power supply. Image in the margin shows the commutator of a real motor.

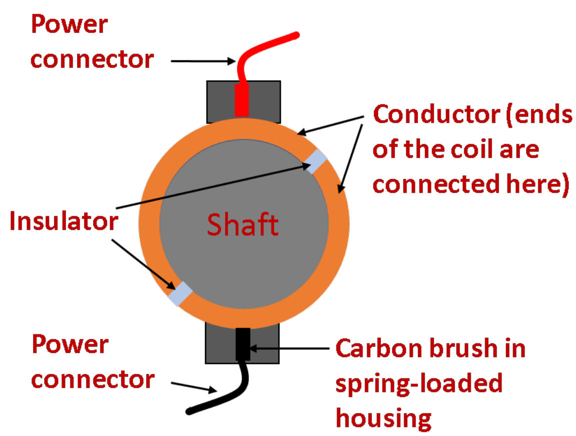


Figure 16: Construction of a two-segment commutator. Commutator used in practice has many segments.

DC motor datasheet

Parts of a sample datasheet of a PMDC motor are given in Figure 17 (graphs) and Figure 18 (parameters).

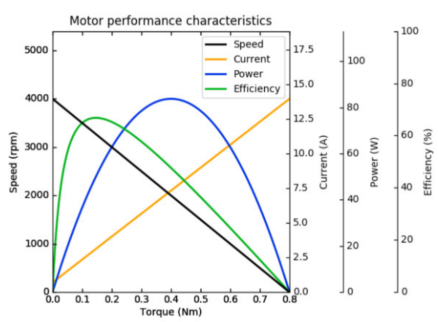


Figure 17: Datasheet often provide motor performance characteristics using graphs.

There are four curves in this chart: speed, current, power, and efficiency on four different y axes, each plotted against torque on the same x axis. These are the "characteristic curves" of a motor.

The speed versus torque graph (black coloured) has already been explained. It is perhaps the most important of these curves. It shows that the relationship between speed and torque of the motor, which is linear for PMDC motor. The no-load speed of this particular motor is 4000 RPM which is equal to

$$\omega_{nl} = 2\pi\frac{4000}{60} = 133.3\pi \; rad/s,$$

and the stall torque is 0.8 Nm. You have seen that the no-load speed and the stall torque are different for different voltage (V_a) applied to the motor. The performance graphs given in a datasheet are obtained by running the motor at the **rated operating voltage**, the terminology that will be explained soon.

The current graph (orange coloured) tells that the torque is proportional to armature current. However, you may notice that the **no-load current**, *i.e.*, current at zero torque is not zero. The graph is obtained by running the motor without any load attached to the shaft. However, the motor needs to produce a small amount of torque to overcome the friction torque of the bearing. The other end of the current graph gives the **stall current** when the shaft is not rotating. This is also called the **starting current** because, when motor starts moving from rest, its speed is zero and the torque is at its maximum.

When the motor first starts, it momentarily draws the maximum amount of current regardless of the physical load attached to it. However, this inrush lasts for a small amount of time, usually on the order of milliseconds.

The power curve (blue coloured) shows that the power output of the PMDC motor reaches the maximum at half the stall torque. The developed power is

$$P_{dev} = T_e \times \omega.$$

Power is zero at no-load speed ($T_e = 0$), and at stalled condition ($\omega = 0$).

The efficiency curve (green coloured) shows how efficiently the motor converts electrical power to the mechanical power.

The datasheet also provides different parameters of the motor as shown in Figure 18.

Parameter	**Units**	**Value**
Nominal voltage	V	24
No-load speed	rpm	4000
No-load current	A	0.7
Rated speed	rpm	3270
Rated torque	Nm	0.15
Rated current	A	3.1
Stall torque	Nm	0.80
Starting current	A	14.0
Max power	W	84
Max efficiency	%	67
Terminal resistance	Ω	1.7
Torque constant	Nm/A	0.057
Speed constant	rpm/V	167
Back-emf constant	V/rpm	0.006

Figure 18: Different parameters of the motor. By now, you are familiar with the parameters **terminal resistance** or armature resistance, **torque constant**, and **back-emf constant**. They are not explained again in this section.

The parameter **speed constant** (RPM/V) tells how the speed varies with varying applied voltage.

Nominal voltage

Although a motor will work with a range of input voltages (*operating range*), the manufacturer specifies a voltage (known as the *nominal voltage*) which will give optimal performance. In this datasheet shown, the nominal voltage is mentioned but not the operating range. Some manufacturers specify the operating range as well, and suggest not to run the motor outside that range.

If the applied voltage is outside the specified voltage range, motor will draw high current and more heat will be generated. If operated in this way, the lifespan of the motor will be decreased.

The values of **no-load speed**, **no-load current**, **stall torque**, and **starting current** given in the datasheet are measured by running the motor at the nominal voltage.

If the shaft is prevented from rotating while it is connected to the source, the torque produced is the stall torque.

Keeping a motor stalled for prolonged period should be avoided as the $I_a^2 R_a$ loss will damage the motor. If an application requires the motor to be operated in the stall condition, the thermal design should take that into consideration so that the heat is adequately dissipated.

The parameters **rated speed**, **rated torque**, and **rated current** refer to the point at which the motor can run continuously without damaging to itself. These are decided based on the thermal properties of the motor. Usually, the rated values happen to be at or near peak efficiency as the motor generates the least excess heat at peak efficiency.

If an application does not require continuous operation, it may be possible to safely operate at a higher torque/lower speed/higher current.

Wound field DC motors

In PMDC motor, K_t and K_e remain constant regardless of the operating point. But that is not true for some wound field DC motors.

In the wound-field motors, the stator coil is powered by an electrical source. The stator flux (Φ_f), hence the stator flux density B, depends on the magnitude of the stator current I_f. The flux density B directly influences the torque produced and the back emf developed. So, instead of using $T_e = K_t I_a$ and $V_e = K_e \omega$, the stator flux Φ_f should be included in these relationships.

Assuming that the parameters related to the rotor, *i.e.*, rotor diameter, rotor length, and number of conductor remain the same, the torque formula and the back emf formula for wound-field motors are

$$
\begin{aligned}
T_e &= K\Phi_f I_a, \\
V_e &= K\Phi_f \omega,
\end{aligned}
$$

For a wound field motor, the manufacturers often specify torque constant in unit of $\frac{Nm}{Weber}$ where Weber is the SI unit of magnetic flux. The torque is not linearly proportional to current.

and, the stator flux Φ_f can be different at different operating points depending on the way the stator coils are energized. There are five different ways of connecting the stator to the power supply as shown in Figure 19.

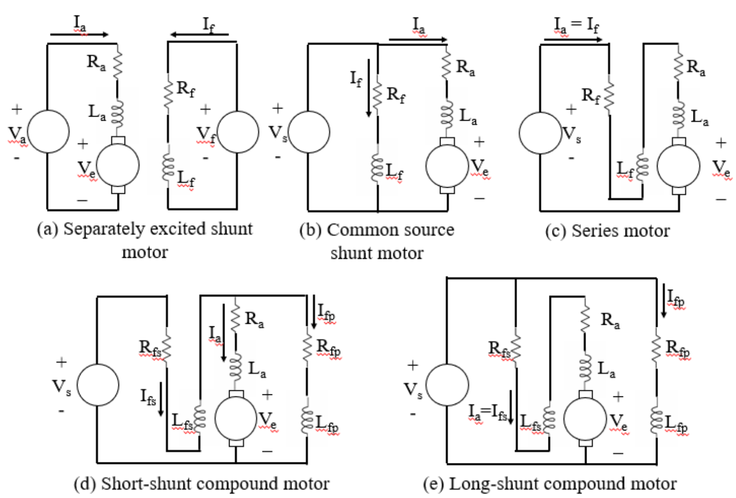


Figure 19: Types of wound-field DC motors. Symbols used are R a: armature resistance, L a: armature inductance, R f: stator field coil resistance, L f: stator field coil inductance, V a: voltage source connected to armature only, V f: voltage source connected to stator coil only, V s: voltage source connected to both windings, V e: back emf, I a: armature current, I f: field current.

1. **Shunt motor:** Stator winding is parallel to the rotor winding. The stator winding in this configuration can be powered in two ways:
   (a) **Separately excited** (schematic drawing of Figure 19 (a): The rotor and the stator are powered by two separate sources.
   (b) **Common source** (Figure 19 (b)): The rotor and the stator are powered by the same source.

   In both cases, the armature circuit is identical to that of the PMDC motor and is independent of the stator coil circuit. Therefore, like in PMDC, the speed-torque graph is a straight line with negative slope. However, unlike in PMDC, the B field can be varied. It gives greater flexibility in regulation of speed and torque. For the common source case, the stator current is varied by inserting a variable resistor in series with the stator. Independent control of the stator field and the armature current makes these motors suitable for variable speed drives.

2. **Series motor:** (Figure 19 (c)): The rotor winding and the stator winding are connected in series, and therefore,

$$I_a = I_f.$$

   Unlike in PMDC or shunt motors, any change in the load torque will not only change the armature current but also the stator current and, hence the magnitude of B. This leads to a nonlinear speed-torque graph. At the starting, when the armature current is high momentarily, the stator current is also very high resulting in a large value of B. So, series motors produce high starting torque.

Details of the speed-torque graph of series motor are out of the scope of EPP1.

3. **Compound motor:** The stator coils are separated into two sets - one set is connected like a shunt motor and the other set like a series motor, combining the advantages of both shunt motor (good speed regulation) and series motor (high starting torque).
   A. Short-shunt compound motor (Figure 19 (d)), and
   B. Long-shunt compound motor (Figure 19 (e)).

Details of the speed-torque graph of compound motor are out of the scope of EPP1.

Special types of DC motor

1. Gear Motor

Many manufacturers supply DC motor with geared speed reducer. The shaft speed of a small DC motor can be several thousand RPM but it produces very little torque. In a gear motor (Figure 20), series of gears is used to make the load rotate slower than the motor's shaft. As a result of the geared down speed, torque at the output shaft is higher than T_e produced by the motor.

Figure 20: Gear motor (source: http://www.robotshop.com)

2. Brushless DC (BLDC) motor

BLDC motors are powered by DC source but the principle of operation is similar to a permanent magnet synchronous motor, which is an AC motor. An electronic circuit is needed to produce alternating current in the motor coil from a DC source. An external difference between DC motor and BLDC motor is the number of wires. DC motors have two wires to connect to the DC source while BLDC motors come with three wires.

Figure 21: A DC motor (image source: http://www.robotshop.com) comes with two wires for connecting it to the source but a BLDC motor (image source: http://www.dys.hk) has three terminals.

3. Stepper motor

Stepper motor is a DC motors that moves in discrete steps. The rotor consists of a set of permanent magnets. The very basic design of a stepper motor is shown in Figure 22, where, the four coils are placed in the stator at intervals of 90°. The coils are activated in cyclic order, one after another. Each time the next coil is energized, the shaft is rotated by 90°.

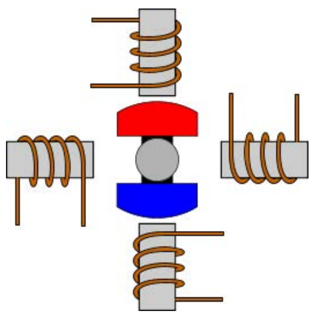


Figure 22: A simple stepper motor

In practice, a stepper motor comes with many alternating magnetic poles in the rotor and many stator coils organized in groups or phases. With a computer controlled stepping, very precise positioning and/or speed control can be achieved. For this reason, stepper motors are the motor of choice for many precision motion control applications.

Figure 23: Internal construction of a stepper motor. Image credit to Wikimedia Commons, Mortlingas - Own work, CC BY-SA 3.0.

Motor Sizing

Motor sizing refers to the process of picking the right motor for a given task. It is important because:

- Too small motor cannot produce sufficient torque to start the load and to run it up to the correct speed. Even if it can get the load up to the speed, it will overheat and burn out.
- If the motor is too large, then money has been wasted in purchasing such a large motor.
- Motor's efficiency varies with varying load. If the motor does not run with good efficiency, operating cost will increase.

Objective of motor sizing is to enable the designer to find the motor that meets the requirements of the load in terms of power, torque, and speed.

Choice of the operating voltage

To provide certain mechanical power, a DC motor with higher voltage rating will draw less current than a motor with lower voltage rating. A 12V DC motor needs about half the current than a 6V DC motor would need. But safety may be a concern with higher voltage rating. You may not get shock at 6V or 12V, but 100V supply can give you shock and possibly injury.

Physical dimensions

The torque produced by a motor is proportional to the volume of the rotor,

$$T_e \propto Volume_{rotor}.$$

Two motors with identical rotor volume, one long and thin the other short and fat, can produce identical amount of torque.

All motors are made from iron and copper, and differ only in the way these materials are disposed. Hence, there is a fairly close relationship between the overall volume and the rotor volume, for motors producing comparable torque. We can therefore make the generally accurate statement that the *overall volume of a motor is determined by the torque it needs to produce*. There are of course exceptions to this rule, but as a general guideline for motor selection, it is extremely useful.

We know that the developed power is

$$P_{dev} = \omega T_e,$$

and bigger motor produces larger torque. So, to get a specific power output, we can choose between

- a large motor (capable of giving high torque) operating at low-speed, or
- a small motor (producing low torque) operating at high-speed.

For a handheld appliances, *for example*, electric hand drills, a small motor is preferred. But a small motor produces small torque. We can choose a small motor running at high speed (thousands of RPM) so that the output power is large. Drill machines need high torque and need not run at high speed. So, in drill machines, the speed is geared down for the final drive that turns the drill bit. In such applications, a direct drive (without gear) motor is not a good choice. Gear motors are also common in mobile robots.

The drawback of mechanical gear is acoustic noise. (You might have heard the noise produced while drilling.) A small high-speed motor with power transmission gear produces higher noise than a bulky direct drive motor giving the same amount of power. If noise is of concern (for example, in fans), a direct drive motor is preferred, despite its larger size.

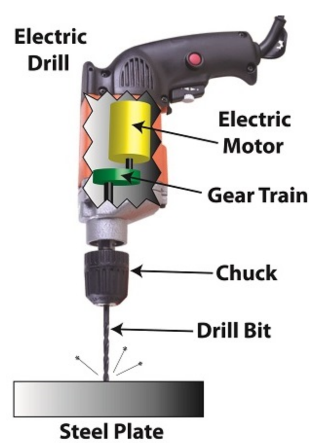


Figure 24: Smaller, high RPM motors are used in electric drills.

Speed Control of DC Motor

The torque produced by a motor must satisfy the condition for equilibrium of torque. The required amount of torque depends on the mechanical load. We often need to alter the speed. Two wheels of a mobile robot run at different speeds during turning. This chapter explains briefly the techniques used for controlling speed of a DC motor.

You have learnt in the previous chapter that the steady state speed of a PMDC motor is given by the equation

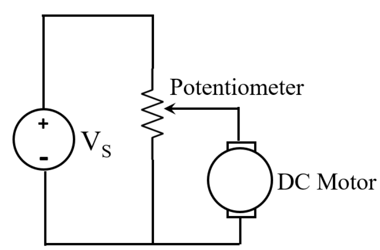
$$\omega = \frac{V_a}{K_e} - \frac{R_r}{K_t K_e} T_e. \tag{19}$$

This is also true for shunt motors.

For varying the speed while keeping T_e unchanged, one needs to vary the applied voltage V_a. In a laboratory setup, this can be done using a variable voltage DC power supply.

You will do this in the lab while characterizing a DC motor.

In many applications, the available DC voltage is fixed. In such a case, voltage divider (figure in the margin) gives a simple solution for varying the voltage applied to the motor, but it is not efficient and has limitation.



- I^2R loss in the potentiometer makes it inefficient.
- Motor is driven in one direction only as the polarity of the applied voltage cannot be changed by the potentiometer.

Pulse width modulation (PWM) is a widely used technique for controlling power flow efficiently. Basic concepts of PWM was introduced in EPP1 when you made DC-to-DC Buck converter. PWM can be used for speed control of DC motor. The buck converter has capacitor and inductor to get smoother (less fluctuation) output voltage. In DC motor control, the motor itself acts as a filter. However, PWM alone cannot give bidirectional control; we need what is called an **H-Bridge** (explained later) for that.

Pulse Width Modulation (PWM)

You are familiar with <u>modulation</u> of human voice by which parameters such as pitch of the sound signal is changed to suggest emotions. In signal processing, **modulation** is a technique by which property of one signal is varied in proportion to another given signal. In PWM, the signal to be modulated is a sequence of **rectangular pulses** at regular interval, and the property to be varied is the **width** of the pulse.

Rectangular pulse

A pulse is a signal with a rapid, transient change in the amplitude from a baseline value to a higher or lower value, followed by a rapid return to the baseline value. A pulse can be of different shapes. PWM uses rectangular pulse, which is shown in Figure 25.

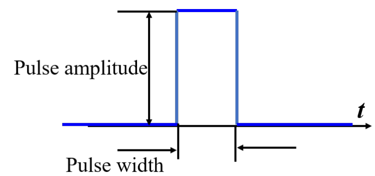


Figure 25: Rectangular pulse

A series of rectangular pulses is shown in Figure 26. In PWM, such a signal is modulated by varying the width of the pulse while keeping the amplitude and the period (repetition interval) unchanged. It is widely used for controlling analog devices, *e.g.*, changing brightness of LED and changing motor speed.

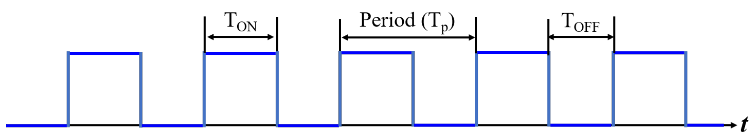


Figure 26: PWM uses a periodic signal consisting of rectangular pulses; the pulse width is modulated.

PWM parameters

Two parameters define the behaviour of PWM.

- **Duty cycle** is the fraction of the period when the signal is ON. It is typically expressed as a ratio or percentage. Referring to Figure 26,

$$Duty\ cycle = \frac{T_{ON}}{T_p},$$

$$Duty\ cycle = 100 \times \frac{T_{ON}}{T_p}\%.$$

- **Frequency** is the rate at which the PWM signal is repeated:

$$Frequency = \frac{1}{T_p}\ \ [Hz].$$

Control LED brightness using PWM

Consider the LED circuit shown in Figure 27, where a PWM signal is used to control the ON/OFF status of the electronic switch and, hence, to control the power delivered to the LED. Voltage applied to the resistor+LED will alternate between V_S and 0, as shown in the waveform below the circuit.

A semiconductor device called MOSFET (Metal Oxide Semiconductor Field Effect Transistor) is often used as an electronic switch.

The vision of a normal human eye lasts $\frac{1}{16}$ second. If something changes faster than this limit, human brain and eye cannot decipher the change. If the LED is turned ON and OFF at a slow rate (less than 16 times per second),

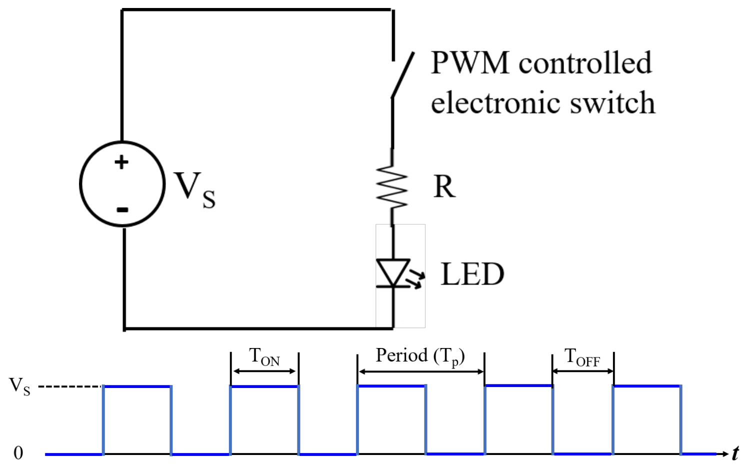


Figure 27: LED brightness control using PWM (top) and the voltage across the resistor and the LED (bottom)

human eye will notice that the LED is flickering. If the LED is turned ON and OFF at intervals $T_p < \frac{1}{16}s = 0.06\ s$ (frequency 16 Hz), human vision cannot detect the flickering. Instead, the average brightness of the LED will be perceived.

Human eye and brain act as a low pass filter. Signals less than 1 Hz signal are detected but greater than 16 Hz are not.

Average DC voltage applied to the Resistor+LED is

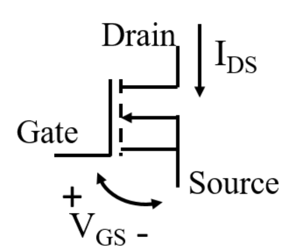
$$\frac{T_{ON} \times V_S + T_{OFF} \times 0}{T_p} = DV_S,$$

where D is the duty cycle of the PWM signal. Higher the duty cycle, brighter the LED will be. Brightness can be controlled by varying the duty cycle.

If you apply PWM signal to connect an RC circuit of appropriate time constant to the voltage source, the output (capacitor voltage) will be a DC voltage equal to duty cycle times the voltage level at the ON state.

MOSFET switch

MOSFET (Metal Oxide Semiconductor Field Effect Transistor) is a three-terminal semiconductor device. There are different types of MOSFET. The circuit symbol shown in the margin is of an N-channel MOSFET. The terminals are named as Gate (G), drain (D), and source (S). Flow of current I_{DS} from drain to source is controlled by a voltage V_{GS} applied between gate and source.



- If V_{GS} is less than some threshold voltage V_T, the current $I_{DS} \approx 0$. The switch is turned **OFF**.
- If $V_{GS} \geq V_T$, current I_{DS} flows, and $V_{DS} \approx 0$. The switch is turned **ON**.

A typical value of V_T is 0.7 volt, but it can be varied by the manufacturing process.

DC motor speed control using PWM

Similar to the LED brightness control, the speed of a DC motor can be controlled using PWM (Figure 28). When the switch is closed, voltage applied to the motor is V_S. When it is open, applied voltage is zero.

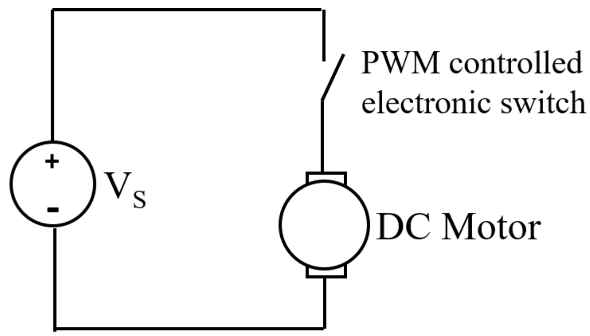


Figure 28: Speed control of DC motor using PWM

The motor is an inductive load. Unlike in the case of LED, the current in motor cannot be changed instantaneously. Rotor inductance smooths out the current and, the motor inertia (mechanical time constant) serves to smooth out the speed. As a result, the motor behaves as if it is being driven by a pure DC voltage. The frequency of PWM signal determines whether the effect of repetitive ON and OFF can be observed.

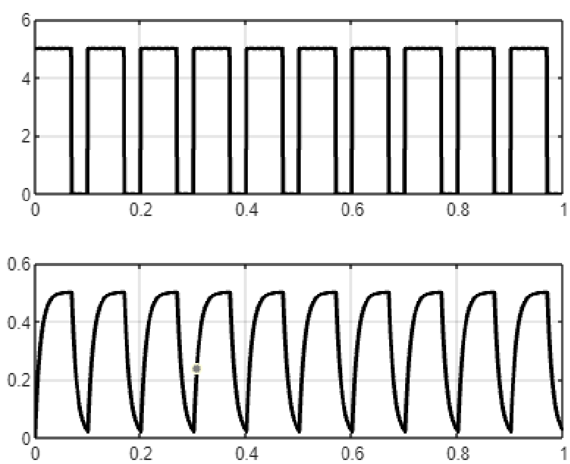


Figure 29: If the frequency of PWM is very low, the motor current reaches the steady state value every time the switch is changed from OFF to ON, and vice versa. In these graphs, the horizontal axis is time in seconds. So, the PWM frequency is 10 Hz.

For low PWM frequency (large T_p), the motor current and hence the torque will reach the steady state in every cycle (Figure 29). Motion of the motor will be visibly jerky and, at very low frequency, the motor may even stop during OFF state of PWM.

If frequency is increased, there is not enough time for the current to reach the steady state. Instead, there will be ripple in the current as well as in the torque. The ripple magnitude decreases with increasing frequency of the PWM as shown in Figure 30.

Torque ripple makes the motor vibrate producing acoustic noise. The effect of torque ripple is diminished to some extent by the mechanical time constant (low pass filter). Higher the frequency of the ripple, greater is the attenuation by the mechanical filter. That means high PWM frequency is desirable. However, if the frequency is very high, *i.e.*, T_{ON} is very small, the

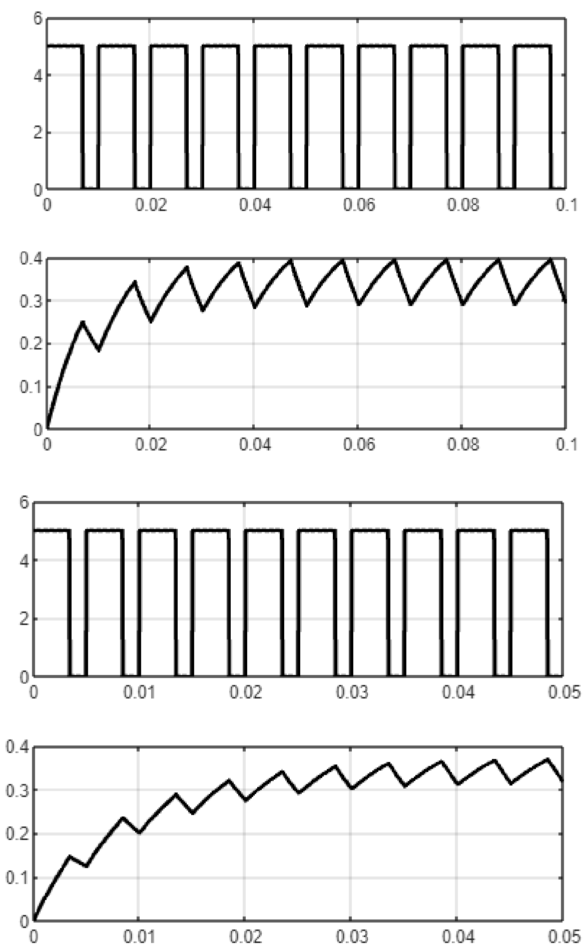


Figure 30: Ripple in the motor current and hence in the torque is diminished with increasing frequency of PWM. Top: 100 Hz, Bottom: 200 Hz.

motor may not produce enough torque required to start moving.

The speed of the motor can be varied using the scheme shown in Figure 28 where the switch is controlled with a PWM signal. But the motor will run in one direction only. For bi-directional control, an H-bridge is used which is explained in the next section.

The PWM frequency cannot be too low or too high. The frequency is often set at 20 kHz which is near the upper bound of human hearing.

H-Bridge

An H-bridge consists of four electronic switches arranged in two branches (Figure 31).

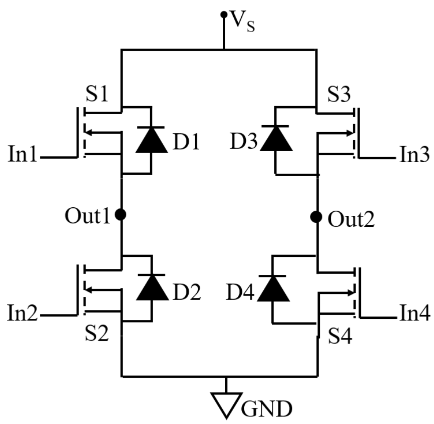


Figure 31: H-bridge

Four MOSFET switches, labelled S1, S2, S3 and S4, can be controlled, respectively, by control inputs In1, In2, In3 and In4. The load, for example, a DC motor, is connected between the points Out1 and Out2.

If switches S1 and S4 are turned ON while switches S2 and S3 are kept OFF, current flows through S1, the motor, and S4, as shown in Figure 32. We can keep S4 always ON and apply PWM signal to S1 to vary the speed. Alternatively, S1 can be always ON while PWM signal controls S4.

To reverse the direction, keep S1 and S4 OFF, and control motor speed using S2 and S3 (Figure 33).

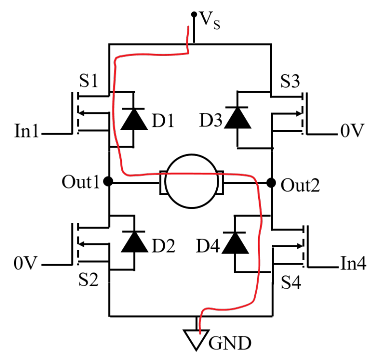


Figure 32: Current flows through the motor from Out1 to Out2.

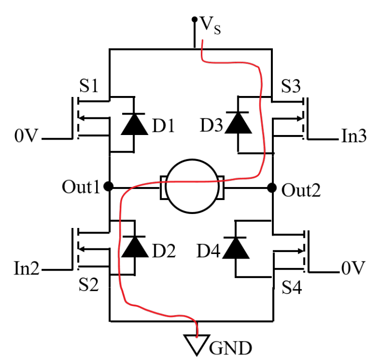


Figure 33: Current flows through the motor from Out2 to Out1.

DC Power Supply

Most electricity generation today is AC, which won the famous **battle of the currents** in the late 1880s and early 1890s. The transmission and distribution of electricity is also done in an AC power network. The AC power network (utility grid) provides a time-varying, sinusoidal voltage with fixed RMS value and frequency. For Singapore, the utility at home and offices is 230V/50Hz.

However, many electrical loads (or devices) like computers, LED lights, and battery chargers require DC voltages. A DC power supply takes in the AC power from the utility grid and converts it to a DC output. In this chapter, you will learn the basics of how to make a DC power supply.

Building a DC power supply

Three essential stages of making a DC power supply:

1. Change in amplitude: the amplitude voltage at the utility power point is large (230 V RMS) compared to the magnitude of DC voltage required. Stepping down from 230V AC to low voltage (below 15V) AC is done using a *transformer*.

2. Controlling polarity: the polarity of AC voltage continuously alternates but the DC supply has a fixed polarity. A *rectifier* is used to convert AC to DC.

3. Time-varying versus Constant: The output of the rectifier is DC but a fluctuating DC. A capacitor used as a *filter* at the output of the rectifier helps to produce a nearly constant DC voltage.

Transformer

Transformers are used in AC power systems to change the voltage and current levels as shown in Figure 34.

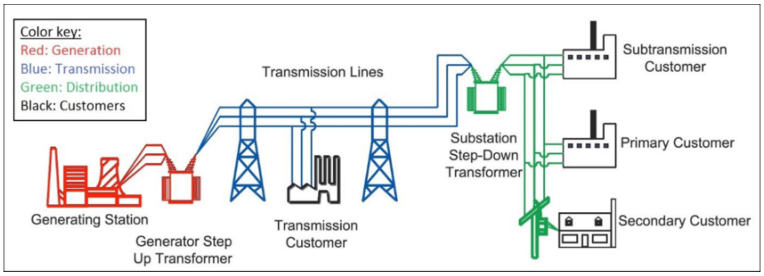


Figure 34: Power system schematic showing generation, transmission and distribution (Source: `en.wikipedia.org`)

Why?

Transmission lines are needed to bring electricity from power stations to the consumers. Transmission lines goes over long geographical distances.

Resistance of the transmission cables will have power losses equal to

$$I^2 R_{line}$$

where, R_{line} is the resistance of the transmission line. This power loss can be reduced (a) by decreasing the resistance and (b) by decreasing the magnitude of current. Decreasing the resistance leads to high cost of the infrastructure, and is not yet a practical.

If it is required to transmit P watts of power, which is equal to $V \times I$, increasing the voltage V would reduce the required line current I. And thus the transmission loss ($I^2 R_{line}$) will be reduced.

Increasing the voltage level at the generator before transmission is done using a **step-up transformer**. At the consumer end, the voltage level must be reduced to a level that is safer, 230 V in Singapore. **Step-down transformers** are used to bring the voltage level down from a higher level to a lower level. In practice, the change of voltage levels is done at different stages. Step-up and step-down transformers are essential parts of every power grid.

Time-varying current is needed for the operation of transformer commonly seen. Hence such a transformer cannot be used to change voltage levels with DC supply.

Ideal transformer

A transformer is a device that couples two AC circuits magnetically rather than through any direct conduction. A transformer consists of two or more coils of wire used to transfer electrical energy by means of a changing magnetic field as shown in Figure 35.

The AC voltage connected to the primary winding produces a time-varying magnetic field in the common magnetic core. According to *Faraday's law* of magnetic induction, voltage is induced in the secondary winding.

For a transformer with N_p turns in the primary winding and N_s turns in the secondary winding, the ratio between the secondary side voltage amplitude and the primary side voltage amplitude is

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

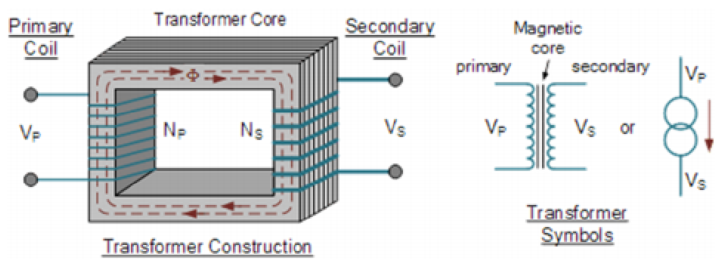


Figure 35: Basic structure of a transformer and circuit symbol used to represent a transformer

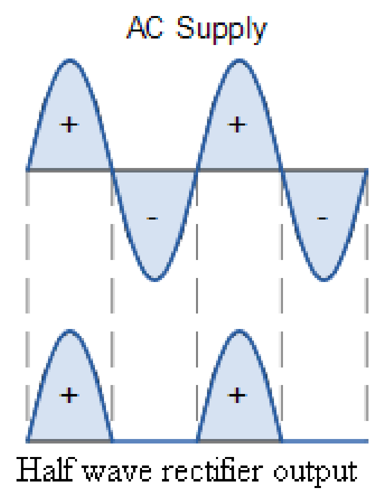
If the **turns-ratio** $\frac{N_s}{N_p}$ is less than 1, amplitude of the secondary side voltage is less than the amplitude on the primary side. The transformer is a step-down transformer. For step-up, the turns-ratio $\frac{N_s}{N_p}$ is greater than 1.

Here the assumption is that all the flux created by the primary winding are restricted within the magnetic core and thus are linked to the secondary coil.

If a load is connected to the secondary side winding, current will flow. Considering an ideal case, *i.e.*, no power is lost in the winding or in the core, the power is fully transferred from the primary side to the secondary side. Then,

$$\begin{aligned} V_p \times I_p &= V_s \times I_s, \\ \frac{V_s}{V_p} &= \frac{I_p}{I_s}. \end{aligned}$$

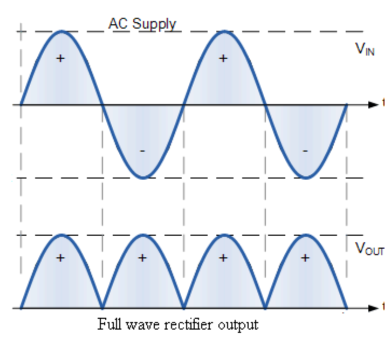
$$\frac{I_s}{I_p} = \frac{N_p}{N_s}.$$



AC-DC rectifier

A rectifier converts the AC voltage to DC voltage. The rectifier keeps the polarity on the DC side fixed by doing one of the following:

- Allow power to flow to the load when AC voltage has one polarity, and block power flow when the polarity is opposite. Such a rectifier is known as *half wave rectifier*. (Top waveform in the margin)
- Allow power flow to the load during both positive half and negative half of the AC voltage but channel the current in the same direction through the load. This type of rectifier is known as *full wave rectifier*. (Bottom waveform in the margin)



Diode

All diodes including LED are semiconductor devices with a PN junction. A diode conducts when applied voltage is positive on the P-side of the diode. This is called **forward bias**. If the applied voltage is positive on the N-side of the diode, ideally no current flows and the diode is said to be **reverse biased**. In practical diode, the reverse bias current is not zero but infinitesimally small which can be ignored, and the forward voltage must exceed certain threshold for the diode to conduct.

Half wave rectifier (HWR)

Only one diode is used. The load does not receive power when the diode is reverse biased. As power is delivered to the load during half-cycle only, the rectification efficiency is quite low.

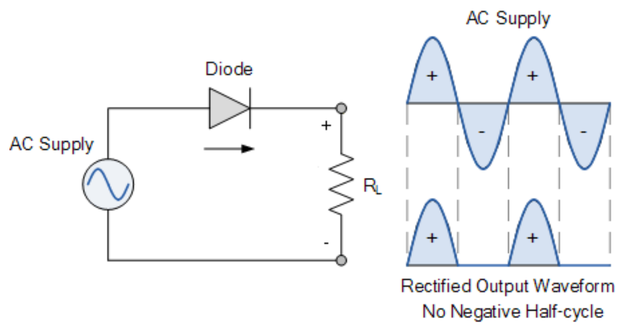


Figure 36: Half wave rectifier: as current flows in one direction only, the load does not get power during the negative half cycle of AC voltage.

Though the voltage appearing across the load has one polarity, it is not constant. The **ripple voltage**, *i.e.*, the fluctuation in the DC voltage is equal to the amplitude (V_m) of the AC voltage. The ripple can be reduced by adding a capacitor at the output of the rectifier (parallel to the load). The capacitor acts as a filter and does not allow fast-changing voltage at the output of the rectifier.

How capacitor filter helps to reduce ripple is explained later in the context of full wave rectification. However, the same explanation is valid for half wave rectifier.

Full wave rectifier (FWR)

There are different ways to make a full wave rectifier. But only **diode bridge rectifier** is included in this note. Four diodes are connected to form a bridge (Figure 37). At any instant, only two diodes are forward biased while the other two remain reverse biased.

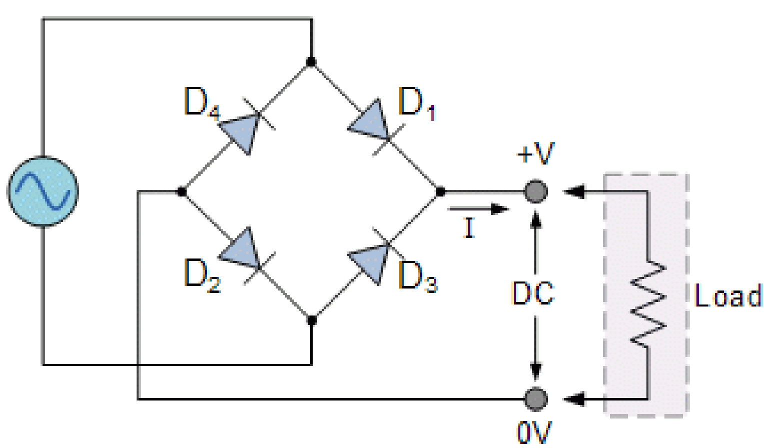


Figure 37: When the resistor is connected, current will flow in both cycles of the AC voltage. However, in both cycles, the flow of current is in the same direction as shown by the arrow.

During the positive half-cycle of the AC supply, diodes D1 and D2 conduct and AC supply is directly connected to the load (positive end of the supply to the positive end of the load and the negative to negative) as shown in Figure 38 (left). During the negative half cycle, diodes D3 and D4 conduct, effectively swapping the terminals of the AC supply and hence the load sees a positive voltage as shown in Figure 38 (right).

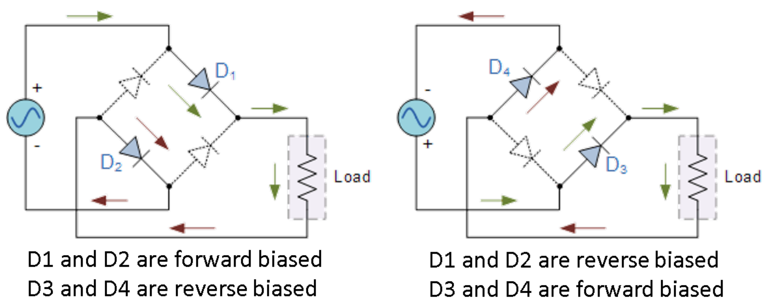


Figure 38: Flow of current during the positive half cycle (left) and negative half cycle (right) of AC voltage.

Note that the voltage at the load is DC with ripple of amplitude equal to V_m. This large ripple can be minimized by adding a capacitor as filter.

Average voltage of the FWR output:

Let the AC voltage be

$$v_{in}(t) = V_m \sin \omega t.$$

Assuming the diodes to be ideal, *i.e.*, no voltage drop across a conducting diode, the average voltage of the FWR output:

$$\begin{aligned} V_{o,avg} &= \frac{1}{T/2}\int_0^{T/2} V_m \sin(\omega t)\, dt, \\ &= \frac{2}{T}\frac{1}{\omega}[-V_m \cos(\omega t)]_0^{T/2}, \\ &= \frac{2V_m}{\omega T}[-\cos(\omega T/2) + \cos 0], \\ &= \frac{V_m}{\pi}[1+1], \\ &= \frac{2V_m}{\pi}. \end{aligned}$$

Average DC voltage is about 63% of V_m.

Diode bridge rectifier with capacitor filter

If a capacitor is connected at the output, as shown in Figure 39, the output voltage becomes smoother.

When the input AC voltage is higher than the capacitor voltage, one pair of diodes conduct. The load receives power directly from the AC supply, and simultaneously the capacitor gets charged, *i.e.*, energy is stored in the

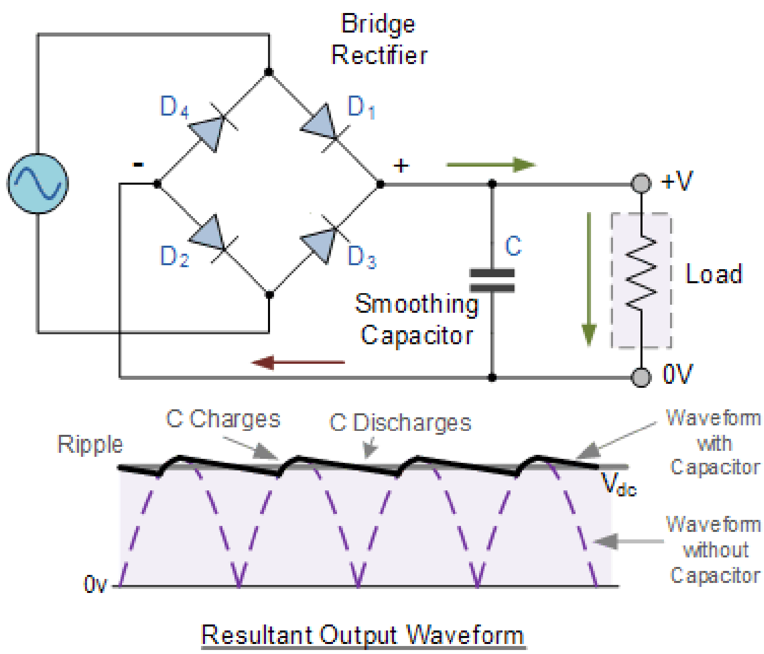


Figure 39: Diode bridge rectifier (FWR) with smoothing capacitor.

capacitor. The AC voltage changes continuously and, at some instant, falls below the capacitor voltage. When that happens, the diodes stop conducting and the load resistor gets power from the energy stored in the capacitor. Discharging of the capacitor causes its voltage to decrease. It continues to decrease until capacitor voltage falls below the AC voltage.

Fluctuation is still present in the output voltage, but the ripple is much smaller than that seen without the capacitor. The magnitude of the *peak-to-peak ripple voltage* depends on the values of the capacitance, the resistance, the output voltage level, and the period of the AC supply.

When the capacitor supplies energy to the resistor (diodes not conducting),

$$C\frac{dv}{dt} = \frac{V}{R},$$

$$\frac{dv}{dt} = \frac{V}{RC},$$

Larger is the value of the product RC, longer it takes for the capacitor to be discharged. We can select a large enough C so that the capacitor is discharged very slowly and it takes approximately $\frac{T}{2}$ for the capacitor voltage to fall below the AC voltage level. If ΔV is the change in capacitor voltage during this time,

$$\begin{aligned} \frac{\Delta V}{0.5T} &\approx \frac{V}{RC}, \\ \Delta V &= \frac{V}{2fRC}, \end{aligned}$$

where, ΔV is the magnitude of the *ripple voltage*, T is the period of the AC waveform, and $f = \frac{1}{T}$ is its frequency.

For HWR, the capacitor discharge time is $\approx$T. Using T in place of $0.5T$ in this expression, we can show that the ripple magnitude in HWR is

$$\Delta V = \frac{V}{fRC}.$$