Hydraulics application of the Free-surface Lattice Boltzmann method

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***Abstract*—In civil engineering field, large scale hydraulic structures are needed to be investigated accurately and concretely in terms of hydraulics. In this paper, a numerical procedure of Lattice Boltzmann approach for free surface flow is presented and examined with open channel hydraulic problems, e.g. dam break, flow over weir and spillway. It has been addressed that the Lattice Boltzmann method is a simple and very appropriate method for solving flow with a complex geometry, which is essential properties of civil engineering study. Our implemented code of the free-surface Lattice Boltzmann method is validated against a dam break experiment and result has shown reasonable consistency and accuracy. Validated code have been successfully applied to evaluate discharge coefficients for a sharp-crested weir and to investigate flow energy dissipation over a stepped spillway in the latter. Our research demonstration examples imply that the free surface Lattice Boltzmann method can solve complex open channel flow with simple, efficiently and accurately.**

Keywords—free surface flow; hydraulics; Lattice Boltzmann method; spillway

I. Introduction

The problem of fluid flow with free surface often occurs in civil engineering field whether in designing and construction stages of hydraulic structures, which interact with fluid flow, or in natural disaster such as flood inundation, storm surges and Tsunamis. Many afford of mathematical and numerical model on free surface prediction for flow had been made since accurate freely moving interface for two immiscible fluids had practically needed. There are two different approaches used to solving free surface problem among numerical techniques addressed as a single and two phase flow. In this paper, we adopted the single phase approach handled with the Lattice Boltzmann method to demonstrate its availability and accuracy for solving open channel hydraulics problems.

The Lattice Boltzmann method (LBM) has been considered efficient and robust mesoscopic numerical tool for computational fluid dynamics, since it had confirmed with classical fluid dynamic problems. Discretized Boltzmann equation, which can express macroscopic flow structure with kinematic and statistical approach as continuum Navier-Stokes equation, is a basic equation to be solved in the Lattice Boltzmann method. In the following sections, free surface Lattice Boltzmann method will be explained and examined on the specific problems.

II. Numerical procedure

A. Lattice Boltzmann method for fluid flow

In this paper, we use two dimensions with nine directions (D2Q9) lattice configuration [1] and Bhatnagar-Gross-Krook (BGK) collision operator [2] for discretized Boltzmann equation, which is given as,

$$\begin{aligned} &f_i(\mathbf{x}+\mathbf{c}_i\triangle t, t+\triangle t) - f_i(\mathbf{x},t) \\ &= -\frac{(f_i(\mathbf{x},t) - f_i^{eq}(\mathbf{x},t))}{\tau} + \triangle t F_i \end{aligned} \tag{1}$$

where $f_i(\mathbf{x},t)$ is the density distribution function, $\tau(= 3v + 1/2)$ is the relaxation time toward equilibrium state, v is the lattice viscosity term, c_i is the discrete velocities of lattice and F_i is the force term. Equilibrium distribution function, f_i^{eq}, for incompressible fluid flow can be determined by following expansion of Maxwell distribution:

$$f_i^{eq} = w_i\rho\Big[1 + \frac{\mathbf{c}_i\cdot\mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i\cdot\mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}\cdot\mathbf{u}}{2c_s^2}\Big] \tag{2}$$

where w_i and $c_s(= 1/\sqrt{3})$ are the lattice weighting factor (for D2Q9 $w_o = 4/9$,$w_i = 1/9$,$i = 1, 2, 3, 4$, and $w_i = 1/36, i = 5, 6, 7, 8$) and the speed of sound of lattice, ρ and $\mathbf{u}$ are the macroscopic density and velocity, which can be obtained by momentum of distribution function f_i as follows,

$$\rho = \sum_{i=0}^{9} f_i,\ \ \rho\mathbf{u} = \sum_{i=0}^{9} \mathbf{c}_i f_i + \frac{\mathbf{F}\triangle t}{2}. \tag{3}$$

For the force term, we use a scheme proposed by [3] and velocity is shifted by force and force term in (1) is

$$F_i = w_i\Big(1 - \frac{1}{2\tau}\Big)\Big[\frac{\mathbf{c}_i - \mathbf{u}}{c_s^2} + \frac{\mathbf{c}_i(\mathbf{c}_i\cdot\mathbf{u})}{c_s^4}\Big]\cdot\mathbf{F}. \tag{4}$$

Since the open channel flow problem often shows turbulent regime in the nature, so that we need to incorporate the turbulent model in simulation. We use sub-grid scale model introduced in [4], which adjust the relaxation time for stability and contribute the eddy viscosity term to the viscosity. Detailed description of the standard LBM and its theoretical background can be found in [1], [5] or [6] and many other publications about LBM. LBM has two computation steps called streaming and collision, in which free surface tracking algorithm need to be embedded.

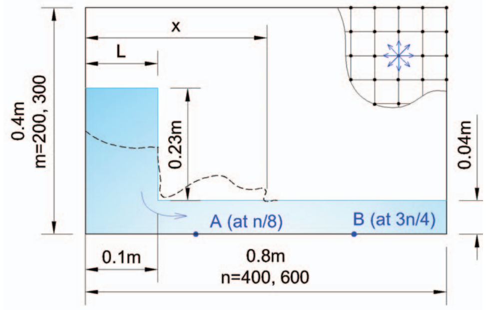

Fig. 1. Schematic sketch of dam break problem with a wet bottom. Lattice grid is depicted on upper right corner of the scheme. Only dimensions of height m and width n for numerical tests had been attached on the scheme.

B. Free-surface algorithm for Lattice Boltzmann method

General procedure of tracking free surface is completed by the lending concept of volume-of-fluid method (VoF). To perform concept of VoF based free surface tracking algorithm for standard LBM, there need to introduce two additional variables related to mass exchange and fluid fraction on a computational cell [7]. Fluid fraction determines cell type as a gas (G), fluid (F) or interface cell (IF). Since mass can be exchanged between cells and they are used to reveal the cell type, the mass and fluid fraction are related to each other as,

$$\epsilon(\mathbf{x},t) = \frac{m(\mathbf{x},t)}{\rho(\mathbf{x},t)}, \tag{5}$$

where ϵ is the fluid fraction, which has value 1 for a fluid and value 0 for a gas state, m is the cell mass and ρ is the density population of lattice node. The mass is evaluated from mass exchange between cells depends on the cell types, which must be updated every time steps regarding on the mass value. For the fluid cell, mass exchange is identical, whilst there is no mass exchange between gas cells or between the gas and the interface cell. Most critical mass exchange, $\triangle m_i(\mathbf{x},t)$, happens between interface cells and the mass exchange can be expressed as,

$$\triangle m_i(\mathbf{x},t+\triangle t) = s_e \frac{\epsilon(\mathbf{x}+\triangle t\mathbf{c}_i,t)+\epsilon(\mathbf{x},t)}{2}, \tag{6}$$

The mass exchange value depends on information about the neighborhood of the current cell and surrounding cells and s_e

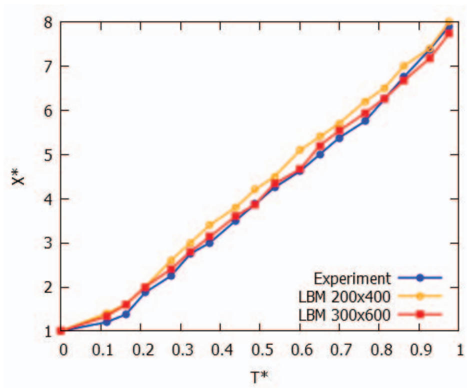

Fig. 2. Water-front positions in experimental and numerical tests.

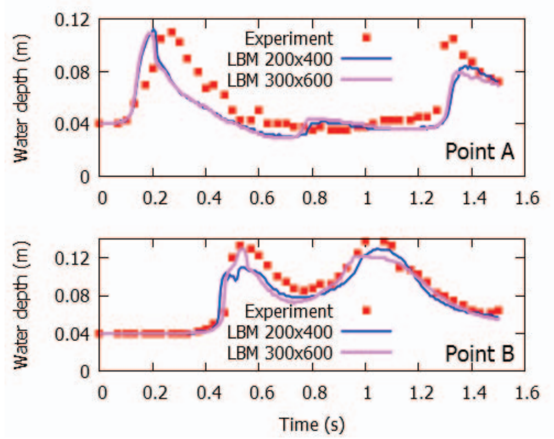

Fig. 3. Time evolution of water level at a control point A and B in experimental and numerical tests.

is given in Table 4.1 of [7] and generally defined by difference of coming and leaving distribution functions on the current cell. Mass at new time step on the cell is expressed by the addition of old cell mass and summation over lattice direction of the mass exchange value obtained by (6) in streaming step:

$$m(\mathbf{x},t+\triangle t) = m(\mathbf{x},t) + \sum_{i=1}^{9} \triangle m_i(\mathbf{x},t+\triangle t). \tag{7}$$

After mass evaluation on the cell, the interface cell might be transformed into the gas or fluid cell due to the conditions in (8), depending on that the neighborhood cells change their states. For instance, if the interface cell is an emptied cell, neighborhood fluid cells have to transform into interface cells.

$$\begin{aligned} &\text{IF}\rightarrow\text{F when } m(\mathbf{x},t+\triangle t) > (1+k)\rho(\mathbf{x},t+\triangle t) \\ &\text{IF}\rightarrow\text{G when } m(\mathbf{x},t+\triangle t) < (-k)\rho(\mathbf{x},t+\triangle t) \end{aligned} \tag{8}$$

where $k(=10^{-3})$ is a offset value avoiding from previously transformed cell is covered again. It is important to note that the interface cell satisfy above condition cannot change its state suddenly, since they need to distribute excess mass among to adjacent interface cells sake of mass conservation after collision step. The excess mass distribution is

$$m(\mathbf{x}+\triangle t\mathbf{c}_i) = m(\mathbf{x}+\triangle t\mathbf{c}_i) + m^{ex}(\eta_i/\eta_{total}), \tag{9}$$

where m^{ex} is negative or positive excess mass on the filled or emptied interface cell, η_{total} is the summation of all weights η_i, each of which is computed as,

$$\begin{aligned} \eta_i &= \begin{cases} \mathbf{n}\cdot\mathbf{c}_i, & \text{if } \mathbf{n}\cdot\mathbf{c}_i > 0 \\ 0, & \text{if otherwise} \end{cases} \quad \text{for filled cells,and} \\ \eta_i &= \begin{cases} -\mathbf{n}\cdot\mathbf{c}_i, & \text{if } \mathbf{n}\cdot\mathbf{c}_i < 0 \\ 0, & \text{if otherwise} \end{cases} \quad \text{for emptied cells,} \end{aligned} \tag{10}$$

where n is the normal vector on the interface cell pointed to the gas state. It can be found by central difference between fluid fraction values of two cells [8]. When the interface cell becomes the fluid cell, neighboring gas cell must convert into the interface cell. For newly generated IF cell, the mass, fluid

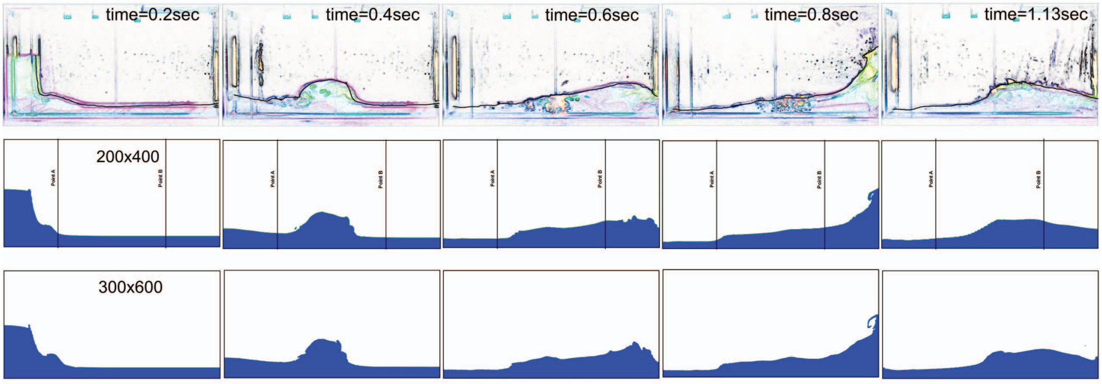

Fig. 4. Time sequence image comparison of experimental and numerical dam break tests with the wet bed.

fraction and distribution functions must be initialized for the next time step of simulation. Distribution functions for the new interface cell can be constructed by equilibrium distribution function using (2), where use an average velocity and density among neighboring nongas cells. Boundary condition in LBM method has to be imposed in terms of the distribution functions. So, after the streaming with mass exchange, unknown distribution function on the interface cell, which satisfies condition that $\mathbf{n} \cdot \mathbf{c}_i > 0$ and distribution function had to be streamed from gas cells, need to be reconstructed according to the free surface boundary condition as follow,

$$f_{\tilde{i}}(\mathbf{x}, t + \triangle t) = f_i^{eq}(\rho_A, \mathbf{u}) + f_{\tilde{i}}^{e}q(\rho_A, \mathbf{u}) - f_i(\mathbf{x}, t), \quad (11)$$

where ρ_A is a gas density and notation $\tilde{i}$ stands on the opposite direction of i. With this condition, gas state can be negligible for simulation since liquid has lower kinematic viscosity than gas.

III. Validation of free surface simulation

A. *Dam break analysis*

First we applied the Free-surface LBM on a dam break benchmark problem to validate capability of the algorithm. Simulated result compared against the experimental test conducted with same geometrical configuration shown in Fig. 1. For the wall, slip boundary condition is imposed. To validate numerical simulation, we measured dimensionless water-front position X^* at dimensionless time T^* [8] as,

$$T^* = t\sqrt{ng/L}, \; X^* = x/L \quad (12)$$

and time evolution of water depth at specific points (A and B) depicted in Fig. 1. In (12), $n(= H/L)$ is the aspect ratio of water column, H and L are the initial height and width of water column, x is the water-front displacement at time t. We conducted two numerical cases, grids are 200x400 and 300x600, to investigate grid resolution independence and time steps $dt_{400} = 0.00007sec$ and $dt_{600} = 0.00006sec$ were used, respectively. The parameters used in simulations are determined through parametrization formula given in [8]. It seemed that the grid resolution has slightly influence in numerical result since curve of case of 300x600 has been plotted very nearly with experimental one in Fig. 2. In the numerical experiment, plate gate, separating water column from the wet bottom in the tank, had not yet included. Effect of the gate removing in the lab experiment appears with water depth evolution on the point A at time $0.2sec$ to $0.4sec$ in Fig. 3, where experiment's time had been delayed. Except some offset, time evolution of water depths have same tendency with experimental one in Fig. 3. Some discrepancies are considered what might be some deficiency of data extraction from video frame of the experiment. Because normal video camera had been used to capture phenomena in laboratory experiment and the image has some perspective representation, which can be seen in Fig. 4. For sake of convenience to printed document, we used edge detecting effect on the each frame of image and black line to express ideal water surface avoiding doubt with perspective surfaces in Fig. 4. Free surface shape for three cases are in good similarity except flying water droplet and splash on the wall. Water splash on the wall and flying droplets are difficult to be captured in small scale LBM simulation since the interface between water and air phase is expressed by continuous single layer of IF cells. Based on the validation process, it can be claimed that single phase simulation of LBM for free surface problem has a substantial capability.

IV. Hydraulics application

A. *Flow over weirs*

Weirs are well studied structures by theoretically and experimentally, but less effort has been made by numerically because of perfection and priority. Matured weirs measure flow discharge very precisely, if a best fit discharge coefficient curve has determined accurately. Advance in the numerical simulation, there exist many opportunity to develop brand-new weir or flume. In this study we simulate sharp-crested rectangular weir in two dimensional space to determine the discharge coefficient and flow pattern over the weir. Weirs and spillways are same hydraulics manner in inflow and outflow a terms of boundary condition. We impose Zou/He boundary condition [5] at the inflow and zero gradient open boundary condition [6] at the outflow. Geometry of simulation is given in Fig. 5 with its numerical results. Discharge equation of

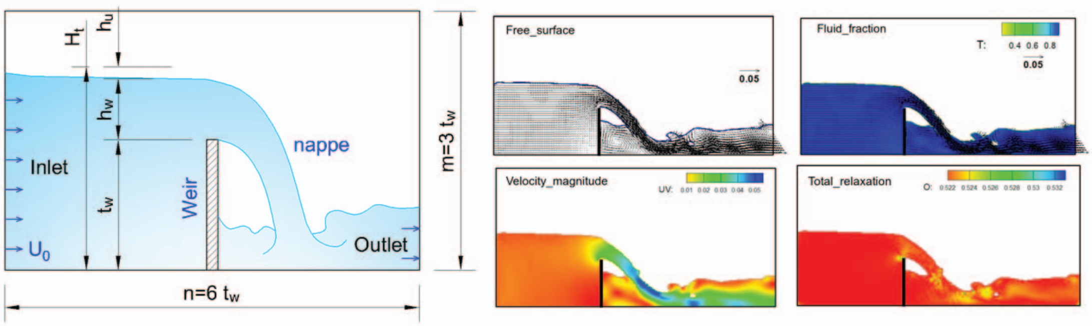

Fig. 5. Sharp-crested weir geometry and numerical simulation result at $t_w = 0.15$ m and unit discharge $q = 0.044\ m^2/s$. Results are shown in terms of lattice unit. To show influence of eddy viscosity term, total relaxation times are plotted.

sharp-crested weir [11] with unit width, mentioned here as unit discharge, can be simplified as,

$$q = \frac{2}{3} C_d h_w^{(3/2)} \sqrt{2g} \tag{13}$$

where C_d and h_w are the discharge coefficient and the static head over the crest. Practically, the discharge coefficient depends on many parameters such as a flow characteristic, channel geometry and ratio of crest height to static head. Ignoring the channel geometry effect on the discharge, we examined discharge coefficients of several discharge cases and compared to result given in [10], where study had conducted by physical experiment and commercial CFD tool, Fluent. Since this research is devoted to demonstrate free surface LBM in open channel hydraulics, we only perform seven numerical simulations on the configuration with crest height $t_w = 0.15$ m, and defined discharge coefficient are plotted with result taken from [10] in Fig. 6. We have used following parameters for time step and grid space, $dt = 0.000074$ sec and $dx = 0.0025$ m. In our simulation, it is observed that ratio of crest height to static head on the crest is main parameter to indicate flow characteristic of sharp-crested weir. If it exceeds over unit, depends on the downstream situation, submerged flow condition can be observed. Closed flow circulation was created between weir and nappe when case of $h_w/t_w > 0.8$, because outlet boundary was first order zero gradient boundary condition. When ratio become $h_w/t_w < 0.2$, nappe flow had totally adhered to the weir surface.

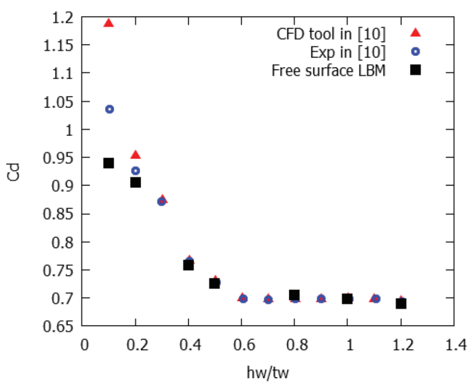

Fig. 6. Comparison of discharge coefficient defined by LBM, CFD tool and experiment for different ratio of h_w/t_w.

B. Spillways

A spillway is a important structure for dam safety and designing and operation of this structure is quite difficult. High energy water flow acting through spillway surface cause a damage for structure like erosion and corrosion. Furthermore, water flow with undissipated energy erode river bed at downstream of such structures. Spillway or significant hydraulic structures are mainly designed by physical model, which has scale effect and requires cost and time. Here, we model small scale stepped spillway to demonstrate free surface LBM to use in investigation of the important hydraulic structures. Two different step configurations are considered to evaluate hydraulic performances, i.e. big stepped and small stepped. This study did not intend to dive into detailed investigation of stepped spillway. Therefore we measured water surface and average velocity though horizontal to expose energy dissipation. Depends on approaching energy, it is appeared that first step, usually designed smaller than other steps, has big void in backward in both simulation cases, which say that the step height must meet with the design procedure of this step. In spillway with big step, nappe flow has starts from first step and continued until last step with quickly appear of skimming flow in the middle, shown in Fig. 7. Whereas, stepped spillway with small steps shows good performance on reducing energy (see Fig. 8) and there skimming flow condition has dominated.

V. Conclusion

In this paper, the Lattice Boltzmann method with free surface tracking simple algorithm is explained and employed on classical open channel problems. Implemented code is validated to experimental study of dam break problem with the wet bed. The numerical result was in good agreement with experimental one and then directly applied to the weir and spillway study. Result of sharp-crested weir flow also compared with result of [10] and accuracy was agreeable. Spillway simulation is fully intended to demonstrate performance of numerical method and goal has achieved with energy dissipation advance in small stepped spillway configuration.

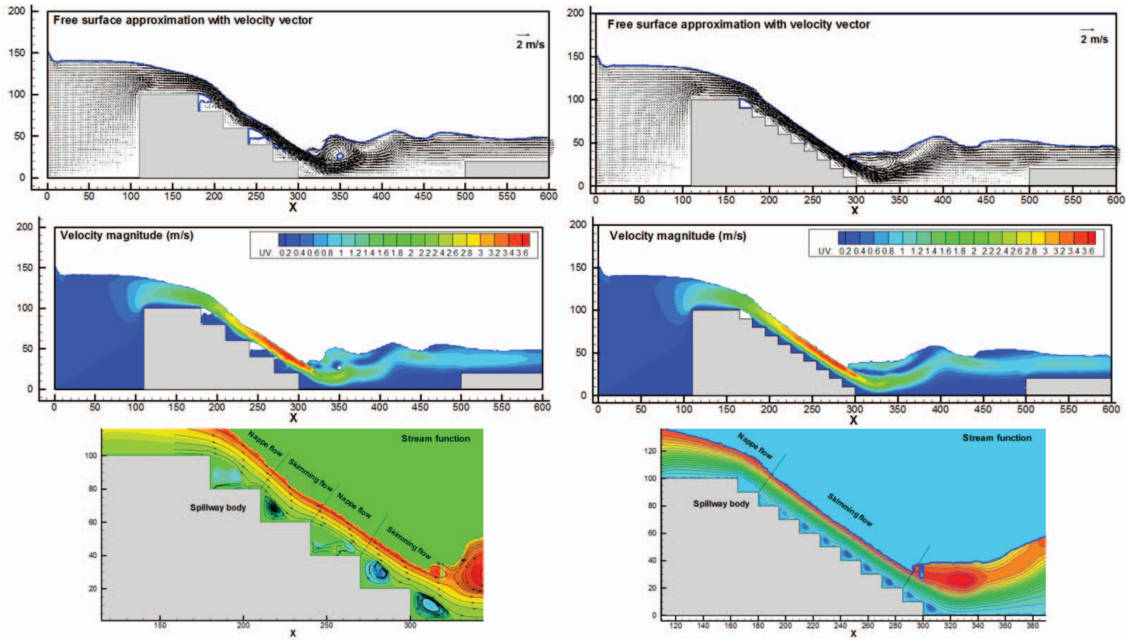

Fig. 7. Two case of stepped spillway simulation results. Spillway heights are 1.0 m and a stilling basin is included to dissipate energy in the simulation. First column shows the result with the step height 0.2 m and the length 0.3 m, while second line exhibits result with the step height 0.1 m and the length 0.2 m.

In our study, it is occurred that serial numerical approach on personal computer does not efficient for memory application in simulation of large-scale hydraulics problem. So in future, we extend our code to parallel operation and exploit inherent nature of the method and the current computational advance.

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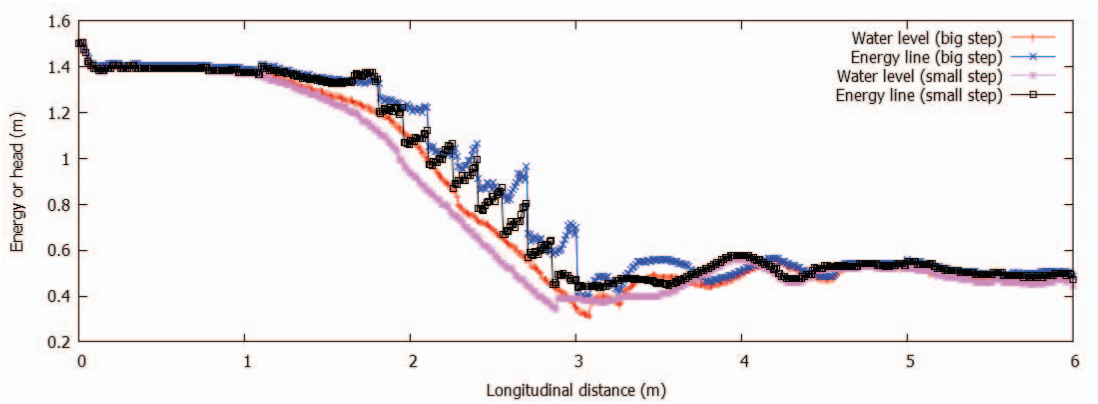

Fig. 8. Energy head over the spillway at two simulation cases. The energy is defined as addition of the static head and the velocity head.