APM466 A3

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Questions - 100 points

1. *(40 points)*

   (a) The three year transition probability matrix is:

   $$P^3 = \begin{pmatrix} 0.405 & 0.281 & 0.148 & 0.166 \\ 0.145 & 0.235 & 0.122 & 0.468 \\ 0.133 & 0.204 & 0.113 & 0.55 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   because: It is obtained by multiplying the 1 year matrix 3 times

   (b) If company X is currently in a "crisis" solvency state, the probability that they will default within the next month is $\frac{1}{40}$, because: by solving for the matrix $P^{\frac{1}{12}}$ we can also notice that within the next month, companies stay in the same state almost surely.

   (c) $\lim_{t\to\infty} P^t = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ Which is obtained by solving $((P_i^G)^\perp - I_{4x4}) \cdot \overrightarrow{\pi} = \overrightarrow{0}$
   These are called the limiting probabilities of P

   (d) Yes/No, because: each probability $p_{ij}^n > 0 for j = 1, 2, 3$ if n is finite. This means that these probabilities $p_{ij}^n - > 0$ for j=1,2,3 $p_{ij}^n - > 1$ for j=4, as n goes to infinity

2. *(40 points)*

   (a) Since we know that $V = \sum_{i=1}^n P_i \cdot e^{-(r)\cdot i} \cdot q_i$; where q is the recovery rate if going through default, setting $V_i^I = e^{-(r)\cdot i} \cdot P_i^I \cdot (q_i + 0.25 \cdot (1 - q_i)$ setting for $V_i^G = P_i^G \cdot e^{(r+h_i)\cdot i}$ we obtain that $P_i^G \cdot e^{-(h_i)\cdot i} = P_i^I \cdot (q_i + 0.25 \cdot (1 - q_i) = P_i^I \cdot (0.75 \cdot q_i + 0.25$
   We then obtained $h_i = \frac{1}{i} \cdot \log(\frac{P_i^G}{P_i^I \cdot (\frac{1}{4} + \frac{3}{4} \cdot q_i)})$

   (b) The 2 state markov chain model is

   $$P^I = \begin{pmatrix} q & (1-q) \\ 0.25 & 0.75 \end{pmatrix}$$

   Further if 0.25¡q¡1 then $P^I \in GL(2, R)$
   $(P^I)^i = \sum_{k=0}^\infty \binom{i}{k} (P^I - I_{2x2})^k$ $(P^I)^i = \sum_{k=0}^\infty \binom{i}{k} \begin{pmatrix} \frac{4}{3}(e^{-h_i\cdot i}) - \frac{4}{3} & \frac{-1}{3} - \frac{4}{3}e^{-h_i\cdot i} \\ 0.25 & -0.25 \end{pmatrix}^k$

(c) Let us define $C_i = (P_i^I - A)^+$ from 2b) $P_i^I = \frac{P_i^G \cdot e^{-h_i \cdot i}}{\frac{1+3q}{4}}$
$<->$ $A = (P_i^I - C_i)^+$ $<->$ $A = (\frac{P_i^G \cdot e^{-h_i \cdot i}}{\frac{1+3q}{4}} - C_i)^+$

(d) . $\partial_{h_i} A = -\frac{P_i^G \cdot i \cdot e^{-h_i \cdot i}}{\frac{1+3q}{4}}$

3. *(20 points)*

   (a) *Simplification 1:* The first simplification made in Merton's Credit risk model is that we are using a european long call and a european short put option to model the asset with the liabilities, when in reality the underlying of the option is not a tradeable asset.

   (b) *Simplification 2:* The second simplification is that the time to maturity, the time when the liabilities are due, is unique, when in reality debt could be due at multiple different dates.

   (c) *Assumption 1:* The first assumption is that the mean and volatility of the underlying assets are constant.

   (d) *Assumption 2:* The second assumption is that the probability measure is risk-neutral, which implies an efficient market and no arbitrage opportunity.