MIT integration bee - 2012

Notación y Consideraciones Generales

A lo largo del documento vamos a utilizar la siguiente notación, en algunos problemas más complejos utilizaremos la notación usual:

$$\{\ f(x)\ \}_a^b := \int_a^b f(x)\ dx \ \ ,\ \ [\ f(x)\ ] := \frac{d}{dx} f(x)$$

Recursos obtenidos de: https://math.mit.edu/~yyao1/integrationbee.html

Herramientas que te pueden ser útiles:

- Calculadora gráfica: https://www.desmos.com/calculator?lang=es
- Calculadora de integrales: https://mathdf.com/es/
- Calculadora de integrales: https://www.wolframalpha.com/
- Motor de busqueda para fórmulas en latex https://approach0.xyz/search/
- Te ayuda con ideas: https://chat.deepseek.com

Ejercicio 1

$$\int \frac{dx}{\sqrt{x}-1}$$

$$--- \text{ Demostración } ---$$

$$\begin{aligned} I &= 2\left\{ \frac{t}{t-1} \right\} && (1) \\ &= 2\,(\,t + \ln(|t-1|)\,) + C \end{aligned}$$

- (1): $\left\{ \begin{array}{rcl} x &=& t^2 \\ dx &=& 2tdt \end{array} \right.$

Ejercicio 2

$$\int x^{1/4} \log(x) dx$$

$$--- \text{ Demostración } ---$$

$$\begin{aligned} I &= \frac{4}{5}x^{5/4}\ln(x) - \frac{4}{5}\left\{\ x^{1/4}\ \right\} && (1) \\ &= \frac{4}{5}x^{5/4}\ln(x) - \frac{16}{25}x^{5/4} + C \end{aligned}$$

- (1): $\left| \begin{array}{c} D \\ \ln(x) \\ \frac{1}{x} \end{array} \right| \begin{array}{c} I \\ x^{1/4} \\ \frac{4}{5}x^{5/4} \end{array} \right|$

Ejercicio 3

$$\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$$

— — — Demostración — — —

$$\begin{aligned}
I &= \left\{ \frac{1}{\sqrt{x}(1+\sqrt{x})\sqrt{1-x}} \right\} \\
&= 2\left\{ \frac{1}{(1+t)\sqrt{1-t^2}} \right\} && (1) \\
&= 2\left\{ \frac{\cos(\theta)}{(\ 1+\sin(\theta)\ )|\cos(\theta)|} \right\} && (2) \\
&= \pm 2\left\{ \frac{1}{1+\sin(\theta)} \right\} \\
&= \pm 2\left\{ \frac{\sec(\theta)}{\sec(\theta)+\tan(\theta)} \right\} \\
&= \pm 2\left\{ \frac{\sec(\theta)(\ \sec(\theta)+\tan(\theta)\ )}{(\ \sec(\theta)+\tan(\theta)\ )^2} \right\} \\
&= \mp 2\frac{1}{\sec(\theta)+\tan(\theta)} + C && (3)\ ,\ (4)
\end{aligned}$$

- (1): $\begin{cases} x &= t^2 \\ dx &= 2t\ dt \end{cases}$
- (2): $\begin{cases} t &= \sin(\theta) \\ dt &= \cos(\theta)\ d\theta \end{cases}$
- (3): $[\sec(\theta)+\tan(\theta)] = \sec(\theta)(\sec(\theta)+\tan(\theta))$
- (4): $\left[\frac{1}{f(x)}\right] = \frac{-f'(x)}{f^2(x)}$

Ejercicio 4

$$\int \frac{dx}{\sqrt{x}(\sqrt[4]{x}+1)^{10}}$$

— — — Demostración — — —

$$\begin{aligned}
I &= 4\left\{ \frac{t}{(t+1)^{10}} \right\} && (1) \\
&= 4\left\{ \frac{u-1}{u^{10}} \right\} && (2) \\
&= 4\left(-\frac{u^{-8}}{8} + \frac{u^{-9}}{9} \right) + C
\end{aligned}$$

- (1): $\begin{cases} x &= t^4 \\ dx &= 4t^3\ dt \end{cases}$
- (2): $\begin{cases} u &= t+1 \\ du &= dt \end{cases}$

Ejercicio 5

$$\int_0^1 \sin(\cos^{-1}(x))dx$$

— — — Demostración — — —

$$
\begin{aligned}
I &= \left\{ \sin^2(\theta) \right\}_0^{\pi/2} && (1)\\
&= \frac{1}{2}\left\{ 1-\cos(2\theta) \right\}_0^{\pi/2} && (2)\\
&= \frac{1}{2}\left(\theta - \frac{\sin(2\theta)}{2} \right)_0^{\pi/2}\\
&= \frac{\pi}{4}
\end{aligned}
$$

- (1): $\left\{ \begin{array}{rcl} x &=& \cos(\theta) \\ dx &=& -\sin(\theta)\, d\theta \end{array} \right.$
- (2): $\cos(2x) = 1 - 2\sin^2(x)$

Ejercicio 6

$$\int \frac{dx}{\sqrt{1-4x-x^2}}$$

$$\color{green}{---\text{ Demostración }---}$$

$$
\begin{aligned}
I &= \left\{ \frac{1}{\sqrt{5-(x+2)^2}} \right\}\\
&= \frac{1}{\sqrt{5}}\left\{ \frac{1}{\sqrt{1-\left(\frac{x+2}{\sqrt{5}}\right)^2}} \right\}\\
&= \arctan\left(\frac{x+2}{\sqrt{5}}\right) + C
\end{aligned}
$$

- (1): $\left\{ \begin{array}{rcl} x &=& \cos(\theta) \\ dx &=& -\sin(\theta)\, d\theta \end{array} \right.$
- (2): $\cos(2x) = 1 - 2\sin^2(x)$

Ejercicio 7

$$\int_{1/4}^{1/2} \left\lfloor \log\left\lfloor \frac{1}{x} \right\rfloor \right\rfloor dx$$

$$\color{green}{---\text{ Demostración }---}$$

$$
\begin{aligned}
I &= \left\{ \left\lfloor \log \left\lfloor \frac{1}{x} \right\rfloor \right\rfloor \right\}_{1/4}^{1/3} + \left\{ \left\lfloor \log \left\lfloor \frac{1}{x} \right\rfloor \right\rfloor \right\}_{1/3}^{1/2} \\
I_1 &= \left\{ \left\lfloor \log \left\lfloor \frac{1}{x} \right\rfloor \right\rfloor \right\}_{1/4}^{1/3} \\
&= \{ \lfloor \ln(3) \rfloor \}_{1/4}^{1/3} \qquad (1) \\
&= \frac{1}{12} \qquad (2) \\
I_2 &= \left\{ \left\lfloor \log \left\lfloor \frac{1}{x} \right\rfloor \right\rfloor \right\}_{1/3}^{1/2} \\
&= \{ \lfloor \ln(2) \rfloor \}_{1/3}^{1/2} \qquad (3) \\
&= 0 \\
I &= \frac{1}{12}
\end{aligned}
$$

- (1): Si $1/4 < x < 1/3$, entonces $3 < \frac{1}{x} < 4$
- (2): $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- (3): Si $1/3 < x < 1/2$, entonces $2 < \frac{1}{x} < 3$

Ejercicio 8

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin(x)}$$

$$--- \text{Demostración} ---$$

$$
\begin{aligned}
I &= \left\{ \frac{\sec(x)}{\sec(x)+\tan(x)} \right\}_0^{\pi/2} \\
&= \left\{ \frac{\sec(x)(\sec(x)+\tan(x))}{(\sec(x)+\tan(x))^2} \right\}_0^{\pi/2} \\
&= -\left(\frac{1}{\sec(x)+\tan(x)} \right)_0^{\pi/2} \\
&= 1
\end{aligned}
$$

- (1): $[\sec(x)+\tan(x)] = \sec(x)(\sec(x)+\tan(x))$
- (2): $\left[\frac{1}{f(x)}\right] = \frac{-f'(x)}{f^2(x)}$

Ejercicio 9

$$\int_1^{2011} \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}} dx$$

$$--- \text{Demostración} ---$$

$$
\begin{aligned}
I &= \left\{ \frac{\sqrt{x}}{\sqrt{\alpha - x} + \sqrt{x}} \right\}_1^{\alpha-1} \\
&= \left\{ \frac{\sqrt{\alpha - t}}{\sqrt{t} + \sqrt{\alpha - t}} \right\}_1^{\alpha-1} && (1) \\
2I &= \left\{ \frac{\sqrt{x}}{\sqrt{\alpha - x} + \sqrt{x}} + \frac{\sqrt{\alpha - x}}{\sqrt{x} + \sqrt{\alpha - x}} \right\}_1^{\alpha-1} \\
&= \alpha \\
I &= \frac{\alpha}{2}
\end{aligned}
$$

- (1): $\left\{ \begin{aligned} x &= \alpha - t \\ dx &= -dt \end{aligned} \right.$

Ejercicio 10

$$
\int \frac{x - 1}{(x+1)\sqrt{x^3 + x^2 + x}} dx
$$

$$
\color{red}{- - - \text{ Demostración } - - -}
$$

$$
\begin{aligned}
I &= 2 \left\{ \frac{t^2 - 1}{(t^2+1)\sqrt{t^4 + t^2 + 1}} \right\} && (1) \\
&= 2 \left\{ \frac{t^2 - 1}{t^2} \frac{t^2}{t^2 + 1} \frac{1}{t\sqrt{t^2 + \frac{1}{t^2} + 1}} \right\} \\
&= 2 \left\{ \left(1 - \frac{1}{t^2}\right) \frac{1}{t + \frac{1}{t}} \frac{1}{\sqrt{(t + \frac{1}{t})^2 - 1}} \right\} \\
&= 2 \left\{ \frac{1}{u\sqrt{u^2 - 1}} \right\} && (2) \\
&= 2 \operatorname{arcsec}(u) + C && (3) \\
&= 2 \operatorname{arcsec} \left(\frac{x + 1}{\sqrt{x}} \right) + C
\end{aligned}
$$

- (1): $\left\{ \begin{aligned} x &= t^2 \\ dx &= 2t\, dt \end{aligned} \right.$
- (2): $\left\{ \begin{aligned} u &= t + \frac{1}{t} \\ du &= (1 - \frac{1}{t^2})\, dt \end{aligned} \right.$

Nota:

1. ¿Cuándo podemos utilizar la sustitución $u = t + \frac{1}{t}$?.
   - $\left\{ \begin{aligned} u &= \frac{t^2+1}{t} \\ du &= \frac{t^2-1}{t^2}\, dt \end{aligned} \right.$
   - $\frac{u}{u'} = t\,\frac{t^2+1}{t^2-1}$, $\frac{u'}{u} = \frac{t^2-1}{t(t^2+1)}$
   - $(t + \frac{1}{t})^2 = t^2 + \frac{1}{t^2} + 2$

Ejercicio 11

$$
\int_{-1}^{0} \frac{x^4 + 4x^3 + 6x^2 + 4x + 1}{x^3 - 3x^2 + 3x - 1} dx
$$

$$--- \text{Demostración} ---$$

$$\begin{aligned}
I &= \left\{ \frac{(x+1)^4}{(x-1)^3} \right\}_{-1}^{0} && (1)\\
&= \left\{ \frac{(t+2)^4}{t^3} \right\}_{-2}^{-1} && (2)\\
&= \left\{ t + 8 + 24t^{-1} + 32t^{-2} + 16t^{-3} \right\}_{-2}^{-1}\\
&= \left(\frac{t^2}{2} + 8t + 24\ln|t| - 32t^{-1} - 8t^{-2} \right)_{-2}^{-1}\\
&= \frac{33}{2} - 24\ln(2)
\end{aligned}$$

- (1): Triángulo de pascal
- (2): $\left\{ \begin{array}{rcl} t &=& x-1 \\ dt &=& dx \end{array} \right.$

Ejercicio 12

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x} \right) dx$$

$$--- \text{Demostración} ---$$

$$I = \sin(x)\ln(x) + C \quad (1)$$

- (1): $[f(x)\ln(x)] = f'(x)\ln(x) + \frac{f(x)}{x}$

Ejercicio 13

$$\int \frac{dx}{x^3 - x}$$

$$--- \text{Demostración} ---$$

$$\begin{aligned}
I &= \left\{ \frac{1}{x(x^2-1)} \right\}\\
&= -\left\{ \frac{1}{\sin(\theta)\cos(\theta)} \right\} && (1)\\
&= -2\left\{ \frac{1}{\sin(2\theta)} \right\} && (2)\\
&= -2\left\{ \csc(2\theta) \right\}\\
&= \ln|\csc(2\theta) + \cot(2\theta)| + C && (3)\\
&= \frac{1}{2}\ln\left|1 - \frac{1}{x^2}\right| + C
\end{aligned}$$

- (1): $\left\{ \begin{array}{rcl} x &=& \sin(\theta) \\ dx &=& \cos(\theta)\, d\theta \end{array} \right.$
- (2): $[\sin(2\theta)] = 2\sin(\theta)\cos(\theta)$
- (3): $[\csc(\theta) + \cot(\theta)] = -\csc(\theta)(\csc(\theta) + \cot(\theta))$

Ejercicio 14

$$\int_0^{1/2} \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}} dx$$

$$\textcolor{green}{---\text{ Demostración }---}$$

$$\begin{aligned} I &= \{ \sin(u)\ u \}_0^{\pi/6} && (1) \\ &= (-u\cos(u) + \sin(u))_0^{\pi/6} && (2) \\ &= \frac{1}{2} - \pi\frac{\sqrt{3}}{12} \end{aligned}$$

- (1): $\left\{ \begin{array}{lcl} u &=& \arcsin(x) \\ du &=& \frac{1}{\sqrt{1-x^2}}\ dx \end{array} \right.$
- (2): $\left| \begin{array}{c|c} D & I \\ u & \sin(u) \\ 1 & -\cos(u) \\ 0 & -\sin(u) \end{array} \right|$

Ejercicio 15

$$\int_0^1 x(1-x)^{99} dx$$

$$\textcolor{green}{---\text{ Demostración }---}$$

$$\begin{aligned} I &= \{ u^{99}(1-u) \}_0^1 && (1) \\ &= \left(\frac{u^{100}}{100} - \frac{u^{101}}{101}. \right)_0^1 \\ &= \frac{1}{10100} \end{aligned}$$

- (1): $\left\{ \begin{array}{lcl} u &=& 1-x \\ du &=& -dx \end{array} \right.$

Ejercicio 16

$$\int_0^{\pi/2} \frac{\sin(4x)}{\sin(x)} dx$$

$$\textcolor{green}{---\text{ Demostración }---}$$

$$\begin{aligned} I &= 2\left\{ \frac{2\sin(2x)\cos(2x)}{\sin(x)} \right\}_0^{\pi/2} && (1) \\ &= 4\left\{ \frac{\sin(x)\cos(x)\cos(2x)}{\sin(x)} \right\}_0^{\pi/2} && (1) \\ &= 4\{ \cos(x)\cos(2x) \}_0^{\pi/2} \\ &= 2\{ \cos(x) + \cos(3x) \}_0^{\pi/2} && (2) \\ &= 2\left(\sin(x) + \frac{\sin(3x)}{3} \right)_0^{\pi/2} \\ &= \frac{4}{3} \end{aligned}$$

- (1): $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- (2): $2\cos(\alpha)\cos(\beta) = \cos(\alpha-\beta) + \cos(\alpha+\beta)$

Ejercicio 17

$$\int \frac{x^{-\frac{1}{2}}}{1+x^{\frac{1}{3}}} dx$$

$$\color{green}{---\ \text{Demostración}\ ---}$$

$$\begin{aligned} I &= 6\left\{ \frac{t^2}{1+t^2} \right\} && (1) \\ &= 6\,(\,t-\arctan(t)\,)+C \end{aligned}$$

- (1): $\left\{ \begin{array}{rcl} x &=& t^6 \\ dx &=& 6t^5\,dt \end{array} \right.$

Ejercicio 18

$$\int \frac{dx}{\sqrt{2x^2-1}}$$

$$\color{green}{---\ \text{Demostración}\ ---}$$

$$\begin{aligned} I &= \left\{ \frac{1}{\sqrt{(\sqrt{2}x)^2-1}} \right\} \\ &= \frac{1}{\sqrt{2}}\left\{ \frac{\sec(\theta)\tan(\theta)}{|\tan(\theta)|} \right\} && (1) \\ &= \frac{1}{\sqrt{2}}\{\,\sec(\theta)\,\} && (2) \\ &= \frac{1}{\sqrt{2}}\ln|\sec(\theta)+\tan(\theta)|+C \\ &= \frac{1}{\sqrt{2}}\ln\left|\sqrt{2}x+\sqrt{2x^2-1}\right|+C \end{aligned}$$

- (1): $\left\{ \begin{array}{rcl} \sqrt{2}x &=& \sec(\theta) \\ \sqrt{2}dx &=& \sec(\theta)\tan(\theta)\,dt \end{array} \right.$
- (2): Dado que $2x^2-1>0$, entonces podemos concluir que $|\sec(\theta)|>1$, en la sustitución (1) se puede tomar a $\theta \in (0,\pi/2)\cup(\pi/2,3\pi/2)$. Así si $\tan(\theta)<0$, entonces $\sec(\theta)<0$ y $\frac{\sec(\theta)\tan(\theta)}{|\tan(\theta)|}=|\sec(\theta)|$. Si $\tan(\theta)>0$, entonces $\sec(\theta)>0$ y $\frac{\sec(\theta)\tan(\theta)}{|\tan(\theta)|}=|\sec(\theta)|$.

Ejercicio 19

$$\int \frac{dx}{\sqrt{e^x-1}}$$

$$\color{green}{---\ \text{Demostración}\ ---}$$

$$\begin{aligned} I &= -2\left\{ \frac{e^t}{\sqrt{1-e^{2t}}} \right\} && (1) \\ &= -2\left\{ \frac{1}{\sqrt{1-u^2}} \right\} && (2) \\ &= -2\arcsin(u)+C \end{aligned}$$

- (1): $\left\{ \begin{array}{rcl} x &=& -2t \\ dx &=& -2\,dt \end{array} \right.$

- (2): $\left\{ \begin{array}{lcl} u & = & e^t \\ du & = & e^t\, dt \end{array} \right.$

Ejercicio 20

$$\int \frac{x}{x^4+4} dx$$

$$— — — \text{ Demostración } — — —$$

$$\begin{aligned} I &= \frac{1}{2}\left\{ \frac{1}{u^2+4} \right\} && (1) \\ &= \frac{1}{8}\left\{ \frac{1}{(u/2)^2+1} \right\} \\ &= \frac{1}{4}\arctan(u/2) + C \end{aligned}$$

- (1): $\left\{ \begin{array}{lcl} u & = & x^2 \\ du & = & 2x\, dx \end{array} \right.$

Ejercicio 21

$$\int \frac{2dx}{(\cos(x)-\sin(x))^2}$$

$$— — — \text{ Demostración } — — —$$

$$\begin{aligned} I &= 2\left\{ \frac{(\cos(x)+\sin(x))^2 - 2\sin(x)\cos(x)}{(\cos(x)-\sin(x))^2} \right\} && (1) \\ &= 2\left\{ \left(\frac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)}\right)^2 - \frac{2\sin(x)\cos(x)}{1-2\sin(x)\cos(x)} \right\} && (1) \\ &= 2\left\{ (\, \tan(\pi/4+x)\,)^2 - \frac{2\sin(x)\cos(x)}{1-2\sin(x)\cos(x)} \right\} && (2) \\ &= 2\left\{ \sec^2(\pi/4+x) - 1 - \frac{2\sin(x)\cos(x)}{1-2\sin(x)\cos(x)} \right\} && (3) \\ &= 2\left\{ \sec^2(\pi/4+x) - \frac{1}{1-2\sin(x)\cos(x)} \right\} \\ &= 2\left\{ \sec^2(\pi/4+x) \right\} - I && (2) \end{aligned}$$

$$I = \tan(\pi/4+x) + C$$

- (1): $(\cos(x) \pm \sin(x))^2 = 1 \pm 2\sin(x)\cos(x)$
- (2): $\tan(\pi/4+x) = \frac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)} = \frac{1+\tan(x)}{1-\tan(x)}$
- (3): $\tan^2(x) + 1 = \sec^2(x)$

Ejercicio 22

$$\int \frac{x\cosh(x)}{\sinh(x)^2} dx$$

$$— — — \text{ Demostración } — — —$$

$$\begin{array}{rcll} I & = & -x\operatorname{csch}(x) + \left\{ \operatorname{csch}(x) \right\} & (1) \\ & = & -x\operatorname{csch}(x) + \left\{ \dfrac{\operatorname{csch}(x)(\ \operatorname{csch}(x) + \coth(x)\ )}{\operatorname{csch}(x) + \coth(x)} \right\} & \\ & = & -x\operatorname{csch}(x) - \ln|\operatorname{csch}(\mathrm{x}) + \coth(\mathrm{x})| + C & (2) \end{array}$$

- (1): $\left| \begin{array}{c|c} D & I \\ x & \frac{\cosh(x)}{\sinh^2(x)} \\ 1 & -\operatorname{csch}(x) \end{array} \right|$ ver nota (2)
- (2): $[\operatorname{csch}(\mathrm{x}) + \coth(\mathrm{x})] = -\operatorname{csch}(x)\coth(x) - \operatorname{csch}^2(x) = -\operatorname{csch}(x)(\ \operatorname{csch}(x) + \coth(x)\ )$

Nota:

1. Propiedades básicas de las funciones trigonometricas hiperbólicas:

$$\left\{ \begin{array}{ccc} \cosh^2(x) - \sinh^2(x) & = & 1 \\ 2\sinh^2(x) & = & \cosh(2x) - 1 \\ 2\cosh^2(x) & = & \cosh(2x) + 1 \\ \sinh^2(x) + \cosh^2(x) & = & \cosh(2x) \\ 2\sinh(x)\cosh(x) & = & \sinh(2x) \\ (\sinh(x) + \cosh(x))^2 & = & \cosh(2x) + \sinh(2x) \end{array} \right.$$

2. Derivadas de las funciones trigonométricas hiperbólicas:

$$\left\{ \begin{array}{ccc} [\sinh(x)] & = & \cosh(x) \\ [\cosh(x)] & = & \sinh(x) \\ [\tanh(x)] & = & \operatorname{sech}^2(x) \\ [\coth(x)] & = & -\operatorname{csch}^2(x) \\ [\operatorname{sech}(x)] & = & -\operatorname{sech}(x)\tanh(x) \\ [\operatorname{csch}(x)] & = & -\operatorname{csch}(x)\coth(x) \end{array} \right.$$

Ejercicio 23

$$\int_0^2 x^5\sqrt{1+x^3}dx$$

$$- - -\ \text{Demostración}\ - - -$$

$$\begin{array}{rcll} I & = & \frac{1}{3}\left\{ (t-1)t^{1/2} \right\}_1^9 & (1) \\ & = & \frac{1}{3}\left\{ t^{3/2} - t^{1/2} \right\}_1^9 & \\ & = & \frac{1}{3}\left(t^{5/2}\frac{2}{5} - t^{3/2}\frac{2}{3} \right)_1^9 & \\ & = & \frac{1192}{45} & \end{array}$$

- (1): $\left\{ \begin{array}{ccc} t & = & x^3 + 1 \\ dt & = & 3x^2\,dx \end{array} \right.$

Ejercicio 24

$$\int_0^1 \frac{x^7 - 1}{\log(x)}dx$$

$$- - -\ \text{Demostración}\ - - -$$

$$\begin{aligned}
I(a) &= \int_0^1 \frac{x^a-1}{\ln(x)}\,dx \\
\frac{d}{da}I(a) &= \int_0^1 \frac{\partial}{\partial a}\left(\frac{x^a-1}{\ln(x)}\right)dx \qquad (1) \\
&= \int_0^1 x^a\,dx \\
&= \frac{1}{a+1} \\
I(a) &= \int \frac{1}{a+1}\,da + C \\
&= \ln(a+1) + C \\
I(0) &= 0 \\
&= 0 + C \\
C &= 0 \\
I(a) &= \ln(a+1)
\end{aligned}$$

- (1): Demostración Regla de Leibniz para derivadas parciales https://math.stackexchange.com/questions/3778739/can-i-replace-the-fracddt-with-frac-partial-partial-t-in-the-leibn?rq=1

$$---\ \text{Demostración}\ ---$$

$$\begin{aligned}
I(a) &= \int_0^1 \frac{x^a-1}{\ln(x)}\,dx \\
&= \int_0^1 \int_0^a x^y\,dy\,dx \qquad (1) \\
&= \int_0^a \int_0^1 x^y\,dx\,dy \qquad (2) \\
&= \int_0^a \left(\frac{x^{y+1}}{y+1}\right)_0^1 dy \\
&= \int_0^a \frac{1}{y+1}\,dy \\
&= (\ln|y+1|)_0^a \\
&= \ln|a+1|
\end{aligned}$$

- (1): $\frac{\partial}{\partial y}\left(\frac{x^y-1}{\ln(x)}\right) = x^y$
- (2): Teorema de Fubini

Ejercicio 25

$$\int \sqrt{\csc(x)-\sin(x)}dx$$

$$---\ \text{Demostración}\ ---$$

$$\begin{aligned}
I &= \left\{ \frac{\cos(x)}{\sqrt{\sin(x)}} \right\} \\
&= 2\sqrt{\sin(x)} + C
\end{aligned}$$

- (1): $[\sqrt{f(x)}] = \frac{1}{2}\frac{f'(x)}{\sqrt{f(x)}}$