Brazilian highways from slime mold's point of view

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INVITED PAPER

Brazilian highways from slime mold's point of view

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Abstract

Purpose – This paper seeks to develop experimental laboratory biological techniques for approximation of existing road networks, optimizing transport links, and designing alternative optimal solutions to current transport problems. It studies how slime mould of *Physarum polycephalum* approximate highway networks of Brazil.

Design/methodology/approach – The 21 most populous urban areas in Brazil are considered and represented with source of nutrients placed in the positions of slime mould growing substrate corresponding to the areas. At the beginning of each experiment slime mould is inoculated in São Paulo area. Slime mould exhibits foraging behavior and spans sources of nutrients (which represent urban areas) with a network of protoplasmic tubes (which approximate vehicular transport networks). The structure of transport networks developed by slime mould are analyzed and compared with families of known proximity graphs. The paper also imitates slime-mould response to simulated disaster.

Findings – It was found that the plasmodium of *P. polycephalum* develops a minimal approximation of a transport network spanning urban areas. *Physarum*-developed network matches man-made highway network very well. The high degree of similarity is preserved even when high-demand constraints are placed on repeatability of links in the experiments. *Physarum* approximates almost all major transport links. In response to a sudden disaster, gradually spreading from its epicenter, the *Physarum* transport networks react by abandoning transport links affected by disaster zone, enhancement of those unaffected directly by the disaster, massive sprouting from the epicenter, and increase of scouting activity in the regions distant to the epicenter of the disaster.

Originality/value – Experimental methods and computer analysis techniques presented in the paper lay a foundation of novel biological laboratory approaches to imitation and prognostication of socio-economical developments.

Keywords Biological techniques, Computer cybernetics, Systems, Transport, Brazil

Paper type Technical paper

1. Introduction

The term "slime mold" refers to a wide variety of microorganisms that have a gelatinous slimy appearance at some stages of their life cycle. Slime molds can be organized as individual unicellular beings, or as in the so-called (acellular) plasmodial form, a "supercell" of cytoplasm containing thousands of individual nuclei. Acellular slime mould *Physarum polycephalum* has quite a sophisticated life cycle (Stephenson and Stempen, 2000), which includes fruit bodied, spores, single-cell amoebas, and syncytium. In its plasmodium stage, *P. polycephalum* consumes microscopic particles,

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and during its foraging behavior the plasmodium spans scattered sources of nutrients with a network of protoplasmic tubes. The plasmodium optimizes its protoplasmic network to cover all sources of nutrients and yet guarantee robust and quick distribution of nutrients in the plasmodium body; for the sake of conciseness, whenever the term plasmodium is used herein as such, it should be understood as a short form for the plasmodium stage of *P. polycephalum*.

Plasmodium's foraging behavior can be interpreted as a computation, when data are represented by spatial of attractants and repellents, and results are represented by structure of protoplasmic network (Adamatzky, 2010a). Plasmodium can solve computational problems with natural parallelism, namely of shortest path (Nakagaki *et al.*, 2001) and hierarchies of planar proximity graphs (Adamatzky, 2008), computation of plane tessellations (Shirakawa *et al.*, 2009), execution of basic logical computing schemes (Tsuda *et al.*, 2004; Adamatzky, 2010b), and natural implementation of spatial logic and process algebra (Schumann and Adamatzky, 2009). See overview of *Physarum*-based computers in Adamatzky (2010a).

Biology, physics and chemistry inspired approximations of shortest paths, so that computation of a transportation network have been already hot applications for non-standard computing scientists. Nature-inspired computing paradigms and experimental implementations are successfully applied to calculation of a minimal-distance path between two given points in a space or a road network. They include ant-based optimization of communication networks load balancing (Dorigo and Stutzle, 2004), approximation of a shortest path in experimental reaction-diffusion chemical systems (Adamatzky *et al.*, 2005), gas-discharge analog systems (Reyes *et al.*, 2002), spatially extended crystallization systems (Adamatzky, 2009), formation of fungi mycelia networks (Jarrett *et al.*, 2006) and indeed plasmodium of *P. polycephalum* (Nakagaki *et al.*, 2001). Amongst all experimental prototypes of path-computing devices, slime mold *P. polycephalum* is the cheapest, most user-friendly and easiest to cultivate and observe the biological substrate. This is the main reason we use it as a computation substrate in present paper.

In 2007, we presented our first experiments on approximation of roads networks in the South-West of England (Adamatzky, 2007), and the results proved to be promising and exciting, and somewhat surprising findings were obtained. We found that *P. polycephalum* indeed approximates existing road networks but the slime mold also takes initiatives in optimizing the existing networks and offering alternative optimal solutions to existing transport problems. In our further studies, we approximated transport hubs in the UK (Adamatzky and Jones, 2010), Mexico (Adamatzky *et al.*, 2010), and Spain and Portugal (Adamatzky and Alonso-Sanz, 2010). In these cases, we have shown that transportation links constructed by plasmodium match man-made motorways, with some pernicious differences. Comparing results for the UK and Mexico, we found that the shape of a country and the spatial configuration of urban areas or cities may determine the behavior of the plasmodium. However, more experiments are necessary to provide generalization and to built a theory of slime mold-based road planning and urban development.

A multitude of issues have to be faced regarding the implementation of the highway network in any country. This is particularly true for a country like Brazil, that did not have a linear progression of development throughout its history, and due to its continental dimension. Hence, in addition to all the technical decisions related to, say, geographical and geophysical considerations, a number of other factors come to place, such as its history of occupation and internal migrations, political pressures of all sorts, imbalanced economic development and population distribution, etc. And on top of all that, Brazil's model of fast economic development from the second half of the last century entailed a progressive dismantling of the railway system, in favor of a total dependence upon its highway system, both for people and industrial production, which ended up becoming unevenly distributed in quantity and quality. So, choosing Brazil as yet another test-bed for *Physarum*-based road planning clearly represents an experimental instance that has not had a similar or even comparable counterpart in our previous efforts. We hope that our study might lead to tools that would help coping with the enormous demand put on the vehicular transport network of the country, which has undergone continuous restructuring throughout the recent decades and that will require even stronger efforts in the years to come.

The paper is structured as follows. Experimental setup is presented in Section 2. Protoplasmic networks developed by slime mold on a set of oat-flake-urban-areas are analyzed in Section 3 and a series of *Physarum* graph is constructed. In Section 4, we compare generalized graphs derived from slime mold networks with highway graphs derived from man-made transport networks. We evaluate position of *Physarum* and highway graphs in the family of planar proximity graphs in Section 5. Restructure of *Physarum*-built transport networks in response to disasters is studied in Section 6. Overview and comparative analysis of results is given in Section 7.

2. Methods

Plasmodium of *P. polycephalum* is cultivated in plastic container, on paper kitchen towels moistened with still water, and fed with oat flakes. For experiments, we use 120 × 120 mm polystyrene square Petri dishes and 2 percent agar gel (select agar, by Sigma Aldrich) as a substrate. Agar plates, about 2-3 mm in depth, are cut in a shape of Brazil.

We consider the 21 most populous urban areas in Brazil **U** (Figure 1(a)), shown below in descending order of population size:

(1) São Paulo.
(2) Rio de Janeiro.
(3) Belo Horizonte.
(4) Porto Alegre.
(5) Salvador.
(6) Recife.
(7) Fortaleza.
(8) Brasília.
(9) Belém.
(10) Manaus.
(11) São Luís.
(12) Teresina.
(13) Cuiabá.
(14) Petrolina-Juazeiro.
(15) Curitiba.

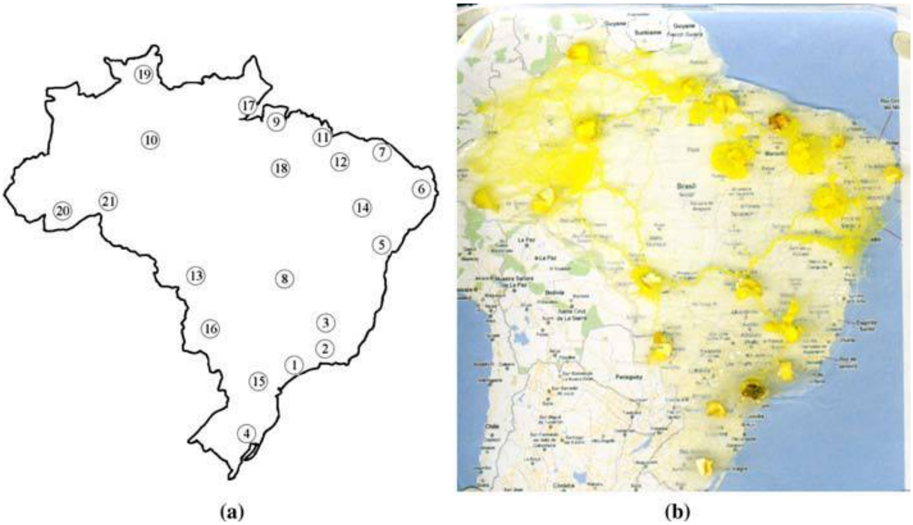


Figure 1. Experimental setup

Notes: (a) Outline map of Brazil with major urban areas **U** shown by encircled numbers; (b) urban areas, represented by oat flakes, are colonized by slime mold of *P. polycephalum* (yellowish veins on the image

(16) Campo Grande.
(17) Macapá.
(18) Imperatriz.
(19) Boa Vista.
(20) Rio Branco.
(21) Porto Velho.

For the sake of standard geographical analyses, Brazil is divided into terms of five macroregions (Regions, for short; Regions of Brazil, 2011), which are also used for the analyses herein; the relation of these regions to the urban areas shown in Figure 1(a) is as follows: North (9, 10, 17, 19, 20, 21), Northeast (5, 6, 7, 11, 12, 14, 18), Central-West (8, 13, 16), Southeast (1, 2, 3) and South (4, 15).

To represent areas of **U**, we place oat flakes in the positions corresponding to the areas. At the beginning of each experiment, an oat flake colonized by plasmodium is placed in São Paulo area.

The Petri dishes with plasmodium are kept in darkness, at temperature 22-25°C, except for observation and image recording. Periodically, the dishes are scanned (with an Epson Perfection 4490 scanner), and the images enhanced for higher visibility, with saturation increased to 55, and contrast to 40. We undertook 53 experiments.

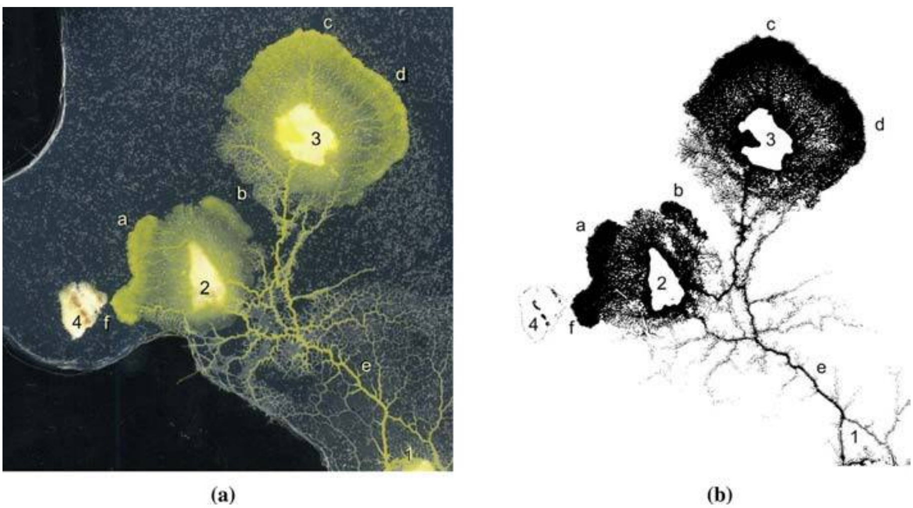
To study reaction of *Physarum*-imitated transport networks on major disasters, we placed crystals of sodium chloride in the approximate positions of Angra Nuclear Power Plant (in the Southeast), Guararapes-Gilberto Freyre International Airport (in the North-eastern city of Recife) and Brasília International Airport (in the center of the country); naturally, this should be regarded as an illustration, since the Brazilian map in the dish does not have resolution for precisely positioning these landmarks.

Six scoping experiments have been undertaken for each epicenter of imitated disaster. Sodium chloride is a strong repellent for *P. polycephalum*. The salt diffuses in the substrate outwards its original application site (epicenter of disaster). It can therefore imitate radioactive and/or chemical pollution (Angra Nuclear Power Plant), and terrorist or enemy attacks on airports (Guararapes-Gilberto Freyre and Brasília International Airports) and subsequent perturbation spreading along transport networks.

To ease readability of experimental images, we provide complementary binary version of each image, where appropriate. Each pixel of a color image is assigned black color if red R and green G components of its RGB color exceed some specified thresholds ($R > \theta_R, G > \theta_G$) and blue component B does not exceed some threshold value $B < \theta_B$; otherwise, the pixel is assigned white color. Exact values of the thresholds are indicated in the figure captions as $\Theta = (\theta_R, \theta_G, \theta_B)$.

3. *Physarum*-built transport links

When placed on a substrate, the plasmodium starts its foraging activity. It follows gradients of chemo-attractants and repellents. On non-nutrient agar gel, as used in the experiments, the propagating part of the plasmodium (also called an active zone) is morphologically and behaviorally similar to localized wave-fragments in sub-excitable chemical medium (Adamatzky *et al.*, 2008). An example of *Physarum* local activity is shown in Figure 2. Active parts of the plasmodium are seen as domains with high-concentration of cytoplasm, visible as dense black zones in Figure 2(b). Initially, slime mold colonized southern flake "1". The mold then propagated towards north-west and north, and colonized flakes "2" and "3", respectively. The flakes "2" and "3" become connected with flake "1" by protoplasmic tubes e. When propagating towards flakes "2" and "3" *Physarum* splits into two active zones. The active zone moving north-west reached oat flake "2" early than the active zone moving north



Notes: Scanned image (a) and its binarized version (b); oat flakes are marked by 1-4, active parts of wave fronts by a-d and f, and protoplasmic tube by e

Figure 2. Local propagation activity and decision making by slime mold

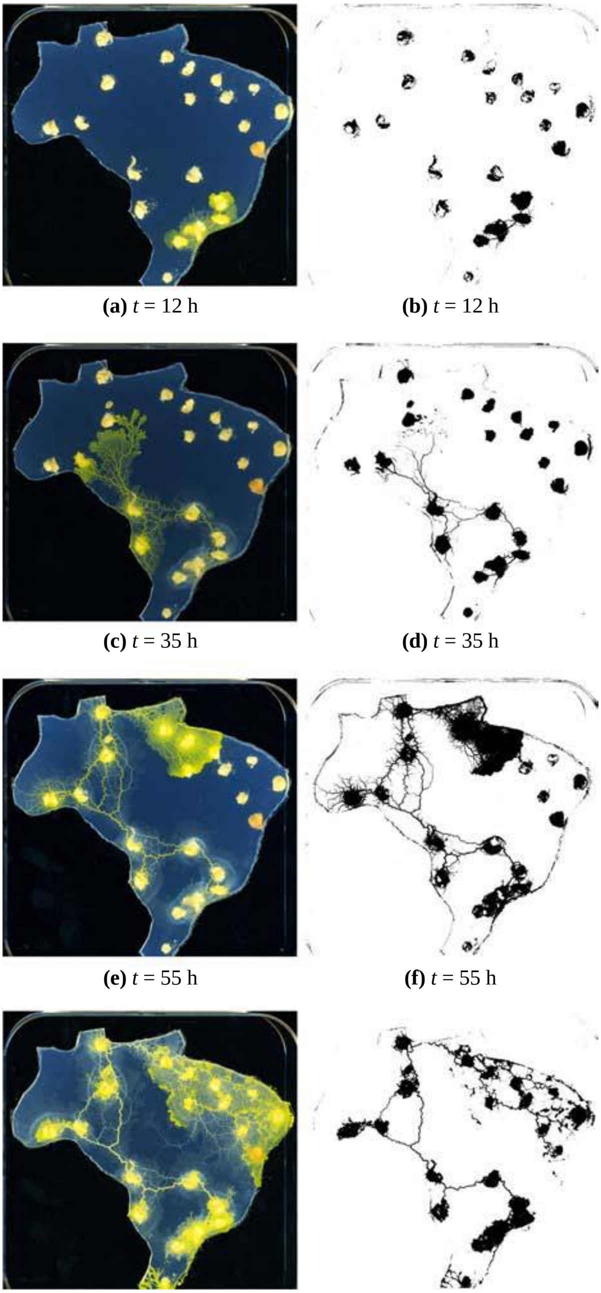
reached flake “3”. Part of the plasmodium in flake “2” then initially started propagating to flake “3” and “4”. However, when it reached flake “3”, a coordinate action occurred and the part propagating from flake “2” to flake “3” (this part is marked by “b” in Figure 2) cancels its movement. Thus, the following active parts of the plasmodium areactualized: zone “f” propagating to flake “4”, exploratory zone “a” propagating north-west (this zone will be cancelled later), and exploratory zones “c” and “d”. As a rule of thumb, plasmodium connecting two oat flakes takes a form of protoplasmic tube/vein, and plasmodium in exploratory mode has a shape of two-dimensional dissipative soliton or a wave-fragment.

A typical scenario of *Physarum* behavior on a Brazil-shaped agar plate with oat flakes representing cities is shown in Figure 3. We placed an oat flake colonized by plasmodium in São Paulo. In 12 h, plasmodium occupies Rio de Janeiro, Belo Horizonte and Curitiba (Figure 3(a) and (b)). In further 23 h, the plasmodium spreads from Belo Horizonte towards Brasília, and then from Brasília to Cuiabá and Campo Grande. Active zone from Cuiabá travels towards Porto Velho and Manaus. Porto Velho becomes colonized by *Physarum* by the 35th hour from slime mold’s inoculation in São Paulo (Figure 3(c) and (d)).

In next 20 h, the following action of events unfolds (Figure 3(e) and (f)). Plasmodium branches from Porto Velho to Rio Branco and Manaus. The plasmodium that colonized Rio Branco stops its exploratory activity while the slime mold based in Manus spreads to Boa Vista and then from Boa Vista to Macapá and then to Belém. The final branching at this stage occurs after Belém, when the plasmodium occupies São Luís and Imperatriz. In one more day, all urban areas **U** are colonized by plasmodium (Figure 3(e) and (f)). Note that Porto Alegre, despite its proximity to the initial inoculation site – São Paulo – is colonized only at final stages of spanning the experimental space by slime mold. This demonstrates that in some case the plasmodium makes its decisions on propagating based not only on the distance between current and next sources of nutrients.

Plamodium’s behavior is determined by many factors and slightly randomized by interactions between myriad of bio-chemical oscillators in its cytoplasm. No two experiments produce exactly the same results. To generalize our experimental results, we constructed a probabilistic *Physarum* graph. A *Physarum* graph is a tuple $\mathbf{P} = \langle \mathbf{U}, \mathbf{E}, w \rangle$, where **U** is the set of 21 urban areas, **E** is a set edges, and w: $\mathbf{E} \rightarrow [0, 1]$ is a probability-weight of edges from **E**. For every two cities a and b from **U**, there is an edge connecting a and b if a plasmodium’s protoplasmic link is recorded at least in one of k experiments, and the edge (ab) has a probability calculated as a ratio of experiments where protoplasmic link (ab) occurred to the total number of experiments k. We do not take into account the exact configuration of the protoplasmic tubes, but merely their existence; for instance, protoplasmic tubes linking Rio Branco and Cuiabá are always positioned inside Brazilian boundaries, while the corresponding edge in *Physarum* graph represents the tubes by straight line crossing over Bolivia. We also consider a threshold $\theta \in [0, 1]$ in *Physarum* graphs $\mathbf{P}(\theta)$, defined as follows: for $a, b \in \mathbf{U}$, $(ab) \in \mathbf{E}$, if $w(ab) > \theta$.

A selection from a series of graphs $\mathbf{P}(\theta)$, $\theta = 0, \ldots, 42$, is shown in Figure 4. A non-pruned graph $\mathbf{P}(0)$ is non-planar (Figure 4(a)). *Physarum* graph becomes planar when cut-off threshold θ reaches 7/53, i.e. when only edges appearing in over 13 percent of experimental trials are present in graph (Figure 4(b)). Assuming acceptable 10 percent error, we can propose that generalized *Physarum* graph is planar.



(a) t = 12 h **(b)** t = 12 h

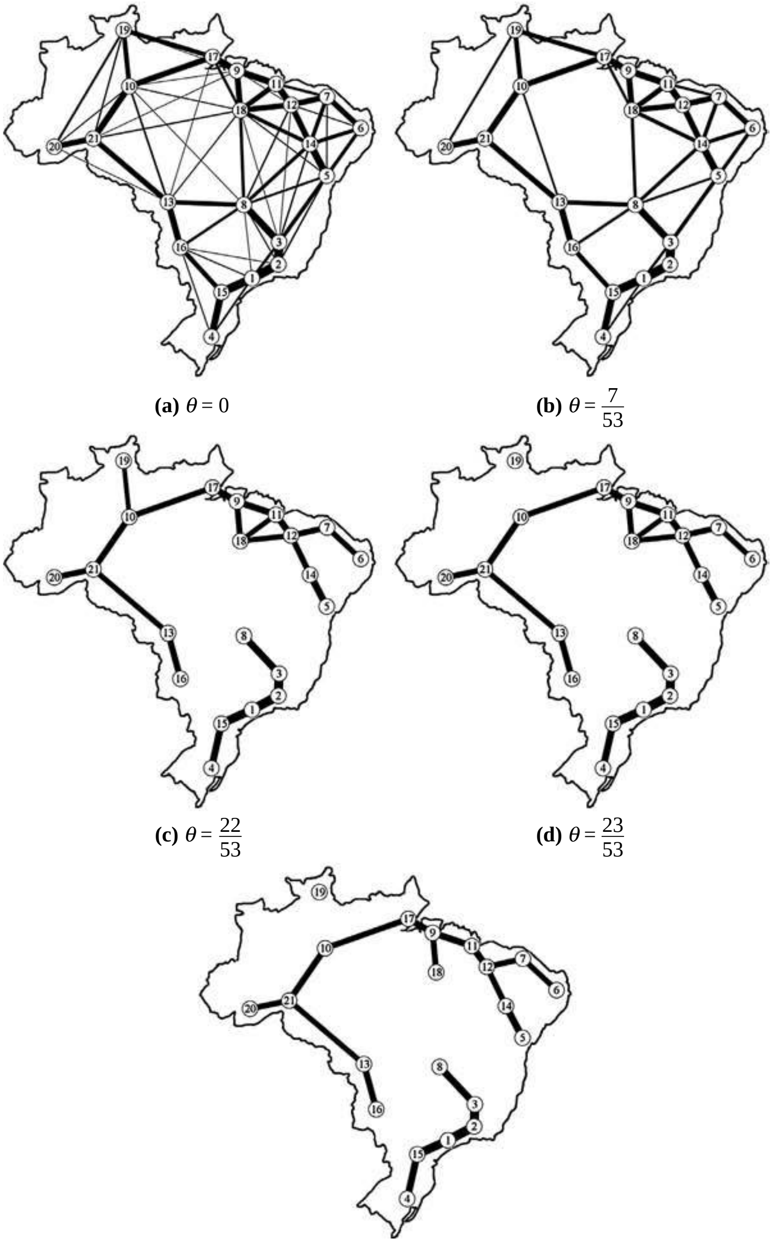
(c) t = 35 h **(d)** t = 35 h

(e) t = 55 h **(f)** t = 55 h

(g) t = 79 h **(h)** t = 79 h

Note: (a), (c), (e) and (g) are the scanned image of experimental Petri dish, and (b), (d), (f) and (h) the binarized images

Figure 3.
Illustrative example of plasmodium development on configuration of cities **U** represented by oat flakes, with elapsed time from inoculation shown in the sub-figure captions, and $\Theta = (150, 150, 150)$



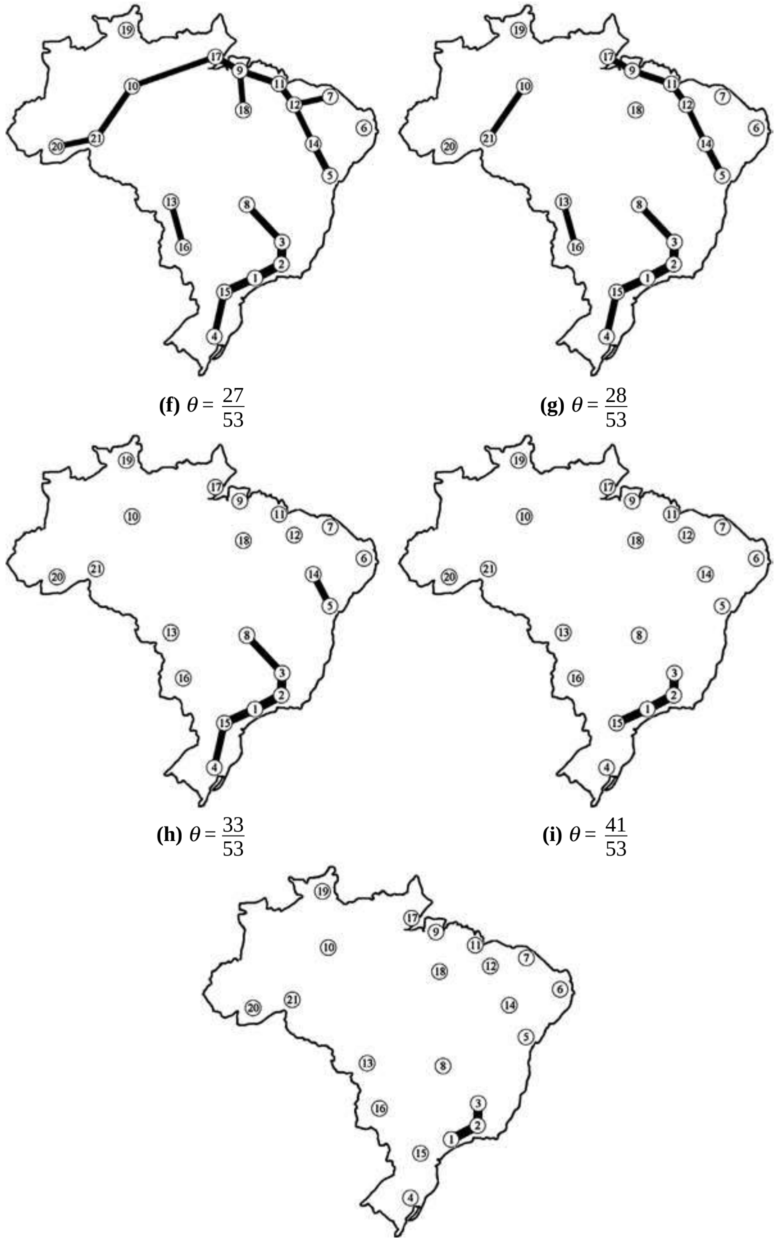
(a) $\theta = 0$ (b) $\theta = \frac{7}{53}$

(c) $\theta = \frac{22}{53}$ (d) $\theta = \frac{23}{53}$

(e) $\theta = \frac{26}{53}$

Figure 4.
Configurations of threshold *Physarum*-graph $\mathbf{P}(\theta)$ for critical values of θ

(*continued*)



(f) $\theta = \frac{27}{53}$

(g) $\theta = \frac{28}{53}$

(h) $\theta = \frac{33}{53}$

(i) $\theta = \frac{41}{53}$

(j) $\theta = \frac{42}{53}$

Note: Thickness of an edge is proportional to the edge's weight

Figure 4.

For $\theta = 22/53$, the *Physarum* graph becomes disconnected. Brasília urban area loses all its neighbors but Belo Horizonte (Figure 4(c)). One component of graph **P**(22/53) is a chain $c_1 =$ (Porto Alegre-Curitiba-São Paulo-Rio de Janeiro-Belo Horizonte-Brasília). The chain c_1 remains stable up to $\theta = 33/53$, in over 62 percent of experiments (Figure 4(d)-(h)).

The second component c_2 of **P**(22/53) is comprised of all other urban areas **U** minus vertices of c_1 (Figure 4(c)). The component c_2 has three cycles, spanning Belém, São Luís, Teresina and Imperatriz. Cyclic parts of c_2 break down when θ reaches 26/53 (Figure 4(e)). At $\theta = 26/53$, *Physarum* graphs consists of the chain c_1, the tree c_3 (derived from component c_2) and an isolated vertex – Boa Vista (Figure 4(e)). Further increase of θ leads to complete dissociation of tree c_3 and formation of isolated vertices (Figure 4(f)-(j)). A single chain (São Paulo-Rio de Janeiro-Belo Horizonte) is present in almost 80 percent of the experiments, i.e. up to $\theta = 42/53$ (Figure 4(j)).

4. Slime mould vs highway network

How well do *Physarum* graphs approximate the highway network? By inspection of the most well-known highway map of the country, freely available in Guia Quatro Rodas (2010) in Portuguese, we constructed the graph **H** of Brazilian highways. But, due to the high imbalance in quality of the Brazilian roads, depending on the region concerned, the highway network we indeed rely upon is a mix of different kinds, from standard motorways (as typical in São Paulo state) down to roads with precarious conditions, as often happens, specially in the North Region.

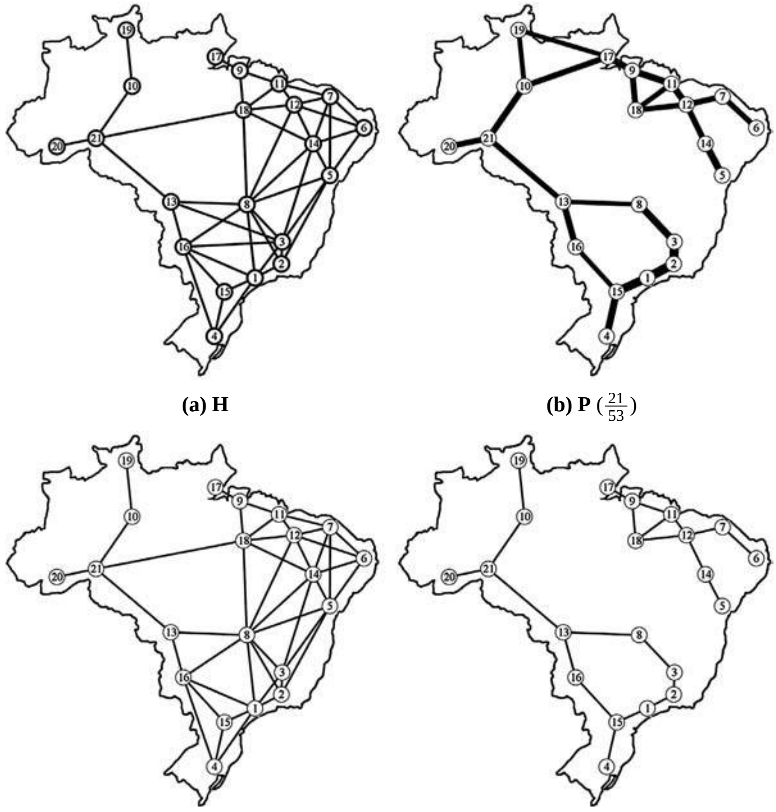
The graph **H** was constructed as follows. Let **U** be a set of urban regions/cities; for any two regions a and b from **U**, the nodes a and b are connected by an edge (ab) if there is a highway starting in vicinity of a, passing in vicinity of b, and not passing in vicinity of any other urban area $c \in \mathbf{U}$. In the case of branching – that is, a highway starts in a, goes in the direction of b and c, and at some point branches towards b and c – we then add two separate edges (ab) and (ac) to the graph **H**. The highway graph is non-planar (Figure 5(a)). Let us compare **H** with the non-planar *Physarum* graph **P**(0), already shown as Figure 5(a), and the highest θ connected *Physarum* graph **P**(21/53), shown as Figure 5(b).

Finding 1

Comparing to existing Brazilian highway network, slime mold *P. polycephalum* overdoes the number of transport links in all urban areas of Brazil, with an emphasis on those in the North Region and its borders.

Highway and *Physarum* graphs have the same labeling so that we can compare them straightforwardly. As Figure 6(a) makes it evident, the distributions of degrees along nodes are almost matching in **H** and **P**(0), with all nodes of highway graph having almost equally less neighbors than nodes of *Physarum* graph. Node-degree distribution in **P**(21/53) does not perfectly match that of **H**, with the best matches happening in the Southern Region and the worse occurring in Manaus and Macapá, in the extreme north.

Spatial distributions of mismatches are shown in Figure 6(b) and (c). Let $d_i^{\mathbf{P}}$ and $d_i^{\mathbf{H}}$ be degrees of node i in graphs **P** and **H**. A node i in Figure 6(b) and (c) has green (light gray) band if $d_i^{\mathbf{P}} > d_i^{\mathbf{H}}$ and red (dark gray) band if $d_i^{\mathbf{P}} < d_i^{\mathbf{H}}$. Width of the band at i is proportional to $|d_i^{\mathbf{P}} - d_i^{\mathbf{H}}|$.



(a) **H** (b) **P** $(\frac{21}{53})$

(c) **P** (0) ∩ **H** (d) **P** $(\frac{21}{53})$ ∩ **H**

Note: Intersections P (θ) ∩ H of *Physarum* P and highways H graphs are shown in (c) for θ = 0 and (d) for θ = 21/53

Figure 5.
Graph **H** of Brazilian highway network is shown in (a), and the highest θ connected *Physarum* graph is shown in (b)

Mismatch between node-degrees of **H** and **P**(0) is pronounced in urban areas of the entire Brazilian North Region and its borders with the Central-West and Northeast Regions. The highest mismatch is observed in Manaus, Macapá, and Imperatriz (Figure 6(b)). In these parts, *Physarum* graphs shows higher connectivity then highway graph. North-east part of Brazil is most characteristic of mismatch between **H** and **P**(21/53) (Figure 6(c)). Urban areas lying in the eastern parts of Brazil, starting in the Central-West Region, have less neighbors in **P** than in **H**. Brasília urban area shows maximal mismatch.

It is also worth noticing that only Curitiba (area 15) has the same connectivity in all three situations shown in Figure 6. This represents a robustness of this southern area for ensuring full connectivity of the country, regardless of how itis conceived.

Finding 2

Physarum matches Brazilian highways almost perfectly, by also creating almost all existing connections.

$$\mathbf{H} = \mathbf{P}(0) \cap \mathbf{H} \cup \{(\textit{Cuiabá-Belo Horizonte}) \cup (\textit{Campo Grande-Belo Horizonte})\}$$

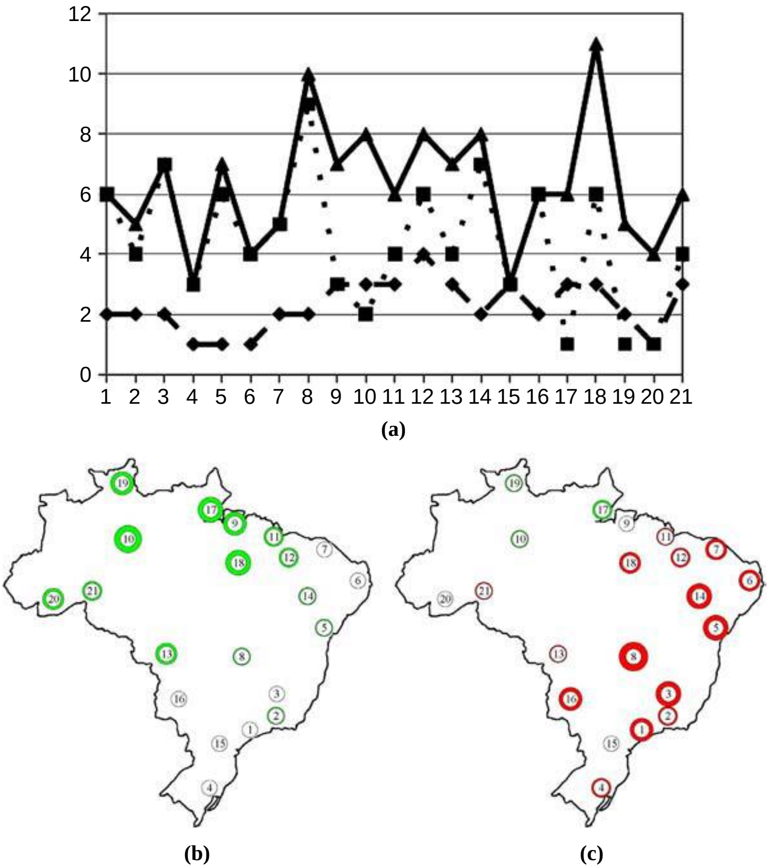


Figure 6. Node-degree based comparison of *Physarum* and highway graphs

Notes: (a) Degrees of nodes in highway graph **H** (dotted line and square markers), *Physarum* graphs **P**(0) (solid line and triangle markers) and **P**(21/53) (dashed line and rhomboid markers); the horizontal axis represents the node labels and the vertical axis their corresponding degrees; (b) mismatch between degrees of **H** and **P**(0) nodes, and (c) **H** and P(21/53)

This comes as a direct observation of Figure 5(a) and (c). In contrast to the somewhat redundant connectivity degree of **P**(0), missing the links from Belo Horizonte to Cuiabá and Campo Grande may indicate that such links do not conform to slime mould's representation of local optimal connectivity.

Finding 3

$$\mathbf{P}\left(\frac{21}{53}\right) = \mathbf{P}\left(\frac{21}{53}\right) \cap \mathbf{H} \cup (\textit{Boa Vista-Macapá}) \cup (\textit{Manaus-Macapá})$$

In the pruned *Physarum* graph **P**(21/53), Macapá plays the same role as Belo Horizonte in the "raw" graph **P**(0). The graph **P**(21/53) ∩ **H** is disconnected and is comprised of two components. South-west component spans the states of Acre, Amazonas, Rondônia, Mato Grosso, Goiás, São Paulo, Minas Gerais, Rio de Janeiro, Paraná, Santa Catarina and Rio Grande do Sul. All states but Pará and Tocantis are spanned by north-east component (Figure 5(d)). South-west component has a single cycle: São Paulo-Rio de Janeiro-Rio de Janeiro-Brasília-Cuiabá-Campo Grande-Curitiba. The three-cycle north-east component of **P**(21/53) $\cap$ **H** is discussed as component c_2 earlier.

5. Comparing with proximity graph

Relative neighborhood graph (RNG), Gabriel graph (GG) and spanning trees (ST) are well-known species of planar proximity graphs used in geographical variational analysis (Gabriel and Sokal, 1969; Matula and Sokal, 1984), simulation of epidemics (Toroczkai and Guclu, 2007), and design of *ad hoc* wireless networks (Li, 2004; Song *et al.*, 2004; Santi, 2005; Muhammad, 2007; Wan and Yi, 2007). The proximity graphs, particularly RNG, are invaluable in simulation of human-made, road networks; these graphs are validated in specially interesting studies of Tsukuba central district road networks (Watanabe, 2005; Watanabe, 2008). The graphs provide a good formal representation of biological transport networks, particularly foraging trails of ants (Adamatzky and Holland, 2002) and protoplasmic networks of slime mold *P. polycephalum* (Adamatzky, 2008; Adamatzky and Jones, 2010). For self-consistency, we provide brief definitions of the graphs below:

- *RNG*. Points a and b are connected by an edge in **RNG** if no other point c is closer to a and b than $dist(a, b)$ (Toussaint, 1980).
- *GG*. Points a and b are connected by edge in **GG** if a disc with diameter $dist(a, b)$, centered in the middle of the segment ab, is empty (Gabriel and Sokal, 1969; Matula and Sokal, 1984).
- *MST*. The Euclidean minimal spanning tree (**MST**) (Nesetril *et al.*, 2001) is a connected acyclic graph which has the minimal possible sum of edges' lengths.
- *Toussaint hierarchy*. The graphs are related as $\mathbf{MST} \subseteq \mathbf{RNG} \subseteq \mathbf{GG}$ (Toussaint, 1980; Matula and Sokal, 1984; Jaromczyk and Toussaint, 1992).

RNG, GG and MST proximity graphs constructed on nodes of **U** are shown in Figure 7. To calculate ST, we used the classical Jaromczyk-Supowit method (Jaromczyk and Kowaluk, 1987; Supowit, 1988). Strictly speaking, a spanning tree rooted in São Paulo (Figure 7(c)) is not a minimum spanning tree. The minimal length trees are those rooted in Fortaleza, Recife or Teresina (Figure 7(d)). However, differences in lengths are negligible: tree rooted in São Paulo is just 4 percent longer then the MST rooted in Fortaleza, Recife or Teresina (Table I). Therefore, further on we will consider the spanning tree rooted in São Paulo as the MST. We will also be referring to **P**(21/53) as simply **P**.

Finding 4

$$\mathbf{MST} + (\textit{Cuiabá-Brasília}) + (\textit{Belém-Sao Luís}) = \mathbf{RNG}$$

This is a direct outcome of Figure 7(a) and (c), which indicates that the configuration of urban areas from **U** allows for an optimal, in a sense of proximity graphs, for construction of transport networks. A minimum spanning tree is just two edges away from the RNG.

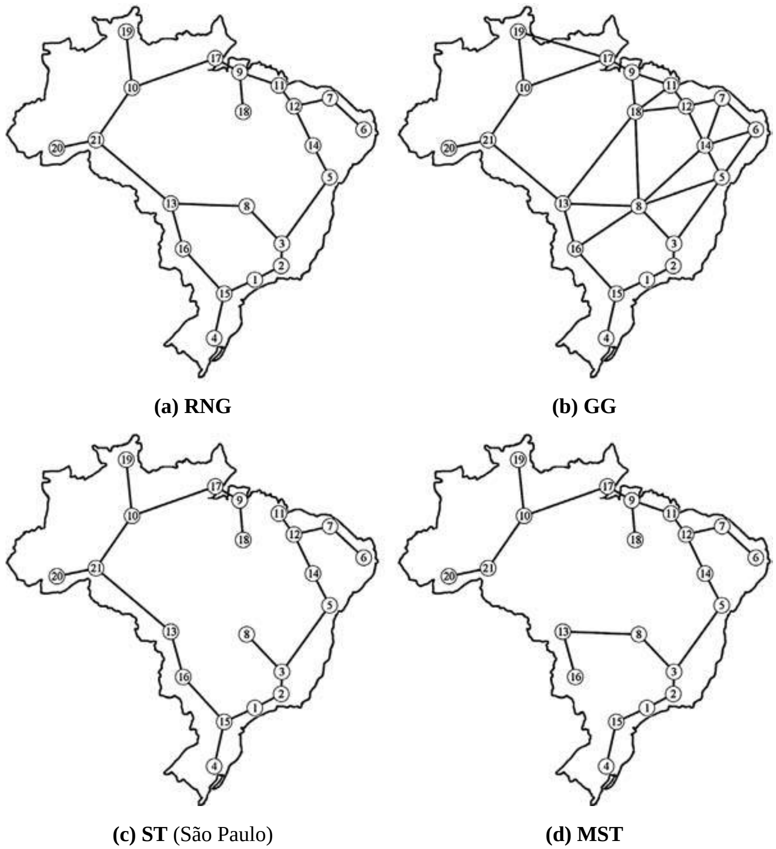


Figure 7.
Proximity graphs constructed on regions **U**

Finding 5

$$\mathbf{MST} - (\mathit{Manaus\text{-}Macap\acute{a}}) \subset \mathbf{H}$$

$$\mathbf{MST} - (\mathit{Belo\ Horizonte\text{-}Salvador}) \subset \mathbf{P}$$

Neither human-made nor *Physarum*-grown transport graphs allow for an optimal, in a sense of the shortest path for transportation. However, removing just one edge (different in each case) from the minimum spanning tree makes the tree embeddable in **H** and **P** (Figures 7(c) and 8(c) and (f)).

Finding 6

$$\mathbf{RNG} - (\mathit{Manaus\text{-}Macap\acute{a}}) \subset \mathbf{H}$$

$$\mathbf{RNG} - (\mathit{Belo\ Horizonte\text{-}Salvador}) \subset \mathbf{P}$$

(Figure 8(a) and (d)).

Node	ST length
São Paulo	1.04
Rio de Janeiro	1.09
Belo Horizonte	1.08
Porto Alegre	1.07
Salvador	1.01
Recife	1.00
Fortaleza	1.00
Brasília	1.09
Belém	1.08
Manaus	1.10
São Luís	1.02
Teresina	1.00
Cuiabá	1.11
Petrolina-Juazeiro	1.04
Curitiba	1.07
Campo Grande	1.12
Macapá	1.05
Imperatriz	1.08
Boa Vista	1.10
Rio Branco	1.09
Porto Velho	1.09

Table I. Ratios between the lengths of the ST rooted in every node of **U** to the length of the minimum spanning tree

Minding that **RNG** is a sub-graph of **GG**, we can propose that (Manaus-Macapá) and (Belo Horizonte-Salvador) are the only transport links that prevent the RNG to be embeddable in human-made and *Physarum*-built transport networks. The next results give us a hint on a possible position of *Physarum* graphs in the hierarchy of proximity graphs.

Finding 7

$$\mathbf{GG} - (\textit{Manaus-Macapá}) - (\textit{Boa Vista-Macapá}) - (\textit{Cuiabá-Imperatriz}) \subset \mathbf{H}$$

$$\mathbf{P} \subset \mathbf{GG}$$

(Figure 8(b) and (e)).

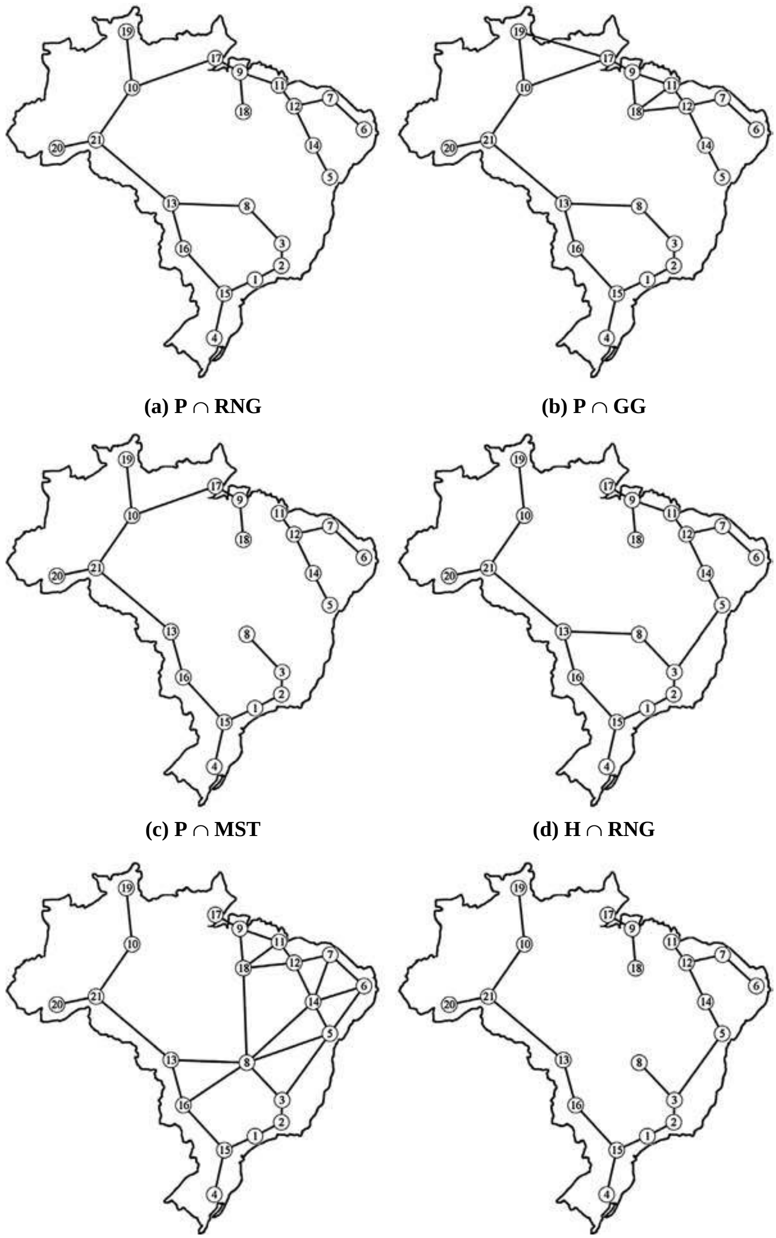
Thus, we can propose that a *Physarum* graph constructed on urban areas **U** is a proximity graph that is neither a sub- nor a super-graph of the minimum spanning graph, but is a sub-graph of the RNG and GG.

6. Network restructure in response to major disasters

Finding 8

In response to a sudden disaster, gradually spreading from its epicenter, the *Physarum* transport networks react by abandoning transport links affected by disaster zone, enhancement of transport links unaffected directly by the disaster, massive sprouting from the epicenter, and increase of scouting activity in the regions distant to the epicenter of the disaster.

We imitate a hypothetical disaster in Angra Nuclear Power Plant by placing a crystal of sodium chloride at the approximate position of the plant (determined indeed by very low resolution of our experimental area, Petri dish). Images of the protoplasmic



(a) P ∩ RNG **(b) P ∩ GG**

(c) P ∩ MST **(d) H ∩ RNG**

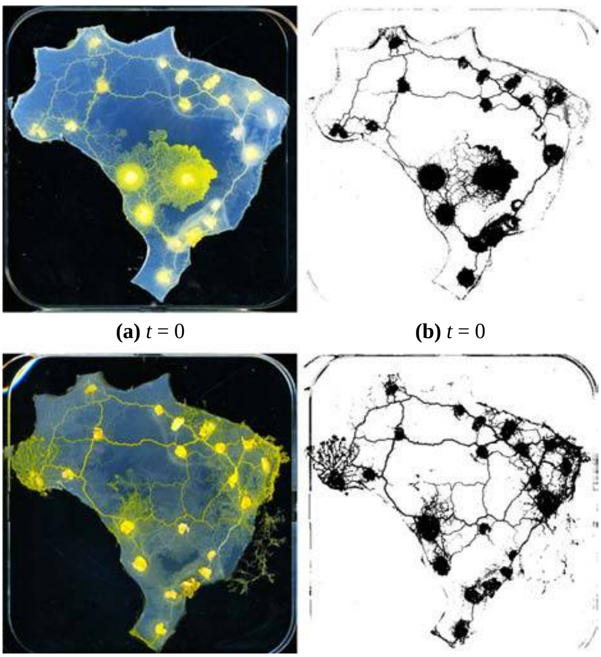
(e) H ∩ GG **(f) H ∩ MST**

Figure 8.
Intersection of *Physarum* graph **P**(21/53) (a), (b) and (c) and highway graph **H** (d), (e) and (f) with proximity graphs **RNG**, **GG** and minimum spanning tree **MST**

network were recorded before the disaster and 24 h later. A typical example of the network restructuring is shown in Figure 9.

Usually, in 24 h, the concentration of diffusing salt (in its repellent effect for plasmodium concentration) propagates as far as eight and four, and almost reaches five. All *Physarum* activity in Brasília, so clearly visible in Figure 9(a) and (b), disappears. Also, the following transport links become destroyed and disfunctional: Curitiba-São Paulo, Curitiba-Campo Grande, São Paulo-Campo Grande, São Paulo-Rio de Janeiro, Rio de Janeiro-Belo Horizonte, Belo Horizonte-Brasília, and Belo Horizonte-Salvador. Plasmodium abandons this protoplasmic tube. The parts of the transport network not affected directly by high concentration of salt increase their functionality (reflected in higher concentration of protoplasm an additional branching), so as to compensate for the lost transport links. Namely, transport arteries connecting Manaus and Imperatriz, Teresina and Petrolina-Juazeiro, Cuiabá and Porto Velho becomes particularly pronounced (Figure 9(c) and (d)). At the same time, plasmodium activates its scouting behavior and explores options of escape and re-colonization. Typically, it tries to relocate in the western parts of the North Region (as Rio Branco) and the Northeast Region (such as Salvador, Fortalezaand Petrolina-Juazeiro areas).

The imitation of a disaster with its epicenter in Guararapes-Gilberto Freyre International Airport is shown in Figure 10, with a sample of two experiments. In 24 h after the disaster, its effect reaches Teresina in the Northeast and Belo Horizonte in the Southeast Regions.

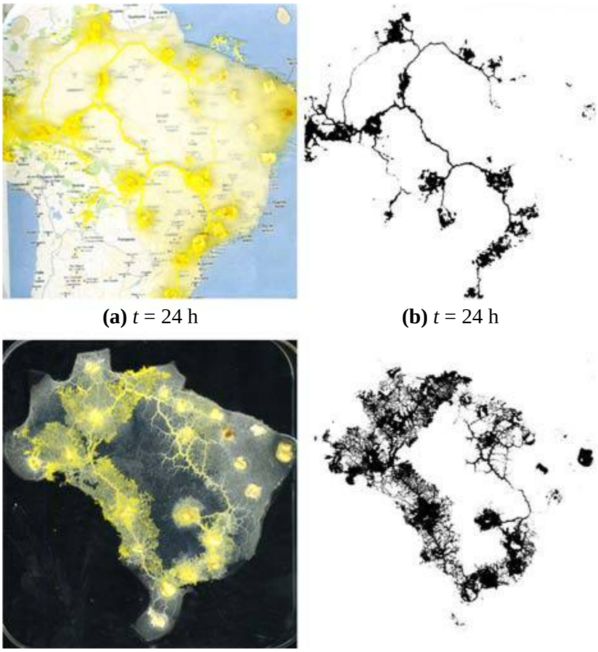


(a) $t = 0$ **(b)** $t = 0$

(c) $t = 24$ h **(d)** $t = 24$ h

Notes: (a) and (c) Scanned images of experimental Petri dish before (a) and 24 h after (c) the imitated disaster; (b) and (d) binarized images of the protoplasmic network; $\Theta = (150, 150, 190)$

Figure 9.
Restructure of the plasmodium network in response to a crystal of sodium chloride placed in the approximate position of Angra Nuclear Power Plant



(a) t = 24 h (b) t = 24 h

(c) t = 24 h (d) t = 24 h

Notes: (a) and (c) Scanned images of experimental Petri dish 24 h after the imitated disaster; (b) and (d) binarized images of the protoplasmic network; $\Theta = (200, 200, 50)$

Figure 10.
Examples of two experiments on restructure of the plasmodium network in response to a crystal of sodium chloride placed at the approximate position of Guararapes-Gilberto Freyre International Airport, in Recife

All transport links inside the disaster zone vanish. The northeastern urban areas of Salvador, Recife, Fortaleza, Teresina and Petrolina-Juazeiro become isolated and disturbed in functioning. Slightly affected or unaffected transport arteries show signs of hypertrophy, such as in the roads connecting Brasília to Cuiabá, and Cuiabá to Manaus in Figure 10. Plasmodium shows exploratory behavior and ventures onto domains not covered by growth substrates in east-most (around Rio Branco) and south-most (around Porto Alegre) parts of Brazil. In some case, we observed a "panic" response of major transport routes, where plasmodium exhibits disorganized multi-source routing. In Figure 10(c) and (d), mini wave fronts of plasmodial activity can be seen sprouting simultaneously from many sites of the main transport link that goes from Porto Alegre up to Rio Branco.

Another validation of Finding 8 was demonstrated in experiments with sodium chloride placed at the approximate position of Brasília International Airport (Figure 11). Typical reaction includes hypertrophy of and "indiscriminate" sprouting from major transport links unaffected directly by the disaster: most remarkably, a very long link connecting all areas of the northern Brazil, stretching from Recife up to Rio Branco, then going to the southern parts of the Central-West Region; and also a short link between São Paulo and Porto Alegre.

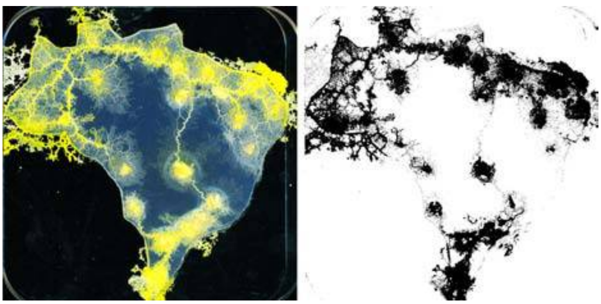
7. Discussion

In laboratory experiments with slime mold *P. polycephalum*, we presented the major urban areas of Brazil with oat flakes, inoculated slime mold in São Paulo, recorded the protoplasmic networks built, and compared them with man-made highwaysand proximity graphs. We found that the plasmodium of *P. polycephalum* develops a minimal approximation of a transport network spanning urban areas **U**. *Physarum*-developed network matches man-made highway network very well. The high degree of similarity is preserved even when we place high-demand constraints on repeatability of links in the experiments (θ threshold). *Physarum* approximates almost all major transport links, apart of (Cuiabá-Belo Horizonte) and (Campo Grande-Belo Horizonte). These nice results are somewhat surprising when recalling the multitude of issues associated to the implementation of the highway network in a country like Brazil, as discussed earlier in the paper. All in all, it seems that the *Physarum* graph sort of "integrates", so to speak, all those aspects into its own set of developmental variables, cancelling out specificities of the original problem, and leading to the empirical observations we could make.

In response to a sudden disaster, gradually spreading from its epicenter, the *Physarum* transport networks react by abandoning transport links affected by disaster zone, enhancement of those unaffected directly by the disaster, massive sprouting from the epicenter, and increase of scouting activity in the regions distant to the epicenter of the disaster. Space-time dynamics of slime mold responding to propagating contamination of the substrate by a repellent complements well existing computer models of spatial and economic impacts of disasters (Okuyama and Chang, 2004; Andersson, 2009; Takada, 2010).

Neither minimum spanning tree constructed on urban areas nor RNG are sub-graph of *Physarum* graph. However, by removing just one edge/link from the spanning tree or RNG, both graphs become embeddable in *Physarum* graph. Moreover, *Physarum* graph is a sub-graph of a GG. We can thus claim that *Physarum* slime mold develops better transport links than that in existing man-made highway network. Under this light, we can reword our first finding as "according to slime mold, Brazilian highway network is excessively redundant in the country's North Region and its borders".

This finding indicates that there are various connectivity options in the north of Brazil, with no particular preference; it is interesting that this is in tune with the true fact that the main connections between many areas of this part of Brazil lead to no particular preference, since all of them may be considered of equally bad quality.



(a) t = 24 h **(b)** t = 24 h

Notes: (a) Scanned image of experimental Petri dish 24 h after the imitated disaster; (b) and (d) binarized image of the protoplasmic network; Θ = (200, 200, 50)

Figure 11.
Experiment on restructure of the plasmodium network in response to a crystal of sodium chloride placed at the approximate position of Brasília International Airport

With regard to optimality and relationships between *Physarum* and highway graphs, it is worth comparing the present results with the outcomes of laboratory experiments with the UK (Adamatzky and Jones, 2010), Mexico (Adamatzky *et al.*, 2010) and the Iberian peninsula (Adamatzky and Alonso-Sanz, 2010):

UK	Mexico	Iberia	Brazil
$\mathbf{H} \subset \mathbf{P}$	$\mathbf{MST} \subset \mathbf{P}$	$\mathbf{RNG} \subset \mathbf{P}$	$\mathbf{P} \subset \mathbf{GG}$
$\mathbf{P} = \mathbf{RNG}$	$\mathbf{RNG} \subset \mathbf{P}$		

This reflects historical, economical and culture differences in vehicular transport networks in the countries studied. More experimental work is required to develop rigorous experimental approaches for validating man-made transport networks and their biological counterparts.

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