**Solutions to hw1**

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Problem1

Q 1.1

True.

From characteristics of convex function, we know that local minimum is also global minimum. Thus, if we use proper learning rate and initialization, gradient descent will reach closer and closer to a minimum until it reaches the minimum.

Q 1.2

False.

Too many layers may lead to overfitting problem.

Q 1.3

False.

Different set of weights may perfectly lead to 100% classification accuracy. Therefore, the final set of weights may be different if the Madaline is initialized with different values.

Q 1.4

False.

It is in the sides of curve not in the middle, where gradient becomes zero. Besides, for a small number of layers, Sigmoid neurons are not likely to die during the training process and they work very well. But for large number of layers, we do not use Sigmoid neurons usually because it is more likely to "die".

Q 1.5

False.

Increasing the height and width of the input feature map will not always lead to a larger output feature map size. For instace, if the kernel size is 5x5, stride is 2, no padding, if the original size of input is 10x10. If we input a larger input like 11x11, the output size will not change.

Problem2

Q2.1

The logic function is AND

| X1 | X2 | s | y |
|---|---|---|---|
| -1 | -1 | -3.5 | -1 |
| -1 | +1 | -1.5 | -1 |
| +1 | -1 | -1.5 | -1 |
| +1 | +1 | 0.5 | +1 |

Q2.2

w0 = -1, w1 = w2 = -1

| X1 | X2 | s | y |
|---|---|---|---|
| -1 | -1 | 1 | +1 |
| -1 | +1 | -1 | -1 |
| +1 | -1 | -1 | -1 |
| +1 | +1 | -3 | -1 |

Q2.3

w0 = 0, w1 = w2 = w3 = 1

| X1 | X2 | X3 | s | y |
|---|---|---|---|---|
| -1 | -1 | -1 | -3 | -1 |
| -1 | -1 | +1 | -1 | -1 |
| -1 | +1 | -1 | -1 | -1 |
| -1 | +1 | +1 | 1 | +1 |
| +1 | -1 | -1 | -1 | -1 |
| +1 | -1 | +1 | 1 | +1 |
| +1 | +1 | -1 | 1 | +1 |
| +1 | +1 | +1 | 3 | +1 |

Q2.4

w21 = w22 = 1, w20 = 1.5

| X1 | X2 | s | y |
|---|---|---|---|
| -1 | -1 | -0.5 | -1 |
| -1 | +1 | 1.5 | +1 |
| +1 | -1 | 1.5 | +1 |
| +1 | +1 | -0.5 | -1 |

Problem3

Q 3.1

$$\frac{\partial x_{l+1}}{\partial x_l} = {W_l}^T x_{l+1} \circ (1 - x_{l+1})$$
$$\frac{\partial x_{l+1}}{\partial W_l} = x_{l+1} \circ (1 - x_{l+1}) x_l^T$$
$$\frac{\partial x_{l+1}}{\partial b_l} = x_{l+1} \circ (1 - x_{l+1})$$

Q 3.2

$$y_1 = W_1 x_1 + b_1$$
$$x_2 = \sigma(y_1)$$
$$x_3 = W_2 x_2 + b_2$$
$$L = \frac{1}{2}(t - x_3)^T (t - x_3)$$

$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial x_3}\frac{\partial x_3}{\partial x_2}\frac{\partial x_2}{\partial y_1}\frac{\partial y_1}{\partial W_1} = ({W_2}^T(x_3 - t)) \circ (x_2 \circ (1 - x_2)){x_1}^T$$

Problem4

4.1

$$\begin{bmatrix} 0 & 0 & -0.25 & -0.5 & 1.75 & -0.5 & -0.25 & 0 & 0 \\ -0.25 & -0.5 & 1.5 & 1.25 & 1 & 1.25 & 1.5 & -0.5 & -0.25 \\ 1.75 & 1.5 & 1 & 1 & 1 & 1 & 1 & 1.5 & 1.75 \\ -0.25 & -0.5 & 1.5 & 1.25 & 1 & 1.25 & 1.5 & -0.5 & -0.25 \\ 0 & 0 & -0.25 & -0.5 & 1.75 & -0.5 & -0.25 & 0 & 0 \end{bmatrix}$$

4.2

From the hint in the problem, I think it is a “sharping” kernel which emphasizes differences in adjacent pixel values. It will make image (input) more vivid. Besides, we can know from the result in 4.1, the difference between adjacent pixels along the edge are larger than before.

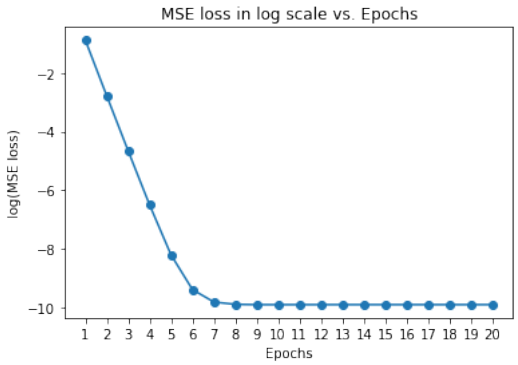
Lab1

(a)

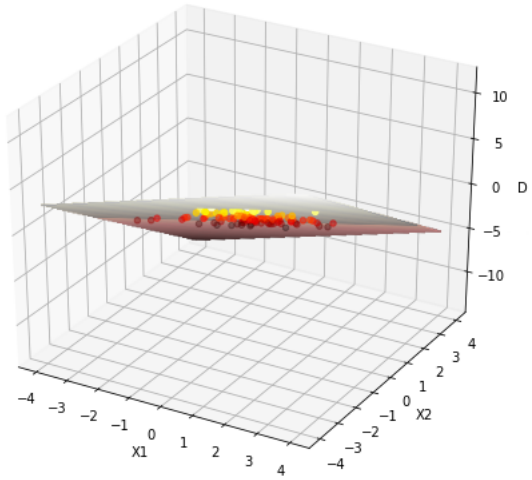
$$W^* = [-0.9993219, \quad 1.00061145, \quad -2.00031968]^T$$
$$MSE = 0.00010079903131736759$$

(b) Run LMS for 20 epochs with learning rate r = 0.01, the weight and MSE loss figure is blelow.

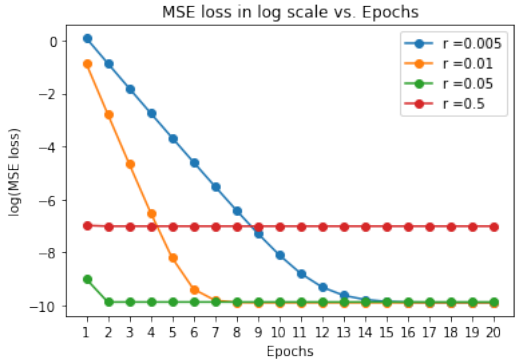
$$W^* = [-0.99925144 \ \ 1.00082858 \ \ -2.00068123]^T$$



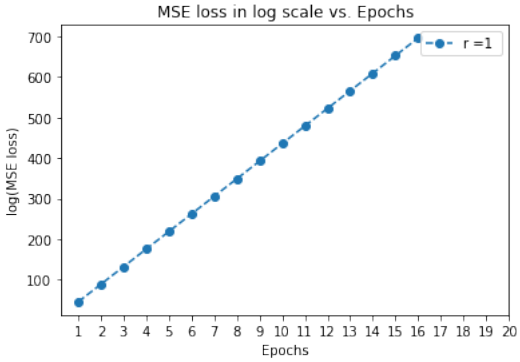

(c) From the 3D figure below, we can find out Both of two linear models fit well. The gray plane is a model in a and the pink plane is b model.



(d) Plot MSE losses with 4 sets of experiments below.




Set r = 1 and plot the MSE loss below.




Quality of the learning process: From the figure above, if the learning rate is very large like 1, it may be impossible for us to get the optimum as it diverges, which will have a bad impact on our training process. If r = 1, the model will diverge from the optimum and reach the infinity. If r = 0.5, we cannot get the optimum either.

Speed of the learning process: from figures above, if the learning rate is not too large to get an optimum, the smaller learning rate is, less times of epochs we need to reach the optimum. In other words, the smaller learning rate we used, the more epochs it takes.

LAB2

(a)

```
# Create the neural network module: LeNet-5
class LeNet5(nn.Module):
    def __init__(self):
        super(LeNet5, self).__init__()
        # Layer definition
        self.conv1 = CONV(3,6,5)    #Your code here
        self.conv2 = CONV(6,16,5)    #Your code here
        self.fc1   = FC(400, 120)    #Your code here
        self.fc2   = FC(120, 84)     #Your code here
        self.fc3   = FC(84, 10)    #Your code here

    def forward(self, x):
        # Forward pass computation
        # Conv 1
        out = F.relu(self.conv1(x))
        # MaxPool
        mp1 = nn.MaxPool2d(2,2)
        out = mp1(out)
        # Conv 2
        out = F.relu(self.conv2(out))
        # MaxPool
        mp2 = nn.MaxPool2d(2,2)
        out = mp2(out)
        # Flatten
        out = out.reshape(out.shape[0],-1)
        # FC 1
        out = F.relu(self.fc1(out))
        # FC 2
        out = F.relu(self.fc2(out))
        # FC 3
        out = self.fc3(out)
        return out

# GPU check
device = 'cuda' if torch.cuda.is_available() else 'cpu'
if device =='cuda':
    print("Run on GPU...")
else:
    print("Run on CPU...")

# Model Definition
net = LeNet5()
net = net.to(device)

# Test forward pass
data = torch.randn(5,3,32,32)
data = data.to(device)

# Forward pass "data" through "net" to get output "out"
out =  net(data) #Your code here

# Check output shape
assert(out.detach().cpu().numpy().shape == (5,10))
print("Forward pass successful")
```

```
Run on CPU...
Forward pass successful
```

# Create the neural network module: LeNet-5
class LeNet5(nn.Module):
    def __init__(self):
        super(LeNet5, self).__init__()
        # Layer definition
        self.conv1 = CONV(3,6,5)    #Your code here
        self.conv2 = CONV(6,16,5)    #Your code here
        self.fc1   = FC(400, 120)    #Your code here
        self.fc2   = FC(120, 84)     #Your code here
        self.fc3   = FC(84, 10)    #Your code here

```
    def forward(self, x):
        # Forward pass computation
        # Conv 1
        out = F.relu(self.conv1(x))
        # MaxPool
        mp1 = nn.MaxPool2d(2,2)
        out = mp1(out)
        # Conv 2
        out = F.relu(self.conv2(out))
        # MaxPool
        mp2 = nn.MaxPool2d(2,2)
        out = mp2(out)
        # Flatten
        out = out.reshape(out.shape[0],-1)
        # FC 1
        out = F.relu(self.fc1(out))
        # FC 2
        out = F.relu(self.fc2(out))
        # FC 3
        out = self.fc3(out)
        return out

# GPU check
device = 'cuda' if torch.cuda.is_available() else 'cpu'
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out =  net(data) #Your code here

# Check output shape
assert(out.detach().cpu().numpy().shape == (5,10))
print("Forward pass successful")
```

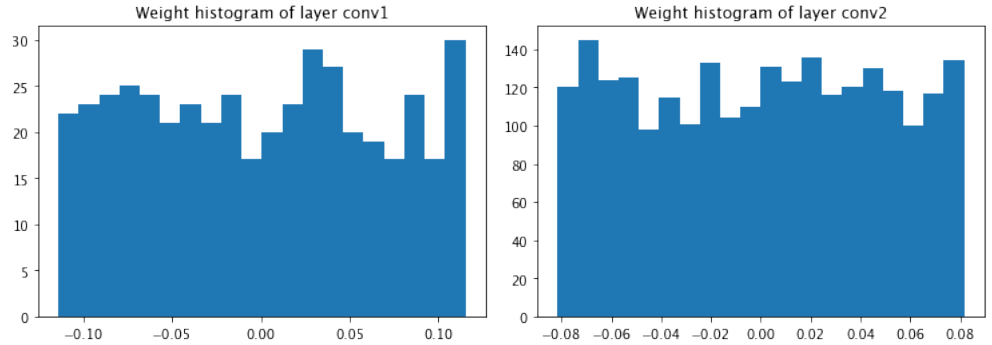
(b) Results to shape size

Table2

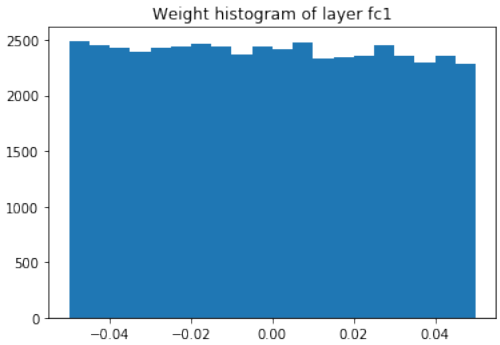
| Layer | Input shape | Output shape | Weight shape | # Param | # MAC |
|---|---|---|---|---|---|
| Conv 1 | (1, 3, 32, 32) | (1, 6, 28, 28) | (6, 3, 5, 5) | 450 | 352800.0 |
| Conv 2 | (1, 6, 14, 14) | (1, 16, 10, 10) | (16, 6, 5, 5) | 2400 | 240000.0 |
| FC1 | (1, 400) | (1, 120) | (120, 400) | 48000 | 48000.0 |
| FC2 | (1, 120) | (1, 84) | (84, 120) | 10080 | 10080.0 |
| FC3 | (1, 84) | (1, 10) | (10, 84) | 840 | 840.0 |

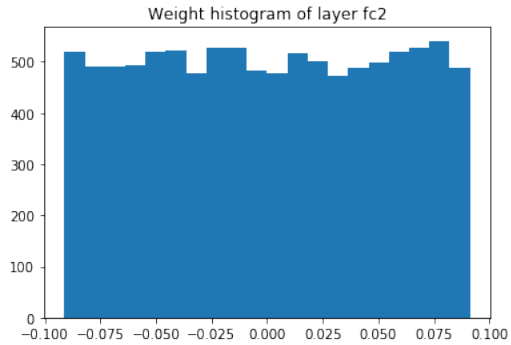
LAB3

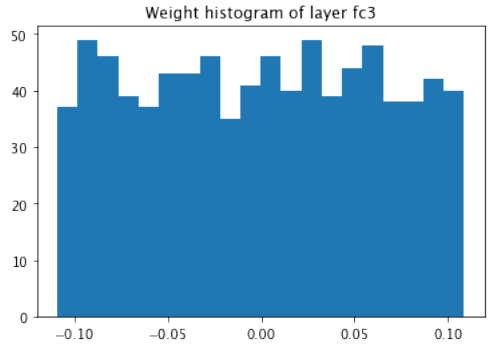
(a)


Weight histogram of layer conv1
30
25
20
15
10
5
0
−0.10
−0.05
0.00
0.05
0.10

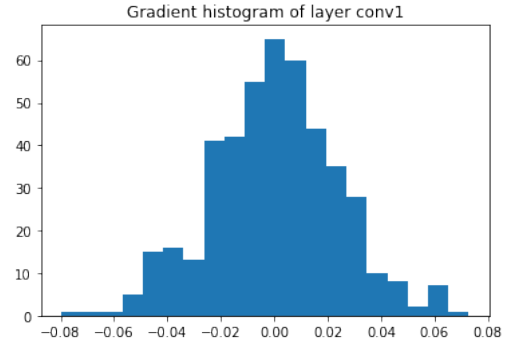

Weight histogram of layer conv2
140
120
100
80
60
40
20
0
−0.08
−0.06
−0.04
−0.02
0.00
0.02
0.04
0.06
0.08

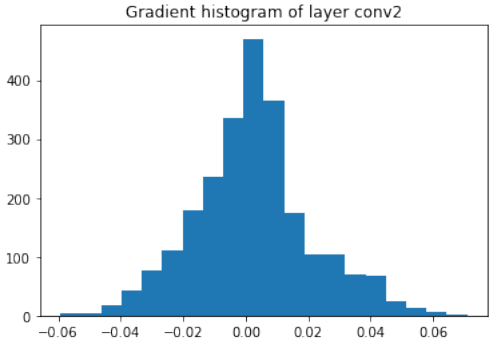

Weight histogram of layer fc1
2500
2000
1500
1000
500
0
−0.04
−0.02
0.00
0.02
0.04

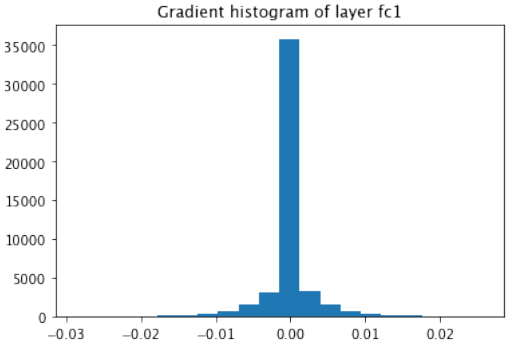

Weight histogram of layer fc2
500
400
300
200
100
0
−0.100
−0.075
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100

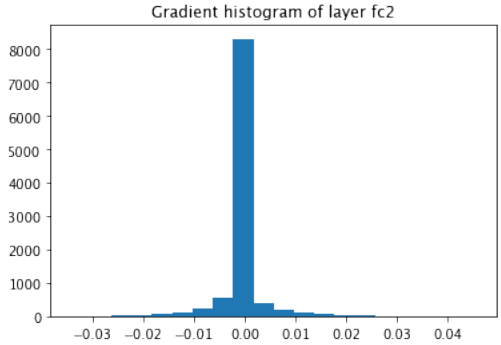

Weight histogram of layer fc3
50
40
30
20
10
0
−0.10
−0.05
0.00
0.05
0.10

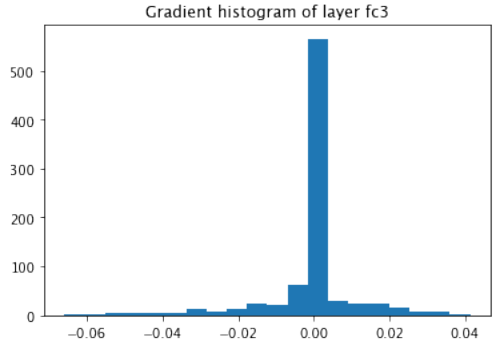
(b)


Gradient histogram of layer conv1
60
50
40
30
20
10
0
−0.08
−0.06
−0.04
−0.02
0.00
0.02
0.04
0.06
0.08


Gradient histogram of layer conv2
400
300
200
100
0
−0.06
−0.04
−0.02
0.00
0.02
0.04
0.06


Gradient histogram of layer fc1
35000
30000
25000
20000
15000
10000
5000
0
−0.03
−0.02
−0.01
0.00
0.01
0.02


Gradient histogram of layer fc2
8000
7000
6000
5000
4000
3000
2000
1000
0
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03
0.04


Gradient histogram of layer fc3
500
400
300
200
100
0
−0.06
−0.04
−0.02
0.00
0.02
0.04

(c) Comparison: Histograms in (c), almost all gradients in other layers are zeros while we have all kinds of values of gradients in (b).
Analysis: This is because we set all weights to 0 in c(c). The process of backpass is chain rule and once the weights are zeros, it will pass zeros to each layer one by one. It is not good for us to set all weights to zeros in CNN model as the results in (c) show because our training process may be dead forever.