14.27
ECONOMICS OF E-COMMERCE
PROBLEM SET 1

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1. Problem 1

Problem: Suppose that demand in an industry is given by:

$$\log(Q) = a + b\log(P) \tag{1}$$

with $b < 0$. What is the price elasticity of demand? What markup (Lerner index) would a firm with a positive marginal cost choose? What happens if the firm is selling information or some other product with zero marginal cost? Comment on the practical importance of this conclusion.

Solution: First, we know that price elasticity is given by $\frac{dQ/Q}{dP/P}$ which becomes $\frac{d\log Q}{d\log P} = b$. Therefore, price elasticity of demand is b. Since we know that a firm with a positive marginal cost will have a lerner index equal to $-QP'(Q)/P(Q) = 1/\epsilon$, we know that a firm would have a markup of $1/b$. If the firm is selling a product with zero marginal cost, then they will maximize:

$$\max_Q P(Q) * Q \tag{2}$$

Since $P(Q) = e^{-a/b}Q^{1/b}$, we see that $P'(Q) = \frac{e^{-a/b}Q^{\frac{1}{b}-1}}{b}$, so that first order conditions give:

$$\frac{e^{-a/b}Q^{\frac{1}{b}-1}}{b}Q + e^{-a/b}Q^{\frac{1}{b}} = 0e^{-a/b}Q^{\frac{1}{b}}(\frac{1}{b}+1) = 0 \tag{3}$$

Since the firm has no marginal cost, the firm can extract as much profit as $\square$