The SKEW Index: Extracting What Has Been Left

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Abstract

This study disentangles the information related to downward movements from the information associated with upward movements of an equity index in a model-free manner. We decompose the implied skew measure into a positive side and a negative side, focusing more on the importance of the negative SKEW index extracted from the Put options. This is a more prudent measure of the implied tail market risk and more informative when added to asset pricing models. The positive SKEW side extracted from the Call options is driven by the market sentiment. We also identify the Granger causality between equity, volatility and skewness.

Keywords: Implied Volatility, Implied Skew, Market Sentiment, Tail Risk, Asset Pricing

JEL: G13,G15,C12

1 Introduction

Do implied volatility indexes reveal the full information contained in options data? Is there anything *left out* that can be useful to investors? In this paper we are showing that a more refined directional construction of volatility *and* skewness indexes may enrich the information extracted from option prices series.

The usefulness of implied volatility measures extracted in a model-free way from equity indexes options data has been widely advocated in the literature (see Whaley, 2000; Carr and Wu, 2006; Jiang and Tian, 2005). Du and Kapadia (2014) pointed out that the volatility indexes underestimate the real stock market volatility when the period is bearish and jumpy. After the Black Monday 1987, the fear of other possible crashes led to a higher weighting for events in the left side of equity return distributions. Furthermore, the 2008 financial crisis has accentuated even more the interest in tail events and outliers. Barberis (2013) advocated that investors are used to over-estimate extreme events when they have a suitable set of information or memories of some similar event still present in their mind. When the information set is limited or when similar tail events have never taken place before, investors may underestimate their likelihood. The S&P 500 Puts are more expensive than the S&P 500 Calls (Bondarenko, 2014) and there is more demand for them during negative times. A possible explanation is that VIX changes are driven more by the negative volatility component (S&P 500 Puts) in comparison to the positive volatility part (S&P 500 Calls).

If the volatility indexes are not able solely to capture adequately the tail risk, new tools should be developed. Recently, different measures have been proposed to gauge this missing risk on the equity market (see Du and Kapadia, 2014; Kelly and Jiang, 2014; Bollerslev and Todorov, 2011). The CBOE proposed a SKEW Index computed from the S&P 500 options following the methodology outlined in Bakshi et al. (2003), henceforth (BKM). While the VIX is the market measure reflecting the "likely", the SKEW measure is more related to the extreme market fear measurement reflecting the "unlikely". A Euro SKEW Index from EUREX has not been developed yet, so we develop it in this paper for the first time applying the same BKM methodology to EUROSTOXX 50 options.

The tail risk literature and the higher risk-neutral moments literature are getting intertwined, better explaining the overall risk in the financial markets. The seminal papers by Bakshi and Madan (2000) and Bakshi et al. (2003) opened a new area of research focused on extracting valuable information about moments of the risk-neutral distribution from option prices. Harvey and Siddique (2000), Conrad et al. (2013) and Mitton and Vorkink (2007) linked the higher risk neutral moments to asset pricing and portfolio management. Xing et al. (2010) found that the implied volatility smirk has a predictive power for future equity returns. Other studies such as Han (2008) and Rehman and Vilkov (2012) concluded that higher moments can increase the accuracy of future asset prices' estimation. Christoffersen et al. (2013) surveyed the available methods for extracting forward looking information from option markets for forecasting purposes and Chang et al. (2012) demonstrated that implied volatility and implied skewness forecast well future realized beta.

The literature on skewness is spanned by two main approaches[1] of calculation as described by Liu (2016). First, there is the raw approach applied by Dennis and Mayhew (2002), Conrad et al. (2013) and Bali and Murray (2013), using the data without filtering or modification. The above studies found that the skew is more negative in period of high market volatility and that the skew for index options tends to be more negative than the skew for individual stock options. The second approach (see Hansis et al., 2010) works with smoothed data, either by data interpolation between OTM Puts with lowest strike and OTM Calls with highest strike or by data extrapolation between the highest and lowest strike prices.

In this paper we decompose the volatility indexes, but we go a step further and decompose the skewness index as well in order to have a more refined understanding of the information revealed by option prices. Our aim is to extract useful forward looking information about tail risk, and also to link it with downside risk area and market sentiment. We separate the implied skewness into a *positive* SKEW index and a *negative* SKEW index, calculated from the relevant class of equity indexes options, Calls and Puts, respectively. Looking to the negative tail risk side can help investors to be prudent and avoid to under-estimate this real, even if rare, risk. For this reason, the tail risk index proposed in this paper focuses on the left side of the distribution and it will be called, from now on, SKEW^- for the U.S. and ESKEW^- for the Eurozone. The views of investors in Calls correspond to another new index, called henceforth, SKEW^+ for the U.S. and ESKEW^+ for the Eurozone. Disentangling the market implied skew indicates that the negative skew side contains more helpful information for explaining future stock returns and tail risk averse investors behaviour. The positive skew most of the time is related to bullish stock market sentiment.

For the implied volatility indexes, we define the negative volatility part, derived from S&P 500 Puts, as VIX^- and the positive volatility part, derived from S&P 500 Calls, as VIX^+. Segal et al. (2015) argue that "bad volatility" comprehends all the news that increase the market tension and decrease the options' underlying value, such as a decline in market productivity, increase of unemployment or market depression. Conversely, "good volatility" regards news impacting underlying assets in a positive way, such as future growth, rising investments and increase in production and consumption.

Moreover, the indexes and their sub-parts will be evaluated in a unified framework using cointegration and Granger causality to further grasp possible relations among them. Comparing the U.S. and the Eurozone equity markets can be also beneficial to extract information on the tail events perceptions across the two main economies. For the volatility market, evidence of the superiority of the negative parts driving the total

[1] Another methodology that has been applied to calculate the skew is the non-parametric approach applied by Xing et al. (2010) and Mixon (2011).

indexes is found. An analysis focused on the Brexit vote is conducted to understand the behavior of the implied volatility and skewness indexes around this event.

Our study will show that the optimistic market sentiment impacting on the Calls side may be sufficient to contaminate the total tail risk index, SKEW, with positive and misleading contributions. Information coming from the "fear index" is not influencing the SKEW$^-$ instead. The negative SKEW and ESKEW seem to be the ones closer to fundamentals. We will confirm our hypothesis that positive and negative skew sides are carrying different information and that they are not linked. We will also show how our SKEW$^-$ index for U.S. and ESKEW$^-$ for Eurozone, are more informative about the equity market returns. They seem to establish also a stronger information channel between U.S. and Eurozone markets compared to the aggregate skew indexes.

The superiority of our proposed negative skew index for predicting portfolios' returns is further confirmed through an asset pricing exercise showing that, for the U.S., adding the aggregate SKEW to the Fama and French (2017) five-factors model does not improve the adjusted R^2 measure while adding SKEW$^-$ improves the adjusted R^2. In the Eurozone, the model that better performs in terms of adjusted R^2 is the one including both implied volatility and skewness indexes extracted from the Puts. Splitting the market in positive and negative scenarios we can observe how SKEW$^+$ and ESKEW$^+$ are more informative in bullish times, while SKEW$^-$ and ESKEW$^-$ in bearish times.

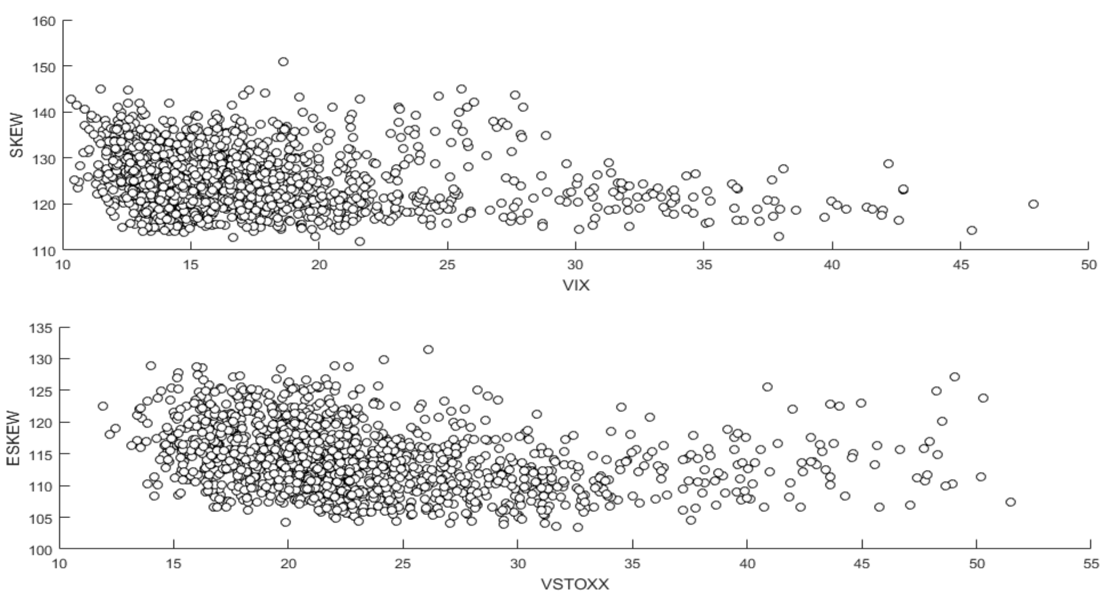
The reminder of this paper is organized as follows. Section 2 describes the methodology behind our analysis, for volatility and skewness. Section 3 presents the evolution of the moment indexes extracted from option prices and it will link skewness with diversification, while Section 4 makes a connection with sentiment indexes. Section 5 contains the econometric analysis testing the main research hypotheses. A more refined analysis around the Brexit vote is discussed in Section 6. Section 7 includes further robustness test results. Section 8 shows the results from the asset pricing exercise and the last section concludes the study.

2 Extracting the Positive and Negative Volatility and Skewness

Jiang and Tian (2005) observed that the VIX is not sensitive enough to the tails of the distribution, thus missing vital information, while Du and Kapadia (2014) argued that the VIX seems not to take into account stock returns jumps, missing relevant information for investors during these situations. They argue that the BKM methodology for computing the second moment of S&P 500 returns is more accurate on average and, especially, during volatile times. The interest in the tail risk has increased after the 2008 financial crisis when investors and academics realized the wrong habit of under-estimating extreme events. Bollerslev and Todorov (2011) proposed a new tail risk measure, called the Investor Fears Index, (FI), using high-frequency intra-day data to extract the expected jump tails in the market and linking them with the corresponding variance jump risk premium that is decomposed into a positive and a negative part. They interpret the difference between the negative and the positive variance risk premium as a measure of investor fear (FI).

The CBOE introduced the CBOE SKEW index as a benchmark providing additional market risk information (see Chicago Board Options Exchange, 2011). This index measures the slope of S&P 500 implied volatility options prices. The equity skew level has increased in the recent years after the financial crisis, suggesting that options traders expect a higher tail risk. The SKEW index is calculated with a methodology similar to the VIX index following BKM. Even if they are both calculated from the same S&P 500 options, the two indexes carry different information and they have a low correlation. When VIX spikes, SKEW remains around its average and vice-versa (see Figure 1). Another measure from the CBOE that has been drawn close to the tail risk concept is the VVIX, the CBOE index of volatility of volatility, calculated by applying the same VIX methodology to a wide range of OTM VIX options. Park (2015) argued that the volatility of the stochastic volatility could be a different way to measure the tail risk and that VVIX, as a mixture of volatility of volatility and also volatility jump risk, is more reliable than VIX in measuring market extreme events.

Figure 1: **VIX VERSUS SKEW and VSTOXX VERSUS ESKEW Scatter Plots**



Notes: These two scatter plots illustrate the relation between VIX & SKEW and VSTOXX & ESKEW. The information provided by the volatility and skew indexes are different, especially during outliers. SKEW and ESKEW are computed following BKM methodology from S&P 500 and EUROSTOXX 50 options. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

Du and Kapadia (2014) proposed the Jump and Tail Index (JTIX), a new index of extreme fear derived from high moments such as skewness and kurtosis of returns and jumps distribution. This index can be replicated by a short position in a risk reversal[2]. Taking a different approach by working with returns at the firm level Kelly and Jiang (2014) developed a measure of tail risk called TAIL Index. This measure is constructed under the assumption that the tail distribution is a time varying power law probability distribution. Hence, they assume that even if different firms are subjected to different risk, they can be aggregated into a single process, assuming similarity in the risk dynamics. The relation between tail risk and individual stock returns is positive under their model, and this is coherent with the fact that investors are tail risk averse. TAIL has also strong predictive power for future market returns.

Our paper is similar in spirit to Feunou et al. (2012) who explicitly models upside and downside volatilities[3] and to Bollerslev et al. (2015) who separated the jump tail risk into a left tail component and a right tail component.

2.1 CBOE VIX and EUREX VSTOXX

We decompose both the VIX and the VSTOXX into their positive and negative sides, using the BKM model-free approach applied daily to our sets of S&P 500 and EUROSTOXX50 daily options[4]. For the current CBOE VIX methodology see Appendix A. The same formula used for the total VIX is applied here in order to compute only the positive or only the negative part. The difference is in the choice of OTM options considered in the computation. For VIX^+ we keep a Call if $K_i \geq K_0$ only, while for VIX^- we keep a Put only when $K_i \leq K_0$. Applying these model-free methodologies we obtain three indexes series: VIX, VIX^+ and VIX^-, daily from the same set of S&P 500 Options, applying the selection above. We are interested to understand how the two volatility indicators VIX^+ and VIX^- behave over different market periods and how they react to investors' sentiment.

The EUROSTOXX 50 index represents the best 50 companies in Europe. The volatility benchmark equivalent to VIX is, in the case, the VSTOXX[5]. See Appendix B for VSTOXX computation details. To compute the negative and positive VSTOXX variants, from now on VSTOXX$^-$ and VSTOXX$^+$, we apply exactly the same OTM options selection and filter rules as for the VIX in order to screen out options.

[2]Risk reversal is an option position of short OTM puts and long OTM Calls.

[3]Downside volatilities have been proposed both for realized volatility as proposed by Barndorff-Nielsen et al. (2010) and also as a model-free option implied volatility as discussed in Andersen and Bondarenko (2007). Earlier research by Bekaert and Wu (2000) advocated that volatility in equity markets is asymmetric in the sense that returns and conditional volatility are negatively correlated.Patton and Sheppard (2015) and Bollerslev et al. (2017) use high-frequency intra-day data to decompose asset return realized volatility into good and bad volatility associated with positive and negative price increments, respectively.

[4]VIX and VSTOXX computed in here might be slightly different from the VIX, VSTOXX historical close prices due to options filters, interest rates, truncation error and hour of computation. This is, however, not affecting the result of this study.

[5]VSTOXX is computed as a 30-days interpolation between the two nearest and available sub-Indexes VSTOXX 1M and VSTOXX 2M equivalent to, respectively, the VIN and VIF for the VIX. In some particular days the two are rolled and the VSTOXX 3M is considered as a far term. The free-risk rate is used is the EURIBOR.

2.2 The BKM Methodology

For the calculation of the cubic risk-neutral moment, we employ the following result from Bakshi et al. (2003); Bakshi and Madan (2000) that underpins our methodology for extracting implied volatility and skewness from options prices:

Theorem 1: Under all martingale pricing measures, the following contract prices can be recovered from the market prices of OTM European Calls and Puts: The t-period risk neutral return skewness, SKEW(t,T) is given by:

$$SKEW(t,T) \equiv \frac{\mathbb{E}_t[(R(t,T) - \mathbb{E}_t[R(t,T)])^3]}{[\mathbb{E}_t(R(t,T) - \mathbb{E}_t[R(t,T)])^2]^{3/2}} = \frac{e^{rt}W(t,T) - 3\mu(t,T)e^{rt}V(t,T) + 2\mu(t,T)^3}{(e^{rt}V(t,T) - \mu(t,T)^2)^{3/2}} \tag{1}$$

The price of volatility and cubic contracts will be, respectively:

$$V(t,T) = 2\int_{S_t}^{\infty} \frac{1 - ln(\frac{K}{S_t})}{K^2} C(t,T;K)dK + 2\int_0^{S_t} \frac{1 + ln(\frac{S_t}{K})}{K^2} P(t,T;K)dK \tag{2}$$

$$W(t,T) = \int_{S_t}^{\infty} \frac{6(ln(\frac{K}{S_t})) - 3(ln(\frac{K}{S_t}))^2}{K^2} C(t,T;K)dK - \int_0^{S_t} \frac{6(ln(\frac{S_t}{K})) + 3(ln(\frac{S_t}{K}))^2}{K^2} P(t,T;K)dK \tag{3}$$

2.3 SKEW and ESKEW

CBOE has introduced the SKEW Index as complementary and supporting to the VIX, computed from the same range of options. The final formula that is applied is the following:

$$SKEW = 100 - 10S \tag{4}$$

where S is the statistical skew $S = \mathbb{E}[(\frac{R-\mu}{\sigma})^3]$ and R is the 30 days log-return of S&P 500 , μ is its expected value and σ is its standard deviation. Following Bakshi et al. (2003), S is calculated starting from a portfolio of S&P 500 options with a pay off reflecting the skewness payoff:

$$S = \frac{\mathbb{E}[R^3] - 3\mathbb{E}[R]\mathbb{E}[R^2] + 2\mathbb{E}[R]^3}{(\mathbb{E}[R^2] - \mathbb{E}^2[R])^{3/2}} \tag{5}$$

Simplifying the notation[6] $S = \frac{P_3 - 3P_1P_2 + 2P_1^3}{(P_2 - P_1^2)^{3/2}}$ the calculation of P_1, P_2 and P_3 from the options market is as follows:

[6]Here we follow CBOE SKEW White Paper: https://www.cboe.com/micro/skew/documents/skewwhitepaperjan2011.pdf

$$P_1 = \mathbb{E}[R_t] = -e^{rT} \sum_i \frac{Q_K \delta_K}{K_i^2} + \varepsilon_1 \tag{6}$$

$$P_2 = \mathbb{E}[R_t^2] = 2e^{rT} \sum_i \frac{(1 - \ln(\frac{K_i}{F_0}))Q_K \delta_K}{K_i^2} + \varepsilon_2 \tag{7}$$

$$P_3 = \mathbb{E}[R_t^3] = 3e^{rT} \sum_i \frac{\ln\left(\frac{K_i}{F_0}\right) - \ln^2\left(\frac{K_i}{F_0}\right) Q_K \delta_K}{K_i^2} + \varepsilon_3 \tag{8}$$

where all the variables are as for the VIX computation, except ε_i that represents adjustments for the difference between the reference price and the forward price given by:

$$\varepsilon_1 = -\left(1 + \ln(\frac{F_0}{K_0}) - \frac{F_0}{K_0}\right). \tag{9}$$

$$\varepsilon_2 = 2\ln\left(\frac{K_0}{F_0}\right)\left(\frac{F_0}{K_0} - 1\right) + \frac{1}{2}\ln^2\left(\frac{K_0}{F_0}\right) \tag{10}$$

$$\varepsilon_3 = 3\ln^2\left(\frac{K_0}{F_0}\right)\left(\frac{1}{3}\ln\left(\frac{K_0}{F_0}\right) - 1 + \left(\frac{F_0}{K_0}\right)\right) \tag{11}$$

These equations are applied both for the near term expiration and also for the far term expiration date. The target is to interpolate these two expirations around 30-days, as for the VIX. The same set of S&P 500 options used for volatility indexes will also be used for SKEW; filters' rules are the same: Puts or Calls with bid price equal to 0 are removed, the same for the options after and before two consecutive double zero bid prices. As in Dennis and Mayhew (2002), we use a trapezoidal approximation, consisting of a discrete sum of our available options prices instead of the integrals in formulae (6), (7) and (8).

For the purpose of this study, in order to compute our negative and positive SKEW indexes, also in here the total SKEW index is split in two sub indexes: the positive SKEW computed from S&P 500 Calls, called SKEW$^+$ and the negative SKEW index computed only from S&P 500 Puts, SKEW$^-$. We introduce an implied SKEW index for the EUREX market as well, using the same BKM methodology:

$$ESKEW = 100 - 10\Phi \tag{12}$$

where Φ is the statistical skewness $\Phi = \mathbb{E}[(\frac{R_{EU} - \mu_{EU}}{\sigma_{EU}})^3]$ where, R_{EU} is the 30 days log return of EUROSTOXX 50, μ_{EU} is its expected value and σ_{EU} is its standard deviation. As before, if we expand the previous relation we will obtain:

$$\Phi = \frac{\mathbb{E}[R_{EU}^3] - 3\mathbb{E}[R_{EU}]\mathbb{E}[R_{EU}^2] + 2\mathbb{E}[R_{EU}]^3}{(\mathbb{E}[R_{EU}^2] - \mathbb{E}^2[R_{EU}])^{3/2}} \tag{13}$$

Simplifying the notation as before $\Phi = \frac{EP_3 - 3EP_1EP_2 + 2EP_1^3}{(EP_2 - EP_1^2)^{3/2}}$ the calculation of EP_1, EP_2 and EP_3 from the options market is as follows:

$$EP_1 = \mathbb{E}[R_{EU,t}] = -e^{rT} \sum_i \frac{EQ_K \delta_K}{K_i^2} + \varepsilon_4 \tag{14}$$

$$EP_2 = \mathbb{E}[R_{EU,t}^2] = 2e^{rT} \sum_i \frac{(1 - \ln(\frac{K_i}{F_0}))EQ_K \delta_K}{K_i^2} + \varepsilon_5 \tag{15}$$

$$EP_3 = \mathbb{E}[R_{EU,t}^3] = 3e^{rT} \sum_i \frac{(2\ln(\frac{K_i}{F_0}) - \ln^2(\frac{K_i}{F_0}))EQ_K \delta_K}{K_i^2} + \varepsilon_6 \tag{16}$$

where r is the EURIBOR and all the other variables are the same used for the VSTOXX computation. For example ε_i, with $i = 4, 5, 6$ are calculated with formulae 9-11. The ESKEW will be then split into positive and negative parts: ESKEW$^+$ and ESKEW$^-$.

2.4 Data and Sample Period

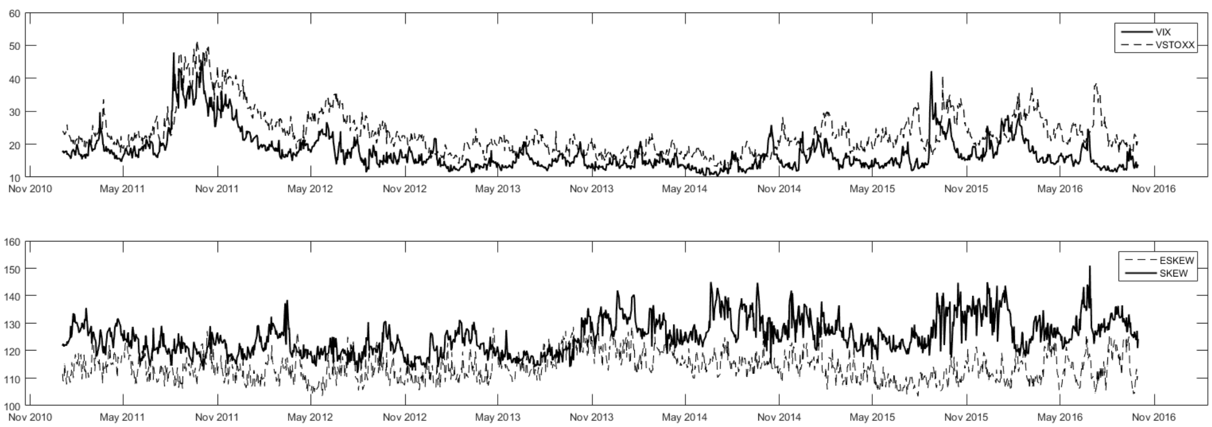
In order to compute our VIX and SKEW time-series we have collected daily S&P 500 Options prices from Bloomberg, both with weekly (SPXW) and also monthly expiration, for the time period January 2011 to September 2016, according to the data availability[7] and closest expiration dates as required by the VIX formula (see Appendix A). Likewise, we collected from Bloomberg daily EUROSTOXX 50 options, with monthly expiration, over the same time period. S&P 500 daily prices were collected from CBOE while EUROSTOXX 50 daily prices were collected from Bloomberg. Daily interest rates for USD and EURO are obtained, respectively, from the Federal Reserve and EMMI.

Options are filtered leaving out options with *BID Price*= 0, options with prices below or above two consecutive zero BID prices and options with less than 2 days expiration (Chicago Board Options Exchange, 2009). After 2014, the U.S. volatility and skewness indexes are calculated following the methodology described above, as a weighted average of the prices of S&P 500 options with weekly expirations. VSTOXX and ESKEW calculations are slightly different from the calculations of VIX and SKEW because weekly expiration options are unavailable. Thus, monthly EUROSTOXX 50 options have been used, with the closest interpolated maturity to 30 days. Usually the weighted average in this case is between 1-month and 2-month expirations. When less than 2 days are left to the expiration date the 3rd month options are used instead of the first month. The option maturity selected changes every month in correspondence to the third Friday of the month.

[7] Before 2014 even if SPXW Options were available their volume and range was not enough to use them in the VIX and SKEW computations. For this reason before 2014 Monthly S&P 500 Options are considered.

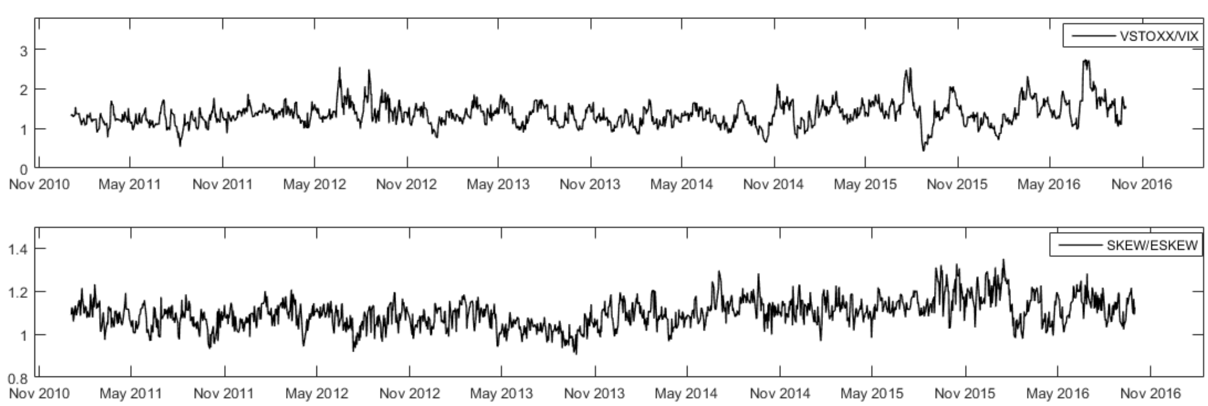
The plots in Figure 2 show the interplay between the U.S. and the Eurozone volatility and skewness indexes between 3-01-2011 and 30-09-2016. For the implied volatility we notice that the VSTOXX is most of the time above the VIX especially when uncertainty hits mostly Europe. Brexit and Grexit are the most evident cases. At a first glance, when some event impacts on the VIX index, VSTOXX seems to follow the trend and there is an immediate reaction. The situation does not seem to follow also in reverse. For the skewness market the U.S. SKEW is mainly higher than ESKEW.

Figure 2: **The VIX versus VSTOXX and SKEW versus ESKEW Relationships**



Notes: The graph shows the relation between volatility indexes (top panel) and skewness indexes (bottom panel) both for the U.S. and the Eurozone. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

Figure 3: **VSTOXX/VIX and SKEW/ESKEW Spread**



Notes: The graph shows the spread between the U.S. and the Eurozone volatility market (top panel) as VSTOXX on VIX ratio and the spread between the U.S. and the Eurozone skewness markets as, instead, SKEW on ESKEW. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

The plots in Figure 3 are illustrating the volatility and skewness indexes spreads between the U.S. and the

Eurozone. The spread is computed as the highest index on the lowest. For the volatility market is VSTOXX on VIX and for the skewness is SKEW on ESKEW. In the volatility market the spread is almost always above 1 and it reached one of its global maximum just before the Brexit due to the uncertainty in Europe around that event. The skewness spread was high during autumn 2015 and beginning of 2016, reflecting in the U.S. tail fear events like Grexit and China Yuan turbulence during the summer of 2015.

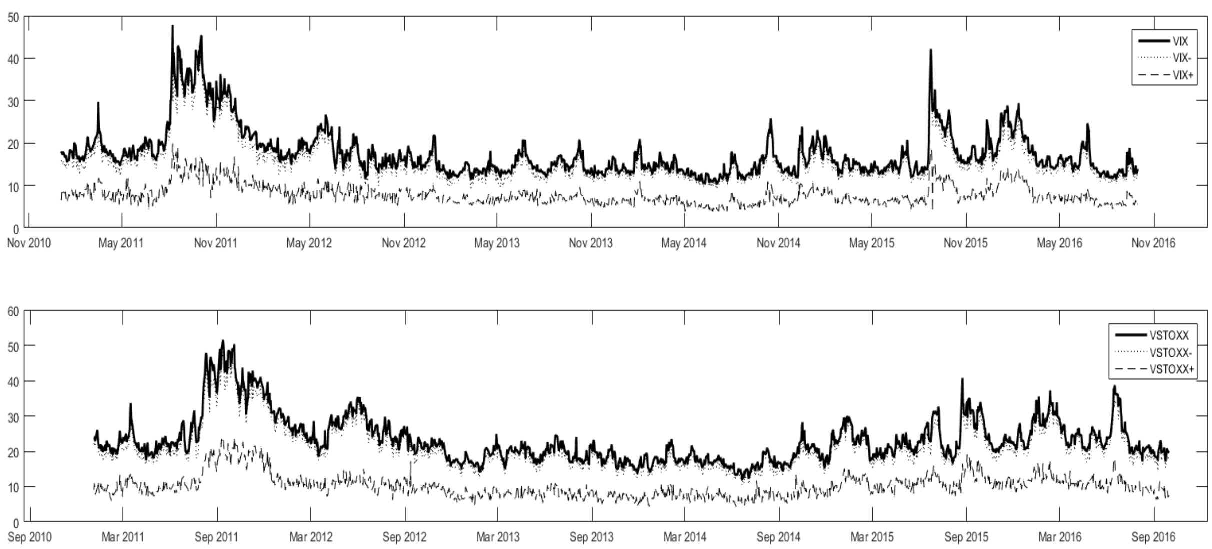
3 Time Series Decomposition: Preliminary Findings

In this section we look at the indexes' decomposition and associated distributions. A full econometric analysis is detailed later on in Section 5.

3.1 Volatility Decomposition

The VIX and the VSTOXX decompositions are dominated by their negative components, VIX^- and $VSTOXX^-$. Figure 4 reveals the prevalent role of the negative indexes side computed from the equity Puts market in the total volatility index aggregate, in line with the conclusion in Ang et al. (2005) that investors weigh differently downside losses versus upside gains. Hence, VIX^- and $VSTOXX^-$ drive the total volatility indexes

Figure 4: **Volatility Indexes U.S. and EURO Comparison**



Notes: This graph shows the decomposed volatility indexes series: VIX, VIX^- and VIX^+ for the U.S. (top panel) and VSTOXX, $VSTOXX^-$ and $VSTOXX^+$ for the Eurozone (bottom panel). The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

the most. This can be attributed to the volume of hedging activity of the investors looking to protect their equity portfolios by trading in S&P 500 or EUROSTOXX 50 protective Puts strategies.

In addition, VIX^+ and $VSTOXX^+$ computed from Calls can be seen as an "insurance premium" charged by volatility indexes' providers. Investors that are long in equity pay this "fee" in order to protect their portfolios against equity market drops or volatility spikes[8]. The positive volatility indexes are reflecting the part of volatility that is not dangerous for long equity investors (Segal et al., 2015) and that can be interpreted even as "euphoria" (Bollerslev et al., 2015). Fear-type volatility could scare the investors, making them less flexible and inclined to speculative equity strategies. The positive out-looking volatility helps the market attenuate the fear relative to equity shortfall and market losses.

3.2 Skewness and Diversification

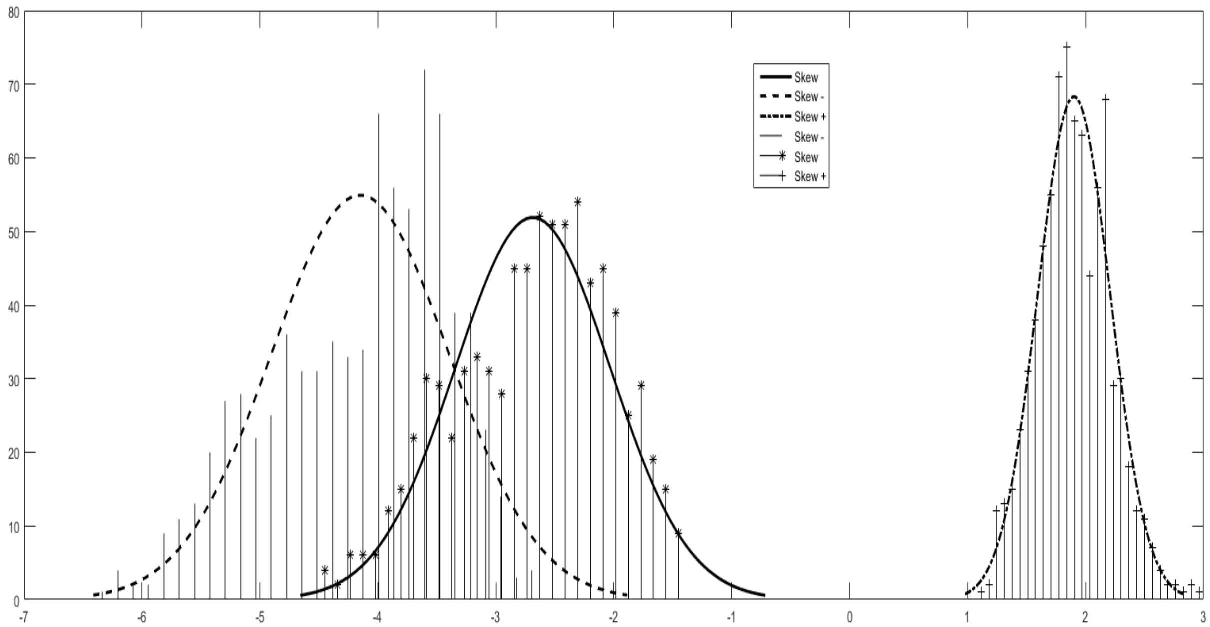
The same decomposition is conducted for skewness, that is the main focus in this paper. The total skewness, computed following the BKM methodology, is a difference between an equity Calls portfolio and an equity Puts portfolio (see Formula (3)). For the aggregate equity index this relation seems to be mostly negative or, in other words, left skewed. This may be justified from the higher impact and weight the Puts portfolio has in comparison to Calls. The prevalent left skewed Puts distribution drags the total skewness to the left as well, as it is illustrated in Figures 5 and 6 for the U.S. and the Eurozone, respectively. The total Skew is never positive and even far away from zero. The $Skew^+$, represents more the long Calls investors.

A comparison between the U.S. and the Eurozone skewness distributions, Skew and ESkew, shows how the U.S. equity market has a more left skewed distribution in comparison to the Eurozone equity market. The same is true for the negative skewness distributions: the U.S. $Skew^-$ is more left skewed than the Eurozone equivalents. Conversely, the U.S. $Skew^+$ is more right skewed than the European $Eskew^+$ (see Figure 7). One possible explanation is the degree of diversification that is very different between S&P 500 and EUROSTOXX 50. Another explanation is the options trading volume, that is larger for the S&P 500 (see Harvey and Siddique, 2000; Mitton and Vorkink, 2007).

These findings allow us to link our analysis to diversification. Harvey and Siddique (2000) points out that decreasing the portfolio skewness will command higher expected returns due to the higher risk of the portfolio, and vice-versa. Considering that a portfolio can be considered properly diversified when the number of components is greater than 100, the S&P 500 is properly diversified, but the EUROSTOXX 50 is under-diversified. Mitton and Vorkink (2007) advocates that returns of under-diversified portfolios are more positively skewed than those of diversified portfolios. According to them, diversification is a two edged sword: it eliminates undesired volatility but, at the same time, it eliminates also desired skewness. The latter is the right skewness that is reduced when stocks are added, so when diversification is amplified the

[8] See Whaley (2009) about the negative relation between equity market and volatility market.

Figure 5: Skew, Skew$^-$ and Skew$^+$ Relationship



Notes: The three bells plot illustrates the decomposed statistical skew distributions for the U.S. The total statistical skew is computed using Formula 5: $S = \frac{E[R^3]-3E[R]E[R^2]+2E[R]^3}{(E[R^2]-E^2[R])^{3/2}}$. The distribution of Skew$^-$ is computed from Puts options only, while the distribution of Skew$^+$ is computed from Calls options only. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

stocks are contributing in a negative way to the portfolio co-skewness.

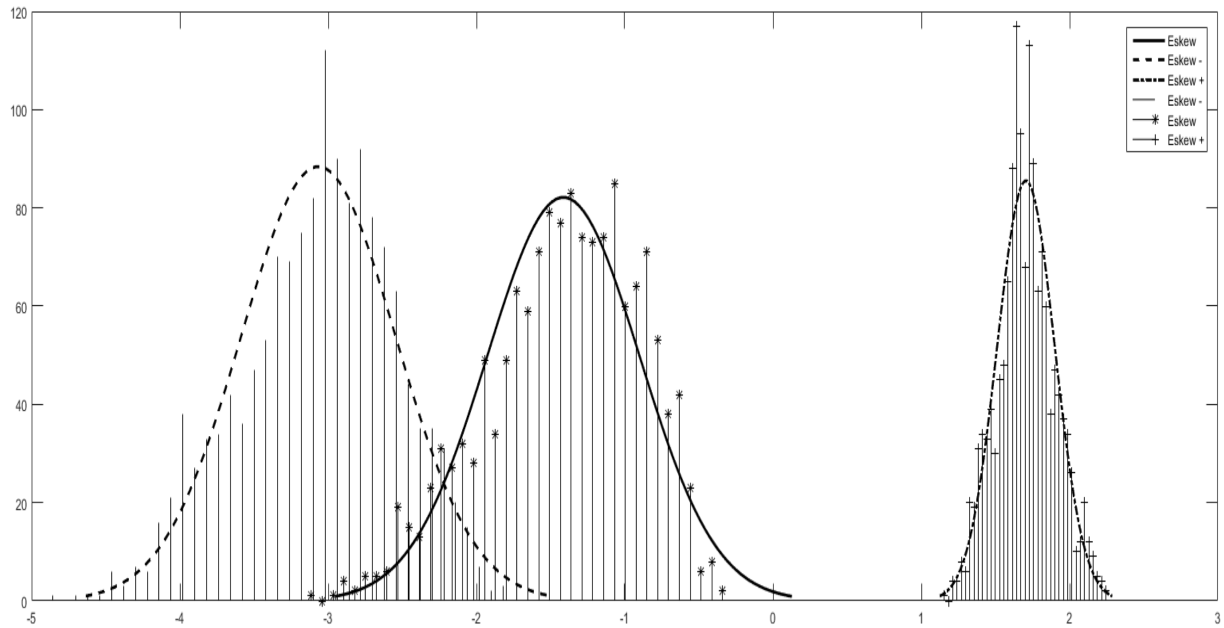
4 Tail Indexes and Market Sentiment

4.1 Market Sentiment

Baker and Wurgler (2000) and Baker and Wurgler (2007) asserted how the presence of investor sentiment is actually impacting on the stock prices. Han (2008) confirmed that there is a link between risk neutral skewness and market sentiment; when the investor sentiment is bearish the index option volatility smile is steeper and the risk neutral skewness is more negative. Conversely, when the investor sentiment is bullish, the volatility smile is flatter and the skewness less negative. Thus, the market sentiment affects equity index options' prices, changing market volatility and skewness.

One of the earliest sentiment measures has been developed by Baker and Wurgler (2006), applying a principal components analysis to six underlying sentiment proxies they selected. Recently, Huang et al. (2015) improved this measure extracting the most relevant information for the stock returns from the same

Figure 6: Eskew, Eskew^{-} and Eskew^{+} Relationship



Notes: The three bells plot illustrates the decomposed statistical skew distributions for the Eurozone. The total statistical skew is computed through Formula 13: $\Phi = \frac{E[R_{EU}^3]-3E[R_{EU}]E[R_{EU}^2]+2E[R_{EU}]^3}{(E[R_{EU}^2]-E^2[R_{EU}])^{3/2}}$. The distribution of Eskew^{-} is computed from Puts options only, while the distribution of Eskew^{+} is computed from Calls options only. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

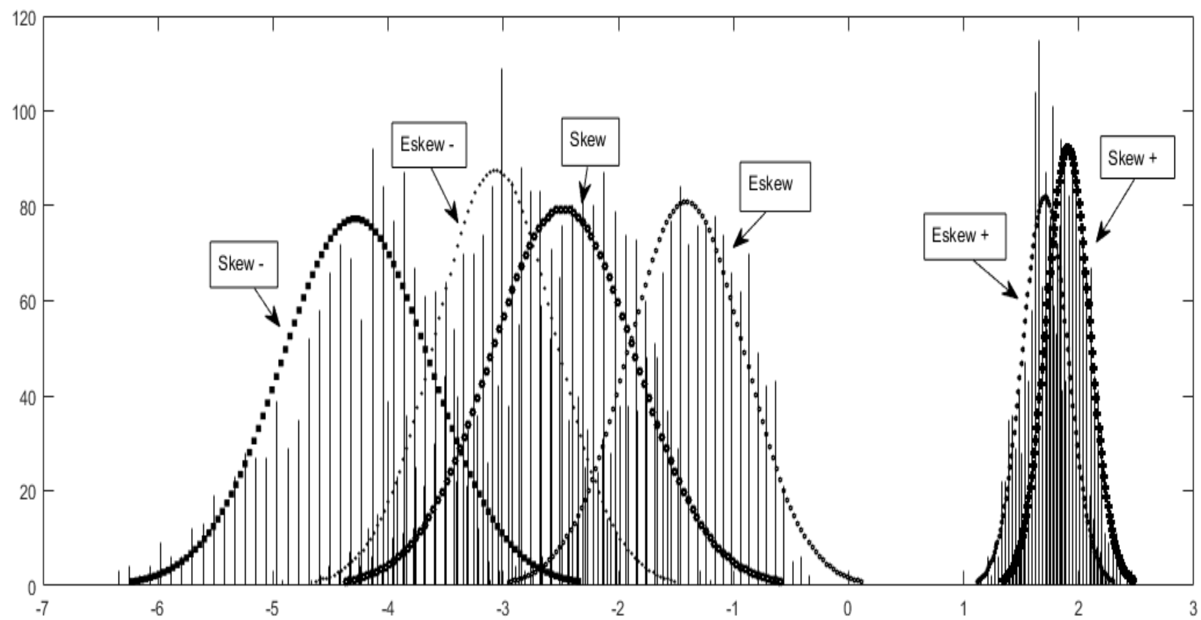
six sentiment proxies, discarding their approximation errors[9]. The relationship between the skew and market sentiment was tested by Dennis and Mayhew (2002), in the form of a link between the skew and the Put-Call ratio, finding a positive, but not significant relation between the two.

Here we will analyse how implied moments are affected by market sentiment. Investors[10] are trading index options based on their directional information (Ni et al., 2008). Index OTM Puts are well known as insurance assets against equity market drops (see Bollen and Whaley, 2004; Han, 2008; Bondarenko, 2014)) and their trading is driven mainly by hedging demand from institutional investors (Lakonishok et al., 2007).

[9]Other common measure of market sentiment are the CBOE Put-Call Ratio (see Dennis and Mayhew, 2002), the same CBOE VIX as a "fear gauge" and the American Association of Individual Investors (AAII)'s survey about investors' market opinions (Bullish, Bearish, Neutral) and measured as the difference between bearish opinion and bullish opinion. It has been considered as a proxy of institutional investors sentiment due to the panel of interviewed people.

[10]It is common in the literature to distinguish between two groups of investors, influenced by opposite market expectations. According to Brown and Cliff (2005), investors are divided into "fundamentalist", trading according to the asset fundamentals, and "speculators", more influenced by market sentiment. Another investors categorisation by Lemmon and Ni (2014) is between institutional investors and "noise traders" or individual investors. The latter are referred to be more influenced by sentiment and behavioural biases compared to the first.

Figure 7: Skews and Eskews Comparison



Notes: This graph shows the comparison between the statistical decomposed skew distributions for the U.S. and the Eurozone. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

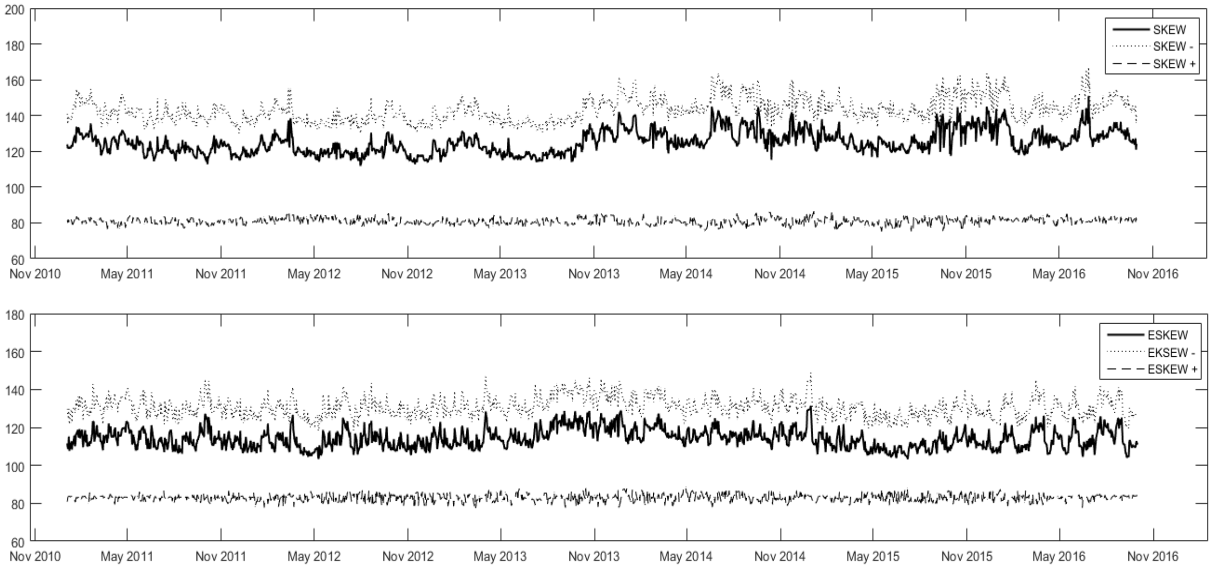
4.2 The Impact of Sentiment on Skewness

Figure 8 illustrates the relationship between total SKEW (ESKEW) indexes and their positive and negative parts. What is clear is that SKEW$^-$ plays a main role in determining the trend for the SKEW series. The same conclusion is confirmed for the Eurozone market. SKEW$^+$ and ESKEW$^+$ become important only occasionally. The negative SKEW index is more conservative in risk management terms, being above the total index in both economies. The positive side lays, similar to the volatility market, below the total index.

The descriptive statistics and sample correlation analysis of the volatility and skewness time-series are reported in Table 1 and 2. The main descriptive statistics for the U.S. and the Eurozone market show an average total VIX (VSTOXX) equal to 17.53 (23.22) and an average total SKEW (ESKEW) close to 125 (114). VIX$^-$ (VSTOXX$^-$) have the largest contribution to the main volatility index, equal to 15.82 (21.26), while VIX$^+$ (VSTOXX$^+$) have a marginal contribution, equal to 7.85 (10.21). The SKEW$^-$ (ESKEW$^-$) average is higher then the total SKEW (ESKEW) index, reaching 143.18 (130.66). The SKEW$^+$ (ESKEW$^+$) is equal to 80.98 (82.94). Interestingly, the Eurozone volatility indexes are more volatile than the corresponding ones for the U.S. while the opposite is true for the skewness indexes.

Table 2 suggests that the correlation between SKEW and SKEW$^-$ (ESKEW and ESKEW$^-$) is highly positive 0.96 (0.93). The correlation between SKEW and SKEW$^+$ (ESKEW and ESKEW$^+$) is not significant, being equal to 0.02 (-0.01). SKEW$^-$ and SKEW$^+$ (ESKEW$^-$ and ESKEW$^+$) have negative correlation equal to -0.12 (-0.23), carrying different information about tail risk expectations.

Figure 8: **Comparison of SKEW Indexes for the U.S. and the Eurozone**



Notes: This graph shows the decomposed SKEW indexes, namely, SKEW, SKEW$^-$ and SKEW$^+$ for the U.S. (upper panel) and ESKEW, ESKEW$^-$ and ESKEW$^+$ for the Eurozone (bottom panel). The SKEW indexes for the U.S. are computed through formula (4): $SKEW = 100 - 10S$. The ESKEW indexes for the Eurozone are computed through Formula (12): $ESKEW = 100 - 10\Phi$. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

Table 1: **Descriptive Statistics for the U.S. and the Eurozone Markets**

	S&P 500	VIX	VIX$^-$	VIX$^+$	SKEW	SKEW$^-$	SKEW$^+$
Mean	1710.31	17.53	15.82	7.85	125.16	143.18	80.98
Median	1771.95	15.94	14.28	7.29	124.33	142.39	80.96
Maximum	2190.15	47.84	42.98	19.87	150.97	167.19	86.55
Minimum	1099.23	10.31	9.16	3.58	111.90	129.98	74.96
Std. Dev.	325.27	5.66	5.25	2.39	6.40	6.55	1.85
Skewness	-0.15	2.00	2.10	1.41	0.58	0.62	-0.03
Kurtosis	1.48	7.62	8.04	5.53	2.93	3.01	2.77
Jarque-Bera	145.01	2260.88	2601.36	870.27	83.31	93.30	3.16
Observations	1447	1447	1447	1447	1447	1447	1447
	EUROSTOXX 50	VSTOXX	VSTOXX$^-$	VSTOXX$^+$	ESKEW	ESKEW$^-$	ESKEW$^+$
Mean	2890.82	23.22	21.26	10.21	114.13	130.66	82.94
Median	2947.31	21.61	19.62	9.67	113.66	130.16	82.96
Maximum	3828.78	51.49	47.99	23.98	131.43	148.90	88.53
Minimum	1995.01	11.89	10.51	4.22	103.35	118.11	77.44
Std. Dev.	397.37	6.70	6.20	3.29	5.13	5.23	1.94
Skewness	-0.05	1.46	1.41	1.29	0.44	0.33	-0.07
Kurtosis	2.43	5.38	5.23	5.10	2.69	2.75	2.85
Jarque-Bera	19.68	864.29	782.87	675.51	52.63	30.00	2.73
Observations	1447	1447	1447	1447	1447	1447	1447

Notes: This table presents the main descriptive statistics for both the U.S. and the Eurozone series for the selected period from 03-01-2011 to 30-09-2016, at daily frequency.

Decomposing the SKEW indexes can be useful in order to have an insight into the implied market view and associated sentiment. Our idea of tail risk index is to rely only on the SKEW$^-$ (ESKEW$^-$) part,

Table 2: **Correlation Analysis for the U.S. and the Eurozone Markets**

	S&P 500	VIX	VIX$^-$	VIX$^+$	SKEW	SKEW$^-$	SKEW$^+$
S&P 500	1.00						
VIX	-0.47	1.00					
VIX^-	-0.46	0.99	1.00				
VIX^+	-0.42	0.87	0.83	1.00			
$SKEW$	0.50	-0.20	-0.19	-0.19	1.00		
$SKEW^-$	0.46	-0.16	-0.16	-0.16	0.96	1.00	
$SKEW^+$	-0.01	0.09	0.09	0.11	0.02	-0.12	1.00
	EUROSTOXX 50	VSTOXX	VSTOXX$^-$	VSTOXX$^+$	ESKEW	ESKEW$^-$	ESKEW$^+$
$EUROSTOXX50$	1.00						
$VSTOXX$	-0.51	1.00					
$VSTOXX^-$	-0.52	0.99	1.00				
$VSTOXX^+$	-0.34	0.90	0.88	1.00			
$ESKEW$	0.02	-0.26	-0.24	-0.26	1.00		
$ESKEW^-$	0.02	-0.23	-0.21	-0.23	0.93	1.00	
$ESKEW^+$	0.01	-0.01	-0.02	0.00	-0.01	-0.23	1.00

Notes: This table presents the correlation analysis between the U.S. and the Eurozone series for the selected period from 03-01-2011 to 30-09-2016, at daily frequency.

removing the information coming from the Calls side representing the optimistic views. This is a more reliable and prudent measure of extreme market fear, tail risk aversion and investors hedging willingness. It seems to be more robust and less attached to the investors' sentiment (see Lemmon and Ni, 2014; Andreou et al., 2016). While there is evidence in the literature about the key role of equity index OTM Puts as shelter against equity market drop (see Dennis and Mayhew, 2002; Han, 2008; Bondarenko, 2014), the role of equity indexes OTM Calls is unclear in the literature.

We have conducted a simple regression for testing the link between market sentiment and options activity behind the skew indexes. To this end we related some of the most common investor sentiment indexes in the literature with our SKEW (ESKEW) series running the simple regression:

$$SKEW_n^i = c + \beta_1 MarkSent_j + \epsilon \tag{17}$$

where for the dependent variable $i = TotalIndex, -, +$ and $n = U.S.\quad and \quad Eurozone$, while $MarkSent_j$, is a market sentiment proxy, with $j = Zhou, \quad BW, \quad PCRatio, \quad AAII, \quad VIX$[11].

Table 3 reports the regressions results. For the U.S. we can confirm that market sentiment is impacting in a positive direction the OTM Calls trade; the coefficients are positive and significant in 3 cases out of 5. Total SKEW and SKEW$^-$ are impacted negatively and slightly by market sentiments in 2 cases out of 5, including the VIX computed from the same S&P 500 options. Where significant, the negative coefficients decrease from SKEW to SKEW$^-$ meaning the first can be seen as a market sentiment weighted average from both the options sides, as concluded also by Brown and Cliff (2005). The coefficient of AAII is positive for all the three indexes since it is a sentiment proxy from the institutional investors. Our results also confirm

[11] Zhou is the investor sentiment index by Huang et al. (2015), BW is the Baker and Wurgler (2006) sentiment index, PCRatio is the Put-Call ratio from CBOE, commonly used as a sentiment proxy (see Dennis and Mayhew, 2002); AAII is the sentiment index from the American Association of Individual Investors's survey. VIX is the volatility "fear gauge".

Table 3: **Regression Analysis Investor Sentiment Indexes on Skewness Indexes**

Variables	Zhou	BW	PCRatio	AAII	VIX
SKEW	-7.082	-0.362*	-0.03	0.196	-0.227*
	(0.201)	(0.013)	(0.991)	(0.176)	(0.000)
$SKEW^-$	-6.942	-0.026*	0.216	0.173	-0.192*
	(0.232)	(0.012)	(0.738)	(0.258)	(0.000)
$SKEW^+$	0.122	3.963*	1.489*	0.041	0.316*
	(0.943)	(0.046)	(0.007)	(0.343)	(0.000)
ESKEW	0.013	-8.299	-1.208	0.033	-0.138*
	(0.997)	(0.283)	(0.173)	(0.799)	(0.000)
$ESKEW^-$	-0.036	-11.647	-1.248*	-0.065	-0.119*
	(0.992)	(0.143)	(0.015)	(0.638)	(0.000)
$ESKEW^+$	-0.962	0.192	-0.075	(0.015)	0.010*
	(0.608)	(0.947)	(0.876)	(0.754)	(0.040)

Notes: This table presents the output of the regression analysis between our dependent variables: SKEW, $SKEW^-$ and $SKEW^+$ for the U.S. and ESKEW, $ESKEW^-$ and $ESKEW^+$ for the Eurozone and the most common market sentiment indexes in the literature as independent variables. The simple regression that is considered is equation 17: $SKEW^i = c + \beta_1 MarkSent_j + \epsilon$ with $i = Tot, -, +$ and $j = Zhou, BW, PutCall, AAII, VIX$. The sentiment indexes which are selected are the sentiment index by Huang et al. (2015) (Zhou), the one by Baker and Wurgler (2006) (BW), Put-Call ratio (PCRatio), American Association or Individual Investors' survey index (AAII) and the volatility fear gauge, VIX. For Zhou, BW and AAII monthly data are used from 01-2011 to 10-2014, due to sentiment index data unavailability afterwards. For the PCRatio and VIX the series are daily with the same length of our data set, from 03-01-2011 to 30-09-2016. The values are the regression coefficients and their respective p-values. Significant coefficients at 5% are marked with *.

the result in Dennis and Mayhew (2002) since the Put-Call ratio is not significant in explaining the total skew[12]. For the Eurozone the picture is less clear and the coefficients are mainly not significant but that should not be surprising since most of the sentiment proxies are U.S. based. Interestingly, the effect of the VIX is significant for all the ESKEW indexes and the Put-Call ratio is significant for explaining the flow of OTM Puts trade in the Eurozone.

The regression coefficients indicate how Puts and Calls portfolios react, mostly in an opposite direction to market sentiment reflecting pessimistic and optimistic views, respectively, and there is a negative correlation between $SKEW^+$ ($ESKEW^+$) and $SKEW^-$ ($ESKEW^-$). SKEW and $SKEW^-$ for U.S. and ESKEW and $ESKEW^-$ for the Eurozone are closer in calm periods. Their distance becomes wider in high market sentiment times.

For the investors long in index Calls, $SKEW^+$ ($ESKEW^+$) is representative of the bullish view about the markets. Optimistic investors could either go long OTM index Calls or short OTM index Puts. Especially in high sentiment market, when the information provided by the tail risk index should be more reliable, OTM index Puts are well known to be expensive (see Bondarenko, 2014; Han, 2008). For this reason investors with a bullish expectation in equity market could go long in OTM index Calls. Investors who are "end-users" have a net long position in S&P 500 options, (Garleanu et al., 2009). Our regression result reveal how $SKEW^+$ can be seen as the part of the index influenced the most by the market sentiment. It can have an unwanted effect in the overall market risk, dragging it towards the "bright" side thus creating an *optimistic illusion* carried from investors' positive expectations.

[12]Put-Call ratio is considered as a proxy of market sentiment and trading pressure. We have found also the same discordance in the coefficient sign. We expected it to be positive, but we found negative. Dennis and Mayhew (2002) expected negative coefficient ex ante and they found positive. They used statistical skewness so the opposite relation is true (see Formula 4).

5 Econometric Analysis

In this section we analyse the interconnections among our volatility and skewness time-series. Our variables can be divided into three main groups: equity levels (S&P 500 and EUROSTOXX 50), volatility (VIX, VIX$^-$, VIX$^+$ and VSTOXX, VSTOXX$^-$, VSTOXX$^+$) and skewness (SKEW, SKEW$^-$, SKEW$^+$ and ESKEW, ESKEW$^-$, ESKEW$^+$). Performing not only a cross-signs comparison within our risk-neutral moments, but also across different economies, the U.S. and the Eurozone, we would like to understand if volatility and skewness provide any information about investors' behavior and their risk perception. Furthermore, we would like to test if the negative implied skewness is more informative and helpful as a tail risk measure.

We perform a unit root test, the Johansen cointegration test and the Granger causality test to investigate the relation and co-dependency between volatility and skewness market in a long-term period. Our unit root analysis indicates the presence of non stationarity in levels, but the first differences of all the series are stationary. Moreover, a pairwise Johansen cointegration test has been performed for all the series in our study as well, taking into account also the negative and positive parts of indexes and the two, U.S. and Eurozone, markets. A study of the Granger causality relation through a VAR model will be first considered. VECM will be tested for a robustness analysis for all the series featuring cointegration evidence. We are interested in the presence of a causality relationship regardless of the evidence on cointegration[13].

The following research hypotheses will be tested through Granger causality tests. **Hypothesis 1**: *$SKEW^+$ and $ESKEW^+$ do not Granger cause $SKEW^-$ and $ESKEW^-$*. The former are more based on investors' sentiment while the latter more on fundamentals and hence the negative implied skewness indexes should be more reliable as tail risk benchmark. **Hypothesis 2**: *There is a relationship between $SKEW$ ($ESKEW$) and the equity market index S&P 500 ($EUROSTOXX$50).* Chang et al. (2013) investigated the empirical relationship between the innovation in option-implied market skewness and the stock market return and they found a significant negative relationship between the innovation in option-implied market skewness and the stock market return. From a diversification point of view, S&P 500 and SKEW should have a higher β coefficient in comparison to EUROSTOXX 50 and ESKEW (Harvey and Siddique (2000) and Mitton and Vorkink (2007)). **Hypothesis 3**: *VIX^- and $VSTOXX^-$ are driving VIX and $VSTOXX$, respectively.* Moreover, we are searching if the same relation is true also for the skewness market. The pairwise Granger relations for each pair will be tested with the following model:

$$\triangle Y_{i,t} = c_i + \sum_{j=1}^{n} \alpha_{i,j} \triangle Y_{t-j} + \sum_{j=1}^{n} \theta_{i,j} \triangle X_{t-j} + \epsilon_{i,t} \tag{18}$$

[13]Pre-testing for cointegration can introduce over-rejections of the no-Granger causality hypothesis, see Clarke and Mirza (2006). VECM will then be used only for the series we have found to be cointegrated and as a robustness comparison with VAR.

where, $i = 1, ...p$ is the indicator for selecting the p variable to test as dependent variable in VAR, j is the lag indicator, t is the last day in our sample. The regressors are the lag Y dependent variables and the lag X independent variables and ϵ is distributed as $N(0, \sigma^2)$.

5.1 Econometric Results

The Johansen cointegration test indicates at least one cointegrated vector between S&P 500 , VIX and SKEW. There is also evidence of cointegration for EUROSTOXX 50, VSTOXX and ESKEW. The analysis of all the U.S. and the Eurozone series shows at least a cointegration of rank two. The same is true for the cross U.S. - Eurozone test between the equity, volatility and skewness levels (see Table 4).

Table 4: **Johansen Cointegration Test Multi-Series**

Obs	Test Type	No Intercept - No Trend	Intercept - No Trend
	S&P 500 , VIX, VIX^-, VIX^+, SKEW, $SKEW^-$, $SKEW^+$		
1411	Trace	2	6
	Max-Eig	1	2
	S&P 500 , VIX, SKEW		
1418	Trace	1	2
	Max-Eig	1	2
	EUROSTOXX 50, VSTOXX, $VSTOXX^-$, $VSTOXX^+$, ESKEW, $ESKEW^-$, $ESKEW^+$		
1416	Trace	2	4
	Max-Eig	2	3
	EUROSTOXX 50, VSTOXX, ESKEW		
1438	Trace	1	1
	Max-Eig	1	1
	S&P 500 , EUROSTOXX 50, VIX, VSTOXX, SKEW, ESKEW		
1427	Trace	2	2
	Max-Eig	2	2

Notes: This table shows the Johansen Cointegration test multi-series for all the U.S. and all the Eurozone series. The test is performed at critical level 0.05, both for No-Intercept-No Trend and also for Intercept-No Trend. Trace Test and Max-Eigenvalue Test are performed. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

The pairwise cointegration test in Table 5 shows that the three U.S. aggregate variables reveal at least one range of cointegration at 0.05 level between S&P 500 and VIX and between SKEW and VIX. There is no evidence of cointegration between S&P 500 and SKEW. The pairwise analysis for the Eurozone shows that EUROSTOXX 50 - VSTOXX and VSTOXX - ESKEW are, at least, one vector cointegrated.

Evidence of pairwise cointegration is found between the levels of equity market and the volatility market, and between second and third moments both in the U.S. and in the Eurozone. There is no evidence of cointegration between equity levels and skewness index. Non-significant results in term of cointegration are found also between $SKEW^+$ for the U.S. and $VSTOXX^+$, $ESKEW^+$ for the Eurozone.

When the U.S. and the Eurozone markets are compared, evidence of at least one cointegration rank is present for the volatility and the skewness markets. However, we could not detect any cointegration between S&P 500 and EUROSTOXX 50 (see Table 6).

A Granger causality test (GCT) is also performed in order to test our hypotheses. Firstly, a pairwise cross-moments GCT between the U.S. series and then between the Eurozone series is conducted, with Granger

Table 5: **Pairwise Johansen Cointegration Test U.S. & Eurozone Market**

U.S. Market					Eurozone Market				
Series	Obs	Lags	NI - NT	I - NT	Series	Obs	Lags	NI - NT	I - NT
$S\&P500$, VIX	1433	13	2 2	1 1	$EUROSTOXX50$, $VSTOXX$	1436	10	1 1	1 1
$S\&P500$, VIX^-	1433	13	2 2	1 1	$EUROSTOXX50$, $VSTOXX^-$	1436	10	1 1	0 1
$S\&P500$, VIX^+	1434	12	2 2	1 1	$EUROSTOXX50$, $VSTOXX^+$	1432	14	0 0	0 1
$S\&P500$, $SKEW$	1439	7	0 0	1 1	$EUROSTOXX50$, $ESKEW$	1432	14	0 1	1 0
$S\&P500$, $SKEW^-$	1433	13	0 0	1 1	$EUROSTOXX50$, $ESKEW^-$	1428	18	0 0	1 1
$S\&P500$, $SKEW^+$	1422	24	2 2	1 1	$EUROSTOXX50$, $ESKEW^+$	1431	15	0 0	1 1
VIX, $SKEW$	1423	23	1 1	2 2	$VSTOXX$, $ESKEW$	1433	13	1 1	2 2
VIX, $SKEW^-$	1436	10	1 1	2 2	$VSTOXX$, $ESKEW^-$	1433	13	1 1	2 2
VIX, $SKEW^+$	1431	15	0 0	2 2	$VSTOXX$, $ESKEW^+$	1433	13	0 1	2 2
VIX^-, $SKEW$	1439	7	1 1	2 2	$VSTOXX^-$, $ESKEW$	1431	15	1 1	2 2
VIX^-, $SKEW^-$	1428	18	1 1	2 2	$VSTOXX^-$, $ESKEW^-$	1432	14	1 1	2 2
VIX^-, $SKEW^+$	1429	17	0 0	1 1	$VSTOXX^-$, $ESKEW^+$	1431	15	0 1	2 2
VIX^+, $SKEW$	1431	15	1 1	2 2	$VSTOXX^+$, $ESKEW$	1431	15	0 0	2 2
VIX^+, $SKEW^-$	1431	15	1 1	2 2	$VSTOXX^+$, $ESKEW^-$	1433	13	0 0	2 2
VIX^+, $SKEW^+$	1430	16	0 0	2 2	$VSTOXX^+$, $ESKEW^+$	1431	15	0 1	2 2

Notes: This table shows the pairwise Johansen Cointegration test for both the U.S. and the Eurozone series. The test is performed at critical level 0.05, assuming No Intercept (NI) - No Trend (NT) and Intercept (I) - No Trend (NT) cases. Trace Test (first row) and Max-Eigenvalue Test (second row) are performed. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

causality (GC) results reported in Table 7. Secondly, a pairwise GCT cross-markets and cross-signs, but within the same moment, has been conducted both for volatility and skewness markets. This Granger causality (GC) evidence is depicted in Table 8.

For the U.S. we uncover unidirectional Granger causality from the equity market to the total, negative and positive volatility indexes. However, we could not find any Granger causality between $\triangle$ SKEW and $\triangle$ S&P 500 . Hence, the hypothesis 2 is not confirmed for the U.S. market. Interestingly, $\triangle$ SKEW$^-$ is more informative than the total $\triangle$ SKEW in carrying information and explaining future stock returns. $\triangle$ S&P 500 and $\triangle$ VIX cause movements in $\triangle$ SKEW$^+$ reflecting the influence of market sentiment on the Calls' side (See 4 Section). A possible explanation is that investors would like to take a long equity position when VIX is low and S&P 500 looks bullish. Conversely, there is no Granger causality between $\triangle$ VIX and $\triangle$ SKEW$^-$ implying that the hedgers' trading strategies are more related to fundamentals, neglecting the information carried from the fear index. Hypothesis 1 is confirmed for U.S. market. Surprisingly, no Granger causality is found between the volatility and the skewness market, even though they are cointegrated[14].

For the Eurozone the same unilateral causality from the equity market to the volatility is revealed. There is Granger causality between equity level and implied skewness index, but in the opposite sense compared to the U.S, being stronger for $\triangle$ ESKEW$^-$. Therefore, for Eurozone the hypothesis 2 seems to be confirmed. We find a bidirectional Granger causality between $\triangle$ VIX and $\triangle$ VIX$^-$ at 10% critical level and the same

[14]Pesaran and Smith (1998) argued that cointegration and causality analysis may not be comparable, the first identifying a long run relation while the second is more a short term relationship.

Table 6: **Pairwise Johansen Cointegration Test Cross-Markets, Cross-Signs, Same Moment**

Volatility Market					Skewness Market				
Series	Obs	Lags	NI - NT	I - NT	Series	Obs	Lags	NI - NT	I - NT
VIX, VIX^-	1425	21	1 1	2 2	$SKEW$, $SKEW^-$	1423	23	1 1	2 2
VIX, VIX^+	1425	21	1 1	2 2	$SKEW$, $SKEW^+$	1420	26	1 1	2 2
VIX^-, VIX^+	1424	22	1 1	2 2	$SKEW^-$, $SKEW^+$	1423	23	1 1	2 2
$VSTOXX$, $VSTOXX^-$	1425	22	1 1	2 2	$ESKEW$, $ESKEW^-$	1428	18	1 1	2 2
$VSTOXX$, $VSTOXX^+$	1433	13	1 1	1 1	$ESKEW$, $ESKEW^+$	1422	24	1 1	2 2
$VSTOXX^-$, $VSTOXX^+$	1429	17	1 1	1 1	$ESKEW^-$, $ESKEW^+$	1426	20	1 1	2 2
VIX, $VSTOXX$	1424	22	1 1	2 2	$SKEW$, $ESKEW$	1426	20	1 1	2 2
VIX, $VSTOXX^-$	1425	21	1 1	2 2	$SKEW$, $ESKEW^-$	1426	20	1 1	2 2
VIX, $VSTOXX^+$	1424	22	1 1	1 1	$SKEW$, $ESKEW^+$	1425	21	0 0	2 2
VIX^-, $VSTOXX$	1425	21	1 1	2 2	$SKEW^-$, $ESKEW$	1426	20	1 1	2 2
VIX^-, $VSTOXX^-$	1425	21	1 1	2 2	$SKEW^-$, $ESKEW^-$	1423	23	1 1	2 2
VIX^-, $VSTOXX^+$	1424	22	1 1	2 2	$SKEW^-$, $ESKEW^+$	1423	23	0 0	1 1
VIX^+, $VSTOXX$	1424	22	1 1	1 1	$SKEW^+$, $ESKEW$	1426	20	0 0	1 1
VIX^+, $VSTOXX^-$	1425	21	1 1	1 1	$SKEW^+$, $ESKEW^-$	1428	18	0 0	1 1
VIX^+, $SKEW^+$	1424	22	1 1	1 1	$SKEW^+$, $ESKEW^+$	1426	20	1 1	2 2

Notes: This table shows the pairwise Johansen Cointegration test for both the volatility market and for the skewness market. The level series are: VIX, VIX$^-$, VIX$^+$, VSTOXX, VSTOXX$^-$, VSTOXX$^+$ for the volatility market and SKEW, SKEW$^-$, SKEW$^+$, ESKEW, ESKEW$^-$, ESKEW$^+$ for the skewness market. The test is performed at critical level 0.05, assuming No Intercept (NI) - No Trend (NT) and Intercept (I) - No Trend (NT) cases. Trace Test (first row) and Max-Eigenvalue Test (second row) are performed. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

between $\triangle$ VSTOXX and $\triangle$ VSTOXX$^-$ at 5% critical level (hypothesis 3). Conversely, uni-directional causality between total volatility indexes and their positive side is also obtained.

Regarding skewness, there is unidirectional Granger causality from $\triangle$ SKEW to $\triangle$ SKEW$^-$ and from $\triangle$ ESKEW to $\triangle$ ESKEW$^-$. This evidence rejects hypothesis 3, but it is coherent with the bi-directional causality we have found for volatility market. Moreover, there is some evidence of unidirectional Granger causality between the positive SKEW side to the total SKEW. This link seems to highlight the fact that the positive SKEW, the Calls investors side, is actually playing a role on the total SKEW index. This can be seen as the *optimistic contamination* we would like to disregard when measuring tail risk. Furthermore, there is no causality link from the U.S. skewness to the European one, although $\triangle$ SKEW$^-$ Granger causes the European $\triangle$ ESKEW$^-$ and $\triangle$ ESKEW$^-$ Granger causes the total U.S. $\triangle$ SKEW. This further shows the usefulness of our tail index compared to the total SKEW index.

In the next section we would like to see the impact of an external shock such as the Brexit vote in the U.K. on the relationships between U.S. and Eurozone implied volatility and skewness.

6 Brexit Analysis

Here we would like to gauge the potential changes in Granger causality across the U.S. and the Eurozone triggered by Brexit. We decided to focus on this event for several reasons. First, looking at Figure 3,

Table 7: **Pairwise Cross-Moments VAR Granger Causality Test**

Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
U.S. Market						
1434	$\triangle S\&P500$ **does not GC** $\triangle VIX$	**4.369**	**7.E-07**	$\triangle VIX$ does not GC $\triangle S\&P500$	1.277	0.225
1434	$\triangle S\&P500$ **does not GC** $\triangle VIX^-$	**5.551**	**2.E-09**	$\triangle VIX^-$ does not GC $\triangle S\&P500$	1.089	0.364
1436	$\triangle S\&P500$ **does not GC** $\triangle VIX^+$	**8.839**	**3.E-14**	$\triangle VIX^+$ does not GC $\triangle S\&P500$	1.218	0.274
1439	$\triangle S\&P500$ does not GC $\triangle SKEW$	1.662	0.114	$\triangle SKEW$ does not GC $\triangle S\&P500$	0.783	0.600
1432	$\triangle S\&P500$ does not GC $\triangle SKEW^-$	1.504	0.095	$\triangle SKEW^-$ **does not GC** $\triangle S\&P500$	**1.762**	**0.034**
1423	$\triangle S\&P500$ **does not GC** $\triangle SKEW^+$	**2.267**	**0.000**	$\triangle SKEW^+$ does not GC $\triangle S\&P500$	1.343	0.128
1424	$\triangle VIX$ does not GC $\triangle SKEW$	1.328	0.141	$\triangle SKEW$ does not GC $\triangle VIX$	0.667	0.880
1438	$\triangle VIX$ does not GC $\triangle SKEW^-$	1.169	0.275	$\triangle SKEW^-$ does not GC $\triangle VIX$	0.815	0.689
1430	$\triangle VIX$ **does not GC** $\triangle SKEW^+$	**2.224**	**0.001**	$\triangle SKEW^+$ does not GC $\triangle VIX$	0.968	0.496
1439	$\triangle VIX^-$ does not GC $\triangle SKEW$	0.635	0.748	$\triangle SKEW$ does not GC $\triangle VIX^-$	0.534	0.831
1427	$\triangle VIX^-$ does not GC $\triangle SKEW^-$	1.130	0.312	$\triangle SKEW^-$ does not GC $\triangle VIX^-$	0.781	0.731
1430	$\triangle VIX^-$ **does not GC** $\triangle SKEW^+$	**2.237**	**0.001**	$\triangle SKEW^+$ does not GC $\triangle VIX^-$	1.0697	0.376
1432	$\triangle VIX^+$ **does not GC** $\triangle SKEW$	**2.114**	**0.009**	$\triangle SKEW$ does not GC $\triangle VIX^+$	1.519	0.096
1432	$\triangle VIX^+$ does not GC $\triangle SKEW^-$	1.662	0.057	$\triangle SKEW^-$ does not GC $\triangle VIX^+$	1.229	0.246
1430	$\triangle VIX^+$ does not GC $\triangle SKEW^+$	0.609	0.901	$\triangle SKEW^+$ does not GC $\triangle VIX^+$	1.134	0.308
Eurozone Market						
Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
1437	$\triangle STOXX50$ **does not GC** $\triangle VSTOXX$	**5.050**	**1.E-06**	$\triangle VSTOXX$ does not GC $\triangle STOXX50$	0.637	0.765
1437	$\triangle STOXX50$ **does not GC** $\triangle VSTOXX^-$	**8.572**	**1.E-12**	$\triangle VSTOXX^-$ does not GC $\triangle STOXX50$	0.803	0.612
1433	$\triangle STOXX50$ **does not GC** $\triangle VSTOXX^+$	**5.410**	**1.E-09**	$\triangle VSTOXX^+$ does not GC $\triangle STOXX50$	1.075	0.376
1433	$\triangle STOXX50$ **does not GC** $\triangle ESKEW$	**1.911**	**0.012**	$\triangle ESKEW$ does not GC $\triangle STOXX50$	1.229	0.251
1429	$\triangle STOXX50$ **does not GC** $\triangle ESKEW^-$	**2.615**	**0.001**	$\triangle ESKEW^-$ does not GC $\triangle STOXX50$	1.036	0.413
1432	$\triangle STOXX50$ does not GC $\triangle ESKEW^+$	0.498	0.935	$\triangle ESKEW^+$ does not GC $\triangle STOXX50$	1.462	0.117
1434	$\triangle VSTOXX$ does not GC $\triangle ESKEW$	1.539	0.103	$\triangle ESKEW$ does not GC $\triangle VSTOXX$	1.304	0.209
1434	$\triangle VSTOXX$ does not GC $\triangle ESKEW^-$	1.457	0.126	$\triangle ESKEW^-$ does not GC $\triangle VSTOXX$	1.313	0.197
1434	$\triangle VSTOXX$ does not GC $\triangle ESKEW^+$	0.907	0.538	$\triangle ESKEW^+$ does not GC $\triangle VSTOXX$	1.347	0.185
1432	$\triangle VSTOXX^-$ does not GC $\triangle ESKEW$	1.185	0.279	$\triangle ESKEW$ does not GC $\triangle VSTOXX^-$	1.513	0.098
1433	$\triangle VSTOXX^-$ does not GC $\triangle ESKEW^-$	1.400	0.144	$\triangle ESKEW^-$ does not GC $\triangle VSTOXX^-$	1.544	0.088
1432	$\triangle VSTOXX^-$ does not GC $\triangle ESKEW^+$	1.416	0.137	$\triangle ESKEW^+$ does not GC $\triangle VSTOXX^-$	1.385	0.152
1432	$\triangle VSTOXX^+$ does not GC $\triangle ESKEW$	0.781	0.690	$\triangle ESKEW$ **does not GC** $\triangle VSTOXX^+$	**1.952**	**0.018**
1434	$\triangle VSTOXX^+$ does not GC $\triangle ESKEW^-$	0.966	0.478	$\triangle ESKEW^-$ does not GC $\triangle VSTOXX^+$	1.746	0.052
1432	$\triangle VSTOXX^+$ does not GC $\triangle ESKEW^+$	0.802	0.683	$\triangle ESKEW^+$ does not GC $\triangle VSTOXX^+$	1.256	0.224

Notes: This table shows the pairwise cross-moments VAR Granger Causality test performed for the U.S. and the Eurozone. First difference of the series is considered: S&P 500 , VIX, VIX^-, VIX^+, SKEW, $SKEW^-$, $SKEW^+$ for the U.S. and EUROSTOXX50 (STOXX50 in the Table), VSTOXX, $VSTOXX^-$, $VSTOXX^+$, ESKEW, $ESKEW^-$, $ESKEW^+$ for the Eurozone. The null hypothesis is: X does not Granger cause (GC) Y. In bold the Granger Causality relations are found significant at 5% level. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

around the Brexit vote the VSTOXX/VIX spread was one of the highest in the history of these two indexes. Different structural break tests for this spread have been conducted for better selecting the months around Brexit event that deserve to be analysed. The multiple breakpoints test (Bai and Perron, 2003) shows there were between two and three break points within the periods before and after Brexit together, all significant at 5%. The Bai-Perron test for global breaks selected three significant breaks. The global information criteria, Schwarz criterion, determined the same number of three breaks. The estimated breaks dates are the 17-05-2016, 01-06-2016 and 24-06-2016. Secondly, the impact of this political event was greater on the Eurozone market, because if the U.K., and only later it showed its consequences also in the U.S. economy and market[15] (see Figures 2 and 3). Due to the structural break test and to the U.S. volatility spike lag compared to Eurozone after Brexit, the period we have decided to select in our study on the Brexit event is three months of daily data, around the 23th June 2016, from 02-05-2016 to the 29-07-2016.

Starting from the U.S., for the three main series S&P 500 , VIX and SKEW no cointegration is detected

[15] Mixon (2009) argues that implied volatility is responsive to realized volatility shocks.

Table 8: **Pairwise Cross-Markets, Cross-Signs, Same Moment VAR Granger Causality Test**

Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
Volatility Market						
Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
1426	$\triangle VIX$ **does not GC** $\triangle VIX^-$	**2.797**	**4.E-05**	$\triangle VIX^-$ **does not GC** $\triangle VIX$	**1.713**	**0.005**
1426	$\triangle VIX$ **does not GC** $\triangle VIX^+$	**6.688**	**7.E-18**	$\triangle VIX^+$ does not GC $\triangle VIX$	1.516	0.066
1426	$\triangle VIX^-$ **does not GC** $\triangle VIX^+$	**5.978**	**2.E-15**	$\triangle VIX^+$ **does not GC** $\triangle VIX^-$	**2.664**	**9.E-05**
1426	$\triangle VSTOXX$ **does not GC** $\triangle VSTOXX^-$	**6.057**	**1.E-15**	$\triangle VSTOXX^-$ **does not GC** $\triangle VSTOXX$	**1.782**	**0.017**
1434	$\triangle VSTOXX$ **does not GC** $\triangle VSTOXX^+$	**9.404**	**1.E-17**	$\triangle VSTOXX^+$ does not GC $\triangle VSTOXX$	1.622	0.079
1430	$\triangle VSTOXX^-$ **does not GC** $\triangle VSTOXX^+$	**7.556**	**3.E-17**	$\triangle VSTOXX^+$ **does not GC** $\triangle VSTOXX^-$	**2.701**	**0.000**
1425	$\triangle VIX$ **does not GC** $\triangle VSTOXX$	**7.866**	**8.E-23**	$\triangle VSTOXX$ does not GC $\triangle VIX$	1.025	0.427
1426	$\triangle VIX$ **does not GC** $\triangle VSTOXX^-$	**6.978**	**7.E-19**	$\triangle VSTOXX^-$ does not GC $\triangle VIX$	1.228	0.220
1425	$\triangle VIX$ **does not GC** $\triangle VSTOXX^+$	**5.124**	**5.E-13**	$\triangle VSTOXX^+$ does not GC $\triangle VIX$	1.496	0.068
1425	$\triangle VIX^-$ **does not GC** $\triangle VSTOXX$	**8.096**	**1.E-23**	$\triangle VSTOXX$ does not GC $\triangle VIX^-$	1.129	0.308
1425	$\triangle VIX^-$ **does not GC** $\triangle VSTOXX^-$	**7.080**	**5.E-20**	$\triangle VSTOXX^-$ does not GC $\triangle VIX^-$	1.287	0.172
1425	$\triangle VIX^-$ **does not GC** $\triangle VSTOXX^+$	**5.421**	**4.E-14**	$\triangle VSTOXX^+$ does not GC $\triangle VIX^-$	1.508	0.065
1426	$\triangle VIX^+$ **does not GC** $\triangle VSTOXX$	**4.630**	**7.E-11**	$\triangle VSTOXX$ **does not GC** $\triangle VIX^+$	**2.027**	**0.004**
1426	$\triangle VIX^+$ **does not GC** $\triangle VSTOXX^-$	**4.150**	**3.E-09**	$\triangle VSTOXX^-$ **does not GC** $\triangle VIX^+$	**2.190**	**0.001**
1425	$\triangle VIX^+$ **does not GC** $\triangle VSTOXX^+$	**3.426**	**3.E-07**	$\triangle VSTOXX^+$ does not GC $\triangle VIX^+$	1.411	0.100
Skewness Market						
Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
1424	$\triangle SKEW$ **does not GC** $\triangle SKEW^-$	**1.709**	**0.021**	$\triangle SKEW^-$ does not GC $\triangle SKEW$	1.265	0.185
1421	$\triangle SKEW$ does not GC $\triangle SKEW^+$	1.295	0.150	$\triangle SKEW^+$ does not GC $\triangle SKEW$	1.449	0.070
1424	$\triangle SKEW^-$ does not GC $\triangle SKEW^+$	1.212	0.226	$\triangle SKEW^+$ does not GC $\triangle SKEW^-$	0.850	0.663
1430	$\triangle ESKEW$ **does not GC** $\triangle ESKEW^-$	**4.839**	**1.E-09**	$\triangle ESKEW^-$ does not GC $\triangle ESKEW$	1.362	0.152
1424	$\triangle ESKEW$ does not GC $\triangle ESKEW^+$	1.048	0.399	$\triangle ESKEW^+$ does not GC $\triangle ESKEW$	1.475	0.072
1424	$\triangle ESKEW^-$ does not GC $\triangle ESKEW^+$	1.163	0.271	$\triangle ESKEW^+$ does not GC $\triangle ESKEW^-$	0.985	0.479
1427	$\triangle SKEW$ does not GC $\triangle ESKEW$	1.375	0.128	$\triangle ESKEW$ does not GC $\triangle SKEW$	1.315	0.163
1427	$\triangle SKEW$ does not GC $\triangle ESKEW^-$	1.326	0.156	$\triangle ESKEW^-$ **does not GC** $\triangle SKEW$	**1.916**	**0.010**
1427	$\triangle SKEW$ does not GC $\triangle ESKEW^+$	1.431	0.102	$\triangle ESKEW^+$ does not GC $\triangle SKEW$	1.011	0.444
1427	$\triangle SKEW^-$ does not GC $\triangle ESKEW$	1.418	0.108	$\triangle ESKEW$ does not GC $\triangle SKEW^-$	1.381	0.125
1427	$\triangle SKEW^-$ **does not GC** $\triangle ESKEW^-$	**1.884**	**0.012**	$\triangle ESKEW^-$ does not GC $\triangle SKEW^-$	1.178	0.267
1424	$\triangle SKEW^-$ does not GC $\triangle ESKEW^+$	1.245	0.199	$\triangle ESKEW^+$ does not GC $\triangle SKEW^-$	1.309	0.153
1427	$\triangle SKEW^+$ does not GC $\triangle ESKEW$	1.369	0.132	$\triangle ESKEW$ does not GC $\triangle SKEW^+$	1.317	0.161
1429	$\triangle SKEW^+$ does not GC $\triangle ESKEW^-$	0.880	0.597	$\triangle ESKEW^-$ does not GC $\triangle SKEW^+$	0.931	0.535
1425	$\triangle SKEW^+$ does not GC $\triangle ESKEW^+$	0.761	0.769	$\triangle ESKEW^+$ does not GC $\triangle SKEW^+$	0.868	0.636

Notes: This table shows the pairwise Cross-Markets, Cross-Signs, Same Moment VAR Granger Causality test performed both for the volatility market and also for the skewness market. First difference of the following series is considered: VIX, VIX$^-$, VIX$^+$, VSTOXX, VSTOXX$^-$, VSTOXX$^+$ for volatility market cross signs and cross U.S. - Eurozone and SKEW, SKEW$^-$, SKEW$^+$, ESKEW, ESKEW$^-$, ESKEW$^+$ for skewness market cross signs and cross U.S. - Eurozone. The null hypothesis is: X does not Granger cause (GC) Y. In bold the Granger Causality relations are found significant at 5% level. The selected period is from 03-01-2011 to 30-09-2016, at daily frequency.

and this is also true for all pairwise relations among VIX$^-$ - S&P 500 , VIX$^-$ - SKEW and S&P 500 - SKEW. There is cointegration between the equity market and SKEW$^-$ instead. For the European market there is at least one cointegrated vector among all three EUROSTOXX 50, VSTOXX and ESKEW and no evidence of cointegration in the pairwise between the implied skewness index and the equity index (except for ESKEW$^-$), and between the implied skewness and volatility indexes (see Table 9).

Table 10 shows that, for short-periods, the Granger causality test provides more robust results. For the U.S., there is bidirectional Granger causality between $\triangle$ SKEW and $\triangle$ VIX and between $\triangle$ SKEW$^-$ and $\triangle$ VIX. However, in terms of cointegration VIX and SKEW are not related, possibly because the period is short and volatile. For the Eurozone many findings are quite different compared to the U.S. market (see Table 11). There is evidence of Granger causality from the equity levels to the corresponding implied volatility. In addition, we find bilateral Granger causality between $\triangle$ EUROSTOXX 50 and $\triangle$ ESKEW$^+$. This can be interpreted as an intense EUROSTOXX 50 Calls trading activity during these three months before and after

Table 9: **Pairwise Johansen Cointegration Test U.S. & Eurozone Market: Brexit Analysis**

U.S. Market					Eurozone Market				
Series	Obs	Lags	NI - NT	I - NT	Series	Obs	Lags	NI - NT	I - NT
$S\&P500$, VIX	55	7	0 0	0 0	$EUROSTOXX50$, $VSTOXX$	56	6	0 0	1 1
$S\&P500$, VIX^-	55	7	0 0	0 0	$EUROSTOXX50$, $VSTOXX^-$	58	4	0 0	0 0
$S\&P500$, VIX^+	58	4	0 0	0 0	$EUROSTOXX50$, $VSTOXX^+$	58	4	0 0	0 0
$S\&P500$, $SKEW$	52	10	0 0	0 0	$EUROSTOXX50$, $ESKEW$	58	4	0 0	0 0
$S\&P500$, $SKEW^-$	48	14	1 1	1 1	$EUROSTOXX50$, $ESKEW^-$	58	4	0 0	1 1
$S\&P500$, $SKEW^+$	57	5	0 0	0 0	$EUROSTOXX50$, $ESKEW^+$	55	7	0 0	0 0
VIX, $SKEW$	58	4	0 0	0 0	$VSTOXX$, $ESKEW$	52	10	0 0	1 1
VIX, $SKEW^-$	56	6	0 0	0 0	$VSTOXX$, $ESKEW^-$	58	4	0 0	0 0
VIX, $SKEW^+$	59	3	0 0	0 0	$VSTOXX$, $ESKEW^+$	58	4	0 0	0 0
VIX^-, $SKEW$	52	10	1 1	1 0	$VSTOXX^-$, $ESKEW$	58	4	0 0	0 0
VIX^-, $SKEW^-$	56	6	0 0	0 0	$VSTOXX^-$, $ESKEW^-$	58	4	0 0	0 0
VIX^-, $SKEW^+$	60	2	0 0	0 0	$VSTOXX^-$, $ESKEW^+$	53	9	0 1	1 1
VIX^+, $SKEW$	58	4	0 0	0 0	$VSTOXX^+$, $ESKEW$	58	4	0 0	0 0
VIX^+, $SKEW^-$	55	7	0 0	0 0	$VSTOXX^+$, $ESKEW^-$	59	3	0 0	0 0
VIX^+, $SKEW^+$	56	6	0 0	0 0	$VSTOXX^+$, $ESKEW^+$	56	6	0 0	2 0

Notes: This table shows the pairwise Johansen Cointegration test for both the U.S. and the Eurozone series during the Brexit period from 02-05-2016 to 29-07-2016. The test is performed at critical level 0.05, assuming No Intercept (NI) - No Trend (NT) and Intercept (I) - No Trend (NT) cases. Trace Test (first row) and Max-Eigenvalue Test (second row) are performed.

Brexit with investors trying to exploit news and political announcements. Most of the time, options trading is higher in the U.S. than Europe especially due to hedging pressures, but during Brexit a high pressure on Calls trading is found. For the U.S. we confirm hypothesis 2 that △ SKEW, and △ SKEW$^-$ cause △ S&P 500 . Hence, there is flow of information from the implied skew to the equity levels.

In terms of the volatility level, there is bidirectional Granger causality between △ VIX and △ VIX$^-$ and between △ VIX and △ VIX$^+$, only unidirectional from n △ VSTOXX to the △ VSTOXX$^-$, which is unusual, hence rejecting the third hypothesis. Regarding the skew measure, there is Granger causality from the total index to the negative side in the U.S., while in the Eurozone there is bidirectional causality between △ ESKEW and △ ESKEW$^-$, unidirectional from △ ESKEW$^+$ to △ ESKEW and from △ ESKEW$^+$ and △ ESKEW$^-$. In this short volatile period the investment sentiment component seems to impact more on the Eurozone equity index Puts. For the U.S. there is still no Granger causality between △ SKEW$^-$ and △ SKEW$^+$. Hence, the first hypothesis is partially confirmed.

Because △ ESKEW$^-$ Granger causes △ SKEW$^-$ we can conclude that S&P 500 Puts investors paid attention more to the hedging strategies related to Eurozone in that period. This conclusion further confirms that the two markets are mostly linked through the equity index Puts market channel. Thus, our prudent tail risk index reflects better the pessimistic investors view and the tail risk perception. The Eurozone and the U.S. markets seem to react differently to the Brexit voting outcome, especially in terms of options activities. As revealed by Figure 2, only *after* the Brexit the U.S. equity market has reacted as well and its volatility has increased subsequently.

Table 10: **Pairwise Cross-Moments VAR Granger Causality Test: Brexit Analysis**

Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
U.S. Market						
Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
55	$\triangle S\&P500$ does not GC $\triangle VIX$	1.441	0.201	$\triangle VIX$ does not GC $\triangle S\&P500$	1.482	0.215
57	$\triangle S\&P500$ does not GC $\triangle VIX^{-}$	1.639	0.168	$\triangle VIX^{-}$ does not GC $\triangle S\&P500$	1.718	0.149
58	$\triangle S\&P500$ does not GC $\triangle VIX^{+}$	0.339	0.849	$\triangle VIX^{+}$ does not GC $\triangle S\&P500$	0.775	0.546
52	$\triangle S\&P500$ does not GC $\triangle SKEW$	1.436	0.211	$\triangle SKEW$ **does not GC** $\triangle S\&P500$	**3.071**	**0.008**
58	$\triangle S\&P500$ does not GC $\triangle SKEW^{-}$	1.476	0.223	$\triangle SKEW^{-}$ **does not GC** $\triangle S\&P500$	**3.910**	**0.007**
56	$\triangle S\&P500$ does not GC $\triangle SKEW^{+}$	0.234	0.963	$\triangle SKEW^{+}$ does not GC $\triangle S\&P500$	0.580	0.743
56	$\triangle VIX$ **does not GC** $\triangle SKEW$	**4.948**	**0.000**	$\triangle SKEW$ **does not GC** $\triangle VIX$	**1.969**	**0.091**
56	$\triangle VIX$ **does not GC** $\triangle SKEW^{-}$	**4.146**	**0.002**	$\triangle SKEW^{-}$ **does not GC** $\triangle VIX$	**2.204**	**0.060**
57	$\triangle VIX$ does not GC $\triangle SKEW^{+}$	0.649	0.663	$\triangle SKEW^{+}$ does not GC $\triangle VIX$	0.942	0.463
56	$\triangle VIX^{-}$ **does not GC** $\triangle SKEW$	**5.176**	**0.000**	$\triangle SKEW$ **does not GC** $\triangle VIX^{-}$	**2.547**	**0.033**
56	$\triangle VIX^{-}$ **does not GC** $\triangle SKEW^{-}$	**3.846**	**0.003**	$\triangle SKEW^{-}$ **does not GC** $\triangle VIX^{-}$	**2.485**	**0.037**
57	$\triangle VIX^{-}$ does not GC $\triangle SKEW^{+}$	0.716	0.614	$\triangle SKEW^{+}$ does not GC $\triangle VIX^{-}$	0.909	0.483
56	$\triangle VIX^{+}$ does not GC $\triangle SKEW$	0.620	0.712	$\triangle SKEW$ **does not GC** $\triangle VIX^{+}$	**2.285**	**0.052**
56	$\triangle VIX^{+}$ does not GC $\triangle SKEW^{-}$	1.027	0.420	$\triangle SKEW^{-}$ does not GC $\triangle VIX^{+}$	1.704	0.143
57	$\triangle VIX^{+}$ does not GC $\triangle SKEW^{+}$	0.390	0.852	$\triangle SKEW^{+}$ does not GC $\triangle VIX^{+}$	0.546	0.740
Eurozone Market						
Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
59	$\triangle STOXX50$ **does not GC** $\triangle VSTOXX$	**3.514**	**0.021**	$\triangle VSTOXX$ does not GC $\triangle STOXX50$	2.100	0.111
59	$\triangle STOXX50$ **does not GC** $\triangle VSTOXX^{-}$	**8.801**	**8.E-05**	$\triangle VSTOXX^{-}$ does not GC $\triangle STOXX50$	1.219	0.311
58	$\triangle STOXX50$ does not GC $\triangle VSTOXX^{+}$	2.479	0.056	$\triangle VSTOXX^{+}$ does not GC $\triangle STOXX50$	0.437	0.781
57	$\triangle STOXX50$ does not GC $\triangle ESKEW$	1.103	0.371	$\triangle ESKEW$ does not GC $\triangle STOXX50$	1.492	0.208
60	$\triangle STOXX50$ does not GC $\triangle ESKEW^{-}$	1.992	0.146	$\triangle ESKEW^{-}$ does not GC $\triangle STOXX50$	1.089	0.343
53	$\triangle STOXX50$ **does not GC** $\triangle ESKEW^{+}$	**3.109**	**0.007**	$\triangle ESKEW^{+}$ **does not GC** $\triangle STOXX50$	**2.955**	**0.010**
59	$\triangle VSTOXX$ does not GC $\triangle ESKEW$	2.352	0.082	$\triangle ESKEW$ does not GC $\triangle VSTOXX$	0.689	0.562
59	$\triangle VSTOXX$ does not GC $\triangle ESKEW^{-}$	1.762	0.165	$\triangle ESKEW^{-}$ does not GC $\triangle VSTOXX$	0.470	0.703
59	$\triangle VSTOXX$ **does not GC** $\triangle ESKEW^{+}$	**3.876**	**0.014**	$\triangle ESKEW^{+}$ does not GC $\triangle VSTOXX$	1.752	0.167
58	$\triangle VSTOXX^{-}$ does not GC $\triangle ESKEW$	1.569	0.197	$\triangle ESKEW$ does not GC $\triangle VSTOXX^{-}$	0.590	0.671
59	$\triangle VSTOXX^{-}$ does not GC $\triangle ESKEW^{-}$	1.690	0.180	$\triangle ESKEW^{-}$ does not GC $\triangle VSTOXX^{-}$	0.496	0.686
58	$\triangle VSTOXX^{-}$ does not GC $\triangle ESKEW^{+}$	2.372	0.065	$\triangle ESKEW^{+}$ does not GC $\triangle VSTOXX^{-}$	0.536	0.709
58	$\triangle VSTOXX^{+}$ does not GC $\triangle ESKEW$	0.280	0.889	$\triangle ESKEW$ does not GC $\triangle VSTOXX^{+}$	1.181	0.330
60	$\triangle VSTOXX^{+}$ does not GC $\triangle ESKEW^{-}$	0.410	0.665	$\triangle ESKEW^{-}$ does not GC $\triangle VSTOXX^{+}$	1.917	0.156
56	$\triangle VSTOXX^{+}$ does not GC $\triangle ESKEW^{+}$	0.899	0.504	$\triangle ESKEW^{+}$ does not GC $\triangle VSTOXX^{+}$	1.283	0.285

Notes: This table shows the pairwise Cross-Moments (Equity, Volatility, Skewness) VAR Granger Causality Test performed both for the U.S. and also for the Eurozone around Brexit. First difference of the following series is considered: S&P 500 , VIX, VIX^{-}, VIX^{+}, SKEW, $SKEW^{-}$, $SKEW^{+}$ for the U.S. and EUROSTOXX50 (STOXX50 in the Table), VSTOXX, $VSTOXX^{-}$, $VSTOXX^{+}$, ESKEW, $ESKEW^{-}$, $ESKEW^{+}$ for the Eurozone. The null hypothesis is: X does not Granger cause (GC) Y. In bold the Granger Causality relations are found significant at 5% level. The selected period is from 02-05-2016 to 29-07-2016, at daily frequency.

7 Robustness Test

In this section we describe some further tests that have been performed as robustness checks to confirm our previous findings. The first robustness test is a Pairwise Granger Causality through a Vector Error Correction Model, VECM. Only the time series that showed evidence of cointegration in the Johansen cointegration analysis have been considered here. Indeed, this methodology should provide theoretically a better interpretation both for long-term and also for short-term relations among the cointegrated vectors[16] in our study. We restrict certain VAR's coefficients (VECM) in order to make it more efficient in terms of estimation compared to the simple VAR. The Granger causality is retested[17] and it shows that the causality relation between series identified as being causally related with the VAR is now stronger.

The cross-moments analysis for the U.S. market indicates evidence of unidirectional Granger Causality. In some cases it becomes even a bidirectional relation, such as $\triangle$ S&P 500 - $\triangle$ VIX and $\triangle$ VIX^{+} - $\triangle$ SKEW.

[16]It is simply a VAR (Vector Auto-regression) model representation when forms of cointegration are found, see Granger Representation Theorem, Engle and Granger (1987).

[17]The cointegration term present in the selected series will be our model error correction.

Table 11: **Pairwise Cross-Markets, Cross-Signs, Same Moment VAR Granger Causality Test: Brexit Analysis**

Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
			Volatility Market			
52	$\triangle VIX$ **does not GC** $\triangle VIX^-$	**3.827**	**0.001**	$\triangle VIX^-$ **does not GC** $\triangle VIX$	**3.638**	**0.002**
57	$\triangle VIX$ **does not GC** $\triangle VIX^+$	**2.555**	**0.040**	$\triangle VIX^+$ **does not GC** $\triangle VIX$	**2.829**	**0.026**
57	$\triangle VIX^-$ does not GC $\triangle VIX^+$	1.909	0.111	$\triangle VIX^+$ does not GC $\triangle VIX^-$	0.887	0.477
58	$\triangle VSTOXX$ **does not GC** $\triangle VSTOXX^-$	**3.801**	**0.009**	$\triangle VSTOXX^-$ does not GC $\triangle VSTOXX$	0.784	0.541
53	$\triangle VSTOXX$ **does not GC** $\triangle VSTOXX^+$	**2.616**	**0.020**	$\triangle VSTOXX^+$ does not GC $\triangle VSTOXX$	1.346	0.250
58	$\triangle VSTOXX^-$ does not GC $\triangle VSTOXX^+$	2.533	0.051	$\triangle VSTOXX^+$ does not GC $\triangle VSTOXX^-$	1.140	0.348
57	$\triangle VIX$ does not GC $\triangle VSTOXX$	2.359	0.054	$\triangle VSTOXX$ does not GC $\triangle VIX$	0.526	0.754
57	$\triangle VIX$ does not GC $\triangle VSTOXX^-$	1.098	0.374	$\triangle VSTOXX^-$ does not GC $\triangle VIX$	0.963	0.450
56	$\triangle VIX$ does not GC $\triangle VSTOXX^+$	1.050	0.406	$\triangle VSTOXX^+$ does not GC $\triangle VIX$	0.753	0.610
57	$\triangle VIX^-$ does not GC $\triangle VSTOXX$	2.329	0.057	$\triangle VSTOXX$ does not GC $\triangle VIX^-$	0.553	0.735
57	$\triangle VIX^-$ does not GC $\triangle VSTOXX^-$	0.974	0.444	$\triangle VSTOXX^-$ does not GC $\triangle VIX^-$	0.938	0.465
59	$\triangle VIX^-$ does not GC $\triangle VSTOXX^+$	0.564	0.641	$\triangle VSTOXX^+$ does not GC $\triangle VIX^-$	0.025	0.994
57	$\triangle VIX^+$ does not GC $\triangle VSTOXX$	0.179	0.969	$\triangle VSTOXX$ does not GC $\triangle VIX^+$	0.659	0.655
58	$\triangle VIX^+$ does not GC $\triangle VSTOXX^-$	0.080	0.988	$\triangle VSTOXX^-$ does not GC $\triangle VIX^+$	0.696	0.597
57	$\triangle VIX^+$ does not GC $\triangle VSTOXX^+$	0.500	0.773	$\triangle VSTOXX^+$ does not GC $\triangle VIX^+$	1.353	0.259
			Skewness Market			
Obs	Null Hypothesis	F-Stat	P-Value	Null Hypothesis	F-Stat	P-Value
58	$\triangle SKEW$ **does not GC** $\triangle SKEW^-$	**2.628**	**0.045**	$\triangle SKEW^-$ does not GC $\triangle SKEW$	1.325	0.273
57	$\triangle SKEW$ does not GC $\triangle SKEW^+$	0.114	0.988	$\triangle SKEW^+$ does not GC $\triangle SKEW$	0.308	0.905
57	$\triangle SKEW^-$ does not GC $\triangle SKEW^+$	0.110	0.989	$\triangle SKEW^+$ does not GC $\triangle SKEW^-$	0.923	0.474
57	$\triangle ESKEW$ **does not GC** $\triangle ESKEW^-$	**4.608**	**0.001**	$\triangle ESKEW^-$ **does not GC** $\triangle ESKEW$	**2.498**	**0.044**
57	$\triangle ESKEW$ does not GC $\triangle ESKEW^+$	0.727	0.606	$\triangle ESKEW^+$ **does not GC** $\triangle ESKEW$	**3.716**	**0.006**
57	$\triangle ESKEW^-$ does not GC $\triangle ESKEW^+$	0.756	0.585	$\triangle ESKEW^+$ **does not GC** $\triangle ESKEW^-$	**3.331**	**0.011**
57	$\triangle SKEW$ does not GC $\triangle ESKEW$	0.602	0.698	$\triangle ESKEW$ does not GC $\triangle SKEW$	1.205	0.321
58	$\triangle SKEW$ does not GC $\triangle ESKEW^-$	0.287	0.884	$\triangle ESKEW^-$ does not GC $\triangle SKEW$	0.037	0.997
60	$\triangle SKEW$ **does not GC** $\triangle ESKEW^+$	**4.872**	**0.011**	$\triangle ESKEW^+$ does not GC $\triangle SKEW$	0.635	0.533
60	$\triangle SKEW^-$ does not GC $\triangle ESKEW$	0.916	0.406	$\triangle ESKEW$ does not GC $\triangle SKEW^-$	0.822	0.444
55	$\triangle SKEW^-$ does not GC $\triangle ESKEW^-$	0.996	0.448	$\triangle ESKEW^-$ **does not GC** $\triangle SKEW^-$	**2.523**	**0.030**
55	$\triangle SKEW^-$ does not GC $\triangle ESKEW^+$	2.114	0.064	$\triangle ESKEW^+$ does not GC $\triangle SKEW^-$	1.699	0.136
51	$\triangle SKEW^+$ does not GC $\triangle ESKEW$	1.895	0.084	$\triangle ESKEW$ does not GC $\triangle SKEW^+$	1.158	0.357
57	$\triangle SKEW^+$ does not GC $\triangle ESKEW^-$	2.046	0.089	$\triangle ESKEW^-$ does not GC $\triangle SKEW^+$	0.746	0.593
57	$\triangle SKEW^+$ does not GC $\triangle ESKEW^+$	0.934	0.467	$\triangle ESKEW^+$ does not GC $\triangle SKEW^+$	1.653	0.165

Notes: This table shows the pairwise cross-markets, cross-signs, same moment VAR Granger causality test performed both for the volatility and also for the skewness markets during the Brexit period. First difference of the following series is considered: VIX, VIX^-, VIX^+, VSTOXX, $VSTOXX^-$, $VSTOXX^+$ for volatility and SKEW, $SKEW^-$, $SKEW^+$, ESKEW, $ESKEW^-$, $ESKEW^+$ for skewness. The null hypothesis is: X does not Granger cause (GC) Y. In bold the Granger Causality relations are found significant at 5% level. The selected period is from 02-05-2016 to 29-07-2016, at daily frequency.

In the Eurozone, with the VEC model we detect Granger causality from $\triangle$ ESKEW to $\triangle$ VSTOXX and from $\triangle$ VSTOXX to $\triangle$ $ESKEW^+$, opposite to the direction identified based on the VAR model. Moreover, we identify two bilateral relations between $\triangle$ $VSTOXX^+$ - $\triangle$ $ESKEW^-$ and $\triangle$ $VSTOXX^+$ - $\triangle$ $ESKEW^+$. For the Brexit analysis, the few series that are cointegrated show a stronger evidence of causality, especially where it has already been detected with the VAR Model Granger Test.

Another simple test is the regression applied to the two SKEW sides: $SKEW^-$ and $SKEW^+$ and our empirical evidence indicate a negative relationship between them, as expected. When $\triangle$ $SKEW^+$ decreases, $\triangle$ $SKEW^-$ increases, becoming more left skewed. This relationship was also hinted by the simple correlation analysis in Table 2 and by the corresponding Granger causality test. In the literature there are also contrasting views about the relation and interaction between volatility and skewness. Our regression analysis confirms the conclusions in Han (2008), Dennis and Mayhew (2002) and Conrad et al. (2013), highlighting a negative relationship between $\triangle$ VIX and $\triangle$ SKEW, but also between $\triangle$ VIX and $\triangle$ $SKEW^-$. When

Table 12: **Pairwise Cross-Moments VEC Granger Causality Test**

Obs	Null Hypothesis	P-Value	Null Hypothesis	P-Value
U.S. Market				
Obs	Null Hypothesis	P-Value	Null Hypothesis	P-Value
1434	$\triangle S\&P500$ **does not GC** $\triangle VIX$	**0.000**	$\triangle VIX$ **does not GC** $\triangle S\&P500$	**0.011**
1434	$\triangle S\&P500$ **does not GC** $\triangle VIX^-$	**0.000**	$\triangle VIX^-$ does not GC $\triangle S\&P500$	0.134
1436	$\triangle S\&P500$ **does not GC** $\triangle VIX^+$	**0.000**	$\triangle VIX^+$ does not GC $\triangle S\&P500$	0.280
1423	$\triangle S\&P500$ **does not GC** $\triangle SKEW^+$	**0.000**	$\triangle SKEW^+$ does not GC $\triangle S\&P500$	0.368
1424	$\triangle VIX$ **does not GC** $\triangle SKEW$	**0.000**	$\triangle SKEW$ does not GC $\triangle VIX$	0.404
1438	$\triangle VIX$ **does not GC** $\triangle SKEW^-$	**0.000**	$\triangle SKEW^-$ does not GC $\triangle VIX$	0.479
1439	$\triangle VIX^-$ **does not GC** $\triangle SKEW$	**0.000**	$\triangle SKEW$ does not GC $\triangle VIX^-$	0.454
1427	$\triangle VIX^-$ **does not GC** $\triangle SKEW^-$	**0.000**	$\triangle SKEW^-$ does not GC $\triangle VIX^-$	0.606
1432	$\triangle VIX^+$ **does not GC** $\triangle SKEW$	**0.004**	$\triangle SKEW$ **does not GC** $\triangle VIX^+$	**0.000**
1432	$\triangle VIX^+$ **does not GC** $\triangle SKEW^-$	**0.018**	$\triangle SKEW^-$ does not GC $\triangle VIX^+$	0.212
Eurozone Market				
Obs	Null Hypothesis	P-Value	Null Hypothesis	P-Value
1437	$\triangle STOXX50$ **does not GC** $\triangle VSTOXX$	**0.000**	$\triangle VSTOXX$ does not GC $\triangle STOXX50$	0.583
1437	$\triangle STOXX50$ **does not GC** $\triangle VSTOXX^-$	**0.000**	$\triangle VSTOXX^-$ does not GC $\triangle STOXX50$	0.405
1433	$\triangle STOXX50$ **does not GC** $\triangle VSTOXX^+$	**0.000**	$\triangle VSTOXX^+$ does not GC $\triangle STOXX50$	0.462
1434	$\triangle VSTOXX$ does not GC $\triangle ESKEW$	0.136	$\triangle ESKEW$ **does not GC** $\triangle VSTOXX$	**0.011**
1434	$\triangle VSTOXX$ does not GC $\triangle ESKEW^-$	0.068	$\triangle ESKEW^-$ **does not GC** $\triangle VSTOXX$	**0.029**
1434	$\triangle VSTOXX$ **does not GC** $\triangle ESKEW^+$	**0.007**	$\triangle ESKEW^+$ does not GC $\triangle VSTOXX$	0.382
1432	$\triangle VSTOXX^-$ does not GC $\triangle ESKEW$	0.341	$\triangle ESKEW$ **does not GC** $\triangle VSTOXX^-$	**0.004**
1433	$\triangle VSTOXX^-$ **does not GC** $\triangle ESKEW^-$	**0.000**	$\triangle ESKEW^-$ **does not GC** $\triangle VSTOXX^-$	**0.003**
1432	$\triangle VSTOXX^-$ **does not GC** $\triangle ESKEW^+$	**0.000**	$\triangle ESKEW^+$ does not GC $\triangle VSTOXX^-$	0.311
1432	$\triangle VSTOXX^+$ **does not GC** $\triangle ESKEW^+$	**0.000**	$\triangle ESKEW^+$ **does not GC** $\triangle VSTOXX^+$	**0.042**

Notes: This table shows the pairwise cross-moments (Equity, Volatility, Skewness) Granger Causality test performed through VECM assuming the No-Intercept and No-Trend case. Only the first difference of series presenting cointegration (see Table 5) have been considered for causality test. Same lags of VAR model are selected. P-Value is considered for comparison purposes, F-Statistic N.A., under the null hypothesis: X does not Granger cause (GC) Y. In bold the Granger Causality relations are found significant at 5% level. The sample period is from 03-01-2011 to 30-09-2016, at daily frequency.

Table 13: **Pairwise Cross-Markets, Cross-Signs, Same Moment VEC Granger Causality Test**

Obs	Null Hypothesis	P-Value	Null Hypothesis	P-Value
Volatility Market				
Obs	Null Hypothesis	P-Value	Null Hypothesis	P-Value
1426	$\triangle VIX$ **does not GC** $\triangle VIX^-$	**0.000**	$\triangle VIX^-$ **does not GC** $\triangle VIX$	**0.003**
1426	$\triangle VIX$ **does not GC** $\triangle VIX^+$	**0.000**	$\triangle VIX^+$ **does not GC** $\triangle VIX$	**0.005**
1426	$\triangle VIX^-$ **does not GC** $\triangle VIX^+$	**0.000**	$\triangle VIX^+$ **does not GC** $\triangle VIX^-$	**0.000**
1426	$\triangle VSTOXX$ **does not GC** $\triangle VSTOXX^-$	**0.000**	$\triangle VSTOXX^-$ **does not GC** $\triangle VSTOXX$	**0.014**
1434	$\triangle VSTOXX$ **does not GC** $\triangle VSTOXX^+$	**0.000**	$\triangle VSTOXX^+$ **does not GC** $\triangle VSTOXX$	**0.010**
1430	$\triangle VSTOXX^-$ **does not GC** $\triangle VSTOXX^+$	**0.000**	$\triangle VSTOXX^+$ **does not GC** $\triangle VSTOXX^-$	**0.000**
1425	$\triangle VIX$ **does not GC** $\triangle VSTOXX$	**0.000**	$\triangle VSTOXX$ **does not GC** $\triangle VIX$	**0.000**
1426	$\triangle VIX$ **does not GC** $\triangle VSTOXX^-$	**0.000**	$\triangle VSTOXX^-$ **does not GC** $\triangle VIX$	**0.001**
1425	$\triangle VIX$ **does not GC** $\triangle VSTOXX^+$	**0.000**	$\triangle VSTOXX^+$ **does not GC** $\triangle VIX$	**0.000**
1425	$\triangle VIX^-$ **does not GC** $\triangle VSTOXX$	**0.000**	$\triangle VSTOXX$ **does not GC** $\triangle VIX^-$	**0.000**
1425	$\triangle VIX^-$ **does not GC** $\triangle VSTOXX^-$	**0.000**	$\triangle VSTOXX^-$ **does not GC** $\triangle VIX^-$	**0.000**
1425	$\triangle VIX^-$ **does not GC** $\triangle VSTOXX^+$	**0.000**	$\triangle VSTOXX^+$ **does not GC** $\triangle VIX^-$	**0.000**
1425	$\triangle VIX^+$ **does not GC** $\triangle VSTOXX^+$	**0.000**	$\triangle VSTOXX^+$ **does not GC** $\triangle VIX^+$	**0.000**
Skewness Market				
Obs	Null Hypothesis	P-Value	Null Hypothesis	P-Value
1424	$\triangle SKEW$ **does not GC** $\triangle SKEW^-$	**0.000**	$\triangle SKEW^-$ does not GC $\triangle SKEW$	0.212
1421	$\triangle SKEW$ **does not GC** $\triangle SKEW^+$	**0.014**	$\triangle SKEW^+$ does not GC $\triangle SKEW$	0.083
1424	$\triangle SKEW^-$ **does not GC** $\triangle SKEW^+$	**0.000**	$\triangle SKEW^+$ does not GC $\triangle SKEW^-$	0.364
1430	$\triangle ESKEW$ **does not GC** $\triangle ESKEW^-$	**0.000**	$\triangle ESKEW^-$ does not GC $\triangle ESKEW$	0.168
1424	$\triangle ESKEW$ **does not GC** $\triangle ESKEW^+$	**0.000**	$\triangle ESKEW^+$ **does not GC** $\triangle ESKEW$	**0.000**
1424	$\triangle ESKEW^-$ **does not GC** $\triangle ESKEW^+$	**0.000**	$\triangle ESKEW^+$ does not GC $\triangle ESKEW^-$	0.229
1427	$\triangle SKEW$ **does not GC** $\triangle ESKEW$	**0.000**	$\triangle ESKEW$ **does not GC** $\triangle SKEW$	**0.000**
1427	$\triangle SKEW$ **does not GC** $\triangle ESKEW^-$	**0.000**	$\triangle ESKEW^-$ **does not GC** $\triangle SKEW$	**0.000**
1427	$\triangle SKEW^-$ **does not GC** $\triangle ESKEW$	**0.001**	$\triangle ESKEW$ **does not GC** $\triangle SKEW^-$	**0.000**
1427	$\triangle SKEW^-$ **does not GC** $\triangle ESKEW^-$	**0.000**	$\triangle ESKEW^-$ **does not GC** $\triangle SKEW^-$	**0.002**
1425	$\triangle SKEW^+$ **does not GC** $\triangle ESKEW^+$	**0.000**	$\triangle ESKEW^+$ **does not GC** $\triangle SKEW^+$	**0.011**

Notes: This table shows the pairwise Cross-Markets (U.S.-Eurozone), Cross-Signs, within the same moment Granger Causality Test performed through VECM assuming the No-Intercept and No-Trend case. Only the first difference of series presenting cointegration (see Table 6) have been considered for causality test. Same lags of VAR model are selected. P-Value is considered for comparison purposes, F-Statistic N.A., under the null hypothesis: X does not Granger cause (GC) Y. In bold the Granger Causality relations are found significant at 5% level. The sample period is from 03-01-2011 to 30-09-2016, at daily frequency.

market volatility is high the negative skewness for S&P 500 is reduced. Furthermore, there is a positive relation between $\triangle$ VIX and $\triangle$ SKEW$^+$. When VIX spikes the positive SKEW side does the same. We have already explained some possible interpretations of these findings, especially, when the market sentiment element is taken into account.

8 Skew Indexes, Volatility Indexes and Asset Pricing

In this section we relate stock portfolios returns with the implied volatility and skewness indexes and their positive and negative components. We expand the Fama and French (2017) five-factors model by testing the explanatory power of the first and second moments indexes when added next to them[18]. The five factors include market excess return, as a difference between market portfolio return and risk-free rate of 1m T-bills, the average return on value portfolios minus average return on growth portfolios (HML), the average return on small portfolios minus average return on big portfolios (SMB), the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios (RMW) and the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios (CMA)[19]. More specifically we want to test whether or not the negative skewness indexes (SKEW$^-$ and ESKEW$^-$) are more informative than the aggregate indexes in explaining our selected portfolios returns. We also conduct the same test looking at the informative power of the volatility indexes, especially coming from Puts (VIX^- and $VSTOXX^-$). The portfolios we have selected are the daily bivariate U.S. and Europe 25 stocks Portfolios sorts on Size and Book-to-Market. Thus, the main hypothesis we want to test is the following - **Hypothesis 4:** *The indexes computed from the Put portfolios might perform better in asset pricing terms than the aggregate measures, especially in stock markets bearish times.*

This section is based on a series of studies advocating the role of stock returns higher moments in asset pricing models in the literature. A positive link between stock returns volatility and asset returns and expected market risk premium has been found (e.g. French et al., 1987). The strand of literature incorporating skewness in asset pricing models is also vast and it has a long history[20]. In particular, Harvey and Siddique (2000) has included a conditional skewness measure in asset pricing model to better understand the cross-sectional variation of asset returns. Among the most recent, Conrad et al. (2013) has focused on

[18] According to Fama and French (2017), this model is found to perform better than the traditional Fama and French (1993) three-factors model where market excess return, return of value portfolios minus average return on growth portfolios (HML) and return on small portfolios minus average return on big portfolios (SMB) are considered.

[19] For further details on the methodology for computing these five factors see Kenneth R. French Data Library at: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html from where the data have been collected.

[20] This literature has started from the seminal work by Kraus and Litzenberger (1976) and has been progressively expanded, see Friend and Westerfield (1980); Kraus and Litzenberger (1983); Harvey and Siddique (2000); Brunnermeier et al. (2007); Mitton and Vorkink (2007); Rehman and Vilkov (2012); Conrad et al. (2013)

both individual securities' risk neutral volatility and skewness and their linkages with future returns and Rehman and Vilkov (2012) have used a set of information that is similar to our paper since ex ante forward looking skewness extracted from options were considered[21].

The role of the skew and volatility indexes is tested by performing the Fama and MacBeth (1973) two steps regression analysis[22] with the aim of finding the premium that compensates investors from their exposure to these factors. Finally, we explore how fundamental factors affect average stock returns under different market conditions by dividing the whole sample into positive times and negative times based on S&P 500 and EUROSTOXX50 returns. The first step involves the system of equations where there are the n selected portfolios returns and the factors we want to test on the left and the right side, respectively, as:

$$
\begin{aligned}
R_{1,t} &= \alpha_1 + \beta_{1,F_1} F_{1,t} + \beta_{1,F_2} F_{2,t} + ... + \beta_{1,F_m} F_{m,t} + \epsilon_{1,t} \\
R_{2,t} &= \alpha_2 + \beta_{2,F_1} F_{1,t} + \beta_{2,F_2} F_{2,t} + ... + \beta_{2,F_m} F_{m,t} + \epsilon_{2,t} \\
&\vdots \\
R_{n,t} &= \alpha_n + \beta_{n,F_1} F_{1,t} + \beta_{n,F_2} F_{2,t} + ... + \beta_{n,F_m} F_{m,t} + \epsilon_{n,t}
\end{aligned}
\tag{19}
$$

where every equation represents a regression for a different $R_{i,t}$ return of portfolio or asset i (from 1 to n) at time t and $F_{j,t}$ is the factor j (from 1 to m) at time t, with β_{i,F_m} are the factor loadings expressing the exposure of each asset or portfolio of assets to that specific factor, and t goes from 1 to T.The second step of the methodology is to compute T cross-sectional regressions of the returns on the m estimates of the βs ($\beta*$) calculated from the first step.[23]

$$
\begin{aligned}
R_{i,1} &= \gamma_{1,0} + \gamma_{1,1} \hat{\beta}_{i,F_1} + \gamma_{1,2} \hat{\beta}_{i,F_2} + ... + \gamma_{1,m} \hat{\beta}_{i,F_m} + \epsilon_{i,t} \\
R_{i,2} &= \gamma_{2,0} + \gamma_{2,1} \hat{\beta}_{i,F_1} + \gamma_{2,2} \hat{\beta}_{i,F_2} + ... + \gamma_{2,m} \hat{\beta}_{i,F_m} + \epsilon_{i,2} \\
&\vdots \\
R_{i,T} &= \gamma_{T,0} + \gamma_{n,1} \hat{\beta}_{i,F_1} + \gamma_{n,2} \hat{\beta}_{i,F_2} + ... + \gamma_{n,m} \hat{\beta}_{i,F_m} + \epsilon_{i,T}
\end{aligned}
\tag{20}
$$

where the $R_{i,t}$ returns are the same as before and γ are the coefficients measuring the risk premium expressed by each factor, with i from 1 to n. Replacing the general equations with our factors we get the following

[21] Many studies have analysed this relation using historic expected skewness measures, e.g. Boyer et al. (2009) and Conrad et al. (2013).

[22] The Fama and MacBeth (1973) allows us to test which are the factors that can better explain stock market returns. Thus, we test whether our risk factors namely decomposed second and third moments can provide better results when added to the standard factors by increasing the significance, coefficients size and sign for both the U.S. and the Eurozone (I step). Then, in the II step, we test what is the risk premium required by these factors on the asset returns.

[23] Each regression uses the same βs from the first step with the aim of detecting the exposure of the n returns to the m factor loadings over time.

equations. Next is the simple Fama and French (1993) three-factor model:

$$R_{i,t,j} = \beta_0 + \beta_1(R_{M,t,j} - r_{f,t,j}) + \beta_2 HML_{t,j} + \beta_3 SMB_{t,j} + \epsilon_{i,t} \tag{21}$$

where $R_{i,t,j}$ is the daily portfolio return i for day t, for market and selected portfolio j where it is based on Size and Book-to-Market[24], r_f is the risk free rate and R_M the equal weighed market portfolio.

The following equation (22) shows the new Fama and French (2017) five-factor model, which includes also profitability (RMW) and investment (CMA) factors:

$$R_{i,t,j} = \beta_0 + \beta_1(R_{M,t,j} - r_{f,t,j}) + \beta_2 HML_{t,j} + \beta_3 SMB_{t,j} + \beta_4 RMW_{t,j} + \beta_5 CMA_{t,j} + \epsilon_{i,t} \tag{22}$$

Volatility and skewness have also been widely advocated as important variables in explaining the variations of stock returns. In addition to that, we also test the models taking into account the positive/negative volatility and skewness indexes:

$$R_{i,t,j} = \beta_0 + \beta_1(R_{M,t,j} - r_{f,t,j}) + \beta_2 HML_{t,j} + \beta_3 SMB_{t,j} + \beta_4 RMW_{t,j} + \beta_5 CMA_{t,j} + \beta_z SKEW^q_{t,j} + \epsilon_{i,t} \tag{23}$$

where $SKEW^q_{t,j}$ is the skewness index either for the U.S. (SKEW) or for the Eurozone (ESKEW) with q indexed for $Tot, -, +$ signs and z taking values from 6 to 8 according to the SKEW sign, and

$$R_{i,t,j} = \beta_0 + \beta_1(R_{M,t,j} - r_{f,t,j}) + \beta_2 HML_{t,j} + \beta_3 SMB_{t,j} + \beta_4 RMW_{t,j} + \beta_5 CMA_{t,j} + \beta_k IV^q_{t,j} + \epsilon_{i,t} \tag{24}$$

where $IV^q_{t,j}$ is the implied volatility index either for the U.S. (VIX) or for the Eurozone ($VSTOXX$) with q indexed for $Tot, -, +$ signs and k taking values from 9 to 11 according to the implied volatility factor sign. Finally the first and second moments indexes are tested together in the same regression according to their signs having a seven factors model as:

$$R_{i,t,j} = \beta_0 + \beta_1(R_{M,t,j} - r_{f,t,j}) + \beta_2 HML_{t,j} + \beta_3 SMB_{t,j} + \beta_4 RMW_{t,j} + \beta_5 CMA_{t,j} + \beta_z SKEW^q_{t,j} + \beta_k IV^q_{t,j} + \epsilon_{i,t} \tag{25}$$

The empirical results show that the six or seven factors models based on the five common factors plus the decomposed implied moments measures can better explain the variations of stock returns than the original Fama and French (1993) three-factors and than the more recent Fama and French (2017) five-factors models.

[24] All the selected portfolios for Size and Book-to-Market are taken as average value weighted returns. For more details see Kenneth R. Frenchs data library at $http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$.

Table 14: **Fama-MacBeth Regression Results for the U.S. and the Eurozone**

Factor	3-Factors	5-Factors	5-Factors & SKEW			5-Factors & VIX			Full 7-Factors		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
γ_0	0.048***	0.017	0.023	0.025	0.025	0.025	0.032	0.016	0.032	0.041*	0.025
	(0.010)	(0.016)	(0.016)	(0.015)	(0.017)	(0.020)	(0.023)	(0.016)	(0.022)	(0.023)	(0.017)
γ_{Mkt-rf}	0.002	0.033**	0.026*	0.024	0.025	0.024	0.017	0.033*	0.017	0.008	0.024
	(0.010)	(0.014)	(0.014)	(0.015)	(0.017)	(0.020)	(0.024)	(0.016)	(0.022)	(0.023)	(0.017)
γ_{HML}	-0.005	-0.007**	-0.007**	-0.007**	-0.007**	-0.007**	-0.007**	-0.008**	-0.007**	-0.007*	-0.008***
	(0.004)	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)
γ_{SMB}	-0.005*	-0.002	-0.001	-0.001	-0.002	-0.001	-0.002	-0.002	-0.001	-0.001	-0.003
	(0.002)	(0.001)	(0.001)	(0.002)	0.001	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)
γ_{RMW}		0.017	0.018	0.016	0.018*	0.016	0.014	0.015	0.017	0.013	0.017
		(0.012)	(0.012)	(0.010)	(0.010)	(0.011)	(0.011)	(0.013)	(0.012)	(0.010)	(0.010)
γ_{CMA}		-0.005	-0.003	-0.002	-0.001	-0.004	-0.006	-0.008	-0.003	-0.003	-0.002
		(0.012)	(0.011)	(0.011)	(0.006)	(0.012)	(0.013)	(0.012)	(0.011)	(0.012)	(0.007)
γ_{SKEW}			-0.006						-0.006		
			(0.006)						(0.006)		
γ_{SKEW-}				-0.007						-0.007	
				(0.004)						(0.005)	
γ_{SKEW+}					0.011						0.014
					(0.009)						(0.009)
γ_{VIX}						0.005			0.007		
						(0.016)			(0.017)		
γ_{VIX-}							0.011			0.011	
							(0.016)			(0.016)	
γ_{VIX+}								-0.029			-0.042***
								0.021			(0.011)
R^2	0.164	0.421	0.445	0.480	0.502	0.426	0.437	0.440	0.452	0.486	0.512
Adj R^2	0.085	0.229	0.261	0.293	0.298	0.235	0.249	0.253	0.237	0.284	0.318
P-value	0.275	0.048	0.068	0.043	0.031	0.086	0.076	0.073	0.113	0.076	0.030

Factor	3-Factors	5-Factors	5-Factors & ESKEW			5-Factors & VSTOXX			Full 7-Factors		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
γ_0	0.069**	0.083***	0.015	0.053	0.085**	-0.015	0.014	0.070*	-0.043	-0.032	0.072*
	(0.029)	(0.032)	(0.034)	(0.027)	(0.033)	(0.044)	(0.044)	(0.037)	(0.041)	(0.045)	(0.039)
γ_{Mkt-rf}	0.029	0.044	0.017	0.016	0.047*	0.044	0.017	0.032	0.070*	0.062	0.035
	(0.027)	(0.027)	(0.034)	(0.025)	(0.023)	(0.043)	(0.042)	(0.035)	(0.040)	(0.045)	(0.037)
γ_{HML}	-0.015	-0.014*	-0.017***	-0.013**	-0.014*	-0.012***	-0.011**	-0.015**	-0.015***	-0.010**	-0.015**
	(0.006)	(0.007)	(0.006)	(0.006)	(0.007)	(0.003)	(0.004)	(0.006)	(0.004)	(0.004)	(0.007)
γ_{SMB}	0.001	0.001	0.006	0.002	0.001	0.009*	0.008	0.002	0.012**	0.008	0.002
	(0.012)	(0.012)	(0.007)	(0.011)	(0.012)	(0.004)	(0.007)	(0.010)	(0.005)	(0.005)	(0.010)
γ_{RMW}		0.007	0.023	0.020	0.008	0.003	0.002	0.008	0.016	0.013	0.008
		(0.024)	(0.027)	(0.025)	(0.024)	(0.017)	(0.017)	(0.023)	(0.018)	(0.019)	(0.024)
γ_{CMA}		-0.013	-0.009	-0.003	-0.012	-0.007	-0.010	-0.012	-0.006	-0.006	-0.011
		(0.020)	(0.015)	(0.014)	(0.019)	(0.013)	(0.017)	(0.020)	(0.012)	(0.011)	(0.019)
γ_{ESKEW}			-0.056**						-0.044**		
			(0.021)						(0.014)		
γ_{ESKEW-}				-0.039*						-0.048***	
				(0.016)						(0.016)	
γ_{ESKEW+}					0.007						0.006
					(0.012)						(0.015)
γ_{VSTOXX}						0.060**			0.053***		
						(0.023)			(0.018)		
$\gamma_{VSTOXX-}$							0.063**			0.078**	
							0.017			(0.031)	
$\gamma_{VSTOXX+}$								0.067			0.064
								(0.104)			(0.106)
R^2	0.058	0.080	0.303	0.295	0.124	0.311	0.287	0.093	0.439	0.425	0.136
Adj R^2	0.032	0.044	0.211	0.208	0.082	0.226	0.186	0.042	0.346	0.359	0.093
P-value	0.420	0.376	0.009	0.085	0.378	0.007	0.057	0.098	0.000	0.000	0.362

Notes: This table shows the results of the second stage of the Fama-MacBeth regression approach in which the portfolios' returns are regressed on the β factor loadings computed in the first step and expressing the exposure of each portfolio of assets to that specific factor as in (20). γ coefficients measure the risk premium associated with each selected factor. The regressions' outcome we report are for the simple 3-factors model (1) (Equation (21)), for the 5-factors model (2) (Equation (22)), for the 6-factors model with skewness indexes (3), (4), and (5) (Equation (23)) and with volatility indexes (6), (7) and (8) (Equation (24)) and the full model with both skewness and volatility indexes, same sign, (9), (10) and (11) (Equation (25)). The results are reported for both the U.S. (upper panel) and for the Eurozone (bottom panel). 25 Portfolios based on Size and Book-to-Value are chosen. The γ coefficients and their standard errors are reported. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The sample period is from 03-01-2011 to 30-09-2016, at daily frequency for a total of 1446 observations.

Table 14 shows the results of the second step Fama-MacBeth methodology for both the U.S. and the Eurozone. Several models are considered in order to check whether or not the regression significance measured by the adjusted R^2 coefficients improves by adding the implied decomposed higher moments indexes to the five-factor model. The traditional three factor model is considered as well for comparison.

In relation to the U.S., the aggregate SKEW incorporation in the five-factors model does not improve the adjusted R^2, while adding the negative SKEW$^-$ in the same model *does* improve the adjusted R^2. However, the adjusted R^2 is only slightly higher when SKEW$^+$ is added to the same model. While SKEW and SKEW$^-$ are negatively related with the portfolios' returns, SKEW$^+$ has a positive impact. Interestingly, the inclusion of decomposed implied skewness indexes improves the regression significance more than the addition of decomposed implied volatility indexes, of the same sign. Overall, what we can observe is how the decomposed SKEW indexes improve the regression models performance more than the aggregate one and more than the implied volatility factors confirming our hypothesis.

For the Eurozone, the results show that the aggregate and negative ESKEW have similar informative power when added to the five-factors model improving the adjusted R^2. They have a negative and significant relation with the Eurozone portfolios' returns. Call portfolios are, on the other hand, not as informative as we found for the U.S. and this might be due to time selection, considering the uncertain and volatile events that have occurred in the Eurozone in the selected period. Precious information is carried out, in the Eurozone market, also from the implied volatility aggregate and negative indexes that emerged as significant with positive coefficients. When the two decomposed indexes are considered together in the seven-factor model as in equation (25), the best model in terms of adjusted R^2 is the one considering information coming from the Puts.

Tables 15 and 16 report the results when the market information is split into positive scenario and negative scenario, both for the U.S. and the Eurozone. Bullish or bearish times are filtered according to their stock market indexes performance, S&P 500 and EUROSTOXX 50, respectively. We can check whether or not the informative power of the additional decomposed indexes varies with the market scenario[25].

Table 15 shows that for the U.S. adding SKEW and SKEW$^-$ in positive market times does not actually increase the five-factors model performance performing similarly as, also, VIX and VIX^-. What appears to be more helpful in this scenario is the set of information coming from the Calls (SKEW$^+$ and VIX^+).

In relation with the negative market scenario, we have found that the opposite is true. The negative SKEW$^-$ and the information contained in Put portfolios considerably increases the regression models' adjusted R^2. SKEW$^-$ emerged also as significant and positively related with the portfolios' returns. Moreover, in this case, the model in which SKEW$^+$ is added shows a poor performance even lower than the models considering the aggregate SKEW. The best model we found in this scenario is the full seven-factors model containing information extracted from the Puts. Surprisingly, VIX^+ has emerged as significant and positively related reflecting, possibly, the speculating activity might have been present in bearish times in the U.S. market.

Table 16 depicts the results in positive and negative times for the Eurozone. We found that models considering both the decomposed ESKEW$^-$ and ESKEW$^+$ perform better than the initial five-factors model and better than the model with the aggregate ESKEW. The best model in explaining the portfolios' returns

[25]For instance, since the role of Calls has found to increase the regression model's R^2 in the full-period case, we test whether or not it is due to the calm period we have selected.

Table 15: **U.S. Fama-MacBeth Regression Results: Positive and Negative Scenario**

U.S. Positive Scenario											
Factor	3-Factors	5-Factors	5-Factors & SKEW			5-Factors & VIX			Full 7-Factors		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
γ_0	0.053***	0.056***	0.055***	0.056***	0.059***	0.061***	0.055***	0.068***	0.061***	0.056***	0.067***
	(0.010)	(0.014)	(0.014)	(0.014)	(0.014)	(0.012)	(0.012)	(0.015)	(0.012)	(0.012)	(0.016)
γ_{Mkt-rf}	0.068	0.111	0.123	0.114	0.231*	0.080	0.119	0.353***	0.115	0.116	0.359***
	(0.098)	(0.104)	(0.120)	(0.107)	(0.127)	(0.106)	(0.148)	(0.098)	(0.106)	(0.162)	(0.106)
γ_{HML}	0.103**	0.119**	0.121**	0.121**	0.123**	0.105*	0.121*	0.058	0.109*	0.121*	0.062
	(0.042)	(0.045)	(0.052)	(0.051)	(0.044)	(0.056)	(0.065)	(0.044)	(0.059)	(0.067)	(0.047)
γ_{SMB}	-0.051	-0.011	-0.002	-0.006	0.001	-0.018	-0.010	0.088	-0.013	-0.006	0.084
	(0.134)	(0.124)	(0.122)	(0.121)	(0.130)	(0.129)	(0.118)	(0.123)	(0.138)	(0.128)	(0.126)
γ_{RMW}		0.054	0.046	0.050	0.044	0.089	0.049	-0.007	0.072	0.049	-0.005
		(0.079)	(0.085)	(0.080)	(0.084)	(0.068)	(0.072)	(0.063)	(0.078)	(0.074)	(0.062)
γ_{CMA}		0.053	0.048	0.050	0.030	0.052	0.052	-0.074	0.036	0.050	-0.070
		(0.033)	(0.042)	(0.041)	(0.036)	(0.035)	(0.035)	(0.061)	(0.044)	(0.042)	(0.072)
γ_{SKEW}			0.001						0.002		
			(0.003)						(0.002)		
γ_{SKEW-}				0.009						0.002	
				(0.005)						(0.002)	
γ_{SKEW+}					0.006**						0.002
					(0.003)						(0.004)
γ_{VIX}						-0.006			-0.008		
						(0.012)			(0.013)		
γ_{VIX-}							-0.005			-0.004	
							(0.023)			(0.029)	
γ_{VIX+}								0.077			0.072
								(0.029)			(0.045)
R^2	0.162	0.250	0.251	0.251	0.299	0.274	0.251	0.371	0.284	0.251	0.372
Adj R^2	0.083	0.113	0.112	0.111	0.165	0.102	0.104	0.231	0.175	0.143	0.244
P-value	0.381	0.215	0.219	0.225	0.164	0.182	0.221	0.110	0.184	0.223	0.102
U.S. Negative Scenario											
Factor	3-Factors	5-Factors	5-Factors & SKEW			5-Factors & VIX			Full 7-Factors		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
γ_0	-0.054***	-0.077***	-0.043	-0.017	-0.076***	-0.078***	-0.086***	-0.094***	-0.028	-0.006	-0.094***
	(0.013)	(0.019)	(0.027)	(0.027)	(0.020)	(0.021)	(0.026)	(0.020)	(0.036)	(0.037)	(0.020)
γ_{Mkt-rf}	0.256*	0.450**	0.219	0.031	0.478**	0.451**	0.469*	0.216	0.145	0.020	0.199
	(0.147)	(0.202)	(0.227)	(0.244)	(0.225)	(0.212)	(0.236)	(0.193)	(0.248)	(0.268)	(0.243)
γ_{HML}	-0.413***	-0.373**	-0.328**	-0.254*	-0.348*	-0.363	-0.305	-0.349**	-0.405*	-0.279*	-0.358**
	(0.129)	(0.160)	(0.168)	(0.140)	(0.167)	(0.242)	(0.227)	(0.133)	(0.218)	(0.152)	(0.140)
γ_{SMB}	-0.011	-0.230	-0.122	-0.074	-0.154	-0.210	-0.091	-0.136	-0.263	-0.133	-0.161
	(0.173)	(0.182)	(0.197)	(0.182)	(0.187)	(0.342)	(0.316)	(0.202)	(0.331)	(0.290)	(0.173)
γ_{RMW}		-0.095	-0.094	-0.054	-0.080	-0.086	-0.034	-0.003	-0.166	-0.083	-0.006
		(0.168)	(0.137)	(0.098)	(0.162)	(0.221)	(0.212)	(0.169)	(0.205)	(0.151)	(0.172)
γ_{CMA}		-0.062	-0.021	0.008	-0.052	-0.058	-0.046	-0.061	-0.048	0.007	-0.064
		(0.082)	(0.095)	(0.083)	(0.082)	(0.119)	(0.100)	(0.071)	(0.109)	(0.082)	(0.081)
γ_{SKEW}			0.022						0.029		
			(0.015)						(0.018)		
γ_{SKEW-}				0.032***						0.035**	
				(0.010)						(0.014)	
γ_{SKEW+}					-0.007						-0.001
					(0.010)						(0.008)
γ_{VIX}						0.029			0.021		
						(0.046)			(0.060)		
γ_{VIX-}							0.053			0.020	
							(0.048)			(0.053)	
γ_{VIX+}								0.161***			0.167**
								(0.044)			(0.059)
R^2	0.272	0.337	0.420	0.543	0.349	0.338	0.365	0.446	0.441	0.579	0.507
Adj R^2	0.168	0.163	0.227	0.391	0.133	0.117	0.153	0.335	0.211	0.464	0.361
P-value	0.077	0.134	0.093	0.016	0.200	0.223	0.172	0.055	0.128	0.011	0.032

Notes: This table shows the results of the second stage of the Fama-MacBeth regression approach in which the portfolios' returns are regressed on the β factor loadings computed in the first step and expressing the exposure of each portfolio of assets to that specific factor as in (20). γ coefficients measure the risk premium associated with each selected factor. The regressions' outcome we report are for the simple 3-factors model (1) (Equation (21)), for the 5-factors model (2) (Equation (22)), for the 6-factors model with skewness indexes (3), (4), and (5) (Equation (23)) and with volatility indexes (6), (7) and (8) (Equation (24)) and the full model with both skewness and volatility indexes, same sign, (9), (10) and (11) (Equation (25)). The results are reported for the U.S. positive scenario (upper panel) and for the U.S. negative scenario (bottom panel) according to our market proxy S&P 500 . 25 Portfolios based on Size and Book-to-Value are chosen. The γ coefficients and their standard errors are reported. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The sample period is from 03-01-2011 to 30-09-2016, at daily frequency for a total of 779 observations for the positive scenario and 667 observations for the negative scenario.

in bullish time is the one taking into account the seven-factor model with the addition of the aggregate $VSTOXX$. In the negative scenario, we found similar results as for the U.S. with Put portfolios being more informative for the implied second and third moment performing better in terms of adjusted R^2.

Table 16: **Eurozone Fama-MacBeth Regression Results: Positive and Negative Scenario**

Eurozone Positive Scenario											
Factor	3-Factors	5-Factors	5-Factors & ESKEW			5-Factors & VSTOXX			Full 7-Factors		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
γ_0	0.194***	0.091**	0.101**	0.064	0.082**	0.024	-0.024	0.071***	0.013	0.036	0.069***
	(0.062)	(0.014)	(0.040)	(0.046)	(0.033)	(0.023)	(0.031)	(0.024)	(0.025)	(0.038)	(0.024)
γ_{Mkt-rf}	0.218***	0.212***	0.223***	0.196	0.198***	0.211	0.458*	0.171***	0.314	0.401	0.105***
	(0.011)	(0.038)	(0.036)	(0.139)	(0.235)	(0.106)	(0.239)	(0.026)	(0.221)	(0.320)	(0.026)
γ_{HML}	0.174	0.183	0.070**	0.273**	0.474	0.105*	0.503	0.464	0.357	0.545	0.420
	(0.870)	(0.479)	(0.054)	(0.157)	(0.284)	(0.056)	(0.373)	(0.436)	(0.317)	(0.378)	(0.452)
γ_{SMB}	-0.131	-0.315	-0.428	-0.098	-0.185	-0.018	-0.040	-0.220	-0.069	-0.148	-0.199
	(0.266)	(0.370)	(0.481)	(0.618)	(0.351)	(0.222)	(0.306)	(0.290)	(0.210)	(0.384)	(0.292)
γ_{RMW}		0.116*	0.046	0.120**	0.111*	0.132**	0.126***	0.137***	0.130***	0.134***	0.108**
		(0.059)	(0.085)	(0.059)	(0.060)	(0.058)	(0.027)	(0.040)	(0.025)	(0.040)	(0.043)
γ_{CMA}		0.052	0.048	0.035	0.089**	0.063	0.002	-0.027	0.015	0.047	-0.016
		(0.048)	(0.042)	(0.037)	(0.038)	(0.047)	(0.030)	(0.035)	(0.033)	(0.030)	(0.046)
γ_{ESKEW}			0.019						0.015		
			(0.034)						(0.013)		
γ_{ESKEW-}				0.043						0.028	
				(0.051)						(0.021)	
γ_{ESKEW+}					-0.055**						-0.021
					(0.023)						(0.016)
γ_{VSTOXX}						-0.149***			0.145***		
						(0.015)			(0.016)		
$\gamma_{VSTOXX-}$							-0.193***			-0.192***	
							(0.027)			(0.025)	
$\gamma_{VSTOXX+}$								0.452***			0.449***
								(0.029)			(0.117)
R^2	0.729	0.845	0.847	0.853	0.850	0.963	0.940	0.897	0.965	0.943	0.898
Adj R^2	0.591	0.728	0.725	0.732	0.729	0.858	0.832	0.783	0.859	0.833	0.781
P-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Eurozone Negative Scenario											
Factor	3-Factors	5-Factors	5-Factors & ESKEW			5-Factors & VSTOXX			Full 7-Factors		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
γ_0	0.020	0.319***	0.233***	0.331***	0.348***	0.377***	0.384***	0.414***	0.362***	0.387***	0.416***
	(0.007)	(0.065)	(0.063)	(0.006)	(0.056)	(0.016)	(0.016)	(0.042)	(0.020)	(0.019)	(0.043)
γ_{Mkt-rf}	0.410	0.450**	0.921	0.588	1.161	0.337	0.008	0.633	0.135	0.034	0.938
	(0.237)	(0.969)	(1.091)	(1.007)	(1.039)	(0.303)	(0.373)	(0.743)	(0.391)	(0.268)	(0.768)
γ_{HML}	-0.433***	-0.268**	-0.192***	-0.271***	-0.273***	-0.051	-0.139	-0.831*	-0.081	-0.149	-0.104**
	(0.102)	(0.062)	(0.043)	(0.063)	(0.058)	(0.332)	(0.350)	(0.451)	(0.218)	(0.347)	(0.038)
γ_{SMB}	0.217***	0.138**	0.721	0.140	0.907	0.619	0.751**	0.195	0.657**	-0.744**	0.114
	(0.075)	(0.068)	(0.603)	(0.685)	(0.620)	(0.268)	(0.289)	(0.472)	(0.289)	(0.288)	(0.461)
γ_{RMW}		-0.150***	-0.105**	-0.149***	-0.108**	-0.034	-0.023	-0.050	-0.035*	-0.024	-0.042
		(0.048)	(0.043)	(0.049)	(0.162)	(0.023)	(0.029)	(0.040)	(0.020)	(0.030)	(0.040)
γ_{CMA}		-0.257***	-0.218***	0.261***	-0.255***	-0.119***	-0.125***	-0.211***	-0.117***	0.126***	-0.214***
		(0.039)	(0.036)	(0.041)	(0.034)	(0.014)	(0.013)	(0.036)	(0.014)	(0.014)	(0.035)
γ_{ESKEW}			0.098***						0.023***		
			(0.034)						(0.008)		
γ_{ESKEW-}				0.018*						0.002	
				(0.009)						(0.009)	
γ_{ESKEW+}					-0.086**						-0.039
					(0.041)						(0.031)
γ_{VSTOXX}						0.172***			0.170***		
						(0.010)			(0.010)		
$\gamma_{VSTOXX-}$							0.243***			0.243***	
							(0.017)			(0.017)	
$\gamma_{VSTOXX+}$								0.623***			0.558***
								(0.099)			(0.097)
R^2	0.635	0.832	0.827	0.842	0.832	0.980	0.981	0.911	0.971	0.981	0.916
Adj R^2	0.511	0.713	0.708	0.712	0.703	0.877	0.877	0.799	0.868	0.876	0.802
P-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: This table shows the results of the second stage of the Fama-MacBeth regression approach in which the portfolios' returns are regressed on the β factor loadings computed in the first step and expressing the exposure of each portfolio of assets to that specific factor as in (20). γ coefficients measure the risk premium expressed by each selected factor. The regressions' outcome we report are for the simple 3-factors model (1) (Equation (21)), for the 5-factors model (2) (Equation (22)), for the 6-factors model with skewness indexes (3), (4), and (5) (Equation (23)) and with volatility indexes (6), (7) and (8) (Equation (24)) and the full model with both skewness and volatility indexes, same sign, (9), (10) and (11) (Equation (25)). The results are reported for the Eurozone positive scenario (upper panel) and for the Eurozone negative scenario (bottom panel) according to the Eurozone market proxy EUROSTOXX50. 25 Portfolios based on Size and Book-to-Value are chosen. The γ coefficients and their standard errors are reported. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The sample period from 03-01-2011 to 30-09-2016, at daily frequency for a total of 741 observations for the positive scenario and 707 observations for the negative scenario.

9 Discussion and Conclusion

This paper contributes to the growing literature in finance on tail risk and higher risk neutral moments. Continuing the line of some recent studies, we focused on the limitation of the volatility indexes and their downside bias. We studied this problem looking at financial market information extracted from option prices. Extracting additional information from the third risk neutral moment market, the SKEW index has been decomposed in its positive and negative side. A step further was to recognize that, most of the time, the aggregate CBOE SKEW index has been influenced by the positive information flows coming from the Calls market side, leading to a possible bias in the measurement of the market tail risk.

In our paper we proposed the tail risk index, SKEW^-, as the skew part extracted from the Puts market. This index is more prudent, conservative and representative of the pessimistic view in the financial market. The necessity of a skew index is very useful in the Eurozone where investors did not have an appropriate implied skewness index. Thus, we constructed the Eurozone skew index ESKEW.

This research also shed some new light on the relationship between equity, volatility and skewness indexes. The econometric analysis conducted in this paper confirms some of the *ex ante* hypotheses extracted from the literature and other financial economics considerations. The first result is that the negative skewness parts, SKEW^- and ESKEW^-, are more informative for the equity market returns and more related to fundamentals. SKEW^+ and ESKEW^+ are found, instead, to be more influenced by market sentiment. An asset pricing exercise further confirms the superior informative power for SKEW^- and ESKEW^- compared to the aggregate indexes. When negative indexes are incorporated the asset pricing models' performance increases especially in bearish times, while it is similar to models including only the aggregate measures in bullish times. Evidence that our proposed tail index, represented by SKEW^- and ESKEW^-, links the U.S. and the Eurozone tail risk perception, more than the aggregate skew index, is also found. The dominance of the negative component, extracted from the Puts market, in the overall index is also confirmed for volatility.

The econometric analysis points out that there is a weak link between volatility and skewness in terms of cointegration, in short and volatile periods, while they are cointegrated in the long-run. However, in terms of Granger causality the opposite is true. There is no causality between the total indexes of the two major economies in the full sample, but there is a bidirectional Granger causality during the Brexit period.

Implied skewness can provide further information that investors may want to consider. There is something *left* to discover about tail risk after taking out the implied volatility: the interaction of implied skewness with implied volatility and market sentiment. Our findings reveal that this missing and informative part seems to lie in the *left* side of the risk-neutral distribution of equity market indexes.

Appendix A CBOE VIX Computation

The following formula is used in order to calculate the implied variance for both the near and also the far term expiration:

$$\sigma^2_{VIX} = \frac{2}{T} \sum \frac{\Delta(K_i)}{K_i^2} e^{rT} Q_t(K_i) - \frac{1}{T} \left[\frac{F_t}{K_0} - 1 \right]^2 \tag{26}$$

where T is the expiration date, F_t is the forward of S&P 500 calculated from the Put-Call parity as:

$$F_t = e^{rT} [c(K,T) - p(K,T)] + K \tag{27}$$

K_0 (*Reference Price*) is the first exercise price less or equal to the forward level $F_t(K_0 \leq F_t)$ and K_t is the strike price of i-out of the money used in the calculation. This is a Call if $K_i > K_0$, Put if $K_i < K_0$ and both Call and Put if $K_i = K_0$. $Q_t(K_i)$ is the average bid-ask of OTM options with exercise price equal to K_i. If $K_i = K_0$ it will be equal to the average between ATM Call and Put price, relative to that strike price. r is the risk free rate with expiration T, and $\Delta(K_i)$ is the sum divided by two of the two nearest prices to the exercise price K_0.

The formula (26) is based on the *Variance Swap* formula approximation:

$$\sum_{i=1}^{n} \frac{\Delta(K_i)}{K_i}^2 e^{rT} Q(K_i) \tag{28}$$

In general $\Delta(K_i)$ is equal to $\frac{(K_{i+1} - K_{i-1})}{2}$ for $2 \leq i \leq n-1$. $Q(K_i)$ is a generic price of a European Call or Put with strike price respectively above or below K_0, the first strike price below F_0. When K_i is equal to K_0, $Q(K_i)$ is equal to the average Call and Put. To calculate the expected variance, the adjustment term is added[26]:

$$\frac{1}{T} \left[\frac{F_0}{K_0} - 1 \right]^2 \tag{29}$$

The VIX calculation is given from the interpolation of the Near Term Variance and the Far Term Variance. These are the closest expirations to 30 days target in which monthly or weekly S&P 500 options are traded. [27]. VIX calculation's aim is to better track the 30-days implied volatility in the equity market (this aim is

[26]It represents the adjustment needed in order to convert ITM Calls in OTM Puts

[27]Rule of the thumb: Weekly S&P 500 options to be selected must have an expiration $\geq$ 23days, $\leq$ 37 days. When Monthly S&P 500 Options are considered and before 2014, expiration $\geq$ 7days since less than 7 days the impact of volatility and volume can misdirect the computation.

improved with the introduction of Weekly S&P 500 since 2014). This allows to better calculate the VIX: expiration dates better tracks the 30 days market news and events.

More specifically an interpolation between $\sigma^2_{VIX_1}(T_1)$ and $\sigma^2_{VIX_2}(T_2)$ is used. Finally, the VIX is computed as:

$$VIX = \sqrt{\frac{365}{30}} \left[T_1 \sigma^2_{VIX_1}(T_1) \frac{N_2 - 30}{N_2 - N_1} + T_2 \sigma^2_{VIX_2}(T_2) \frac{30 - N_1}{N_2 - N_1} \right] 100 \tag{30}$$

Appendix B EUREX VSTOXX Computation

VSTOXX is computed as a rolling index with an expiration date of 30-days through the interpolation of the two nearest available Sub-Indexes. The Sub-Indexes are 8 and their range include expirations from 1-month to 2-years. So, basically, VSTOXX is computed as interpolation of, generally, the first two months Sub-Indexes, VSTOXX 1M and VSTOXX 2M. When the first month is not available or few days are left for its expiration, the considered Sub-Indexes are rolled to the next expiration, and the 3M VSTOXX is taken into account as far term index. The formulas we have used in order to compute the VSTOXX are the following[28]:

$$VSTOXX = 100 \sqrt{\left(\frac{T_1}{T_{365}} \sigma_1^2 \frac{T_2 - T_{30}}{T_2 - T_1} + \frac{T_2}{T_{365}} \sigma_2^2 \frac{T_{30} - T_1}{T_2 - T_1} \right) \frac{T_{365}}{T_{30}}} \tag{31}$$

where T_i with $i = 1, 2$ are the considered Sub indexes expiration days, T_{365} and T_{30} are, respectively, the number of days in one year and in one month, σ_i with $i = 1, 2$ are the considered Sub indexes.

The *Sub Indexes* are calculated as follow:

$$\sigma_i^2 = \frac{2}{\frac{T_i}{T_{365}}} \sum_j \frac{\triangle K_{i,j}}{K_{i,j}^2} e_i^r \frac{T_i}{T_{365}} M(K_{i,j}) - \frac{1}{\frac{T_i}{T_{365}}} \left(\frac{F_i}{K_{i,0}} - 1 \right)^2 \tag{32}$$

where F_i is the forward price derived as $F_i = K_{min} + R_i(C - P)$, K_{min} is the strike price corresponding to the min difference between calls and puts, $K_{i,j}$ is the exercise price of each options in the range, $K_{i,0}$ is the exercise price below the forward price F_i, $\triangle K_{i,j}$ is the average of the distance between two consecutive options strike prices and r_i is the risk free rate for the selected expiration date.

[28]STOXX Strategy Index Guide.
Available at: https://www.stoxx.com/document/Indices/Common/Indexguide/stoxx_dvp_calculation_guide.pdf

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