GEKKO Python Example Applications

GEKKO is optimization software for mixed-integer and differential algebraic equations. It is coupled with large-scale solvers for linear, quadratic, nonlinear, and mixed integer programming (LP, QP, NLP, MILP, MINLP). Modes of operation include data reconciliation, real-time optimization, dynamic simulation, and nonlinear predictive control. See the [GEKKO documentation (http://gekko.readthedocs.io/en/latest/overview.html)](http://gekko.readthedocs.io/en/latest/overview.html) for additional information.

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In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

In [2]:
```
# package information
from gekko import GEKKO
!pip show gekko
```

```
Name: gekko
Version: 0.0.4rc3
Summary: Optimization software for differential algebraic equations
Home-page: https://github.com/BYU-PRISM/GEKKO
Author: BYU PRISM Lab
Author-email: john_hedengren@byu.edu
License: MIT
Location: c:\programdata\anaconda3\lib\site-packages
Requires: numpy, flask
Required-by:
```

1: GEKKO solver selection

Solve $y^2 = 1$ with APOPT solver

In [3]:
```
m = GEKKO()            # create GEKKO model
y = m.Var(value=2)     # define new variable, initial value=2
m.Equation(y**2==1)    # define new equation
m.options.SOLVER=1     # change solver (1=APOPT,3=IPOPT)
m.solve(disp=False)    # solve locally (remote=False)
print('y: ' + str(y.value)) # print variable value
```

```
y: [1.0]
```

2: Solve Linear Equations

$3\,x + 2\,y = 1$

$x + 2\,y = 0$

In [4]:
```
m = GEKKO()             # create GEKKO model
x = m.Var()             # define new variable, default=0
y = m.Var()             # define new variable, default=0
m.Equations([3*x+2*y==1, x+2*y==0])  # equations
m.solve(disp=False)     # solve
print(x.value,y.value) # print solution
```

```
[0.5] [-0.25]
```

3: Solve Nonlinear Equations

$x + 2\,y = 0$

$x^2 + y^2 = 1$

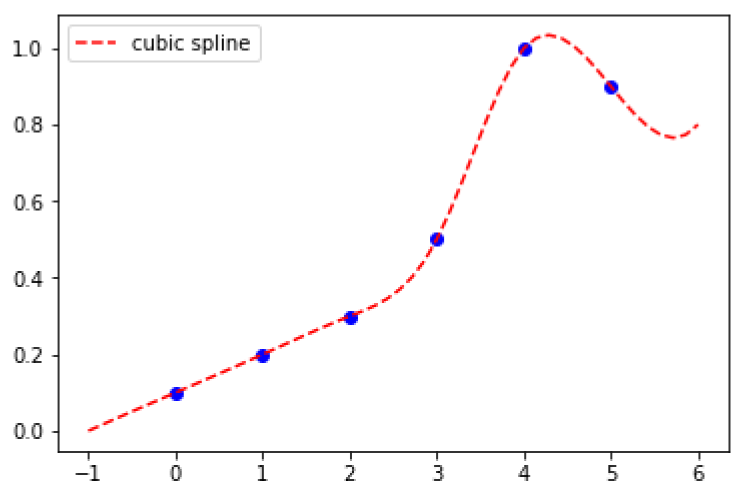
In [5]:
```
m = GEKKO()             # create GEKKO model
x = m.Var(value=0)      # define new variable, initial value=0
y = m.Var(value=1)      # define new variable, initial value=1
m.Equations([x + 2*y==0, x**2+y**2==1]) # equations
m.solve(disp=False)     # solve
print([x.value[0],y.value[0]]) # print solution
```

```
[-0.8944272, 0.4472136]
```

4: Interpolation with Cubic Spline

In [6]:
```
xm = np.array([0,1,2,3,4,5])
ym = np.array([0.1,0.2,0.3,0.5,1.0,0.9])

m = GEKKO()             # create GEKKO model
m.options.IMODE = 2     # solution mode
x = m.Param(value=np.linspace(-1,6)) # prediction points
y = m.Var()             # prediction results
m.cspline(x, y, xm, ym) # cubic spline
m.solve(disp=False)     # solve

# create plot
plt.plot(xm,ym,'bo')
plt.plot(x.value,y.value,'r--',label='cubic spline')
plt.legend(loc='best')
```

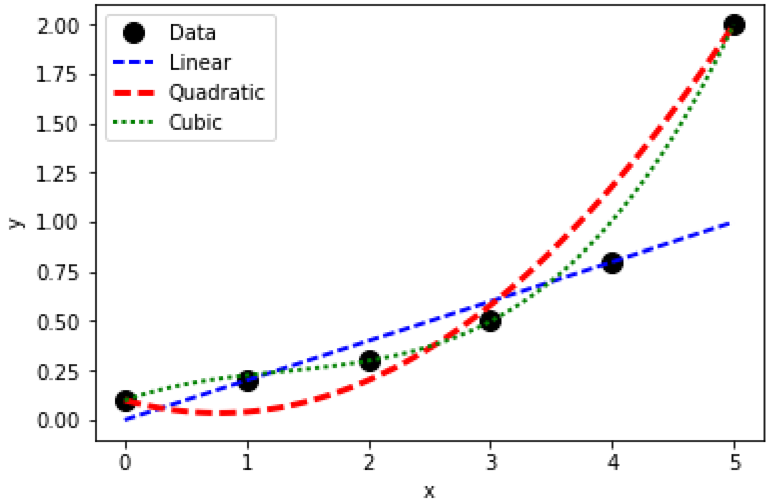
Out[6]:
```
<matplotlib.legend.Legend at 0x23e45d01f98>
```



5: Linear and Polynomial Regression

In [7]:
```python
xm = np.array([0,1,2,3,4,5])
ym = np.array([0.1,0.2,0.3,0.5,0.8,2.0])

#### Solution
m = GEKKO()
m.options.IMODE=2
# coefficients
c = [m.FV(value=0) for i in range(4)]
x = m.Param(value=xm)
y = m.CV(value=ym)
y.FSTATUS = 1
# polynomial model
m.Equation(y==c[0]+c[1]*x+c[2]*x**2+c[3]*x**3)

# linear regression
c[0].STATUS=1
c[1].STATUS=1
m.solve(disp=False)
p1 = [c[1].value[0],c[0].value[0]]

# quadratic
c[2].STATUS=1
m.solve(disp=False)
p2 = [c[2].value[0],c[1].value[0],c[0].value[0]]

# cubic
c[3].STATUS=1
m.solve(disp=False)
p3 = [c[3].value[0],c[2].value[0],c[1].value[0],c[0].value[0]]

# plot fit
plt.plot(xm,ym,'ko',markersize=10)
xp = np.linspace(0,5,100)
plt.plot(xp,np.polyval(p1,xp),'b--',linewidth=2)
plt.plot(xp,np.polyval(p2,xp),'r--',linewidth=3)
plt.plot(xp,np.polyval(p3,xp),'g:',linewidth=2)
plt.legend(['Data','Linear','Quadratic','Cubic'],loc='best')
plt.xlabel('x')
plt.ylabel('y')
```

```
Out[7]: <matplotlib.text.Text at 0x23e45df35f8>
```

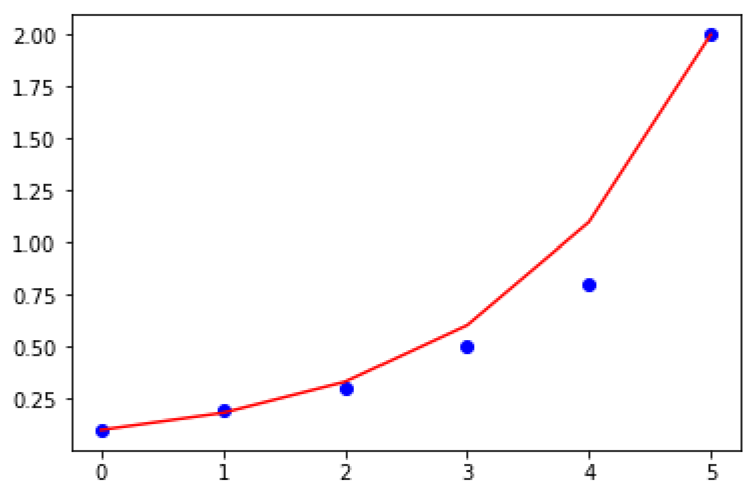


6: Nonlinear Regression

In [8]:
```
# measurements
xm = np.array([0,1,2,3,4,5])
ym = np.array([0.1,0.2,0.3,0.5,0.8,2.0])

# GEKKO model
m = GEKKO()

# parameters
x = m.Param(value=xm)
a = m.FV()
a.STATUS=1

# variables
y = m.CV(value=ym)
y.FSTATUS=1

# regression equation
m.Equation(y==0.1*m.exp(a*x))

# regression mode
m.options.IMODE = 2

# optimize
m.solve(disp=False)

# print parameters
print('Optimized, a = ' + str(a.value[0]))

plt.plot(xm,ym,'bo')
plt.plot(xm,y.value,'r-')
```

```
Optimized, a = 0.5990964
```

Out[8]:
```
[<matplotlib.lines.Line2D at 0x23e45dec710>]
```



7: Machine Learning

Approximate $y = sin(x)$ with an Artificial Neural Network

Trigonometric Function (trig=True)

- Input: x
- Layer 1: linear layer, 1 node, $l1 = w1\ x$
- Layer 2: nonlinear layer, 1 node, cosine function, $l2 = \cos(w2a + w2b\ l1)$
- Layer 3: linear layer, 1 node, $l3 = w3\ l2$
- Output: $y = \sum l3$

Artificial Neural Network Description (trig=False)

- Input: x
- Layer 1: linear layer, 2 nodes, $l1 = w1\ x$
- Layer 2: nonlinear layer, 2 nodes, hyperbolic tangent activation function, $l2 = \tanh(w2a + w2b\ l1)$
- Layer 3: linear layer, 2 nodes, $l3 = w3\ l2$
- Output: $y = \sum l3$

See [Online Neural Network Demo (https://playground.tensorflow.org)](https://playground.tensorflow.org) with TensorFlow.

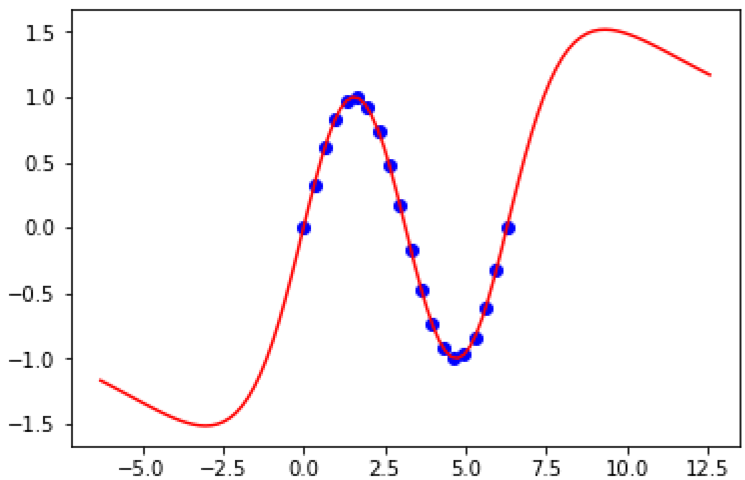
In [9]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
from gekko import GEKKO

# generate training data
x = np.linspace(0.0,2*np.pi,20)
y = np.sin(x)

# option for fitting function
select = False # True / False
if select:
    # Size with cosine function
    nin = 1  # inputs
    n1 = 1   # hidden layer 1 (linear)
    n2 = 1   # hidden layer 2 (nonlinear)
    n3 = 1   # hidden layer 3 (linear)
    nout = 1 # outputs
else:
    # Size with hyperbolic tangent function
    nin = 1  # inputs
    n1 = 2   # hidden layer 1 (linear)
    n2 = 2   # hidden layer 2 (nonlinear)
    n3 = 2   # hidden layer 3 (linear)
    nout = 1 # outputs

# Initialize gekko
train = GEKKO()
test = GEKKO()

model = [train,test]

for m in model:
    # input(s)
    m.inpt = m.Param()

    # layer 1
    m.w1 = m.Array(m.FV, (nin,n1))
    m.l1 = [m.Intermediate(m.w1[0,i]*m.inpt) for i in range(n1)]

    # layer 2
    m.w2a = m.Array(m.FV, (n1,n2))
    m.w2b = m.Array(m.FV, (n1,n2))
    if select:
        m.l2 = [m.Intermediate(sum([m.cos(m.w2a[j,i]+m.w2b[j,i]*m.l1[j])
 \
                                for j in range(n1)])) for i in range(n2)]
    else:
        m.l2 = [m.Intermediate(sum([m.tanh(m.w2a[j,i]+m.w2b[j,i]*m.l1[j])
 \
                                for j in range(n1)])) for i in range(n2)]

    # layer 3
    m.w3 = m.Array(m.FV, (n2,n3))
    m.l3 = [m.Intermediate(sum([m.w3[j,i]*m.l2[j] \
            for j in range(n2)])) for i in range(n3)]
```

```python
    # output(s)
    m.outpt = m.CV()
    m.Equation(m.outpt==sum([m.l3[i] for i in range(n3)]))

    # flatten matrices
    m.w1 = m.w1.flatten()
    m.w2a = m.w2a.flatten()
    m.w2b = m.w2b.flatten()
    m.w3 = m.w3.flatten()

# Fit parameter weights
m = train
m.inpt.value=x
m.outpt.value=y
m.outpt.FSTATUS = 1
for i in range(len(m.w1)):
    m.w1[i].FSTATUS=1
    m.w1[i].STATUS=1
    m.w1[i].MEAS=1.0
for i in range(len(m.w2a)):
    m.w2a[i].STATUS=1
    m.w2b[i].STATUS=1
    m.w2a[i].FSTATUS=1
    m.w2b[i].FSTATUS=1
    m.w2a[i].MEAS=1.0
    m.w2b[i].MEAS=0.5
for i in range(len(m.w3)):
    m.w3[i].FSTATUS=1
    m.w3[i].STATUS=1
    m.w3[i].MEAS=1.0
m.options.IMODE = 2
m.options.SOLVER = 3
m.options.EV_TYPE = 2
m.solve(disp=False)

# Test sample points
m = test
for i in range(len(m.w1)):
    m.w1[i].MEAS=train.w1[i].NEWVAL
    m.w1[i].FSTATUS = 1
    print('w1['+str(i)+']: '+str(m.w1[i].MEAS))
for i in range(len(m.w2a)):
    m.w2a[i].MEAS=train.w2a[i].NEWVAL
    m.w2b[i].MEAS=train.w2b[i].NEWVAL
    m.w2a[i].FSTATUS = 1
    m.w2b[i].FSTATUS = 1
    print('w2a['+str(i)+']: '+str(m.w2a[i].MEAS))
    print('w2b['+str(i)+']: '+str(m.w2b[i].MEAS))
for i in range(len(m.w3)):
    m.w3[i].MEAS=train.w3[i].NEWVAL
    m.w3[i].FSTATUS = 1
    print('w3['+str(i)+']: '+str(m.w3[i].MEAS))
m.inpt.value=np.linspace(-2*np.pi,4*np.pi,100)
m.options.IMODE = 2
m.options.SOLVER = 3
m.solve(disp=False)
```

```
plt.figure()
plt.plot(x,y,'bo')
plt.plot(test.inpt.value,test.outpt.value,'r-')
```

```
w1[0]: 0.821782978
w1[1]: 0.728869969
w2a[0]: -1.41731735
w2b[0]: 0.548984509
w2a[1]: -0.131957576
w2b[1]: 0.592683118
w2a[2]: -0.330742133
w2b[2]: 0.144440713
w2a[3]: -2.92831117
w2b[3]: 0.66823565
w3[0]: -1.5171649
w3[1]: -1.5171649
w3[2]: 1.62870093
w3[3]: 1.62870093
```

Out[9]: [<matplotlib.lines.Line2D at 0x23e480091d0>]



8: Solve Differential Equation(s)

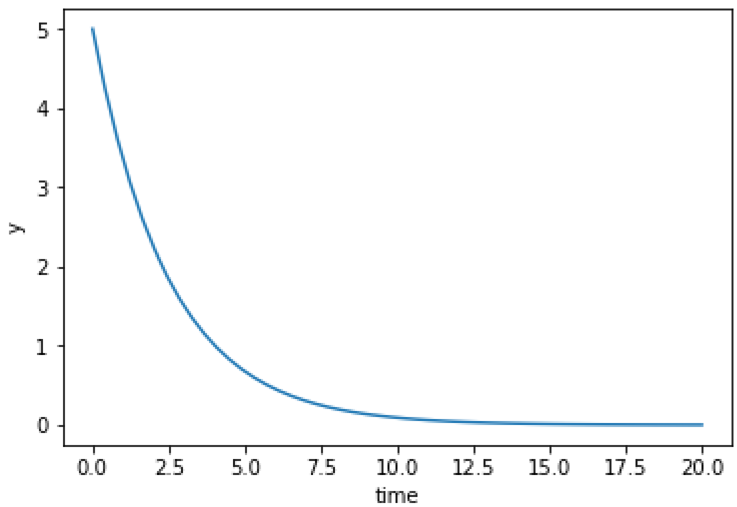
Solve the following differential equation with initial condition $y(0) = 5$:

$k \, \frac{dy}{dt} = -t \, y$

where $k = 10$. The solution of $y(t)$ should be reported from an initial time 0 to final time 20. Create of plot of the result for $y(t)$ versus t.

In [10]:
```
m = GEKKO() # Initialize gekko
a = 0.4
m.time=np.linspace(0,20)
y = m.Var(value=5.0)
m.Equation(y.dt()==-a*y)
m.options.IMODE=4
m.options.NODES=3
m.solve(disp=False)

plt.plot(m.time,y.value)
plt.xlabel('time')
plt.ylabel('y')
```

Out[10]: `<matplotlib.text.Text at 0x23e45d76b70>`



9: Nonlinear Programming Optimization

Solve the following nonlinear optimization problem:

$$\min x_1 x_4 \left(x_1 + x_2 + x_3\right) + x_3$$

$$\mathrm{s.t.} \quad x_1 x_2 x_3 x_4 \geq 25$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 40$$

$$1 \leq x_1, x_2, x_3, x_4 \leq 5$$

with initial conditions:

$$x_0 = (1, 5, 5, 1)$$

In [11]:
```
m = GEKKO() # Initialize gekko
# Use IPOPT solver (default)
m.options.SOLVER = 3
# Change to parallel linear solver
m.solver_options = ['linear_solver ma97']
# Initialize variables
x1 = m.Var(value=1,lb=1,ub=5)
x2 = m.Var(value=5,lb=1,ub=5)
x3 = m.Var(value=5,lb=1,ub=5)
x4 = m.Var(value=1,lb=1,ub=5)
# Equations
m.Equation(x1*x2*x3*x4>=25)
m.Equation(x1**2+x2**2+x3**2+x4**2==40)
m.Obj(x1*x4*(x1+x2+x3)+x3) # Objective
m.options.IMODE = 3 # Steady state optimization
m.solve(disp=False) # Solve
print('Results')
print('x1: ' + str(x1.value))
print('x2: ' + str(x2.value))
print('x3: ' + str(x3.value))
print('x4: ' + str(x4.value))
print('Objective: ' + str(m.options.objfcnval))
```

```
Results
x1: [1.0]
x2: [4.743]
x3: [3.82115]
x4: [1.379408]
Objective: 17.0140171
```

10: Mixed Integer Nonlinear Programming

In [12]:
```
m = GEKKO() # Initialize gekko
m.options.SOLVER=1  # APOPT is an MINLP solver

# optional solver settings with APOPT
m.solver_options = ['minlp_maximum_iterations 500', \
                    # minlp iterations with integer solution
                    'minlp_max_iter_with_int_sol 10', \
                    # treat minlp as nlp
                    'minlp_as_nlp 0', \
                    # nlp sub-problem max iterations
                    'nlp_maximum_iterations 50', \
                    # 1 = depth first, 2 = breadth first
                    'minlp_branch_method 1', \
                    # maximum deviation from whole number
                    'minlp_integer_tol 0.05', \
                    # covergence tolerance
                    'minlp_gap_tol 0.01']

# Initialize variables
x1 = m.Var(value=1,lb=1,ub=5)
x2 = m.Var(value=5,lb=1,ub=5)
# Integer constraints for x3 and x4
x3 = m.Var(value=5,lb=1,ub=5,integer=True)
x4 = m.Var(value=1,lb=1,ub=5,integer=True)
# Equations
m.Equation(x1*x2*x3*x4>=25)
m.Equation(x1**2+x2**2+x3**2+x4**2==40)
m.Obj(x1*x4*(x1+x2+x3)+x3) # Objective
m.solve(disp=False) # Solve
print('Results')
print('x1: ' + str(x1.value))
print('x2: ' + str(x2.value))
print('x3: ' + str(x3.value))
print('x4: ' + str(x4.value))
print('Objective: ' + str(m.options.objfcnval))
```

```
Results
x1: [1.358909]
x2: [4.599279]
x3: [4.0]
x4: [1.0]
Objective: 17.5322673
```

11: Optimal Control with Integral Objective

Original Form

$\min_u \frac{1}{2} \int_0^2 x_1^2(t)\, dt$

subject to

$\frac{dx_1}{dt} = u$

$x_1(0) = 1$

$-1 \leq u(t) \leq 1$

Equivalent Form for GEKKO with new Variable x_2

$\min_u x_2\left(t_f\right)$

subject to

$\frac{dx_1}{dt} = u$

$\frac{dx_2}{dt} = \frac{1}{2} x_1^2(t)$

$x_1(0) = 1$

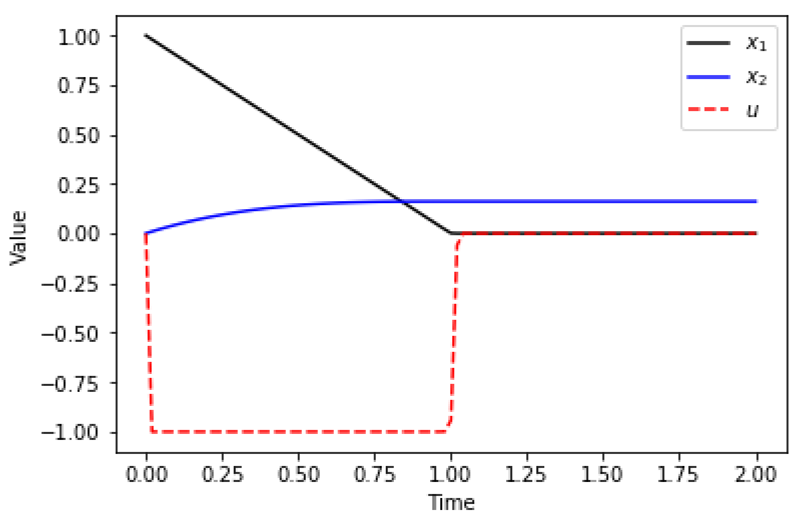
$x_2(0) = 0$

$t_f = 2$

$-1 \leq u(t) \leq 1$

In [13]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
from gekko import GEKKO
m = GEKKO() # initialize gekko
nt = 101
m.time = np.linspace(0,2,nt)
# Variables
x1 = m.Var(value=1)
x2 = m.Var(value=0)
u = m.Var(value=0,lb=-1,ub=1)
p = np.zeros(nt) # mark final time point
p[-1] = 1.0
final = m.Param(value=p)
# Equations
m.Equation(x1.dt()==u)
m.Equation(x2.dt()==0.5*x1**2)
m.Obj(x2*final) # Objective function
m.options.IMODE = 6 # optimal control mode
m.solve(disp=False) # solve
plt.figure(1) # plot results
plt.plot(m.time,x1.value,'k-',label=r'$x_1$')
plt.plot(m.time,x2.value,'b-',label=r'$x_2$')
plt.plot(m.time,u.value,'r--',label=r'$u$')
plt.legend(loc='best')
plt.xlabel('Time')
plt.ylabel('Value')
```

Out[13]: <matplotlib.text.Text at 0x23e481265f8>



12: Optimal Control with Economic Objective

Original Form

$\max_{u(t)} \int_0^{10} \left(E - \frac{c}{x}\right) u \, U_{max} \, dt$

subject to

$\frac{dx}{dt} = r \, x(t) \left(1 - \frac{x(t)}{k}\right) - u \, U_{max}$

$x(0) = 70$

$0 \leq u(t) \leq 1$

$E = 1, \; c = 17.5, \; r = 0.71$

$k = 80.5, \; U_{max} = 20$

Equivalent Form for GEKKO

$\min_{u(t)} -J\left(t_f\right)$

subject to

$\frac{dx}{dt} = r \, x(t) \left(1 - \frac{x(t)}{k}\right) - u \, U_{max}$

$\frac{dJ}{dt} = \left(E - \frac{c}{x}\right) u \, U_{max}$

$x(0) = 70$

$J(0) = 0$

$0 \leq u(t) \leq 1$

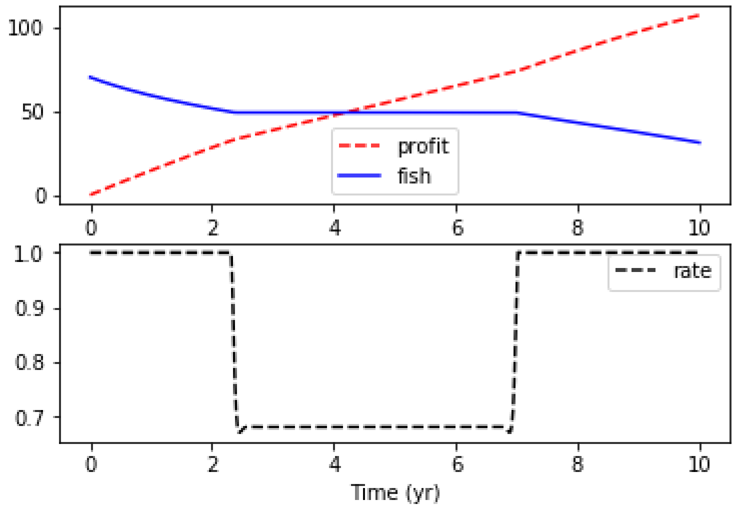
$t_f = 10, \; E = 1, \; c = 17.5$

$r = 0.71, \; k = 80.5, \; U_{max} = 20$

In [14]:
```
from gekko import GEKKO
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
# create GEKKO model
m = GEKKO()
# time points
n=501
m.time = np.linspace(0,10,n)
# constants
E,c,r,k,U_max = 1,17.5,0.71,80.5,20
# fishing rate
u = m.MV(value=1,lb=0,ub=1)
u.STATUS = 1
u.DCOST = 0
x = m.Var(value=70) # fish population
# fish population balance
m.Equation(x.dt() == r*x*(1-x/k)-u*U_max)
J = m.Var(value=0) # objective (profit)
Jf = m.FV() # final objective
Jf.STATUS = 1
m.Connection(Jf,J,pos2='end')
m.Equation(J.dt() == (E-c/x)*u*U_max)
m.Obj(-Jf) # maximize profit
m.options.IMODE = 6  # optimal control
m.options.NODES = 3  # collocation nodes
m.options.SOLVER = 3 # solver (IPOPT)
m.solve(disp=False) # Solve
print('Optimal Profit: ' + str(Jf.value[0]))
plt.figure(1) # plot results
plt.subplot(2,1,1)
plt.plot(m.time,J.value,'r--',label='profit')
plt.plot(m.time,x.value,'b-',label='fish')
plt.legend()
plt.subplot(2,1,2)
plt.plot(m.time,u.value,'k--',label='rate')
plt.xlabel('Time (yr)')
plt.legend()
```

```
Optimal Profit: 106.9061
```

```
Out[14]: <matplotlib.legend.Legend at 0x23e484c5630>
```



13: Optimal Control: Minimize Final Time

Original Form

$\min_{u(t)} \;\; t_f$

subject to

$\frac{dx_1}{dt} = u$

$\frac{dx_2}{dt} = \cos(x_1(t))$

$\frac{dx_3}{dt} = \sin(x_1(t))$

$x(0) = [\pi/2, 4, 0]$

$x_2\left(t_f\right) = 0$

$x_3\left(t_f\right) = 0$

$-2 \leq u(t) \leq 2$

Equivalent Form for GEKKO

$\min_{u(t), t_f} \;\; t_f$

subject to

$\frac{dx_1}{dt} = t_f \, u$

$\frac{dx_2}{dt} = t_f \; \cos(x_1(t))$

$\frac{dx_3}{dt} = t_f \; \sin(x_1(t))$

$x(0) = [\pi/2, 4, 0]$

$x_2\left(t_f\right) = 0$

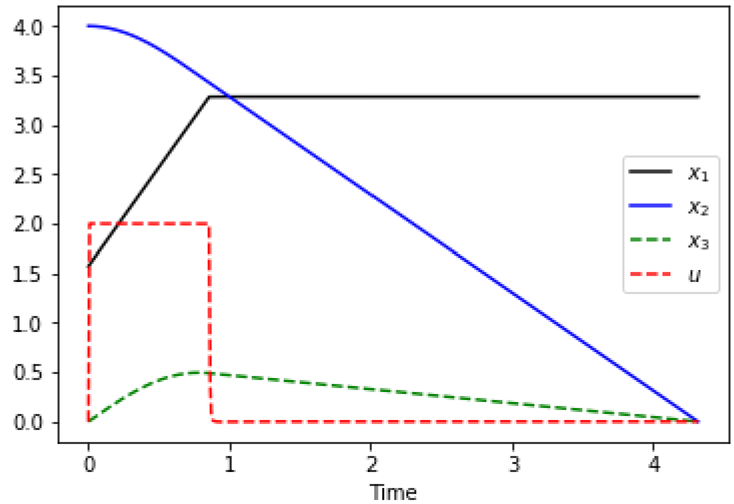
$x_3\left(t_f\right) = 0$

$-2 \leq u(t) \leq 2$

In [15]:
```
import numpy as np
from gekko import GEKKO
import matplotlib.pyplot as plt
%matplotlib inline
m = GEKKO() # initialize GEKKO
nt = 501
m.time = np.linspace(0,1,nt)
# Variables
x1 = m.Var(value=np.pi/2.0)
x2 = m.Var(value=4.0)
x3 = m.Var(value=0.0)
p = np.zeros(nt) # final time = 1
p[-1] = 1.0
final = m.Param(value=p)
# optimize final time
tf = m.FV(value=1.0,lb=0.1,ub=100.0)
tf.STATUS = 1
# control changes every time period
u = m.MV(value=0,lb=-2,ub=2)
u.STATUS = 1
m.Equation(x1.dt()==u*tf)
m.Equation(x2.dt()==m.cos(x1)*tf)
m.Equation(x3.dt()==m.sin(x1)*tf)
m.Equation(x2*final<=0)
m.Equation(x3*final<=0)
m.Obj(tf)
m.options.IMODE = 6
m.solve(disp=False)
print('Final Time: ' + str(tf.value[0]))
tm = np.linspace(0,tf.value[0],nt)
plt.figure(1)
plt.plot(tm,x1.value,'k-',label=r'$x_1$')
plt.plot(tm,x2.value,'b-',label=r'$x_2$')
plt.plot(tm,x3.value,'g--',label=r'$x_3$')
plt.plot(tm,u.value,'r--',label=r'$u$')
plt.legend(loc='best')
plt.xlabel('Time')
```

```
Final Time: 4.316256
```

```
Out[15]: <matplotlib.text.Text at 0x23e48557c18>
```



14: PID Control Tuning

A PID Controller (https://en.wikipedia.org/wiki/PID_controller) has proportional, integral, and derivative terms to determine the controller output (OP) based on the set point (SP) and process variable (PV). A standard PID form has constants K_c, τ_I, and τ_D.

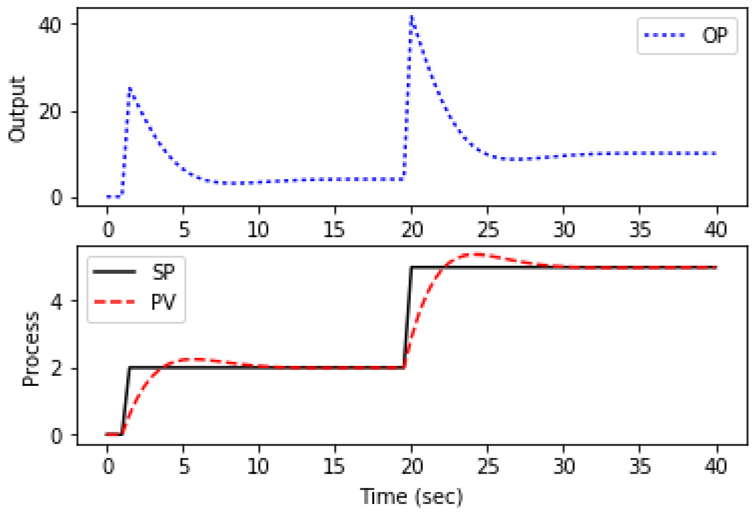
$$err = SP - PV$$

$$OP = OP_0 + K_c \, err + \frac{K_c}{\tau_I} \int err \, dt - K_c \, \tau_D \frac{d\,PV}{dt}$$

The effect of the tuning constants is shown with the PID Tuning Notebook (http://nbviewer.jupyter.org/url/apmonitor.com/pdc/uploads/Main/pid_widget.ipynb). This example is an alternative implementation in GEKKO.

In [16]:
```
from gekko import GEKKO
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline

m = GEKKO()
tf = 40
m.time = np.linspace(0,tf,2*tf+1)
step = np.zeros(2*tf+1)
step[3:40] = 2.0
step[40:]  = 5.0

# Controller model
Kc = 15.0                     # controller gain
tauI = 2.0                   # controller reset time
tauD = 1.0                   # derivative constant
OP_0 = m.Const(value=0.0)   # OP bias
OP = m.Var(value=0.0)       # controller output
PV = m.Var(value=0.0)       # process variable
SP = m.Param(value=step)    # set point
Intgl = m.Var(value=0.0)    # integral of the error
err = m.Intermediate(SP-PV) # set point error
m.Equation(Intgl.dt()==err) # integral of the error
m.Equation(OP == OP_0 + Kc*err + (Kc/tauI)*Intgl - PV.dt())

# Process model
Kp = 0.5                     # process gain
tauP = 10.0                  # process time constant
m.Equation(tauP*PV.dt() + PV == Kp*OP)

m.options.IMODE=4
m.solve(disp=False)

plt.figure()
plt.subplot(2,1,1)
plt.plot(m.time,OP.value,'b:',label='OP')
plt.ylabel('Output')
plt.legend()
plt.subplot(2,1,2)
plt.plot(m.time,SP.value,'k-',label='SP')
plt.plot(m.time,PV.value,'r--',label='PV')
plt.xlabel('Time (sec)')
plt.ylabel('Process')
plt.legend()
```

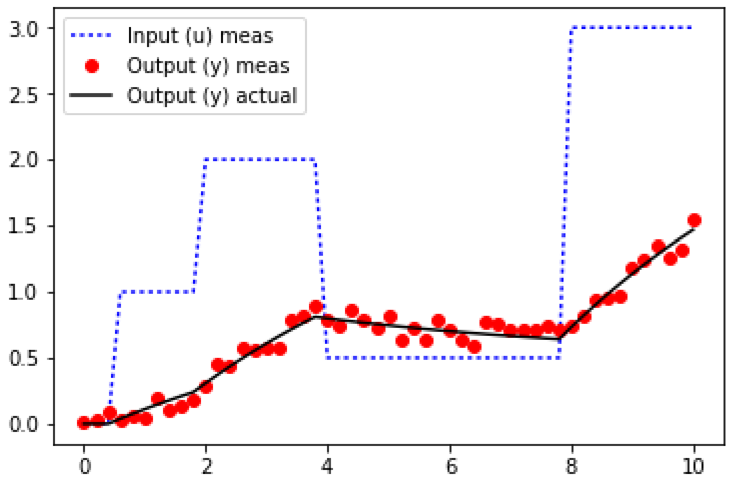
```
Out[16]: <matplotlib.legend.Legend at 0x23e4969b940>
```



15: Process Simulator

In [17]:
```
import numpy as np
from gekko import GEKKO
import matplotlib.pyplot as plt
%matplotlib inline

# Generate "data" with process simulation
nt = 51
# input steps
u_meas = np.zeros(nt)
u_meas[3:10] = 1.0
u_meas[10:20] = 2.0
u_meas[20:40] = 0.5
u_meas[40:] = 3.0
# simulation model
p = GEKKO()
p.time = np.linspace(0,10,nt)
n = 1 #process model order
# Parameters
steps = np.zeros(n)
p.u = p.MV(value=u_meas)
p.u.FSTATUS=1
p.K = p.Param(value=1) #gain
p.tau = p.Param(value=5) #time constant
# Intermediate
p.x = [p.Intermediate(p.u)]
# Variables
p.x.extend([p.Var() for _ in range(n)])  #state variables
p.y = p.SV() #measurement
# Equations
p.Equations([p.tau/n * p.x[i+1].dt() == -p.x[i+1] + p.x[i] for i in range(n)])
p.Equation(p.y == p.K * p.x[n])
# Simulate
p.options.IMODE = 4
p.solve(disp=False,remote=False)
# add measurement noise
y_meas = (np.random.rand(nt)-0.5)*0.2
for i in range(nt):
    y_meas[i] += p.y.value[i]
plt.plot(p.time,u_meas,'b:',label='Input (u) meas')
plt.plot(p.time,y_meas,'ro',label='Output (y) meas')
plt.plot(p.time,p.y.value,'k-',label='Output (y) actual')
plt.legend()
```

Out[17]: <matplotlib.legend.Legend at 0x23e49746978>



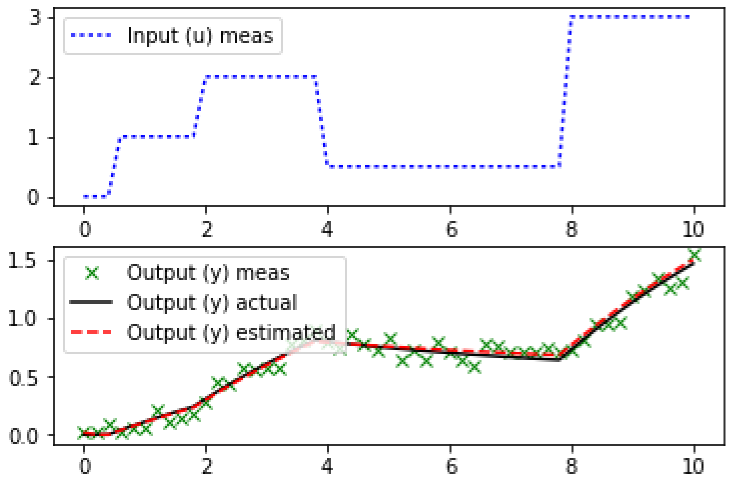
16: Moving Horizon Estimation

Run the Process Simulation cell above to generate the data. The MHE application uses a first order model while the process simulation is a second order system. This is done to emulate a realistic case with model mismatch and measurement noise.

This demonstrates just one cycle of an MHE application. Typical MHE applications receive an additional measurements, re-optimize parameters and states, and re-inject the parameters into a controller.

In [18]:
```
import numpy as np
from gekko import GEKKO
import matplotlib.pyplot as plt
%matplotlib inline

# Estimator Model
m = GEKKO()
m.time = p.time
# Parameters
m.u = m.MV(value=u_meas) #input
m.K = m.FV(value=1, lb=1, ub=3)    # gain
m.tau = m.FV(value=5, lb=1, ub=10) # time constant
# Variables
m.x = m.SV() #state variable
m.y = m.CV(value=y_meas) #measurement
# Equations
m.Equations([m.tau * m.x.dt() == -m.x + m.u,
             m.y == m.K * m.x])
# Options
m.options.IMODE = 5 #MHE
m.options.EV_TYPE = 1
# STATUS = 0, optimizer doesn't adjust value
# STATUS = 1, optimizer can adjust
m.u.STATUS = 0
m.K.STATUS = 1
m.tau.STATUS = 1
m.y.STATUS = 1
# FSTATUS = 0, no measurement
# FSTATUS = 1, measurement used to update model
m.u.FSTATUS = 1
m.K.FSTATUS = 0
m.tau.FSTATUS = 0
m.y.FSTATUS = 1
# DMAX = maximum movement each cycle
m.K.DMAX = 2.0
m.tau.DMAX = 4.0
# MEAS_GAP = dead-band for measurement / model mismatch
m.y.MEAS_GAP = 0.25

# solve
m.solve(disp=False)

# Plot results
plt.subplot(2,1,1)
plt.plot(m.time,u_meas,'b:',label='Input (u) meas')
plt.legend()
plt.subplot(2,1,2)
plt.plot(m.time,y_meas,'gx',label='Output (y) meas')
plt.plot(p.time,p.y.value,'k-',label='Output (y) actual')
plt.plot(m.time,m.y.value,'r--',label='Output (y) estimated')
plt.legend()
```

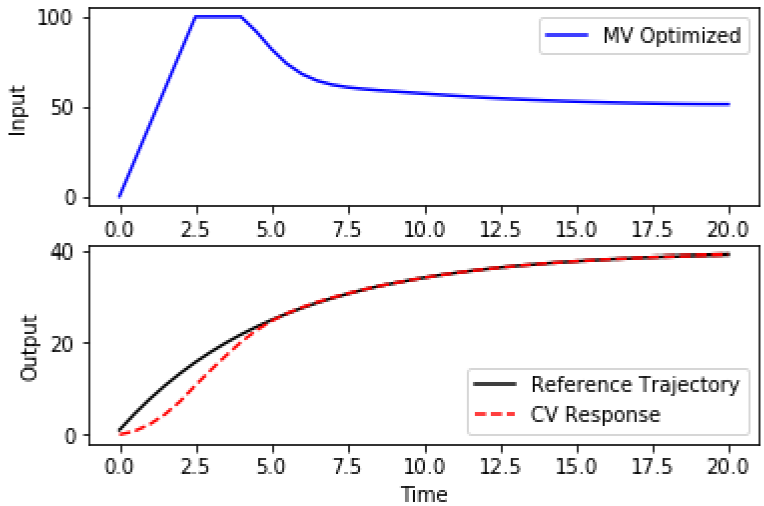
Out[18]: <matplotlib.legend.Legend at 0x23e49816588>



17: Model Predictive Control

In [19]:
```
import numpy as np
from random import random
from gekko import GEKKO
import matplotlib.pyplot as plt

m = GEKKO()
m.time = np.linspace(0,20,41)

# Parameters
mass = 500
b = m.Param(value=50)
K = m.Param(value=0.8)

# Manipulated variable
p = m.MV(value=0, lb=0, ub=100)
p.STATUS = 1  # allow optimizer to change
p.DCOST = 0.1 # smooth out gas pedal movement
p.DMAX = 20   # slow down change of gas pedal

# Controlled Variable
v = m.CV(value=0)
v.STATUS = 1  # add the SP to the objective
m.options.CV_TYPE = 2 # squared error
v.SP = 40     # set point
v.TR_INIT = 1 # set point trajectory
v.TAU = 5     # time constant of trajectory

# Process model
m.Equation(mass*v.dt() == -v*b + K*b*p)

m.options.IMODE = 6 # control
m.solve(disp=False)

# get additional solution information
import json
with open(m.path+'//results.json') as f:
    results = json.load(f)

plt.figure()
plt.subplot(2,1,1)
plt.plot(m.time,p.value,'b-',label='MV Optimized')
plt.legend()
plt.ylabel('Input')
plt.subplot(2,1,2)
plt.plot(m.time,results['v1.tr'],'k-',label='Reference Trajectory')
plt.plot(m.time,v.value,'r--',label='CV Response')
plt.ylabel('Output')
plt.xlabel('Time')
plt.legend(loc='best')
```

```
Out[19]: <matplotlib.legend.Legend at 0x23e499465c0>
```



18: Debugging Resources

In [20]:
```
from gekko import GEKKO

m = GEKKO()               # create GEKKO model

print(m.path)             # source file path

# test application
u = m.FV(value=5)         # define fixed value
x = m.SV()                # define state variable
m.Equation(x==u)          # define equation
m.options.DIAGLEVEL = 1   # diagnostic level (0-10)
m.options.SOLVER = 1      # change solver (1=APOPT,3=IPOPT)
m.options.MAX_ITER = 500  # adjust maximum iterations
m.options.SENSITIVITY = 1 # sensitivity analysis
m.solve(disp=True)        # solve locally (remote=False)
print('x: ' + str(x.value)) # print variable value
```

```
C:\Users\johnh\AppData\Local\Temp\tmpahyxbtw1gk_model18
apm 67.60.251.55_gk_model18 <br><pre> ------------------------------------
----------------------------
 APMonitor, Version 0.8.1
 APMonitor Optimization Suite
 ----------------------------------------------------------------


 Run id   : 2018y05m19d10h40m28.596s

 COMMAND LINE ARGUMENTS
 coldstart:           0
 imode    :           3
 dbs_read :  T
 dbs_write:  T
 specs    :  T

 rto selected
 Called files(          35 )
 READ info FILE FOR VARIABLE DEFINITION: 67.60.251.55_gk_model18.info
 SS MODEL INIT            0
 Parsing model file 67.60.251.55_gk_model18.apm
 Read model file (sec):  1.999999993131496E-003
 Initialize constants (sec):  0.000000000000000E+000
 Determine model size (sec):  4.000000044470653E-004
 Allocate memory (sec):  9.999999019782990E-005
 Parse and store model (sec):  4.000000044470653E-004

 --------- APM Model Size ------------
 Each time step contains
   Objects      :           0
   Constants    :           0
   Variables    :           2
   Intermediates:           0
   Connections  :           0
   Equations    :           1
   Residuals    :           1

 Error checking (sec):  1.000000047497451E-004
 Compile equations (sec):  9.999999019782990E-005
 Check for uninitialized intermediates (sec):  0.000000000000000E+000
 ------------------------------------------------------
 Total Parse Time (sec):  3.099999987171032E-003
 SS MODEL INIT            1
 SS MODEL INIT            2
 SS MODEL INIT            3
 SS MODEL INIT            4
 Called files(          31 )
 READ info FILE FOR PROBLEM DEFINITION: 67.60.251.55_gk_model18.info
 Called files(           6 )
 Files(6): File Read rto.t0 F
 files: rto.t0 does not exist
 Called files(          51 )
 Read DBS File defaults.dbs
 files: defaults.dbs does not exist
 Called files(          51 )
 Read DBS File 67.60.251.55_gk_model18.dbs
```

```
 files: 67.60.251.55_gk_model18.dbs does not exist
 Called files(          51 )
 Read DBS File measurements.dbs
 files: measurements.dbs does not exist
 Called files(          51 )
 Read DBS File overrides.dbs
 Number of state variables:              1
 Number of total equations: -            1
 Number of slack variables: -            0
 ---------------------------------------
 Degrees of freedom       :              0

 ----------------------------------------------
 Steady State Optimization with APOPT Solver
 ----------------------------------------------

 Iter    Objective  Convergence
    0  0.00000E+00  5.00000E+00
    1  0.00000E+00  5.00000E+00
 Successful solution

 ---------------------------------------------------
 Solver         :  APOPT (v1.0)
 Solution time  :   1.359999999112915E-002 sec
 Objective      :   0.000000000000000E+000
 Successful solution
 ---------------------------------------------------

 Called files(           2 )
 Called files(          52 )
 WRITE dbs FILE
 Called files(          56 )
 WRITE json FILE

 Generating Sensitivity Analysis
 Writing apm_sens_A_matrix.txt
 Writing sensitivity.txt
 Writing sensitivity.htm
Timer #     1       0.02/       1 =       0.02 Total system time
Timer #     2       0.01/       1 =       0.01 Total solve time
Timer #     3       0.00/       2 =       0.00 Objective Calc: apm_p
Timer #     4       0.00/       1 =       0.00 Objective Grad: apm_g
Timer #     5       0.00/       2 =       0.00 Constraint Calc: apm_c
Timer #     6       0.00/       0 =       0.00 Sparsity: apm_s
Timer #     7       0.00/       0 =       0.00 1st Deriv #1: apm_a1
Timer #     8       0.00/       1 =       0.00 1st Deriv #2: apm_a2
Timer #     9       0.00/       1 =       0.00 Custom Init: apm_custom_in
it
Timer #    10       0.00/       1 =       0.00 Mode: apm_node_res::case 0
Timer #    11       0.00/       1 =       0.00 Mode: apm_node_res::case 1
Timer #    12       0.00/       1 =       0.00 Mode: apm_node_res::case 2
Timer #    13       0.00/       1 =       0.00 Mode: apm_node_res::case 3
Timer #    14       0.00/       9 =       0.00 Mode: apm_node_res::case 4
Timer #    15       0.00/       4 =       0.00 Mode: apm_node_res::case 5
Timer #    16       0.00/       0 =       0.00 Mode: apm_node_res::case 6
Timer #    17       0.00/       2 =       0.00 Base 1st Deriv: apm_jacobi
an
```

```
Timer #    18       0.00/       1 =       0.00 Base 1st Deriv: apm_conden
sed_jacobian
Timer #    19       0.00/       2 =       0.00 Non-zeros: apm_nnz
Timer #    20       0.00/       0 =       0.00 Count: Division by zero
Timer #    21       0.00/       0 =       0.00 Count: Argument of LOG10 n
egative
Timer #    22       0.00/       0 =       0.00 Count: Argument of LOG neg
ative
Timer #    23       0.00/       0 =       0.00 Count: Argument of SQRT ne
gative
Timer #    24       0.00/       0 =       0.00 Count: Argument of ASIN il
legal
Timer #    25       0.00/       0 =       0.00 Count: Argument of ACOS il
legal
Timer #    26       0.00/       1 =       0.00 Extract sparsity: apm_spar
sity
Timer #    27       0.00/      17 =       0.00 Variable ordering: apm_var
_order
Timer #    28       0.00/       1 =       0.00 Condensed sparsity
Timer #    29       0.00/       0 =       0.00 Hessian Non-zeros
Timer #    30       0.00/       2 =       0.00 Differentials
Timer #    31       0.00/       0 =       0.00 Hessian Calculation
Timer #    32       0.00/       0 =       0.00 Extract Hessian
Timer #    33       0.00/       2 =       0.00 Base 1st Deriv: apm_jac_or
der
Timer #    34       0.01/       1 =       0.01 Solver Setup
Timer #    35       0.00/       1 =       0.00 Solver Solution
Timer #    36       0.00/      20 =       0.00 Number of Variables
Timer #    37       0.00/       9 =       0.00 Number of Equations
Timer #    38       0.01/      14 =       0.00 File Read/Write
Timer #    39       0.00/       0 =       0.00 Dynamic Init A
Timer #    40       0.00/       0 =       0.00 Dynamic Init B
Timer #    41       0.00/       0 =       0.00 Dynamic Init C
Timer #    42       0.00/       1 =       0.00 Init: Read APM File
Timer #    43       0.00/       1 =       0.00 Init: Parse Constants
Timer #    44       0.00/       1 =       0.00 Init: Model Sizing
Timer #    45       0.00/       1 =       0.00 Init: Allocate Memory
Timer #    46       0.00/       1 =       0.00 Init: Parse Model
Timer #    47       0.00/       1 =       0.00 Init: Check for Duplicates
Timer #    48       0.00/       1 =       0.00 Init: Compile Equations
Timer #    49       0.00/       1 =       0.00 Init: Check Uninitialized
Timer #    50       0.00/       3 =       0.00 Evaluate Expression Once
Timer #    51       0.00/       1 =       0.00 Sensitivity Analysis: LU F
actorization
Timer #    52       0.00/       1 =       0.00 Sensitivity Analysis: Gaus
s Elimination
Timer #    53       0.00/       1 =       0.00 Sensitivity Analysis: Tota
l Time
x: [5.0]
```