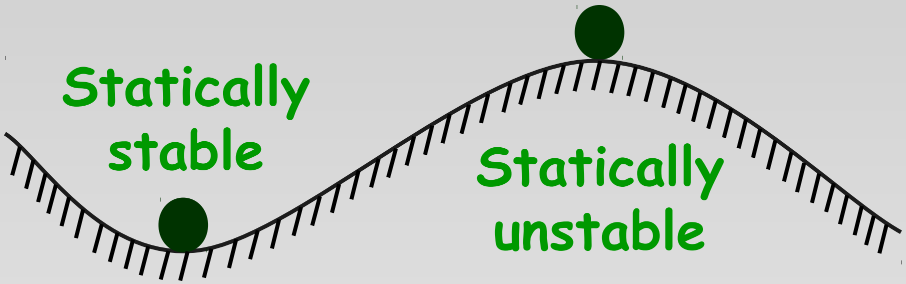
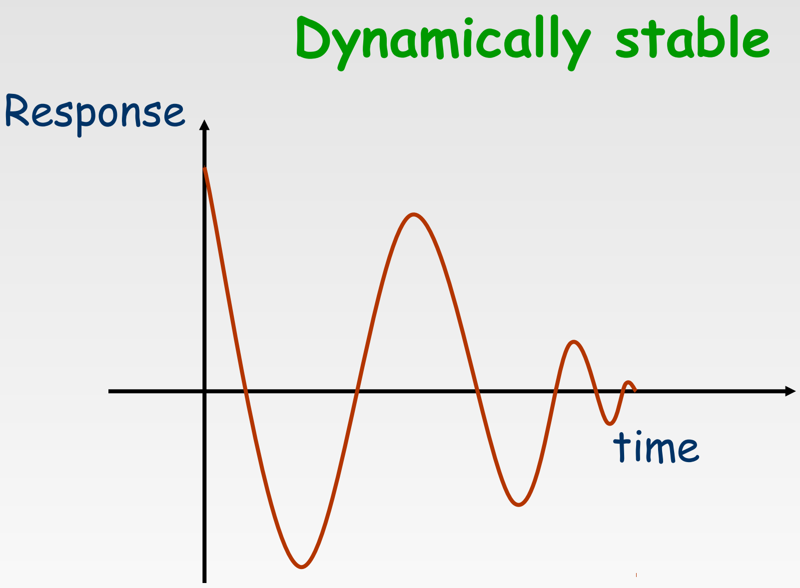
Stability and Control analysis

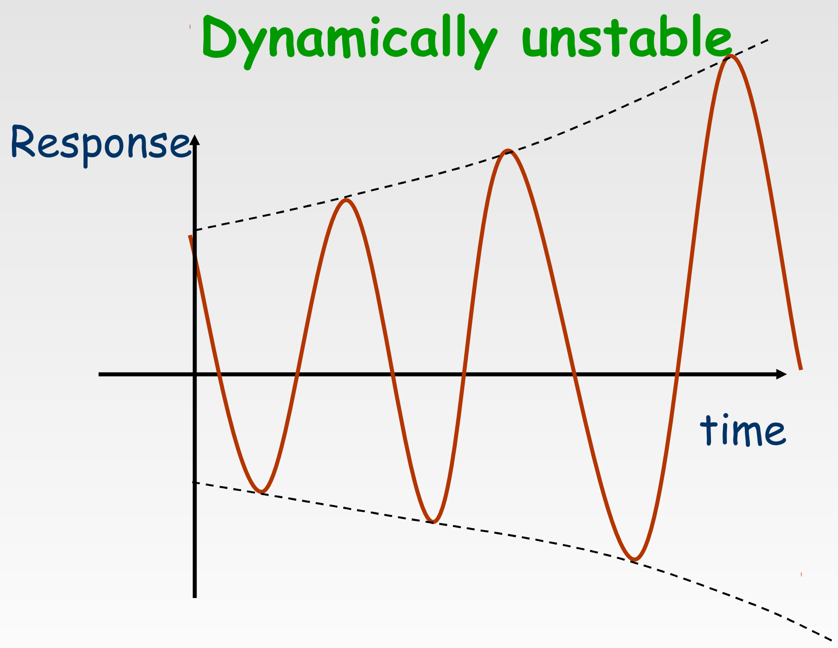
What it's all about

- Our model aircraft needs to be adjusted for performance, but needs also to be stable and controllable.
  - Stability analysis is a characteristic of "hands-off controls" flight
  - Control analysis measures the plane's reactions to pilot instruction
- To some extent, this can be addressed by simulation
- An option has been added in XFLR5 v6 for this purpose

Static and Dynamic stability



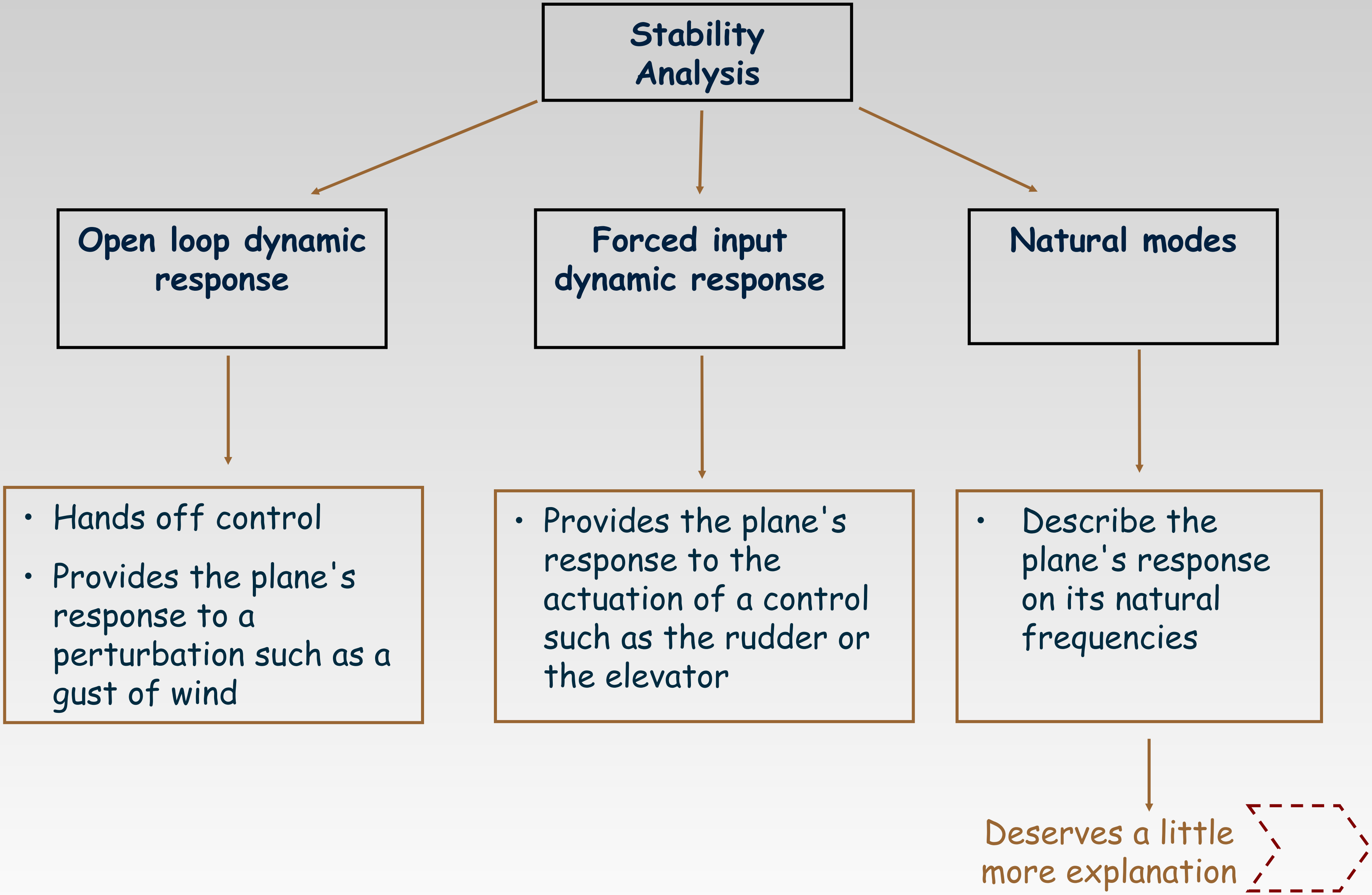




Sailplane stability

- A steady "static" state for a plane would be defined as a constant speed, angle of attack, bank angle, heading angle, altitude, etc.
- Difficult to imagine
- A gust of wind, an input from the pilot will inevitably disturb the plane
- The purpose of Stability and Control analysis is to evaluate the dynamic stability and time response of the plane for such a perturbation
- In the following slides, we refer only to dynamic stability

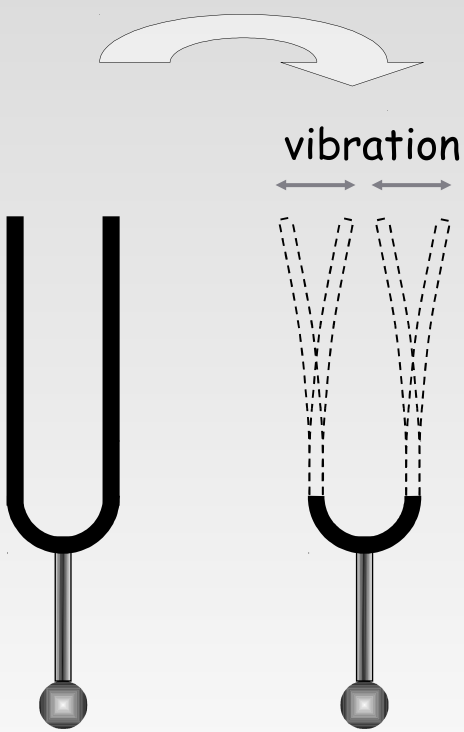
One analysis, three output

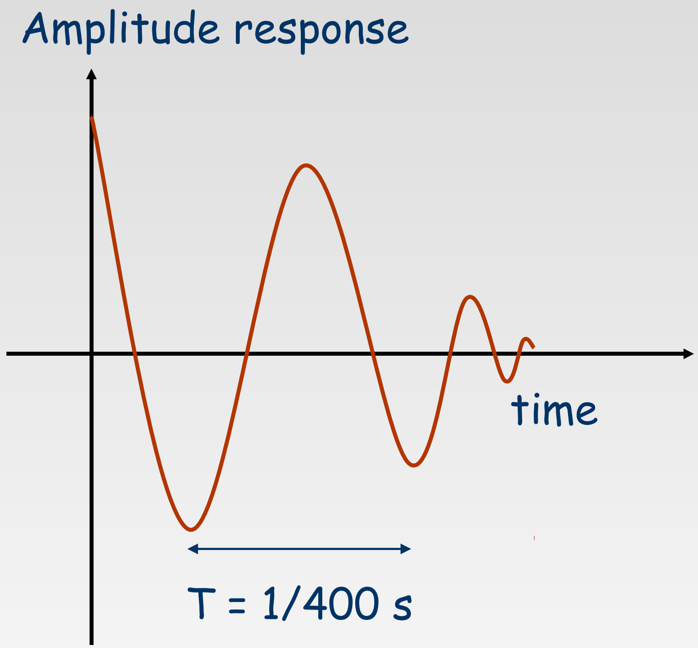


Natural modes - Mechanical

- **Example of the tuning fork**

Shock perturbation
→ preferred response on A note
= 440 Hz

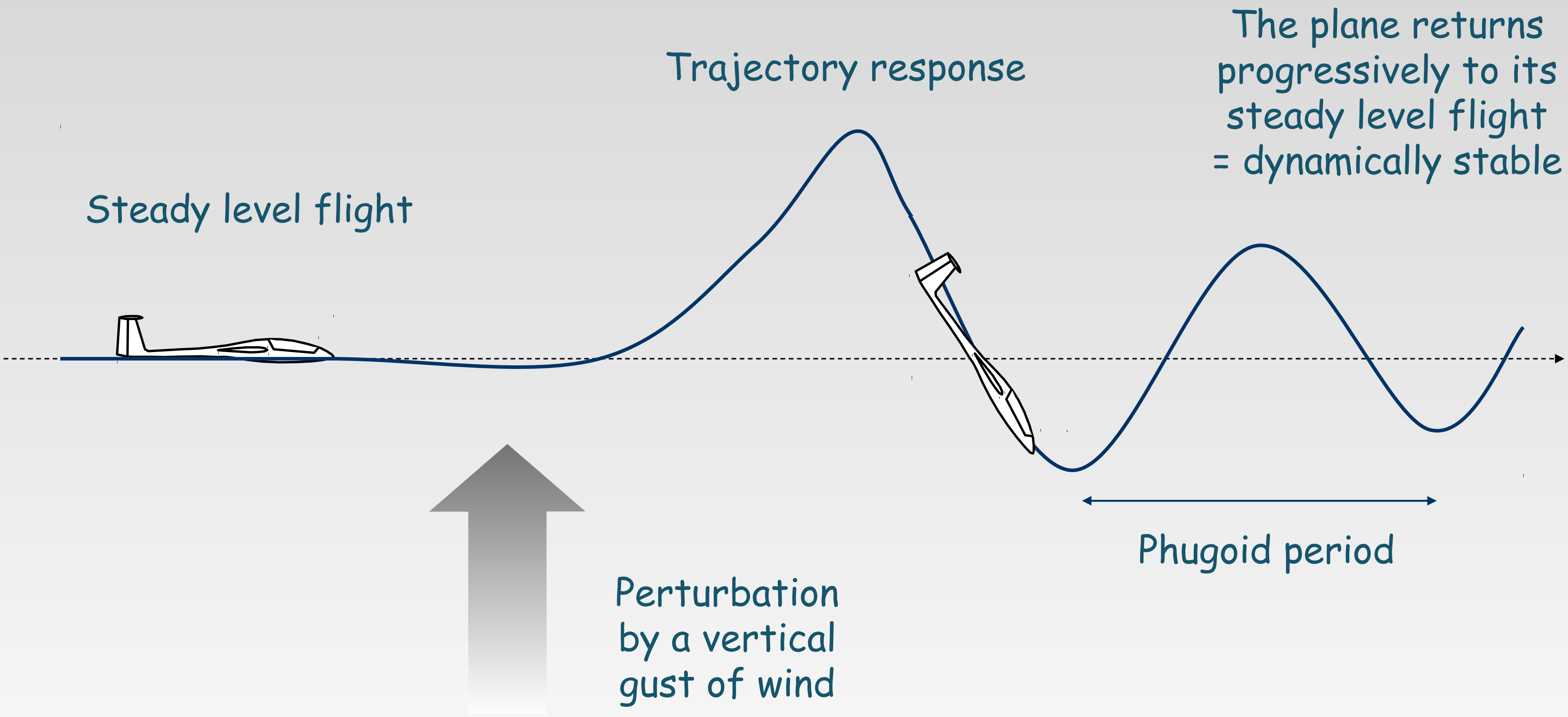




The sound decays with time
The fork is dynamically stable… not really a surprise

Natural modes - Aerodynamic

- **Example of the phugoid mode**



The 8 aerodynamic modes

- A well designed plane will have 4 natural longitudinal modes and 4 natural lateral modes

Longitudinal

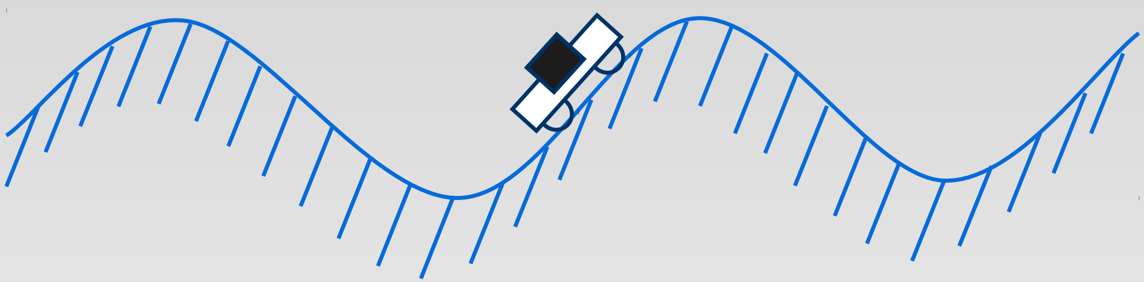
2 symmetric phugoid modes
2 symmetric short period modes

Lateral

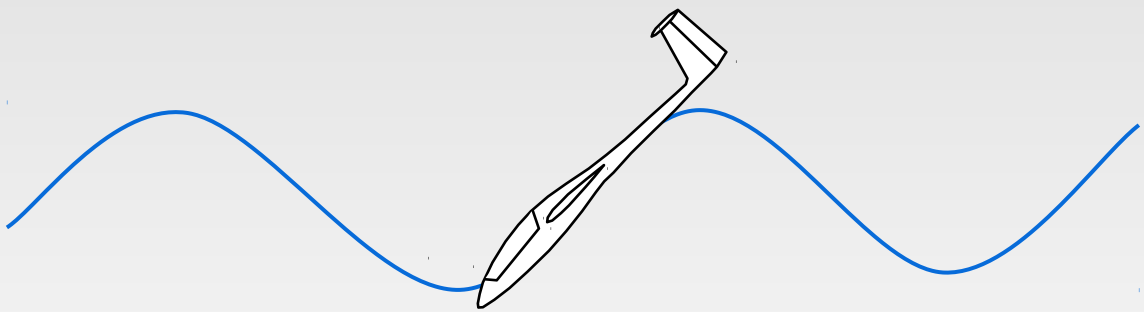
1 spiral mode
1 roll damping mode
2 Dutch roll modes

The phugoid

… is a macroscopic mode of exchange between the Kinetic and Potential energies



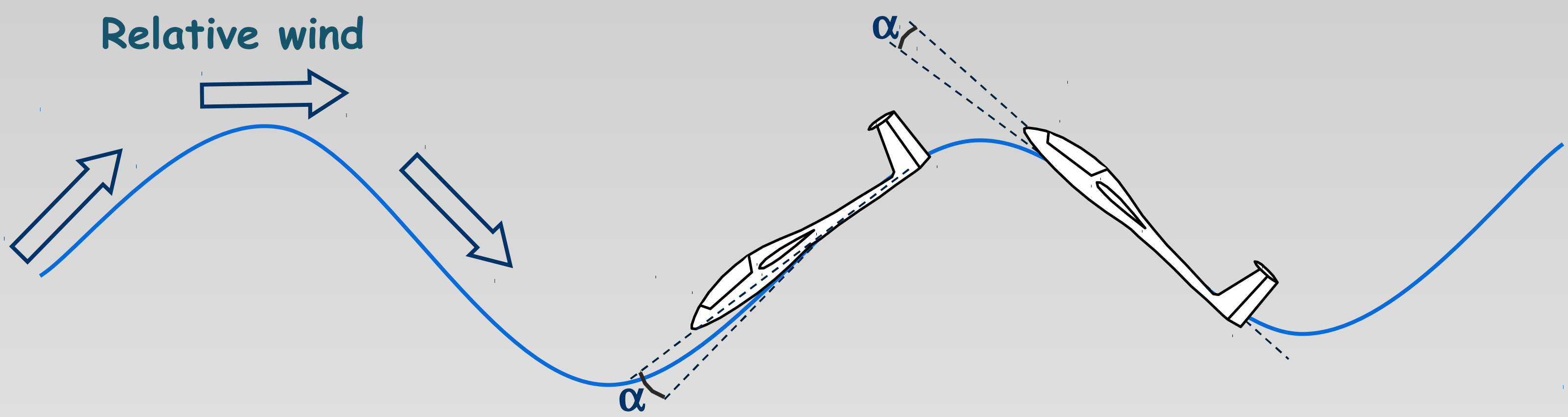
Russian Mountains :
Exchange is made by the contact force



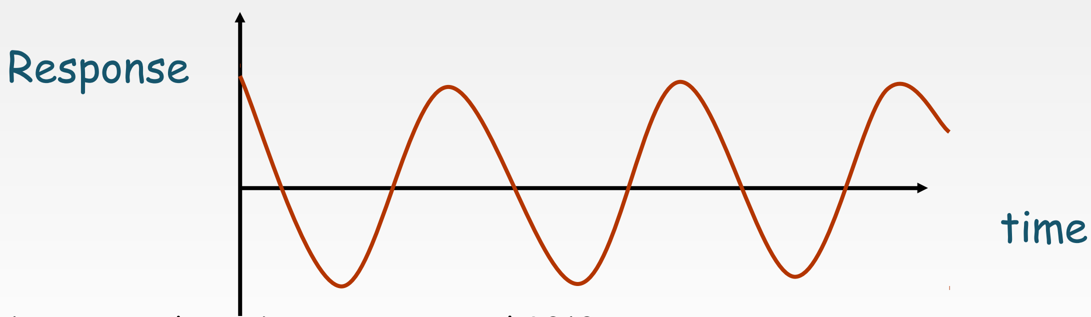
Aerodynamic :
Exchange is made by the lift force

Slow, lightly damped, stable or unstable

The mechanism of the phugoid

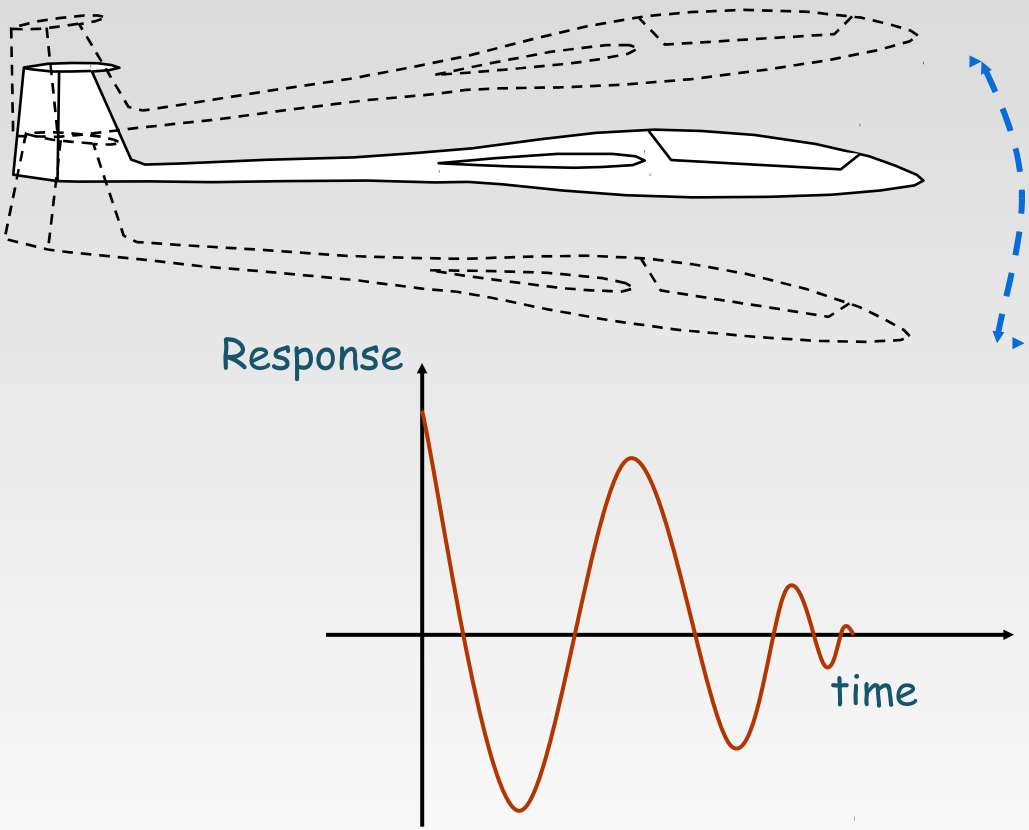


Dive → The sailplane accelerates → α Constant → Cl constant → At iso-Cl, The lift increases as the square power of the speed $L = \frac{1}{2} \rho S V^2 Cl$ → The sailplane rises and slows down → The lift decreases → Dive



The short period mode

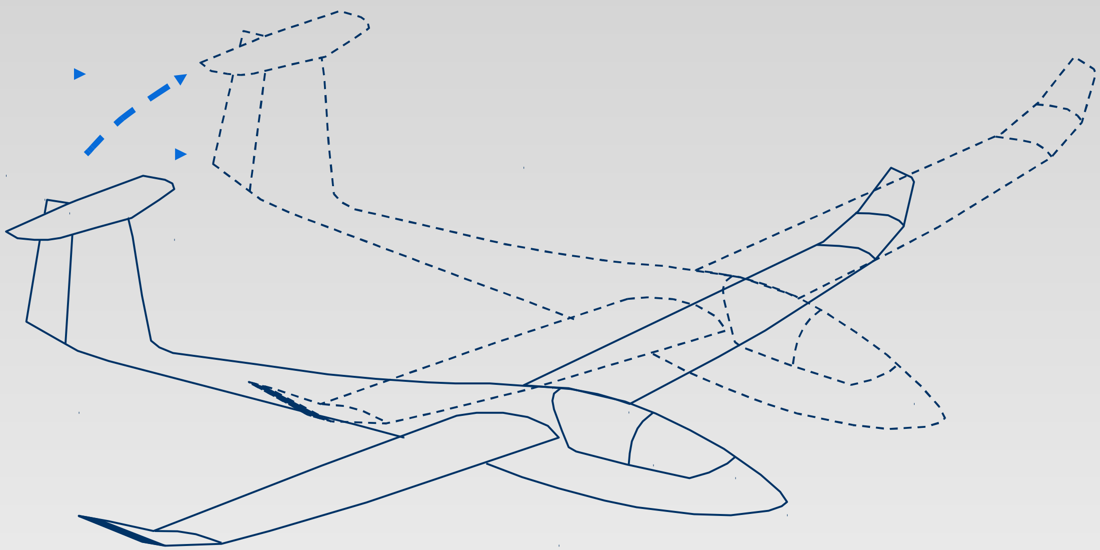
- Primarily vertical movement and pitch rate in the same phase, usually high frequency, well damped



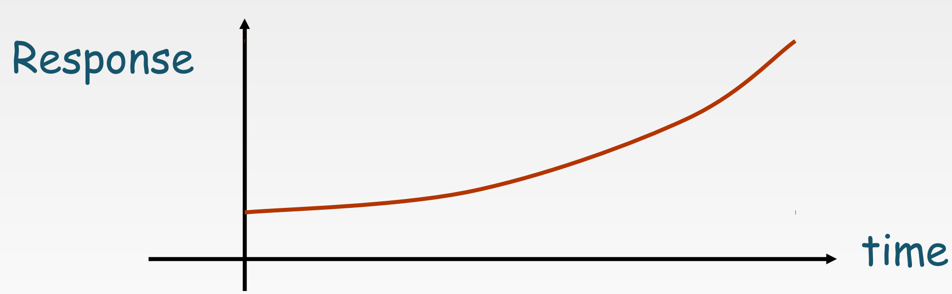
The mode's properties are driven by the stiffness of the negative slope of the curve $Cm=f(\alpha)$

Spiral mode

- Primarily heading, non-oscillatory, slow, generally unstable



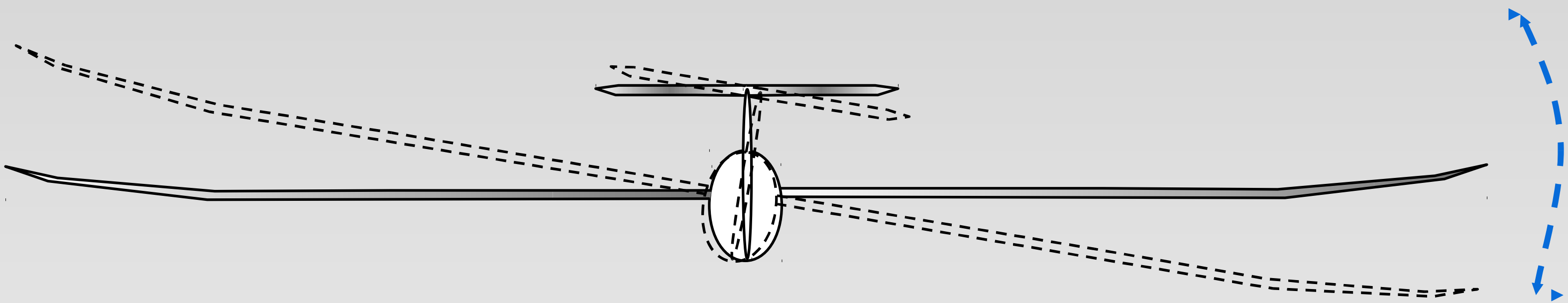
The mode is initiated by a rolling or heading disturbance
This creates a positive a.o.a. on the fin, which tends to increase the yawing moment



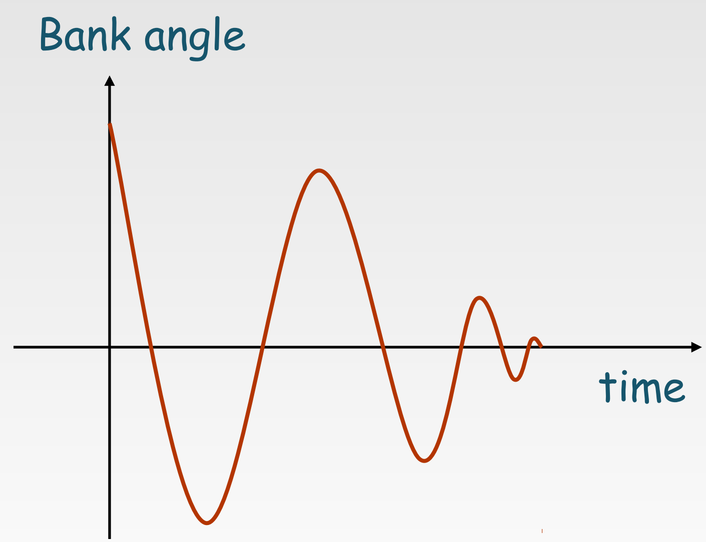
Requires pilot input to prevent divergence !

Roll damping

- **Primarily roll, stable**

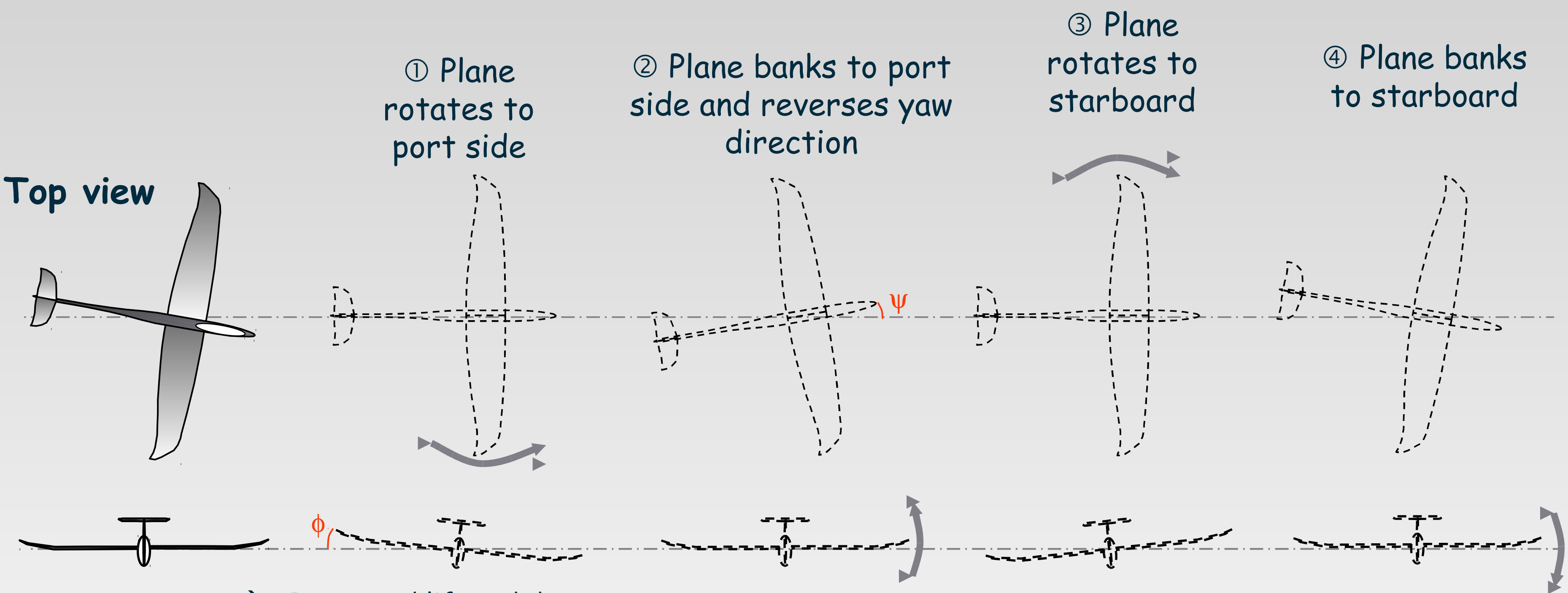


1. Due to the rotation about the x-axis, the wing coming down sees an increased a.o.a., thus increasing the lift on that side. The symmetric effect decreases the lift on the other side.
2. This creates a restoring moment opposite to the rotation, which tends to damp the mode



Dutch roll

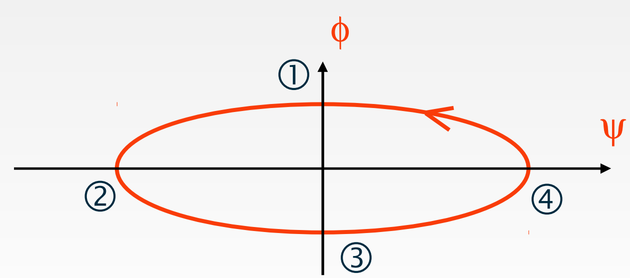
- The Dutch roll mode is a combination of yaw and roll, phased at 90°, usually lightly damped



→ Increased lift and drag on starboard side, decreased lift and drag on port side

→ The lift difference creates a bank moment to port side

→ The drag difference creates a yawing moment to starboard

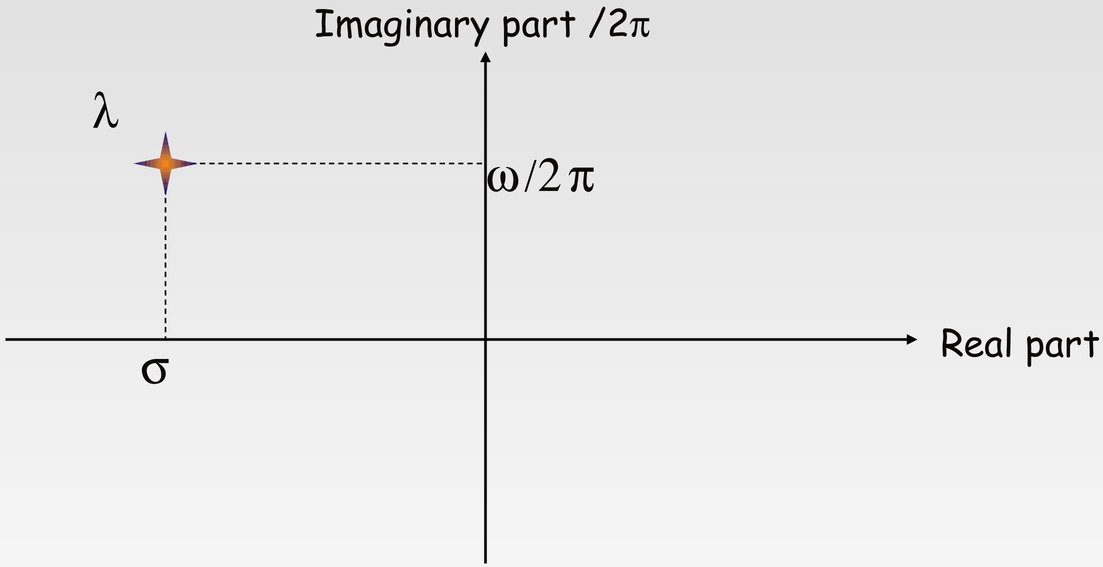


Pre-requisites for the analysis

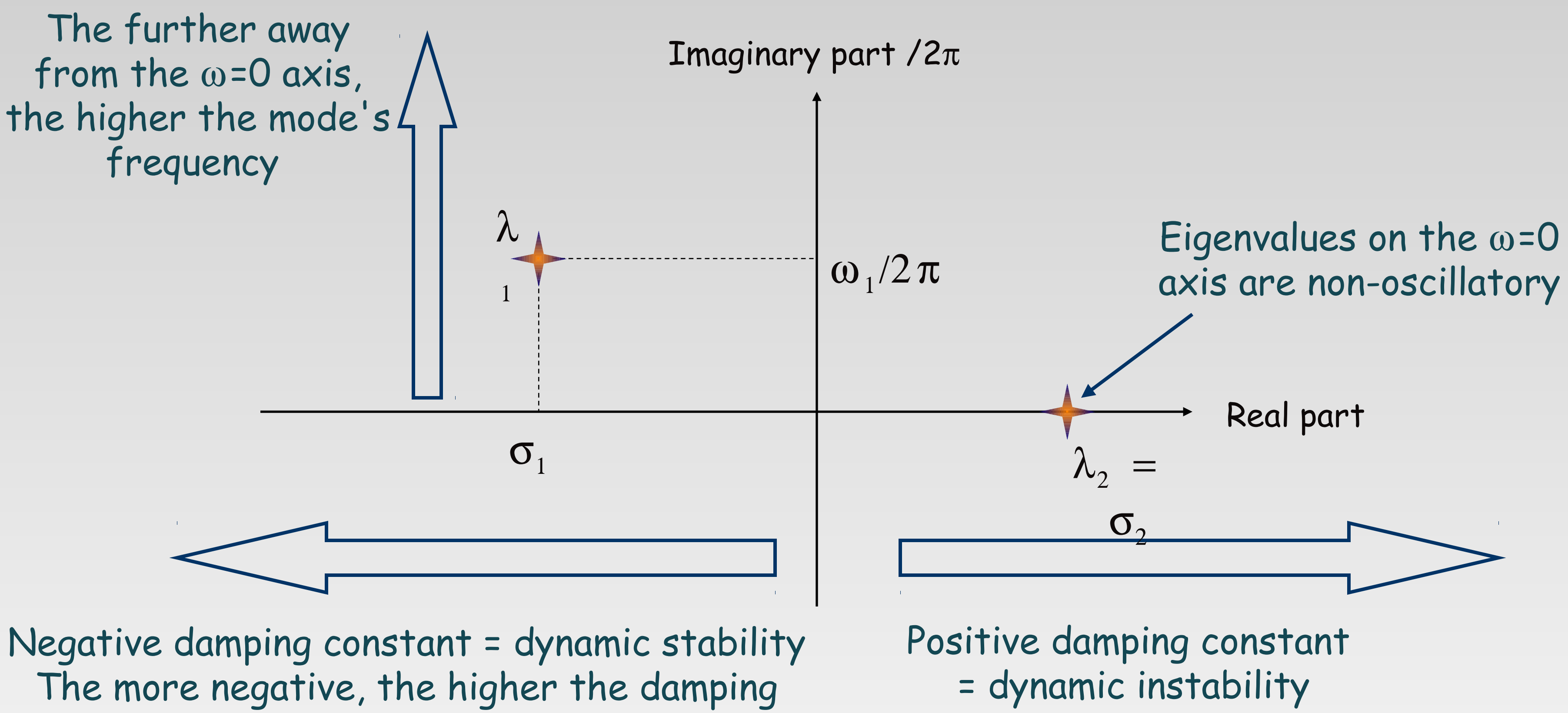
- The stability and control behavior analysis requires that the inertia properties have been defined
- The evaluation of the inertia requires a full 3D CAD program
- Failing that, the inertia can be evaluated approximately in XFLR5 by providing
  - The mass of each wing and of the fuselage structure
  - The mass and location of such objects as nose lead, battery, receiver, servo-actuators, etc.
- XFLR5 will evaluate roughly the inertia based on these masses and on the geometry
- Once the data has been filled in, it is important to check that the total mass and CoG position are correct

The root locus view

- This graphic view provides a visual interpretation of the frequency and damping of a mode with eigenvalue $\lambda = \sigma + i\omega$
- The time response of a mode component such as u,w, or q, is $f(t) = k.e^{\lambda t} = ke^{(\sigma + i\omega)t}$
- σ is the damping constant, ω is the natural circular frequency and $\omega/2\pi$ is the mode's natural frequency
- The eigenvalue is plotted in the $(\sigma,\ \omega/2\pi)$ axis, i.e. the root locus graph

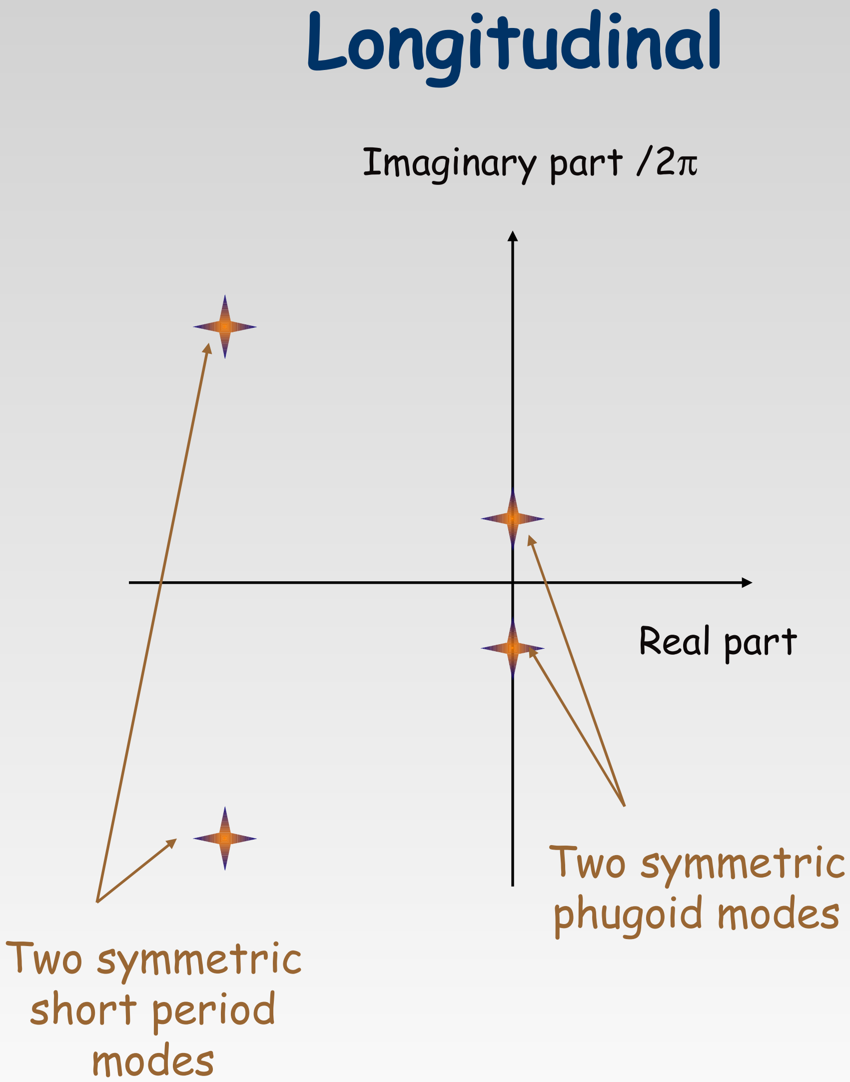


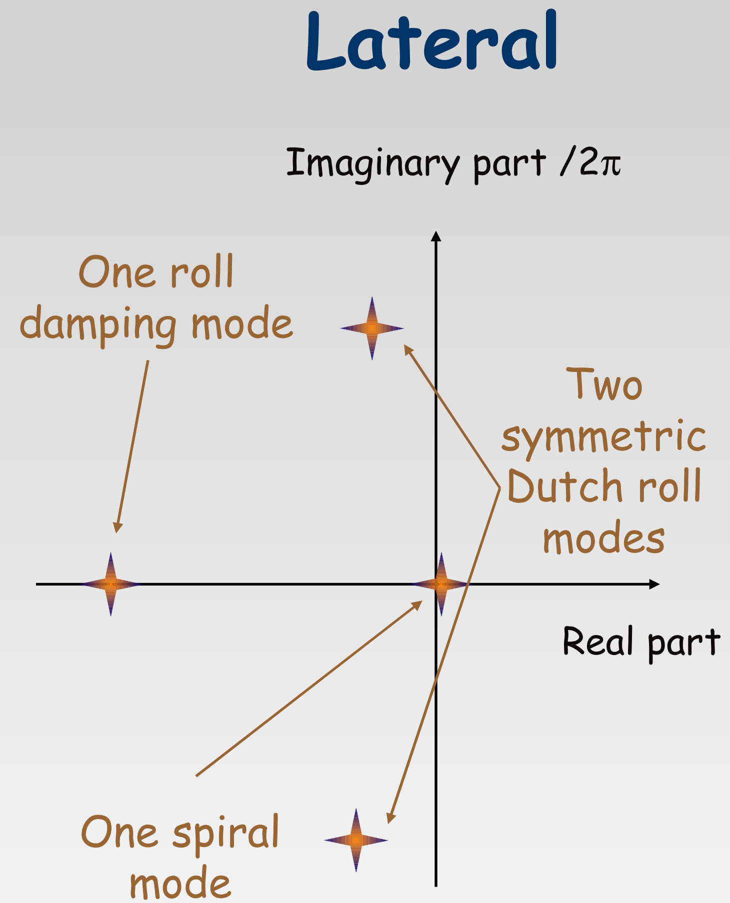
The root locus interpretation



- **λ_1 corresponds to a damped oscillatory mode**
- **λ_2 corresponds to an un-damped, non-oscillatory mode**

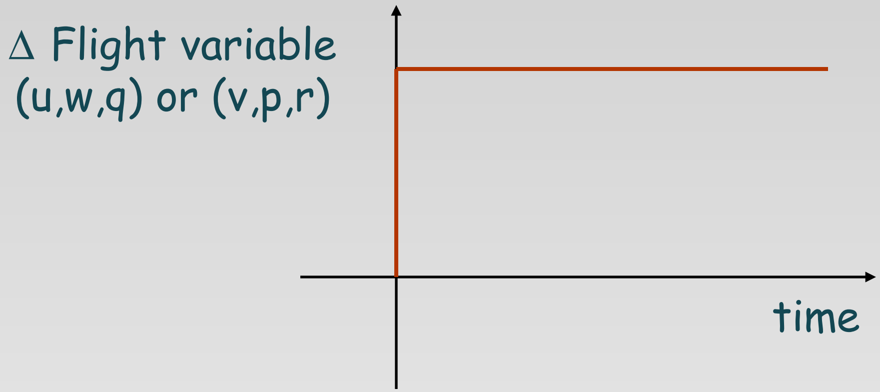
The typical root locus plots



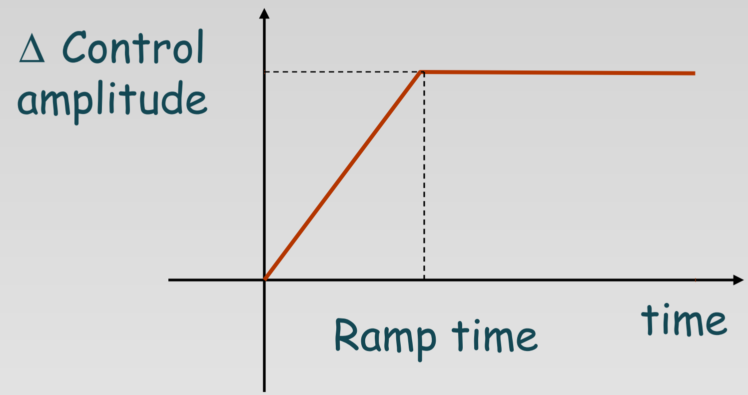


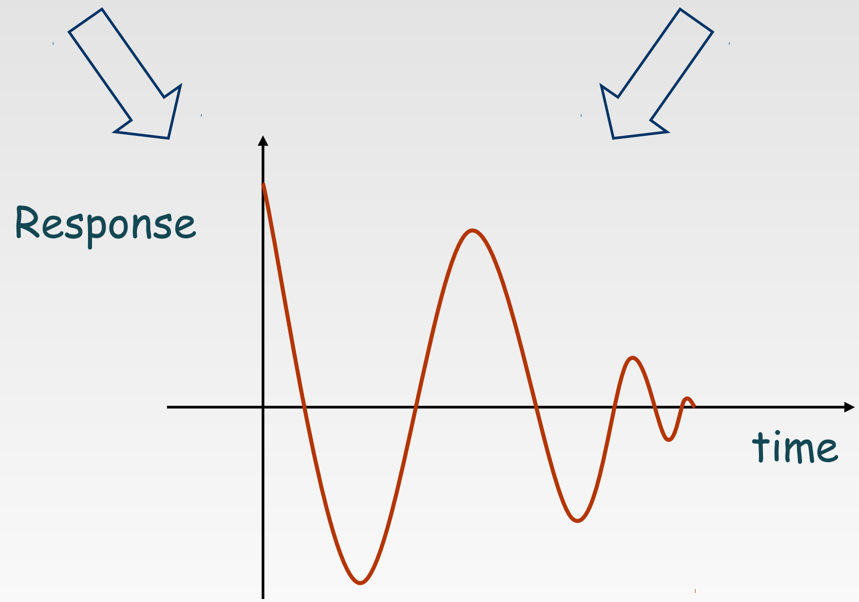
The time response view : two type of input

① Perturbation



② Control actuation





The 3D mode animation

- The best way to identify and understand a mode shape ?
- Note :
  - The apparent amplitude of the mode in the animation has no physical significance.
  - A specific mode is never excited alone in flight – the response is always a combination of modes.

Example of Longitudinal Dynamics analysis

First approximation for the "Short Period Mode"

$$F_1 = \frac{1}{2\pi}\sqrt{\frac{K}{J}}$$

1. The coefficient K can be evaluated with XFLR5 :

$$K = \frac{1}{2}\rho V^2 \, S_{wing} \; M.A.C.\frac{\partial C_m}{\partial \alpha}$$

Negative slope of the curve
Cm = f(α)

2. J is the inertia for pitching: J = I_{xx}

3. This assumes that the pitching behaviour is independent of the vertical velocity… not necessarily a valid assumption
→ need for a more sophisticate approximation

Second approximation for the Short Period Mode

- **Taking into account the dependency to the vertical velocity leads to a more complicated expression**

$$t^{\text{¿}} = \frac{MAC}{2u_0} \qquad \hat{I}_y = \frac{8I_y}{\rho . S . MAC^3} \qquad \mu = \frac{2m}{\rho . S . MAC}$$

u_0 = horizontal speed

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \qquad C_{z_\alpha} = \frac{\partial C_z}{\partial \alpha}$$

$C_{m\alpha}$ and $C_{z\alpha}$ are the slopes of the curves Cm = f(α) and Cz = f(α). The slopes can be measured on the polar graphs in XFLR5

$$B = \frac{C_{z_\alpha}}{2t^{\text{¿}}\mu} \qquad C = -\frac{C_{m_\alpha}}{t^{\text{¿}2}\hat{I}_y}$$

$$F_2 = \frac{1}{2\pi}\sqrt{-B^2 + 4C}$$

Despite their complicated appearance, these formula can be implemented in a spreadsheet, with all the input values provided by XFLR5

Lanchester's approximation for the Phugoid

- **The phugoid's frequency is deduced from the balance of kinetic and potential energies, and is calculated with a very simple formula**

$$F_{ph} = \frac{1}{\pi\sqrt{2}}\frac{g}{u_0}$$

g is the gravitational constant, i.e. *g* = 9.81 m/s

Numerical example – from a personal model sailplane

- Plane and flight Data

MAC =	0.1520	m²
Mass =	0.5250	kg
Iyy =	0.0346	kg.m²
S =	0.2070	m²
ρ =	1.225	kg/m3

u0 =	16.20	m/s
α =	1.05	°
q =	160.74	Pa

Cx =	0.0114	
Cz =	0.1540	
dCm/dα =	-1.9099	
dCz/dα =	-5.3925	

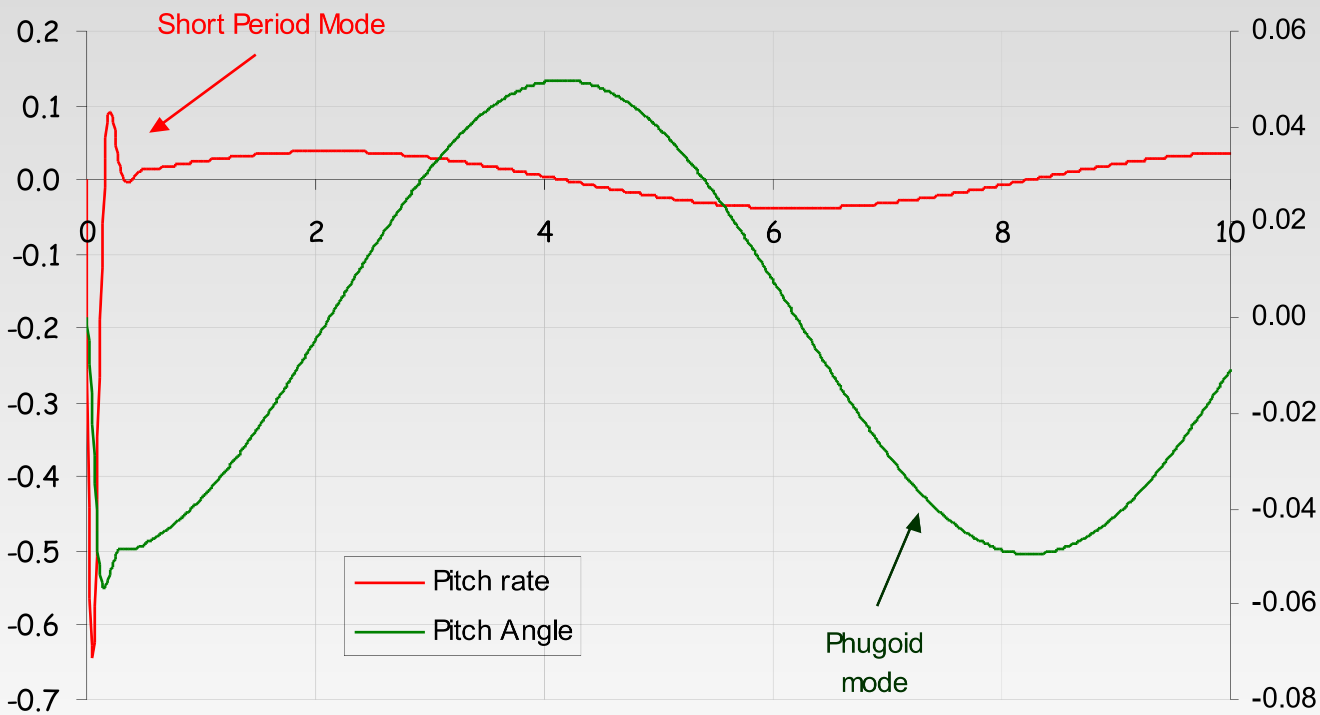
- Results

	Short Period			Phugoid	
	F1	F2	XFLR5 v6	Fph	XFLR5 v6
Frequency (Hz) =	4.45	4.12	3.86	0.136	0.122
Period (s) =	0.225	0.243	0.259	7.3	8.2

Graphic Analysis →

Analysis

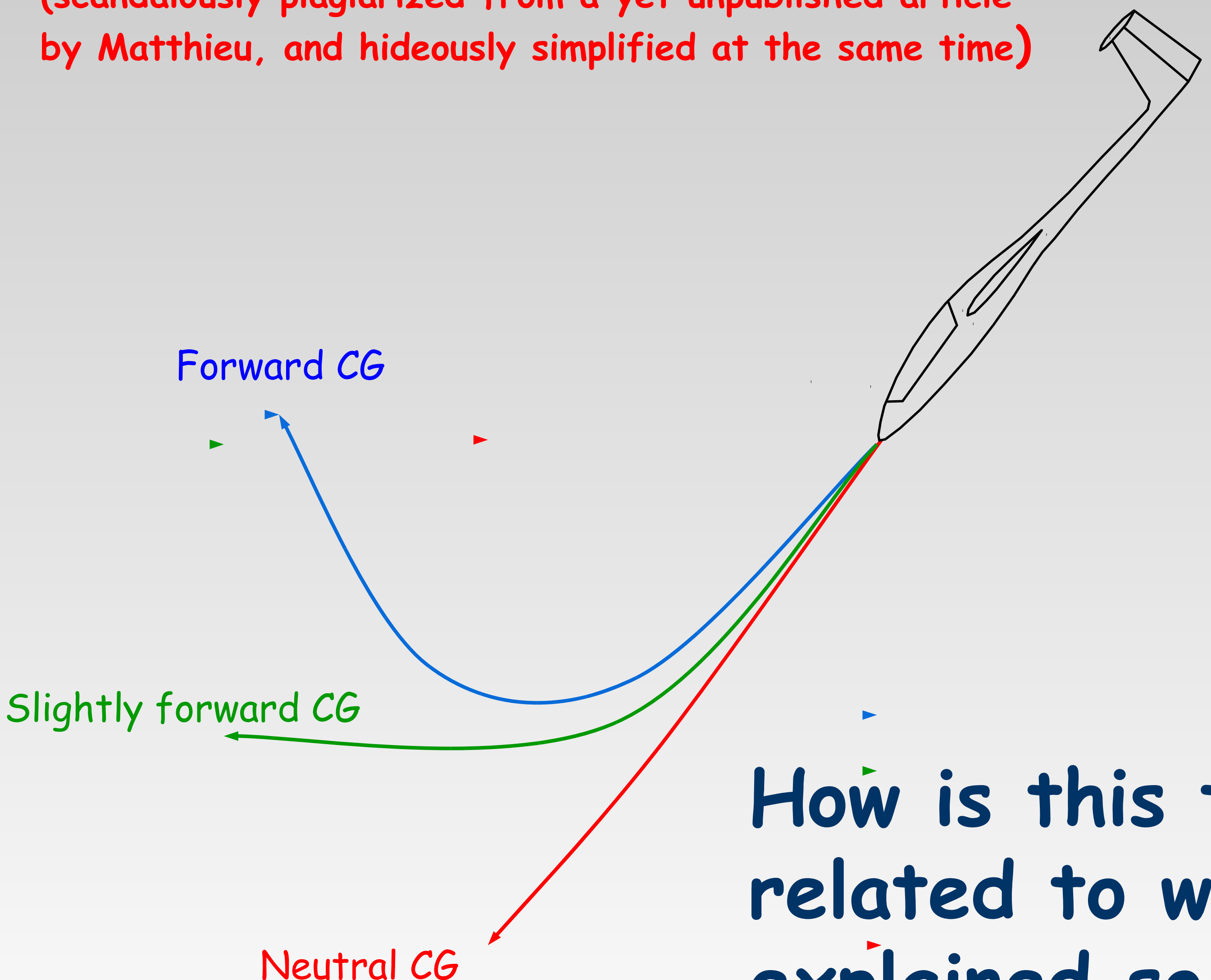
- There is factor 10x between the numerical frequencies of both modes, which means the plane should be more than stable
- A time response analysis confirms that the two modes do not interact



About the Dive Test

About the dive test

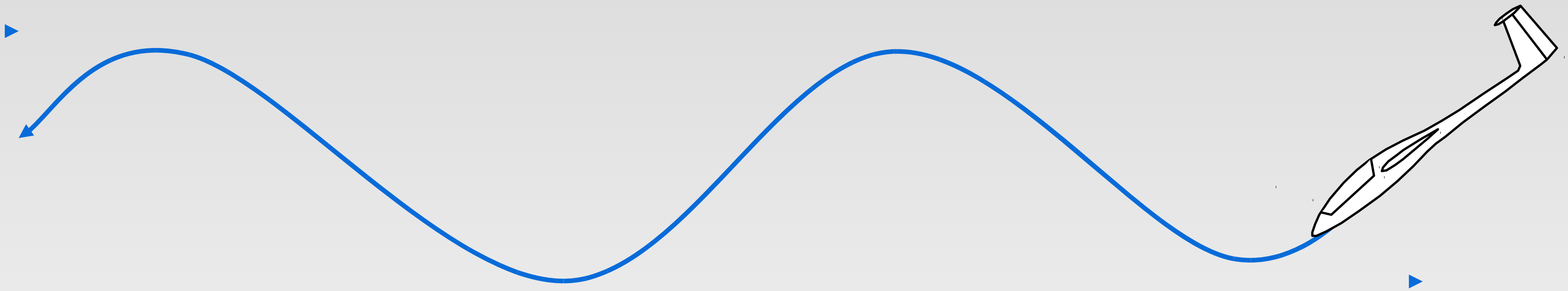
(scandalously plagiarized from a yet unpublished article by Matthieu, and hideously simplified at the same time)



How is this test related to what's been explained so far?

Forward CG

- If the CG is positioned forward, the plane will enter the phugoid mode

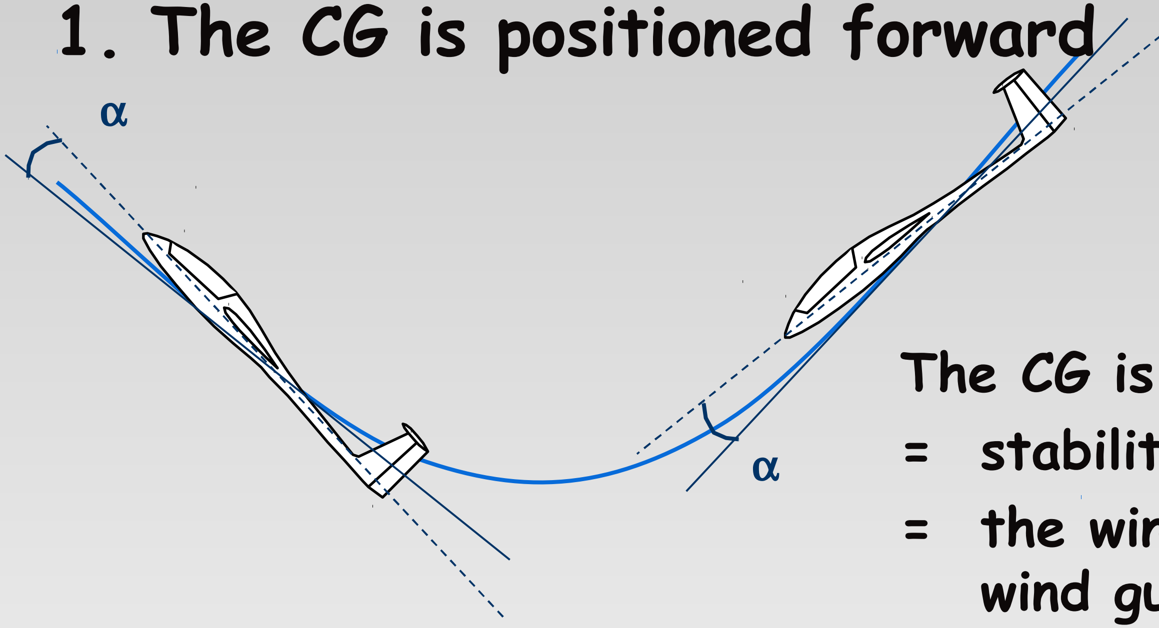


Stick to the phugoid

- **As the plane moves along the phugoid, the apparent wind changes direction**
- **From the plane's point of view, it's a perturbation**
- **The plane can react and reorient itself along the trajectory direction, providing**
  - That the slope of the curve $Cm = f(\alpha)$ is stiff enough
  - That it doesn't have too much pitching inertia

Summarizing :

1. The CG is positioned forward



The CG is positioned forward

= stability

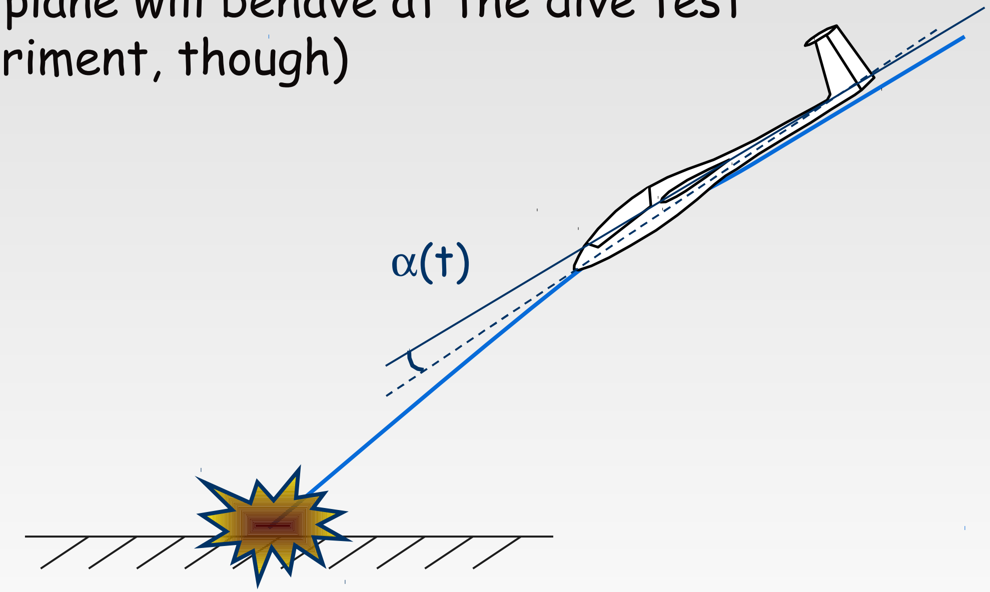
= the wind vane which follows the wind gusts

- The two modes are un-coupled
- The relative wind changes direction along the phugoid…
- … but the plane maintains a constant incidence along the phugoid, just as the chariot remains tangent to the slope
- The sailplane enters the phugoid mode

2. The CG is positioned aft

Remember that backward CG = instability = the wind vane which amplifies wind gusts

- The two modes are coupled
- The incidence oscillation $\alpha(t)$ amplifies the phugoid,
- The lift coefficient is not constant during the phugoid
- The former loop doesn't work any more
- The phugoid mode disappears
- No guessing how the sailplane will behave at the dive test
  (It's fairly easy to experiment, though)



That's all for now

Good design and nice flights ☺