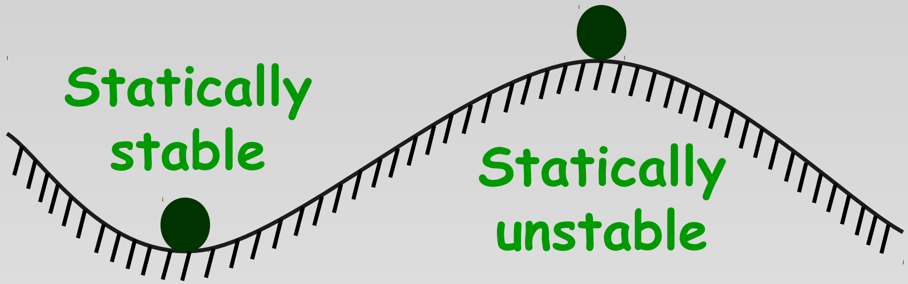
Stability and Control analysis

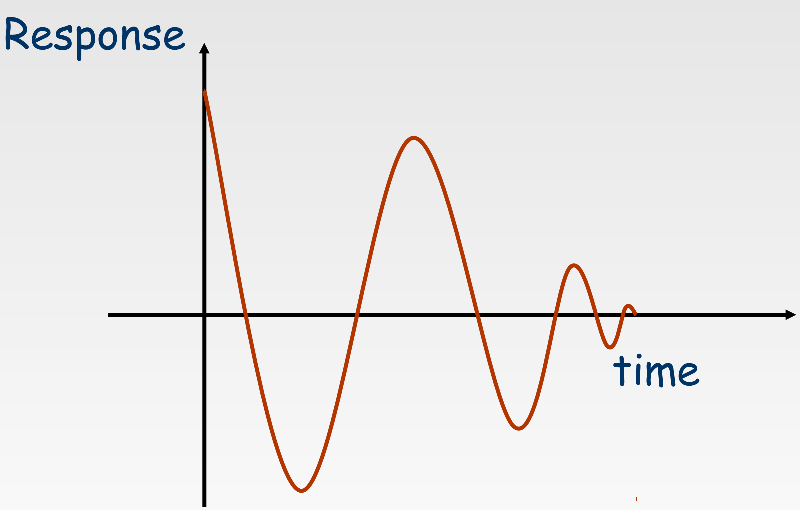
What it's all about

- Our model aircraft needs to be adjusted for performance, but needs also to be stable and controllable.
  - Stability analysis is a characteristic of "hands-off controls" flight
  - Control analysis measures the plane's reactions to the pilot's instruction
- To some extent, this can be addressed by simulation
- An option has been added in XFLR5 v6 for this purpose

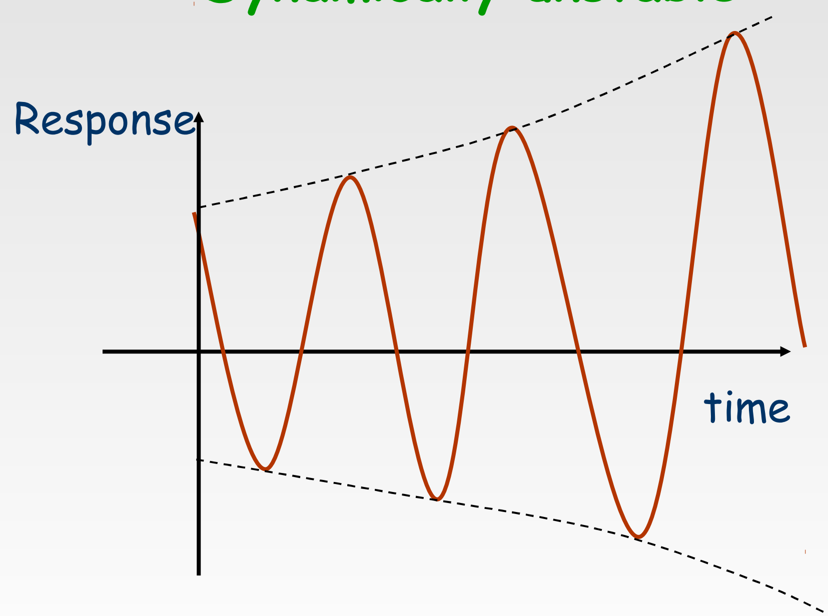
Static and Dynamic stability



Dynamically stable



Dynamically unstable



Sailplane stability

- A steady "static" state for a plane would be defined as a constant speed, angle of attack, bank angle, heading angle, altitude, etc.
- Difficult to imagine
- Inevitably, a gust of wind, an input from the pilot will disturb the plane
- The purpose of Stability and Control Analysis is to evaluate the dynamic stability and time response of the plane for such a perturbation
- In the following slides, we refer only to dynamic stability

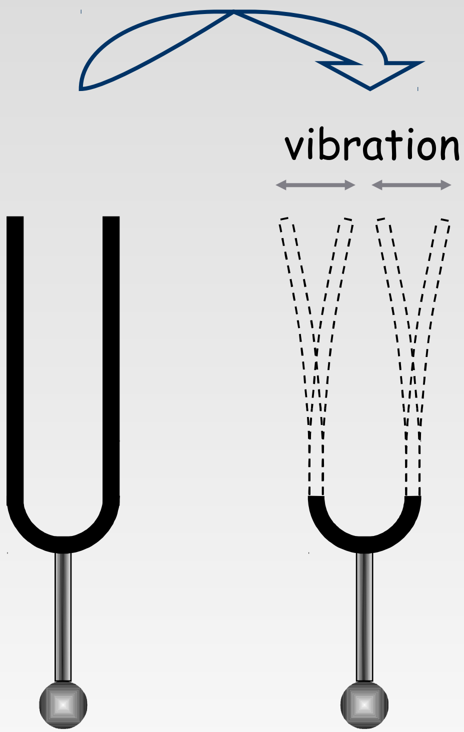
Natural modes

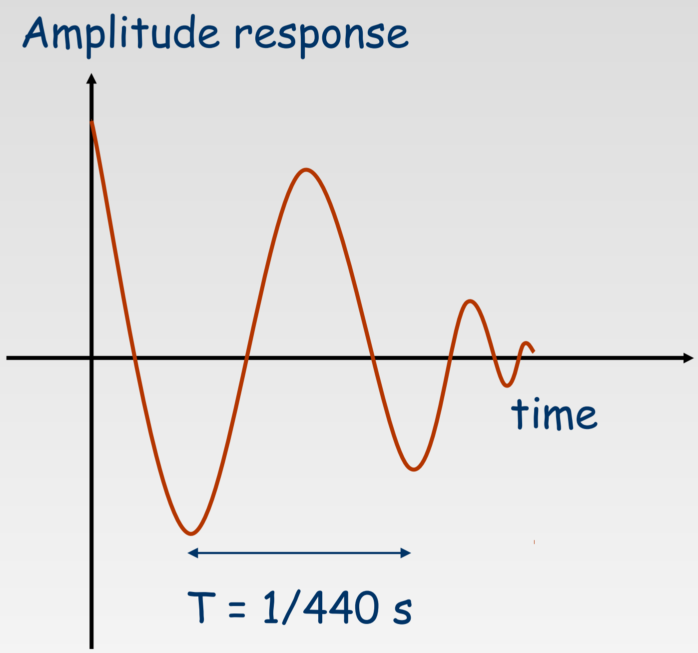
- Physically speaking, when submitted to a perturbation, a plane tends to respond on "preferred" flight modes
- From the mathematic point of view, these modes are called "Natural modes" and are described by
  - an eigenvector, which describes the modal shape
  - an eigenvalue, which describes the mode's frequency and its damping

Natural modes - Mechanical

- Example of the tuning fork

Shock perturbation
→ preferred response on A note
= 440 Hz

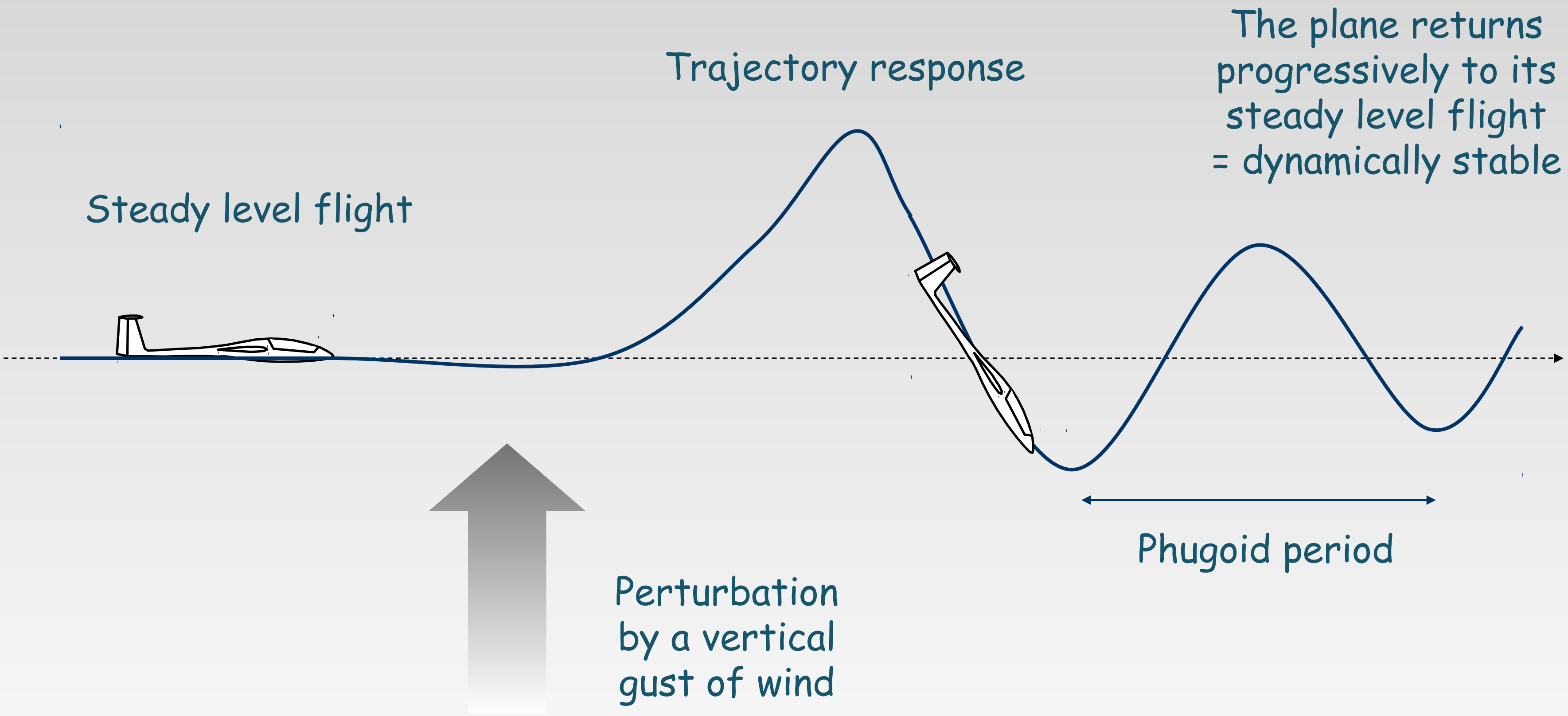




The sound decays with time
The fork is dynamically stable… not really a surprise

Natural modes - Aerodynamic

- Example of the phugoid mode



The 8 aerodynamic modes

- **A well designed plane will have 4 natural longitudinal modes and 4 natural lateral modes**

Longitudinal

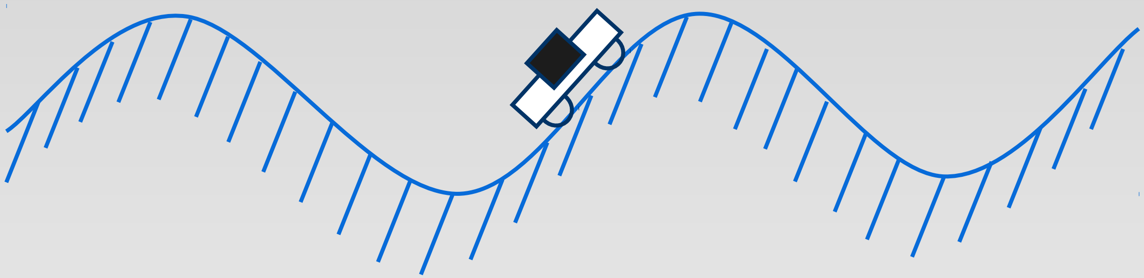
2 symmetric phugoid modes
2 symmetric short period modes

Lateral

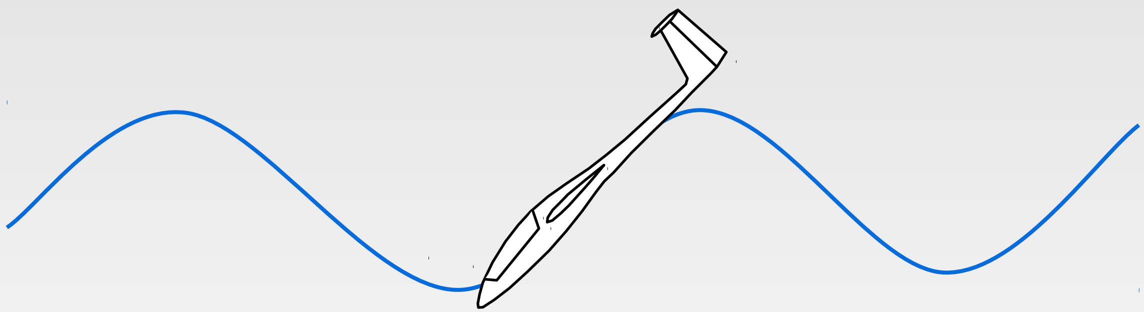
1 spiral mode
1 roll damping mode
2 Dutch roll modes

The phugoid

… is a macroscopic mode of exchange between the Kinetic and Potential energies



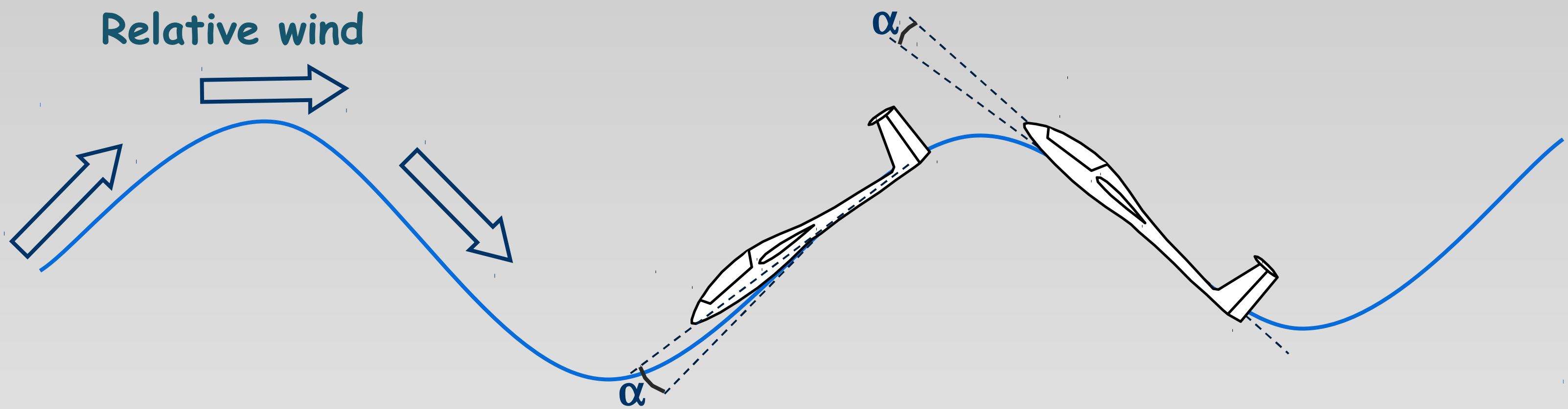
Russian Mountains : Exchange is made by the contact force



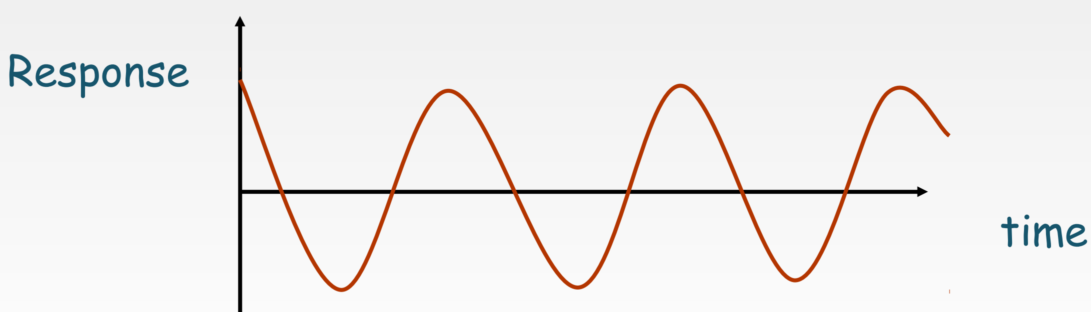
Aerodynamic : Exchange is made by the lift force

Slow, lightly damped, stable or unstable

The mechanism of the phugoid

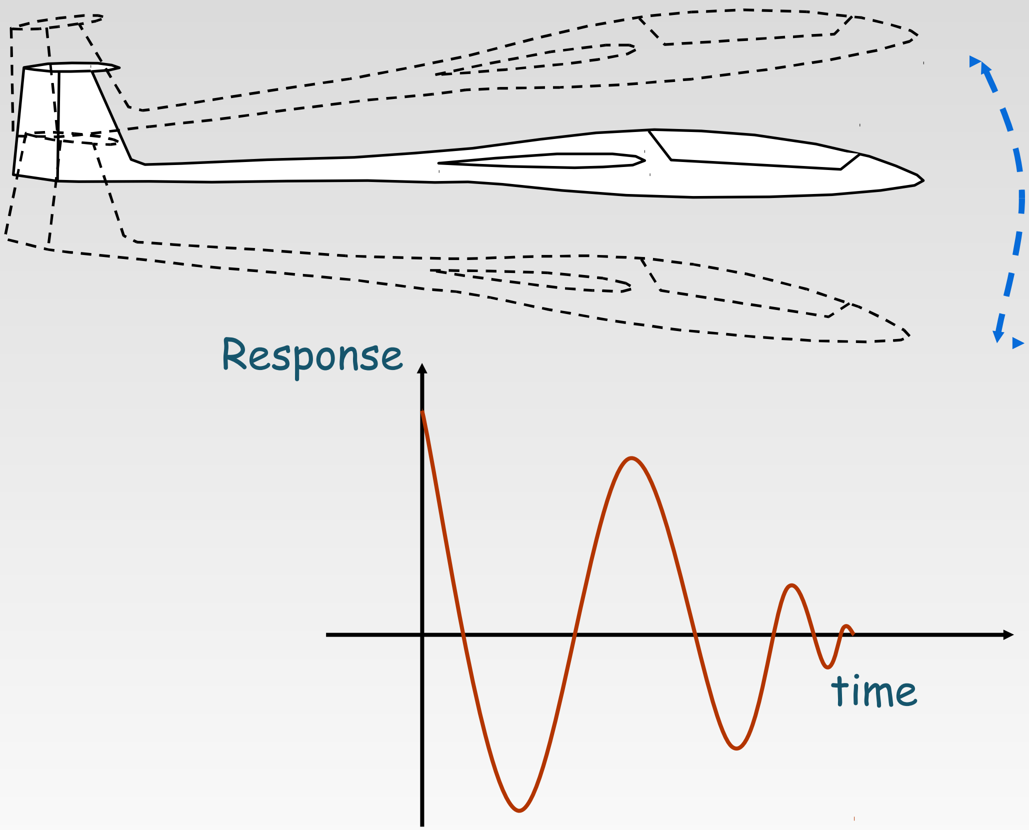


Dive → The sailplane accelerates → α Constant → Cl constant → At iso-Cl, The lift increases as the square power of the speed $L = \frac{1}{2} \rho \, S \, V^2 \, Cl$ → The sailplane rises and slows down → The lift decreases → Dive



The short period mode

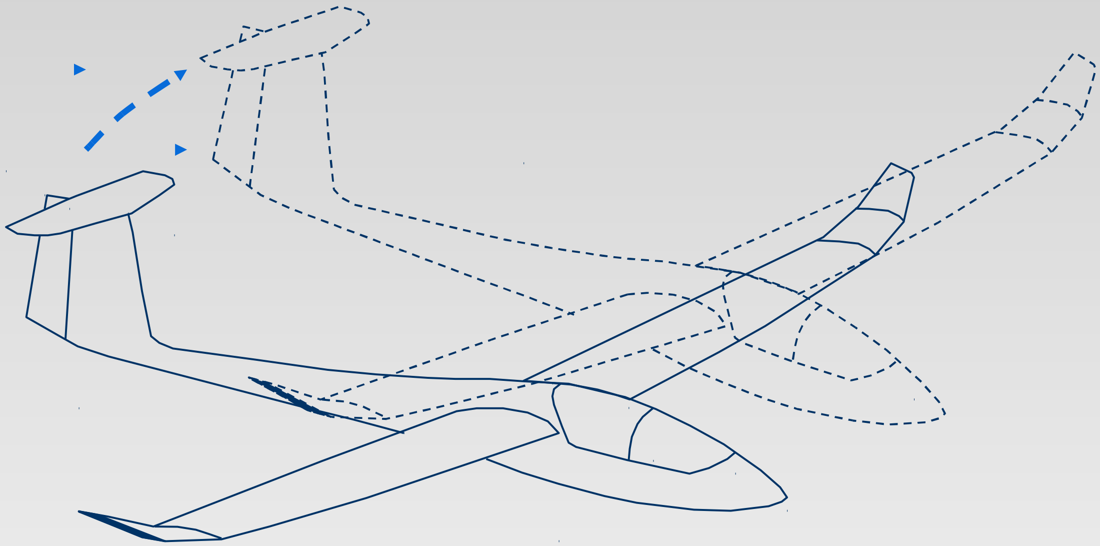
- Primarily vertical movement and pitch rate in the same phase, usually high frequency, well damped



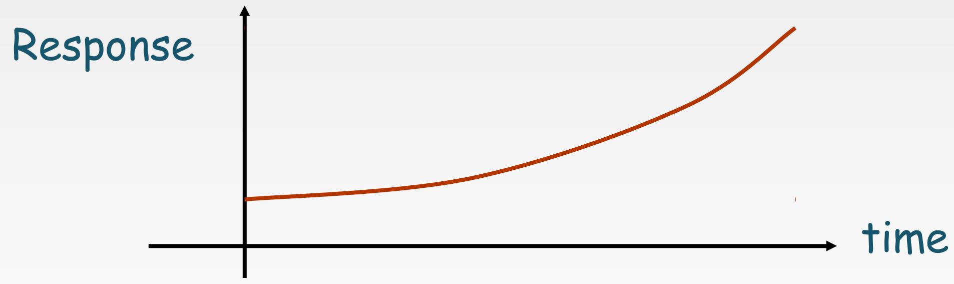
The mode's properties are primarily driven by the stiffness of the negative slope of the curve Cm=f(α)

Spiral mode

- Primarily heading, non-oscillatory, slow, generally unstable



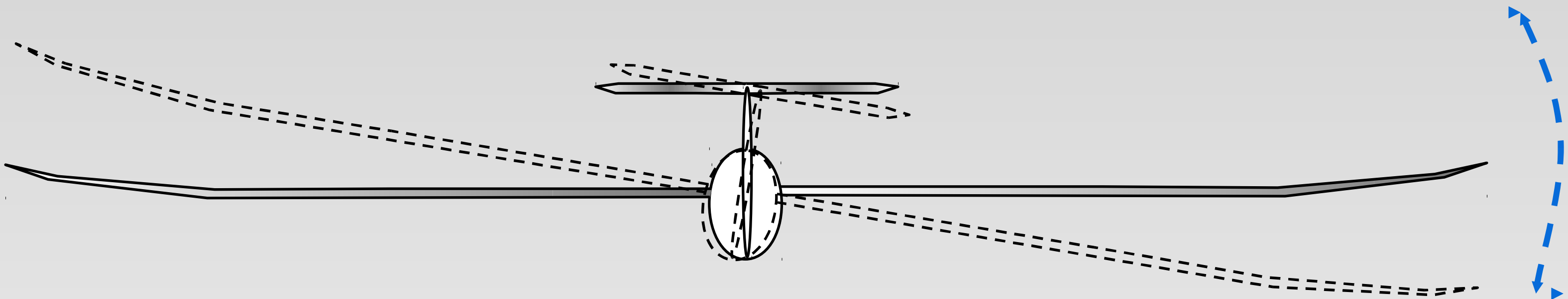
The mode is initiated by a rolling or heading disturbance. This creates a positive a.o.a. on the fin, which tends to increase the yawing moment



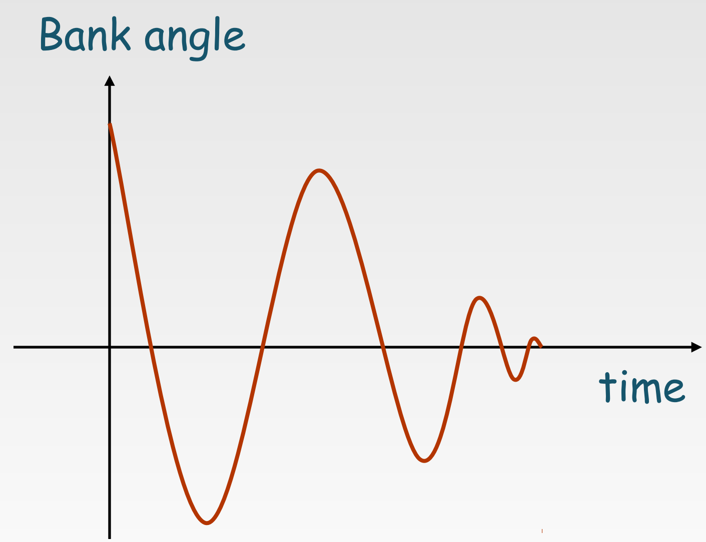
Requires pilot input to prevent divergence !

Roll damping

➢ **Primarily roll, stable**

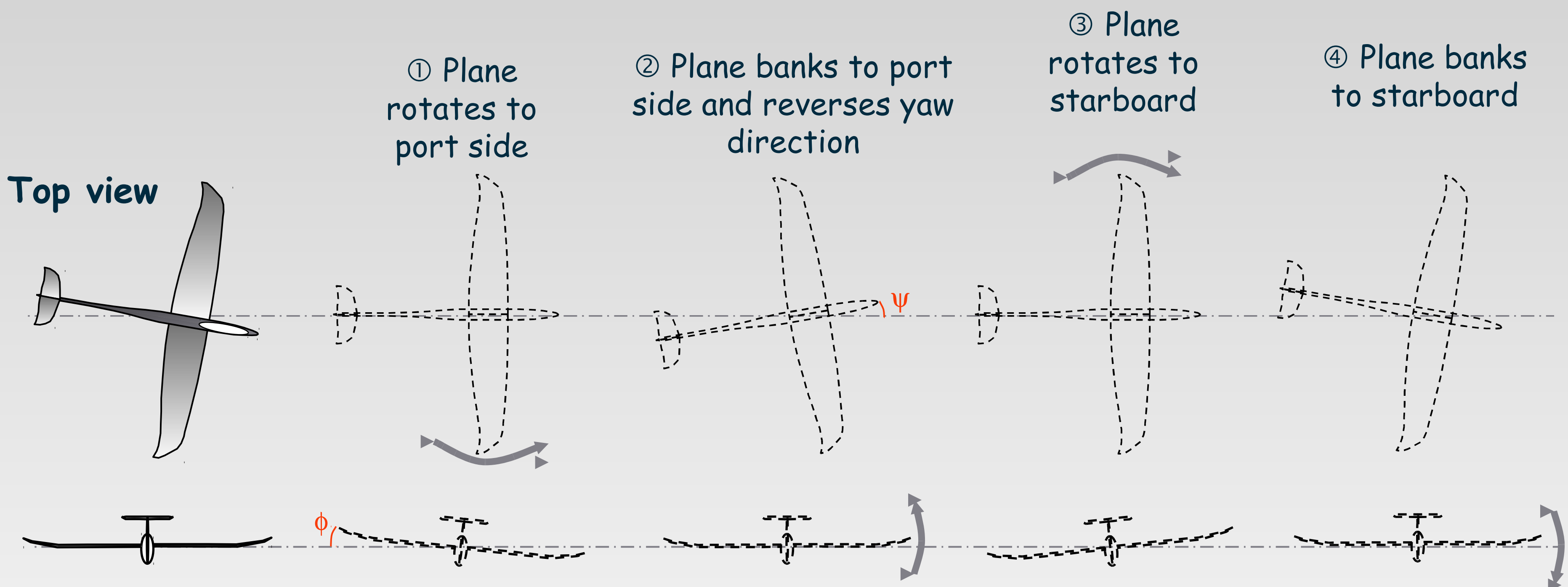


1. Due to the rotation about the x-axis, the wing coming down sees an increased a.o.a., thus increasing the lift on that side. The symmetric effect decreases the lift on the other side.
2. This creates a restoring moment opposite to the rotation, which tends to damp the mode



Dutch roll

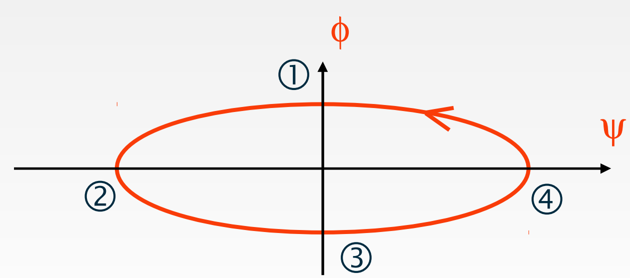
- The Dutch roll mode is a combination of yaw and roll, phased at 90°, usually lightly damped



→ Increased lift and drag on starboard side, decreased lift and drag on port side

→ The lift difference creates a bank moment to port side

→ The drag difference creates a yawing moment to starboard



Modal response for a reduced scale plane

- During flight, a perturbation such as a control input or a gust of wind shall excite all modes in different proportions :
  - Usually, the response on the short period and the roll damping modes, which are well damped, disappear quickly
  - The response on the phugoid and Dutch roll modes are visible to the eye
  - The response on the spiral mode is slow, and low in magnitude compared to other flight factors.
    It isn't visible to the eye, and is corrected unconsciously by the pilot

Modal behaviour

- **Some modes are oscillatory in nature…**
  - Phugoid,
  - Short period
  - Dutch roll

Defined by

1. a "mode shape" or eigenvector
2. a natural frequency
3. a damping factor

- **…and some are not**
  - Roll damping
  - Spiral

Defined by

1. a "mode shape" or eigenvector
2. a damping factor

The eigenvector

- In mathematical terms, the eigenvector provides information on the amplitude and phase of the flight variables which describe the mode,
- In XFLR5, the eigenvector is essentially analysed visually, in the 3D view

- A reasonable assumption is that the longitudinal and lateral dynamics are independent and are described each by four variables

The four longitudinal variables

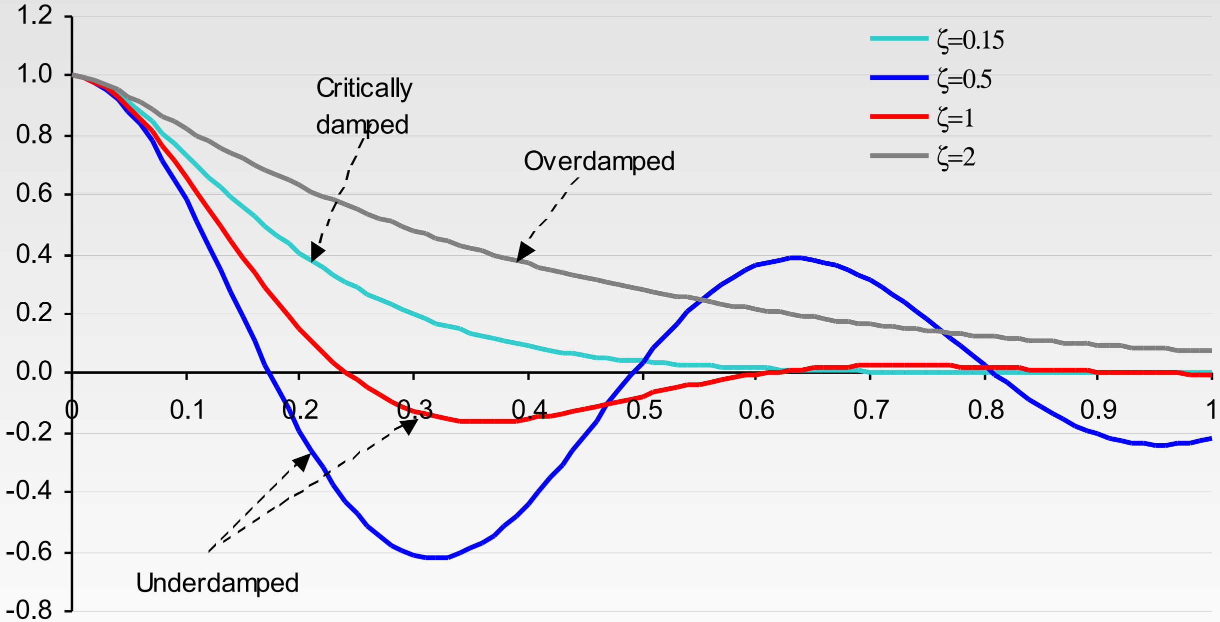
- The longitudinal behaviour is described by
  - The axial and vertical speed variation about the steady state value $V_{inf} = (U_0,0,0)$
    - $u = dx/dt - U_0$
    - $w = dz/dt$
  - The pitch rate $q = d\theta/dt$
  - The pitch angle θ
- Some scaling is required to compare the relative size of velocity increments "u" and "w" to a pitch rate "q" and to an angle "θ "
- The usual convention is to calculate
  - $u' = u/U_0$, $w' = w/U_0$, $q' = q/(2U_0/mac)$,
  - and to divide all components such that $\theta = 1$

The four lateral variables

- The longitudinal behaviour is described by four variables
  - The lateral speed variation $v = dy/dt$ about the steady state value $V_{inf} = (U_0,0,0)$
  - The roll rate $p = d\phi/dt$
  - The yaw rate $r = d\psi/dt$
  - The heading angle ψ
- For lateral modes, the normalization convention is
  - $v' = u/U_0$, $p' = p/(2U_0/span)$, $r' = r/(2U_0/span)$,
  - and to divide all components such that $\psi = 1$

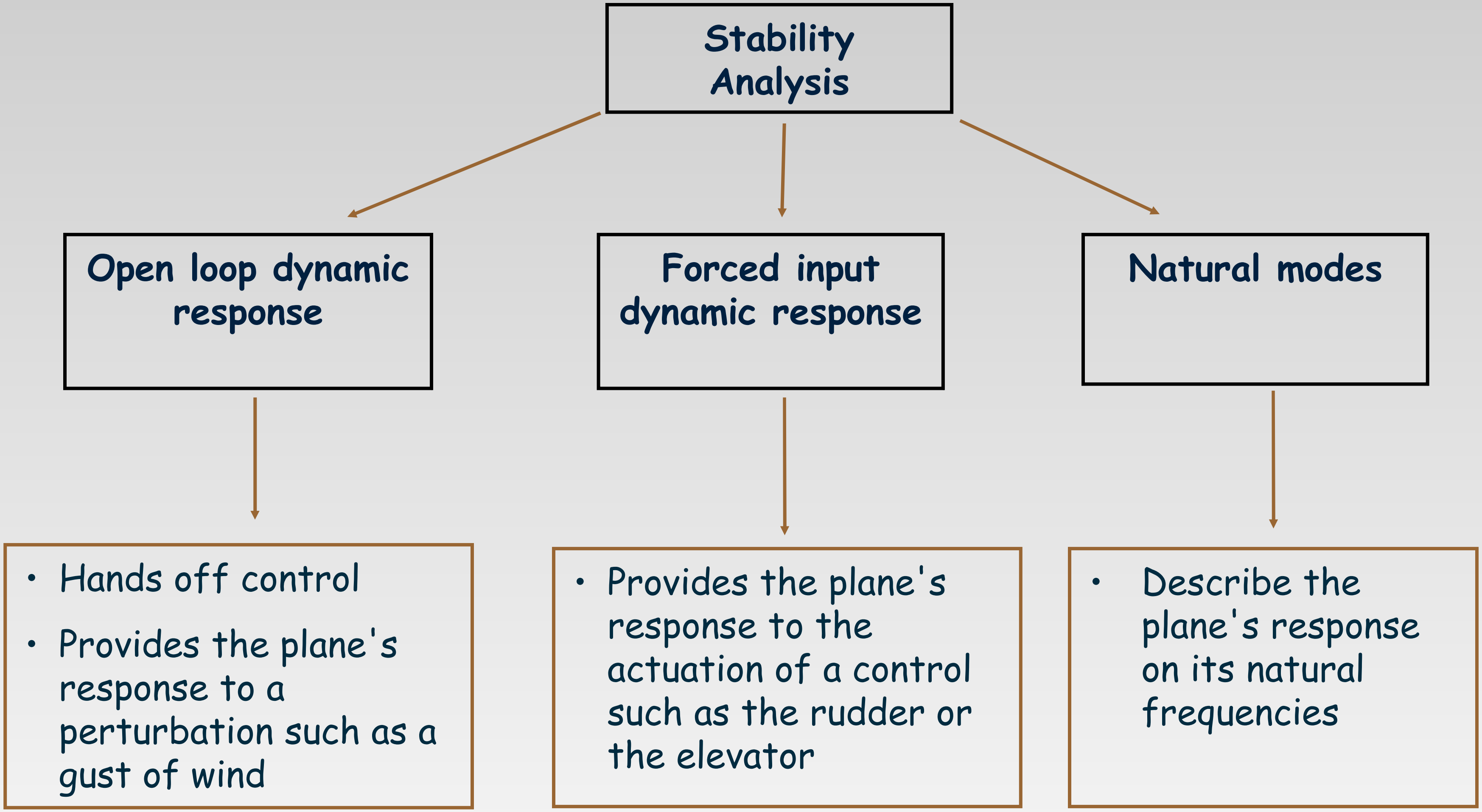
Frequencies and damping factor

- The damping factor ζ is a non-dimensional coefficient
- A critically damped mode, ζ = 1, is non-oscillating, and returns slowly to steady state
- Under-damped (ζ < 1) and over-damped (ζ > 1) modes return to steady state slower than a critically damped mode
- The "natural frequency" is the frequency of the response on that specific mode
- The "undamped natural frequency" is a virtual value, if the mode was not damped
- For very low damping, i.e. ζ << 1, the natural frequency is close to the undamped natural frequency



Modal analysis in XFLR5

One analysis, three output

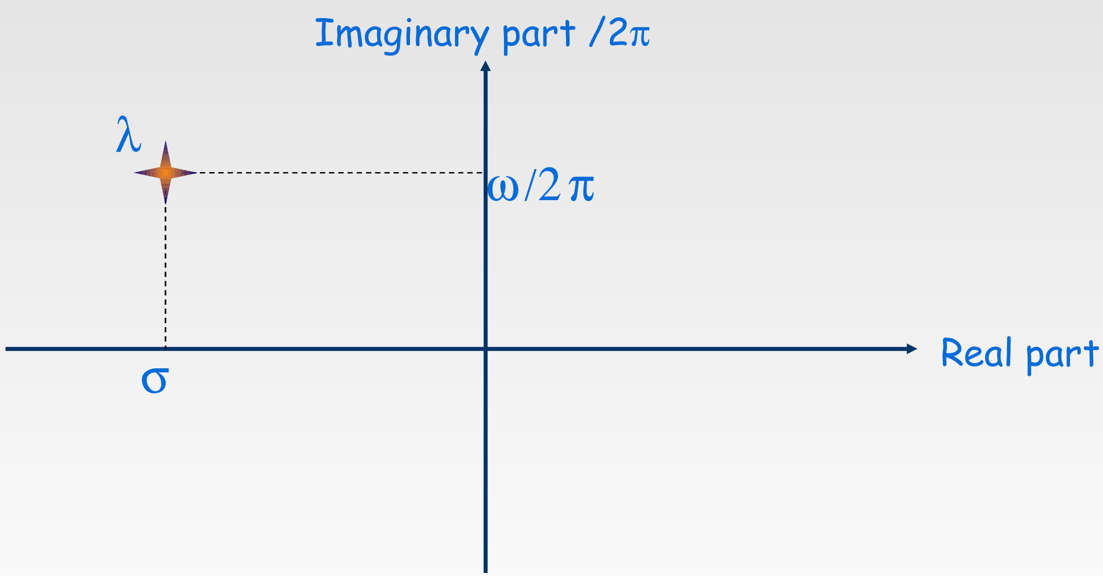


Pre-requisites for the analysis

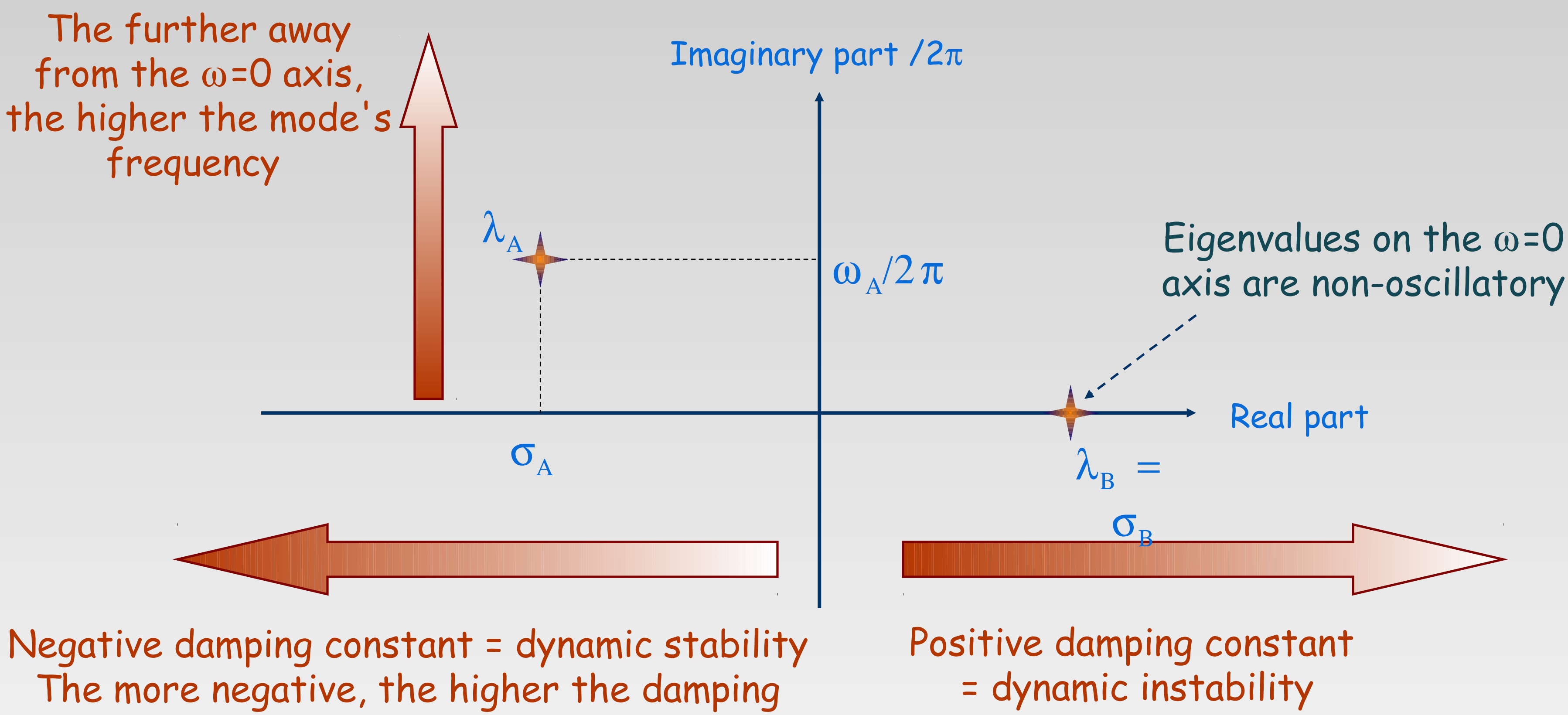
- The stability and control behavior analysis requires that the inertia properties have been defined
- The evaluation of the inertia requires a full 3D CAD program
- Failing that, the inertia can be evaluated approximately in XFLR5 by providing
  - The mass of each wing and of the fuselage structure
  - The mass and location of such objects as nose lead, battery, receiver, servo-actuators, etc.
- XFLR5 will evaluate roughly the inertia based on these masses and on the geometry
- Once the data has been filled in, it is important to check that the total mass and CoG position are correct

The root locus view

- This graphic view provides a visual interpretation of the frequency and damping of a mode with eigenvalue $\lambda = \sigma_1 + i\omega_N$
- The time response of a mode component such as u,w, or q, is $f(t) = k.e^{\lambda t} = ke^{(\sigma_1 + i\omega_N)t}$
- ω_N is the natural circular frequency and $\omega_N/2\pi$ is the mode's natural frequency
- $\omega_1 = \sqrt{\sigma_1{}^2 + \omega_N{}^2}$ is the undamped natural circular frequency
- σ_1 is the damping constant and is related to the damping ratio by $\sigma_1 = -\omega_1\zeta$
- The eigenvalue is plotted in the $(\sigma_1,\ \omega_N/2\pi)$ axes, i.e. the root locus graph



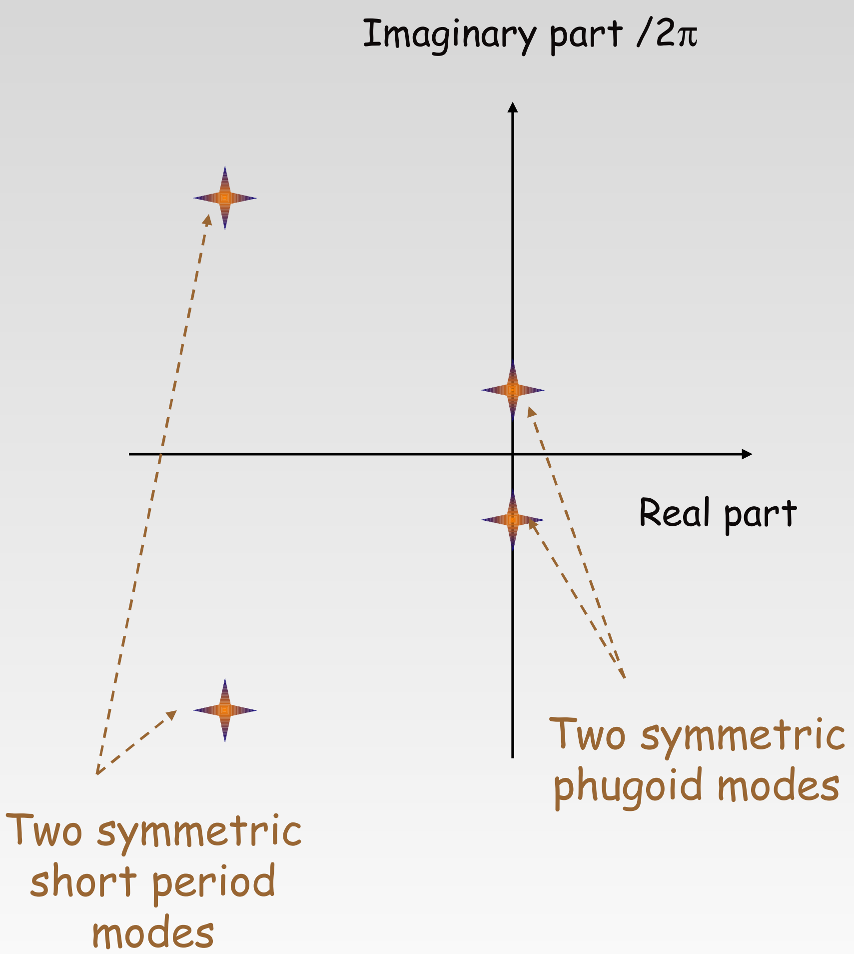
The root locus interpretation



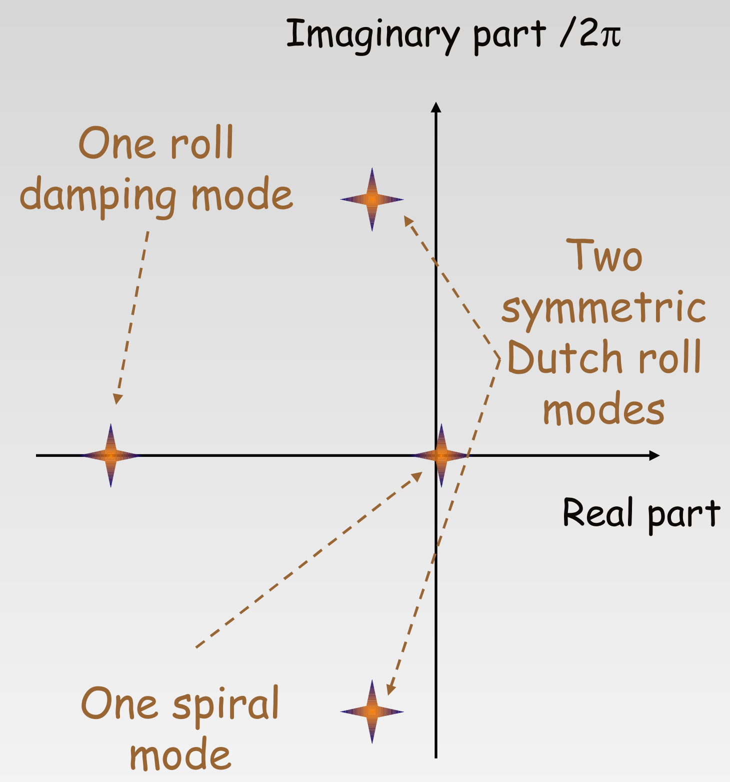
- λ_A corresponds to a damped oscillatory mode
- λ_B corresponds to an un-damped, non-oscillatory mode

The typical root locus plots

Longitudinal

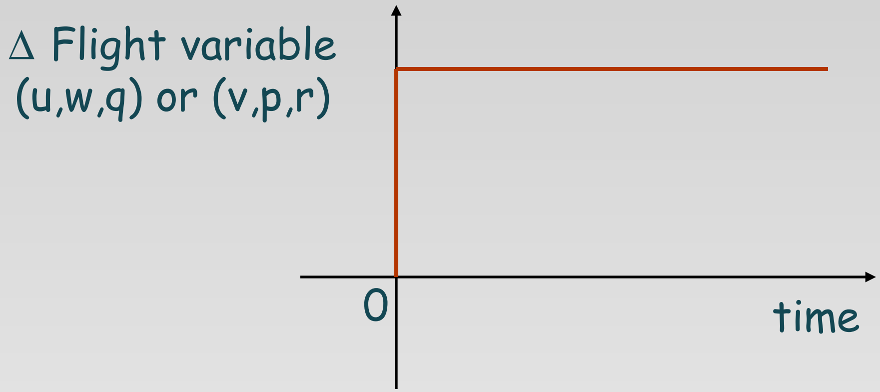


Lateral

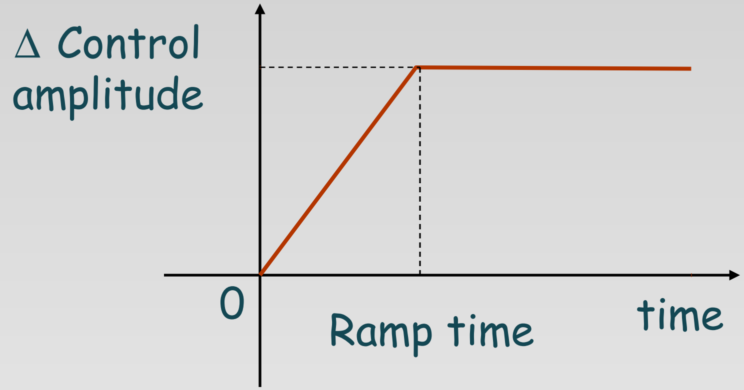


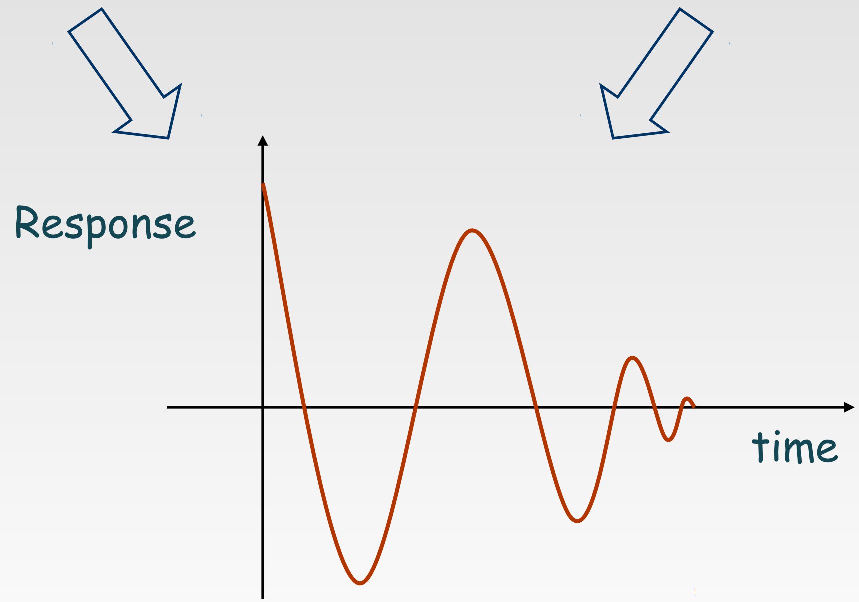
The time response view : two type of input

① Perturbation



② Control actuation





The 3D mode animation

- The best way to identify and understand a mode shape ?
- Note :
  - The apparent amplitude of the mode in the animation has no physical significance.
  - A specific mode is never excited alone in flight – the response is always a combination of modes.

Example of Longitudinal Dynamics analysis

First approximation for the "Short Period Mode"

$$F_1 = \frac{1}{2\pi}\sqrt{\frac{K}{J}}$$

1. The coefficient K can be evaluated with XFLR5 :

$$K = \frac{1}{2}\rho V^2 \, S_{wing} \; M.A.C. \frac{\partial C_m}{\partial \alpha}$$

Negative slope of the curve
$Cm = f(\alpha)$

2. J is the inertia for pitching: $J = I_{xx}$

3. This assumes that the pitching behaviour is independent of the vertical velocity… not necessarily a valid assumption
→ need for a more sophisticate approximation

Second approximation for the Short Period Mode

- Taking into account the dependency to the vertical velocity leads to a more complicated expression

$$t^* = \frac{MAC}{2u_0} \qquad \hat{I}_y = \frac{8I_y}{\rho.S.MAC^3} \qquad \mu = \frac{2m}{\rho.S.MAC}$$

u_0 = horizontal speed

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \qquad C_{z_\alpha} = \frac{\partial C_z}{\partial \alpha}$$

$C_{m\alpha}$ and $C_{z\alpha}$ are the slopes of the curves Cm = f(α) and Cz = f(α). The slopes can be measured on the polar graphs in XFLR5

$$B = \frac{C_{z_\alpha}}{2t^*\mu} \qquad C = -\frac{C_{m_\alpha}}{t^{*2}\hat{I}_y}$$

$$F_2 = \frac{1}{2\pi}\sqrt{-B^2 + 4C}$$

Despite their complicated appearance, these formula can be implemented in a spreadsheet, with all the input values provided by XFLR5

Lanchester's approximation for the Phugoid

- **The phugoid's frequency is deduced from the balance of kinetic and potential energies, and is calculated with a very simple formula**

$$F_{ph} = \frac{1}{\pi\sqrt{2}}\frac{g}{u_0}$$

g is the gravitational constant, i.e. g = 9.81 m/s
u_0 is the plane's speed

Numerical example – from a personal model sailplane

- Plane and flight Data

MAC =	0.1520	m
Mass =	0.5250	kg
Iyy =	0.0346	kg.m²
S =	0.2070	m²
ρ =	1.225	kg/m3

u0 =	16.20	m/s
α =	1.05	°
q =	160.74	Pa

Cx =	0.0114	
Cz =	0.1540	
dCm/dα =	-1.9099	
dCz/dα =	-5.3925	

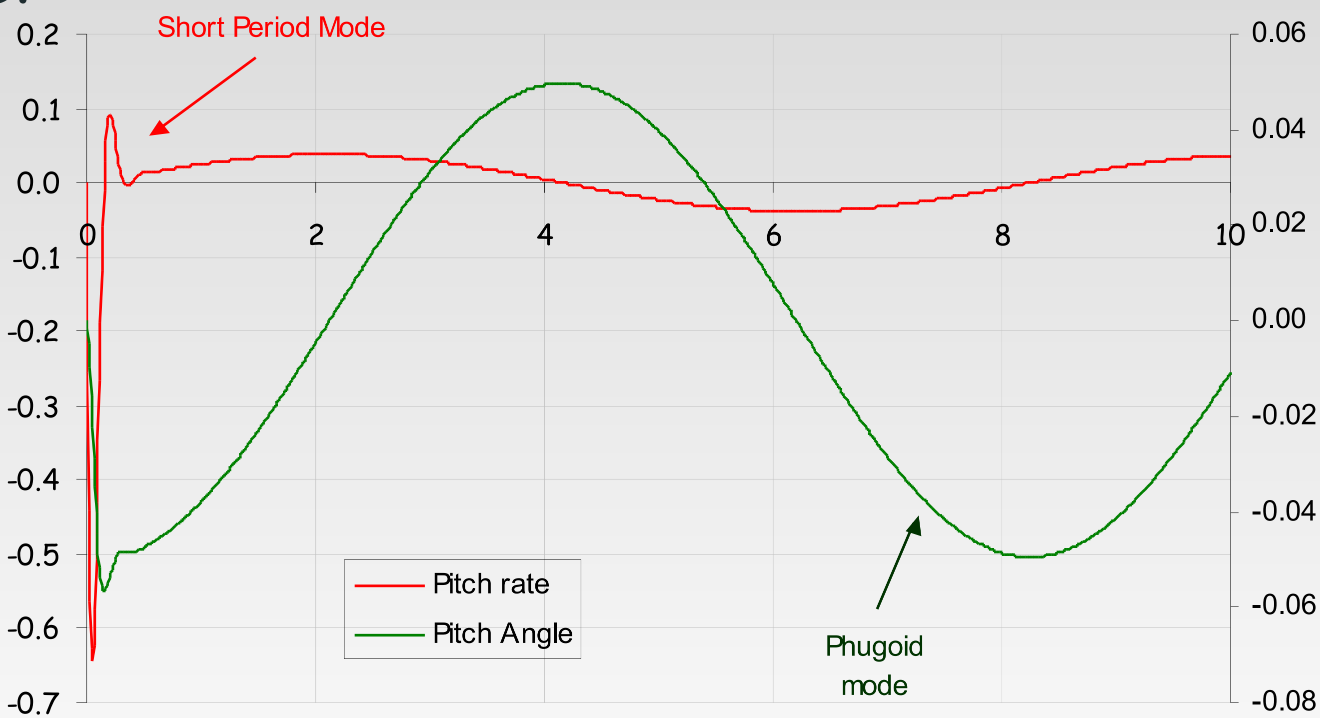
- Results

	Short Period			Phugoid	
	F1	F2	XFLR5 v6	Fph	XFLR5 v6
Frequency (Hz) =	4.45	4.12	3.86	0.136	0.122
Period (s) =	0.225	0.243	0.259	7.3	8.2

Graphic Analysis →

Time response

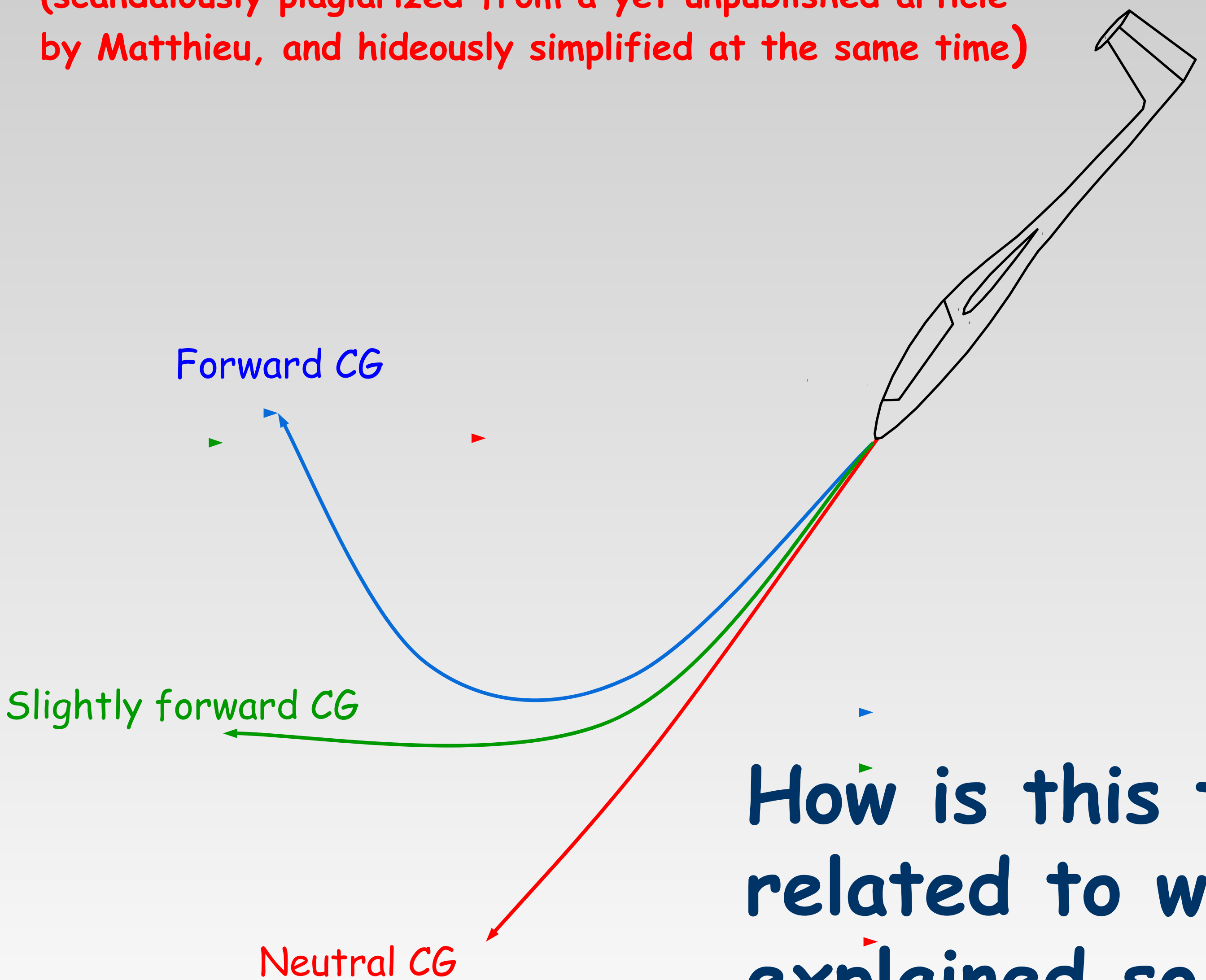
- There is factor 40x between the numerical frequencies of both modes, which means the plane should be more than stable
- A time response analysis confirms that the two modes do not interact



About the Dive Test

About the dive test

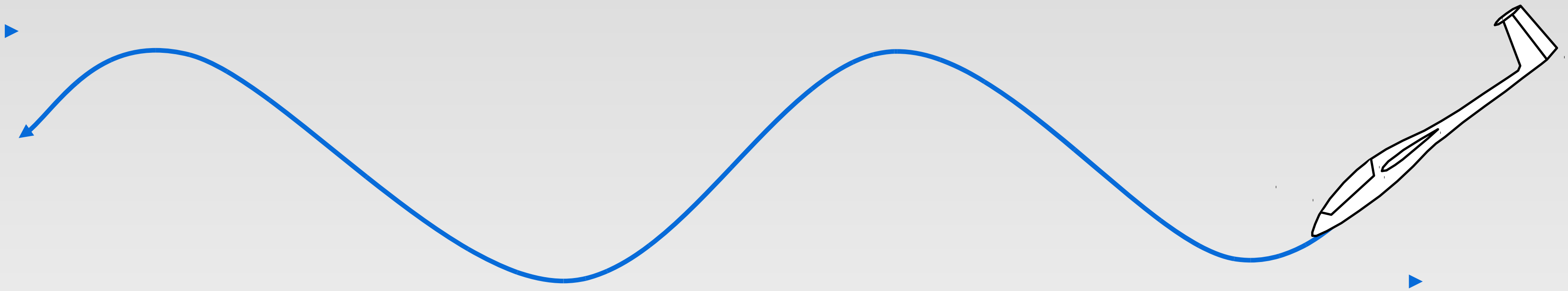
(scandalously plagiarized from a yet unpublished article by Matthieu, and hideously simplified at the same time)



How is this test related to what's been explained so far?

Forward CG

- **If the CG is positioned forward, the plane will enter the phugoid mode**

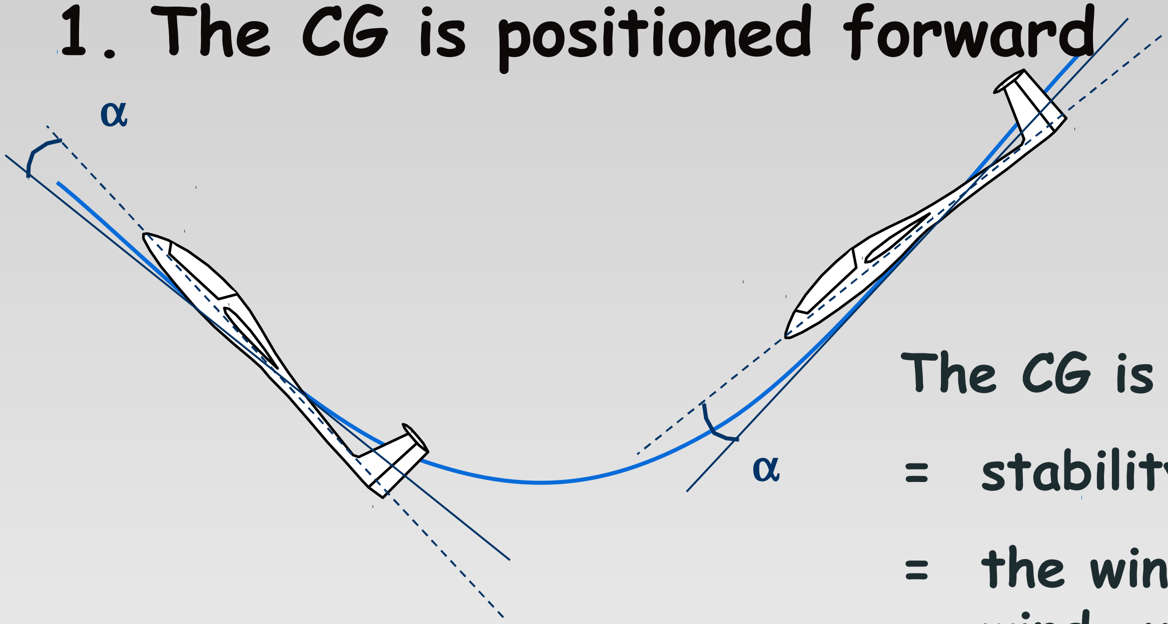


Stick to the phugoid

- As the plane moves along the phugoid, the apparent wind changes direction
- From the plane's point of view, it's a perturbation
- The plane can react and reorient itself along the trajectory direction, providing
  - That the slope of the curve $Cm = f(\alpha)$ is stiff enough
  - That it doesn't have too much pitching inertia

Summarizing :

1. The CG is positioned forward



The CG is positioned forward

= stability

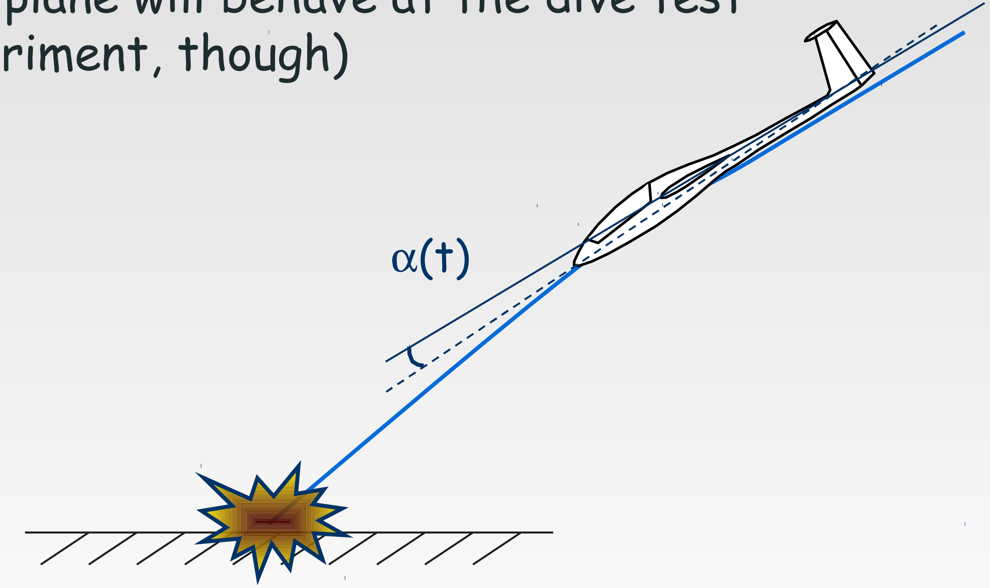
= the wind vane which follows the wind gusts

- The two modes are un-coupled
- The relative wind changes direction along the phugoid…
- … but the plane maintains **a constant incidence** along the phugoid, just as the chariot remains tangent to the slope
- The sailplane enters the phugoid mode

2. The CG is positioned aft

Remember that backward CG = instability = the wind vane which amplifies wind gusts

- The two modes are coupled
- The incidence oscillation $\alpha(t)$ amplifies the phugoid,
- The lift coefficient is not constant during the phugoid
- The former loop doesn't work any more
- The phugoid mode disappears
- No guessing how the sailplane will behave at the dive test
  (It's fairly easy to experiment, though)



That's all for now

Good design and nice flights ☺