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Computers & Industrial Engineering 49 (2005) 463–503

computers & industrial engineering

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Scheduling in dynamic assembly job-shops to minimize the sum of weighted earliness, weighted tardiness and weighted flowtime of jobs

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Received 13 March 2003; revised 1 June 2005; accepted 2 June 2005
Available online 18 October 2005

Abstract

In many manufacturing systems, jobs that are completed early are held as finished-goods inventory until their due-dates, and hence we incur earliness costs. Similarly, jobs that are completed after their due-dates incur penalty. The objective in such situations would, therefore, be to meet the due-dates of the respective jobs as closely as possible, and consequently minimize the sum of earliness and tardiness of jobs because earliness and tardiness of jobs greatly influence the performance of a schedule with respect to cost. In addition, a job incurs holding cost from the time of its arrival until its completion. Most studies on scheduling in such manufacturing systems assume unit earliness cost, unit tardiness cost and unit holding cost of a job. However, in reality such an assumption need not always hold and it is quite possible that there exist different costs of earliness, tardiness and holding for different jobs. In addition, most studies on job-shop scheduling assume that jobs are independent and that no assembly operations exist. The current study addresses the problem of scheduling in dynamic assembly job-shops (i.e. shops that manufacture multi-level jobs) with the consideration of jobs having different earliness, tardiness and holding costs. An attempt is made in this paper to present dispatching rules by incorporating the relative costs of earliness, tardiness and holding of jobs in the form of scalar weights. In the first phase of the study, relative costs (or weights for) earliness and tardiness of jobs are considered, and the dispatching rules are presented in order to minimize the sum of weighted earliness and weighted tardiness of jobs. In the second phase of the study, the objective considered is the minimization of the sum of weighted earliness, weighted tardiness and weighted flowtime of jobs, and the dispatching rules are presented by incorporating the relative costs of earliness, tardiness and flowtime of jobs. Simulation studies have been conducted separately for both phases of the current study, the performance of the scheduling rules have been observed independently, and the results of the simulation study have been reported.

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doi:10.1016/j.cie.2005.06.005

The proposed rules are found to be effective in minimizing the mean and maximum values of the measures of performance.

Keywords: Dynamic assembly job-shop; Weighted earliness; Weighted tardiness; Weighted flowtime; Dispatching rules; Simulation

1. Introduction

Most research on scheduling in dynamic manufacturing systems has dealt with the problem of scheduling in dynamic job-shops. A survey of dispatching rules in dynamic shops was presented by Day and Hottenstein (1970); Blackstone, Phillips, and Hogg (1982); Haupt (1989), and Ramasesh (1990). Of late, efficient dispatching rules in dynamic job-shops and flow-shops have been proposed by Holthaus and Ziegler (1997), Holthaus and Rajendran (1997, 2000, 2002), Jayamohan and Rajendran (2000), and Framinan, Ruiz-Usano, and Leisten (2000). Researchers have attempted to evaluate the performance of dispatching rules in an assembly job-shop environment where assembly operations take place. In an assembly job-shop, the operations of an item are carried out serially by following the precedence relationships, while those of another item belonging to the same assembly may be carried out in parallel (unlike non-assembled or serial jobs where all the operations are performed in series). In this context, an item here may refer to a component, a sub-assembly, or a sub-sub-assembly. As a result, in addition to waiting for a resource, an item may wait for the processing of its mating items, before the required assembly can take place. Moreover, the jobs can have a very simple structure involving just one level of assembly, or can be complicated with several levels of assemblies. Irrespective of the nature of the job structure, processing of components, sub-assemblies and sub-sub-assemblies must be completed in such a way to make the schedule and assembly feasible. This makes the scheduling problem in assembly job-shops quite challenging, when compared to the conventional job-shop (Adam, Bertrand, & Surkis, 1987).

Unlike the job-shop, the amount of research work done in the assembly job-shop environment is rather limited. The measures of performance, used by most of the researchers in the area of dynamic assembly job-shops, to evaluate the performance of the dispatching rules include job flowtime, tardiness, percentage of tardy jobs and assembly delay. Initially, simple job structures were considered, and the performance of simple and composite rules (with the consideration of information about the shop status, job progress in the form of remaining job-time, and slack) was evaluated with respect to the measures of mean flowtime and mean tardiness (e.g. Maxwell & Mehra, 1968; Sculli, 1980, 1987). The good performance of the job due-date rule (Goodwin & Goodwin, 1982) and the non-performance of the other milestones, namely, the operation and assembly due-dates (Phillipoom, Markland, & Fry, 1989) for complex job structures were reported with respect to the minimization of mean flowtime, mean tardiness and percentage of tardy jobs. New dispatching rules, namely, relative remaining operations (RRO), relative remaining processing time (RRP) and importance ratio (IR), were developed with due consideration given to pacing, acceleration and structural complexity (Adam et al., 1987; Phillipoom, Russell, & Fry, 1991) which resulted in minimizing the staging delay. A tie-breaking rule is used when two or more jobs wait in the queue and have the same priority value. The importance of tie-breaking was observed by Adam et al. (1987) when they used the TWKR (total work content remaining or total processing times of remaining operations on the job) rule with their proposed tie-breaking rules, namely, relative remaining operations (RRO) and relative remaining processing times (RRP). Later, Phillipoom et al. (1991) evaluated the use of the importance ratio (IR) rule for tie-breaking. Recently, Reeja and Rajendran (2000a,b) came up with the operation synchronization date (OSD) rule, and found the rule to be better than the RRP and IR rules for tie-breaking when the TWKR rule is the primary one. The effect of product structures on the performance of the dispatching rules (Fry, Oliff, Minor, & Leong, 1989) and dynamic assignment of due-date without the use of parameters (Adam, Bertrand, Morehead, & Surkis, 1993) are other important contributions in the assembly job-shop environment.

In real-life situations, since the strength of the relationship between the customer and the firm depends on a variety of factors, it seems appropriate for the manufacturer to reflect these priorities in their scheduling decisions. Accordingly, different tardiness and earliness penalties need to be assigned to different customers. Likewise, the costs of holding different jobs after completion inside the system will also be different. It is therefore important that the scheduling decisions reflect these priorities (or costs) with respect to earliness, tardiness and holding in the process of dispatching jobs in assembly job-shops. In such instances, it is appropriate to associate weights for earliness, tardiness and flowtime of jobs, and gauge the performance of rules by employing the weighted measures of performance (Scudder & Hoffmann, 1987; Jensen, Philipoom & Malhotra, 1995). It also seems appropriate at this juncture to consider briefly the work done in the case of job-shop scheduling with the consideration of weights for earliness/tardiness/flowtime of jobs. Conway, Johnson, and Maxwell (1960); Rowe (1960) are perhaps the first to consider the value-based dispatching of jobs in the case of job-shop scheduling. Other studies that are related to job-shop scheduling and those directly use cost based information in developing dispatching priorities include those of Aggarwal and McCarl (1974); Hoffmann and Scudder (1983). Vepsalainen and Morton (1987) first demonstrated the weighted cost over time (COVERT) rule's superiority over the weighted shortest processing time (SPT) rule in the case of dynamic job-shop environment. The authors further developed the average-tardiness-cost (ATC) rule that uses a complex weighted criterion. The ATC rule has been shown to perform better than the weighted SPT and weighted COVERT rules for normalized weighted measures of tardiness. Kutanoglu and Sabuncuoglu (1999) showed that the priority rules which make use of operational information such as operation due-dates and operation processing times perform consistently better than their job-based counterparts and that the weighted SPT rule is more robust to changes in experimental settings. Kanet and Christy (1984), in their work, have stressed the importance of the problem of forbidden early shipment in a manufacturing system. A significant amount of work has been carried out in the area of single-machine scheduling involving early/tardy problem (Baker & Scudder, 1990). However, there appears to be no research work in the environment of multi-machines where serial operations are performed on the job (i.e. job-shops) with the consideration of earliness, tardiness and holding costs.

Scudder and Hoffmann (1989), for randomly routed and flow shop environment, developed a two-class active and inactive queue system at each work center with jobs in the active queue having their operations start date reached. The authors found the performance of the time based rules, namely, the critical ratio and operation critical ratio rules, better than the value-based rules. The next important contribution in this area was due to Rohleder and Scudder (1992). The authors used net present value (NPV) and system inventory measures to evaluate a number of rules and found job-based rules performing well for both measures. In another study by the same authors (Rohleder & Scudder, 1993) in a dynamic job-shop environment, a new rule was developed to minimize the early/tardy costs ratio, which is a modified version of the rule (called EXPET rule) proposed by Ow and Morton (1989) for a single machine early/tardy problem. The authors concluded that the proposed rule clearly dominated the WCOVERT, earliest due date (EDD) and first come first served (FCFS) rules with respect to minimizing the early/tardy cost ratio. The importance of total cost criterion for evaluating the performance of the shop floor dispatching rules was highlighted by Yang and Sum (1994). They tested the time-based, value-based and slack-based rules proposed in the prior studies in a job-shop environment and proposed new composite dispatching rules. Their proposed rules are shown to perform better than the earlier rules in environments where early shipment of completed jobs is forbidden. In a recent study, Thiagarajan and Rajendran (2003) proposed dispatching rules with the primary measure of performance being the minimization of the total scheduling cost consisting of the sum of weighted flowtime and weighted tardiness of jobs, and the secondary measures of performance being the minimization of weighted mean flowtime, weighted mean tardiness, maximum weighted flowtime, maximum weighted tardiness and so on, considered separately. Their rules considered the weights for holding and tardiness, apart from considering the factors such as total work content remaining in the job, process time of the imminent operation, job due-date, earliest completion time of the job, and time-in-system.

In the current study, the motivation for the development of new dispatching rules that incorporates weights for earliness, tardiness and flowtime has been derived from the findings of the survey on the available literature. Almost all researchers have so far assumed that the cost of earliness, the cost of tardiness and the cost of holding per unit time are the same, whereas in real-life situations, it need not be so. Further, the performance criteria used in a majority of such studies are time-based measures and only a few are cost-based measures of performance. It appears from the literature review that no work has been done with the consideration of earliness, tardiness and flowtime costs in an assembly shop environment where multi-level jobs are processed. Hence the current work is undertaken in two phases. In the first phase, the cost-consideration has included the weighted earliness and weighted tardiness of jobs, while in the second phase, the cost-consideration has included the weighted earliness, weighted tardiness and weighted flowtime of jobs. The current work can therefore be regarded as a follow-up work of Thiagarajan and Rajendran (2003). We first present the details of the first phase of our work, and then the details of the second phase of our work.

2. Notations

- i job index, or equivalently, job-order index (in the context of job arrivals in the shop).
- j operation index.
- T current time at which dispatching decision is to be made.
- h_i holding cost per unit time for job i (or weight relating to flowtime of job i).
- e_i earliness cost per unit time for job i (or weight relating to earliness of job i).
- r_i tardiness penalty per unit time for job i (or weight relating to tardiness of job i).
- D_i due-date of job i.
- QT_{ij} time at which an item of job i enters the queue/waiting line for its operation j.
- C_i completion time of job i, i.e. completion time of the final assembly of job i.
- A_i arrival time of job i in the shop.
- T_i tardiness of job i, given by max $\{C_i - D_i; 0\}$.
- E_i earliness of job i, given by max $\{D_i - C_i; 0\}$.
- cp_i sum of the processing times of operations of job i along the critical path.

t_{ij} processing time of operation j on job i.
O_{ij} set of all remaining operations on job i, including the current operation j.
O'_{ij} set of remaining operations on job i, including current operation j, along the path from operation j up to the final assembly.
TWKR_i total processing times of all remaining operations on job i.
Z_{ij} priority index of the item on which operation j of job i is to be performed at the instant T (the item with the least priority value is chosen for loading).
Z'_{ij} priority index of item on which operation j of job i is to be performed at the instant T (the item with the largest priority value is chosen for loading).
c due-date allowance factor.
n number of jobs scheduled during the steady state period in the simulation run.

3. Measures of performance considered in phase 1 and phase 2

In the first phase of the study, the total scheduling cost is considered to be the sum of weighted earliness and weighted tardiness of jobs. The dispatching rules are evaluated with respect to the mean, maximum and variance of the performance measure under consideration. The mean value is an indicator of the performance of a rule over a long run, i.e. long run average of the performance of the rule. The maximum value gives the worst case performance of a rule with respect to some select set of job-orders. Obviously, a rule that is capable of minimizing both the mean and the maximum values will be a better choice for selection. The variance measure indicates the spread of the values around the mean value. In other words, it gives the risk undertaken by the scheduler while selecting a particular rule. A rule that provides a high variance on the chosen performance measure has a higher probability of disrupting the schedule. In practice, a scheduler must have the knowledge of performance of a rule with respect to a number of measures of performance and under a variety of shop-floor conditions. Hence we have considered the performance-evaluation of all these rules for a number of different conditions so that a scheduler has a complete picture in respect of the performance of every dispatching rule. The measures of performance in this phase correspond to the Eqs. (1)–(9).

$$\text{Weighted mean earliness } (\mathrm{WME}^e) = \frac{\sum_{i=1}^{n} \max[D_i - C_i; 0] e_i}{\sum_{i=1}^{n} e_i} \tag{1}$$

$$\text{Weighted mean tardiness } (\mathrm{WMT}^r) = \frac{\sum_{i=1}^{n} \max[C_i - D_i; 0] r_i}{\sum_{i=1}^{n} r_i} \tag{2}$$

$$\text{Variance of earliness } (\mathrm{VE}^e) = \frac{\sum_{i=1}^{n} [\max[D_i - C_i; 0] - (\mathrm{WME}^e)]^2 e_i}{\sum_{i=1}^{n} e_i} \tag{3}$$

$$\text{Variance of tardiness } (\mathrm{VT}^r) = \frac{\sum_{i=1}^{n} [\max[C_i - D_i; 0] - (\mathrm{WMT}^r)]^2 r_i}{\sum_{i=1}^{n} r_i} \tag{4}$$

$$\text{Weighted mean scheduling cost } (\mathrm{WMSC}^{e,r}) = \frac{\sum_{i=1}^{n} \max[D_i - C_i; 0] e_i}{\sum_{i=1}^{n} e_i} + \frac{\sum_{i=1}^{n} \max[C_i - D_i; 0] r_i}{\sum_{i=1}^{n} r_i} \tag{5}$$

$$\text{Mean scheduling cost } (\mathrm{MSC}^{e,r}) = \frac{\sum_{i=1}^{n} \max[D_i - C_i; 0] e_i + \sum_{i=1}^{n} \max[C_i - D_i; 0] r_i}{n} \tag{6}$$

Maximum total scheduling cost ($\mathrm{MaxTSC}^{e,r}$)

$$= \max_{1 \le i \le n} \{\max[D_i - C_i; 0] e_i + \max[C_i - D_i; 0] r_i\} \tag{7}$$

$$\text{Weighted variance of total scheduling cost } (\mathrm{WVTSC}^{e,r}) = \mathrm{VE}^e + \mathrm{VT}^r \tag{8}$$

Variance of total scheduling cost ($\mathrm{VTSC}^{e,r}$)

$$= \frac{\sum_{i=1}^{n} [\max[D_i - C_i; 0] e_i + \max[C_i - D_i; 0] r_i - \mathrm{MSC}^{e,r}]^2}{n} \tag{9}$$

In the second phase of the study, the total scheduling cost is defined as the sum of weighted earliness, weighted tardiness and weighted flowtime of jobs. Let h_i denote the holding cost per unit time for job i (or weight relating to flowtime of job i). Expressions related to various measures of performance for this phase of study are given in Eqs. (10)–(16).

$$\text{Weighted mean flowtime } (\mathrm{WMF}^h) = \frac{\sum_{i=1}^{n} (C_i - A_i) h_i}{\sum_{i=1}^{n} h_i} \tag{10}$$

$$\text{Variance of flowtime } (\mathrm{VF}^h) = \frac{\sum_{i=1}^{n} [(C_i - A_i) - (\mathrm{WMF}^h)]^2 h_i}{\sum_{i=1}^{n} h_i} \tag{11}$$

$$\text{Weighted mean scheduling cost } (\mathrm{WMSC}^{e,r,h}) = \mathrm{WME}^e + \mathrm{WMT}^r + \mathrm{WMF}^h \tag{12}$$

Mean scheduling cost ($\mathrm{MSC}^{e,r,h}$)

$$= \frac{\sum_{i=1}^{n} \max[D_i - C_i; 0]e_i + \sum_{i=1}^{n} \max[C_i - D_i; 0]r_i + \sum_{i=1}^{n}(C_i - A_i)h_i}{n} \tag{13}$$

Maximum total scheduling cost ($\mathrm{MaxTSC}^{e,r,h}$)

$$= \max_{1 \le i \le n}\{\max[D_i - C_i; 0]e_i + \max[C_i - D_i; 0]r_i + (C_i - A_i)h_i\} \tag{14}$$

$$\text{Weighted variance of total scheduling cost } (\mathrm{WVTSC}^{e,r,h}) = \mathrm{VE}^e + \mathrm{VT}^r + \mathrm{VF}^h \tag{15}$$

Variance of total scheduling cost ($\mathrm{VTSC}^{e,r,h}$)

$$= \frac{\sum_{i=1}^{n} [\max[D_i - C_i; 0]e_i + (C_i - A_i)h_i + \max[C_i - D_i; 0]r_i - \mathrm{MSC}^{e,r,h}]^2}{n} \tag{16}$$

The rules considered in the present study are now presented. We wish to mention here that the rules are not intended to address the multi-criteria optimization in assembly job-shops; rather a given rule is primarily intended to address a single measure of performance, and the performance of the rule with respect to other measures is also observed in order to get a complete picture of the performance of the rule.

4. Rules under investigation in phase 1 and phase 2

In the first phase of the study, the un-weighted form of the rules are converted into the weighted form by the use of the factor (r_i/e_i). This factor is designed due to the following two reasons: (i) the costs of earliness and tardiness are conflicting in nature, and hence, we have the term '(r_i/e_i)', and (ii) a job having larger r_i and smaller e_i needs to be given higher priority than the job having smaller r_i and larger e_i. In the second phase, the un-weighted form of the rules are converted into the weighted form by the use of the factor ($h_i + (r_i/e_i)$). This factor is designed due to the following reasons: (i) the costs of earliness and tardiness are conflicting in nature, and hence we have the term '(r_i/e_i)' and (ii) a job with larger 'h_i' and 'r_i' values needs to be given higher priority.

$$\text{Let } w_i = \frac{r_i}{e_i}, \tag{17}$$

when the objective function involves the consideration of weighted earliness and weighted tardiness of jobs.

$$\text{Let } w_i = (h_i + (r_i/e_i)), \tag{18}$$

when the objective function involves the consideration of weighted earliness, weighted tardiness and weighted flowtime of jobs.

When $w_i=(r_i/e_i)$, the notation $W^{e,r}$ is used as a prefix for the dispatching rules, while $W^{e,r,h}$ is used as a prefix when $w_i=(h_i+(r_i/e_i))$. For example, in the case of SPT rule, when the objective function involves the consideration of weighted earliness and weighted tardiness of jobs in the first phase of the current work, the rule is denoted as $W^{e,r}$(SPT) and given by t_{ij}/w_i, where $w_i=(r_i/e_i)$; and when the objective function involves the consideration of weighted earliness, weighted tardiness and weighted flowtime of jobs in the second phase, the rule is denoted as $W^{e,r,h}$(SPT) and given by t_{ij}/w_i, where $w_i=(h_i+(r_i/e_i))$. Totally, 11 rules in the first phase and 12 rules in the second phase have been investigated in this study. The mechanics of some rules are illustrated in Appendix A.

4.1. W(TWKR) rule

The TWKR (total work content of all operations remaining to be done on items of the job) rule is modified in the current study to incorporate the weights. The priority index of the item of job i for operation j is given as follows:

$$Z_{ij} = \mathrm{TWKR}_i/w_i. \tag{19}$$

The item with the minimum value of Z_{ij} is taken up for loading. Since the basic TWKR rule is found to be good in minimizing mean tardiness (in the case of shops with high utilization level and tight due-date setting), an attempt is made in this study to evaluate its performance when weights are considered. With respect to tie-breaking, performance analysis has been made in this study by making use of importance ratio (IR) (Phillipoom et al., 1991), relative remaining operations (RRO) and relative remaining processing times (RRP) (Adam et al., 1987), and operation synchronization date (OSD) (Reeja & Rajendran, 2000a) as tie-breaking rules in assembly job-shops. It is found after pilot simulation experiments that the OSD rule performs better than the IR, RRO and RRP rules. The superior performance of the OSD rule is attributed to the proper pacing and coordination of items and sub-assemblies, thereby reducing the staging-delay component in the waiting time of items. Hence, the OSD rule is used for tie-breaking in this study. See Appendix A for an illustration of OSD calculations.

4.2. W(SPT) rule

This rule is widely used in job-shops and assembly job-shops. In job-shops, the SPT rule, in weighted form, is shown to be quite effective in minimizing the weighted mean flowtime, especially under highly loaded conditions (Vepsalainen & Morton, 1987). The rule is also found to be robust to changes in the levels of experimental factors and perform well in minimizing the average weighted tardiness (Kutanoglu & Sabuncuoglu, 1999). The priority index of the item of job i for operation j is given as follows:

$$Z_{ij} = t_{ij}/w_i \tag{20}$$

The item with the minimum value of Z_{ij} is chosen for loading. In case of any tie, choose the item with the least OSD value.

4.3. W(JDD) rule

The JDD (job due-date) rule is found to perform well in assembly job-shops, and the job due-date calculation is based on the job arrival time and critical path time (i.e. the sum of the processing times of items along the critical path) (Adam et al., 1993; Goodwin & Goodwin, 1982; Russell & Taylor, 1985). This rule is otherwise also called as EDD (earliest due date) rule in the literature on job-shop scheduling. In the current study, the term 'JDD' is modified by the weights. The priority index of the item of job i for operation j is given as follows:

$$Z_{ij} = D_i/w_i. \tag{21}$$

The item with the minimum value of Z_{ij} is taken up for loading. Since all items of the same job have the same value of Z_{ij}, we use the OSD value for tie-breaking.

4.4. W(ECT) rule

This rule makes use of the completion time of the final assembly (i.e. job completion time). The earliest completion time (ECT) of a job is computed by considering arrival time and processing times of operations along the critical path of the job (see Appendix A for details of ECT computation). The priority index of the item of job i for operation j is given as follows:

$$Z_{ij} = \mathrm{ECT}_i/w_i. \tag{22}$$

The item with the minimum value Z_{ij} is taken up for loading. This rule is used with the OSD as the tie-breaker.

4.5. W(PT-BY-TIS) rule

In the case of assembly job-shops, the well-known shortest processing time (SPT) rule is not found to perform quite well in its un-weighted form for minimizing the mean values of flowtime. Hence we modify the SPT rule by considering the time spent in the system by the job up to the current operation, and the rule is then weighted by the weights. According to this rule, a job that has a lower operation-processing time, and, at the same time, resident for a longer time in the system, in relation to other jobs in the queue, is given a higher priority.

The W(PT-BY-TIS) rule is explained below:

Time-in-system $(\mathrm{TIS}_i) = (T - A_i)$, and the priority index of the item of job i for operation j is given by

$$Z_{ij} = (t_{ij}/\,\mathrm{TIS}_i) \times (1/w_i). \tag{23}$$

The item with the minimum value of Z_{ij} is taken up for loading. Ties are broken by choosing the item with the minimum OSD value. This rule seeks to minimize the mean, maximum and variance of the objective function.

4.6. W(TWKR-BY-TIS) rule

The total work remaining (TWKR) rule in its un-weighted form performs well with respect to mean values of flowtime and tardiness in some cases. Hence it is modified by considering the time spent by the job in the system up to the current operation and by the relevant weights.

$$\mathrm{TWKR}_i = \sum_{k \in O_{ij}} t_{ik},$$

The W(TWKR-BY-TIS) rule works as follows:

Time-in-system $(\mathrm{TIS}_i) = (T - A_i)$, and

$$Z_{ij} = \frac{\mathrm{TWKR}_i}{\mathrm{TIS}_i} \times \frac{1}{w_i}. \tag{24}$$

The item with the minimum value of Z_{ij} is taken up for loading. Ties are broken by choosing the item with the minimum OSD value. This rule attempts to minimize the objective function with respect to its mean, maximum and variance values, by bringing the term ‘TIS’ in the denominator.

4.7. W(LBE+LBT) rule (used in the first phase of the current study)

This rule computes the lower bound on the completion time of job i, and hence computes the lower bounds on the earliness cost (or weighted earliness) and the tardiness cost (or weighted tardiness) of job i. The lower bound on the completion time of job i, computed with respect to operation j, (LBCT_{ij}) at instant T is given as follows:

$\mathrm{LBCT}_{ij} = T + \mathrm{ECT}_i - \mathrm{LST}_{ij}$, where LST_{ij} is the latest start time of operation j of job i (see Appendix A for computational details for latest start times). Hence the lower bound on the sum of weighted earliness and weighted tardiness of job i (computed with the consideration of operation j) ($\mathrm{LBSC}_{ij}^{e,r}$) at instant T is given as follows:

$$\mathrm{LBSC}_{ij}^{e,r} = \max\{(LBCT_{ij} - D_i); 0\} r_i - \max\{D_i - \mathrm{LBCT}_{ij}; 0\} e_i.$$

We set

$$Z_{ij}^{\prime} = \mathrm{LBSC}_{ij}^{e,r}. \tag{25}$$

The item with the largest value of $Z_{ij}^{\prime}$ is taken up for loading. Ties are broken by choosing the item with the minimum OSD value. This rule attempts to minimize the maximum and variance values of the sum of weighted earliness and weighted tardiness of jobs.

4.8. W(LBE+LBT+LBF) rule (used in the second phase of the current study)

This rule computes the lower bound on the completion time of job i, and hence computes the lower bounds on the earliness cost (or weighted earliness), the tardiness cost (or weighted tardiness) and the holding cost (or weighted flowtime) of job i. The lower bound on the completion time of job i, computed with respect to operation j, at instant T is given as follows:

$$\mathrm{LBCT}_{ij} = T + \mathrm{ECT}_i - \mathrm{LST}_{ij}.$$

Hence the lower bound on the sum of weighted earliness, weighted tardiness and weighted flowtime of job i (computed with the consideration of operation j) ($\mathrm{LBSC}_{ij}^{e,r,h}$) at instant T is given as follows:

$$\mathrm{LBSC}_{ij}^{e,r,h} = \max\{(\mathrm{LBCT}_{ij} - D_i); 0\}r_i - \max\{D_i - \mathrm{LBCT}_{ij}; 0\}e_i + (\mathrm{LBCT}_{ij} - A_i)h_i.$$

We set

$$Z_{ij}^{l} = \mathrm{LBSC}_{ij}^{e,r,h}. \tag{26}$$

The item with the largest value of Z_{ij}^{l} is taken up for loading. Ties are broken by choosing the item with the minimum OSD value. This rule attempts to minimize the maximum and variance values of the sum of weighted earliness, weighted tardiness and weighted flowtime of jobs.

4.9. FIFO rule

This rule is often used in job-shop scheduling. The FIFO (first in first out) rule works as follows:

$$Z_{ij} = \mathrm{QT}_{ij}. \tag{27}$$

The item with the minimum value of Z_{ij} is taken up for loading. Ties are broken by choosing the item with the minimum OSD value.

4.10. W(COVERT) rule

The basic COVERT (cost over time) rule works as follows: if an item has zero or negative slack, then the expected cost per unit time is related to the tardiness cost; if the slack exceeds an estimate of the remaining waiting time over remaining operations, the expected cost is zero; or if slack is between these two extremes, the cost goes up linearly as slack decreases. In the context of the current problem statement, when the objective function involves the consideration of weighted earliness and weighted tardiness of jobs, we modify the rule and define the priority index of the item of job i for operation j as follows:

$$a = \left\{ \begin{array}{ll} 1/e_i, & \text{if } s_{ij} \geq \sum_{\ell \in O'_{ij}} kw_{i\ell}, \\ \left\{ \left(\sum_{\ell \in O'_{ij}} kw_{i\ell} - s_{ij} \right) (r_i/e_i) / \sum_{\ell \in O'_{ij}} kw_{i\ell} \right\}, & \text{if } 0 \leq s_{ij} < \sum_{\ell \in O'_{ij}} kw_{i\ell}, \text{ and} \\ r_i, & \text{if } s_{ij} < 0. \end{array} \right\},$$

and

$$Z_{ij}^{l} = a/t_{ij}, \tag{28}$$

When the objective function involves the consideration of weighted earliness, weighted tardiness and weighted flowtime of jobs, we modify the rule and define the priority index of the item of job i for operation j as follows:

$$a = \left\{ \begin{array}{ll} h_i + 1/e_i, & \text{if } s_{ij} \geq \sum_{\ell \in O'_{ij}} kw_{i\ell}, \\ \left\{ \left(\sum_{\ell \in O'_{ij}} kw_{i\ell} - s_{ij} \right) \times (r_i/e_i) / \sum_{\ell \in O'_{ij}} kw_{i\ell} \right\} + h_i, & \text{if } 0 \leq s_{ij} < \sum_{\ell \in O'_{ij}} kw_{i\ell}, \text{ and} \\ (h_i + r_i), & \text{if } s_{ij} < 0. \end{array} \right\},$$

and

$$Z'_{ij} = a/t_{ij}, \tag{29}$$

In both the cases, the item with the largest Z'_{ij} is chosen for loading. In the above, w_{il} denotes the estimated waiting time of job i for operation l, s_{ij} denotes the slack of job i with respect to operation j and parameter k is used to factor in the estimated waiting time. The term O'_{ij} refers to the set of remaining operations on job i, including the current operation j, along the path from operation j up to the final assembly.

We have the slack of job i for operation j, given as follows:

$$s_{ij} = D_i - \text{RWK}_{ij} - T, \text{ where } \text{RWK}_{ij} = \text{ECT}_i - \text{LST}_{ij}.$$

The modification is done in view of the fact that a job with larger e_i will have less priority than that with less e_i, and that a job with larger r_i will have more priority than that with less r_i. Also, when the job's slack is found to be negative, the term 'a' consists of the weight for tardiness, and weights for tardiness and flowtime in the first and second phases of the study, respectively. Similarly, when the job is likely to be early, the term 'a' consists of the weight for earliness, and weights for earliness and flowtime in the first and second phases of the study, respectively. When the job's slack is positive but less than the estimated waiting time, then the term 'a' includes weights for tardiness and earliness, and weights for tardiness, earliness and flowtime in the first and second phases of the study, respectively. In the above, RWK_{ij} denotes the total work content (or processing times) of items of job i, including current operation j, along the path from operation j to final assembly. The estimated waiting time w_{il} for operation l of a job i is set equal to the dynamically calculated average waiting time of items in the queue of the machine (Russell, Dar-El, & Taylor, 1987). The estimated waiting time is calculated for the item along the path from its current operation up to the final assembly. In the current study, we have found the value of $k=1$ performing the best, as opposed to the values of $k=0.5$, 1.5, and 2.0, after pilot runs.

4.11. W(CR) rule

The critical ratio (CR) of a job is calculated as the difference between the job due-date and the current time divided by its total remaining processing time. Composite rules, incorporating the CR values, hourly total cost (HTC) and a threshold parameter, p, have been proposed by Yang and Sum (1994). In the current study, weights for earliness, tardiness and flowtime are included in the rule proposed by Yang and Sum. Also, the total remaining work content (or processing times) of items of job i, including current operation j, is calculated along the path from operation j to final assembly, and is used in estimating the remaining work content of the job, RWK_{ij}. The threshold parameter, p, has been tested for different values (namely, 1, 1.5 and 2), and the value of 1.0 is found to be giving good results for the assembly shop environment. When the objective function involves the consideration of weighted earliness and weighted tardiness of jobs, we compute

$$\mathrm{CR}_i = \frac{D_i - T}{\mathrm{RWK}_{ij}}, \qquad \mathrm{HTC}'_i = (r_i/e_i), \qquad \mathrm{HTC}_i = r_i, \qquad \mathrm{WCR}_i = \frac{\mathrm{CR}_i}{\mathrm{HTC}'_i} \tag{30}$$

When the objective function involves the consideration of weighted earliness, weighted tardiness and weighted flowtime of jobs, we compute

$$\mathrm{CR}_i = \frac{D_i - T}{\mathrm{RWK}_{ij}}, \qquad \mathrm{HTC}'_i = (h_i + (r_i/e_i)), \qquad \mathrm{HTC}_i = (h_i + r_i), \qquad \mathrm{WCR}_i = \frac{CR_i}{HTC'_i} \tag{31}$$

For items with CR values less than p, the item with the largest total cost (HTC_i) is scheduled first. Items having CR values greater than or equal to p are scheduled according to the weighted CR (i.e. WCR_i) value.

The rule works as follows: form a set of items for which CR $< p$; if the item set is not a null set, then choose the item from the item set with the largest value of HTC; else choose the item (from the waiting line) with the least WCR value.

4.12. W(EXPET) rule

This rule is similar in basic form to COVERT rule in the sense that it uses slack and cost information. It also includes early cost information and a look-ahead exponential function. The primary feature of this rule is to recognize approaching tardiness and adjust job priorities accordingly. When the objective function involves the consideration of weighted earliness and weighted tardiness of jobs, we define the priority index of the item of job i for operation j as follows:

Set

$$a = \left\{ \exp\left[-\frac{(r_i + e_i)}{e_i \times t_{avg}} \max\{0, (D_i - \mathrm{RWK}_{ij} - T)\} \right] \right\},$$

and

$$Z'_{ij} = \frac{a r_i}{t_{ij}}. \tag{32}$$

When the objective function involves the consideration of weighted earliness, weighted tardiness and weighted flowtime of jobs, the rule is modified and defined as follows:

Set

$$a = \left\{ \exp\left[-\frac{(r_i + e_i)}{e_i \times t_{\mathrm{avg}}} \max\{0, (D_i - \mathrm{RWK}_{ij} - T)\} \right] \right\},$$

and

$$Z'_{ij} = \frac{h_i + a r_i}{t_{ij}}. \tag{33}$$

The item with the largest Z'_{ij} is chosen for loading. The priority index of the item of job i is calculated by considering the scaled weight for tardiness (scaling being done by taking into account the weights for both tardiness and earliness). The term t_{avg} is the average of processing times of all jobs in the queue considered.

4.13. W(ATC) rule

The apparent-tardiness-cost (ATC) rule (Vepsalainen and Morton, 1987) is similar to the COVERT rule with two main differences: the slack is considered to be the local resource-constrained slack which takes into account the waiting times on downstream machines and the decay function is exponential rather than linear. When the objective function involves the consideration of weighted earliness, weighted tardiness and weighted flowtime of jobs, we define the priority index of the item of job i for operation j and modify the rule as follows:

Set

$$a = \exp\left[-\max\left\{\left(D_i - \sum_{l\in O'_{i,j+1}}(w_{il}+t_{il}) - T - t_{ij}\right)/(k'\times t_{avg}); 0\right\}\right],$$

and

$$Z'_{ij} = \begin{Bmatrix} (h_i + ar_i)/t_{ij}, & \text{if } a=1 \\ (h_i + (ar_i/e_i))/t_{ij}, & \text{otherwise} \end{Bmatrix} \tag{34}$$

The item with the largest Z'_{ij} is chosen for loading. The term t_{avg} represents the average processing time of all jobs waiting to be processed at the machine where operation j of job i is to be performed. The exponential look-ahead parameter k' is set at 3 in this study (as in the work by Vepsalainen & Morton, 1987). It is also found from the pilot runs carried out in the current study that the rule with $k'=3$ performs the best, as against other settings of k', in the assembly job-shop environment as well. Similarly, we have set $w_{il}=2t_{il}$ (as recommended by Vepsalainen and Morton, and the setting being subsequently validated in our pilot runs). The rationale for the modification of the basic ATC rule in this study is that when the job is likely to be tardy, the numerator in Eq. (34) consists of the sum of weights for flowtime and tardiness, and consists of the sum of the weights for flowtime and the ratio of the scaled weight for tardiness and earliness, otherwise.

It is to be noted that the existing rules such as TWKR and SPT rules have been modified in this current study (denoted by W(TWKR), W(SPT), W(JDD), W(ECT), FIFO, W(COVERT), W(CR), W(EXPET) and W(ATC)), while new rules have also been proposed in this work (see rules W(PT-BY-TIS), W(TWKR-BY-TIS), W(LBE+LBT) and W(LBE+LBT+LBF)).

5. Simulation model of the assembly shop

From the available literature on the simulation study of job-shops, it is seen that the number of machines considered in the case of a hypothetical shop is typically small, usually of the order of 10 (Blackstone et al., 1982). Certain actual real-life shops models have considered more than 100 machines (Rowe, 1960), but there is no evidence that the number of machines more than seven has a crucial influence on the performance of dispatching rules (Baker & Dzeilinski, 1960). Though the finding by Baker and Dzeilinski was based on a simulation study concerning job-shops, it can be considered to hold good for assembly job-shops as well. In fact, many simulation studies deal with models of assembly job-shops considering about eight machines (e.g. Adam et al., 1987, 1993). The hypothetical assembly job-shop model considered in this study is an open-shop configuration consisting of ten work-centers with two identical machines in each work-center. The model considers one assembly station, but the number of sub-assembly stations varies with the product structure. The assumptions followed in this study are standard ones (e.g. no pre-emption of processing, no alternate machine routings, and no machine breakdowns) that are present in other studies as well (e.g. Adam et al., 1993). The job order arrivals are assumed to follow Poisson distribution. The inter-arrival times of the jobs are exponentially distributed with a mean so chosen to yield the desired utilization level of the shop. The process times for operations at various levels are uniformly distributed in the range [1, 20] for all the three different types of product structures. The due-date allowance factor is set at 1, 2 and 3, representing tight, medium and loose due-date settings, respectively. The due-date of a job is calculated by multiplying the due-date allowance factor with sum of processing times on the critical path and adding it to the arrival time of the job (Adam et al., 1993). In the first phase of this study, the weights (or relative cost rates) of earliness and tardiness of a job are sampled from uniform distributions in the range [1, 10] and [10, 20], respectively. Thus on an average, the weight for tardiness of a job is approximately three times the weight for earliness of the job, in view of the fact that the weight for tardiness of a job is more than the weight for earliness of the job in most situations.

In the second phase of this study, the experimental settings utilized, such as utilization levels, job structures and allowance factors, are the same as those used in the first phase of the current study. The weights relating to tardiness of jobs are sampled from a uniform distribution in the range [10, 20], the weights relating to holding cost of jobs are sampled from a uniform distribution in the range [1, 10], and the weights relating to earliness cost of jobs are sampled from a uniform distribution in the range [sampled holding cost, 10]. The rationale in the setting of these costs is as follows: (a) the mean weight for tardiness is approximately three times the mean weight for flowtime, the reason being that the penalty for tardiness is very high in real-life situations, and (b) likewise, the weight for earliness is more than the weight for flowtime, but less than that for tardiness, because earliness is associated with the finished-goods inventory.

The product structures considered are as follows:

single-level (Fig. 1(a)): the lowest level consists of a set of components and the next highest level consists of the final assembly;

two-level (Fig. 1(b)): the lowest level consists of the components; the intermediate level consists of the sub-assemblies; the highest level consists of the final assembly;

three-level (Fig. 1(c)): the lowest level consists of the components; next higher level consists of the sub-sub-assemblies; next higher level consists of the sub-assemblies; and the highest level consists of the final assembly.

For the different product structures, the number of items at different levels varies according to the uniform distribution in a given range, except for the final assembly. In the current study, it is to be noted that the term ‘item’ refers to either a component, or a sub-assembly, or a sub-sub-assembly, or the final assembly. The number of operations and the processing times for each operation of the items for the three different product structures are assumed to be uniformly distributed. A higher-level item is assembled from the immediate lower-level items, and hence its processing can start only when the processing of all its lower level items is completed. The three types of job structures considered in this study are single-level structure [3–8], two-level structure [4–7][3–6] and three-level structure [3–6][3–6]

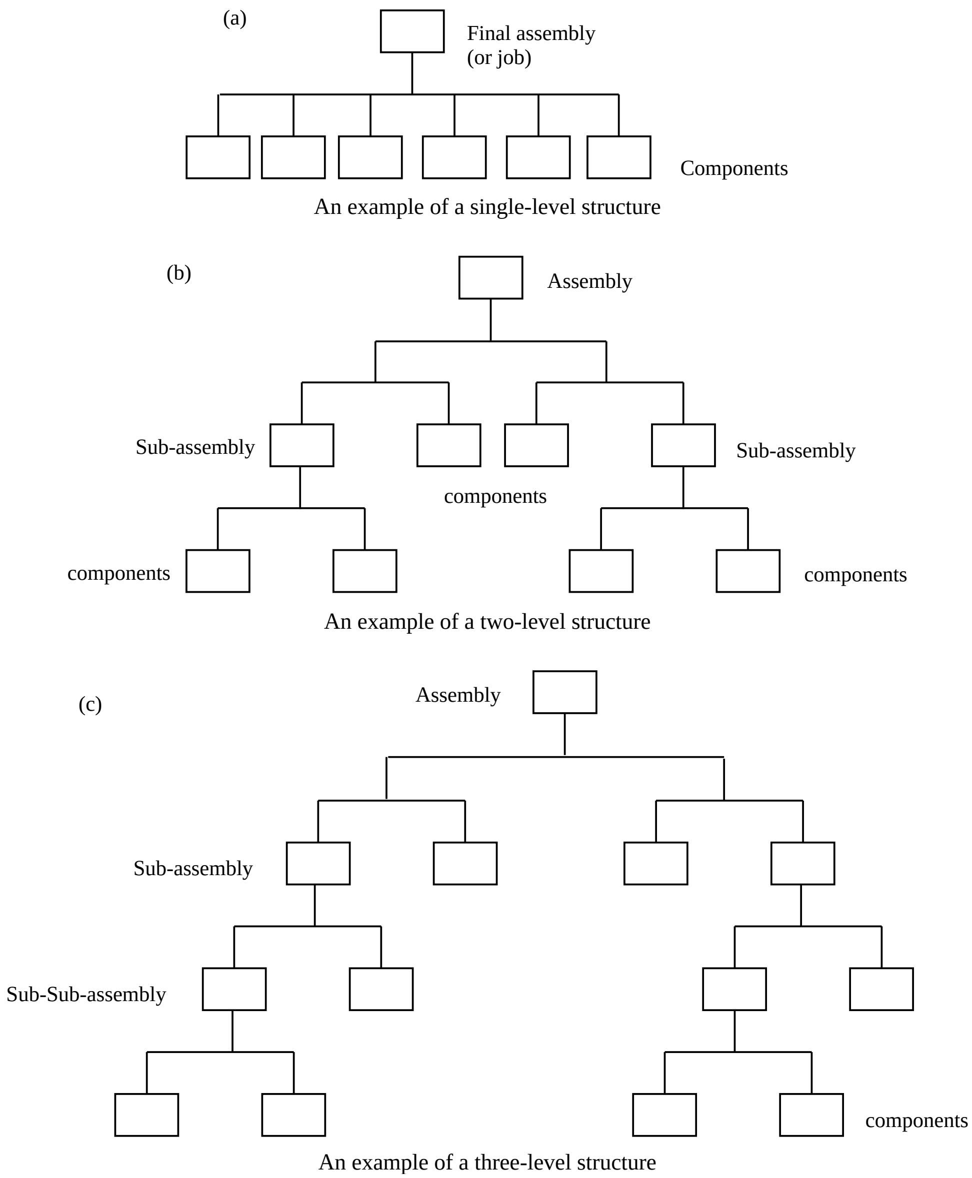


Fig. 1. (a) An example of a single-level structure. (b) An example of a two-level structure. (c) An example of a three-level structure.

[3–6]. To explain further, the two-level structure [4–7][3–6] means that the job has three levels of operations. The number of items at the lowest level is sampled in the range [3–6]. The number of items at the middle level is sampled in the range [4–7]. The highest level denotes the final assembly. The number of items and the number of operations per item are sampled from discrete uniform distributions. The routings for items are independent and generated randomly, with every work-center having the same probability of being chosen. The utilization levels considered in this study are 85 and 95%. In all, we have three different job structures, two different utilization levels and three due-date settings, thereby making 18 simulation experiments for every dispatching rule evaluated in the study.

The experimental settings chosen for the current study are similar to those chosen by Adam et al. (1993); Reeja and Rajendran (2000a), and Thiagarajan and Rajendran (2003). On the basis of the pilot runs, is has been found that such job structures with the chosen distribution of the number of components/sub-assemblies and the distribution of the process time yield values of earliness/tardiness of jobs in most experimental settings, such that the settings bring out the distinction clearly between the rules under investigation. Machine-utilization levels and moving averages of job flowtimes are observed to ascertain when the system reaches the steady state. In this study, it is observed that steady state is attained after the arrival of 1000 job orders. Typically, the total sample size of a simulation study is of the order of thousands of job completions (Blackstone et al., 1982; Conway et al., 1960). For a given sample size, it is preferable to have a smaller number of replications and a larger run length (Law & Kelton, 1984). Following these guidelines, the number of replications (runs) is fixed at 30 in each simulation experiment, and the run length for each replication is fixed to cover 2000 job completions (i.e. $n=2000$). In each replication, the jobs (i.e. job-orders) are numbered on arrival. Statistics are collected for jobs that are numbered from 1001 to 3000 at the time of entry into the shop. The shop is continuously loaded with jobs until the completion of these 2000 jobs is over, in order to overcome the problems of censored data (Conway et al., 1960).

In this work antithetic random numbers are used between two consecutive replications for the same rule while the common random numbers approach is used across different rules (Law & Kelton, 2000; Narsingh Deo, 1979). The statistical analysis of the experimental data with single factor ANOVA with randomised block design (Lorenzen & Anderson, 1993; Montgomery, 1991) has shown that this sample size (of 60,000 jobs or job-orders) yields a variance that results in a Type I error of at most 1%.

The simulation experiments are performed to evaluate different dispatching rules considering the three different product structures for all experimental configurations. The mean of every measure of performance is obtained for each replication. The mean of the values of the measures of performance for the first phase of the study (Eqs. (1)–(9)) over 30 replications are observed and reported in Tables 1–5. In all, in the first phase of this current work, a total of 17.82 millions of jobs (or equivalently, job-orders) (corresponding to 11 scheduling rules, 3000 jobs per replication, 30 replications, three job structures, two utilization levels and three due-date settings), with every job requiring several operations corresponding to items and sub-assemblies, have been generated in the simulation experiments of this phase of this study. The mean of the values of the measures of performance for the second phase of the study (Eqs. (10)–(16)) over 30 replications are observed and reported in Tables 6–10. In all, in the second phase of this current work, a total of 19.44 millions of jobs (or equivalently, job-orders) (corresponding to 12 scheduling rules, 3000 jobs per replication, 30 replications, three job structures, two utilization levels and three due-date settings), with every job requiring several operations corresponding to items and sub-assemblies, have been generated in the simulation experiments of this phase of this study.

Duncan's multiple range test has been used to identify the best subset of rules, second best subset of rules, and so on. This is done after conducting the test on the absolute values of performance measures yielded by the rules with respect to every performance measure. Among the rules under evaluation, '1' indicates the set of best performing rules, '2' indicates the set of next best rules, and '3' indicates the set of the following best set of rules (Tables 1–10). For the sake of

Table 1
Performance of rules with respect to $WMSC^{e,r}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r}$(TWKR)	287.14[1]	552.18[1]	144.82	374.43[1]	202.87	340.07
$W^{e,r}$(LBE+LBT)	297.04[2]	612.49[3]	103.29[1]	402.03[2]	111.24[2]	220.05[1]
$W^{e,r}$(TWKR-BY-TIS)	289.33[1]	592.86[2]	111.88[2]	378.63[1]	157.43[3]	237.50[2]
$W^{e,r}$(PT-BY-TIS)	298.95[2]	610.94[3]	127.45[3]	402.02[2]	161.77[3]	277.69
$W^{e,r}$(ECT)	372.98	761.78	235.70	595.36	268.97	567.76
$W^{e,r}$(JDD)	372.98	761.78	234.78	597.55	266.83	572.85
$W^{e,r}$(COVERT)	363.30	824.36	103.37[1]	524.46	113.79[2]	276.04
$W^{e,r}$(EXPET)	340.61[3]	815.77	109.07[2]	494.98	194.99	257.95[3]
FIFO	405.52	911.29	203.79	692.77	159.40[3]	492.58
$W^{e,r}$(SPT)	371.00	830.75	219.61	649.08	240.45	587.42
$W^{e,r}$(CR)	359.86	779.60	107.12[2]	478.97[3]	105.36[1]	236.46[2]
(b) For two-level job structures						
$W^{e,r}$(TWKR)	328.25[2]	630.89[1]	154.96	394.83[3]	300.24	395.36
$W^{e,r}$(LBE+LBT)	329.44[2]	685.55[3]	97.98[1]	385.81[2]	182.13[2]	177.06[1]
$W^{e,r}$(TWKR-BY-TIS)	308.49[1]	641.44[2]	107.00[2]	344.95[1]	290.04	209.32[2]
$W^{e,r}$(PT-BY-TIS)	358.76[3]	736.71	138.06[3]	443.79	254.30	291.86[3]
$W^{e,r}$(ECT)	561.68	1215.03	344.97	964.52	367.03	899.01
$W^{e,r}$(JDD)	564.11	1213.13	346.73	966.65	366.98	902.22
$W^{e,r}$(COVERT)	571.82	1317.72	318.04	1040.91	199.97[3]	856.72
$W^{e,r}$(EXPET)	556.44	1311.04	294.22	1023.29	161.68[1]	698.23
FIFO	559.80	1366.96	272.36	1059.16	213.05	761.95
$W^{e,r}$(SPT)	566.92	1316.66	331.66	1047.00	334.64	939.74
$W^{e,r}$(CR)	587.83	1309.12	344.42	1042.85	187.22[2]	781.23
(c) For three-level job structures						
$W^{e,r}$(TWKR)	709.39[3]	1083.36[2]	259.14[3]	552.67[3]	586.60	631.36
$W^{e,r}$(LBE+LBT)	643.00[2]	1080.62[2]	161.26[1]	481.42[2]	369.29[1]	290.57[1]
$W^{e,r}$(TWKR-BY-TIS)	632.73[1]	1027.71[1]	167.48[1]	435.36[1]	631.85	426.60[2]
$W^{e,r}$(PT-BY-TIS)	722.26	1260.21[3]	245.38[2]	669.33	553.05	524.52[3]
$W^{e,r}$(ECT)	1393.97	2452.08	839.29	1867.97	737.01	1631.70
$W^{e,r}$(JDD)	1393.97	2452.08	839.16	1865.32	739.22	1627.18
$W^{e,r}$(COVERT)	1299.44	2672.21	731.22	2076.49	466.72[3]	1686.84
$W^{e,r}$(EXPET)	1248.18	2714.29	675.15	2113.76	388.50[2]	1675.07
FIFO	1126.84	2386.26	506.35	1747.73	380.60[2]	1197.28
$W^{e,r}$(SPT)	1295.90	2658.99	742.02	2066.78	665.81	1802.40
$W^{e,r}$(CR)	1412.33	2513.15	854.44	1937.53	512.70	1508.15

simplifying the presentation of the results, we do not indicate the further set of worse rules. The presence of a set of numbers against a given rule indicates that there is no significant difference between the rule and the sets of rules indicated by the corresponding set of numbers marked against the rule. For the sake of making the paper concise, we do not indicate the evaluation of rules with respect to all measures of performance mentioned in Section 3; we have rather chosen to include the most important ones in the tables.

All programs are coded in C++ and run on LINUX cluster of machines.

Table 2
Performance of rules with respect to $MSC^{e,r}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r}$(TWKR)	4307.43^{1}	8282.90^{1}	1901.40	5496.06^{1}	1607.60	4301.85
$W^{e,r}$(LBE+LBT)	4455.89^{3}	9187.69	1473.56^{3}	6027.95	726.81^{2}	3193.80^{1}
$W^{e,r}$(TWKR-BY-TIS)	4340.25^{2}	8893.05^{2}	1523.14	5666.95^{2}	1061.79	3296.13^{2}
$W^{e,r}$(PT-BY-TIS)	4484.48	9164.47^{3}	1749.65	6007.24^{3}	1204.84	3846.63
$W^{e,r}$(ECT)	5595.41	11427.61	3259.56	8775.58	2750.59	7671.92
$W^{e,r}$(JDD)	5595.41	11427.61	3246.56	8808.93	2721.98	7751.49
$W^{e,r}$(COVERT)	5449.82	12366.30	1398.17^{1}	7825.83	707.90^{1}	3895.66
$W^{e,r}$(EXPET)	5109.46	12237.72	1427.71^{2}	7411.06	1161.64	3563.82
FIFO	6082.67	13668.10	2973.34	10386.73	1476.21	7293.52
$W^{e,r}$(SPT)	5565.28	12461.68	3068.93	9635.67	2419.05	8173.38
$W^{e,r}$(CR)	5398.34	11694.55	1523.01	7181.12	735.40^{3}	3444.35^{3}
(b) For two-level job structures						
$W^{e,r}$(TWKR)	4923.92^{2}	9464.71^{1}	1804.75	5697.81^{2}	1995.88	4516.31
$W^{e,r}$(LBE+LBT)	4941.76^{3}	10284.74^{3}	1281.76^{2}	5783.36^{3}	1085.23^{1}	2409.88^{1}
$W^{e,r}$(TWKR-BY-TIS)	4627.46^{1}	9622.31^{2}	1156.00^{1}	5144.28^{1}	1658.94^{3}	2485.28^{2}
$W^{e,r}$(PT-BY-TIS)	5381.45	11051.96	1736.98^{3}	6623.45	1619.92^{2}	3833.58^{3}
$W^{e,r}$(ECT)	8426.14	18228.91	4867.98	14302.91	3895.12	12465.90
$W^{e,r}$(JDD)	8462.56	18200.30	4898.17	14337.14	3906.27	12513.33
$W^{e,r}$(COVERT)	8578.10	19768.72	4712.36	15589.01	2685.05	12794.18
$W^{e,r}$(EXPET)	8347.63	19669.15	4356.07	15350.11	1824.47	10454.55
FIFO	8396.84	20505.83	3987.68	15888.08	1916.83	11378.42
$W^{e,r}$(SPT)	8504.63	19752.79	4738.32	15607.58	3565.19	13340.22
$W^{e,r}$(CR)	8818.28	19640.12	5148.68	15645.87	2592.19	11719.10
(c) For three-level job structures						
$W^{e,r}$(TWKR)	10641.26^{3}	16251.95^{3}	3095.09	7889.07^{3}	3659.87	6085.65
$W^{e,r}$(LBE+LBT)	9644.92^{2}	16209.77^{2}	1988.62^{2}	7139.85^{2}	2161.82^{1}	3013.21^{1}
$W^{e,r}$(TWKR-BY-TIS)	9490.96^{1}	15415.54^{1}	1716.71^{1}	6335.54^{1}	3508.97	3426.46^{2}
$W^{e,r}$(PT-BY-TIS)	10834.37	18904.57	2998.79^{3}	9875.32	3345.33^{3}	5756.27^{3}
$W^{e,r}$(ECT)	20911.60	36785.58	12305.09	27853.36	8717.62	22911.00
$W^{e,r}$(JDD)	20911.60	36785.58	12304.63	27816.84	8747.94	22843.78
$W^{e,r}$(COVERT)	19493.19	40086.47	10882.90	31101.10	6523.33	25139.51
$W^{e,r}$(EXPET)	18725.12	40718.40	10112.95	31707.04	5421.75	25101.55
FIFO	16902.55	35793.03	7501.71	26206.96	3170.55^{2}	17521.92
$W^{e,r}$(SPT)	19440.21	39887.66	10845.18	30868.68	7514.52	25645.08
$W^{e,r}$(CR)	21186.44	37702.19	12807.59	29065.80	7414.71	22608.70

6. Results and discussion of simulation study on performance evaluation of rules: phase 1

The results of the performance evaluation of various rules are presented in Tables 1–5.

6.1. Weighted mean scheduling cost ($WMSC^{e,r}$) (Table 1a–c)

The $W^{e,r}$(TWKR-BY-TIS) rule performs very well in most cases in minimizing the weighted mean sum of weighted earliness and weighted tardiness of jobs. The reason is due to the presence of the terms 'TIS'

Table 3
Performance of rules with respect to $\mathrm{MaxTSC}^{e,r}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r}$(TWKR)	36340.54	93941.47	32884.87	90690.23	29511.13	87487.47
$W^{e,r}$(LBE+LBT)	10418.67[1]	17703.97[1]	6950.70[1]	14407.57[1]	4032.40[1]	11494.07[1]
$W^{e,r}$(TWKR-BY-TIS)	17362.77[2]	36681.30[2]	13385.57[2]	32700.13[2]	9796.53	28820.67[2]
$W^{e,r}$(PT-BY-TIS)	21304.50[3]	41555.70[3]	17738.50[3]	37969.90[3]	14458.53	34526.43[3]
$W^{e,r}$(ECT)	56951.50	147189.27	54382.70	144700.41	51878.33	142211.53
$W^{e,r}$(JDD)	56951.50	147189.27	53296.37	143177.83	49333.57	144411.53
$W^{e,r}$(COVERT)	57481.27	170064.83	38719.20	160986.30	15945.60	124872.66
$W^{e,r}$(EXPET)	43273.07	135485.53	24521.97	119026.47	7352.30[2]	73019.03
FIFO	22607.37	43787.93	18271.23	39143.60	14237.93	34682.57
$W^{e,r}$(SPT)	55785.17	170684.67	52366.50	167044.36	49092.20	163427.73
$W^{e,r}$(CR)	56247.67	183654.80	25928.70	134564.27	8003.83[3]	77560.03
(b) For two-level job structures						
$W^{e,r}$(TWKR)	39409.80	126383.03	34966.33	122649.60	30690.80	118946.37
$W^{e,r}$(LBE+LBT)	13451.87[1]	21924.50[1]	8601.10[1]	17755.13[1]	5521.70[1]	12459.30[1]
$W^{e,r}$(TWKR-BY-TIS)	18835.00[2]	35925.27[2]	13406.10[2]	30614.93[2]	9041.87[2]	25798.43[2]
$W^{e,r}$(PT-BY-TIS)	24061.83[3]	42905.77[3]	19063.27[3]	37635.30[3]	14556.00[3]	32932.90[3]
$W^{e,r}$(ECT)	79674.07	261504.23	75717.77	258224.47	71780.73	254960.50
$W^{e,r}$(JDD)	76432.34	258678.77	74366.60	259743.64	67977.16	256918.67
$W^{e,r}$(COVERT)	72155.43	262696.72	70457.23	238251.97	60533.00	239894.30
$W^{e,r}$(EXPET)	59256.10	203762.27	54893.73	193473.41	40988.17	174789.30
FIFO	30024.90	62725.43	24229.33	56704.37	18659.43	50848.10
$W^{e,r}$(SPT)	73927.40	252102.94	69849.27	248307.50	65947.87	244528.27
$W^{e,r}$(CR)	78656.97	311411.53	75793.50	298567.13	58620.80	273899.00
(c) For three-level job structures						
$W^{e,r}$(TWKR)	67632.40	235877.53	59336.50	228712.09	51344.60	221586.00
$W^{e,r}$(LBE+LBT)	26046.40[1]	45492.77[1]	14937.60[1]	35043.00[1]	10970.07[1]	25058.57[1]
$W^{e,r}$(TWKR-BY-TIS)	33361.90[2]	62739.07[2]	22005.30[2]	52445.70[2]	13935.17[2]	42746.27[2]
$W^{e,r}$(PT-BY-TIS)	49562.07[3]	90661.13[3]	39029.80[3]	80122.00[3]	29555.20[3]	70443.47[3]
$W^{e,r}$(ECT)	163529.67	497903.94	155442.44	491178.41	147452.27	484452.88
$W^{e,r}$(JDD)	163529.67	497903.94	154829.09	499251.22	147768.47	482221.28
$W^{e,r}$(COVERT)	151503.50	531961.38	147217.44	510290.94	136773.70	485102.16
$W^{e,r}$(EXPET)	126999.50	455204.94	120364.34	429691.91	106091.37	416968.53
FIFO	60808.96	141715.06	48459.83	129136.60	36625.04	116782.87
$W^{e,r}$(SPT)	161464.44	502941.69	153250.44	495406.53	145050.77	487871.38
$W^{e,r}$(CR)	170838.80	533663.00	161884.20	517388.03	138634.23	505877.56

and 'w_i' (Eq. 24) in the denominator. The factor 'TIS' does not allow jobs to reside in the system for a long time thereby reducing their chances of getting tardy, and thereby leading to adherence of job completion to job due-date. The factor 'w_i' reckons with the weights for earliness and tardiness, and hence the weighted mean of the sum of weighted earliness and weighted tardiness of jobs is minimized by the $W^{e,r}$(TWKR-BY-TIS) rule. The good performance of this rule is more pronounced for complex job structures. This rule is followed by the rules $W^{e,r}$(TWKR), $W^{e,r}$(LBE+LBT) and $W^{e,r}$(PT-BY-TIS). The $W^{e,r}$(LBE+LBT) rule

Table 4
Performance of rules with respect to $WVTSC^{e,r}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r}$(TWKR)	75003.42	546171.72	60266.70	516173.40	50350.30	454410.22
$W^{e,r}$(LBE+LBT)	14588.89[1]	55250.74[1]	9444.63[1]	46491.22[1]	11584.48[1]	35766.63[1]
$W^{e,r}$(TWKR-BY-TIS)	24884.55[2]	120129.97[2]	18932.19[2]	114155.90[2]	18234.83	90820.49[2]
$W^{e,r}$(PT-BY-TIS)	35996.06[3]	176097.17	28411.06	168395.25	25093.49	136821.94[3]
$W^{e,r}$(ECT)	243775.04	1900568.21	220146.31	1855237.50	189741.35	744814.90
$W^{e,r}$(JDD)	243775.04	1900568.21	215621.90	1878423.05	181317.77	1814898.65
$W^{e,r}$(COVERT)	177823.96	1750083.87	60487.49	1467200.67	14258.79[2]	737862.37
$W^{e,r}$(EXPET)	123087.73	1320830.42	24839.92[3]	739621.76	14874.88[3]	268407.65
FIFO	36908.48	152814.48[3]	32841.34	149044.70[3]	25987.41	139758.48
$W^{e,r}$(SPT)	179275.41	1778142.64	158211.18	1732856.83	131435.82	1621257.85
$W^{e,r}$(CR)	209893.04	2242668.55	30357.40	989161.11	15065.93	280094.91
(b) For two-level job structures						
$W^{e,r}$(TWKR)	79301.74	722456.21	62102.80	681824.27	60829.08	598927.85
$W^{e,r}$(LBE+LBT)	18035.57[1]	60826.61[1]	10521.85[1]	48203.81[1]	22710.87[1]	31427.97[1]
$W^{e,r}$(TWKR-BY-TIS)	24003.52[2]	111519.08[2]	17257.51[2]	105275.34[2]	25785.74[2]	75190.66[2]
$W^{e,r}$(PT-BY-TIS)	45137.50[3]	209344.84[3]	34429.83[3]	199283.95[3]	37238.82[3]	150546.90[3]
$W^{e,r}$(ECT)	448502.96	5110449.41	410144.12	5033229.27	348280.74	4798020.57
$W^{e,r}$(JDD)	449148.08	5093415.81	414657.44	5111278.75	345726.64	4904308.23
$W^{e,r}$(COVERT)	361340.43	4579887.04	326257.86	4370050.89	186416.99	4073443.71
$W^{e,r}$(EXPET)	296254.27	3832755.66	245001.57	3735723.18	80040.27	2578217.55
FIFO	62348.50	273523.01	57742.17	273143.75	45401.25	267859.62
$W^{e,r}$(SPT)	356057.42	4408486.70	324370.48	4345586.56	268215.05	4117357.60
$W^{e,r}$(CR)	462819.81	6433864.86	371489.23	5986522.24	132830.03	4328041.85
(c) For three-level job structures						
$W^{e,r}$(TWKR)	197666.10	1480570.77	158579.28	1406459.57	159765.61	1218898.00
$W^{e,r}$(LBE+LBT)	46691.73[1]	231970.83[1]	29709.74[1]	172575.80[1]	76926.19[2]	115237.62[1]
$W^{e,r}$(TWKR-BY-TIS)	52310.75[2]	266016.80[2]	38022.55[2]	243712.76[2]	60824.22[1]	188427.15[2]
$W^{e,r}$(PT-BY-TIS)	144564.79[3]	711624.22[3]	112474.78[3]	669244.44[3]	124268.14[3]	505782.14[3]
$W^{e,r}$(ECT)	1821448.49	4751293.45	1727225.83	14603753.53	1431069.90	13848763.59
$W^{e,r}$(JDD)	1821448.49	14751293.45	1742229.79	14902188.11	1439276.95	13790837.29
$W^{e,r}$(COVERT)	1424293.32	17289972.93	1338427.46	17331721.27	901402.37	15716539.36
$W^{e,r}$(EXPET)	1219286.33	16865134.10	1071768.41	16641067.13	583500.85	14869983.08
FIFO	190379.06	1557417.44	181126.19	1553521.73	135877.27	1459117.34
$W^{e,r}$(SPT)	1428630.96	17067261.88	1342357.77	16917511.36	1091373.34	16099496.81
$W^{e,r}$(CR)	1902165.54	16170515.62	1655958.03	15511792.44	843590.62	12520711.21

differentiates between earliness and tardiness of jobs, and in the process minimizes the deviation of the completion times of jobs from their due-dates, thereby minimizing the mean sum of weighted earliness and weighted tardiness of jobs as well. In case of job-shops, the basic COVERT rule and the basic EXPET rule are shown to perform well in minimizing the measures of weighted tardiness and the early/tardy cost ratio, respectively (Rohleder & Scudder, 1993; Vepsalainen & Morton, 1987). In our study, we observe that for single-level job structures, the rules $W^{e,r}$(COVERT) and $W^{e,r}$(EXPET) perform well particularly when

Table 5
Performance of rules with respect to $VTSC^{e,r}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r}$(TWKR)	14631525.17	99651007.95	10785966.67	94191127.90	5498982.57	77624355.41
$W^{e,r}$(LBE+LBT)	2807634.05[1]	9120469.81[1]	1848936.41[1]	8956209.35[1]	348762.05[1]	6860426.78[1]
$W^{e,r}$(TWKR-BY-TIS)	5661293.75[2]	25530835.90[2]	3540084.38[2]	23539204.57[2]	1049783.63	16881490.99[2]
$W^{e,r}$(PT-BY-TIS)	7600459.15[3]	35311733.73[3]	5270415.29	33411637.40[3]	1856000.95	24782802.18[3]
$W^{e,r}$(ECT)	42561305.69	309487090.96	37689653.02	304814402.49	27349595.80	279151409.27
$W^{e,r}$(JDD)	42561305.69	309487090.96	36773090.04	308327460.39	25814815.66	290912802.79
$W^{e,r}$(COVERT)	34489145.06	323184016.89	11479908.07	271516308.26	1260293.24	136556054.39
$W^{e,r}$(EXPET)	20620135.75	218623800.09	4293503.61[3]	127944299.78	1042158.50	46598337.74
FIFO	10310203.57	44179395.36	7685556.89	39785736.53	3044731.53	34205860.11
$W^{e,r}$(SPT)	34769844.76	326628689.58	29741078.45	319602964.96	20065583.49	292411042.99
$W^{e,r}$(CR)	36683656.83	365027077.68	5544383.30	163378491.33	553818.11[2]	47073161.93
(b) For two-level job structures						
$W^{e,r}$(TWKR)	15287833.03	125226444.64	10106407.90	118337220.99	4742801.37	94189229.13
$W^{e,r}$(LBE+LBT)	3982888.04[1]	10905011.56[1]	1810411.35[1]	10227411.79[1]	611596.32[1]	5414781.18[1]
$W^{e,r}$(TWKR-BY-TIS)	5726866.22[2]	24325089.26[2]	2480319.23[2]	21723890.04[2]	1403733.79[2]	11552382.65[2]
$W^{e,r}$(PT-BY-TIS)	9735648.36[3]	42246985.01[3]	5882028.82[3]	39504777.85[3]	1976759.11[3]	25322619.22[3]
$W^{e,r}$(ECT)	78790098.63	818725855.21	71140946.36	815230850.12	50802666.97	763637722.86
$W^{e,r}$(JDD)	78856397.39	814476608.17	71862291.58	826034231.47	50381296.51	779917416.77
$W^{e,r}$(COVERT)	66791525.17	767891784.88	60677731.93	743895265.60	32761773.88	692422640.99
$W^{e,r}$(EXPET)	41363499.22	525672848.39	37408371.13	534800755.62	10819662.80	379549892.69
FIFO	17664531.18	82864010.98	13538718.80	75354550.51	5103120.10	67861485.47
$W^{e,r}$(SPT)	65999436.66	744532858.77	59183813.84	742091854.13	40218665.07	691901855.11
$W^{e,r}$(CR)	81340066.83	1027487183.86	65689118.39	967627078.02	22599411.88	698752293.16
(c) For three-level job structures						
$W^{e,r}$(TWKR)	42082515.77	264781778.33	27394621.24	249496366.34	11350651.86	183796030.15
$W^{e,r}$(LBE+LBT)	12296892.32[1]	48857741.93[1]	4906602.53[1]	37898304.28[1]	2589271.28[1]	13754098.83[1]
$W^{e,r}$(TWKR-BY-TIS)	15184784.37[2]	64188879.01[2]	5191347.90[2]	51594863.97[2]	5164177.74[2]	20030871.12[2]
$W^{e,r}$(PT-BY-TIS)	33158323.57[3]	149085722.04[3]	19590820.40[3]	135229042.76[3]	7071576.75[3]	76923207.60[3]
$W^{e,r}$(ECT)	335143639.15	2439666996.43	317738025.92	2448291059.42	228128798.51	2280791482.16
$W^{e,r}$(JDD)	335143639.15	2439666996.43	319742268.63	2487926060.65	230048133.40	2263837637.13
$W^{e,r}$(COVERT)	270990733.50	2982216123.55	254644430.04	3014674527.26	164461705.67	2740320174.49
$W^{e,r}$(EXPET)	171902387.88	2365317869.33	165773440.66	2424985115.64	89886129.01	2210376501.82
FIFO	56935515.01	420068708.94	43632743.30	393136480.62	14591186.95	342836113.11
$W^{e,r}$(SPT)	271769027.66	2940255497.48	253234536.07	2947507320.64	172549094.04	2765404999.72
$W^{e,r}$(CR)	350166324.17	2675209979.36	306730464.72	2589841696.61	153683741.34	2096227150.57

the shop is less congested and with loose due-date settings. Only some jobs are tardy at low utilization levels with loose due-date setting. This behaviour is similar to that of the COVERT and ATC rules in job-shops, as observed by Kutanoglu and Sabuncuoglu (1999). But the performance of these rules becomes poor when the job structure complexity increases. This is so because proper pacing of items and sub-assemblies is not

Table 6
Performance of rules with respect to $WMSC^{e,r,h}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r,h}$(TWKR)	729.60[1]	1168.83[1]	600.50[3]	995.02[1]	703.53[3]	1009.91[2]
$W^{e,r,h}$(LBE+LBT+LBF)	823.23[3]	1469.31[3]	628.60	1256.40	683.09[2,3]	1079.70[3]
$W^{e,r,h}$(TWKR-BY-TIS)	782.33[2]	1348.10[2]	603.42[3]	1131.56[2]	672.11[2]	985.63[1]
$W^{e,r,h}$(PT-BY-TIS)	777.77[2]	1361.61[2]	604.95[3]	1146.80[2]	677.71[2]	1017.56[2,3]
$W^{e,r,h}$(ECT)	870.79	1518.73	759.63	1375.15	831.42	1395.65
$W^{e,r,h}$(JDD)	870.79	1518.73	758.33	1373.18	830.24	1393.99
$W^{e,r,h}$(COVERT)	865.79	1601.53	567.23[1]	1200.37[3]	669.37[2]	998.04[1]
FIFO	1030.38	2042.37	828.61	1823.85	784.12	1623.65
$W^{e,r,h}$(SPT)	886.63	1608.88	741.82	1422.66	801.19	1393.66
$W^{e,r,h}$(CR)	852.15	1528.26	596.39[2,3]	1246.09	653.39[1]	1021.59[2,3]
$W^{e,r,h}$(ATC)	902.45	1652.78	572.90[1]	1345.67	685.46[2,3]	1100.65
$W^{e,r,h}$(EXPET)	931.15	1964.71	608.19	1490.43	669.39[2]	1125.36
(b) For two-level job structures						
$W^{e,r,h}$(TWKR)	880.07[1]	1374.94[1]	736.51[2]	1151.28[1]	940.64[3]	1239.49[3]
$W^{e,r,h}$(LBE+LBT+LBF)	973.58[3]	1704.81[3]	757.66[3]	1397.55[3]	936.79[3]	1213.71[2]
$W^{e,r,h}$(TWKR-BY-TIS)	903.05[2]	1546.42[2]	705.53[1]	1244.25[2]	919.25[1]	1112.40[1]
$W^{e,r,h}$(PT-BY-TIS)	974.84[3]	1700.20[3]	761.20[3]	1398.64[3]	926.29[2]	1247.89
$W^{e,r,h}$(ECT)	1304.57	2387.05	1116.22	2160.85	1191.50	2161.46
$W^{e,r,h}$(JDD)	1308.61	2380.55	1121.46	2168.47	1194.92	2151.86
$W^{e,r,h}$(COVERT)	1330.48	2505.30	1119.63	2240.35	1174.69	2188.86
FIFO	1427.64	3042.02	1140.20	2734.21	1080.86	2436.90
$W^{e,r,h}$(SPT)	1327.31	2500.50	1100.67	2226.92	1160.34	2170.92
$W^{e,r,h}$(CR)	1354.83	2490.56	1197.00	2307.16	1203.63	2186.09
$W^{e,r,h}$(ATC)	1380.43	2680.54	1300.65	2256.78	1182.89	2401.34
$W^{e,r,h}$(EXPET)	1486.43	3227.17	1246.91	2942.56	1134.23	2522.83
(c) For three-level job structures						
$W^{e,r,h}$(TWKR)	1896.23[2]	2479.99[1]	1479.87[3]	1974.43[1]	1930.23[2]	2219.47[3]
$W^{e,r,h}$(LBE+LBT+LBF)	1934.69[3]	2828.99[3]	1462.07[2]	2194.21[3]	1935.36[2]	2171.96[2]
$W^{e,r,h}$(TWKR-BY-TIS)	1876.52[1]	2641.90[2]	1415.74[1]	2045.79[2]	1918.30[1]	2068.86[1]
$W^{e,r,h}$(PT-BY-TIS)	1941.77	2958.07	1491.68	2364.74	1915.50[1]	2292.32
$W^{e,r,h}$(ECT)	3180.72	4915.85	2653.26	4353.13	2657.77	4233.70
$W^{e,r,h}$(JDD)	3180.72	4915.85	2654.36	4351.39	2650.56	4229.69
$W^{e,r,h}$(COVERT)	2990.47	5130.84	2478.99	4563.11	2564.98	4464.04
FIFO	2895.06	5414.34	2274.61	4775.80	2148.55[3]	4225.18
$W^{e,r,h}$(SPT)	2995.63	5154.84	2452.21	4560.19	2483.02	4376.25
$W^{e,r,h}$(CR)	3183.06	4965.45	2750.42	4522.10	2747.90	4381.13
$W^{e,r,h}$(ATC)	3009.93	5190.03	2464.04	4582.00	2504.25	4425.44
$W^{e,r,h}$(EXPET)	3274.79	6624.26	2759.04	6086.99	2682.90	5726.10

achieved by these rules in the case of tall job structures. These rules appear to be more suitable for scheduling jobs in job-shops than in assembly job-shops. The behaviour of the $W^{e,r}$(CR) rule is also similar to the $W^{e,r}$(COVERT) and $W^{e,r}$(EXPET) rules. The performance of the other rules, namely, $W^{e,r}$(SPT), $W^{e,r}$(ECT), FIFO, and $W^{e,r}$(JDD) is not encouraging.

Table 7
Performance of rules with respect to $MSC^{e,r,h}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r,h}$(TWKR)	6965.72[1]	12154.64[1]	4709.32	9384.30[1]	5125.87	8777.24
$W^{e,r,h}$(LBE+LBT+LBF)	7613.05	14328.03	4631.98	11146.91	4454.95	8364.44
$W^{e,r,h}$(TWKR-BY-TIS)	7329.71[2]	13454.05[2]	4504.46	10196.32[2]	4529.62	7748.65
$W^{e,r,h}$(PT-BY-TIS)	7345.83[3]	13734.11[3]	4589.37	10496.14[3]	4658.50	8238.11[2]
$W^{e,r,h}$(ECT)	8817.70	16858.91	6758.77	14427.30	6805.99	13874.70
$W^{e,r,h}$(JDD)	8817.70	16858.91	6744.39	14397.40	6795.56	13851.84
$W^{e,r,h}$(COVERT)	8567.20	17315.01	4198.08[1]	11850.89	4303.21[2]	8228.17[1]
FIFO	9832.05	20454.32	6744.89	17173.97	5470.21	14105.00
$W^{e,r,h}$(SPT)	8789.97	17380.69	6351.43	14467.75	6259.38	13350.68
$W^{e,r,h}$(CR)	8551.48	16915.76	4464.17	12149.21	4280.24[1]	8249.11[3]
$W^{e,r,h}$(ATC)	8673.98	17412.10	4390.06[2]	12567.03	4508.96	8356.38
$W^{e,r,h}$(EXPET)	8637.73	19113.78	4438.20[3]	13377.29	4431.83[3]	8802.82
(b) For two-level job structures						
$W^{e,r,h}$(TWKR)	8142.00[1]	14086.23[1]	5408.74[3]	10428.55[1]	6683.43	10353.05
$W^{e,r,h}$(LBE+LBT+LBF)	8802.99[3]	16412.70[3]	5228.26[2]	11856.80[3]	6122.24[1]	8640.47[2]
$W^{e,r,h}$(TWKR-BY-TIS)	8162.41[2]	15115.17[2]	4814.18[1]	10562.20[2]	6204.43[2]	7964.66[1]
$W^{e,r,h}$(PT-BY-TIS)	9014.03	16944.88	5478.71	12398.58	6303.47[3]	9579.49[3]
$W^{e,r,h}$(ECT)	13209.89	26722.15	9951.55	23028.74	9721.59	21927.03
$W^{e,r,h}$(JDD)	13257.01	26633.20	10011.87	23116.17	9760.72	21800.26
$W^{e,r,h}$(COVERT)	13313.28	27650.81	9655.53	23428.64	8634.44	21208.14
FIFO	13604.46	30558.93	9221.43	25941.36	7467.05	21444.66
$W^{e,r,h}$(SPT)	13281.90	27576.35	9590.41	23346.90	9182.39	21595.44
$W^{e,r,h}$(CR)	13764.91	27926.58	10529.66	24424.27	8857.68	21022.04
$W^{e,r,h}$(ATC)	13567.90	27785.66	9832.12	24592.01	8843.46	22356.15
$W^{e,r,h}$(EXPET)	13900.59	31169.16	10067.62	26832.77	7799.30	21404.53
(c) For three-level job structures						
$W^{e,r,h}$(TWKR)	17312.34[2]	24217.72[1]	10283.53[3]	16158.15[2]	13239.85	16732.84
$W^{e,r,h}$(LBE+LBT+LBF)	17395.06[3]	26700.46[3]	9722.71[2]	17271.86[3]	12639.32[1]	14575.55[2]
$W^{e,r,h}$(TWKR-BY-TIS)	16847.56[1]	25028.50[2]	9311.34[1]	15932.84[1]	12877.15[2]	13968.64[1]
$W^{e,r,h}$(PT-BY-TIS)	17718.29	28849.41	10303.49	19783.77	12954.86[3]	16702.98[3]
$W^{e,r,h}$(ECT)	32259.52	53554.79	23938.86	44829.94	21706.12	41158.52
$W^{e,r,h}$(JDD)	32259.52	53554.79	23948.94	44806.33	21617.51	41106.05
$W^{e,r,h}$(COVERT)	29895.78	55689.63	21586.34	46848.13	19293.64	42573.28
FIFO	27512.17	53962.17	18136.74	44378.19	14458.56	35806.50
$W^{e,r,h}$(SPT)	29948.32	56002.40	21448.80	46917.20	19530.03	42513.71
$W^{e,r,h}$(CR)	32274.68	54159.28	24665.34	46474.59	21149.15	41085.84
$W^{e,r,h}$(ATC)	29198.33	54961.45	20598.48	45668.22	18303.03	40996.95
$W^{e,r,h}$(EXPET)	30885.51	64181.81	22652.75	55672.97	19305.42	49400.83

6.2. Mean scheduling cost ($MSC^{e,r}$) (Table 2a–c)

The rules $W^{e,r}$(TWKR-BY-TIS), $W^{e,r}$(LBE+LBT) and $W^{e,r}$(TWKR) continue to perform the best with respect to this measure of performance as well. The performance of the most of the rules for this measure of performance is similar to that of the rules with respect to the minimization of weighted mean scheduling cost (Section 6.1).

Table 8
Performance of rules with respect to $MaxTSC^{e,r,h}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r,h}$(TWKR)	53553.70	144196.94	49519.80	140324.44	45510.60	136465.27
$W^{e,r,h}$(LBE+LBT+LBF)	19469.77[1]	33678.54[1]	16053.90[1]	30132.97[1]	13434.70[2]	26525.27[1]
$W^{e,r,h}$(TWKR-BY-TIS)	24065.37[2]	47071.13[2]	19674.30[2]	42837.70[2]	15543.80[3]	38680.07[2]
$W^{e,r,h}$(PT-BY-TIS)	26454.87[3]	57997.30[3]	22425.13[3]	54126.90[3]	18615.33	50345.93[3]
$W^{e,r,h}$(ECT)	86698.43	226708.73	82801.47	223195.91	78983.80	219686.14
$W^{e,r,h}$(JDD)	86698.43	226708.73	82984.47	236146.59	79522.77	231306.50
$W^{e,r,h}$(COVERT)	76558.34	227348.36	52817.73	213471.30	21488.37	176712.44
FIFO	34462.90	65134.93	30292.60	60698.63	26215.23	56387.37
$W^{e,r,h}$(SPT)	77610.47	220458.94	73591.20	216738.80	69810.50	213088.59
$W^{e,r,h}$(CR)	81583.50	256493.09	39743.87	188500.06	13084.43[1]	121627.93
$W^{e,r,h}$(ATC)	78300.07	228654.32	63478.89	228567.94	22845.54	177234.43
$W^{e,r,h}$(EXPET)	71604.37	232770.23	44289.23	180773.93	16510.30	133858.23
(b) For two-level job structures						
$W^{e,r,h}$(TWKR)	59280.46	200472.50	54308.10	195345.23	49506.13	190245.91
$W^{e,r,h}$(LBE+LBT+LBF)	24527.30[2]	40418.73[1]	19280.83[2]	34118.60[1]	15761.23[2]	29978.20[1]
$W^{e,r,h}$(TWKR-BY-TIS)	22898.73[1]	46094.30[2]	17386.80[1]	40777.46[2]	14203.57[1]	35760.54[2]
$W^{e,r,h}$(PT-BY-TIS)	30921.40[3]	61204.67[3]	25448.83[3]	55885.83[3]	20437.53[3]	50648.04[3]
$W^{e,r,h}$(ECT)	119222.87	393751.00	113898.16	388757.94	108685.07	383789.03
$W^{e,r,h}$(JDD)	117600.57	395988.44	110301.97	386515.03	106242.07	396033.22
$W^{e,r,h}$(COVERT)	101610.20	375059.84	99508.53	358045.63	88856.73	352471.19
FIFO	46610.10	93068.34	40839.83	87311.37	35427.67	81616.03
$W^{e,r,h}$(SPT)	102375.03	348059.41	97117.66	342614.91	91899.70	337221.09
$W^{e,r,h}$(CR)	121386.16	441055.91	115685.30	424693.91	81591.77	368356.94
$W^{e,r,h}$(ATC)	113456.78	367098.32	101435.87	362341.08	94523.80	365231.28
$W^{e,r,h}$(EXPET)	106838.17	367760.03	97191.50	368053.00	76535.87	335710.28
(c) For three-level job structures						
$W^{e,r,h}$(TWKR)	95674.20	352855.50	85693.57	342944.03	75848.00	333161.72
$W^{e,r,h}$(LBE+LBT+LBF)	46822.40[2]	79826.93[2]	34858.27[2]	68742.07[2]	30344.63[2]	55974.20[2]
$W^{e,r,h}$(TWKR-BY-TIS)	43378.93[1]	76189.10[1]	31908.83[1]	64738.27[1]	25354.37[1]	53901.20[1]
$W^{e,r,h}$(PT-BY-TIS)	59400.67[3]	121513.34[3]	48292.57[3]	110960.43[3]	37610.77[3]	100630.73[3]
$W^{e,r,h}$(ECT)	243569.23	739504.25	232861.80	729192.00	222168.30	718879.88
$W^{e,r,h}$(JDD)	243569.23	739504.25	237463.59	722365.75	224594.00	726109.19
$W^{e,r,h}$(COVERT)	222600.36	733686.13	214640.06	747375.25	193830.23	757884.81
FIFO	92868.03	209273.20	81158.34	197361.44	69673.43	185509.64
$W^{e,r,h}$(SPT)	220374.17	742004.94	209977.97	730936.00	199718.00	719898.25
$W^{e,r,h}$(CR)	253541.50	795215.19	240276.47	780545.19	197122.77	744094.25
$W^{e,r,h}$(ATC)	222886.50	744483.31	222050.17	743397.63	200740.53	715648.94
$W^{e,r,h}$(EXPET)	237691.50	835369.06	226637.36	789996.75	205657.67	775949.00

Table 9
Performance of rules with respect to $WVTSC^{e,r,h}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r,h}$(TWKR)	125374.83	967100.20	112839.40	942328.33	100584.54	879515.74
$W^{e,r,h}$(LBE+LBT+LBF)	32025.21[1]	117698.68[1]	26428.37[1]	100707.45[1]	33804.51[2]	91928.33[1]
$W^{e,r,h}$(TWKR-BY-TIS)	39682.05[2]	160037.33[2]	35419.11[2]	155855.55[2]	35043.33[3]	142857.09[2]
$W^{e,r,h}$(PT-BY-TIS)	48251.60[3]	221971.80[3]	43880.67[3]	219326.12[3]	41880.31	203709.72[3]
$W^{e,r,h}$(ECT)	398278.62	3299507.85	373909.60	3249660.54	339137.57	3125094.50
$W^{e,r,h}$(JDD)	398278.62	3299507.85	372996.84	3268168.29	335191.31	3170926.66
$W^{e,r,h}$(COVERT)	284590.41	2690974.56	97096.93	2174070.71	30960.13[1]	1084349.21
FIFO	76541.40	311647.83	72475.17	307878.88	65624.66	298595.90
$W^{e,r,h}$(SPT)	291143.12	2633849.58	273772.58	2604175.82	246066.32	2505372.09
$W^{e,r,h}$(CR)	343960.08	3363423.34	56535.69	1550757.82	36781.83	489657.60
$W^{e,r,h}$(ATC)	275345.90	2756871.87	94523.01	2045310.09	37561.82	1009034.00
$W^{e,r,h}$(EXPET)	268572.69	2883622.02	61210.05	1637605.79	35086.94	606522.74
(b) For two-level job structures						
$W^{e,r,h}$(TWKR)	127721.14	1349197.96	112936.79	1315087.92	104135.25	1219043.14
$W^{e,r,h}$(LBE+LBT+LBF)	38629.13[2]	130909.70[1]	29469.71[1]	102547.19[1]	47462.75[2]	90105.59[1]
$W^{e,r,h}$(TWKR-BY-TIS)	35428.92[1]	139226.78[2]	30591.20[2]	137098.81[2]	36498.93[1]	120943.21[2]
$W^{e,r,h}$(PT-BY-TIS)	62089.73[3]	271412.85[3]	54720.24[3]	269273.08[3]	56138.73[3]	243099.84[3]
$W^{e,r,h}$(ECT)	740174.06	8804970.26	701103.76	8719990.90	632529.87	8453650.42
$W^{e,r,h}$(JDD)	748844.71	8705017.45	712074.55	8685200.38	640686.81	8314067.33
$W^{e,r,h}$(COVERT)	565672.44	7090999.43	508241.58	6888242.29	298745.90	6080751.90
FIFO	126627.51	549508.22	122020.01	549128.98	109730.60	543818.18
$W^{e,r,h}$(SPT)	581405.18	6863729.64	555258.19	6823831.99	496671.26	6607966.80
$W^{e,r,h}$(CR)	783481.02	9665262.86	598147.26	9127540.54	224911.54	6318734.99
$W^{e,r,h}$(ATC)	553467.36	6435132.50	500278.04	6923201.65	276210.43	6678206.12
$W^{e,r,h}$(EXPET)	655354.31	8962640.91	567292.58	8683007.54	211570.91	6170095.71
(c) For three-level job structures						
$W^{e,r,h}$(TWKR)	255863.05	2531560.30	227672.93	2468789.64	216177.90	2267012.12
$W^{e,r,h}$(LBE+LBT+LBF)	99785.06[2]	487761.86[2]	71148.99[2]	390427.01[2]	140110.21[2]	308344.68 [1]
$W^{e,r,h}$(TWKR-BY-TIS)	79671.53[1]	379927.09[1]	69002.61[1]	363918.26[1]	86003.27[1]	323916.45 [2]
$W^{e,r,h}$(PT-BY-TIS)	175342.90[3]	961091.23[3]	152694.30[3]	934273.37[3]	158150.72[3]	815656.54 [3]
$W^{e,r,h}$(ECT)	3111843.60	23959025.53	3017868.13	23806670.95	2713503.27	23002859.28
$W^{e,r,h}$(JDD)	3111843.60	23959025.53	3033662.06	23746292.01	2678077.25	23002699.13
$W^{e,r,h}$(COVERT)	2320600.07	24595164.84	2153183.18	24886968.27	1449492.79	23497351.54
FIFO	385910.51	3122999.60	376671.55	3119104.95	331532.72	3024614.57
$W^{e,r,h}$(SPT)	2317438.84	25277331.11	2246677.31	25175224.18	1999476.60	24428545.99
$W^{e,r,h}$(CR)	3116926.49	25585627.00	2726543.68	28037868.80	1395745.30	18702537.63
$W^{e,r,h}$(ATC)	2343125.22	25395265.65	2264383.04	24688441.86	1858789.21	23757871.39
$W^{e,r,h}$(EXPET)	2754923.56	39217352.06	2520175.58	39566849.75	1429997.75	34602899.34

Table 10
Performance of rules with respect to $VTSC^{e,r,h}$

Rule	$c=1$		$c=2$		$c=3$	
	85%	95%	85%	95%	85%	95%
(a) For single-level job structures						
$W^{e,r,h}$(TWKR)	21852990.87	177926559.06	16620423.45	168526069.43	11447291.55	143358140.24
$W^{e,r,h}$(LBE+LBT+LBF)	8524176.31[1]	26606396.17[1]	7108848.30[3]	25858617.90[1]	4760456.16	23120910.28[2]
$W^{e,r,h}$(TWKR-BY-TIS)	8836122.75[3]	30582467.68[2]	6000311.34[1]	27426158.31[2]	4454068.51[1]	20468433.98[1]
$W^{e,r,h}$(PT-BY-TIS)	8788072.74[2]	37452690.33[3]	6191125.94[2]	34958576.02[3]	4720014.20	27225577.64[3]
$W^{e,r,h}$(ECT)	70036298.33	616138518.21	60571618.55	597340333.26	45675434.37	547562711.15
$W^{e,r,h}$(JDD)	70036298.33	616138518.21	60591733.29	602815744.40	45425934.89	559359283.03
$W^{e,r,h}$(COVERT)	51419152.54	513079956.36	16965670.64	409611345.17	4945327.34	203090949.57
FIFO	22873633.56	92621210.09	19196514.82	87547207.50	11273386.04	79705321.68
$W^{e,r,h}$(SPT)	52644690.52	497605588.62	45316429.80	486388415.16	33031010.99	446072220.57
$W^{e,r,h}$(CR)	60327310.61	636304601.90	9440522.54	281149712.98	4532892.88[3]	84989426.91
$W^{e,r,h}$(ATC)	44328907.43	547600231.76	15673421.08	412356432.76	4765301.78	243576534.01
$W^{e,r,h}$(EXPET)	50732578.86	516815878.10	12438919.86	303629105.39	4523075.36[2]	114056895.60
(b) For two-level job structures						
$W^{e,r,h}$(TWKR)	21904832.85	240936380.55	15946804.90	228613114.33	12706323.17	192306142.41
$W^{e,r,h}$(LBE+LBT+LBF)	11831681.47[2]	32753050.09[2]	8685121.88[3]	30571892.27[2]	7504644.14[2]	24337108.44[2]
$W^{e,r,h}$(TWKR-BY-TIS)	9907860.45[1]	28177217.58[1]	5855082.60[1]	24859387.15[1]	7236694.06[1]	16050820.97[1]
$W^{e,r,h}$(PT-BY-TIS)	12592087.83[3]	45267676.96[3]	8146656.22[2]	42032780.71[3]	7810106.56[3]	29720114.09[3]
$W^{e,r,h}$(ECT)	128246815.22	1634763707.13	112563435.11	1603411076.74	83121112.57	1497811627.46
$W^{e,r,h}$(JDD)	130028748.43	1607764732.58	114118150.48	1590552960.18	83874645.38	1458945061.69
$W^{e,r,h}$(COVERT)	97066014.03	1311108514.16	85076071.52	1268318697.81	47248627.79	1100608297.02
FIFO	39392876.10	173204333.12	34004166.63	165344957.64	19770729.18	156106538.79
$W^{e,r,h}$(SPT)	100535096.95	1264822598.04	89529492.91	1250455373.50	64685277.33	1164698008.42
$W^{e,r,h}$(CR)	135415136.27	1793471396.83	100741549.80	1680846216.21	35981010.42	1135005406.91
$W^{e,r,h}$(ATC)	99864231.09	1546723987.08	96432190.43	1567843256.33	53249108.55	1345672109.67
$W^{e,r,h}$(EXPET)	110611044.48	1420626355.57	101703136.92	1417304858.27	39087296.69	1024166463.50
(c) For three-level job structures						
$W^{e,r,h}$(TWKR)	53881169.29	455463576.02	37773262.24	428694065.93	35947590.08	351116177.52
$W^{e,r,h}$(LBE+LBT+LBF)	39072218.48[2]	126993963.42[2]	26437596.23[2]	109827545.55[2]	30898535.01[3]	66397049.26[2]
$W^{e,r,h}$(TWKR-BY-TIS)	35179210.35[1]	104315448.15[1]	22494416.03[1]	87558330.28[1]	28961912.58[1]	49077987.29[1]
$W^{e,r,h}$(PT-BY-TIS)	43142507.03[3]	183895661.69[3]	27809816.40[3]	164028759.49[3]	30782865.12[2]	100868082.65[3]
$W^{e,r,h}$(ECT)	538539812.60	4405699052.42	496171078.69	4347390714.53	364304899.97	4021233796.64
$W^{e,r,h}$(JDD)	538539812.60	4405699052.43	499171930.26	4324662998.47	357440748.99	4012766569.06
$W^{e,r,h}$(COVERT)	406928562.32	4542164983.77	366584139.78	4605591422.61	237783692.14	4313027913.28
FIFO	131595446.76	862535584.30	115929731.53	834679118.85	67591990.82	763766880.34
$W^{e,r,h}$(SPT)	406243425.09	4698930423.29	372691274.22	4659168188.54	265792818.16	4356002052.93
$W^{e,r,h}$(CR)	537676450.59	4707110423.02	461242064.32	4583863230.80	228735611.73	3366039444.13
$W^{e,r,h}$(ATC)	402844387.77	4616181590.97	376743845.71	4501970301.11	257516306.13	4239752836.56
$W^{e,r,h}$(EXPET)	472158500.16	6262495517.90	463187983.50	6470952803.94	283517259.31	5716301392.53

6.3. Maximum total scheduling cost ($MaxTSC^{e,r}$) (Table 3a–c)

The $W^{e,r}$(LBE+LBT) rule performs the best, followed by the $W^{e,r}$(TWKR-BY-TIS) and $W^{e,r}$(PT-BY-TIS) rules, in all experimental settings for all levels of job structures. The $W^{e,r}$(LBE+LBT) rule gives importance to jobs that are expected to be tardy, and yet, the rule postpones the early completion of jobs by giving importance to jobs that have lower weighted earliness. In other words, the expected completion times of the jobs with respect to their respective due-dates determines the priority for selection. Since this rule tries to bring the completion times of the jobs closer to their due-dates, this rule minimizes the maximum sum of weighted earliness and weighted tardiness of jobs. The rule $W^{e,r}$(TWKR-BY-TIS) is found to perform quite well in minimizing the maximum sum of weighted earliness and weighted tardiness of jobs due to the presence of the term 'TIS' in the denominator, apart from the presence of the term 'w_i'. It is noteworthy that the $W^{e,r}$(TWKR-BY-TIS) rule also performs quite well in minimizing the weighted mean of the sum of weighted earliness and weighted tardiness of jobs (due to the presence of TWKR in the numerator) (Sections 6.1 and 6.2).

It is noteworthy that the $W^{e,r}$(TWKR-BY-TIS) rule that attempts to minimize many measures of performance simultaneously. The $W^{e,r}$(TWKR) and $W^{e,r}$(SPT) rules do not perform well in minimizing the maximum values for this measure of performance. The performance of these rules is similar to the performance of the un-weighted SPT rule in the case of conventional job-shops (see Holthaus & Rajendran, 1997, in terms of the performance of the un-weighted SPT rule with respect to the minimization of maximum flowtime in conventional job-shops). The reason is that the process-time based rules do not perform well with respect to the minimization of maximum values. We also find that the other rules, namely, $W^{e,r}$(COVERT), $W^{e,r}$(EXPET) and $W^{e,r}$(CR) rules are not effective in minimizing the maximum values. The performance of the rules such as FIFO, $W^{e,r}$(JDD) and $W^{e,r}$(ECT) rules is not encouraging as well. The reason is that these rules are basically static in nature. In the case of $W^{e,r}$(ECT) and $W^{e,r}$(JDD) rules, the terms 'ECT' and 'JDD' do not capture the work content of remaining items of a job dynamically (as opposed to $W^{e,r}$(TWKR), $W^{e,r}$(TWKR-BY-TIS), and $W^{e,r}$(LBE+LBT) rules) nor do they capture the probable tardiness/earliness of a job dynamically (as opposed to the $W^{e,r}$(LBE+LBT) rule).

6.4. Weighted variance of total scheduling cost ($WVTSC^{e,r}$) (Table 4a–c)

The rules $W^{e,r}$(LBE+LBT), $W^{e,r}$(TWKR-BY-TIS) and $W^{e,r}$(PT-BY-TIS) perform very well across all experimental settings. These rules are observed to minimize the maximum values of the sum of weighted earliness and weighted tardiness of jobs (Section 6.3), and hence minimize the weighted variance of the sum of weighted earliness and weighted tardiness of jobs as well, as expected. The rules that do not perform well in minimizing the maximum sum of weighted earliness and weighted tardiness of jobs show a poor performance in minimizing the variance of the sum of weighted earliness and weighted tardiness of jobs as well.

6.5. Variance of total scheduling cost ($VTSC^{e,r}$) (Table 5a–c)

The rule $W^{e,r}$(LBE+LBT) performs the best in minimizing the measure of variance of the sum of earliness and tardiness costs in most cases. This is followed by the rules $W^{e,r}$(TWKR-BY-TIS) and $W^{e,r}$(PT-BY-TIS). Since the $W^{e,r}$(LBE+LBT) rule seeks to minimize the maximum values of the sum of weighted earliness and weighted tardiness of jobs, the variance of the sum of weighted earliness and weighted tardiness of jobs is also reduced in the process. The good performance of $W^{e,r}$(TWKR-BY-TIS) and $W^{e,r}$(PT-BY-TIS) rules is due to the reason that these rules do not allow the jobs to stay in the system for a long period, thereby reducing the variance of the sum of weighted earliness and weighted tardiness of jobs.

7. Results and discussion of simulation study on performance evaluation of rules: phase 2

The results of the performance evaluation of various rules are presented in Tables 6–10.

7.1. Weighted mean scheduling cost ($WMSC^{e,r,h}$) (Table 6a–6c)

The $W^{e,r,h}$(TWKR) rule performs the best in the case of tight due-date setting and high shop utilization for all job structures. The reason is due to the presence of the terms 'TWKR' and 'w_i' (Eq. 19) in the rule. The factor 'TWKR' in the case of multi-level job structures is an extension of the factor 'process time' in the case of job-shops, and hence the mean flowtime of jobs is minimized. The factor 'w_i' reckons with the weights of earliness, tardiness and flowtime, and hence the weighted mean of the sum of earliness, tardiness and holding costs is minimized by the $W^{e,r,h}$(TWKR) rule. As opposed to the $W^{e,r,h}$(TWKR) rule, the $W^{e,r,h}$(SPT) rule that performs well in the case of conventional job-shops, does not perform well in the case of assembly job-shops that manufacture multi-level jobs. The reason is that the factor 'shortest process time' is myopic and local in nature in the case of assembly job-shops, in the sense that the process times of other components, sub-sub-assemblies, sub-assemblies, and final assembly are not taken into consideration by the $W^{e,r,h}$(SPT) rule. This finding is similar to the finding by Russell and Taylor (1985). Russell and Taylor investigated the performance of the simple (i.e. un-weighted) SPT rule in assembly job-shops, and found the performance of the un-weighted SPT rule to be non-encouraging. The $W^{e,r,h}$(TWKR-BY-TIS) rule performs better than (or not worse than) all other rules under consideration, except the $W^{e,r,h}$(TWKR) rule, in most cases. In fact, the $W^{e,r,h}$(TWKR) and $W^{e,r,h}$(TWKR-BY-TIS) rules compete with each other. The reason is, once again, due to the presence of the term 'TWKR'. We observe that the rules $W^{e,r,h}$(EXPET), $W^{e,r,h}$(COVERT), $W^{e,r,h}$(ATC) and $W^{e,r,h}$(CR) perform well in the case of single-level job structures, and not well in the case of tall structures, revealing the fact that these rules are more suited for job-shops than assembly job-shops. The performance of the other benchmark rules, namely, $W^{e,r,h}$(ECT), FIFO, and $W^{e,r,h}$(JDD), is not encouraging.

7.2. Mean scheduling cost ($MSC^{e,r,h}$) (Table 7a–c)

The rules $W^{e,r,h}$(TWKR) and $W^{e,r,h}$(TWKR-BY-TIS) continue to perform the best with respect to this measure of performance as well. The performance of the most of the rules for this measure of performance is similar to that of the rules with respect to the minimization of weighted mean scheduling cost (Section 7.1).

7.3. Maximum total scheduling cost ($MaxTSC^{e,r,h}$) (Table 8a–c)

The $W^{e,r,h}$(LBE + LBT + LBF) and $W^{e,r,h}$(TWKR-BY-TIS) rules perform the best in most experimental settings. The $W^{e,r,h}$(LBE + LBT + LBF) rule chooses the item associated with the corresponding maximum sum of weighted earliness, weighted tardiness and weighted flowtime of the job. This rule attempts to give importance to jobs that are likely to be tardy and would stay for a longer period of time in the system, and at the same time tries to postpone the early completion of jobs by giving importance to jobs that have lower earliness cost. Since this rule tries to bring the completion times of the jobs close to their due-dates, and at the same time gives priority to jobs that present in the system for a longer time, this rule minimizes the maximum sum of weighted earliness, weighted tardiness and weighted flowtime of jobs. The rule $W^{e,r,h}$(TWKR-BY-TIS) is found to perform quite well in minimizing the maximum sum of weighted earliness, weighted tardiness and weighted flowtime, due to the presence of the term 'TIS' in the denominator, apart from the presence of the term 'w_i'. The performance of the rule is seen improved when the job structure becomes complex. It is noteworthy that the $W^{e,r,h}$(TWKR-BY-TIS) rule also performs quite well. The $W^{e,r,h}$(TWKR) and $W^{e,r,h}$(SPT) rules do not perform well in minimizing the maximum values for this measure of performance. The performance of these rules is similar to the performance of the un-weighted SPT rule in the case of conventional job-shops (see Holthaus & Rajendran, 1997, for the performance of the un-weighted SPT rule with respect to the minimization of maximum flowtime in conventional job-shops). The reason is that the process-time based rules do not perform well with respect to the minimization of maximum values (Holthaus & Rajendran, 1997). We also find that the other rules, namely, $W^{e,r,h}$(COVERT), $W^{e,r,h}$(ATC), $W^{e,r,h}$(EXPET) and $W^{e,r,h}$(CR) rules, are not effective in minimizing the maximum values. The performance of the rules such as FIFO, $W^{e,r,h}$(JDD) and $W^{e,r,h}$(ECT) rules is not encouraging as well. The reason is that these rules are basically static in nature. In the case of $W^{e,r,h}$(ECT) and $W^{e,r,h}$(JDD) rules, the terms 'ECT' and 'JDD' do not dynamically capture the work content of remaining items of a job (as opposed to $W^{e,r,h}$(TWKR), $W^{e,r,h}$(TWKR-BY-TIS), and $W^{e,r,h}$(LBE + LBT + LBF) rules) nor do they capture the probable tardiness and earliness of a job dynamically (as opposed to the $W^{e,r,h}$(LBE + LBT + LBF) rule).

7.4. Weighted variance of total scheduling cost (WVTSC^{e,r,h}) (Table 9a–c)

Due to the presence of the term 'TIS' in the denominator of the $W^{e,r,h}$(TWKR-BY-TIS) rule, the maximum value and hence the variance of sum of weighted earliness, weighted tardiness and weighted flowtime costs is reduced by this rule. The good performance of the $W^{e,r,h}$(TWKR-BY-TIS) rule in reducing the variance of the total scheduling cost is clearly seen when the job structure becomes complex. Similarly, the $W^{e,r,h}$(PT-BY-TIS) rule shows good performance in minimizing the variance of the total scheduling cost, and the reason for such a performance is due to the presence of the term 'TIS' in the denominator of the rule. The $W^{e,r,h}$(LBE + LBT + LBF) rule performs better in the case of single level and two level structures than in the case of tall structures, because the lower bound on job completion time becomes weak in the case of tall structures. The rules that do not perform well in minimizing the maximum values of the total sum of weighted earliness, weighted tardiness and weighted flowtime costs show a poor performance in minimizing the variance of sum of weighted earliness, weighted tardiness and weighted flowtime of jobs as well.

7.5. Variance of total scheduling cost (VTSC^{e,r,h}) (Table 10a–c)

The rule $W^{e,r,h}$(LBE + LBT + LBF) performs the best in minimizing the variance of sum of weighted earliness, weighted tardiness and weighted flowtime of jobs in most cases. This is followed by the rules $W^{e,r,h}$(TWKR-BY-TIS) and $W^{e,r,h}$(PT-BY-TIS). Since the $W^{e,r,h}$(LBE + LBT + LBF) rule seeks to minimize the maximum values of weighted earliness, weighted tardiness and weighted flowtime of jobs, the variance of sum of weighted earliness, weighted tardiness and weighted flowtime of jobs is also reduced in the process. The good performance of $W^{e,r,h}$(TWKR-BY-TIS) and $W^{e,r,h}$(PT-BY-TIS) rules is due to the reason that these rules do not allow the jobs to stay in the system for a long period, thereby reducing the variance of sum of weighted earliness, weighted tardiness and weighted flowtime of jobs.

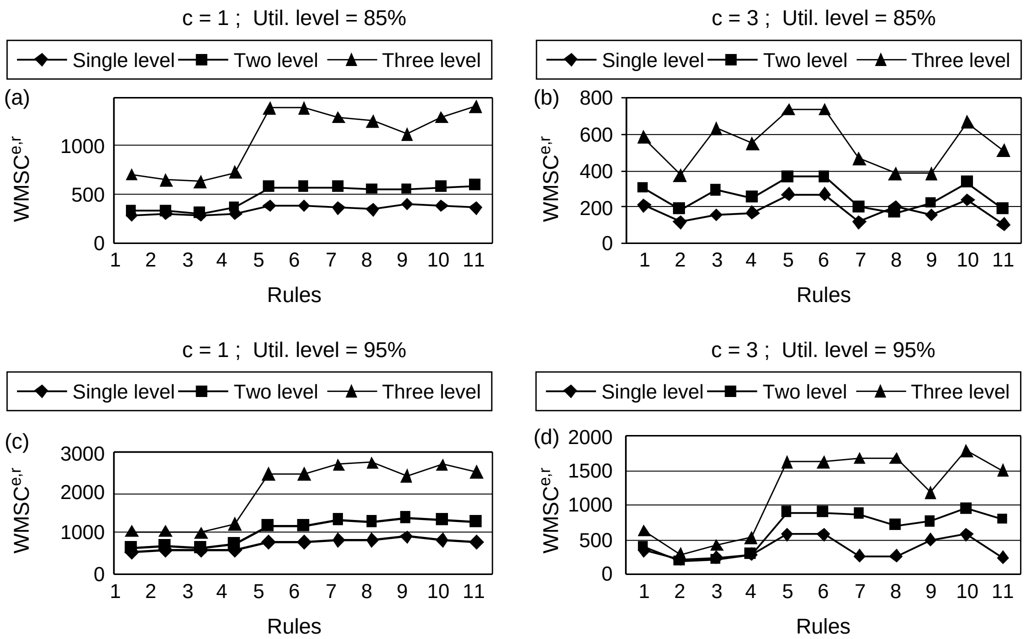


Fig. 2. WMSC^{e,r}—weighted mean scheduling cost (Eq. (5)).

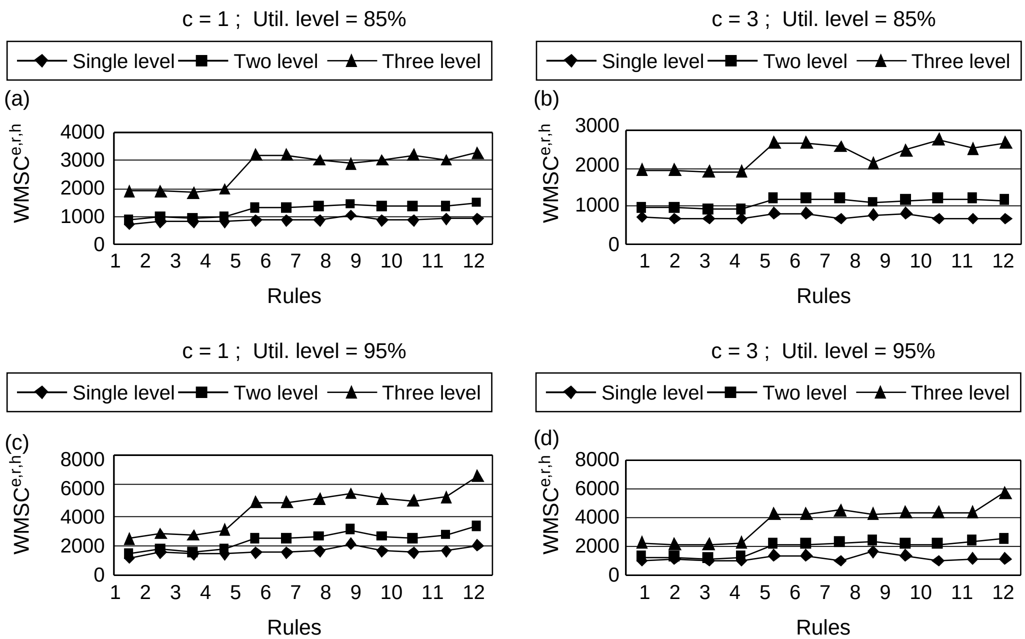


Fig. 3. WMSC^{e,r,h}—weighted mean scheduling cost (Eq. (12)).

8. Further discussions on performance of rules

Relevant figures in the form of graphs are given to depict the performance of rules with respect to WMSC^{e,r} and WMSC^{e,r,h} (Figs. 2 and 3), in order to have a visual comparison of performance of rules. However, we prefer to give the complete set of results in the form of tables because the exact values of measures of performance of rules can be seen in tables only. Apart from the study on the performance of rules presented earlier, we have also studied the effect of interaction of allowance factor, utilization level and job structure, by treating them as factors in the context of design of experiments (DOE). We have also considered the rules as a factor in the DOE. It has been observed from the statistical analysis that apart from the main effects (Tables 1–10), the four factors, namely, dispatching rule, job structure, allowance factor and utilization level, show significant interaction effects, as expected. Future research could possibly look deeper into the aspect of interaction of factors, as the primary objective of this paper is to focus more on the study of main effects.

Interaction or profile plots have been obtained and are shown for some selected combination of the factor levels (Figs. 4 and 5) with respect to WMSC^{e,r} and WMSC^{e,r,h}. These interaction plots are quite useful to visually see the effect of interaction between factors (in terms of job structure, utilization level, and allowance factor) in respect of a given rule. Moreover, we observe that the interaction effects seem to be more pronounced in the case of W(EXPET) rule, as against in the case of other rules. This is so because the W(EXPET) rule makes use of many job-shop characteristics such as slack and cost information, a look-ahead exponential function, recognition of approaching tardiness and adjustment of job priorities accordingly. All these aspects seem to contribute to the pronounced interaction effects in the case of W(EXPET) rule.

As already mentioned, no rule is found to be dominant and hence the selection of any particular rule on the shop floor will warrant separate simulation runs to determine the choice of a dispatching rule, depending upon the job structure, utilization level, allowance factor and measure of performance. We wish to reiterate that our experimental results suggest a select set of best-performing dispatching rules for every measure of performance, on an overall basis, depending upon the job structure, allowance factor and utilization level, and our findings can possibly guide a decision-maker to choose the rule that is appropriate to the shop-floor condition and his/her primary and secondary objectives of scheduling.

At this juncture, we would also like to present some discussions with respect to the choice of the term 'r_i/e_i' (in Eqs. (17) and (18)) in the Expressions (19), (20), (21), (22), and so on (see the presentation of rules in Section 4). Since the measure of earliness is non-regular and the measure of tardiness is regular, and these two objectives are conflicting (in nature in the sense that as we attempt to minimize the earliness of a job, tardiness of the job invariably increases), we have opted for the expression 'r_i/e_i' in our work. Moreover, for the same reason, in the proposed rule W$^{\mathrm{e,r}}$(LBE + LBT), we used the expression '$Z_{ij}^{l} = \mathrm{LBSC}_{ij}^{e,r} = \max\{(\mathrm{LBCT}_{ij} - D_i); 0\}r_i - \max\{D_i - \mathrm{LBCT}_{ij}; 0\}e_i$' where we used the coefficient '-1' with reference to earliness term. Hence we are consistent with respect to the incorporation of weights for earliness and tardiness in all our rules. Nevertheless, we evaluated the performance of rules with the consideration of these weights in forms such as '$e_i + r_i$' and '$e_i \times r_i$'. In respect of results for the W$^{\mathrm{e,r}}$(TWKR) rule with the consideration of the expressions involving e_i and r_i (i.e. TWKR/($e_i + r_i$)), we have found that the performance of the rule improves only when we have tight due-date setting (i.e. when $c = 1$) because almost all jobs are tardy in the case of tight due-date setting, and hence the additive form of weights performs well. On the other hand, the additive form does not perform well when due-date settings are not tight. Likewise, when we use the additive form of weights in the TWKR/TIS rule, there is no appreciable difference in

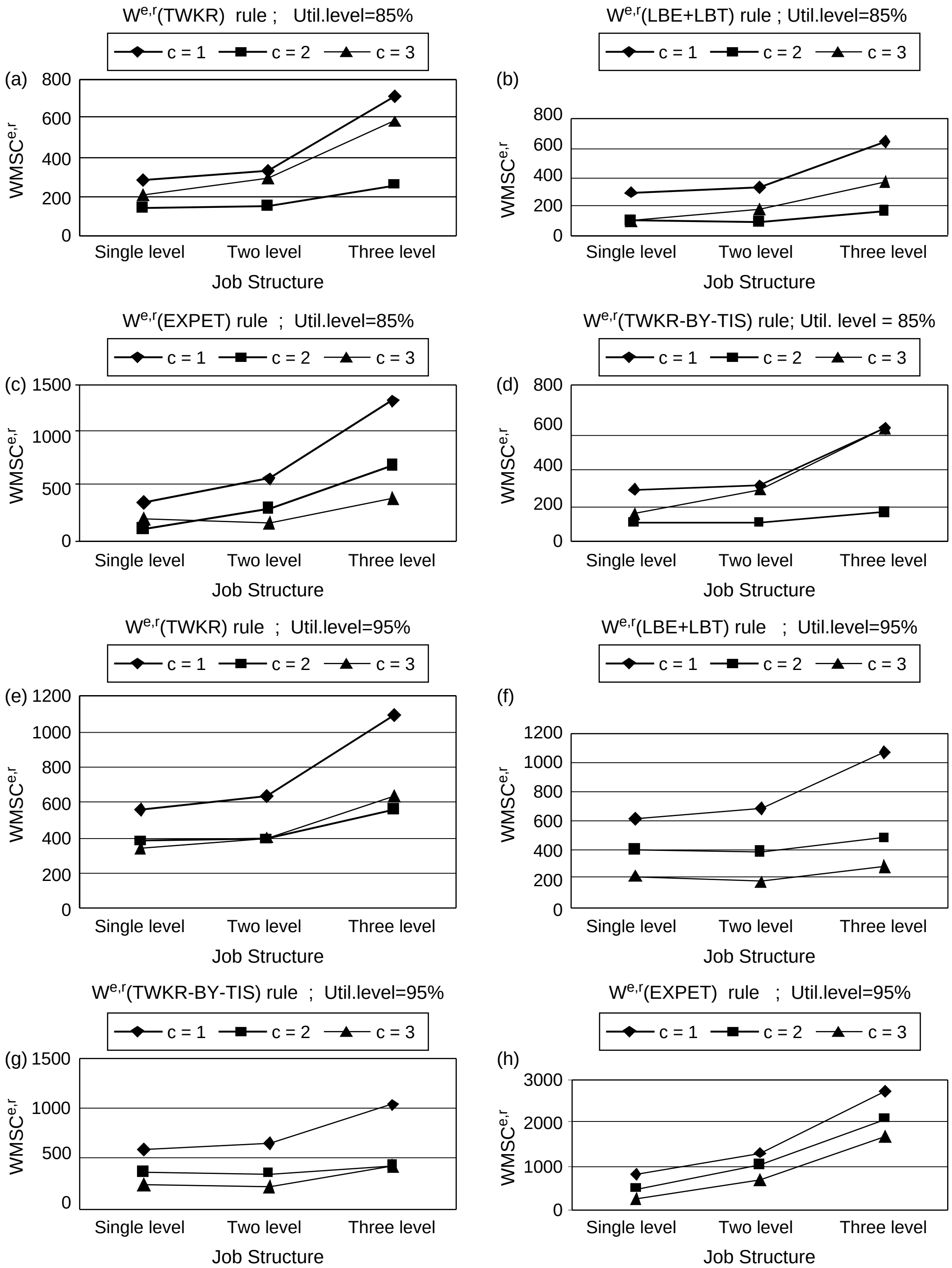


Fig. 4. Interaction plots with respect to $WMSC^{e,r}$.

the performance between the implementation of this form of combining weights and the ratio form given in the paper. The reason is due to the presence of TIS in the denominator, which plays the dominant role in hastening the job completion. In addition, following a similar approach of additive form of weights for tardiness and earliness, we have tested the rule LBE+LBT in the following form as well:

$$Z_{ij}^{l}(\text{additive}) = \text{LBSC}_{ij}^{e,r} = \max\{(\text{LBCT}_{ij} - D_i); 0\}r_i + \max\{D_i - \text{LBCT}_{ij}; 0\}e_i.$$

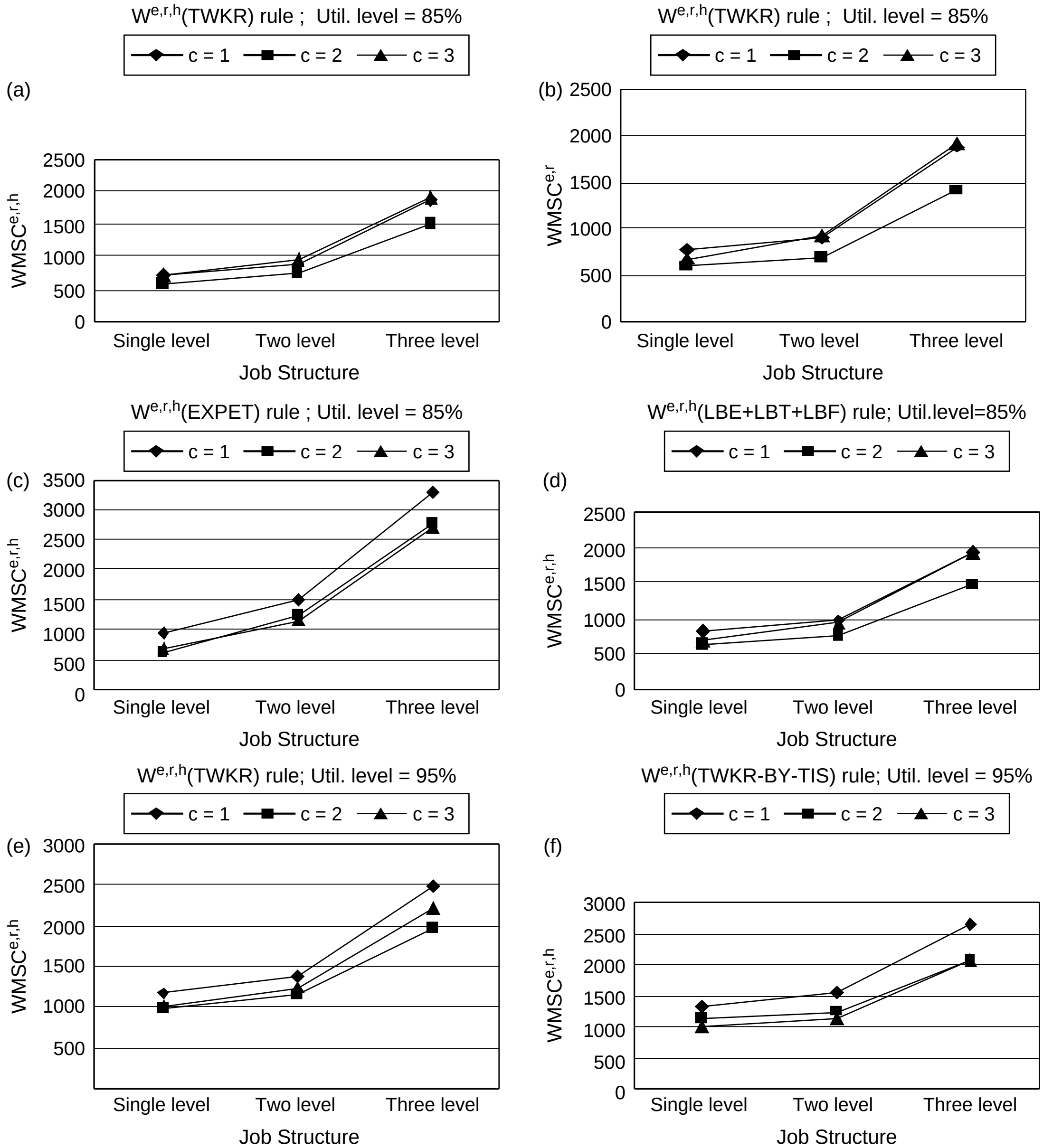
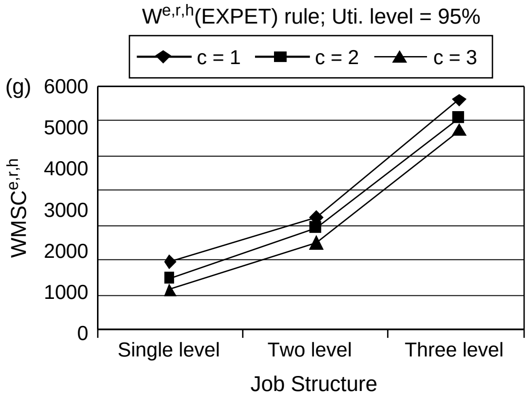


Fig. 5. Interaction plots with respect to $\text{WMSC}^{e,r,h}$.



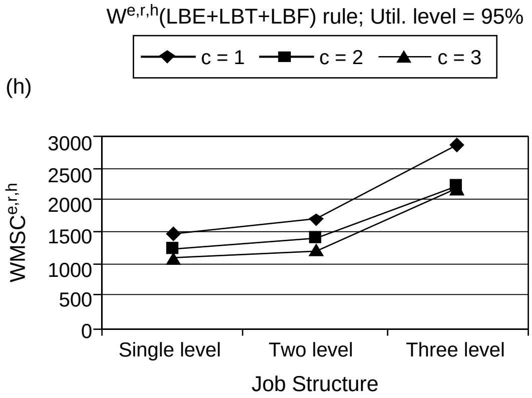


Fig. 5 (*continued*)

This form of LBE+LBT rule performs worse, especially in the case of loose due-date settings (as expected, because the measures of earliness and tardiness are basically conflicting in nature).

As for the multiplicative combination of weights for earliness and tardiness, we have evaluated the performance of the TWKR rule (i.e. TWKR/$(e_i \times r_i)$), with the ties broken by OSD), and have found the performance of this form of rule to be poor. Similar poor performance has also been observed in the case of TWKR/TIS and LBE+LBT rules, when weights are considered in multiplicative form.

In view of the above discussions, we justify the existing form of combining the weights for earliness and tardiness of jobs (i.e. in the form 'r_i/e_i') in our study.

We wish to mention that we have done similar analyses with respect to the evaluation of rules with the combination of weights in forms such as $(h_i+(r_i+e_i))$ and $(h_i+(r_i \times e_i))$. We have also observed similar findings in this phase and hence we have not reported them.

9. Summary

In this study, an investigation of dispatching rules for scheduling in dynamic assembly job-shops with the consideration of different weights for earliness, tardiness and flowtime of jobs has been undertaken. In the first phase of this study, weights for earliness and tardiness have been considered, and the dispatching rules are modified accordingly. In the second phase, weights for flowtime of jobs are also considered, apart from weights for earliness and tardiness of jobs. In both phases, an extensive simulation study for assembly job-shops that manufacture different types of multi-level jobs, with different levels of shop utilization and job due-date settings, has been carried out. All the dispatching rules under consideration have been evaluated with respect to a number of measures of performance.

In the first phase of this study, it has been found that the $W^{e,r}$(TWKR-BY-TIS) rule performs very well with respect to the minimization of weighted mean sum of weighted earliness and weighted tardiness of jobs. As far as the minimization of maximum sum of weighted earliness and weighted tardiness of jobs is concerned, $W^{e,r}$(LBE+LBT) and $W^{e,r}$(TWKR-BY-TIS) rules perform the best.

In addition, these rules also perform very well in minimizing the variance of the sum of weighted earliness and weighted tardiness of jobs as well.

In the second phase of this study, it has been found that the $W^{e,r,h}$ (TWKR) rule mostly performs the best with respect to the minimization of weighted mean sum of weighted earliness, weighted tardiness and weighted flowtime of jobs. The $W^{e,r,h}$(TWKR-BY-TIS) rule performs very well and forms the second best set in most of the cases with respect to the minimization of weighted mean sum of weighted earliness, weighted tardiness and weighted flowtime of jobs. As far as the minimization of maximum sum of weighted earliness, weighted tardiness and weighted flowtime of jobs, $W^{e,r,h}$(LBE + LBT + LBF) and $W^{e,r,h}$(TWKR-BY-TIS) rules perform very well. In addition, these rules also perform very well in minimizing the variance and weighted variance of the sum of weighted earliness, weighted tardiness and weighted flowtime of jobs.

Acknowledgements

The authors are thankful to the three referees for their valuable suggestions to improve the earlier two versions of this paper.

Appendix A. Appendix

A sample calculation for computing the priority indices for some of the rules considered in this study is illustrated using a two-level job structure. The job structure is shown in Fig. 6. The processing times of the various items used in the job are also shown. An item here may refer to a component, or a sub-sub-assembly, or a sub-assembly, or a final assembly.

Item A refers to as the final assembly (or job), items B and C refer to sub-assemblies of assembly A, and items D and E refer to components of sub-assembly B, and similarly, items F and G to components of sub-assembly C.

Item D has two operations d1 and d2, item E has one operation e1, item F has two operations f1 and f2, and item G has one operation g1. Sub-assembly B has two operations b1 and b2, and similarly sub-assembly C has one operation c1. The final assembly has two operations a1 and a2. The processing time for a given operation is indicated within the bracket of the corresponding operation. The final assembly A can be assembled when all operations on sub-assemblies B and C are completed. Similarly, only upon completion of all operations on components D, E, F and G, processing on sub-assembly B and C can be taken up. We assume that the final assembly A is present at level 1, while sub-assemblies B and C are present at level 2, and components D, E, F and G are present at level 3. The due-date of an arriving job i is determined by its critical path length (cp_i), the allowance factor (c) and job arrival time (A_i) (i.e. $D_i = A_i + c \times cp_i$) where the critical path length is the sum of the processing times of operations on the critical path.

Let time at the instant of computing the priority values, $T = 20$;
arrival time of the job i, $A_i = 5$;
earliness cost (or weight for earliness) for job $i = e_i = 8$; and
tardiness cost (or weight for tardiness) for job $i = r_i = 18$.
holding cost (or weight for tardiness) for job $i = h_i = 5$.

Let us assume that operations d1 and d2 on item D, operation e1 on item E, operations f1 and f2 on item F, and operation g1 on item G have been already completed at $T=20$.

We have the critical path length of job $i=\mathrm{cp}_i=(9+4)+(8)+(6+3)=30$.

The priority index of job i with respect to operation c1 is now computed and is shown for the rules.

A.1. $W^{e,r,h}$(TWKR) rule

Total work remaining for job $i=(8)+(5+3)+(3+6)=25$;

Priority value of job i with respect to operation c1 $(Z_{i,c1})=25(1/(h_i+(r_i/e_i)))=25(1/(5+(18/8)))=3.448$.

A.2. $W^{e,r,h}$(TWKR-BY-TIS) rule

Total work remaining for job $i=(8)+(5+3)+(3+6)=25$.

Time-in-system for job i at time instant $T=T-A_i=20-5=15$.

Total work remaining/time-in-system$=25/15=1.67$.

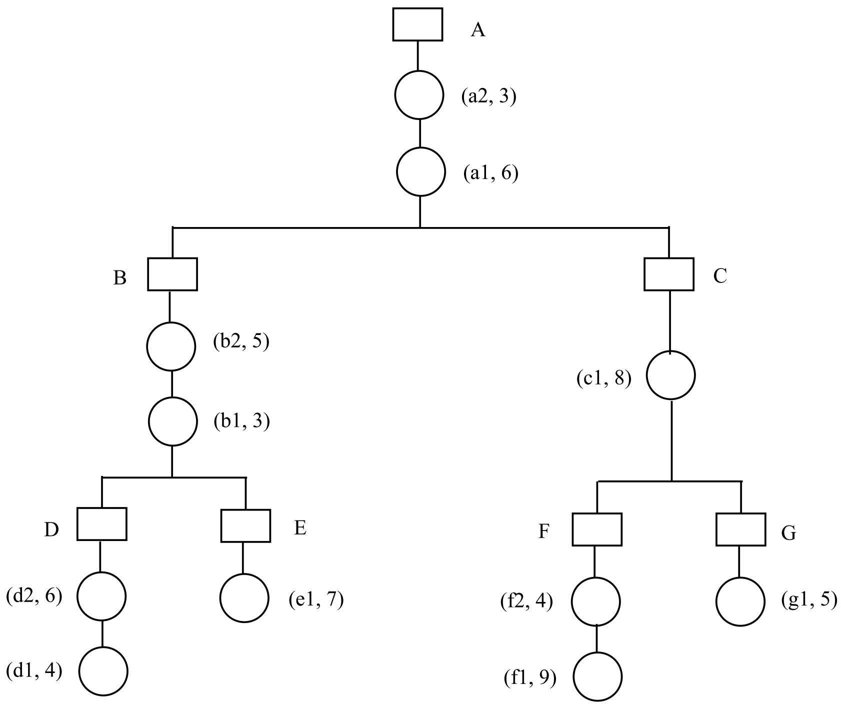


Fig. 6. A two-level structure.

Priority value of job i with respect to operation c1 $(Z_{i,\text{c1}}) = 1.67(1/(h_i + (r_i/e_i)))$= $1.67(1/(5+(18/8))) = 0.230$.

A.3. $W^{e,r,h}$(PT-BY-TIS) rule

Processing time for operation c1 of job $i = 8$.
Time-in-system for job i at time instant $T = T - A_i = 20 - 5 = 15$.
Processing time/time-in-system $= 8/15 = 0.53$.
Priority value of job i for operation c1 $(Z_{i,\text{c1}}) = 0.53(1/(h_i + (r_i/e_i))) = 0.53(1/(5+(18/8))) = 0.073$.

A.4. $W^{e,r,h}$(LBE+LBT+LBF) rule

$\text{EFT}_{i,\text{d1}}$ = earliest finish time of item d1 of job $i = \text{EST}_{i,\text{d1}} + t_{i,\text{d1}} = 5 + 4 = 9$;

$$\text{EFT}_{i,\text{d2}} = \text{EFT}_{i,\text{d1}} + t_{i,\text{d2}} = 9 + 6 = 15;$$

$$\text{EST}_{i,\text{e1}} = \text{EST}_{i,\text{f1}} = \text{EST}_{i,\text{g1}} = 5;$$

$$\text{EFT}_{i,\text{e1}} = \text{EST}_{i,\text{e1}} + t_{i,\text{e1}} = 5 + 7 = 12;$$

$$\text{EFT}_{i,\text{f1}} = \text{EST}_{i,\text{f1}} + t_{i,\text{f1}} = 5 + 9 = 14;$$

$$\text{EFT}_{i,\text{g1}} = \text{EST}_{i,\text{g1}} + t_{i,\text{g1}} = 5 + 5 = 10;$$

$$\text{EFT}_{i,\text{f2}} = \text{EFT}_{i,\text{f1}} + t_{i,\text{f2}} = 14 + 4 = 18;$$

$$\text{EST}_{i,\text{b1}} = \max\{\text{EFT}_{i,\text{d2}}; \text{EFT}_{i,\text{e1}}\} = \max\{15, 12\} = 15;$$

$$\text{EFT}_{i,\text{b1}} = \text{EST}_{i,\text{b1}} + t_{i,\text{b1}} = 15 + 3 = 18;$$

$$\text{EFT}_{i,\text{b2}} = \text{EFT}_{i,\text{b1}} + t_{i,\text{b2}} = 18 + 5 = 23;$$

$$\text{EST}_{i,\text{c1}} = \max\{\text{EFT}_{i,\text{f2}}; \text{EFT}_{i,\text{g1}}\} = \max\{18, 10\} = 18;$$

$$\text{EFT}_{i,\text{c1}} = \text{EST}_{i,\text{c1}} + t_{i,\text{c1}} = 18 + 8 = 26;$$

$$\text{EST}_{i,\text{a1}} = \max\{\text{EFT}_{i,\text{b2}}; \text{EFT}_{i,\text{c1}}\} = \max\{23, 26\} = 26;$$

$$\text{EFT}_{i,\text{a1}} = \text{EST}_{i,\text{a1}} + t_{i,\text{a1}} = 26 + 6 = 32; \text{ and}$$

$$\text{EFT}_{i,\text{a2}} = \text{ECT}_i = \text{EST}_{i,\text{a2}} + t_{i,\text{a2}} = 32 + 3 = 35;$$

latest finish time of item a2 of job $i = \text{LFT}_{i,\text{a2}} = \text{ECT}_\text{i}$ = earliest completion time of job $i = 35$;
latest start time of item a2 of job $i = LST_{i,\text{a2}} = LFT_{i,\text{a2}} - t_{i,\text{a2}} = 35 - 3 = 32$;

$$\mathrm{LFT}_{i,\mathrm{a1}} = \mathrm{LST}_{i,\mathrm{a2}} = 32; 4 \quad \mathrm{LST}_{i,\mathrm{a1}} = \mathrm{LFT}_{i,\mathrm{a1}} - t_{i,\mathrm{a1}} = 32 - 6 = 26;$$

$$\mathrm{LFT}_{i,\mathrm{c1}} = \mathrm{LST}_{i,\mathrm{a1}} = 26; \text{ and } \mathrm{LST}_{i,\mathrm{c1}} = 26 - 8 = 18.$$

$$(T + \mathrm{ECT}_i - \mathrm{LST}_{i,\mathrm{c1}}) = (20 + 35 - 18) = 37.$$

We have

$$\begin{aligned}
&(T + \mathrm{ECT}_i - \mathrm{LST}_{i,\mathrm{c1}} - A_i)h_i - \max\{(D_i - (T + \mathrm{ECT}_i - \mathrm{LST}_{i,\mathrm{c1}})); 0\}e_i + \max\{(T + \mathrm{ECT}_i \\
&\quad - \mathrm{LST}_{i,\mathrm{c1}} - D_i); 0\}r_i \\
&\quad = (20 + 35 - 18 - 5)5 - \max\{(35 - (20 + 35 - 18)); 0\}8 + \max\{(20 + 35 - 18 - 35); 0\}18 \\
&\quad = 196;
\end{aligned}$$

and priority value with respect to operation c1 of job i, $Z^{l}_{i,\mathrm{c1}} = 196$.

A.5. 5. Computation of operation synchronization date (OSD)

We define OSD of job i for a given operation j as follows:

$$\mathrm{OSD}_{ij} = \mathrm{OSD}_{i,\mathrm{successor}} - \{[t_{i,\mathrm{successor}}/(\mathrm{EFT}_{i,\mathrm{successor}} - A_i)] \times (\mathrm{OSD}_{i,\mathrm{successor}} - A_i)\}$$

In the above, $\mathrm{EFT}_{i,\mathrm{successor}}$ denotes the earliest finish time of the successor operation of the operation under consideration, and $t_{i,\mathrm{successor}}$ denotes the process time of the successor operation of the operation under consideration of job i. Consider the two-level job structure in Fig. 6. Note that the earliest start time (EST) is the earliest time at which an operation could be started, and the earliest finish time (EFT) is the earliest time at which an operation could be completed.

As assumed above, we have the time at the instant of computing the priority values, $T=20$; and the arrival time of the job i, $A_i=5$; We now compute the OSD of various operations:

$$\mathrm{OSD}_{i,\mathrm{a2}} = \text{ECT of job } i = \mathrm{ECT}_i = 35;$$

$$\mathrm{OSD}_{i,\mathrm{a1}} = \mathrm{OSD}_{i,\mathrm{a2}} - \{t_{i,\mathrm{a2}} \times (\mathrm{OSD}_{i,\mathrm{a2}} - A_i)/(\mathrm{EFT}_{i,\mathrm{a2}} - A_i)\} = 35 - \{3 \times (35 - 5)/(35 - 5)\} = 32;$$

$$\mathrm{OSD}_{i,\mathrm{b2}} = 32 - \{6 \times (32 - 5)/(32 - 5)\} = 26;$$

$$\mathrm{OSD}_{i,\mathrm{b1}} = 26 - \{5 \times (26 - 5)/(23 - 5)\} = 20.2;$$

$$\mathrm{OSD}_{i,\mathrm{c1}} = 32 - \{6 \times (32 - 5)/(32 - 5)\} = 26;$$

$$\mathrm{OSD}_{i,\mathrm{g1}} = 26 - \{8 \times (26 - 5)/(26 - 5)\} = 18;$$

$$\mathrm{OSD}_{i,\mathrm{f2}} = 26 - \{8 \times (26 - 5)/(26 - 5)\} = 18;$$

$$\mathrm{OSD}_{i,\mathrm{f}1} = 18 - \{4 \times (18-5)/(18-5)\} = 14;$$

$$\mathrm{OSD}_{i,\mathrm{e}1} = 20.2 - \{3 \times (20.2-5)/(18-5)\} = 16.7;$$

$$\mathrm{OSD}_{i,\mathrm{d}2} = 20.2 - \{3 \times (20.2-5)/(18-5)\} = 16.7;$$

$$\mathrm{OSD}_{i,\mathrm{d}1} = 16.7 - \{6 \times (16.7-5)/(15-5)\} = 9.7.$$

Note that the operations that are not the critical path have their OSDs greater than their EFTs. The OSD principle paces the completions of various individual items in such a way so as to synchronize them at subassembly/assembly stages. This pacing is ensured by a tight monitoring of operation completion times of items, with the difference between the critical path time and non-critical path time being appropriately apportioned to the non-critical items (see Reeja & Rajendran, 2000a, for further details).

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