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## ALICE

### Analysis & Learning Iterative Consecutive Executions

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University of Iceland

June 30, 2016



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## Motivation:

- ★ The general goal is to train optimisation algorithms using data.

## Contribution:

- ★ The main contribution of this thesis is towards a better understanding of how this training data should be constructed.

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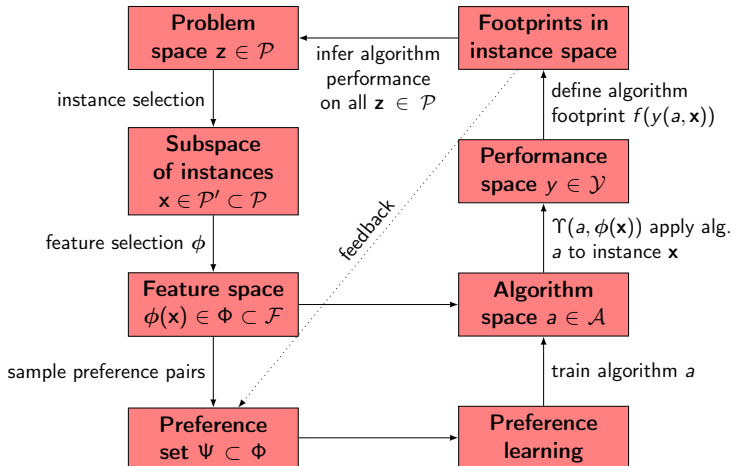
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# Mad Hatter Tea-party

## Definition

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The attending guests: They all have to:

$J_1$ ) Alice

$M_1$ ) have wine or pour tea

$J_2$ ) March Hare

$M_2$ ) spread butter

$J_3$ ) Dormouse

$M_3$ ) get a haircut

$J_4$ ) Mad Hatter.

$M_4$ ) check the time of the broken watch

$M_5$ ) say what they mean.

This can be considered as a typical  $4 \times 5$  job-shop, where:

- ★ our guests are the **jobs**
- ★ their tasks are the **machines**
- ★ objective is to **minimise**  $C_{\max}$  (when Alice can leave).

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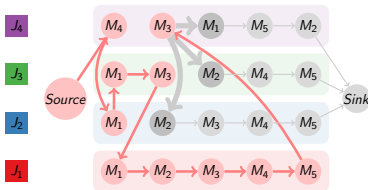
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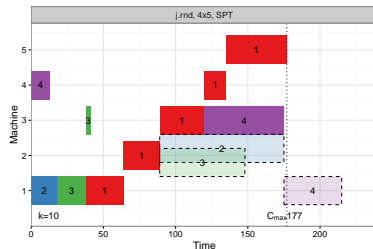
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Midway:  $k = 10$



**Figure:** Disjunctive graph



**Figure:** Gantt chart

# Mad Hatter Tea-party

## $K$ -solutions

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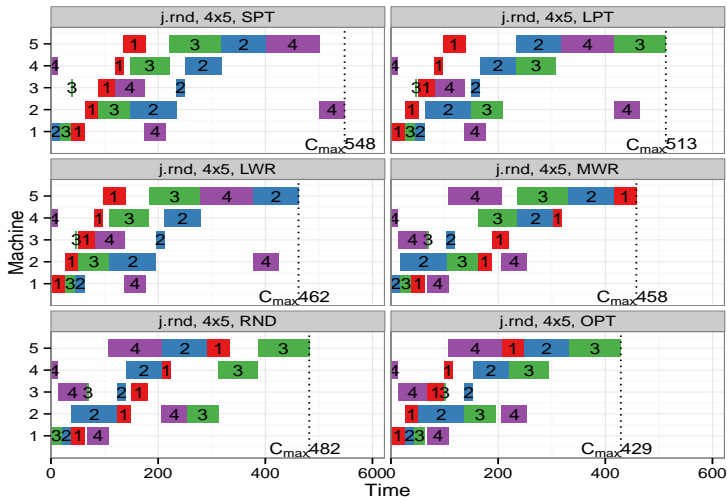
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# Problem Instance Generators

Based on Watson et al. (2002)

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	name	size ( $n \times m$ )	$N_{\text{train}}$	$N_{\text{test}}$	note
JSP	$\mathcal{P}_{j.\text{rnd}}^{6 \times 5}$	$6 \times 5$	500	500	random
	$\mathcal{P}_{j.\text{rndn}}^{6 \times 5}$	$6 \times 5$	500	500	random-narrow
	$\mathcal{P}_{j.\text{rnd}, J_1}^{6 \times 5}$	$6 \times 5$	500	500	random with job variation
	$\mathcal{P}_{j.\text{rnd}, M_1}^{6 \times 5}$	$6 \times 5$	500	500	random with machine variation
	$\mathcal{P}_{j.\text{rnd}}^{10 \times 10}$	$10 \times 10$	300	200	random
	$\mathcal{P}_{j.\text{rndn}}^{10 \times 10}$	$10 \times 10$	300	200	random-narrow
	$\mathcal{P}_{j.\text{rnd}, J_1}^{10 \times 10}$	$10 \times 10$	300	200	random with job variation
	$\mathcal{P}_{j.\text{rnd}, M_1}^{10 \times 10}$	$10 \times 10$	300	200	random with machine variation
	$\mathcal{P}_{\text{JSP}, \text{ORLIB}}$	various	–	82	various
FSP	$\mathcal{P}_{f.\text{rnd}}^{6 \times 5}$	$6 \times 5$	500	500	random
	$\mathcal{P}_{f.\text{rndn}}^{6 \times 5}$	$6 \times 5$	500	500	random-narrow
	$\mathcal{P}_{f.\text{jc}}^{6 \times 5}$	$6 \times 5$	500	500	job-correlated
	$\mathcal{P}_{f.\text{mc}}^{6 \times 5}$	$6 \times 5$	500	500	machine-correlated
	$\mathcal{P}_{f.\text{mxc}}^{6 \times 5}$	$6 \times 5$	500	500	mixed-correlation
	$\mathcal{P}_{f.\text{rnd}}^{10 \times 10}$	$10 \times 10$	300	200	random
	$\mathcal{P}_{\text{FPS}, \text{ORLIB}}$	various	–	31	various

# Feature Space

for job-shop

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job	$\phi_1$	job processing time
	$\phi_2$	job start-time
	$\phi_3$	job end-time
	$\phi_4$	job arrival time
	$\phi_5$	time job had to wait
	$\phi_6$	total processing time for job
	$\phi_7$	total work remaining for job
	$\phi_8$	number of assigned operations for job
machine	$\phi_9$	when machine is next free
	$\phi_{10}$	total processing time for machine
	$\phi_{11}$	total work remaining for machine
	$\phi_{12}$	number of assigned operations for machine
	$\phi_{13}$	change in idle time by assignment
	$\phi_{14}$	total idle time for machine
	$\phi_{15}$	total idle time for all machines
	$\phi_{16}$	current makespan
final makespan	$\phi_{17}$	final makespan using SPT
	$\phi_{18}$	final makespan using LPT
	$\phi_{19}$	final makespan using LWR
	$\phi_{20}$	final makespan using MWR
	$\phi_{\text{RND}}$	final makespans using 100 random rollouts
	$\phi_{21}$	mean for $\phi_{\text{RND}}$
	$\phi_{22}$	standard deviation for $\phi_{\text{RND}}$
	$\phi_{23}$	minimum value for $\phi_{\text{RND}}$
	$\phi_{24}$	maximum value for $\phi_{\text{RND}}$



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Following the **policy**:

- ★  $(\phi^{\text{OPT}})$  expert  $\pi_{\star}$ .
- ★  $(\phi^{\text{SPT}})$  shortest processing time (SPT).
- ★  $(\phi^{\text{LPT}})$  longest processing time (LPT).
- ★  $(\phi^{\text{LWR}})$  least work remaining (LWR).
- ★  $(\phi^{\text{MWR}})$  most work remaining (MWR).
- ★  $(\phi^{\text{RND}})$  random policy (RND).
- ★  $(\phi^{\text{ES}.\rho})$  the policy obtained by optimising with CMA-ES.
- ★  $(\phi^{\text{ALL}})$  union of all of the above.

# Sampled Size of $|\Phi(k)|$

$6 \times 5, N_{train} = 500$

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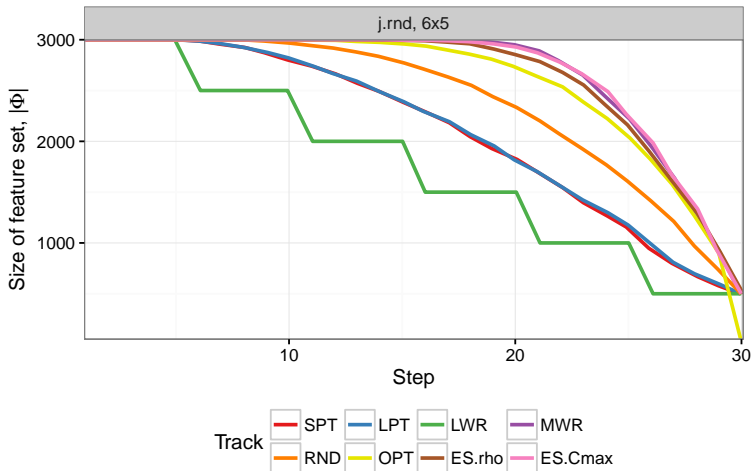
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# Various Methods for Solving JSP

## Based on Jain and Meeran (1999)

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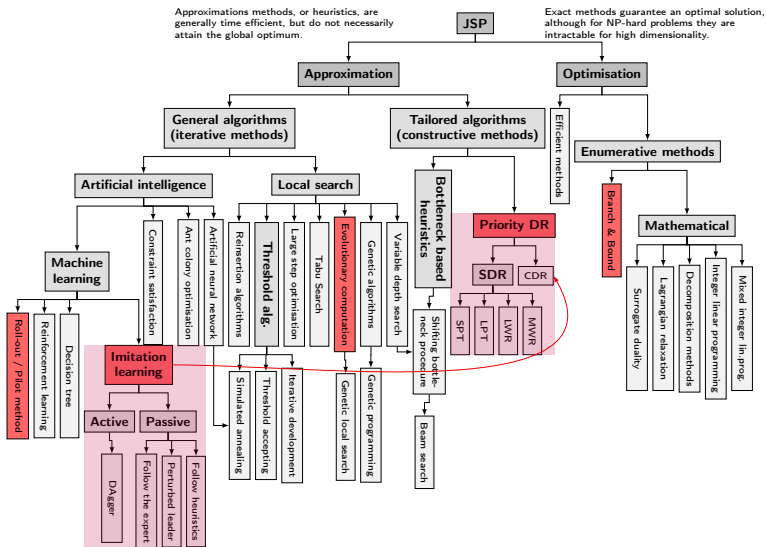
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Performance of policy  $\pi$  compared with its optimal makespan, found using an expert policy,  $\pi_*$ , is the following loss function:

$$\rho = \frac{C_{\max}^{\pi} - C_{\max}^{\pi_*}}{C_{\max}^{\pi_*}} \cdot 100\%$$

The goal is to minimise this discrepancy between **predicted** value and **true** outcome.

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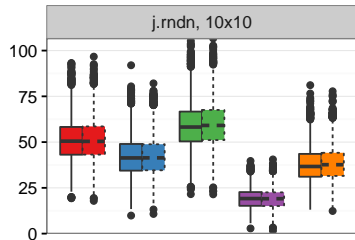
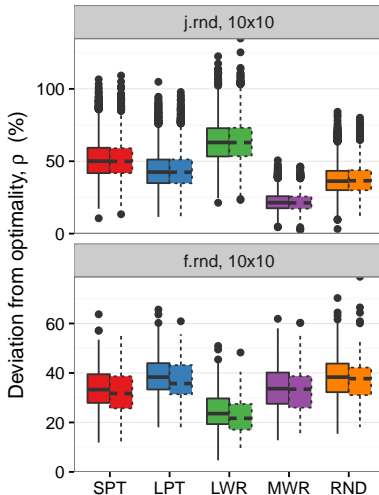
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Simple dispatching rule

- Shortest Processing Time
- Longest Processing Time
- Least Work Remaining
- Most Work Remaining
- Random dispatches

Data set



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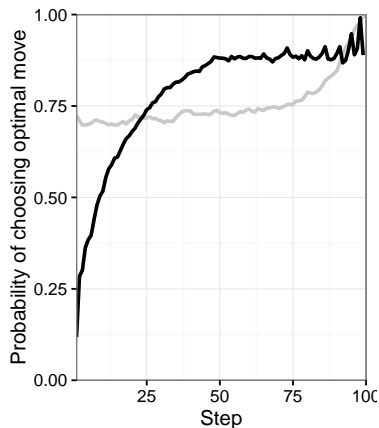
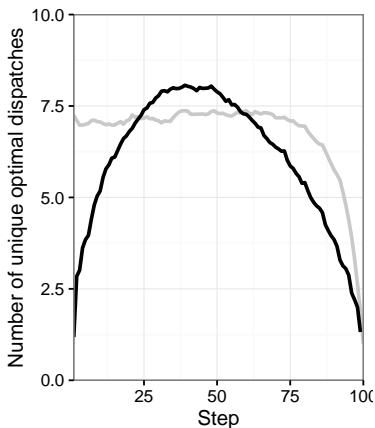
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# Probability of SDR Being Optimal

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$$\xi_{\langle \text{SDR} \rangle}^*$$

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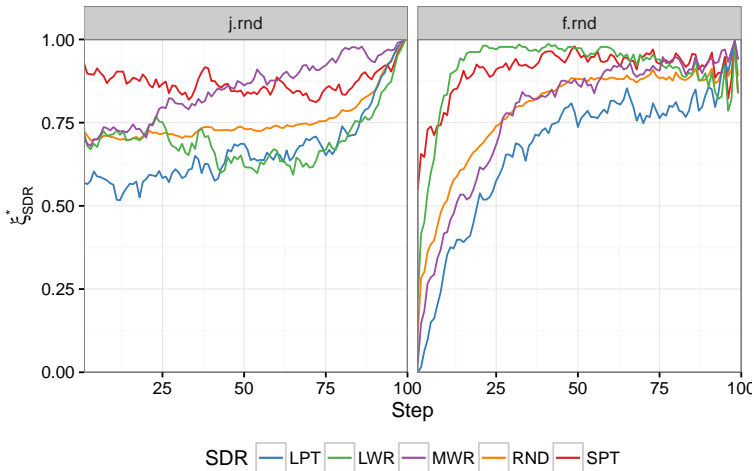
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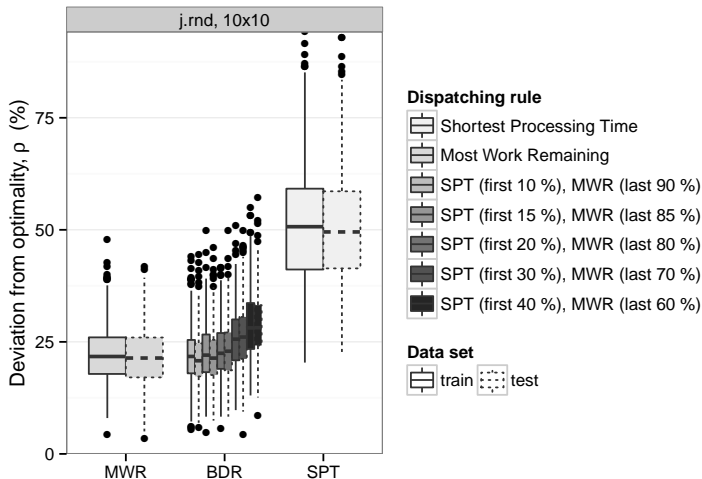
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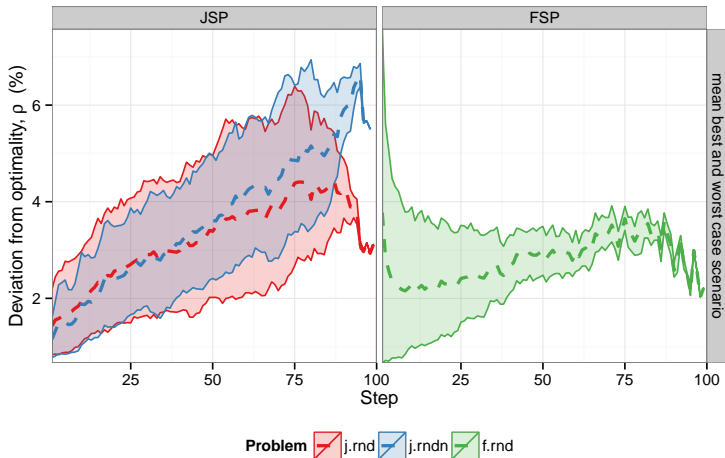
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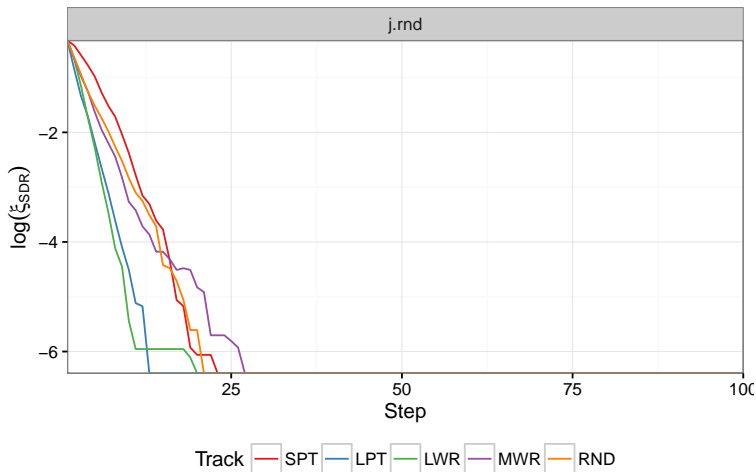
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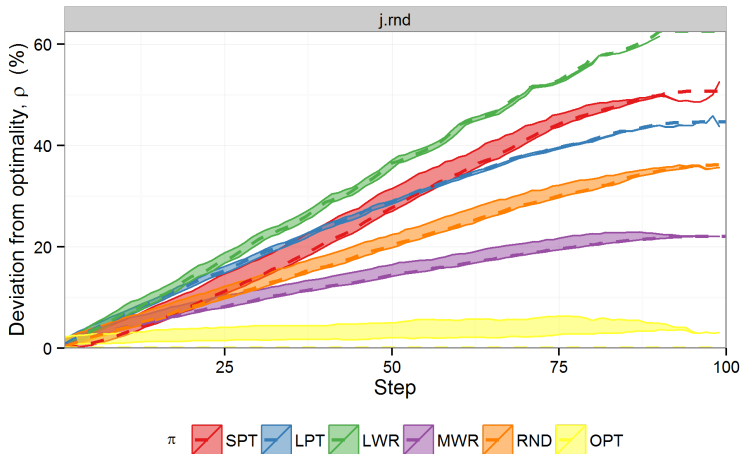
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Generating training data:

- ★ **Generate** feature set,  $\Phi \subset \mathcal{F}$ , both from

- ★ **optimal** solutions,  $\phi^o$

- ★ **suboptimal** solutions,  $\phi^s$

by exploring various **trajectories** within the feature-space  
(where  $\phi^o, \phi^s \in \mathcal{F}$ ).

- ★ Sample  $\Phi$  to **create** training set  $\Psi$  with rank pairs:

- ★ **optimal** decision,  $(z^o, y_o) = (\phi^o - \phi^s, +1)$

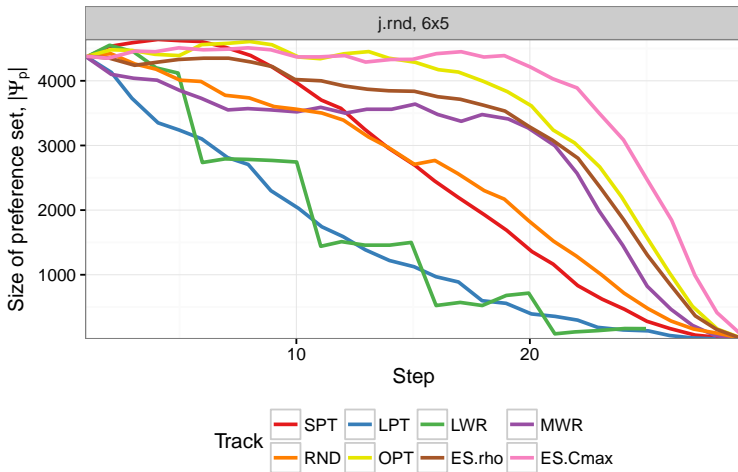
- ★ **suboptimal** decision,  $(z^s, y_s) = (\phi^s - \phi^o, -1)$

using different **ranking** schemes (where  $z^o, z^s \in \Psi$ )

- ★ **Sample**  $\Psi$  using **stepwise bias** for time independent policy.



## Conclusions



# Stepwise Bias Strategies

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$6 \times 5, N_{train} = 500$

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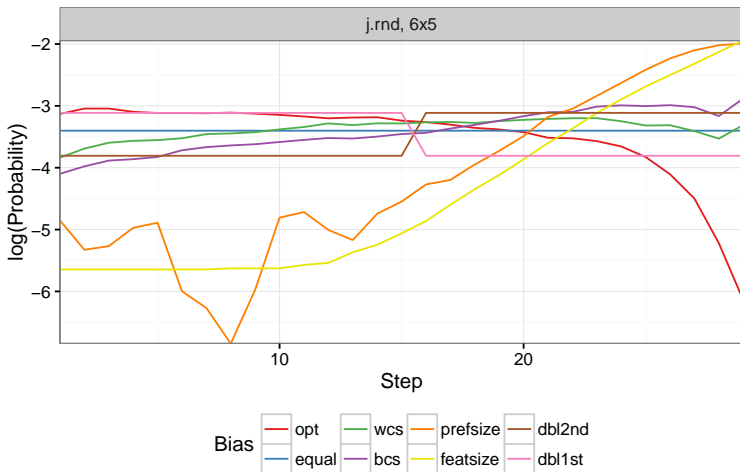
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## Preference learning:

- ★ Mapping of points to ranks:  $\{h(\cdot) : \Phi \mapsto Y\}$  where

$$\phi_o \succ \phi_s \iff h(\phi_o) > h(\phi_s)$$

- ★ The preference is defined by a **linear** function:

$$h(\phi) = \langle \mathbf{w} \cdot \phi \rangle$$

**optimised** w.r.t.  $\mathbf{w}$  based on training data  $\Psi$

- ★ Note: **Limitations** in **approximation** function to capture the complex dynamics incorporated in optimal trajectories.

# Various Methods for Solving JSP

## Based on Jain and Meeran (1999)

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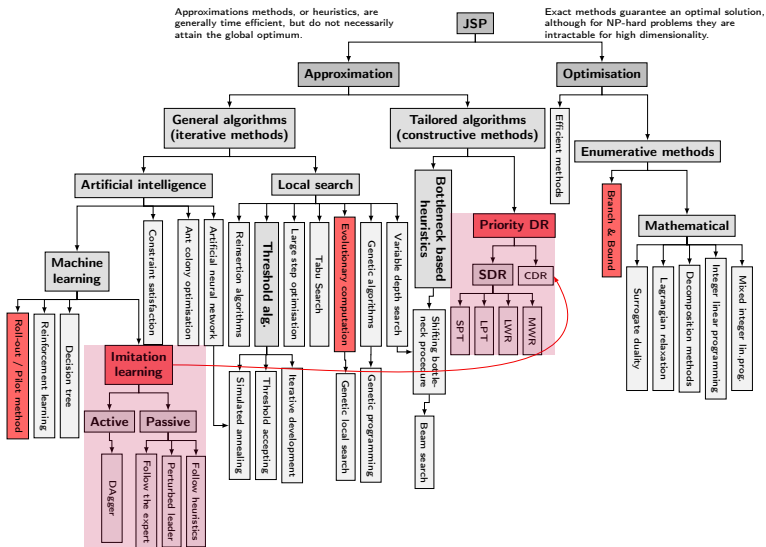
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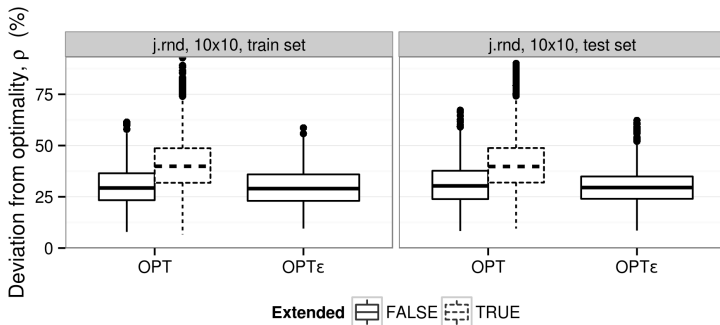
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Passive imitation learning (single pass):

- ★ Prediction with expert advice,  $\pi_\star$
- ★ Follow the perturbed leader ( $\text{OPT}_\epsilon$ )
- ★ Follow a heuristic (e.g.  $\text{SDRs}$ ).



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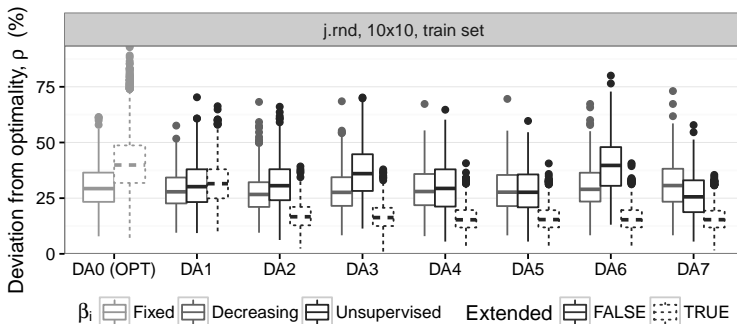
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Active imitation learning (**iterative**):

★ Dataset Aggregation (**DAgger**)

$$\pi_i = \beta_i \pi_{\star} + (1 - \beta_i) \hat{\pi}_{i-1}$$

where  $\hat{\pi}_{i-1}$  is the previous learned model, and  $\hat{\pi}_i$  learns on **aggregated dataset** of all previous iterations.



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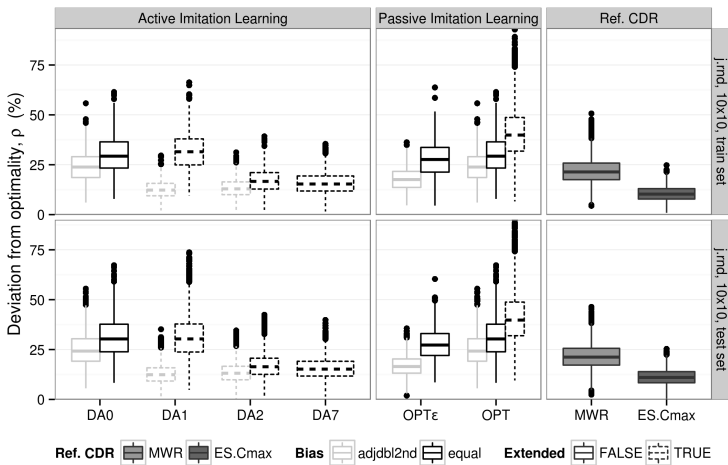
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The thesis introduces a framework for learning (**linear**) composite priority dispatching rule – using **job-shop** as a case-study – with the following guidelines:

- ★ For a given problem domain, use a suitable problem **generator** to **train** and **test** on.
- ★ Define **features** to grasp the essence of visited **k-solutions**
- ★ **Success** is highly dependent on the preference pairs introduced to the system:
  - ★  $\Psi_p$  **reduces** the preference set **without loss** of performance.
  - ★ **Stepwise bias** is needed to balance time dependent  $\Psi_p$  in order to create **time independent** models.

It is **non intuitive** how to go about **collecting** training data.



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Continued from prev. slide:

- ★ Learning **optimal** trajectories predominant in literature. Study showed  $\Phi^{\text{OPT}}$  can result in **insufficient** knowledge.
- ★ Following **sub-optimal** deterministic policies, yet labelling with an optimal solver, **improves** the guiding policy.
- ★ Active update procedure using **DAGger** ensures sample states the **learned model is likely to encounter** is integrated to  $\Psi_p^{\text{DA}i}$ .
- ★ Instead of reusing the same problem instances, extend the training set with **new** instances for **quicker convergence** of DAGger.
- ★ In **sequential** decision making, all future observations are dependent on **previous** operations.

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**Illustrations:** Sir John Tenniel (1820–1914)

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Supplementary material:

- ★ Shiny application
- ★ Github.

