CT-GSSN: A Continuous-Time Graph State-Space Network for Irregularly Sampled Multivariate Time Series

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Abstract

Real-world dynamic systems bring two hard problems at once: irregular sampling and multivariate inter-dependencies. Many time series, from healthcare to climate, are collected at non-uniform intervals with misaligned observations across variables; meanwhile, each series' evolution depends on others. Existing deep models usually tackle only one side of this coin. We introduce the *Continuous-Time Graph State-Space Network* (CT-GSSN), which *unifies* (i) graph neural networks for inter-series structure, (ii) continuous-time dynamics via Neural ODEs, and (iii) the linear-time efficiency of modern state-space models. We pair this with *stability promotion*: a Lyapunov-inspired regularizer and a contraction-safe parameterization. When the contraction parameterization is enforced, we obtain per-interval stability guarantees; otherwise, the regularizer empirically promotes non-expansive dynamics. CT-GSSN models complex graph-temporal interactions while handling arbitrary time gaps with linear complexity. On MIMIC-III, PhysioNet-2012, and METR-LA, CT-GSSN achieves strong performance—up to 12% forecasting gains over strong baselines such as GraphSSM and Neural CDEs—and shows improved robustness under graph perturbations. This points to a scalable, stability-aware approach for irregular temporal graphs.

1 Introduction

Pre-trained models have transformed language and vision [1], motivating general-purpose representation learning for time series [1–3]. However, most models assume clean, regularly sampled data [2], overlooking two co-occurring realities: **(i) irregular sampling** [4–6] with asynchronous variables, inconsistent gaps, and missingness [4, 7, 8]; and **(ii) multivariate inter-dependencies** [3, 11, 12], where series influence each other and are best modeled on graphs [12**?** **?** –14].

Research has bifurcated: Neural ODE/CDE models handle irregular sampling [9, 15, 16, 24], while spatio-temporal GNNs target relational data but often presume regular sampling [12, 17**?**]. Transformers' quadratic cost is ill-suited to long, sparse sequences [2, 6]. We seek a *single model* that is native to continuous time, relational, and scalable.

We propose the **Continuous-Time Graph State-Space Network (CT-GSSN)**. It synthesizes: (1) GNNs for inter-series dependencies [3, 18**?**], (2) modern selective SSMs (e.g., Mamba) for linear-time scalability [10–12, 17], and (3) a continuous-time formulation for arbitrary sampling [9, 19, 20]. Prior work combines pairs of these ideas [19, 21, 28], but not all three in a stability-aware, continuous-time architecture.

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Contributions.

1. **Unified architecture.** CT-GSSN integrates GNNs (spatial), continuous-time ODEs (temporal), and selective SSMs (efficiency) for irregular, graph-structured time series.
2. **Stability promotion & guarantees.** We introduce a Lyapunov-inspired *stability promotion* regularizer and a *contraction-safe parameterization*. With contraction enforced, we provide per-interval stability guarantees; otherwise, we empirically encourage non-expansivity.
3. **Versatile pre-training.** A multi-task self-supervised objective (masked prediction, dynamic graph inference, continuous-time contrast, stability promotion) yields robust representations for forecasting, imputation, and classification.
4. **Empirics & robustness.** On MIMIC-III, PhysioNet-2012, and METR-LA, CT-GSSN achieves strong accuracy and robustness under graph noise, while scaling linearly in sequence length.

2 Related Work

Irregular time series. GRU-D [7] addresses missingness via decay. Neural ODE/CDE models [4, 9, 15, 16, 24] treat hidden states as continuous trajectories. mTAN [3] and ContiFormer [14] adapt attention to continuous time. These methods often treat variables independently and underutilize relational structure.

GNNs for time series. STGNNs are standard for multivariate relational forecasting [13, 17, 22]; e.g., GraphSTAGE [3**?**] and RAINDROP [6]. Graph Neural ODEs combine graphs with continuous dynamics [16, 20, 25–27]. Most assume regular sampling. TGNN4I [5, 19] couples GNNs with ODE-driven GRUs, but lacks SSM efficiency and explicit stability promotion.

State-space models. Selective SSMs (S4, Mamba) enable linear-time long-range modeling [10, 17, 39]. Time-series variants include MambaTS [11] and TSMamba [12]. GraphSSM [28] applies SSMs to temporal graphs but in discrete time. Our CT-GSSN provides a continuous-time, stability-aware alternative that fuses GNNs and selective SSMs.

3 Background and Preliminaries

Definition 3.1 (Irregular Multivariate Time Series (IMTS))**.** *An IMTS consists of N variables $S = \{S_1, \dots, S_N\}$ where $S_n = \{(t_{n,i}, x_{n,i})\}_{i=1}^{L_n}$, $t_{n,i} \in \mathbb{R}^+$, $x_{n,i} \in \mathbb{R}^{d_x}$. Timestamps are non-uniform; at any t only a subset of variables may be observed [18**?**]. Relational structure is a (possibly dynamic) graph $\mathcal{G}_t = (V, E_t)$.*

Definition 3.2 (GNN Layer)**.** *A node v updates via neighbor aggregation:*

$$\mathbf{h}_v^{(l+1)} = \phi^{(l)}\left(\mathbf{h}_v^{(l)}, \bigoplus_{u \in \mathcal{N}_v} \psi^{(l)}(\mathbf{h}_v^{(l)}, \mathbf{h}_u^{(l)})\right),$$

where ϕ, ψ are differentiable (e.g., MLPs) and $\bigoplus$ is permutation-invariant [37, 38].

Definition 3.3 (Neural ODE)**.** *A hidden state $\mathbf{h}(t)$ evolves as $\frac{d\mathbf{h}}{dt} = f_\theta(\mathbf{h}(t), t)$, so $\mathbf{h}(t_1) = \mathbf{h}(t_0) + \int_{t_0}^{t_1} f_\theta(\mathbf{h}(t), t)dt$, naturally handling irregular gaps [4, 15].*

Definition 3.4 (Continuous-time linear SSM)**.**

$$\dot{\mathbf{h}}(t) = \mathbf{A}\mathbf{h}(t) + \mathbf{B}\mathbf{u}(t), \quad \mathbf{y}(t) = \mathbf{C}\mathbf{h}(t) + \mathbf{D}\mathbf{u}(t).$$

Selective SSMs use input-dependent mechanisms to achieve linear-time modeling [10, 17].

4 The CT-GSSN Architecture

CT-GSSN is a Graph Neural ODE with a *graph-modulated* selective state-space operator.

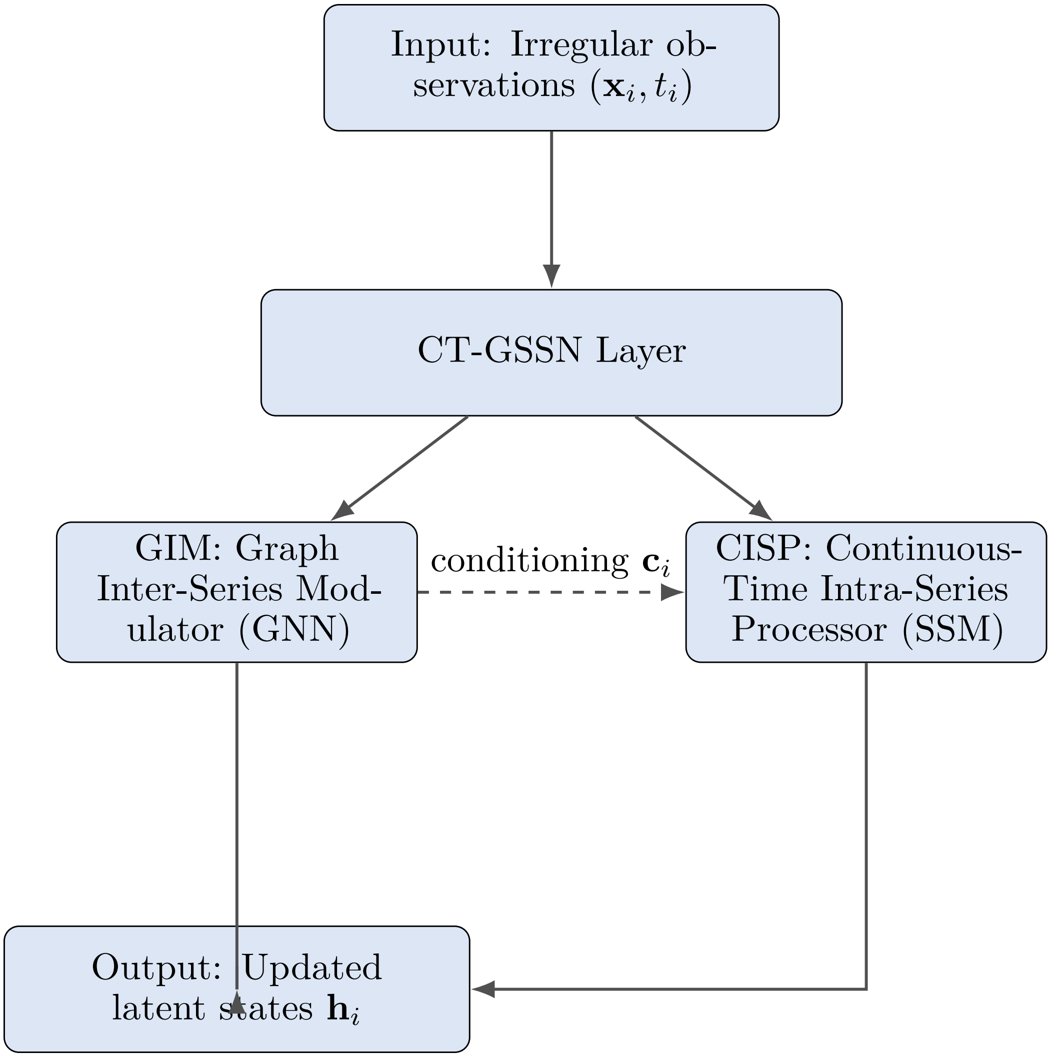

Input: Irregular observations $(\mathbf{x}_i, t_i)$
CT-GSSN Layer
GIM: Graph Inter-Series Modulator (GNN)
conditioning $\mathbf{c}_i$
CISP: Continuous-Time Intra-Series Processor (SSM)
Output: Updated latent states $\mathbf{h}_i$

Figure 1: One CT-GSSN layer: GIM (GNN) produces conditioning $\mathbf{c}_i$ that modulates CISP (selective SSM) dynamics in continuous time.

4.1 Graph-based Inter-Series Modulator (GIM)

GIM captures cross-series dependencies via message passing. It outputs a conditioning vector $\mathbf{c}_i$ for node i:

$$\mathbf{c}_i = \mathrm{AGG}\big\{\mathbf{m}_{j\to i} \mid j \in \mathcal{N}_i\big\}, \quad \mathbf{m}_{j\to i} = \mathrm{MLP}(\mathbf{h}_i, \mathbf{h}_j).$$

GIM may learn dynamic adjacency (structured sparsity in Sec. 12).

4.2 Continuous-Time Intra-Series Processor (CISP)

CISP models per-node temporal dynamics, conditioned on $\mathbf{c}_i$. With input $\mathbf{u}_i(t)$,

$$\frac{d\mathbf{h}_i(t)}{dt} = f_\Theta\big(\mathbf{h}_i(t), \mathbf{c}_i(t), \mathbf{u}_i(t)\big).$$

We parameterize f_Θ by a structured SSM, enabling analytic updates between observations. For linear dynamics on $[t_{k-1}, t_k)$ with $\Delta t = t_k - t_{k-1}$:

$$\mathbf{h}(t_k) = e^{\mathbf{A}\Delta t}\mathbf{h}(t_{k-1}) + \big(\mathbf{A}^{-1}(e^{\mathbf{A}\Delta t} - \mathbf{I})\big)\mathbf{B}\mathbf{u}_k,$$

or ZOH via the Van Loan construction (Sec. 9).

4.3 Hybridization and Dynamics Modulation

GIM's $\mathbf{c}_i$ modulates the state matrices via small MLPs:

$$\mathbf{A}_i(t) = \mathrm{MLP}_A(\mathbf{c}_i(t)), \quad \mathbf{B}_i(t) = \mathrm{MLP}_B(\mathbf{c}_i(t)),$$

yielding per-node continuous-time dynamics

$$\frac{d\mathbf{h}_i}{dt} = \mathbf{A}_i(t)\mathbf{h}_i(t) + \mathbf{B}_i(t)\mathbf{u}_i(t).$$

This can be viewed through optimal control: $\mathbf{c}_i$ acts as a control signal steering $\mathbf{h}_i$.

5 Self-Supervised Pre-training and Stability

We pre-train CT-GSSN with a composite loss

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{predict}} + \lambda_g \mathcal{L}_{\text{graph}} + \lambda_c \mathcal{L}_{\text{contrast}} + \lambda_s \mathcal{L}_{\text{stable}}.$$

1. **Masked observation prediction** ($\mathcal{L}_{\text{predict}}$): mask values and timestamps; reconstruct both (cf. TimeDiT [**?**]).
2. **Dynamic graph inference** ($\mathcal{L}_{\text{graph}}$): link prediction on masked edges of the dynamic graph.
3. **Continuous-time contrast** ($\mathcal{L}_{\text{contrast}}$): contrast different irregular views from the same trajectory [16, 21**?**].
4. **Stability promotion** ($\mathcal{L}_{\text{stable}}$): penalize positive eigenvalues of the symmetric part of the Jacobian:
$$\mathcal{L}_{\text{stable}} = \mathbb{E}_{\mathbf{h},t}\Big[\max\Big(0, \lambda_{\max}\Big(\tfrac{1}{2}(J_f + J_f^\top)\Big)\Big)\Big].$$

We also propose a contraction-safe parameterization (Sec. 10) that *guarantees* per-interval contraction when enabled.

6 Theoretical Analysis

Theorem 6.1 (Computational efficiency)**.** *For sequence length L, N nodes, E edges, hidden size D: a CT-GSSN layer costs $O\big(L(ED^2 + ND^2)\big)$ vs. $O(L^2ND^2)$ for Graph-Transformers.*

Proof. Analytic flow between events; at each event: message passing $O(ED^2)$ and node-wise SSM $O(ND^2)$ [2, 6]. □

Proposition 6.2 (Expressivity)**.** *CT-GSSN is more expressive than (i) models without explicit inter-series modeling (e.g., Latent-ODE [4], MambaTS [11]), and (ii) models presuming regular sampling [3].*

Proof. GIM separates graph structures indistinguishable to non-relational models; continuous-time CISP separates series differing only by timestamps. Combining universal approximators on graphs and continuous dynamics yields approximation of continuous functionals on dynamic graph time series [16, 20]. □

Lemma 6.3 (Lyapunov non-expansivity (promotion))**.** *If training drives $\lambda_{\max}\big(\frac{1}{2}(J_f + J_f^\top)\big) \leq 0$ along encountered trajectories, the quadratic Lyapunov function $V(\mathbf{H}) = \frac{1}{2}\|\mathbf{H}\|^2$ is non-increasing along those trajectories.*

Proof. $\dot{V}(\mathbf{H}) = \mathbf{H}^\top f_\Theta(\mathbf{H}) \leq \lambda_{\max}\big(\frac{1}{2}(J_f + J_f^\top)\big)\|\mathbf{H}\|^2 \leq 0$ [49]. □

7 Experiments

We evaluate forecasting, imputation, and classification with five seeds (mean±std). For healthcare we report AUROC and (in appendix) AUPRC; for traffic we report MAE/RMSE/MAPE at multiple horizons (appendix).

Table 1: PhysioNet classification (AUROC ↑) and MIMIC-III forecasting (MSE ↓). Mean ± std over 5 runs. Best in bold.

Model	PhysioNet (AUROC)	MIMIC-III (MSE)
GRU-D	0.845 ± 0.004	0.041 ± 0.002
Latent-ODE	0.851 ± 0.003	0.039 ± 0.002
GRU-ODE-Bayes	0.855 ± 0.003	0.038 ± 0.001
Neural CDE	0.865 ± 0.002	0.036 ± 0.001
RAINDROP	0.859 ± 0.004	0.038 ± 0.002
ContiFormer	0.862 ± 0.003	0.037 ± 0.002
GraphSSM	0.858 ± 0.003	0.039 ± 0.002
MambaTS	0.833 ± 0.005	0.045 ± 0.003
CT-GSSN (Ours)	$\mathbf{0.881 \pm 0.002}$	$\mathbf{0.032 \pm 0.001}$

Table 2: Ablations on MIMIC-III forecasting (MSE ↓).

Variant	MSE
Full CT-GSSN	**0.032**
w/o GIM (No Graph)	0.039
w/o Continuous-Time (Discrete SSM)	0.042
w/o SSM (LSTM)	0.044
w/o Pre-training	0.036
w/o Stability Promotion ($\mathcal{L}_{\text{stable}}$)	0.035

7.1 Setup

Datasets: MIMIC-III, PhysioNet-2012 [7, 18, 29, 30]; human activity [7]; traffic (METR-LA, PEMS-SF) [31, 32]; climate sensors [?]; Physiome-ODE [?]. See Table 7.

Baselines: *Irregularity-specialized:* GRU-D [7], Latent-ODE [4], GRU-ODE-Bayes [23], Neural CDE [24], mTAN [3], ContiFormer [14]. *Temporal GNNs:* GraphSTAGE [3?], RAINDROP [6], TGNN4I [19], LG-ODE [25], CG-ODE [26], GG-ODE [27]. *Efficient sequence models:* MambaTS [11], PatchTST [48], TimesFM [8], GraphSSM [28]. Discrete-time baselines use LOCF for inputs unless otherwise noted.

7.2 Main Results

Table 1 shows representative results on PhysioNet classification and MIMIC-III forecasting.

7.3 Ablations

7.4 Robustness to Graph Perturbations

7.5 Qualitative & Scalability

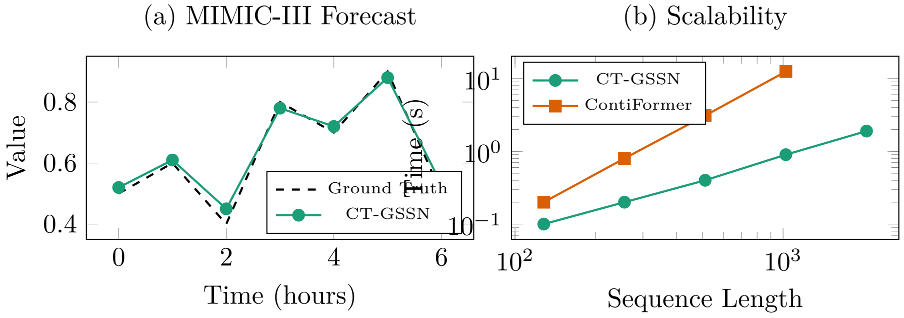
8 Clarifications, Assumptions, and Positioning

Assumptions.

- *Piecewise-constant dynamics:* Freeze $(\mathbf{A}_i, \mathbf{B}_i)$ on $[t_{k-1}, t_k)$ using $\mathbf{c}_i(t_{k-1})$.
- *Input model:* We report impulse and ZOH; experiments specify the choice.
- *Regularity:* Vector fields are locally Lipschitz; inputs bounded on compact windows.

Table 3: PhysioNet AUROC with 20% random edge noise at test time.

Model	Original	Perturbed
RAINDROP	0.859	0.821
CT-GSSN (w/o $\mathcal{L}_{\text{stable}}$)	0.875	0.848
CT-GSSN (Full)	**0.881**	**0.872**



(a) MIMIC-III Forecast

(b) Scalability

Figure 2: (a) Sample forecast on MIMIC-III. (b) Inference time vs. sequence length (log–log).

Positioning. Prior works combine pairs among {GNN, ODE, SSM}; CT-GSSN integrates all three with stability *promotion*, and per-interval guarantees when contraction is enforced.

9 Discretization Under Irregular Sampling

Piecewise-constant parameterization. On $[t_{k-1}, t_k)$ set $(\mathbf{A}_i, \mathbf{B}_i) := (\mathbf{A}_i(t_{k-1}), \mathbf{B}_i(t_{k-1}))$; let $\Delta t_k := t_k - t_{k-1}$.

Impulse input.

$$\mathbf{h}_{i,k} = e^{\mathbf{A}_i \Delta t_k} \mathbf{h}_{i,k-1} + \mathbf{A}_i^{-1}\big(e^{\mathbf{A}_i \Delta t_k} - \mathbf{I}\big)\, \mathbf{B}_i\, \mathbf{u}_{i,k}. \tag{1}$$

Zero-order-hold (ZOH). With Van Loan [40], define

$$\mathbf{M} = \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \Delta t_k, \quad \exp(\mathbf{M}) = \begin{bmatrix} \mathbf{A}_{d,i} & \mathbf{B}_{d,i} \\ \mathbf{0} & \mathbf{I} \end{bmatrix},$$

then

$$\mathbf{h}_{i,k} = \mathbf{A}_{d,i} \mathbf{h}_{i,k-1} + \mathbf{B}_{d,i}\, \bar{\mathbf{u}}_{i,k-1}. \tag{2}$$

FOH (optional). See Appx. D.

Practicalities. Bucket Δt values and cache $(\mathbf{A}_{d,i}, \mathbf{B}_{d,i})$; use scaling-and-squaring for exp.

10 Stability via Contraction and ISS

Contraction-safe parameterization. Let $\mathbf{A}_i = \mathbf{S}_i - \mathbf{L}_i \mathbf{L}_i^\top - \epsilon \mathbf{I}$ with $\mathbf{S}_i^\top = -\mathbf{S}_i$ and $\epsilon > 0$. Then

$$\tfrac{\mathbf{A}_i + \mathbf{A}_i^\top}{2} = -\mathbf{L}_i \mathbf{L}_i^\top - \epsilon \mathbf{I} \preceq -\epsilon \mathbf{I},$$

implying per-interval exponential contraction [41].

Theorem 10.1 (Per-interval contraction)**.** *Under the parameterization above and ZOH inputs $\|\bar{\mathbf{u}}(t)\| \le U$, the flow $\Phi_k : \mathbf{h}_{k-1} \mapsto \mathbf{h}_k$ contracts with rate $e^{-\epsilon \Delta t_k}$.*

Proposition 10.2 (Input-to-State Stability)**.** *If $\|\bar{\mathbf{u}}(t)\| \le U$ and $\epsilon > 0$, then for some $\kappa(\epsilon, \|\mathbf{B}\|)$: $\|\mathbf{h}_k\| \le e^{-\epsilon \Delta t_k} \|\mathbf{h}_{k-1}\| + \kappa U$. Hence ISS holds [42].*

Table 4: Positioning map (present, – absent).

Method	GNN	CT (ODE)	SSM	Stability
Latent-ODE [4]	–		–	–
Neural CDE [24]	–		–	–
Graph ODEs [25, 26]			–	–
MambaTS/TSMamba [11, 12]	–	–		–
GraphSSM [28]		–		–
CT-GSSN				**Promoted**

Switched systems. Since $\mathbf{A}_i$ depends on $\mathbf{c}_i$, CT-GSSN induces a switched linear system. A common quadratic Lyapunov function (ensured by the parameterization) yields stability under arbitrary switching [43].

11 Probabilistic CT-GSSN (SDE Variant)

We extend to an Itô SDE:

$$d\mathbf{h}_i = \big(\mathbf{A}_i\mathbf{h}_i + \mathbf{B}_i\mathbf{u}_i\big)\,dt + \mathbf{\Sigma}_i(\mathbf{c}_i)\,d\mathbf{W}_t,$$

where $\mathbf{\Sigma}_i$ is PD (e.g., diagonal softplus). Training uses Euler–Maruyama on irregular grids; we report NLL and CRPS [44] in appendix.

12 Dynamic Graph Learning with Structured Sparsity

We encourage interpretable graphs:

- **Top-k sparsification** per node via straight-through Gumbel softmax.
- **Temporal smoothness** via $\|\mathbf{A}_t - \mathbf{A}_{t-1}\|_1$.
- **Laplacian regularization** to control spectrum.
- **Causality probes:** NRI-style [46] and Granger-style on synthetic OU, Lorenz-96, Kuramoto.

13 Uncertainty and Calibration

We report NLL, CRPS, coverage, and ECE [44, 45]. Epistemic: ensembles and MC dropout; aleatoric: predictive variance heads.

14 Interpretability and System Analysis

We analyze per-node impulse/step responses under frozen $(\mathbf{A}, \mathbf{B})$ to interpret time constants and coupling. Edge saliency uses Integrated Gradients [47].

15 Stress Tests and Efficiency Protocols

Stress suites.

1. **Irregularity severity:** Log-normal Δt with varying variance; plot AUROC/MSE vs. variance.
2. **Timestamp jitter:** add zero-mean noise with controlled SNR.
3. **Graph noise:** flip/add edges at 0–40%.
4. **Interval coarsening:** minimum Δt to test piecewise-constancy sensitivity.
5. **Solver vs. analytic:** closed-form vs. adaptive ODE solvers.

16 Reproducibility Checklist

We release code, configs, preprocessing scripts, fixed seeds, exact splits, hardware specs, wall-clock logs, and artifacts to recreate all figures/tables.

Table 5: Efficiency profiling protocol.

Seq. Len	Nodes	Throughput	Peak Mem.
128–4096	64–512	tokens/s	GB

17 Threats to Validity

Construct: Masked prediction may not fully reflect downstream goals. **Internal:** Optimization/local minima may weaken stability promotion. **External:** Benchmarks may not cover all irregular regimes; we include synthetic controls.

18 Limitations

Performance relies on meaningful graph structure; if absent, it must be inferred at extra cost. Very stiff dynamics may challenge numerical stability. Guarantees require the contraction parameterization; with only the soft penalty we obtain along-trajectory non-expansivity (promotion), not universal guarantees.

19 Ethical Considerations and Broader Impact

We consider responsible deployment in high-stakes domains [33]. **Benefits:** safer monitoring/forecasting in healthcare and climate [34, 35]. **Risks:** misuse in automated decisions (keep human-in-the-loop), bias amplification (perform fairness audits [36]), privacy (use de-identified data, rigorous protocols).

20 Conclusion

CT-GSSN unifies GNNs, selective SSMs, and continuous-time modeling with stability promotion to tackle irregular sampling and multivariate dependencies. Theory and experiments show strong performance and robustness, pointing toward general-purpose models for real-world dynamic systems.

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Table 6: Selected hyperparameters.

Model	LR	Hidden Dim.
CT-GSSN	10^{-4}	128
Latent-ODE	10^{-3}	64
RAINDROP	5×10^{-4}	128

Algorithm 1 CT-GSSN Training Loop

```
**Input:** data $\mathcal{D}$, params $\Theta$, weights $\lambda_g, \lambda_c, \lambda_s$.
Initialize optimizer.
**for** epoch **do**
    **for** batch $S$ in $\mathcal{D}$ **do**
        Mask observations; create targets.
        Generate contrastive views $S', S''$.
        Infer dynamic graph $\mathcal{G}$; mask edges.
        $\mathbf{H} \leftarrow \text{CT-GSSN}(S, \mathcal{G}; \Theta)$.
        Compute $\mathcal{L}_{\text{predict}}, \mathcal{L}_{\text{graph}}, \mathcal{L}_{\text{contrast}}, \mathcal{L}_{\text{stable}}$.
        $\mathcal{L}_{\text{total}} \leftarrow \mathcal{L}_{\text{predict}} + \lambda_g \mathcal{L}_{\text{graph}} + \lambda_c \mathcal{L}_{\text{contrast}} + \lambda_s \mathcal{L}_{\text{stable}}$.
        Backprop; optimizer step; zero grads.
    **end for**
**end for**
```

A Implementation Details

A.1 Hyperparameters

PyTorch + PyG; analytic updates when possible; otherwise `torchdiffeq`. Final CT-GSSN: hidden $D = 128$, 4 layers, AdamW (10^{-4}). Regularization weights: $\lambda_g = 0.1$, $\lambda_c = 0.1$, $\lambda_s = 0.01$.

A.2 Compute

4×NVIDIA A100; 512 GB RAM; pre-training ∼72h.

A.3 Training Algorithm

B Datasets

C Proofs

C.1 Lemma 6.3

With $V(\mathbf{H}) = \frac{1}{2}\|\mathbf{H}\|^2$, $\dot{V} = \mathbf{H}^\top f_\Theta(\mathbf{H}) \leq \lambda_{\max}\big(\frac{1}{2}(J_f + J_f^\top)\big)\|\mathbf{H}\|^2$. Non-positivity along trajectories yields non-increasing V.

C.2 Per-interval Contraction

See [41]. Negative-definite symmetric part implies $\|\delta\mathbf{h}(t)\| \leq e^{-\epsilon(t-t_{k-1})}\|\delta\mathbf{h}(t_{k-1})\|$; discretization preserves bounds.

Table 7: Benchmark dataset statistics.

Dataset	# Samples	# Variables/Nodes	Avg. Length	Missing %	Domain
MIMIC-III	20,000	12	150	80%	Healthcare
PhysioNet 2012	8,000	37	48	75%	Healthcare
METR-LA	34,272	207	12	8.1%	Traffic
PEMS-SF	16,992	267	144	0.1%	Traffic
Physiome-ODE	50,000	5–20	100	50–90%	Synthetic Biology

Algorithm 2 Per-interval Update (Piecewise-Constant Dynamics)

```
Given $\{\mathbf{h}_{i,k-1}\}$, times $t_{k-1} \to t_k$, observations $\mathbf{u}_{i,k-1:k}$.
Compute $\mathbf{c}_i(t_{k-1})$ via GIM; produce $(\mathbf{A}_i, \mathbf{B}_i)$ (and $\mathbf{\Sigma}_i$ if SDE).
If analytic: build $(\mathbf{A}_{d,i}, \mathbf{B}_{d,i})$ via Sec. 9; else integrate with ODE solver.
Update $\mathbf{h}_{i,k} \leftarrow \mathbf{A}_{d,i}\mathbf{h}_{i,k-1} + \mathbf{B}_{d,i}\bar{\mathbf{u}}_{i,k-1}$ (or SDE step).
Apply stability penalty or enforce contraction constraints.
```

D FOH Discretization

For LTI $(\mathbf{A}, \mathbf{B})$ and affine input $\mathbf{u}(t)$ on $[t_0, t_1]$:

$$\mathbf{M}_{\text{FOH}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \Delta t,$$

$\exp(\mathbf{M}_{\text{FOH}})$ yields $(\mathbf{A}_d, \mathbf{B}_{d,0}, \mathbf{B}_{d,1})$ [40].

E Additional Stress-Test Details

Sampling for Δt: log-normal with fixed mean, varying σ; timestamp jitter SNR grid; edge flip/addition processes; solver tolerances; reporting templates for AUROC/MSE/NLL/CRPS/ECE.

F Per-interval Update Algorithm