TITLE OF THE THESIS

by

Author (CID: ...)

Department of Informatics

King's College London

WC2R 2LS London

United Kingdom

Thesis submitted as part of the requirements for the award of the MSc in Web Intelligence, King's College London, 2016-2017

Contents

Acknowledgements

I would like to thank my supervisor.....

1 Introduction

General introduction.

2 Option pricing

2.1 The fundamental theorem of asset pricing

2.2 The Black-Scholes model

Consider a given probability space $(\Omega, (\mathcal{F})_t, \mathbb{P})$ supporting a Brownian motion $(W_t)_{t\geq 0}$. In the Black-Scholes model, the stock price process $(S_t)_{t\geq 0}$ is the unique strong solution to the following stochastic differential equation:

$$\frac{\mathrm{d}S_t}{S_t} = r\mathrm{d}t + \sigma \mathrm{d}W_t, \qquad S_0 > 0, \tag{2.1}$$

where $r \geq 0$ denotes the instantaneous risk-free interest rate and $\sigma > 0$ the instantaneous volatility.

2.2.1 No interest rates

2.2.2 Including interest rates

A European call price $C_t(S_0, K, \sigma)$ with maturity $t > 0$ and strike $K > 0$ pays at maturity $(S_t - K)_+ = \max(S_t - K, 0)$. When the stock price follows the Black-Scholes SDE (2.1), Black and Scholes [1] proved that its price at inception is worth

$$C_t(S_0, K, \sigma) = S_0 \mathcal{N}(d_+) - K\mathrm{e}^{-rt}\mathcal{N}(d_-),$$

where

$$d_\pm := \frac{\log\left(S_0 \mathrm{e}^{rt}/K\right)}{\sigma\sqrt{t}} \pm \frac{\sigma\sqrt{t}}{2},$$

and where $\mathcal{N}$ denotes the cumulative distribution function of the Gaussian random variable.

Here is an example of how to insert a picture:

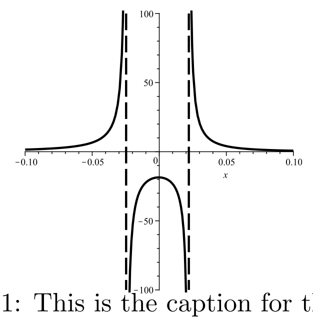

Figure 1: This is the caption for the figure.

or two side-by-side pictures:

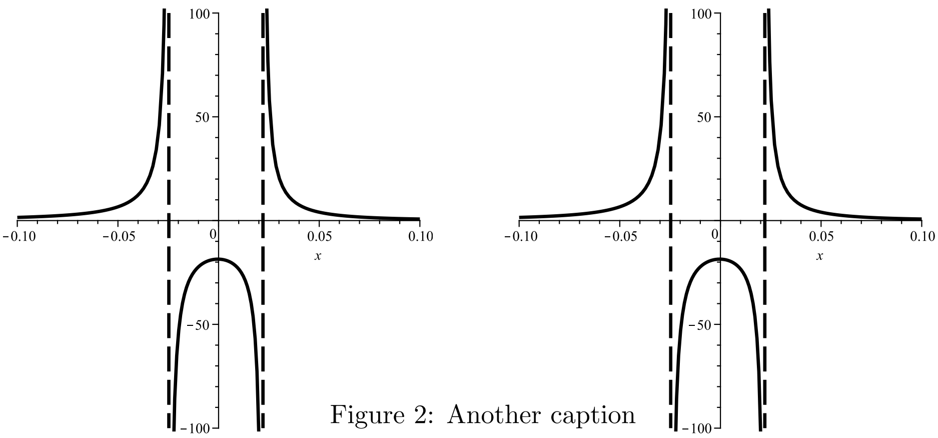

Figure 2: Another caption

2.3 The Heston model

In the Heston model, the stock price is the unique strong solution to the following stochastic differential equation:

$$\begin{aligned} \mathrm{d}S_t &= S_t\sqrt{V_t}\mathrm{d}W_t, & S_0 = s > 0, \\ \mathrm{d}V_t &= \kappa(\theta - V_t)\mathrm{d}t + \xi\sqrt{V_t}\mathrm{d}Z_t, & V_0 = v_0 > 0, \\ \mathrm{d}\langle W, Z\rangle_t &= \rho\mathrm{d}t, \end{aligned} \tag{2.2}$$

where $\kappa, \xi, \theta, v_0, s > 0$ and the correlation parameter ρ lies in $[-1, 1]$.

3 Model calibration

3.1 What is calibration?

Here is an example of a matrix in $A \in \mathcal{M}_n(\mathbb{R})$:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ a_{n1} & \dots & \dots & a_{1n}. \end{pmatrix}$$

3.2 Numerical methods for calibration

...

A Review of stochastic calculus

A.1 Riemann integration

A.2 The Itô integral

B Some technical proofs

Conclusion

Conclusion if needed...

References

[1] F. Black and M. Scholes. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81 (3): 637-659, 1973.

[2] I. Karatzas and S.E. Shreve. Brownian Motion and Stochastic Calculus. Springer-Verlag, 1997.

[3] S. Karlin and H. Taylor. A Second Course in Stochastic Processes. Academic Press, 1981.

[4] P. Tankov. Pricing and hedging in exponential Lévy models: review of recent results. *Paris-Princeton Lecture Notes in Mathematical Finance*, Springer, 2010.

[5] D. Williams. Probability With Martingales. CUP, 1991.

Acknowledgements

I would like to thank my supervisor.....