Estimation of Power System Model Parameters

- Uncertainty Distributions and Confidence Intervals -

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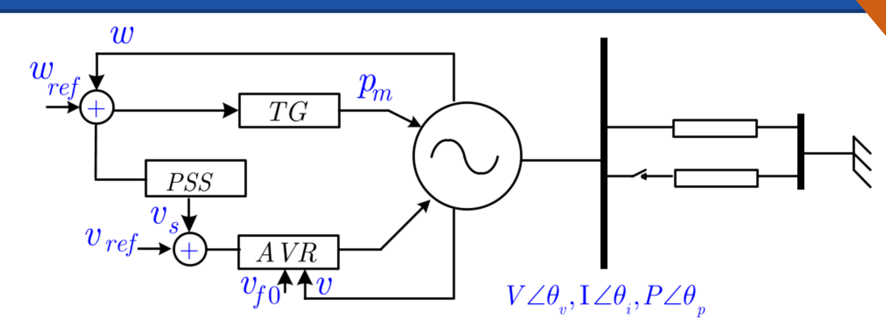
Problem statement 1

To identify the model parameters and their confidence interval.

Assuming: (i) the model structure of a power plant (including a generator with controls) and, (ii) having available measurements from the tests and on the terminal bus.



Generator with controllers connected to an infinite bus

$$\frac{dx}{dt} = f(x(t), \rho, u, \xi) \tag{1}$$

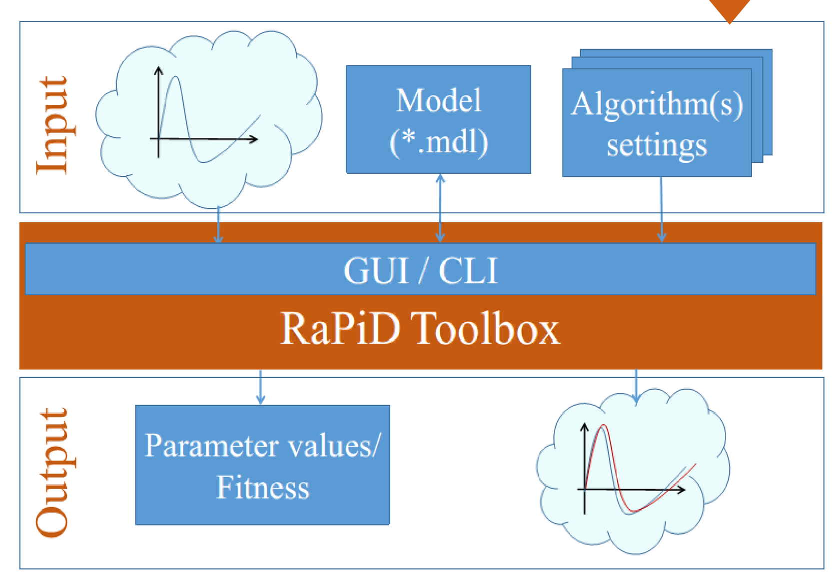
$$y = h(x(t), \rho, u, \sigma), \tag{2}$$

where $\{x, \rho\}$- states/parameters (some of them to be estimated); σ - measurement noise, ξ - process noise; and u - input (e.g. terminal voltage), y - measurements.

Computation Approach 2

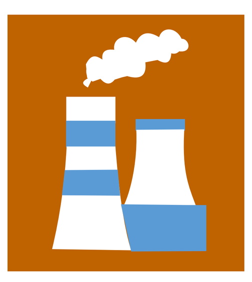
The RaPId Toolbox implemented in MATLAB/Simulink to carry out model validation and parameter identification on models developed using the Modelica language.

The Modelica model inside an "FMU" is used for simulation in MATLAB/Simulink within RaPId's workflow.

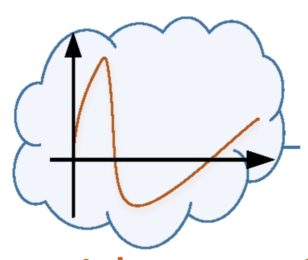


Identification Method 3

Model structure



Measurements



Algorithm settings

$(p_{min}, p_{max}, N_p, \epsilon, n_{it})$

2 Run Particle Filter (PF)

PF recursively estimates a belief (pdf) in the parameters by using all available information about the system's structure and measurements.

Algorithm 1 Particle Filter

1: **procedure** PF($p_{min}, p_{max}, N_p, \epsilon, n_{it}$)
2: **while** $n < n_{it}$ and {stop criteria} **do**
3: Step 1. Initialization (sampling from uniform prior pdf):
4: **for** i = 1 to N_p **do**
5: draw the samples $x_n^{(i)} \propto p(x_0)$
6: $\omega_0^{(i)} \leftarrow 1/N_p$.
7: Step 2. Importance Sampling:
8: **for** i = 1 to N_p **do**
9: draw samples $\hat{x}_n^{(i)} \propto p(x_n|x_{n-1}^{(i)})$,
10: $\hat{x}_n^{(i)} \leftarrow \{x_{0:n-1}^{(i)}, \hat{x}_n^{(i)}\}$
11: Step 3. Weight update with normalization:
12: **for** i = 1 to N_p **do**
13: $RSSD(y,\hat{y}) \leftarrow \frac{1}{M}\sum_i(\frac{y_i-\hat{y}_i}{y_i})^2$
14: $fitness \leftarrow RSSD(y,\hat{y})$
15: $\omega_n^{(i)} \leftarrow 1 - \frac{fitness}{\sum_i fitness}$
16: Step 4. Resampling:
17: Generate/prune particles $x_n^{(i)}$ from $\{\hat{x}_n^{(i)}\}$
18: according to $\omega_n^{(i)}$/(prune if $\omega_n^{(i)} < \epsilon$)
19: to obtain N_p random samples

where $[p_{min}, p_{max}]$ - range of parameters' values which define parameter space, N_p - number of particles used to fill the space of parameter values; Y - real measurements; $\hat{Y}$ - estimate; ϵ - prune threshold (defines which percentage of particles with lowest weight will not survive); {stop criteria} - set of conditions to finish the main procedure cycle; M - number of measurement instances.

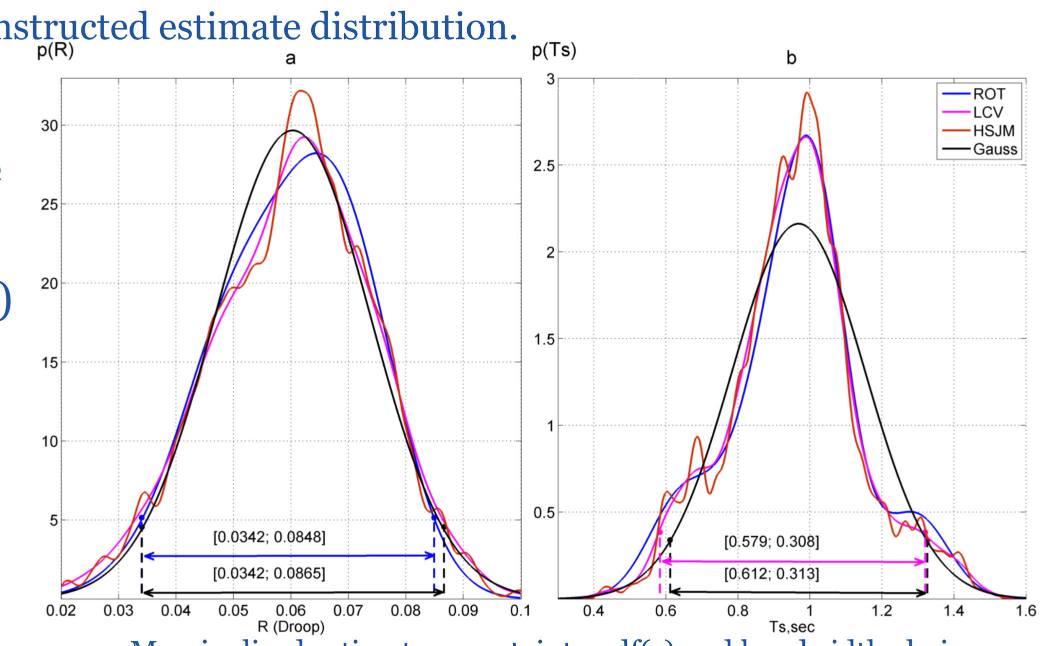
Particles, Fitness

1 Choose the bandwidth for the Gaussian kernel

Covariance in the Gaussian kernel plays the role of a smoothing the parameter that defines the shape of the reconstructed estimate distribution.

Three methods for bandwidth selection are used:

- Rule of Thumb (ROT)
- Least-Squares Cross Validation (LCV)
- Plug-in method (HSJM).



Marginalized estimate uncertainty pdf(s) and bandwidth choice

Particles, weights

Covariance

3 Reconstruct the uncertainty distribution

The posterior distribution that is an output of PF is described by a number of samples (particles) with weights assigned according to a fitness function. The continuous pdf can be constructed from PF by assigning kernel over each particle.

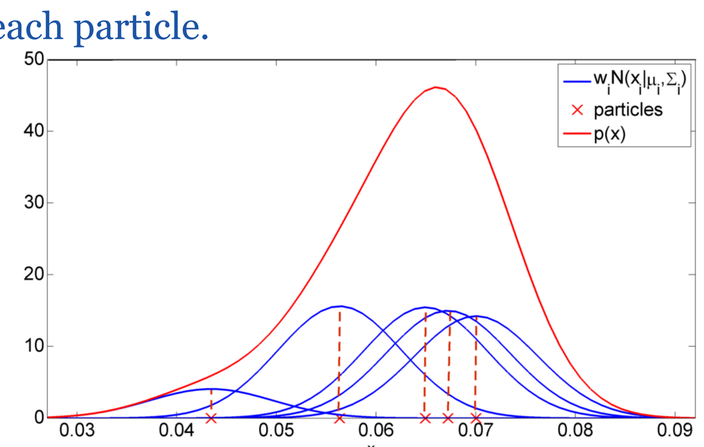
$$p(x) = \sum_{n=1}^{N_p} w_n N(\boldsymbol{x}_n|\boldsymbol{\mu}_n, \Sigma_n), \tag{8}$$

where $\sum_{n=1}^{N_p} w_n = 1$, $0 \leqslant w_n \leqslant 1$ - normalization of N_p individual M-dimentional Gaussian components $N(\boldsymbol{x}_n|\boldsymbol{\mu}_n, \Sigma_n)$, where M - number of parameters.

Each component is a multivariate Gaussian distribution given by:

$$N(\boldsymbol{x}|\boldsymbol{\mu}, \Sigma) \stackrel{def}{=} \frac{1}{(2\pi)^{M/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right) \tag{9}$$

where $\boldsymbol{x} = [x_1 .. x_M]$ - vector of parameters.



Gaussian mixture distribution reconstruction

Uncertainty distribution

4 Estimate the confidence intervals

The required confidence space allocation, and consequently, the confidence intervals allocation, is transformed to the following problem:

Find the cutting surface C: $p(x)=b$ that intersects with the Gaussian mixtures pdf $p(x)$ given by (8) and gives a projection contour area on ($x_1, .., x_M$) coordinates plane equal to S, where S – union of ellipses.

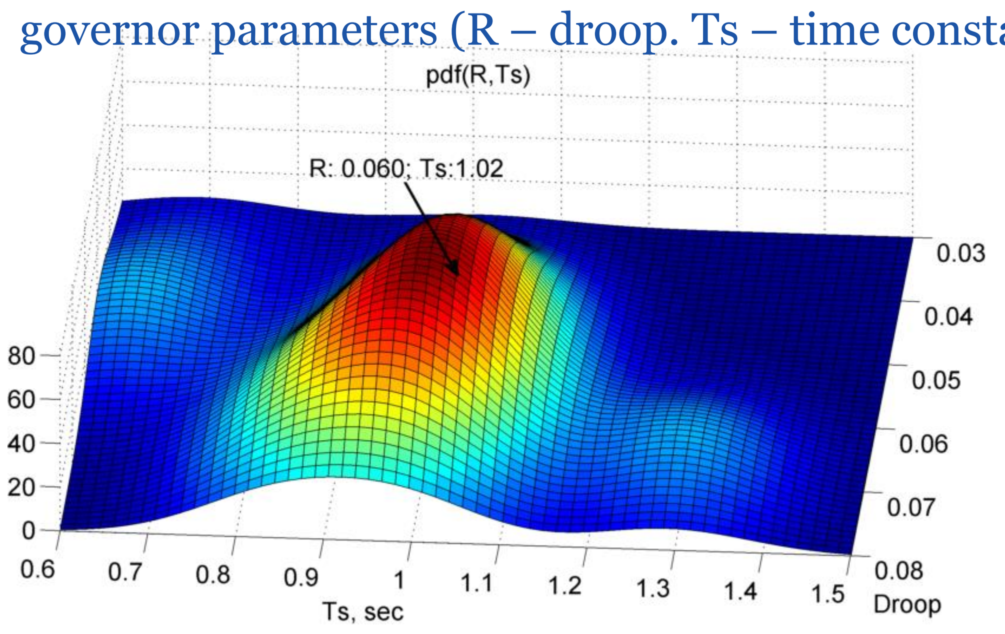
In (9), $\Delta^2 = (\boldsymbol{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu} = \sum_{m=1}^{M}\frac{y_m^2}{\lambda_m} = S_i = a$, where $y_m \stackrel{def}{=} \boldsymbol{u}_m^T(\boldsymbol{x}-\boldsymbol{\mu})$, represents the squared Mahalanobis distance between x and μ. When Δ^2 is equal to a constant (a), it will define ellipsoids that result in locus of the equal density level of each Gaussian component. This means that it defines the confidence contours on the required (predefined) confidence region percentage (e.g. 95% confidence is the most common). Note that the union of the ellipses is given by:

$$S = \bigcup_{i=1}^{N} S_i \tag{10}$$

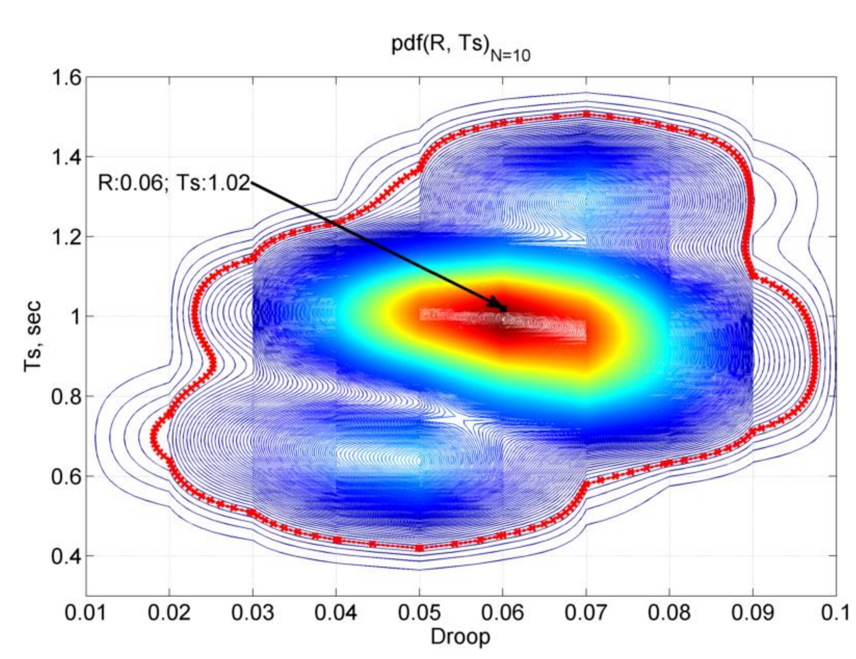
where N is the number of particles.

Parameter values, Uncertainty Distribution, Confidence Region and Intervals

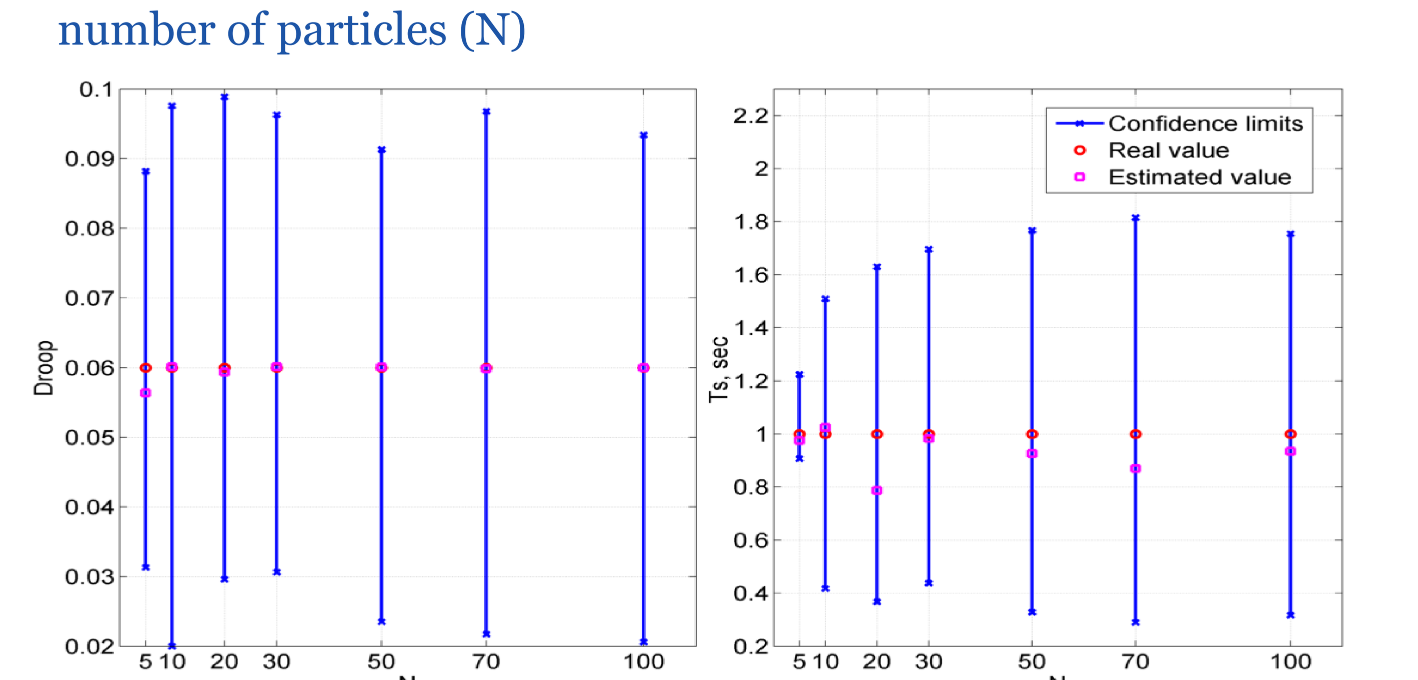
Reconstructed estimate uncertainty probability density function (N=10) of identified turbine-governor parameters (R – droop. Ts – time constant)



95% confidence region of [R, Ts] parameters uncertainty of estimate



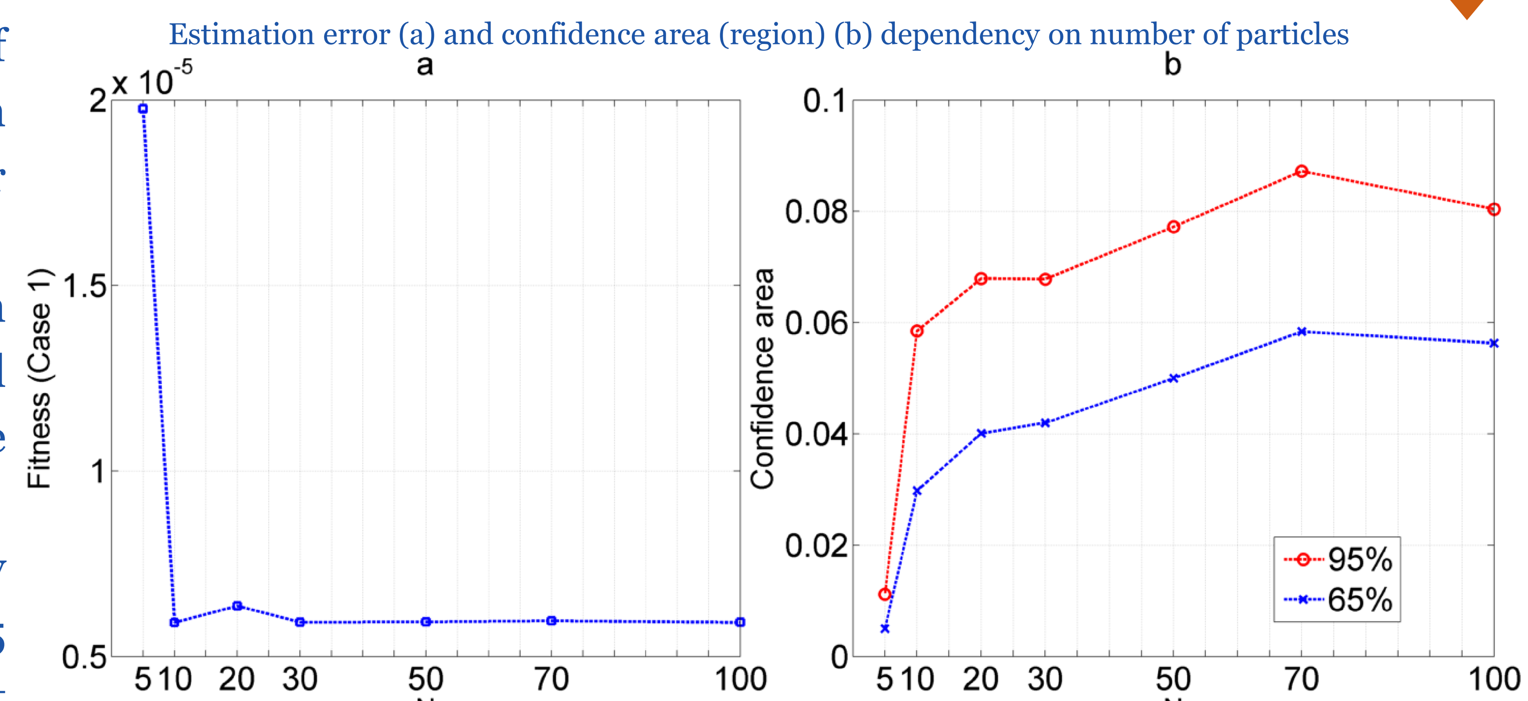
Confidence limits of the 95% probability for each parameter vs the number of particles (N)



Discussion 4

- The choice of the number of particles for PF was based on the minimal estimation error value.
- The sigma was chosen of such method giving the minimal uncertainty (confidence interval).
- The confidence area steeply growing with increase N from 5 to 10. This could mean that when N=5 there is a lack of knowledge about the uncertainty.

Estimation error (a) and confidence area (region) (b) dependency on number of particles



Estimated parameter values of the turbine governor

	Exp. Name	Exp. Setup	Parameter	Estimate	Confidence Region's (95%) boundary	Est. Error (Fitness)
1	TG staged tests	$\Delta\omega = +0.2\,Hz$ $(0.004\,p.u)$	$Droop\ (R)$ $Ts,\ sec$	0.0601 1.0242	[0.02..0.0976] [0.4175..1.5088]	5.91e-06

Conclusions 5

▶ The new methodology and algorithm to estimate parameters and their uncertainty distributions (the parameter confidence intervals) has been presented.

▶ The computationally greedy PF can be compensated for its computing disadvantages by providing it with more information about the estimates.

▶ The technique is recommended to power system analysts to use when performing power system model validation and parameter calibration tasks.

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