

Mohammed Ahsan Adib Murad, ID: 16203295

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University College Dublin

## 1 Hybrid Representations of Power Systems

Continuous behaviour of power systems generally described by Differential-Algebraic Equation (DAEs). But the discrete events in the power systems influence the continuous dynamics, these discrete events are handled as an ad hoc addition to continuous system description. In [1] and [2] a hybrid system representation of power system is proposed by a set of Differential , Switched Algebraic and State-Reset (DSAR) equations, which can capture all the characteristics of hybrid systems. The DSAR model is given by the equations (1)-(5)

$$\dot{x} = f(x, y, z, \lambda) \quad (1)$$

$$\dot{z} = 0 \quad (2)$$

$$0 = g^{(0)}(x, y) \quad (3)$$

$$0 = \begin{cases} g^{i-}(x, y, z, \lambda) & y_{s,i} < 0 \\ g^{i+}(x, y, z, \lambda) & y_{s,i} > 0 \end{cases} \quad i=1, \dots, s \quad (4)$$

$$z^+ = h_j(x^-, y^-, z^-, \lambda) \quad y_{r,j} = 0 \quad j \in 1, \dots, r \quad (5)$$

It can be seen, that (1) describes the differential equations, (3) and (4) describe the switched algebraic equation and (5) the state-reset equations, where

- $x$  are continuous dynamic states
- $y$  are algebraic states
- $z$  are discrete dynamic states
- $\lambda$  are parameters

of the system. The superscript  $-$  stands for pre-event and  $+$  for post event values. Based on this representation a simulation framework is developed in [3].

### 1.1 Example: Simple Power System:

In order to demonstrate the ability of the DSAR structure (1)-(5) a simple power system network is considered with a tap-changing transformer. The power system network is shown in Fig. 1. Same example system can be found also in [1]-[3].

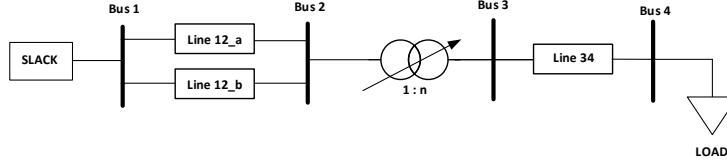


Figure 1: Example Power System.

The Automatic Voltage Regulator (AVR) logic of the tap-changing transformer is for low voltages, i.e., for increasing tap ratio. The model can be represented in the DSAR form as,

$$\begin{aligned}
 \dot{x}_1 &= y_1 y_7 \\
 0 &= y_2 - V_3 + V_{\text{low}} \\
 0 &= y_3 - y_4 + z_1 \\
 0 &= y_6 - n + n_{\text{max}} - n_{\text{step}}/2 \\
 0 &= n V_2 - V_3 \\
 0 &= y_1 - 1 & y_2 < 0 \\
 \left. \begin{aligned} 0 &= y_1 \\ 0 &= y_4 - x_1 \end{aligned} \right\} & y_2 > 0 \\
 0 &= y_7 - 1 & y_6 < 0 \\
 0 &= y_7 & y_6 > 0 \\
 0 &= y_5 - x_1 + z_1 + T_{\text{tap}} & y_3 < 0 \\
 0 &= y_5 - x_1 + y_4 + T_{\text{tap}} & y_3 > 0 \\
 \left. \begin{aligned} z_1^+ &= x_1^- \\ n^+ &= n^- + n_{\text{step}} \end{aligned} \right\} & \text{when } y_5 = 0
 \end{aligned}$$

The continuous dynamics of the real power load in bus 4 is given by

$$\begin{aligned}
 \dot{x}_p &= \frac{1}{T_p} (P_s^0 - P_d) \\
 P_d &= x_p + P_t V_4^2
 \end{aligned}$$

where,  $x_p$  is the load state driving the actual load demand  $P_d$ . In response to voltage disturbance, the load undergoes an initial transient given by the term  $P_t V_4^2$  and rate of recovery is dictated by the load time constant  $T_p$ .

Table 1: Base case parameters.

Component	Parameters
Line 12_a	$R = 0 \quad X = 0.65$
Line 12_b	$R = 0 \quad X = 0.40625$
Line 34	$R = 0 \quad X = 0.80$
Slack	$ V  = 1.05 \quad angle = 0$
Transformer	$V_{low} = 1.04 \quad n_{max} = 1.1 \quad T_{tap} = 20.0 \quad n_{step} = 0.0125$
Load	$P_s^0 = 0.4 \quad P_t = 0.4 \quad T_p = 5$

The data of the test system is given in Table 1. The simulation is carried out by tripping the line (Line 12\_b) at 10s. The initial tap position is set to 1.0375. The simulation results are shown in Fig. 2.

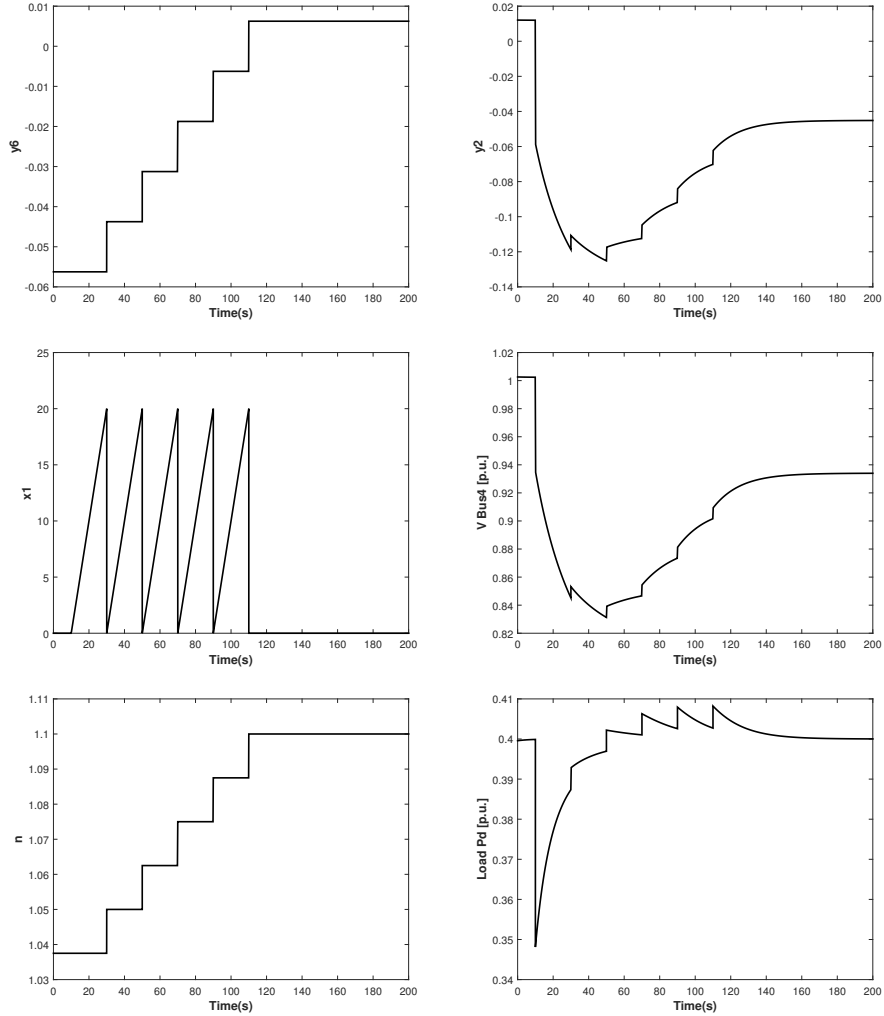


Figure 2: Trajectories of Example power system.

The dynamics of the tap-changing transformer are driven by different events that govern the behaviour of the timer. If the voltage is within the dead-band ( $y_2 < 0$ ) or, the tap position is at the upper limit ( $y_6 > 0$ ) then the timer is blocked. When the voltage is outside the dead-band ( $y_2 < 0$ ), the timer will run and if the timer reaches  $T_{tap}(y_5 = 0)$ , a tap change will occur. After every tap change the timer will be reset, but not necessarily blocked. From the simulation results shown in Fig. 2, it is evident that the proposed DSAR structure with appropriate simulation framework is capable of simulating combined continuous and discrete dynamics of power systems.

## 1.2 Extended version

The updated model is: TransformerDiscrete2. This model considers the voltage deadband: [vlow vhigh] and the tap can go in both direction. Simulation is carried out by tripping the breaker of line (Line 12\_b) at 20s and re-closing the breaker at 200s. The trajectories of voltage at Bus 3 and tap ratio are shown in Fig. 3.

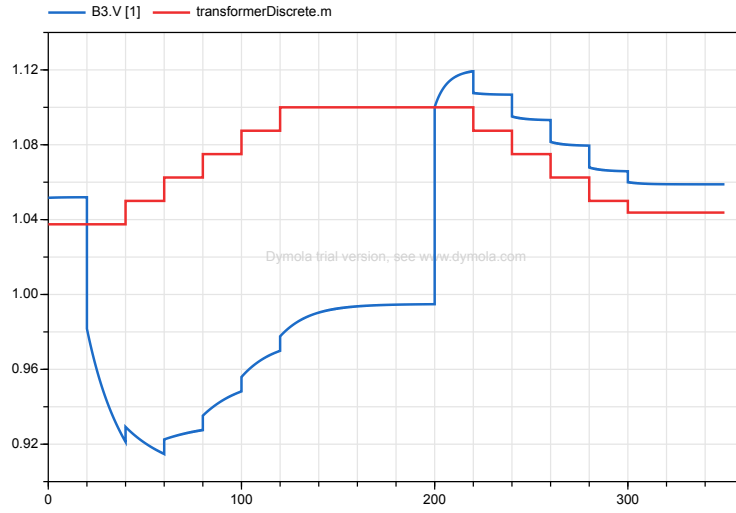


Figure 3: Trajectories of updated ULTC.

## References

- [1] I. A. Hiskens and M. A. Pai, "Hybrid systems view of power system modelling," in 2000 IEEE International Symposium on Circuits and Systems. Emerging Technologies for the 21st Century. Proceedings (IEEE Cat No.00CH36353), vol. 2, pp. 228–231 vol.2, 2000.
- [2] I. A. Hiskens and P. J. Sokolowski, "Systematic modeling and symbolically assisted simulation of power systems," IEEE Transactions on Power Systems, vol. 16, pp. 229–234, May 2001.
- [3] T. Demiray, Simulation of power system dynamics using dynamic phasor models. PhD thesis, Dept. Inf. Technol. Elect. Eng., ETH Zurich, Zurich, 2008.