iTesla

Innovative Tools for Electrical System Security within Large Area

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Author		Approval	
Name	Visa	Name	Date
[KTH] Dr. Rafael Segundo Dr. Luigi Vanfretti			

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Contributions

For the final deliverable the authors acknowledge the collaboration of the following people:

- **RTE**: *Jean-Baptiste*, for his contribution: providing dynamic simulations from the French network.
- **Statnett**: *Emil Hillberg* and *Jukka Turunen*, for their contribution: providing dynamic simulations from the Scandinavian network.
- **RSE**: *Diego Cirio*, for his valuable feedback to improve the indexes.

1 Introduction

The rising number of large-scale power outages in recent years clearly indicates the need for methods determining the likelihood of catastrophic system failures [1].

Dynamic impact assessment of detailed time-domain simulations is part of the off-line analysis. Its aim is to provide stability analysis, and to develop offline criteria to support the online analysis workflow in the iTesla toolbox [2].

After performing a dynamic simulation for a specific contingency, a set of appropriate post-contingency severity indexes will be determined in order to classify the impact of the contingency. To do so, a set of scalars , namely severity indexes, provides a measure of how severe the contingency is given a specific contingency. The measures proposed in the impact assessment allow fast computation because several contingencies have to be evaluated for each sample, and they must provide a good measure of how severe the contingency is.

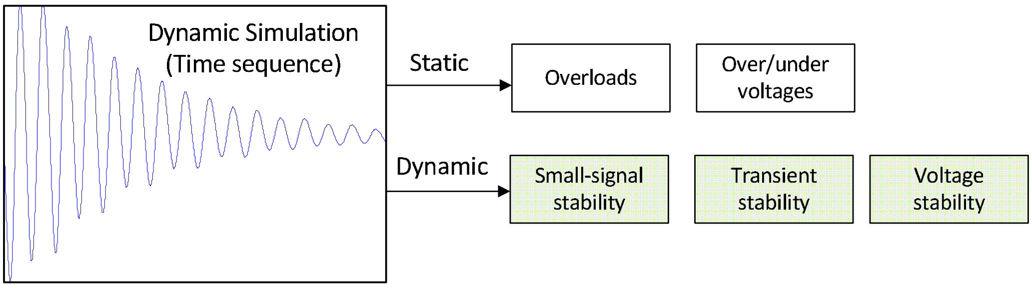

Figure 1: Classification of dynamic simulation

The outcome of a dynamic simulation is typically a sequence (time series) in the state space of the power system. After obtaining it, the main tasks are to, firstly, check whether this sequence will converge to a safe and stable equilibrium point for the system, and secondly, to measure the quality of the stability (or severity of the instability). This is carried out by calculating various indexes from the dynamic simulation's outputs of a contingency. The proposed indexes capture both static (i.e overloads and overvoltages) and dynamic (i.e. small-signal/voltage/transient) stability problems as seen in Figure 1.

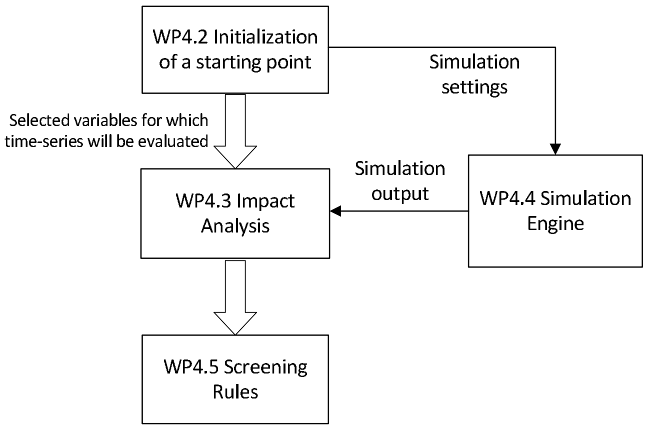

Figure 2: Interaction with other modules in WP4 of iTesla

During the development of the security indexes, we have interacted with other work packages such as WP4.2, WP4.4 and WP4.5 as described in Figure 2, to determine the relationships between each module of the iTesla toolbox, and to provide the required functionalities.

2 Static Classification

An index is a scalar, vector or a matrix of numbers or ratio indicating a specifics characteristic or properties, in this case, of a power system's stability. Before assessing system stability, simulation outputs can be analyzed to determine if a particular contingency will result in an acceptable operating condition.

Static indexes are used to classify the condition of the analyzed system as safe or unsafe through specific operational criteria. In this section, two different static indexes are presented, namely the overload and the under/over voltage index.

2.1 Overload Index

The time series of active and reactive power in the transmission network just after an outage has occurred can be calculated from simulation outputs. This calculations can be used to compare against the capacities of different devices in order to observe if the calculated post-fault time series of active and reactive power through the lines exceeds the capacity of any component in the network. If one or more components of the network are overloaded, the overload index can be used to measure the associated severity of the overload. The equation describing this index is

$$f_x = \sum_{i=1}^{N_l} wf_i \left(\frac{S_{mean,i}}{S_{max,i}} \right)^p \tag{1}$$

Where f_x is the overload performance index for the operating point x, N_l is the number of transmission lines, $S_{mean,i}$ and $S_{max,i}$ are the average and maximum power flows of the ith line, respectively, wf_i is a weighting factor for each transmission line, which can be defined by the best judgment of the system operator, for instance $wf = [1, 1, ..., 1]$ for unitary weight in all the lines. Finally p is an exponent to reduce masking effects, which means that a high value of the exponent will scale the effects of an overload resulting in a higher index value. Table 1 shows the format of each parameter.

Table 1: Overloads Index Format

Variable	Description	Dimension	Units
f_x	Actual overload index	$scalar$	-
N_l	Number of transmission lines	$scalar$	-
S_{mean}	Mean power flow	$\Re^{1\times N_l}$	MVA
S_{max}	Maximum power flow	$\Re^{1\times N_l}$	MVA
wf	Weighting factor of lines	$\Re^{1\times N_l}$	-
p	Exponent to reduce masking effects	$scalar$	-

2.1.1 Overload Index Interpretation

The final value of the overload index f_x is a scalar, and its interpretation is as follows:

$$\begin{aligned} f_x = 1 &\quad \rightarrow \quad \text{All lines are within the limits} \\ f_x > 1 &\quad \rightarrow \quad \text{At least one line has violated its limit} \\ f_x \gg 1 &\quad \rightarrow \quad \text{A sever violation has ocurred} \end{aligned} \tag{2}$$

2.2 Under/Over Voltage Index

Following a disturbance in the power network, e.g. a line outage, the power flow trough the transmission lines is affected causing changes in other variables of the system. For instance, voltages across the system can be depressed or increased. Data from a simulation will include information about faults, this be used to determine if any device has violated the acceptable operational limits. For the case of bus voltages, it is possible to measure the severity ratio of violations (under and over operational limits) using the following approaches.

2.2.1 Approach 1: Based on Mean Value Computation

$$v_x = \sum_{i=1}^{N_b} wv_i \left(\frac{v_{init,i} - v_{mean,i}}{\Delta v_i} \right)^q, \quad \Delta v_i = \frac{v_{max,i} - v_{min,i}}{2} \tag{3}$$

Where v_x is the performance index for the operating point x. It indicates if any bus in the system has surpassed the operational limits. N_b is the number of buses to be analyzed, $v_{init,i}$ is the initial voltage at the ith bus before any disturbance has occurred (pre-fault value), $v_{mean,i}$ is the average voltage of the post-fault data at the ith bus. wv_i is a weighting factor of each bus, which can be defined by the best judgment of the system operator, for instance $wv = [1, 1, ..., 1]$ for unitary weight in all buses. $v_{max,i}$ and $v_{min,i}$ are the upper and lower voltage limits for the ith bus, respectively and q is an exponent to reduce masking effects, which means that a high value of the exponent will scale the effects of violations in the voltage limits resulting in a large index value.

2.2.2 Approach 2: Based on Largest Simulated Value Computation

$$v_x = max \left(\sum_{i=1}^{N_b} wv_i \left(\frac{v_{init,i} - v_{post,i}}{\Delta v_i} \right)^q \right), \quad \Delta v_i = \frac{v_{max,i} - v_{min,i}}{2} \tag{4}$$

As an alternative to the post-fault voltage using the mean value v_{mean} as in (3), the index can be calculated using the full set of post-fault voltage simulation data v_{post}. As a result, the index will be a vector of values, instead of a single constant value, representing the condition for each time stamp of the analyzed post-fault data. The maximum value of this set of data has to be chosen to retrieve the final index ratio, as indicated in equation (4). More details about this index are described in Section 5.2. Table 2 shows the format of each index parameter.

Table 2: Onder/over Voltage Index Format

Variable	**Description**	**Dimension**	**Units**
v_x	Onder/over Voltage index	$scalar$	-
N_b	Number of buses	$scalar$	-
v_{init}	Nominal voltage (pre-fault)	$\Re^{1\times N_b}$	V, p.u.
$v_{mean}*$	Mean voltage (post-fault)	$\Re^{1\times N_l}$	V, p.u.
$v_{post}*$	Post-fault voltages	$\Re^{nt\times N_l}$	V, p.u.
v_{max}	Maximum voltage allowed in bus	$\Re^{1\times N_l}$	V, p.u
v_{min}	Minimum voltage allowed in bus	$\Re^{1\times N_l}$	V, p.u
wv	Weighting factor of buses	$\Re^{1\times N_l}$	-
q	Exponent to reduce masking effects	$scalar$	-

nt = Length of analyzed window of time, $*$ Required depending on the selected approach

2.2.3 Under/Over Voltage Index Interpretation

Regardless the approach used in this index, the final value v_x is a gain, and its interpretation is as follows:

$$\begin{aligned} v_x = 1 \quad &\rightarrow \quad \text{All buses are within the limits} \\ v_x > 1 \quad &\rightarrow \quad \text{At least one bus has violated its limit} \\ v_x \gg 1 \quad &\rightarrow \quad \text{A sever violation has ocurred} \end{aligned} \tag{5}$$

3 Dynamic Classification

3.1 Transient Stability Index (Angular Deviation)

Transient stability is defined as the ability to maintain synchronism in the operation of the system when subjected to sever disturbances that make generators lose synchronism [3]. From the system theory point of view, transient stability is a strongly nonlinear, high-dimensional problem. To assess this problem, numerical integration methods or ”time domain” (TD) simulation methods are used. To analyze time domain solutions, indices such as the integral square generator angle (ISGA) are used [4] as described below

$$J = \frac{1}{M_{tot}T} \int_0^T \sum_{i=1}^{N_m} M_i \left(\delta_i(t) - \delta_{coa}(t)\right)^2 \, dt, \tag{6}$$

where

$$\delta_{coa}(t) = \sum_{i=1}^{N_m} M_i \delta_i(t) \Big/ \sum_{i=1}^{N_m} M_i \quad \text{and} \quad M_{tot} = \sum_{i=1}^{N_m} M_i \tag{7}$$

The index can be used to judge the severity of stable and unstable transient events in simulations. The term $\delta_i(t)$ represent the generator angle of the ith machine as a function of time. The constant M_i is the inertia of the ith machine and $\delta_{coa}(t)$ is the center of angle or inertia of all the machines. Table 3 displays the format of each parameter in the ISGA index.

Table 3: Transient Stability Index Format

Variable	Description	Dimension	Units
J	ISGA index	$scalar$	-
N_m	Number of machines	$scalar$	-
T	Number of simulation seconds	$scalar$	-
M	Inertia of the machines (2*H)	$\Re^{1\times N_m}$	-
M_{tot}	Total Inertia of all machines	$scalar$	-
δ	Generator angles	$\Re^{nt\times N_m}$	p.u.
δ_{coa}	Center of angle or inertia	$\Re^{nt\times 1}$	p.u.

nt = Length of analyzed window of time

3.1.1 Transient Stability Index Interpretation

The final ISGA index value J is a scalar, and its interpretation is as follows:

$$\begin{array}{lcl} J < 1 & \rightarrow & \text{All machines are transient stable} \\ J > 1 & \rightarrow & \text{A group of machines is deviating from its center of inertia, indicating transient instability} \\ J \gg 1 & \rightarrow & \text{Sever transient instability} \end{array} \tag{8}$$

3.2 Small-Signal Stability Index (Damping Ratio)

Small-signal stability is the ability of the power system to maintain synchronism under small disturbances. Such disturbances occur continually on the system because of small variations in loads and generation. Instability mat arise in two forms: a) increase of rotor angle due to lack of sufficient synchronization torque, or b) rotor oscillations of increasing amplitude due to lack of sufficient damping torque. The small-disturbance stability problem is equivalent to ensure sufficient damping in system oscillations [5].

This index is computed from an estimate of the eigenvalues of the system, which are determined using time series from dynamic simulations. The eigenvalues allow to calculate the damping ratio of each system oscillation to provide a measure of stability for a given contingency. Estimating the damping of the system time series is a challenging process; first, appropriate signal processing techniques have to be applied.

3.2.1 Detrending and Energy Calculation

Dynamic simulations provide time series with useful information to assess the behaviour of the system. In order to estimate the eigenvalues of the system following a disturbance, the first step is to extract the signals with highest oscillation content. This can be performed by extracting the energy of each signal and sorting them in descending order. First, the signals are detrended to remove linear trends and put all of them in the same reference (see equation (9)), and then the energy of the time varying signals $x_i(t)$ can be calculated as in (10).

$$\hat{x}_i(t) = x_i(t) - x_{mean,i}(t) \tag{9}$$

$$E_i = \int_{-\infty}^{\infty} |\hat{x}_i(t)|^2 \, dt \tag{10}$$

Where $x_i(t)$ are the signals to be analyzed, $x_{mean,i}$ is the mean value of the ith signal, $\hat{x}_i$ are the detrended signals and E_i are the energies of all signals.

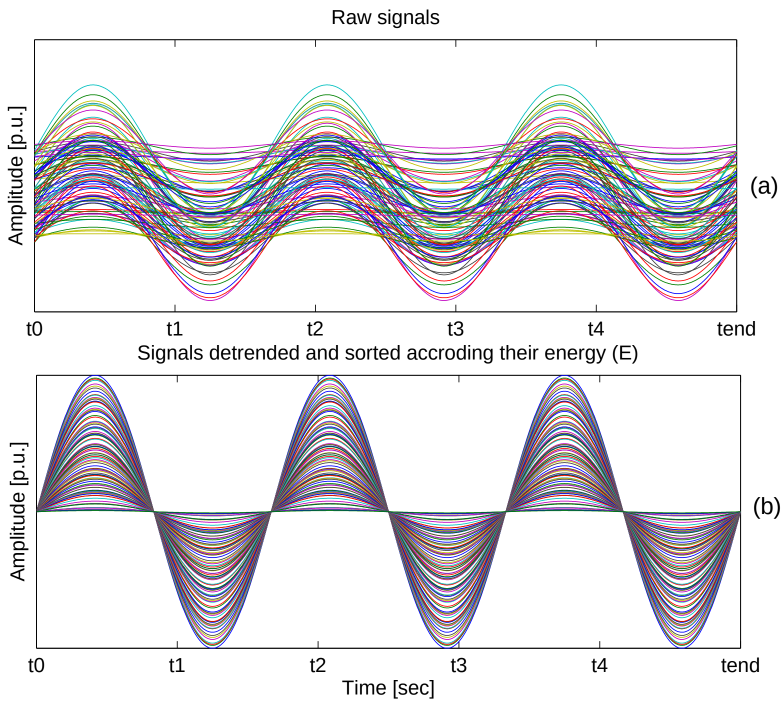

Figure 3: Application of (9) and (10) to a set of signals $x_i(t)$

Figure 3 shows an illustrative example of the first signal processing step required for small-signal analysis. In Figure 3(a), the signals are plotted directly from the dynamic simulations (raw data) and in Figure 3(b) the signals have been detrended and sorted in descending order according to their energy.

3.2.2 Frequency Screening

Applying a filter to identify the frequency of the selected signals is optional and can be useful before estimating the eigenvalues of the system. In large power systems, small-signal stability problems may be either local or global in nature. Local problems involve a small part of the system. Such oscillations are known as ***local***

oscillations and usually have frequencies in the range of 1.0 to 2.0 Hz. Analysis of local small-signal stability requires a detailed representation of a small portion of the complete interconnected system. On the other hand, global small-signal stability problems are caused by interactions among large groups of generator and have widespread effects. They involve oscillations of a group of generators in one area swinging against a group of generators in another area. These oscillations are known as ***inter-area oscillations*** and they can be classified in two groups according to the frequency of the modes. A very low frequency mode involving all the generators in the system, up to 0.1 Hz, and modes involving subgroups of generators swinging against each other, 0.1 to 1.0 Hz.

Because dynamic simulations provide a large number of signals to be analyzed, it is useful to divide the signals into a smaller subgroup to facilitate calculations. Signals can be classified according to their frequency, so only signals within a desired frequency are analyzed. The classification can be carried out using the Fast Fourier Transform (FFT) as described below

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-i2\pi k \frac{n}{N}}, \qquad k = 0, \ldots, N-1. \tag{11}$$

Where N is the size of $x(t)$. $X(k)$ is a vector of complex numbers, the largest magnitudes of the complex vector $|X(k)|$ are the frequencies of the signal $x(t)$. Figure 4 shows an illustrative example of using this filter. In this example, 3 signals with known frequencies have been artificially generated (Fig 4(a)), to produce the signal shown in Figure 4(b).

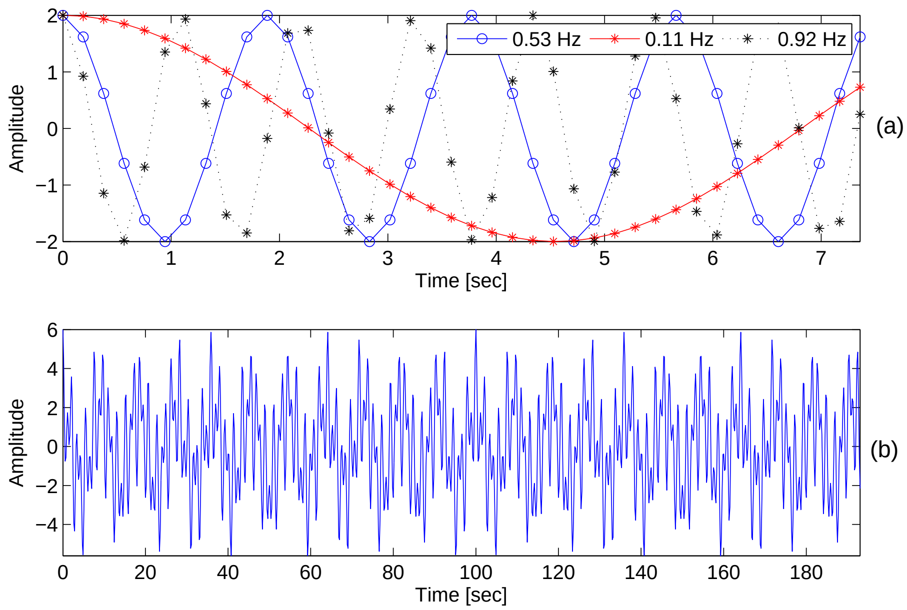

Figure 4: (a) Independent signals, (b) Signal with 3 different frequency modes.

After applying the filter using (11), we retrieve information about 3 frequencies from the analyzed signal, as shown in Figure 5.

The filter can be used to group signals according to their frequency, and consequently, the type of oscillation: local and inter-area.

3.2.3 Ringdown Analysis Methods

The modal analysis problem may be posed as follows: given a set of measurements that vary with time, it is desired to fit a time-varying waveform of pre-specified form (such as (13)) to the actual waveform. Consider

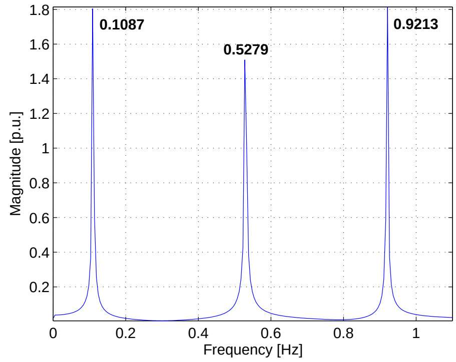

Figure 5: Frequencies found after applying the FFT filter

the following linear system in continuous and discrete time

$$\begin{array}{rl} \dot{x} = & Ax + Bu \\ y = & Cx + Du \end{array}, \quad x(t_0) = x_0 \quad \text{and} \quad \begin{array}{rl} x(k+1) = & Ax(k) + Bu(k) \\ y(k) = & Cx(k) + Du(k) \end{array}, \quad x(k_0) = x_0 \tag{12}$$

where $\dot{x}$ denotes differentiation of x with respect to time and k represents the discrete time interval. Variables u and y are the input and the output of the system, respectively; x, the internal state of the system, is usually taken to be a vector of n elements (n being the order of the system differential equation). These equations, and the system matrices within them, can be rearranged in many different ways to serve specific purposes. Each individual element x_i is be given by:

$$x_i(t) = \sum_{i=1}^{n} r_i x_{i0} e^{\lambda_i t} = \sum_{i=1}^{n} a_i e^{\sigma_i t} \cos(\omega_i t + \theta_i) \tag{13}$$

The parameter r_i is the residue of the mode i, x_{i0} is derived from initial conditions, and λ_i represents the (possibly complex) eigenvalues of A. The estimation of these responses yields modal information about the system that can be used to predict possible unstable behavior, controller design, parametric summaries for damping studies, and modal interaction information.

The primary task in modal identification is to determine the system poles of the system's transfer function or, equivalently, the eigenvalues of A. Transfer function identification must, in addition to the poles, also determine the zeros and the gains along one or more response paths. The system transfer function involves all of the system matrices in (12) and is given by

$$T(s) = \frac{G(s^m + a_{m-1}s^{m-1} + \cdots + a_1 s + a_0)}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0} \quad \text{or} \quad T(k) = \frac{G(s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\cdots(s - p_n)} \tag{14}$$

$$T(s) = \sum_{i=1} \frac{K_i}{s - p_i} \tag{15}$$

where each pole i, p_i, is identical to an eigenvalue λ_i of the matrix A and K_i is the transfer function residue of the associated pole p_i.

3.2.3.1 Prony's Method

Prony's method is designed to directly estimate the parameters for the exponential terms in (13), by fitting a function to an observed record for $y(t)$. In doing this it may also be necessary to model offsets, trends, noise, and other extraneous effects in the signal. The Prony method is a "polynomial" method in that it includes the process of finding the roots of a characteristic polynomial.

Let the record for $y(t)$ consist of N samples $y(t_k)$ that are evenly spaced by an amount Δt. The notation is simplified if (13) is written in exponential form

$$\hat{y}(t) = \sum_{i=1}^{n} A_i e^{\sigma it} \cos(\omega_i t + \theta_i) \tag{16}$$

where $n \leq N$ is the subset of modes to be determined. At the sample times t_k , this can be discretized to

$$\hat{y}(k) = \sum_{i=1}^{n} B_i z_i^k \quad \text{where} \quad z_i = e^{\lambda_i \Delta t} \tag{17}$$

The z_i are the roots of the polynomial

$$z^n - (a_1 z\, n - 1 + a_2 z^{n-2} + \cdots + a_{n-1} z^0) = 0, \tag{18}$$

where the a_i coefficients are unknown and are calculated from the set of measurement vectors. The strategy for obtaining a Prony solution can be summarized as follows:

- Step 1: Assemble the selected elements of the record into a Toeplitz data matrix
- Step 2: Fit the data with a discrete linear prediction model, such as a least squares solution.
- Step 3: Find the roots of the characteristic polynomial (16) associated with the model of Step 1.
- Step 4: Using the roots of Step 3 as the complex modal frequencies for the signal, determine the amplitude and initial phase for each mode.

A Toeplitz matrix is a matrix with a constant diagonal in which each descending diagonal from left to right is constant. The approach to the Toeplitz (or the closely related Hankel) matrix assembly of Step 1 has received the most attention in the literature. The problem can be formulated in many different ways. Here, the problem is formulated as

$$\begin{bmatrix} y_0 & 0 & \cdots & 0 \\ y_1 & y_0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ y_n & y_{n-1} & \cdots & y_0 \\ y_{n+1} & y_n & \cdots & y_1 \\ \vdots & \vdots & & \vdots \\ y_N & y_{N+1} & \cdots & y_{N-n-1} \\ 0 & \cdots & 0 & y_N \end{bmatrix} \begin{bmatrix} -1 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{19}$$

In the majority of practical cases, the Toeplitz matrix is non-square with more rows than columns. The system z_i of equation (17) requires a least squares error solution to find the factors a_1 through an. After the a_i coefficients are obtained, the n roots z_i of the polynomial in (18) can be found by factoring.

Step 4 is also a linear algebra problem. Once the roots z_i are obtained from Step 3, they are substituted into (16) and written in matrix form as

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_n^0 \\ z_1^1 & z_2^1 & \cdots & z_n^1 \\ \vdots & \vdots & & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{N-1} \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} \tag{20}$$

The $N \times n$ matrix in (20) is a Vandermonde matrix and solving it for B_i results in the Vandermonde problem. Once the residue coefficients are found, the estimated signal $\hat{y}(t)$ can be reconstructed from (16) using the roots of (17). The reconstructed signal $\hat{y}(t)$ will usually not fit $y(t)$ exactly. An appropriate measure for the quality of this fit is the signal to noise (SNR) ratio:

$$SNR = 20\ log\frac{\|\hat{y} - y\,\|}{\|y\,\|} \tag{21}$$

where the SNR is given in decibels (dB).

3.2.3.2 Eigensystem Realization Algorithm (ERA)

The Eigensystem Realization Algorithm (ERA) is based on the singular value decomposition of the Hankel matrix H_0 associated with the linear ringdown of the system. A Hankel matrix is a square matrix with constant skew-diagonals. The Hankel matrices are typically assembled using all of the available data such that the top left-most element of H_0 is y_0 and the bottom right-most element of H_1 is y_N. Hankel matrices are assembled such that:

$$H_0 = \begin{bmatrix} y_0 & y_1 & \cdots & y_r \\ y_1 & y_2 & \cdots & y_{r+1} \\ \vdots & \vdots & & \vdots \\ y_r & y_{r+1} & \cdots & y_N \end{bmatrix} \quad H_1 = \begin{bmatrix} y_0 & y_1 & \cdots & y_{r+1} \\ y_2 & y_3 & \cdots & y_{r+2} \\ \vdots & \vdots & & \vdots \\ y_{r+1} & y_{r+2} & \cdots & y_N \end{bmatrix} \quad r = \frac{N}{2} - 1\cdot \tag{22}$$

This choice of r assumes that the number of data points is sufficient such that $r > n$. The ERA formulation begins by separating the singular value decomposition of H_0 into two components according to the relative size of the singular values:

$$H_0 = U\Sigma V^T = \begin{bmatrix} U_n & U_z \end{bmatrix} \begin{bmatrix} \Sigma_n & 0 \\ 0 & \Sigma_z \end{bmatrix} \begin{bmatrix} V_n^T \\ V_z^T \end{bmatrix} \tag{23}$$

where $\Sigma_n = diag(\sigma_1, \sigma_2, \cdots, \sigma_n)$ and $\Sigma_z = diag(\sigma_{n+1}, \sigma_{n+2}, \cdots, \sigma_N)$ are diagonal matrices with their elements ordered by magnitude:

$$\sigma_1 > \sigma_2 > \cdots > \sigma_n > \sigma_{n+1} > \sigma_{n+2} > \cdots > \sigma_N \tag{24}$$

The SVD is a useful tool for determining an appropriate value for n. The ratio of the singular values of contained in Σ can determine the best approximation of n. The ratio of each singular value σ_i to the largest singular value σ_{max} is compared to a threshold value, where p is the number of significant decimal digits in the data:

$$\frac{\sigma_i}{\sigma_{max}} \approx 10^{-p} \tag{25}$$

An example is to set p equal to 3 significant digits, thus any singular values with a ratio below 10^{-3} are assumed to be part of the noise and are not included in the reconstruction of the system. The value of n should be set to the number of singular values with a ratio above the threshold 10^{-p} . It can be shown that for a linear system of order n, the diagonal elements of Σ_z are zero (assuming that the impulse response is free of noise). The practical significance of this result is that the relative size of the singular values provides an indication of the identified system order. If the singular values exhibit a significant grouping such that $\sigma_n >> \sigma_{n+1}$, then from the partitioned representation given in (22), H_0 can be approximated by

$$H_0 \approx U_n \Sigma_n V_n^T \tag{26}$$

The method for obtaining the eigenvalue realization algorithm solution can be summarized as follows:

- ◇ Step 1: Assemble selected elements of the record into a Hankel data matrices H_0 and H_1
- ◇ Step 2: Perform the singular value decomposition of H_0 and estimate the system order n based on the magnitude of the singular values

◇ Step 3: Compute the discrete system matrices as described below

$$\begin{aligned} A = &\ \Sigma_n^{-1/2} U_n^T H_1 V_n \Sigma_n^{-1/2} \\ B = &\ \Sigma_n^{1/2} V_n^T (1:n, 1:n_u) \\ C = &\ Un(1:N, 1:n)\Sigma_n^{1/2} \\ D = &\ y_0 \end{aligned}$$

◇ Step 4: Calculate continuous time system matrices A_c, B_c assuming a zero order hold and sampling interval Δt

$$\begin{aligned} A_c = &\ In\left(\frac{A}{\Delta t}\right) \\ B_c = &\ \left(\int_0^{\Delta t} e^{A\tau}\right)^{-1} B \end{aligned}$$

The reduced system response can then be computed from the continuous time matrices A_c and B_c.

3.2.4 Small Signal Stability Index: SMI, AMI, GMI

In this section the interpretation of the small-signal stability index is given. The final index has three layers. The index provides a measure related to the stability of the system and is based on the damping ratio of the system's modes. The measure is the angular distance (in radians) from each mode to a pre-defined damping ratio, as seen in Figure 6.

- Single Mode Index (SMI), a matrix.
- All Modes Index (AMI), a vector.
- Global Modes Index (GMI), a scalar.

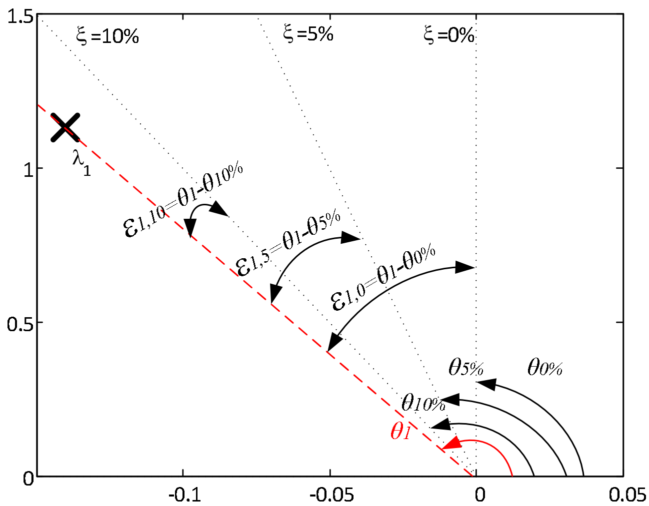

Figure 6: Distance from each mode to a pre-defined damping ratio

The **SMI** is the most detailed source of information, but for a system with multiple number of modes, it is not easy to interpret (matrix of data). It provides the individual distance of each mode to a pre-defined damping ratio, e.g. $\zeta_0 = 0\%$, $\zeta_5 = 5\%$ and $\zeta_{10} = 10\%$. If any of the elements of SMI is negative, this indicates that the mode corresponding to the specific row, has a damping ratio less than the damping ratio for that required column. For instance, less assume that the element (3,2) in SMI is negative ($\varepsilon_{3,5} < 0$), then the mode 3 (λ_3) has a damping ratio less than 5% ($\zeta_3 < 5\%$) and consequently this mode is violating the damping ratios ζ_5 and ζ_{10}, respectively. The SMI is defined as follows

$$SMI = \begin{bmatrix} \varepsilon_{1,0} & \varepsilon_{1,5} & \varepsilon_{1,10} \\ \vdots & \vdots & \vdots \\ \varepsilon_{i,0} & \varepsilon_{i,5} & \varepsilon_{i,10} \end{bmatrix} \tag{27}$$

and

$$\begin{array}{rcl} \varepsilon_{i,\zeta_j} & = & \theta_i - \theta_{\zeta_j} \\ \theta_i & = & cos^{-1}(\zeta_i) \\ \theta_{\zeta_j} & = & \pi - cos^{-1}(\zeta_j) \end{array} \tag{28}$$

where ε_{i,ζ_j} is the (i,j) element of SMI, ζ_i is the damping ratio of the ith mode (λ_i) and ζ_j is the jth pre-defined damping ratio.

AMI facilitates the interpretation of SMI, it gives the minimum distance of the modes with respect to each of the damping ratios. If one value of the AMI row is negative, this means that at least one mode has a damping less than the one required.

$$AMI = \left[\; _{min}|\varepsilon_{i,0}| \quad _{min}|\varepsilon_{i,5}| \quad _{min}|\varepsilon_{i,10}| \; \right] \tag{29}$$

GMI gives a global interpretation of the modes respect to all pre-defined damping ratios, is the minimum distance among all modes respect to all pre-defined ratios.

$$GMI = \left[\; _{min}|\varepsilon_{i,\zeta_j}| \; \right] \tag{30}$$

3.3 Voltage Stability Index (Distance to bifurcation)

Voltage stability is a concern for modern power system security. Both static and dynamic analyses are used to investigate different aspects of voltage instability phenomena. When only time series are available for voltage stability assessment, it is necessary to identify a model from the time series data to perform the assessment. The approach proposed here is to identify a simple equivalent from the time series of the simulations. The choice of the model was made by considering the trade-off between model accuracy, speed to compute the stability limits, and the fact that no other information about the power system other than the time series was available.

Simple equivalent models of the power system and the load at a measurement point are estimated from the data, and then used for calculating power-voltage (PV) curves to predicting the stability limit [6].

3.3.1 Single Voltage Source (SVS) Model

Let us consider the Thevenin equivalent of the power system as viewed from the measurement point. Figure 7 shows a single line diagram of the equivalent system. For simplicity, the network resistance is neglected. Though it is straightforward to extend the results with the equivalent resistance included.

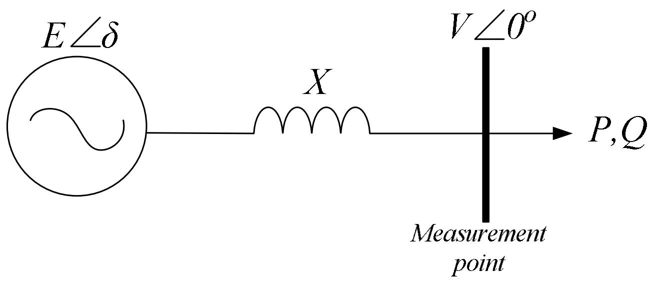

Figure 7: Equivalent model of the power system

Where

$$\begin{array}{rl} P = & \frac{EV}{X}\sin\delta \\ Q = & \frac{V(E\cos\delta - V)}{X} \end{array} \tag{31}$$

Taking the last m samples for any instant of time yields:

$$\left. \begin{array}{rcl} P_iX - V_iE\sin\delta_i & = & 0 \\ Q_iX - V_iE\cos\delta_i + V_i^2 & = & 0 \end{array} \right., \quad i = 1, \cdots, m \cdot \tag{32}$$

With $2m$ equations in (32), only $m+2$ variables are unknown, i.e. the voltage E, the reactance X and all the bus angles δ_i $1,\ldots,m$. They are hence overdetermined, and need to be calculated solving the following least square errors problem

$$\min_{E,X,\delta_i} \left\| \begin{matrix} P_1X - V_1E\sin\delta_1 \\ Q_1X - V_1E\cos\delta_1 + V_1^2 \\ \vdots \\ P_mX - V_mE\sin\delta_m \\ Q_mX - V_mE\cos\delta_m + V_m^2 \end{matrix} \right\|. \tag{33}$$

3.3.2 Load Model

The equation $Q = \alpha + \beta P$ describes the linear P-Q load side model. Using the same data samples as for the system model estimation, α and β are the solution of the following linear least square problem.

$$\min_{\alpha,\beta} \left\| \begin{bmatrix} 1 & P_1 \\ & \vdots \\ 1 & P_m \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} - \begin{bmatrix} Q_1 \\ \vdots \\ Qm \end{bmatrix} \right\| \tag{34}$$

The single voltage source model of Figure 7 can be combined with the linear P-Q load side model to calculate the PV characteristic and the voltage stability limit for the power transfer through the study point by estimating

$$P^2X^2 - E^2V^2 + (\alpha X + \beta PX + V^2)^2 = 0 \tag{35}$$

3.3.3 Voltage stability Index: SBI, ABI, GBI

This index provides a measure of how far the system is from the maximum loadability limit. At least, 3 sets of time series data with the same contingency are required, as seen in Figure 8:

- The loading level is low
- The loading level is acceptable (OK)
- The loading level is at the limit (unacceptable for operational standards)

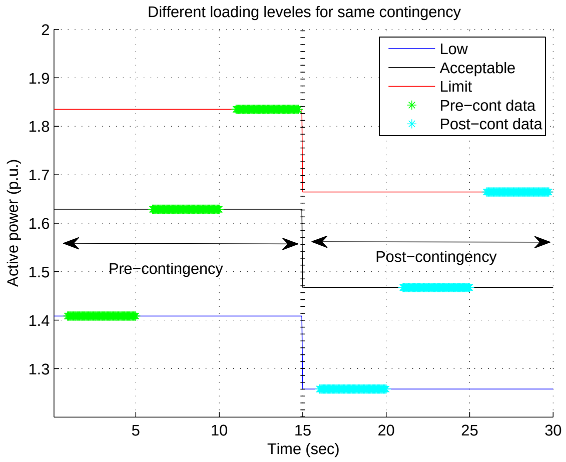

Figure 8: Different loading levels for the same contingency

Combining this data, 2 PV curves can be estimated, the pre-contingency curve (using the green data), and the post-contingency curve (using the light-blue data), as shown in Figure 9. This Figure shows the set of data used, where the pair (P_a, V_a) represents the power and voltage at low loading level in pre-contingency, the pair

$(\hat{P}_a, \hat{V}_a)$ represents the power and voltage at low loading level in post-contingency, the pair (P_{lim}, V_{lim}) represents the power and voltage operational limits in pre-contingency and $(\hat{P}_{lim}, \hat{V}_{lim})$ represents the power and voltage operational limits in post-contingency. Note that the operational voltage limit is the same in pre-and-post-contingency $(\hat{V}_{lim} = V_{lim})$.

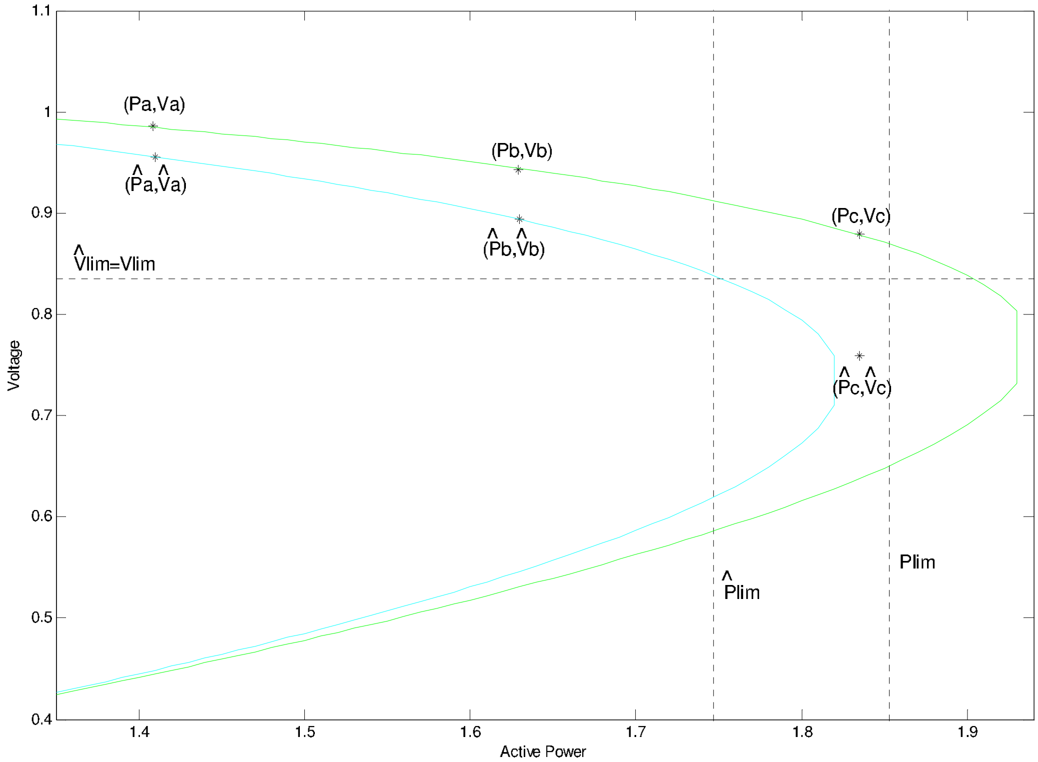

Figure 9: Estimated PV curves using simulation results, where the green curve corresponds to the pre-contingency data and the light-blue curve corresponds to the post-contingency data.

Similarly to the small-signal stability index, the voltage stability index is comprised by 3 layers

- Single Bus Index (SBI)
- All Buses Index (ABI)
- Global Bus Index (GBI)

The **SBI** is a matrix and provides the distance in pre-and-post contingency for each loading level a, b, and c (low, acceptable and limit), respectively, to the loadability (P_{lim}) and voltage limits (V_{lim}) for the ith Bus.

$$SBI = \begin{bmatrix} \Delta P_{1,a} & \Delta P_{1,b} & \Delta P_{1,c} & \Delta V_{1,a} & \Delta V_{1,b} & \Delta V_{1,c} \\ \Delta \hat{P}_{1,a} & \Delta \hat{P}_{1,b} & \Delta \hat{P}_{1,c} & \Delta \hat{V}_{1,a} & \Delta \hat{V}_{1,b} & \Delta \hat{V}_{1,c} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta P_{i,a} & \Delta P_{i,b} & \Delta P_{i,c} & \Delta V_{i,a} & \Delta V_{i,b} & \Delta V_{i,c} \\ \Delta \hat{P}_{i,a} & \Delta \hat{P}_{i,b} & \Delta \hat{P}_{i,c} & \Delta \hat{V}_{i,a} & \Delta \hat{V}_{i,b} & \Delta \hat{V}_{i,c} \end{bmatrix} \tag{36}$$

and

$$\begin{array}{ll} \Delta P_{i,j} = \frac{P_{i,lim} - P_{i,j}}{P_{i,lim}}, & \Delta V_{i,j} = \frac{V_{i,j} - V_{i,lim}}{V_{i,lim}}, \\ \Delta \hat{P}_{i,j} = \frac{\hat{P}_{i,lim} - \hat{P}_{i,j}}{\hat{P}_{i,lim}}, & \Delta \hat{V}_{i,j} = \frac{\hat{V}_{i,j} - \hat{V}_{i,lim}}{\hat{V}_{i,lim}}, \end{array} \quad i = 1, 2, \cdots, n \quad j = a, b, c\cdot \tag{37}$$

If an element of SBI is negative, then the power or the voltage at some loading level has exceed the operational limits, a negative number in SBI means potential voltage instability in pre-or-post-contingency.

All Bus Index **ABI** is a vector that provides the minimum distance among all buses for each loading level a, b, and c (low, acceptable and limit), respectively, to the power (P_{lim}) and voltage limits (V_{lim}), respectively. The

ABI index helps to identify if a loading level is violating a limit in pre-or-post contingency.

$$ABI \quad = \quad \left[\begin{array}{cccccc} \Delta\tilde{P}_{i,a} & \Delta\tilde{P}_{i,b} & \Delta\tilde{P}_{i,c} & \Delta\tilde{V}_{i,a} & \Delta\tilde{V}_{i,b} & \Delta\tilde{V}_{i,c} \end{array}\right] \tag{38}$$

and

$$\Delta\tilde{P}_{i,j} =_{min} \left| \begin{array}{c} \Delta P_{i,j} \\ \Delta\hat{P}_{i,j} \end{array} \right|, \quad \Delta\tilde{V}_{i,j} =_{min} \left| \begin{array}{c} \Delta V_{i,j} \\ \Delta\hat{V}_{i,j} \end{array} \right|, \qquad j = a,b,c\cdot \tag{39}$$

The Global Bus Index **GBI** is a 2 element vector that provides the overall minimum distance to the loadability (P_{lim}) and the over all voltage (V_{lim}) limits respect to all buses. The GBI index indicates if a limit has been violated.

$$GBI \quad = \quad \left[\begin{array}{cc} \Delta\bar{P} & \Delta\bar{V} \end{array}\right] \tag{40}$$

and

$$\begin{array}{rcl} \Delta\bar{P} & = & _{min}\left| \begin{array}{ccc} \Delta\tilde{P}_{i,a} & \Delta\tilde{P}_{i,b} & \Delta\tilde{P}_{i,c} \\ \Delta\tilde{V}_{i,a} & \Delta\tilde{V}_{i,b} & \Delta\tilde{V}_{i,c} \end{array} \right|, \\ \Delta\bar{V} & = & _{min} \end{array} . \tag{41}$$

4 Time Series Handling

This section explains how the time series are handled in the implemented MATLAB functions. For instance, lets assume that we have to assess the signal $x(t)$ described in Figure 10(a). We can observe that the simulation time runs from $t = 0$ to $t = 60$ seconds and that 3 different events have occurred at $t = 2$, $t = 22$ and $t = 42$ seconds, respectively. The time step is $\Delta t = 0.5$ sec so the length of t is 120 samples or $t \in \Re^{120\times 1}$. The functions described in this report have been developed to analyze time series with several signals but for one contingency at the time, this means that, to analyze the 3 events in the signal described in Figure 10(a), it is require to execute the functions for that index 3 times, one for each contingency.

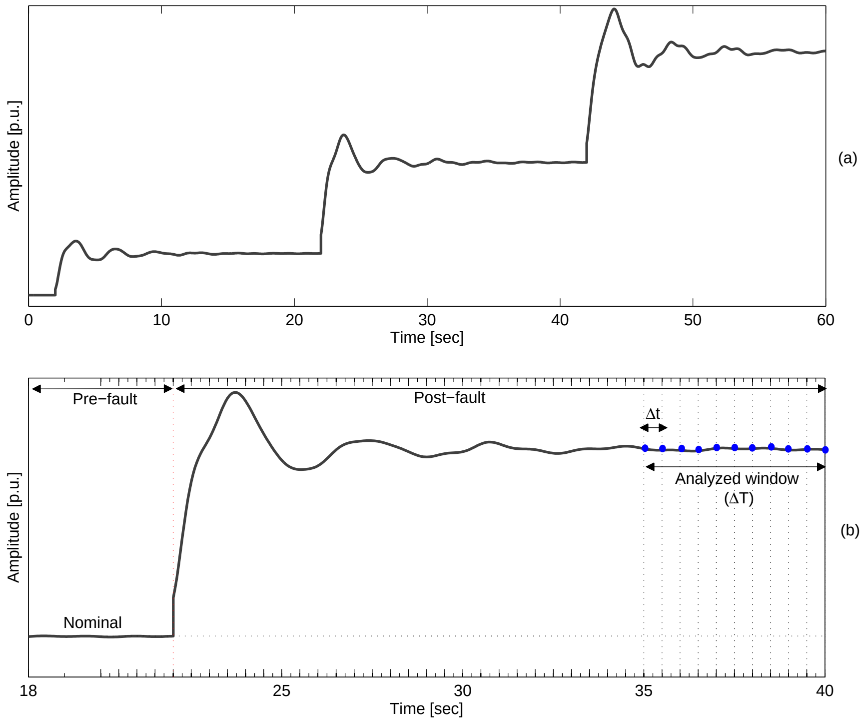

Figure 10: Handling time series in the implemented MATLAB functions

Lets assume that we want to assess the event at 22 seconds in Figure 10(a). To do so, first we define t_s, which is a vector of time specifying the section of the signal to be analyzed and it is defined as $t_s = [t_{init} : \Delta t : t_{end}]^T$, where t_{init} is the initial time, t_{end} is the final time and Δt is the time step. The dimension of t_s is nt, which is the total number of samplings. Some of the functions analyze the post-fault data, therefore it is required to define a number of samplings ΔT prior to t_{end}, this small vector is defined as $t_{\Delta} = [t_s(t_{end} - \Delta T)\ :\ t_s(t_{end})]$. Figure 10(b) depicts a section selected, in this example the initial and final time were defined as $t_{init} = 18$ sec and $t_{end} = 40$ sec, respectively, so $t_s = [18 : 0.5 : 40]^T$ and $nt = 45$. In order to study the post-fault data, the number of samplings prior t_{end} was set to $\Delta T = 11$, so $t_{\Delta} = [35 : 40]$ as seen in Figure 10(b) in blue dots.

Table 4: Time input vector format

Variable	Description	Dimension	Units
t	Simulation time total	$\Re^{n\times 1}$	sec
n	Length of t	$scalar$	-
t_s	Analyzed section of the simulation	$\Re^{nt\times 1}$	sec
nt	Length of t_s	$scalar$	-
t_{init}	Initial time of analyzed section	$scalar$	sec
t_{end}	Final time of analyzed section	$scalar$	sec
Δt	Simulation time step	$scalar$	-
ΔT	Number of sampling times prior t_{end}	$scalar$	-

4.1 Overload and Under/Over Voltage Indexes Time Handling

In the overload index the parameter S_{max} is calculated based on the value of the power flow S_{nom} at time t_{init} where a user defined value ΔS is incremented to set the maximum amount of power allowed in the line $S_{max} = S_{nom} + \Delta S$. In this index, the parameter S_{mean} is the mean value among the elements of t_{Δ}.

In the under/over voltage index, the nominal value v_{init} is set as the value of the signal before the contingency $v_{init} = v(t_{init})$ and the minimum v_{min} and maximum v_{max} limits are set to a user defined value of $\pm\Delta v$ of the nominal value, i.e. $v_{min} = v_{init} - \Delta v_{init}$ and $v_{max} = v_{init} + \Delta v_{init}$. Similar to the overload index, the parameter v_{mean} is the mean value of the voltage in the analyzed window ΔT and v_{post} is the value if the voltage at each point of the analyzed window ΔT, therefore $v_{mean} \in \Re^{1\times 1}$ and $v_{post} \in \Re^{\Delta T\times 1}$.

Let us suppose that we want to apply the overload or under/over voltage index to the signal described in Figure 10 (b), in the functions described in Appendices A.1.1 and A.1.2, we have provided as input the following information

$$\begin{aligned} t_1 &= [t_{init},\ t_{end},\ \Delta T] \\ t_1 &= [18,\ 40,\ 11] \end{aligned} \tag{42}$$

which means that, the nominal value of the signal will be calculated at time $t = 18$ seconds and in order to calculate the post-fault index, the signal will be analyzed from $t = 35$ to $t = 40$ seconds, 11 sampling times prior 40 seconds for a time step of $\Delta t = 0.5$ seconds.

4.2 Transient Stability Index Time Handling

For the transient stability index the time input vector is very simple to understand, the function described in Appendix A.1.3 only requires t_{init} and t_{end} as input. Let us suppose that we want to apply the transient stability index to the signal described in Figure 10 (b), the analyzed time be defined as

$$\begin{aligned} t_1 &= [t_{init},\ t_{end}] \\ t_1 &= [18,\ 40] \end{aligned} \tag{43}$$

from t_{init} to t_{end}, the ISGA index J will be calculated (e.g. from 18 to 40 seconds).

4.3 Small-signal Stability Index Time Handling

This index handles the time input vector slightly different to the previous indexes. To understand the inputs, let us consider again the signal described Figure 10 (b), the input parameters of the function are described by

$$\begin{aligned} t_1 &= [t_{init},\ t_{end},\ t_{small}] \\ t_1 &= [18,\ 40,\ 35] \end{aligned} \tag{44}$$

where t_{init} and t_{end} are the initial and final time of the signal to analyze and t_{small} defines the initial time, within the selected time frame, the ringdown method specified will be applied, which means from $t_{small} = 35$ to $t_{end} = 40$ seconds.

4.4 Voltage Stability Index Time Handling

This index is simple to use once the structure of the input parameters is understood. Since this index requires three set of simulations, the signal in Figure 10 (b) cannot be used as illustrative example, so we recall Figure 8, shown below as Figure 11

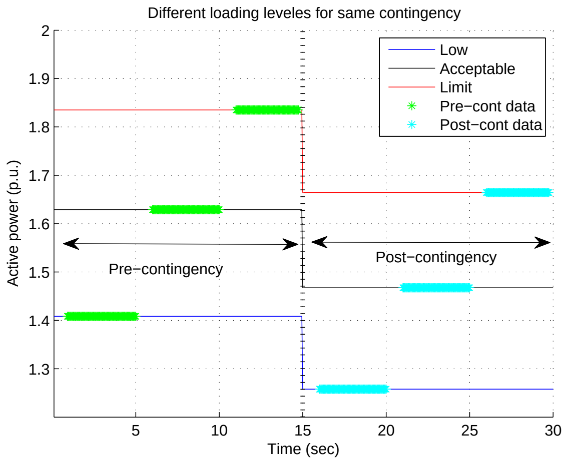

Figure 11: Different loading levels for the same contingency

In Figure 11 three loading levels are shown: low loading level (blue line), the acceptable (OK) loading level (black line) and the stressed loading level (red line). We have to select 2 sections of data, one for pre-contingency and another one for post-contingency for each loading level. The MATLAB function implemented as described in Appendix A.1.5 requires the following information

$$\begin{array}{llcl@{\qquad}lcl} & \multicolumn{3}{c}{\text{Pre-contingecny (green data)}} & \multicolumn{3}{c}{\text{Post-contingecny (blue data)}} \\ \text{Low} & t_{a,pre} & = & [1,\ 5] & t_{a,post} & = & [16,\ 20] \\ \text{Acceptable} & t_{b,pre} & = & [6,\ 10] & t_{b,post} & = & [21,\ 25] \\ \text{Stressed} & t_{c,pre} & = & [11,\ 15] & t_{c,post} & = & [26,\ 30] \end{array} \tag{45}$$

5 Validation

5.1 Overload Index Validation using Statnett's Simulations

In this section we present results of the application of the overload index described in Section 2.1 to the set of time series provided by our partner Statnett, see Figure 12. Three different case studies are shown and a total of 898 lines in the Nordic grid model were analyzed to validate the index.

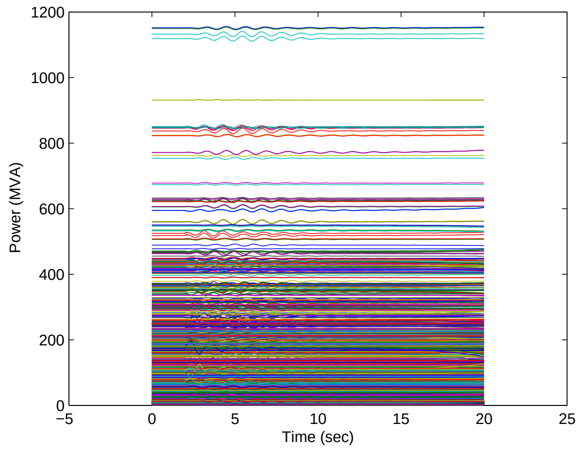

Figure 12: Power of 898 lines in Statnett's simulations

Figure 13 depicts 20 seconds of simulation and a disturbance has been applied at 2 seconds. The overload index uses data prior the disturbance, at 1 second, to calculate the maximum load allowed (black dotted line), similarly the index analyzes some samples of post-fault data to calculate the average loading level (red solid line). After using this information, the index is computed resulting in a scalar, which is grater than one $f = 1.002$, this number indicates that at least one power in the lines has slightly over passed the operational limits, as shown in Figure 13.

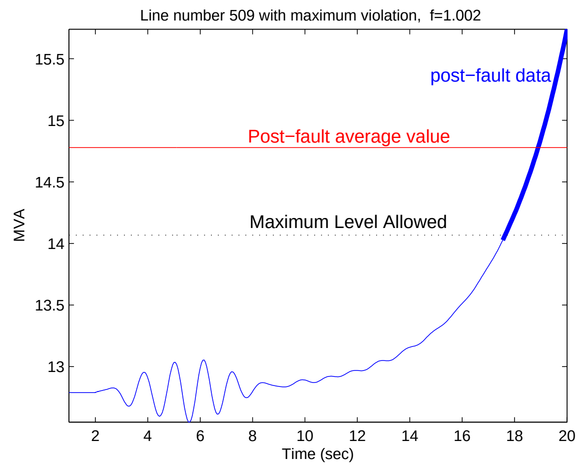

Figure 13: Overload violation, the overload index is $f = 1.002$

Figure 14 shows a large violation. As it can be observed, after the disturbance, the power at line 511 changes from 65 MVA to an average value of approximately 94 MVA. The final index is grater than one and grater than the previous example, $f = 1.008$, indicating that although most of the lines are within their limit, few lines has over passed their limits.

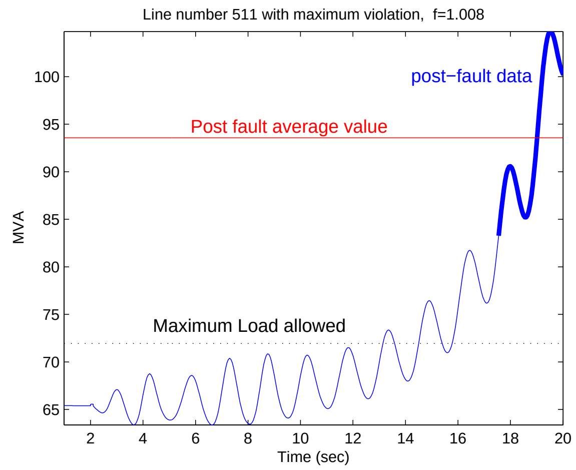

Figure 14: Increase in the overload violation, the overload index grows to $f = 1.008$

In the third case study, the system has suffered several disturbances, Figure 15, shows the power at the line with the largest violation. In this figure, it can be seen how the loading post-fault level around 80 seconds is approximately 550 MVA, which is much grater than the pre-fault value (20 MVA). The index accurately reflects this severe violation by showing a value of much more grater then one, $f = 152.4145$.

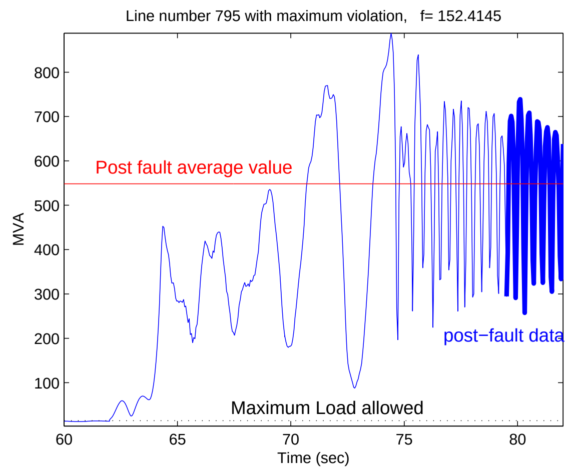

Figure 15: Severe overload violation, the overload index grows to $f = 152.4145$

5.2 Under/Over Voltage Index Validation using RTE Simulations

In this section we present results of the application of the under/over voltage index described in Section 2.2, using approach 1 (mean value computation). Figure 16, describes the voltage of the 48 most significant buses in the French network. At 10 seconds a disturbance has been applied to the network.

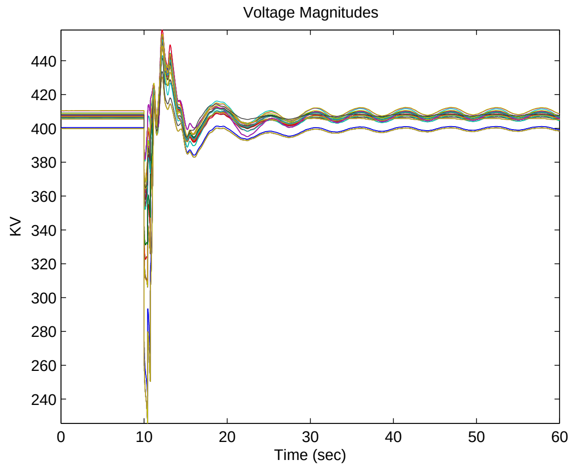

Figure 16: Voltage of 48 Buses in the French network

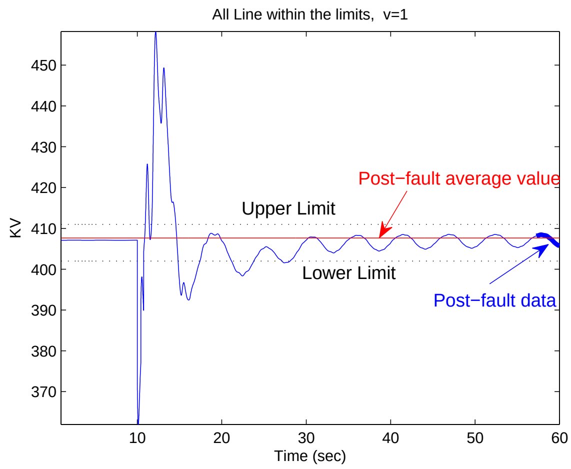

Figure 17: No Violation, the under/over voltage index is $v = 1$

These time series have been used to validate the under/over voltage index. The result is presented in Figure 17, the mean value (red solid line) of the analyzed post-fault data does not overpass neither the upper nor the lower limits (black dash lines). In this case the final value of the index is a hard one, $v = 1$.

In a different case study shown in Figure 18, a mild disturbance has been applied at 10 seconds in the French network. It can be seen how long it takes to the voltage to reach a steady state value (around 500 seconds). Using post-fault data, the under/over index is calculated and the bus with the largest violation is shown in Figure 19. It can be observed that the post-fault mean value (solid red line) overpasses the lower limit (dashed black line). Although the mean value is not too far from the lower limit, several lines have violated their limits. The value of the index is $v = 1.2810$, which confirms this violation.

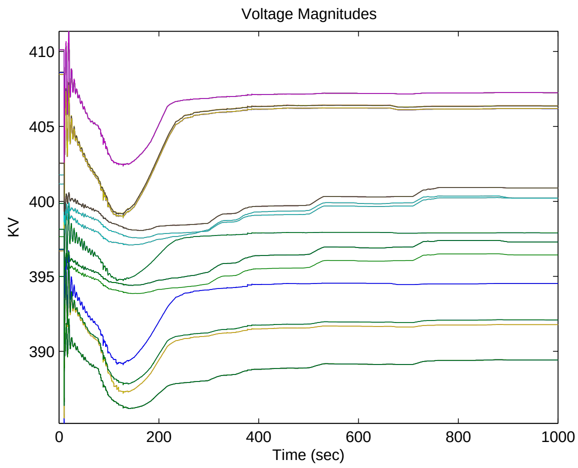

Figure 18: Mild disturbance in the French network

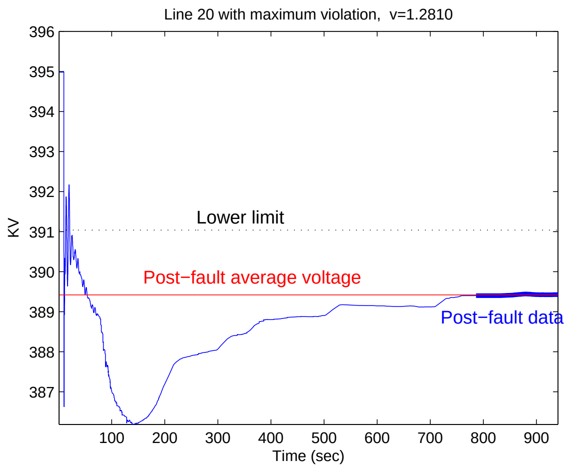

Figure 19: Under voltage violation, the index is $v = 1.2810$

5.3 Transient Stability Index Validation using Statnett Simulations

In this section the validation of the transient stability index developed in Section 3.1 is carried out. Figure 20 depicts the angle of 772 machines in Scandinavia. In this simulation, several disturbances were applied at different time stamps. For the validation, we calculated the index at different times, the objective was to demonstrate that the machine angles deviate from the center of inertia as time increases and that the magnitude of the index grows according to the deviation.

Transient Stability Index

Time (sec)	10	20	30	40	50	60	61	62	63	64	65	66	67	67	68	69
Index J	0.23	0.23	0.24	0.24	0.25	0.26	0.27	0.30	0.49	**1.21**	**3.11**	**6.97**	**13.69**	**23.74**	**37.96**	**56.64**

Table 5: Transient Stability Index (J) at different times

Table 5 summarizes the results of the validation. It can be observed how the index J is slowly growing during the first 60 seconds in a rate of 0.01, indicating that the machines are deviating from the center of inertia. However, at 64 seconds, the index changes drastically from $J = 0.49$ to $J = 1.21$ in one second and continue growing every second (highlighted in bold). The first significant change in the index corresponds to the time in the simulation where several machines deviate from the rest indicating a severe transient instability problem, seen Figure 21. Note that index J is not calculated only for the time shown in Table 5, in fact it is calculated

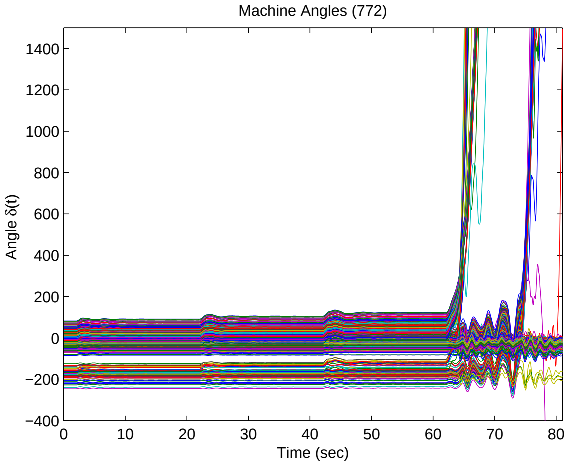

Figure 20: Angle of 772 Machines in Scandinavia, several disturbances applied at different times, a severe transient instability problem arises after 60 seconds

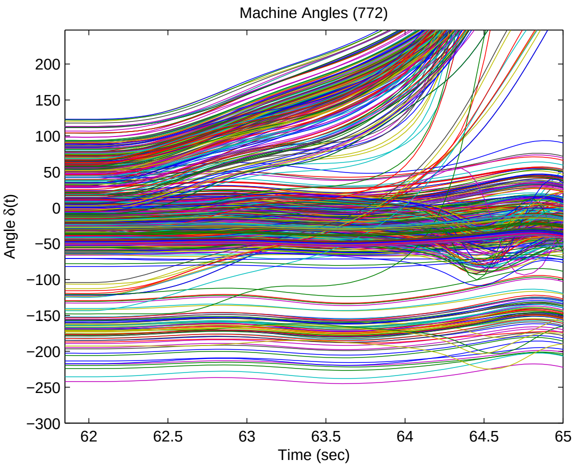

Figure 21: Instant of time where the severe transient stability problem begins

from time $t = 1$ seconds, to the time specified in Table 5, see Section 4.2 for more information.

5.4 Small-Signal Stability Index Validation using Synthetic Simulations

In this section the 3 layer index described in Section 3.2 is validated using an illustrative example. Three different signals have been generated $y_1(t)$, $y_2(t)$ and $y_3(t)$ with pre-defined frequencies $f_1 = 0.35$, $f_2 = 0.4$ and $f_3 = 0.45$ (in Hz), respectively and pre-defined damping ratios (in %). The objective of this index is to estimate the modes λ_i (complex values) of the signals only using the time-series provided. First, the damping of each mode is calculated and then, using the SMI, AMI and GMI indexes described in Section 3.2.4, the distances from each mode to a pre-defined damping ratio are calculated to indicate if any of the estimated modes is violating a specified damping ratio. In this example, the distance of the modes are calculated in relation to the damping ratios $\zeta_0 = 0\%$ $\zeta_5 = 5\%$ and $\zeta_10 = 10\%$, respectively.

5.4.1 Test 1: All modes with high damping ratio (ideal case)

Figure 22 shows the results of applying the small-signal stability index. In Figure 22 (a), the synthetic signals $y_1(t)$, $y_2(t)$ and $y_3(t)$ used in the illustrative example are shown. It can be observed that the signals were generated with high damping ratios (above 10%) and fast settling time are shown. Figure 22 (b) depicts the first step of the index calculation: pre-processing of the signals, a chosen section of the time series is analyzed and then, these signals are sorted according to their energy and detrended. After this, a ringdown analysis method is applied and the results are shown in Figure 22 (c). It can be seen that the signals have been accurately estimated. After this, all existing modes in the signals are known and the SMI, AMI and GMI indexes can be applied. Figure 22 (d) describes the estimated modes in the complex plain, it can be observed that the position of the each mode respect to each of the pre-defined damping ratios $\zeta_0 = 0\%$ $\zeta_5 = 5\%$ and $\zeta_10 = 10\%$, is positive, which means that there is no violation with respect to operational limits.

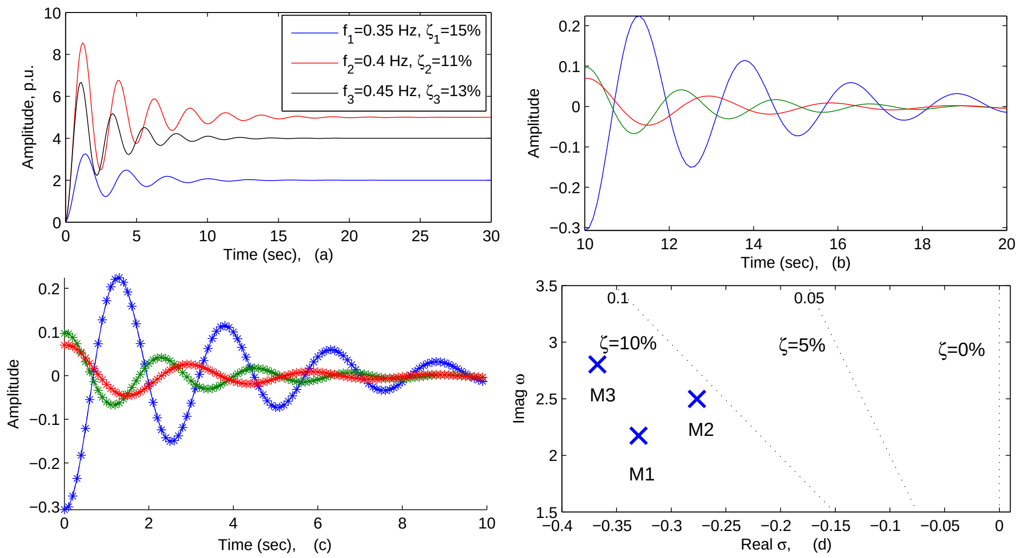

Figure 22: (a) Analyzed time series, (b) Detrended and selected data, (c) Signal estimation from ringdown analysis, (d) Estimated modes

Table 6 summarizes the index results. GMI is positive, which means that all modes have damping ratio above $\zeta_0 = 0\%$ $\zeta_5 = 5\%$ and $\zeta_10 = 10\%$, respectively. In this case it is not necessary to check the AMI and SBI index. However, if we analyze these two indexes, we can emphasize that all values within the indexes are positive indicating that no modes violates the pre-defined damping ratios.

GMI	0.0101

	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
AMI	0.1102	0.0602	0.0101

SMI	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
M1	0.1506	0.1005	0.0504
M2	0.1102	0.0602	0.0101
M3	0.1304	0.0803	0.0302

Table 6: Small-signal stability indexes for synthetic case 1

5.4.2 Test 2: Two modes violating one damping ratio

In this case study, a new set of signals $y_1(t)$, $y_2(t)$ and $y_3(t)$ were generated. The frequency used was the same as in the previous example but the damping ratios have been reduced. The objective was to test the indexes under several scenarios and to illustrate the use of SMI, AMI and GMI. In Figure 23 (a) it can be seen how the generated signals have slower settling time due to the reduction of the damping ratios. The signals are accurately estimated as shown in Figure 23 (c). In this case, only the signal $y_3(t)$ is above all required damping ratios unlike signals $y_1(t)$ and $y_2(t)$ that violate the $\zeta = 10\%$.

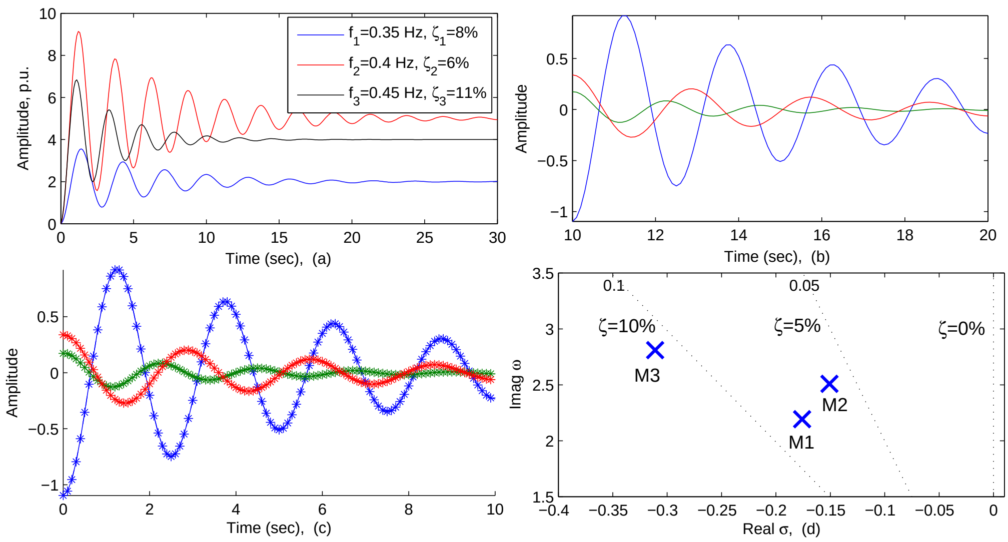

Figure 23: (a) Analyzed time series, (b) Detrended and selected data, (c) Signal estimation from ringdown analysis, (d) Estimated Modes

Table 7 summarizes the results. GMI is negative indicating that at least one mode has violated one of the damping ratios. In this case GMI highlights a problem but it is not enough to complete the analysis and AMI has to be reviewed. The third element of AMI is negative, as seen from Table 7. The information that we can retrieve from this is that at least one mode has a damping below $\zeta = 10\%$, we know this because the negative number is in the third column, which was assigned for the distance of all modes respect to the $\zeta = 10\%$ damping ratio. In order to retrieve more accurate information about which of the 3 modes has violated the measure, we analyze SBI.We can observe that the elements $(3,1)$ and $(3,2)$ in SMI are negative. By looking to SMI, it is clear now that modes 1 and 2 have violated the damping ratios specified and this can be verified in Figure 23 (d).

GMI	-0.0401

	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
AMI	0.0600	0.0100	-0.0401

SMI	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
M1	0.0801	0.0301	-0.0201
M2	0.0600	0.0100	-0.0401
M3	0.1102	0.0602	0.0101

Table 7: Small-signal stability indexes for synthetic case 2

5.4.3 Test 3: Violation of two damping ratios

In this test, we highlight the importance and usefulness of the 3 layer index. Figure 24 shows signals $y_1(t)$, $y_2(t)$ and $y_3(t)$. In this example, we preserve the frequency of all signals but the damping of all of them have been reduced. Figure 24 (a) describes the new set of signals used. After applying the small-signal stability index, three modes were estimated and they are shown in Figure 24 (d). It can be seen that the estimated mode from $y_3(t)$ (M3) has the lowest damping ratio and is below two of the predefined values.

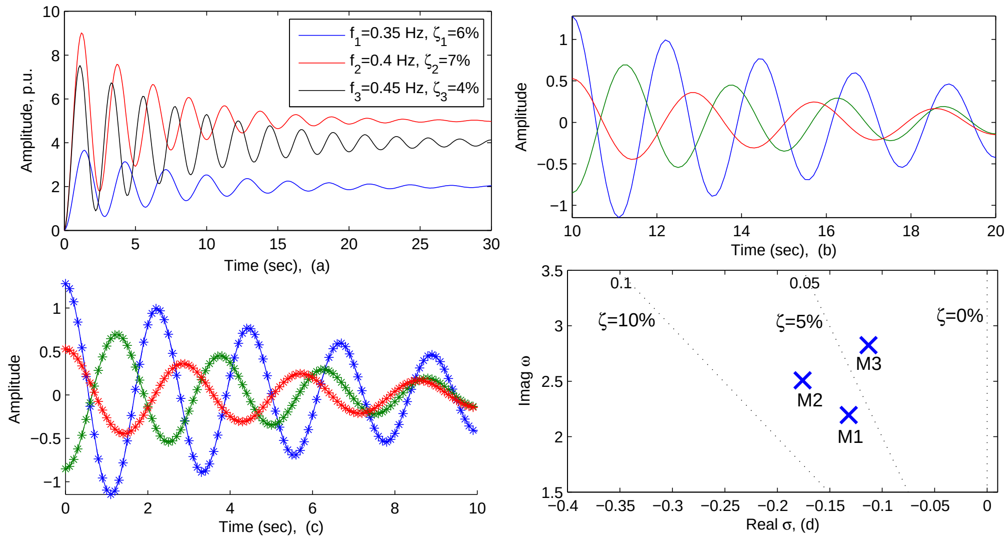

Figure 24: (a) Analyzed time series, (b) Detrended and selected data, (c) Signal estimation from ringdown analysis, (d) Estimated Modes

After observing Table 8 we can notice that GMI is negative and the second and third elements of AMI are negative as well, using this information we can assert that the $\zeta = 10\%$ and $\zeta = 5\%$ ratios have been violated for at least one mode. To enquire which and how many modes are breaking the rules, SMI is evaluated. A close look into SMI points out that mode 3 is breaking the $\zeta = 5\%$ and that all modes are violating the $\zeta = 10\%$ constraint.

GMI	-0.0602

	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
AMI	0.0400	-0.0100	-0.0602

SMI	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
M1	0.0600	0.0100	-0.0401
M2	0.0701	0.0200	-0.0301
M3	0.0400	-0.0100	-0.0602

Table 8: Small-signal stability indexes for synthetic case 3

5.4.4 Test 4: All Modes Violating all damping ratios

Figure 25 presents a case where all pre-defined damping ratios have been violated for at least one mode. In this example it is shown that the 3 layer index correctly reflects the violations even in the worst situation. Figure 25 (a) depicts the set of signals used. In this figure all signals have very low damping ratios and low settling times, even more, signal $y_1(t)$ has negative damping and its oscillation is growing exponentially.

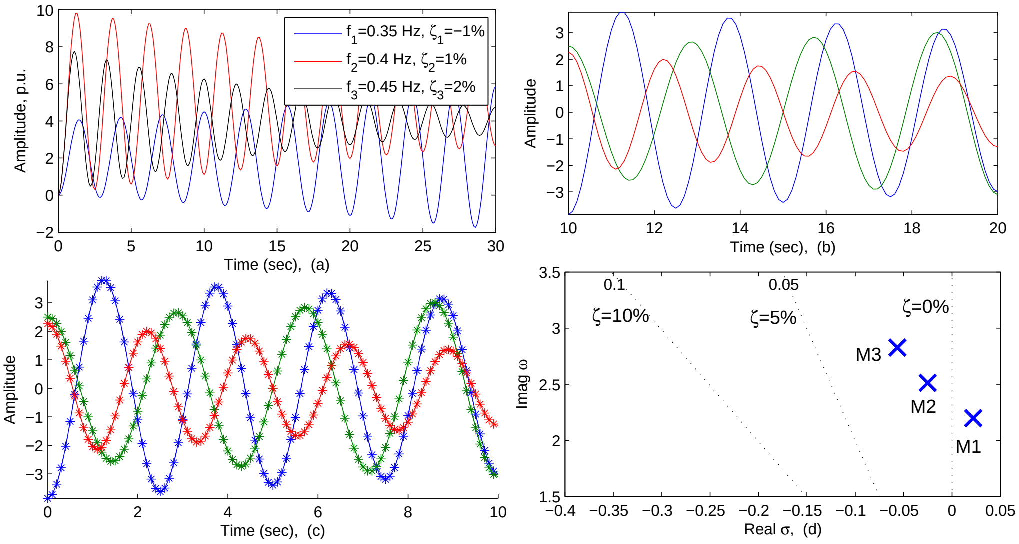

Figure 25: (a) Analyzed time series, (b) Detrended and selected data, (c) Signal estimation from ringdown analysis, (d) Estimated Modes

Table 9 depicts the final indexes for this example. On overview of Table 9 exhibits negative numbers in all columns. It can be asserted that all modes have violated at least one opperational constraint. An inspection to SMI denotes that mode 1 (M1) presents the worst case, the mode has a damping ratio below $\zeta = 0\%$ and is breaking all the required damping operational limits, as seen from the negative numbers in row 1 of Table 9.

GMI	-0.1102

	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
AMI	-0.0100	-0.0600	-0.1102

SMI	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
M1	-0.0100	-0.0600	-0.1102
M2	0.0100	-0.0400	-0.0902
M3	0.0200	-0.0300	-0.0802

Table 9: Small-signal stability indexes for synthetic case 4

5.4.5 Test 5: Signals with multiple modes

In the previous sections, we presented simple examples where there was only one mode per signal, the objective was to illustrate how to interpret the three layer index (SMI, AMI and GMI) according to the case study. In this new test, we demonstrate the potential of the small-signal stability index to identify the frequency of signals with multiple modes.

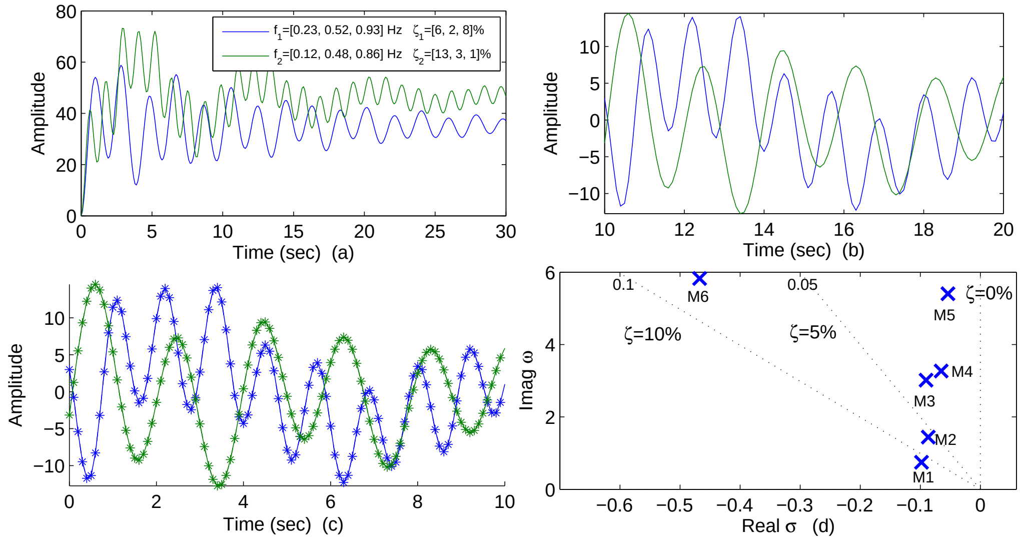

Figure 26: (a) Analyzed time series, (b) Detrended and selected data, (c) Signal estimation from ringdown analysis, (d) Estimated Modes

Figure 26 (a) presents a pair of synthetic time series used, each of the signals shown here have three modes. Hence, there are six modes and six different frequencies. Figure 26 (b) shows the section of the time series that has been analyzed and Figure 26 (c) shows the results after applying the ringdown method. In Figure 26 (d) are presented the identified modes, which are also described in Table 10.

	Mode	Freq (Hz)	Damping (%)
M1	-0.0980 + j 0.7476	0.1190	13.0
M2	-0.0867 + j 1.4425	0.2296	6.0
M3	-0.0905 + j 3.0146	0.4798	3.0
M4	-0.0653 + j 3.2666	0.5199	2.0
M5	-0.0540 + j 5.4033	0.8600	1.0
M6	-0.4675 + j 5.8246	0.9270	8.0

Table 10: Identified modes within frequencies of: 0.10 and 1.00 Hz

Table 11 summarizes the results. The value of the index GMI is negative, indicating that at least one mode has violated one of the pre-defined damping ratios. After a review of AMI, we can observe that elements $(1,2)$ and $(1,3)$ are negative, which clearly indicates that at least one mode has violated the pre-defined damping ratios $\zeta_{10} = 10\%$ and $\zeta_5 = 5\%$, respectively. Finally, in order to retrieve detailed information about each mode, examining SMI we can conclude that 3 modes are violating the $\zeta_5 = 5\%$ damping ratio and that 5 modes are violating the $\zeta_{10} = 10\%$ damping ratio, as indicated by their negative number. The results from Table 11 agree with the location of each mode in Figure 26 (d).

GMI	-0.0902

	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
AMI	0.0100	-0.0400	-0.0902

SMI	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
M1	0.1304	0.0803	0.0302
M2	0.0600	0.0100	-0.0401
M3	0.0300	-0.0200	-0.0702
M4	0.0200	-0.0300	-0.0802
M5	0.0100	-0.0400	-0.0902
M6	0.0801	0.0301	-0.0201

Table 11: Small-signal stability indexes for synthetic case 5

5.5 Small-Signal Stability Index Validation using RTE Simulations

This section presents the validation of the small-signal stability index described in Section 3.2 using simulations from RTE. The example is similar to the synthetic examples shown in the previous subsections, with the main difference of using simulation data from a large scale power system model. Figure 27 depicts the simulations results after applying the small-signal index to the simulations of the French network. The active power in lines (118 lines) has been analyzed as shown in Figure 27 (a). As a difference from the synthetics examples, only one mode with frequency between 0.1 and 1 Hz has been identified, as shown in Figure 27 (b). This mode has an estimated damping ratio of $\zeta_1 = 1.74\%$ and a frequency of $f_1 = 1.8$ Hz, it is presumably a local mode.

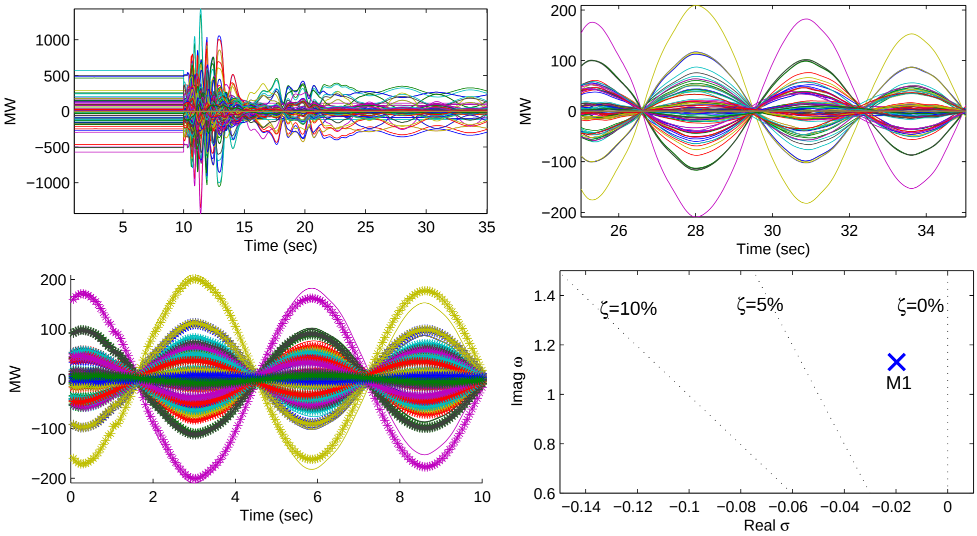

Figure 27: (a) Analyzed time series, (b) Detrended and selected data, (c) Ringdown Analysis, (d) Estimated Modes

After a review of the indexes GMI, AMI and SMI in Table 12, it can be seen that:

- At least one criteria was violated, as indicated by the negative number in GMI.
- At least one mode has broken the $\zeta = 5\%$ and $\zeta = 10\%$ damping requirement, as indicated by the negative numbers in the second and third column of AMI.
- Mode 1 (M1) has damping ration below $\zeta = 5\%$ and $\zeta = 10\%$ and greater than $\zeta = 0\%$
- Since only one mode was identified, SMI is a vector rather than matrix and is equal to AMI (SMI=AMI).

GMI	-0.0827

	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
AMI	0.0174	-0.0326	-0.0827

SMI	$\zeta = 0\%$	$\zeta = 5\%$	$\zeta = 10\%$
M1	0.0174	-0.0326	-0.0827

Table 12: Small-signal stability indexes for synthetic case 4

5.6 Voltage Stability Index Validation using Synthetic Simulations

In this section we validate the voltage stability index described in Section 3.3. Similarly to the small-signal stability index, this index has three layers to facilitate interpretation. The aim is to calculate the distance from different loading levels in the network to maximum operational limits in terms of power and voltage. To compute this index, the minimum information required is a set of 3 different dynamic simulations at different loading levels: very low loading level, acceptable (or OK) loading level and an extreme case where the loading level is unacceptable for operational standards. The aim is to represent 3 different regions of the power- voltage (PV) curve and estimate it accurately using the approach described in Section 3.3.

For this illustrative example, we have simulated a simple system of one machine, two nodes, one line and one load as described in Figure 28. Three different simulations at different loading levels have been performed. On each simulation of 30 seconds length, the reactance (X) of the line was increased at time $t = 15$ sec, simulating the loss of one line in a transfer corridor of real power system. The results are shown in Figure 29.

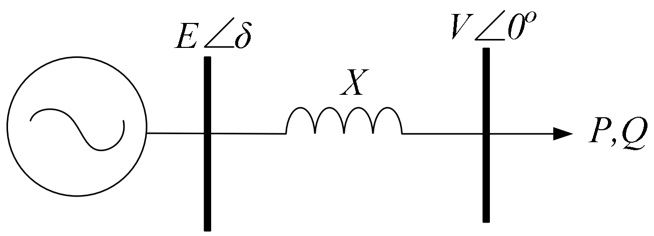

Figure 28: Simple power system model used for validation

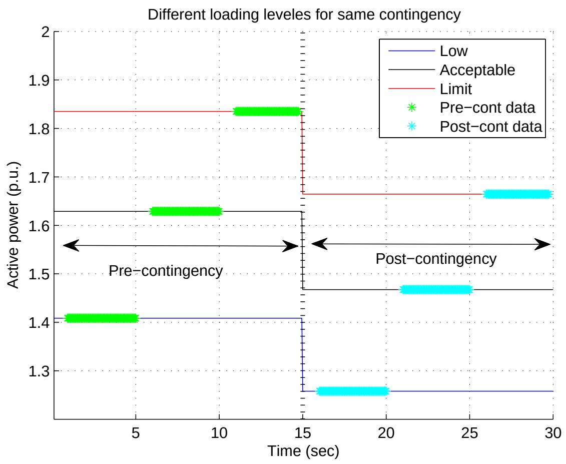

Figure 29: Three sets of simulation data used to validate the index

Using the data shown in Figure 29, two PV curves were estimated; the resulting pre-fault and post-fault curves in Figures 30 and 31, respectively. The pre-contingency curve (blue curve in Figure 30) was estimated using the green data shown in Figure 29, the mean value of each section of data is also shown in Figure 30 (green points). Each green point represents a different loading level: (P_a, V_a) for low loading level, (P_b, V_b) for an OK loading level and (P_c, V_c) for the high loading level of power and voltage, respectively. Figure 30 not only shows the estimated curve but also the distances of each level to the operational power and voltage limits (P_{lim}, V_{lim}) in blue.

On the other hand, curve red in Figure 31 depicts the estimated PV curve in post-contingency. The red curve was estimated using the light blue data from Figure 29. Figure 31 shows the new set of data at different loading levels (P_a, V_a), (P_b, V_b) and (P_c, V_c). Note that all voltage values have shrunk and this is because the post-contingency curve (in red) is smaller than the pre-contingency curve (in blue), the impedance of the system changed following a disturbance in the network. Figure 31 also indicates the distances to the new references

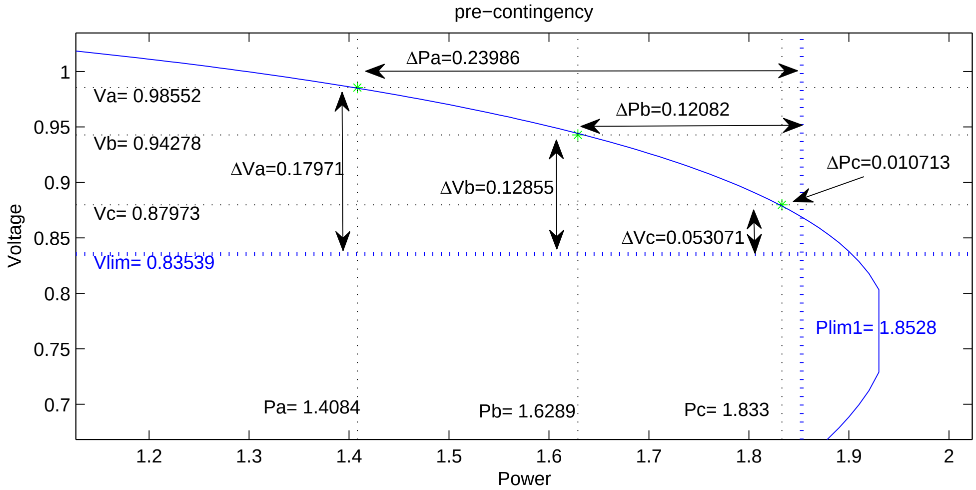

Figure 30: Estimated PV curve using the pre-fault data, distance to power and voltage limits

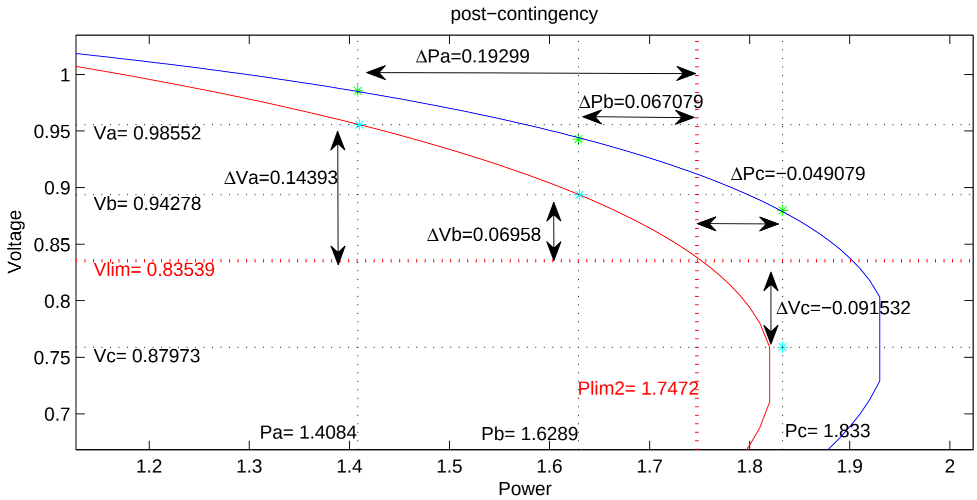

Figure 31: Estimated PV curve using the post-fault data, distance to new power and voltage limits

$(\hat{P}_{lim}, V_{lim})$ in red. It can be seen that the power and voltage at heavy loading levels (P_c, V_C) are outside the range in the post-contingency curve. Based on Figures 30 and 31, we can conclude that operating the system at stressed loading level (P_c, V_c) in pre-contingency, the distance between (P_c, V_c) to the operational limits (P_{lim}, V_{lim}) is small and following a disturbance in such conditions, the system will be subject to a voltage instability.

Table 9 summarizes the results using the GBI, ABI and SBI indexes described in Section 3.3. GBI is used to interpret the overall results. In this case, both elements of GBI are negative, indicating that at least one power and voltage limit has been violated. By looking only to GBI it is not possible to know exactly which loading level is potentially dangerous. ABI is then used and we can observe that both power and voltage corresponding to the stressed loading levels $(\Delta P_c, \Delta V_c)$ are negative. ABI shows that there is a problem at heavy loading level in both the power and the voltage so SBI is reviewed to retrieve more specific information.

In this illustrative example, there is only one bus to analyze so SBI has two rows, one for pre-contingency and one for post-contingency distances, respectively. Studying SBI results, we can observe that there are two negative numbers, both in post-contingency. We can conclude that $(\Delta P_c, \Delta V_c)$ in pre-contingecy is already small and following a disturbance in such conditions, the system will collapse, the values of $(\Delta P_c, \Delta V_c)$ are out of the operational limits $(\hat{P}_{lim}, V_{lim})$.

GBI	$\Delta P's$	$\Delta V's$
	-0.0491	-0.0915

	ΔP_a	ΔP_b	ΔP_c	ΔV_a	ΔV_b	ΔV_c
ABI	0.1930	0.0671	-0.0491	0.1439	0.0696	-0.0915

SBI		ΔP_a	ΔP_b	ΔP_c	ΔV_a	ΔV_b	ΔV_c
Bus 1	Pre	0.2399	0.1208	0.0107	0.1797	0.1286	0.0531
	Post	0.1930	0.0671	-0.0491	0.1439	0.0696	-0.0915

Table 13: Voltage stability indexes for the synthetic case

5.6.1 Bus and Line Selection for Index Calculation

In order to apply the voltage stability index correctly, the bus and line to be studied have to be chosen appropriately. In this section some guidelines to select these parameters are presented. Figure 32 shows a simple one line diagram of a power system, the network is comprised by four buses, four lines, two machines and two loads.

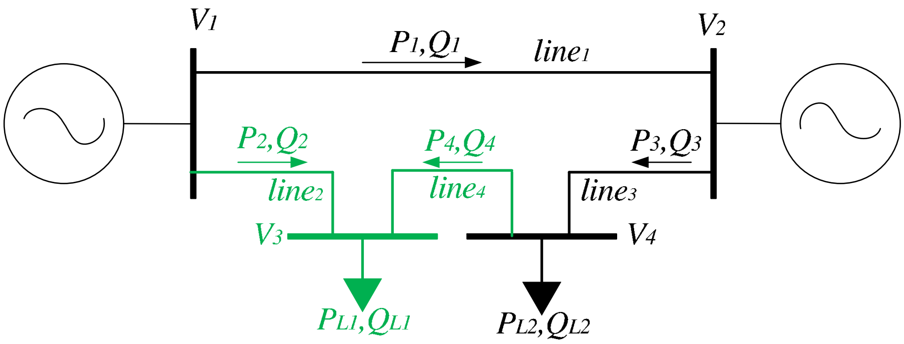

Figure 32: Illustrative power system one line diagram with correct parameter selection

A correct bus and line selection are highlighted in green, in this case, the voltage at bus 3 is selected. This bus, which acts as the measurement point, will be used to identify an equivalent model. The active and reactive powers related to this bus are also required. Lines 2 and 3 are highlighted in greed indicating that will be used to calculate the total active and reactive power from the measurement point as indicated in (46).

$$\begin{aligned} V =& \quad V_3 \\ P =& \quad P_2 + P_4 - P_{L1} \\ Q =& \quad Q_2 + Q_4 - Q_{L1} \end{aligned} \tag{46}$$

Figure 33 depicts another correct selection. In this case the voltage at bus 4 is used to identify the equivalent model, just like the active and reactive powers in lines 3 and 4. The total voltage, active and reactive power are shown in (47).

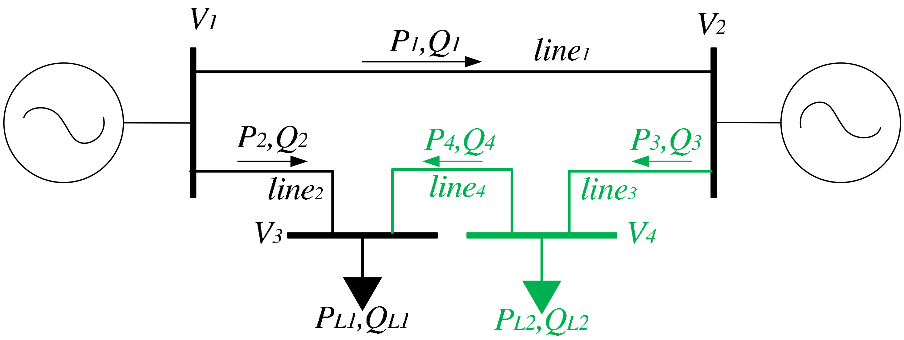

Figure 33: Another correct parameter selection

$$
\begin{aligned}
V &= V_4 \\
P &= P_3 - P_4 - P_{L2} \\
Q &= Q_3 - Q_4 - Q_{L2}
\end{aligned}
\tag{47}
$$

Finally, Figure 34 provides an incorrect parameter selection highlighted in red. In this case the voltage at bus 3 is selected to identify the equivalent model but the active and reactive powers are measured from line 4. From the figure, it is obvious that the parameters are not linked and therefore the index will not provide valid information.

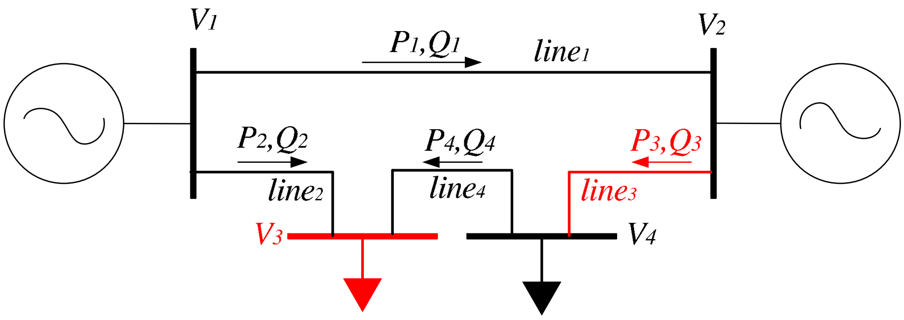

Figure 34: Incorrect parameter selection

A Appendix A

A.1 Input/output data specification for functions execution

This section illustrates how the MATLAB functions for all indexes are called, what are the inputs required and outputs provided. An example using a MATLAB script is given for each function.

A.1.1 Overload Index

The inputs to the overload index are defined as follows

```
%% OVERLOAD Inputs
t;                  % Time vector of size t x 1
t1=[1,40,30];       % Section of Signal to Analyze t1(1)= Initial Time
                    %                              t1(2)= Final time
                    %                              t1(3)= Number of samples prior t1(2)
                    %                                 These make up the window of data
                    %                                 used to compute the index.

Signal = S1;         % Input signal, Apparent Power (S) Matrix of size t x N
nl     = size(S1,2);% Determine the number of Signals N
wf_i   = ones(1,nl);% Uniform weights in all Lines
                     % If information about each bus is provided, then a
                     % similar vector with the weights for each bus needs to
                     % be provided by the user
p      = 3;          % Exponent used to scale the index, value between 1 to "inf"
dev    = 10;         % Maximum Variation allowed from the nominal value
                     % in percent, i.e, 5 for 5%
flag   = 1;          % Plotting results of flag=1, otherwise flag=0
```

Having described the inputs, the MATLAB function is called as follows:

```
[f_x,f_scaled,f_noscaled]=static_overload(t,t1,Signal,wf_i,p,dev,flag);
```

The outputs are described below

```
% f_x        = Overload index, gain
% f_scaled   = Individual overload index of each line affected by the the exponent "p"
% f_noscaled = Individual overload index of each line without exponent "p" effect
```

Note that the final index value is given by f_x and the other outputs f_{scaled} and $f_{noscaled}$ are given for future and potential analysis of any calculated index.

A.1.2 Under/Over Voltage Index

The inputs to the under/over voltage index are defined as follows

```
% UNDER/OVER VOLTAGE inputs
t;                  % time vector of size t x 1
t1 =[1,900,50];     % Section of Signal to Analyze
                    %      t1(1)= Initial Time
                    %      t1(2)= Final time
                    %      t1(3)= Number of samples before time t1(2)
                    %             These make up the window of data
                    %             used to compute the index.

V    = BVm;         % Voltage Magnitudes - Matrix of size t x N
nb   = size(BVm,2);% Determine the number of Buses  N
wv_i = ones(1,nb); % Uniform weights in all Buses
                    % If information about each bus is provided, then a
                    % similar vector with the weights for each bus needs to
                    % be provided by the user
p    = 3;           % Exponent used to scale the index, value between 1 to "inf"
Dev  = 1;           % Maximum voltage variation allowed
                    % (in %) of the nominal value, i.e. 2 for 2%
Method = 0;         % Select the method for time series analysis
                    %     Method=0 average value  (recommended)
                    %     Method=1 largest value
flag =1;            % Plotting results of flag=1, otherwise flag=0
```

Having described the inputs, the MATLAB function is called as follows:

```
[v_x,v_scaled,v_noscaled]=static_voltage(t,t1,BVm,wv_i,p,Dev,Method,flag);
```

The outputs are described below

```
% v_x        = Under/Over Voltage index, gain
% v_scaled   = Individual index of each bus scaled by the the exponent "p"
% v_noscaled = Individual index of each bus without exponent "p" effects
```

Note that the final index value is given by v_x and the other outputs v_{scaled} and $v_{noscaled}$ are given for future and potential analysis of any calculated index.

A.1.3 Transient Stability Index

The inputs to the transient stability index are defined as follows

```
% TRANSIENT STABILITY inputs
t;                   % Time vector size t x 1
t1=[1,68];           % Section of Signal t1(1)= Initial Time in seconds
                     %                   t1(2)= Final time in seconds

delta;  % Angle of the machines size t x N;
M;      % Two times the Inertia (H) of the machines (M=2*H), size 1 x N
```

Having described the inputs, the MATLAB function is called as follows:

```
J=dynamic_transient(t,t1,delta,M);
```

The output is described below

```
% outputs
% J  = Transient stability index, gain
```

A.1.4 Small-Signal Stability Index

The inputs to the small-signal stability index are defined as follows

```
% SMALL-SIGNAL inputs
Method =2;      % Method=1 (ERA analysis)
                % Method=2 (PRONY analysis recommended)
t;              % Time vector size t x 1
SigAn = LP1;    % Signals to analyze (Active power flows at critical lines)
t1=[1,35,25];   % Select window section t1(1)= Initial Time
                                      %  t1(2)= Final Time
                                      %  t1(3)= Time to start Ringdown Analysis
Pcent=100;      % Percent number of signals to  analyze, for all signals use 100%
f=[0.1,1];      % Range of frequencies of modes of interest
                %  f(1) = fmin,
                %  f(2) = fmax both in Hz
Nmodes=[];      % Number of Modes to estimate,
                %     Nmodes = [];  Automatic selection,
                %     Nmodes = [2]; Force Algorithm to estimate only 2 modes.
Damp=[0,5,10]; % Pre-defined Damping ratios,
                % Calculate damping distance from each mode
                % to these predefined damping, ie [0%, 5%, 10%]
flag=1;         % if flag= 1 plot results, other wise flag=0
```

Having described the inputs, the MATLAB function is called as follows:

```
[GMI,AMI,SMI,Poles,Freq,Ener]=smallsignal(Method,t,SigAn,t1,Pcent,f,Nmodes,Damp,flag);
```

The outputs are described below

```
% Outputs
% Ener  Structure with time and signals sorted according to its energy
%       Ener.y  Signals sorted, from Time t1(1) to Time t1(2)
%       Ener.yh Signals sorted, from Time t1(3) to Time t1(2)
%       Ener.t  Time vector, from Time t1(1) to Time t1(2)
%       Ener.th Time vector, from Time t1(2) to Time t1(2)
%
% Freq  All Frequencies found after applying the FFT filter
%
% Poles Structure with ERA or PRONY results
%      if ERA
%      Poles.sys  Is a continous state-space model
%
%      if PRONY
%      Poles.sys Is the "A" matrix of the state-space model
%
% SMI  Single Mode Indicator, Matrix.
% AMI  All Mode Indicator, Vector.
% GMI  Global Mode Indicator, Gain.
```

A.1.5 Voltage Stability Index

The inputs to the voltage stability index are defined as follows

```
% VOLTAGE STABILITY Inputs
% At least 3 set of simulation results has to be provided for 3 different loading
% levels (a=low, b=OK, c=limit), the signals required are:
% Time (t), Voltage (V), Active Power (P) and Reactive Power (Q) in cell array format
% where each element of the cell array, corresponds to a different simulation result
% for a different loading level

t={t20,t60,t80}; % Time
V={V20,V60,V80}; % Voltage at representative buses
P={P20,P60,P80}; % Active Power in representative lines
Q={Q20,Q60,Q80}; % Reactive Power on representative lines

t_pre = {ta_pre,tb_pre,tc_pre};% Intervals of time for different loading levels
                               % in pre-contingency
                               % ta_pre=[t1,t2] t1=Initial time, t2=Final time
                               % tb_pre=[t3,t4] t3=Initial time, t4=Final time
                               % tc_pre=[t5,t6] t5=Initial time, t6=Final time

t_post= {ta_post,tb_post,tc_post};% Intervals of time for different loading levels
                                  % in post-contingency

Nb;             %Number of bus or buses to analyze,
                        %i.e. Nb=[10] for single bus,
                        %     Nb=[5,8,12] for 3 or more buses
Nl;             %Number of line to analyze
Limits;         % Limits=[Plim,Vlim] in (%),  Percent of Power and Voltage
                % from the limits
x01 = pi/8;     % Initial guess for angle deltas
x02 = 1;        % Initial guess for voltage  E
x03 = 0.03;     % Initial guess for reactance X
r01 = 1;  % Initial guess for alpha
r02 = 1;  % Initial guess for beta

x0=[x01;x02;x03]; % Initial conditions
r0=[r01;r02];     % Initial conditions
```

Having described the inputs, the MATLAB function is called as follows:

```
[SBI,ABI,GBI,Pre,Post]=voltagestability(t,V,P,Q,t_pre,t_post,Nb,Nl,Limits,x0,r0);
```

The outputs are described below

```
% Outputs
% SBI - Single Bus Index. Is a matrix and provides:
%       the distance in pre and post contingency for each
%       loading level (low, ok and limit) to  the maximum loadability (Pmax)
%       and voltage (Vlim) limits for a selected bus(i).
% ABI - All Bus Index. Is a vector that provides the minimum distance
%       among all buses for each loading level (low, ok and limit)
%       to the loadability (Pmax) and the voltage (Vlim) limits.
%       The ABI index helps to identify if a loading level is violating
%       a limit in pre or post contingency.
% GBI - Is a 2 element vector that provides the overall minimum distance
%       to  the loadability (Pmax) and the over all voltage (Vmax)
%       limits respect to all buses.
%       The GBI index indicates if a limit has been violated.
%
% Pre & Post - Structures containing the following information.
%     .limits - row 1  Power limits, row 2 Voltage limits
%     .mean   - row 1, Power mean values for each loading level [Pa,Pb,Pb],
%                      where a=low, b=OK, c=limit levels
%               row 2, Voltage mean values for each loading level [Va,Vb,Vc],
%                      where a=low, b=OK, c=limit levels
%     .deltas - same information as SBI
%     .curve  - column 1 estimated Power, useful to reproduce nose curve
%               column 2 estimated Voltage, useful to reproduce nose curve
%                    i.e. pre-contingency PV plot = plot(Pre.curve(:,1),Pre.curve(:,2))
```

A.2 MATLAB Toolboxes Dependencies

Index	Toolbox	Function	Called Functions (Dependency)	Brief description
Under/over voltage	-----	Functions and scripts	**static_voltage***	Calculate index
			horline.m*	Add a horizontal line in plots
Overload	-----	Functions and scripts	**static_overload***	Calculate index
			horline.m*	Add a horizontal line in plots
Transient stability	-----	Functions and scripts	**dynamic_transient***	Calculate index
Voltage stability	Optimization Toolbox Version 6.0 (R2011a)	Least Squares (Curve Fitting) functions	**lsqnonlin.m**	Solve nonlinear least-squares (nonlinear data-fitting) problems
	-----	Functions and scripts	**voltagestability***	Calculate index
			EstimateE_and_X.m**	Estimate the voltage E and the reactance X
			Estimate_a_and_b.m**	Estimate the parameters α and β
			verline.m*	Add a vertical line in plots
Small-signal stability	Control System Toolbox Version 9.1 (R2011a)	Linear Model Identification	**ss.m**	Convert linear models to Control System Toolbox LTI models
			impulse.m	Plot impulse response with confidence interval
		Conversions	**c2d.m**	Convert from continuous- to discrete-time models
			d2c.m	Convert from discrete- to continuous-time models
	-----	Functions and scripts	**smallsignal**	Calculate index
			fiximp.m**	Helper function to convert the 3D output of MATLAB impulse function to 2D socit ver
			era.m**	Simple ERA system identification routine
			prony.m**	Prony analysis program for fitting to a ringdown
			fft_get.m*	Retrieve frequencies from given signals
			energy_sort.m*	Detrend signals and sort according to its energy
MATLAB Version 7.12.0.635 (R2011a) MATLAB License Number: 308809 Operating System: Microsoft Windows 7 Version 6.1 (Build 7601: Service Pack 1) Java VM Version: Java 1.6.0_17-b04 with Sun Microsystems Inc. Java HotSpot(TM) 64-Bit Server VM mixed mode				

* The function was created entirely for iTesla, full code available.

** Background software developed for other project or other researchers. Available only in pre-compiled form (MATLAB .p files).

----- No dependency

Figure 35: MATLAB toolboxes dependencies

References

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[4] G. Li and S. Rovnyak, “Integral square generator angle index for stability ranking and control,” *Power Systems, IEEE Transactions on*, vol. 20, no. 2, pp. 926–934, 2005.

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