Stat580 - Homework 1

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Problem 1

$X_1, X_2, \ldots, X_n \sim Unif(0,1)$.
$(\Sigma_{i=1}^n X_i) mod 1 = \Sigma_{i=1}^n X_i - \lfloor \Sigma_{i=1}^n \rfloor \sim Unif(0,1)$.

Proof:

Problem 2

Let F be a cumulative distribution function and let $F^{-1} = min\{x|F(x) \geq u\}$. If $U \sim Unif(0,1)$ then $F^{-1}(U) \sim F$. We start with the cumulative distribution function for $F^{-1}(U)$: $P(F^{-1}(U) \leq x)$

$$\begin{aligned} \text{Applying F to both sides (F is monotonic): } & P(F^{-1}(U) \leq x) = P(U \leq F(x)) \\ \text{But since U is uniform: } & P(U \leq F(x)) = F(x) \end{aligned}$$

Problem 3

Part a

We know that $U_1, U_2 \sim Unif(0,1)$ and $X = \sqrt{-2log(U_1)}cos(2\pi U_2)$ and $Y = \sqrt{-2log(U_1)}sin(2\pi U_1)$. We will transform U_1 and U_2 using the above functions and show that it yields a normal.

$$\begin{aligned} X = \sqrt{-2log(U_1)}cos(2\pi U_2) & \text{ and } Y = \sqrt{-2log(U_1)}sin(2\pi U_1) \\ X^2 + Y^2 = -2log(U_1) & \longrightarrow U_1 = exp\{\frac{-1}{2}(X^2+Y^2)\} \\ \frac{Y}{X} = tan(2\pi U_2) & \longrightarrow U_2 = \frac{1}{2\pi}tan^{-1}(\frac{Y}{X}) \\ |J| = |det \begin{bmatrix} \frac{\partial U_1}{\partial X} & \frac{\partial U_1}{\partial Y} \\ \frac{\partial U_2}{\partial X} & \frac{\partial U_2}{\partial Y} \end{bmatrix}| & = | \begin{bmatrix} exp\{\frac{-1}{2}(X^2+Y^2)\}(-X) & exp\{\frac{-1}{2}(X^2+Y^2)\}(-Y) \\ \frac{1}{2\pi}\frac{X^2}{X^2+Y^2}\frac{-Y}{X^2} & \frac{1}{2\pi}\frac{X^2}{X^2+Y^2}\frac{1}{X} \end{bmatrix} | \\ & = \frac{1}{2\pi}\frac{X^2}{X^2+Y^2}exp\{-\frac{X^2+Y^2}{2}\}(1+\frac{Y^2}{X^2}) \\ & = \frac{1}{2\pi}exp\{-\frac{X^2+Y^2}{2}\} \\ & = \frac{1}{\sqrt{2\pi}}exp\{-X^2/2\}\frac{1}{\sqrt{2\pi}}exp\{-Y^2/2\} \end{aligned}$$

Thus we have that $f_{X,Y}(x,y) = \frac{1}{\sqrt{2\pi}}exp\{-x^2/2\}\frac{1}{\sqrt{2\pi}}exp\{-y^2/2\}$ thus we have two independent standard normal variables.

Part b

Problem 4

Problem 5

Part a

Part b

Problem 6

Part a

Part b

Part c

Problem 7

Part a

Code available on impact2.stat.iastate.edu under /class/stat580/ashum/homework1/temp.c

Part b

Code available on impact2.stat.iastate.edu under /class/stat580/ashum/homework1/mult.c