A Survey of Facial Modeling and Animation Techniques

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Abstract

Realistic facial animation is achieved through geometric and image manipulations. Geometric deformations usually account for the shape and deformations unique to the physiology and expressions of a person. Image manipulations model the reflectance properties of the facial skin and hair to achieve small-scale detail that is difficult to model by geometric manipulation alone. Modeling and animation methods often exhibit elements of each realm. This paper summarizes the theoretical approaches used in published work and describes their strengths, weaknesses, and relative performance. A taxonomy groups the methods into classes that highlight their similarities and differences.

Introduction

Since the pioneering work of Frederic I. Parke [90] in 1972, many research efforts have attempted to generate realistic facial modeling and animation. The most ambitious attempts perform the modeling and rendering in real time. Because of the complexity of human facial anatomy, and our natural sensitivity to facial appearance, there is no real time system that captures subtle expressions and emotions realistically on an avatar. Although some recent work [43, 102] produces realistic results with relatively fast performance, the process for generating facial animation entails extensive human intervention or tedious tuning. The ultimate goal for research in facial modeling and animation is a system that 1) creates realistic animation, 2) operates in real time, 3) is automated as much as possible, and 4) adapts easily to individual faces.

Recent interest in facial modeling and animation is spurred by the increasing appearance of virtual characters in film and video, inexpensive desktop processing power, and the potential for a new 3D immersive communication metaphor for human-computer interaction. Much of the facial modeling and animation research is published in specific venues that are relatively unknown to the general graphics community. There are few surveys or detailed historical treatments of the subject [84]. This survey is intended as an accessible reference to the range of reported facial modeling and animation techniques.

Facial modeling and animation research falls into two major categories, those based on geometric manipulations and those based on image manipulations (Fig. 1). Each realm comprises several subcategories. Geometric manipulations include key-framing and geometric interpolations [33, 85, 90], parameterizations [21, 87, 88, 89], finite element methods [6, 44, 101], muscle based modeling [69, 95, 100, 105, 106, 108, 120, 129], visual simulation using pseudo muscles [49, 70], spline models [78, 79, 123, 124, 125] and free-form deformations [24, 49]. Image manipulations include image morphing between photographic images [10], texture manipulations [81], image blending [102], and vascular expressions [48]. At the preprocessing stage, a person-specific individual model may be constructed using anthropometry [25], scattered data interpolation [122], or by projecting target and source meshes onto spherical or

cylindrical coordinates. Such individual models are often animated by feature tracking or performance driven animation [12, 35, 83, 92, 131].

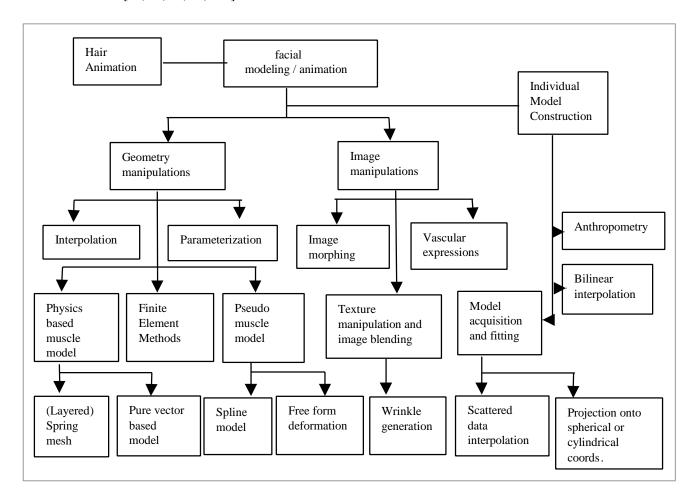


Fig. 1 - Classification of facial modeling and animation methods

This taxonomy in Figure 1 illustrates the diversity of approaches to facial animation. Exact classifications are complicated by the lack of exact boundaries between methods and the fact that recent approaches often integrate several methods to produce better results.

The survey proceeds as follows. Section 1 and 2 introduce the interpolation techniques and parameterizations followed by the animation methods using 2D and 3D morphing techniques in section 3. The Facial Action Coding System, a frequently used facial description tool, is summarized in section 4. Physics based modeling and simulated muscle modeling are discussed in sections 5 and 6, respectively. Techniques for increased realism, including wrinkle generation, vascular expression and texture manipulation, are surveyed in sections 7, 8, and 9. Individual modeling and model fitting are described in section 10, followed by animation from tracking data in section 11. Section 12 describes mouth animation research, followed by general conclusions and observations.

1. Interpolations

Interpolation techniques offer an intuitive approach to facial animation. Typically, an interpolation function specifies smooth motion between two key-frames at extreme positions, over a normalized time interval (Fig. 2).

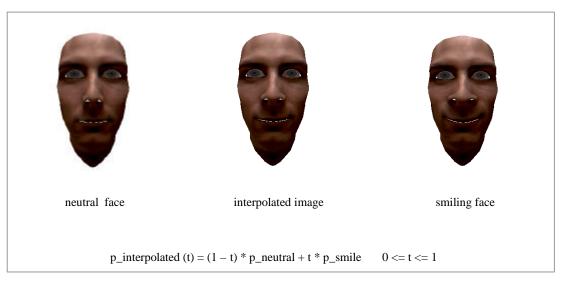


Fig. 2 - Linear interpolation is performed on muscle contraction values

Linear interpolation is commonly used [102] for simplicity, but a cosine interpolation function or other variations can provide acceleration and deceleration effects at the beginning and end of an animation [127]. When four key frames are involved, rather than two, bilinear interpolation generates a greater variety of facial expressions than linear interpolation [89]. Bilinear interpolation, when combined with simultaneous image morphing, creates a wide range of realistic facial expression changes [4].

Interpolated images are generated by varying the parameters of the interpolation functions. Geometric interpolation directly updates the 2D or 3D positions of the face mesh vertices, while parameter interpolation controls functions that indirectly move the vertices. For example, Sera et al. [113] perform a linear interpolation of the spring muscle force parameters, rather than the positions of the vertices, to achieve realistic mouth animation. Figure 2 shows two key frames and an interpolated image using linear interpolation of muscle contraction parameters.

Although interpolations are fast, and they easily generate primitive facial animations, their ability to create a wide range of realistic facial configurations is severely restricted. Combinations of independent face motions are difficult to produce. Interpolation is a good method to produce a small set of animations from a few key-frames.

2. Parameterizations

Parameterization techniques for facial animation [21, 87, 88, 89] overcome some of the limitations and restrictions of simple interpolations. Ideal parameterizations specify any possible face and expression by a combination of independent parameter values [84 pp. 188]. Unlike interpolation techniques, parameterizations allow explicit control of specific facial configurations. Combinations of parameters provide a large range of facial expressions with relatively low computational costs.

As Waters [126] indicates, there is no systematic way to arbitrate between two conflicting parameters to blend expressions that effect the same vertices, hence parameterization rarely produces natural human expressions or configurations when a conflict between parameters occurs. For this reason, parameterizations are designed to only affect specific facial regions, however this often introduces noticeable motion boundaries. Another limitation of parameterization is that the choice of the parameter set depends on the facial mesh topology and, therefore, a complete generic parameterization is not possible. Furthermore, tedious manual tuning is required to set parameter values, and even after that, unrealistic motion or configurations may result. The limitations of parameterization led to the development of diverse techniques such as morphing between images, (pseudo) muscle based animation, and finite element methods.

3. 2D & 3D morphing

Morphing effects a metamorphosis between two target images or models. A 2D image morph consists of a warp¹ between corresponding points in the target images and a simultaneous cross dissolve². Typically, the correspondences are manually selected to suit the needs of the application. Morphs between carefully acquired and corresponded images produce very realistic facial animations. Beier et al. [10] demonstrated 2D morphing between two images with manually specified corresponding features (line segments). The warp function is based upon a field of influence surrounding the corresponding features. Realism, with this approach, requires extensive manual interaction for color balancing, correspondence selection, and tuning of the warp and dissolve parameters. Variations in the target image viewpoints or features complicate the selection of correspondences. Realistic head motions are difficult to synthesize since target features become occluded or revealed during the animation.

To overcome the limitations of 2D morphs, Pighin et al. [103] combine 2D morphing with 3D transformations of a geometric model. Pighin et al. animate key facial expressions with 3D geometric interpolation, while image morphing is performed between corresponding texture maps. This approach achieves viewpoint independent realism, however, animations are still limited to interpolations between pre-defined key-expressions.

The 2D and 3D morphing methods can produce realistic facial expressions, but they share similar limitations with the interpolation approaches. Selecting corresponding points in target images is manually intensive, dependent on viewpoint, and not generalizable to different faces. Also, the animation viewpoint is constrained to approximately that of the target images.

4. Facial Action Coding System

The Facial Action Coding System (FACS) is a description of the movements of the facial muscles and jaw/tongue derived from an analysis of facial anatomy [32]. FACS includes 44 basic action units (AUs). Combinations of independent action units generate facial expressions. For example, combining the AU1 (Inner brow raiser), AU4 (Brow Raiser), AU15 (Lip Corner Depressor), and AU23 (Lip Tightener) creates a sad expression. A table of the sample action units and the basic expressions generated by the actions units are presented in Tables 1 and 2.

AU	FACS Name	AU	FACS Name
1	Inner Brow Raiser	12	Lid Corner Puller
2	Outer Bow Raiser	14	Dimpler
4	Brow Lower	15	Lip Corner Depressor
5	Upper Lid Raiser	16	Lower Lip Depressor
6	Check Raiser	17	Chin Raiser
7	Lid Tightener	20	Lip Stretcher
9	Nose Wrinkler	23	Lip Tightener
10	Upper Lid Raiser	26	Jaw Drop

Basic Expressions	Involved Action Units		
Surprise	AU1, 2, 5, 15, 16, 20, 26		
Fear	AU1, 2, 4, 5, 15, 20, 26		
Disgust	AU2, 4, 9, 15, 17		
	, , , ,		
Anger	AU2, 4, 7, 9, 10, 20, 26		
Happiness	AU1, 6, 12, 14		
Sadness	AU1, 4, 15, 23		

Table 1 - Sample single facial action units.

Table 2 – Example sets of action units for basic expressions

Animation methods using muscle models or simulated (pseudo) muscles overcome the correspondence and lighting difficulties of interpolation and morphing techniques. Physical muscle modeling mathematically

¹ Basic warping maps an image onto a regular shape such as a plane or a cylinder.

² In cross dissolving, one image is faded out while another is simultaneously faded in.

describes the properties and the behavior of human skin, bone, and muscle systems. In contrast, pseudo muscle models mimic the dynamics of human tissue with heuristic geometric deformations. Approaches of either type often parallel the Facial Action Coding System and Action Units developed by Ekman and Friesen [32].

5. Physics Based Muscle Modeling

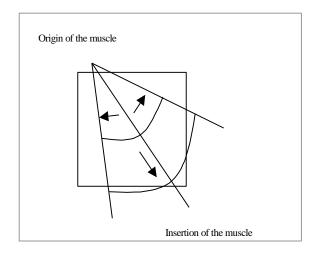
Physics-based muscle models fall into three categories: mass spring systems, vector representations, and layered spring meshes. Mass-spring methods propagate muscle forces in an elastic spring mesh that models skin deformation. The vector approach deforms a facial mesh using motion fields in delineated regions of influence. A layered spring mesh extends a mass spring structure into three connected mesh layers to model anatomical facial behavior more faithfully.

5.1. Spring Mesh Muscle

The work by Platt and Badler [105] is a forerunner of the research focused on muscle modeling and the structure of the human face. Forces applied to elastic meshes through muscle arcs generate realistic facial expressions. Platt's later work [104] presents a facial model with muscles represented as collections of functional blocks in defined regions of the facial structure. Platt's model consists of 38 regional muscle blocks interconnected by a spring network. Action units are created by applying muscle forces to deform the spring network.

5.2. Vector Muscle

A very successful muscle model was proposed by Waters [129]. A delineated deformation field models the action of muscles upon skin. A muscle definition includes the vector field direction, an origin, and an insertion point (Fig. 3). The field extent is defined by cosine functions and fall off factors that produce a cone shape when visualized as a height field. Waters also models the mouth sphincter muscles as a simplified parametric ellipsoid. The sphincter muscle contracts around the center of the ellipsoid and is primarily responsible for the deformation of the mouth region. Waters animated human emotions such as anger, fear, surprise, disgust, joy, and happiness using vector based linear and orbicularis oris muscles implementing the FACS. Figure 4 shows Waters' muscles embedded in a facial mesh.



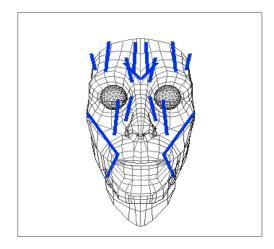


Fig. 3 - Zone of influence of Waters' linear muscle model. Deformation decreases in the directions of the arrows.

Fig. 4 - Waters' linear muscles

The positioning of vector muscles into anatomically correct positions can be a daunting task. No automatic way of placing muscles beneath a generic or person-specific mesh is reported. The process involves

manual trial and error with no guarantee of efficient or optimal placement. Incorrect placement results in unnatural or undesirable animation of the mesh. Nevertheless, the vector muscle model is widely used because of its compact representation and independence of the facial mesh structure. An example of vector muscles is seen in Billy, the baby in the movie "Tin Toy", who has 47 Waters' muscles on his face.

5.3. Layered Spring Mesh Muscles

Terzopoulos and Waters [120] proposed a facial model that models detailed anatomical structure and dynamics of the human face. Their three-layers of deformable mesh correspond to skin, fatty tissue, and muscle tied to bone. Elastic spring elements connect each mesh node and each layer. Muscle forces propagate through the mesh systems to create animation. This model achieves great realism, however, simulating volumetric deformations with three-dimensional lattices requires extensive computation. A simplified mesh system reduces the computation time while still maintaining visual realism (Wu. et al [133]).

Lee et al. [60] presented models of physics-based synthetic skin and muscle layers based on earlier work [120]. The face model consists of three components: a biological tissue layer with nonlinear deformation properties, a muscle layer knit together under the skin, and an impenetrable skull structure beneath the muscle layer. The synthetic tissue is modeled as triangular prism elements that are divided into the epidermal surface, the fascia surface, and the skull surface (Fig. 5). Spring elements connecting the epidermal and fascia layers simulate skin elasticity. Spring elements that effect muscle forces connect the fascia and skull layers. The model achieves spectacular realism and fidelity, however tremendous computation is required, and extensive tuning is needed to model a specific face or characteristic.

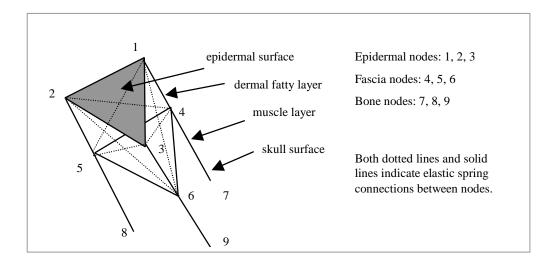


Fig. 5 - Triangular skin tissue prism element

6. Pseudo or Simulated Muscle

Physics-based muscle modeling produces realistic results by approximating human anatomy, but it is daunting to consider the exact modeling and parameters tuning needed to simulate a specific human's facial structure. Simulated muscles offer an alternative approach by deforming the facial mesh in muscle-like fashion, but ignoring the complicated underlying anatomy. Deformation usually occurs only at the thin-shell facial mesh. Muscle forces are simulated in the form of splines [78, 79, 123, 124, 125], wires [114], or free form deformations [24, 49].

6.1. Free form deformation

Free form deformation (FFD) deforms volumetric objects by manipulating control points arranged in a three-dimensional cubic lattice [112]. Conceptually, a flexible object is embedded in an imaginary, clear, and flexible control box containing a 3D grid of control points. As the control box is squashed, bent, or twisted into arbitrary shapes, the embedded object deforms accordingly (Fig. 6). The basis for the control points is a trivariate tensor product Bernstein polynomial. FFDs can deform many types of surface primitives, including polygons; quadric, parametric, and implicit surfaces; and solid models.

Extended free form deformation (EFFD) [24] allows the extension of the control point lattice into a cylindrical structure. A cylindrical lattice provides additional flexibility for shape deformation compared to regular cubic lattices. Rational free form deformation (RFFD) incorporates weight factors for each control point, adding another degree of freedom in specifying deformations. Hence, deformations are possible by changing the weight factors instead of changing the control point positions. When all weights are equal to one, then RFFD becomes a FFD. A main advantage of using FFD (EFFD, RFFD) to abstract deformation control from that of the actual surface description is that the transition of form is no longer dependent on the specifics of the surface itself [67 pp. 175].

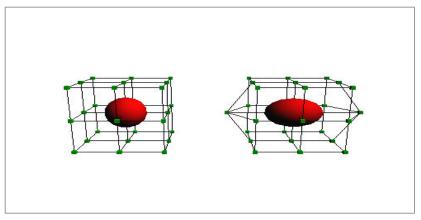


Fig. 6 - Free form deformation. Controlling box and embedded object are shown. When controlling box is deformed by manipulating control points, so is embedded object.

Kalra et al. [49] interactively simulates the visual effects of the muscles using Rational Free Form Deformation (RFFD) combined with region-based approach. To simulate the muscle action on the facial skin, surface regions corresponding to the anatomical description of the muscle actions are defined. A parallelepiped control volume is then defined on the region of interest. The skin deformations corresponding to stretching, squashing, expanding, and compressing inside the volume are simulated by interactively displacing the control points and by changing the weights associated with each control point. Linear interpolation is used to decide the deformation of the boundary points lying within the adjoining regions. Since the computation for the overall deformation is slow, larger regions are defined and stiffness factors associated with each control points are exploited to control the deformation. Displacing a control point is analogous to actuating a physically modeled muscle. Compared to Waters model [129], manipulating the positions or the weights of the control points is more intuitive and simpler than simulating the muscle vectors. However, RFFD (FFD, EFFD) does not provide a precise simulation of the actual muscle and the skin behavior so that it cannot model furrows, bulges, and wrinkles in the skin. Furthermore, since RFFD (FFD, EFFD) is based upon surface deformation, volumetric changes occurring in the physical muscle is not accounted for.

In [49], facial animation is driven by a procedure called an Abstract Muscle Action (AMA) reported by Magnenat-Thalmann et al. [70]. These AMA procedures are similar to the action units of FACS and work on specific regions of the face. Each AMA procedure represents the behavior of a single or a group of

related muscles. Facial expressions are formed by group of AMA procedures. When applied to form facial expression, the ordering of the action unit is important due to the dependency among the AMA procedures.

6.2. Spline Pseudo Muscles

Albeit polygonal models of the face are widely used, they often fail to adequately approximate the smoothness or flexibility of the human face. Fixed polygonal models do not deform smoothly in arbitrary regions, and planar vertices can not be twisted into curved surfaces without sub-division.

An ideal facial model has a surface representation that supports smooth and flexible deformations. Spline muscle models offer a solution. Splines are usually up to C2 continuous, hence a surface patch is guaranteed to be smooth, and they allow localized deformation on the surface. Furthermore, affine transformations are defined by the transformation of a small set of control points instead of all the vertices of the mesh, hence reducing the computational complexity.

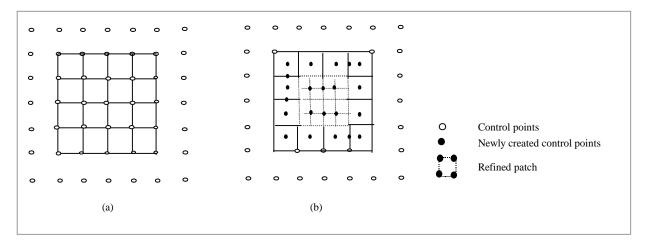


Fig. 7 - (a) shows a 16 patch surface with 49 control points and (b) shows the 4 patches in the middle refined to 16 patches. (Following the original description from [130])

Some spline-based animation can be found in [78, 79, 123]. Pixar used bicubic Catmull-Rom spline³ patches to model Billy, the baby in animation "Tin Toy", and recently, used a variant of Catmull-Clark [19] subdivision surfaces to model Geri, a human character in short film Geri's game. This technique is mainly adapted to model sharp creases on a surface or discontinuities between surfaces [27]. For a detailed description of Catmull-Rom splines and Catmull-Clark subdivision surfaces, refer to [20] and [19] respectively. Eisert and Girod [31] used triangular B-splines to overcome the drawback that conventional B-splines do not refine curved areas locally since they are defined on a rectangular topology.

A hierarchical spline model reduces the number of unnecessary control points. Wang et al. [125] showed a system that integrated hierarchical spline models with simulated muscles based on local surface deformations. Bicubic B-splines are used because they offer both smoothness and flexibility, which are hard to achieve with conventional polygonal models. The drawback of using naive B-splines for complex surfaces becomes clear, however, when a deformation is required to be finer than the patch resolution. To produce finer patch resolution, an entire row or column of the surface is subdivided. Thus, more detail (and control points) are added where none are needed. In contrast, hierarchical splines provide the local refinements of B-spline surfaces and new patches are only added within a specified region (Fig. 7).

³ A distinguishing property of Catmull-Rom splines is that the piecewise cubic polynomial segments pass through all the control points except the first and last when used for interpolation. Another is that the convex hull property is not observed in Catmull-Rom spline.

Hierarchical B-splines are an economical and compact way to represent a spline surface and achieve high rendering speed. Muscles coupled with hierarchical spline surfaces are capable of creating bulging skin surfaces and a variety of facial expressions.

Dubreuil et al. [29] used the animation model called DOGMA (Deformation of Geometrical Model Animated) [8] to define space deformation in terms of displacement constraints. The animation model of DOGMA [8, 9] is a four-dimensional deformation system, that is a subset of generalized n-dimensional model called DOGME [14]. A 4D deformation, where the fourth dimension is time, deforms both space and time, simultaneously. A limited set of spline muscles simulates the effects of muscle contractions without anatomic modeling.

7. Wrinkles

Wrinkles are important for realistic facial animation and modeling. They aid in recognizing facial expressions as well as a person's age. There are two types of wrinkles, temporary wrinkles that appear for a short time in expressions, and permanent wrinkles that form over time as perma nent features of a face [133]. Wrinkles and creases are difficult to model with techniques such as simulated muscles or parameterization, since these methods are designed to produce smooth deformations. Physically based modeling with plasticity or viscosity, and texture techniques like bump mapping are more appropriate.

7.1. Wrinkles with Bump Mapping

Bump mapping produces perturbations of the surface normals that alter the shading of a surface. Arbitrary wrinkles can appear on a smooth geometric surface by defining wrinkle functions [13]. This technique easily generates wrinkles by varying wrinkle function parameters. Bump mapping technique is relatively computationally demanding as it requires about twice the computing effort needed for conventional color texture mapping. A bump mapped wrinkled surface is depicted in figure 8.

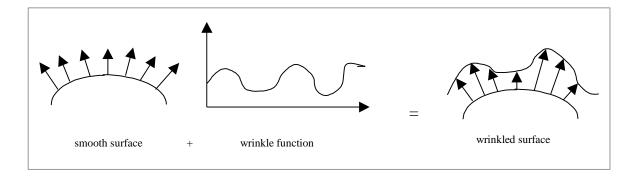


Fig. 8 - Generation of wrinkled surface using bump mapping technique.

*** this figure does not really illustrate bump mapping ***

Moubaraki et al. [76] presented a system using bump mapping to produce realistic synthetic wrinkles. The method synthesizes and animates wrinkles by morphing between wrinkled and unwrinkled textures. The texture construction starts with an intensity map (gray-level wrinkle texture) that is filtered using a simple averaging or Gaussian filter for noise-removal. Bump map gradients are extracted in orthogonal directions and used to perturb the normals of the unwrinkled texture. Correspondence is not a big issue the wrinkled and unwrinkled images are essentially similar. Animations, as with any interpolation or morphing system, are limited to pre-defined target images. Until recently, bump mapping was also difficult to compute in real time.

7.2. Physically Based Wrinkles

Physically based wrinkle models using the plastic-visco-elastic properties of the facial skin and permanent skin aging effects are reported by Wu et al. [133]. Viscosity and plasticity are two of the canonical

inelastic properties. Viscosity is responsible for time dependent deformation while plasticity is for non-invertible permanent deformation that occurs when an applied force goes beyond a threshold. Both viscosity and plasticity add to the simulations of inelasticity that moves the skin surface in smooth facial deformations. For generating immediate expressive wrinkles, the simulated skin surface deforms smoothly from muscle forces until the forces exceed the threshold; plasticity comes into play, reducing the restoring force caused by elasticity, and forming permanent wrinkles. The plasticity does not occur at all points simultaneously, rather it occurs at points that are most stressed by muscle contractions. By the repetition of this inelastic process over time, the permanent expressive wrinkles become increasingly salient on the facial model. The model is a simplified version of head anatomy, however, bones are ignored, and the muscle and fat layers are located according to the skin surface connections by simulated springs.

7.3. Other Wrinkle Approaches

Simpler inelastic models developed by Terzopoulos [121] compute only the visco-elastic property of the face. Spline segments model the bulges for the formation of wrinkles [123]. Moubaraki et al. [77] showed the animation of facial expressions using a time-varying homotopy⁴ based on the homotopy sweep technique [47]. In [77], emphasis was placed on the forehead and mouth motions accounting for the generation of wrinkles.

8. Vascular Expressions

Realistic face modeling and animation demand not only face deformation, but also skin color changes that depend on the emotional state of the person. Not much research is reported on this subject. The first notable computational model of vascular expression was reported by Kalra et al. [48] although simplistic approaches were conceived earlier [91].

Patel [91] added a skin tone effect to simulate the variation of the facial color by changing the color of all the polygons during strong emotion. Kalra et al. [48] developed a computational model of emotion that includes such visual characteristics as vascular effects and their pattern of change during the term of the emotions.

In [48], emotion is defined as a function of two parameters in time, one tied to the intensities of the muscular expressions and the other with the color variations due to vascular expressions. The elementary muscular actions in this system are based on the Minimum Perceptible Actions (MPAs) [50] similar to FACS [32] (see section 4 for details about FACS). The notion of Minimum Perceptible Color Action (MPCA) analogous to MPA is also introduced to change the color attributes due to blood circulation in the different parts of the face. Modeling the color effects directly from blood flow is complicated. Texture maps and pixel valuation offer a simpler means of approximating vascular effects. Pixel valuation computes the parameter change for each pixel inside the Bezier planar patch mask that defines the affected region of an MPCA in the texture image. This pixel parameter modifies the color attributes of the texture image. With this technique, pallor and blushing of the face are demonstrated [48].

9. Texture Manipulation

Synthetic facial images derive color from either shading or texturing. Shading computes a color value for each pixel from the surface properties and a lighting model. Because of the subtlety of human skin coloring, simple shading models do not generally produce adequate realism. Textures enable complex variations of surface properties at each pixel, thereby creating the appearance of surface detail that is absent in the surface geometry. Consequently, textures are widely used to achieve facial image realism.

Using multiple photographs, Pighin et al. [102] developed a photorealistic textured 3D facial model. Both view-dependent and view-independent texture maps exploit weight-maps to blend multiple textures. Weight maps are dependent on factors such as self-occlusion, smoothness, positional certainty, and view similarity.

⁴ Homotopy is the notion that forms the basis of algebraic topology. Readers should refer to [42] for more information on Homotopy theory.

A view-independent fusion of multiple textures often exhibits blurring from sampling and registration errors. In contrast, a view-dependent fusion dynamically adjusts the blending weights for the current view by rending the model repeatedly, each time with different texture maps. The drawback of view-dependent textures is their higher memory and computing requirements. In addition, the resulting images are more sensitive to lighting variation in the original texture photographs.

Oka et al. [81] demonstrates a real time texture mapping system for the synthesis of realistic facial expressions and their animation. When the geometry of the 3D objects or the viewpoint change, new texture mapping occurs for the optimal display. This mapping takes place in real time (30 times per second) due to the simplicity and the efficiency of the proposed algorithm. In the proposed algorithm, a mapping function from the texture plane into the output screen is approximated by a locally linear function on each of the small regions that form the texture plane altogether. Its only constraint is that the mapping function needs to be smooth enough. Realistic facial expressions and their animations are synthesized by interpolation and extrapolation among multiple 3D facial surfaces and the real time texture mapping onto them depending on the viewpoint and geometry changes.

10. Fitting and Model Construction

An important problem in facial animation is to model a specific person, *i.e.*, modeling the 3D geometry of an individual face. A range scanner, digitizer probe, or stereo disparity can measure three -dimensional coordinates. However, the models obtained by those processes are often poorly suited for facial animation. Information about the facial structures is missing; measurement noise produces distracting artifacts; and model vertices are poorly distributed. Also, many measurement methods produce incomplete models, lacking hair, ears, eyes, etc.

An approach to person-specific modeling is to painstakingly prepare a prototype or generic animation mesh with all the necessary structure and animation information. This generic model is fitted or deformed to a measured geometric mesh of a specific person to create a personalized animation model. The geometric fit also facilitates the transfer of texture if it is captured with the measured mesh. If the generic model has fewer polygons than the measured mesh, decimation is implicit in the fitting process.

Person-specific modeling and fitting processes use various approaches such as scattered data interpolations [102, 122], anthropometry techniques [27, 58], and projections onto the cylindrical coordinates incorporated with a positive Laplacian field function [60]. Some methods attempt an automated fitting process, but most require significant manual intervention. Figure 9 depicts the general fitting process.

10.1. Bilinear interpolation

Parke [89] uses bilinear interpolation to create various facial shapes. His assumption is that a large variety of faces can be represented from variations of a single topology. He creates ten different faces by changing the conformation parameters of a generic face model. Parke's parametric model is restricted to the ranges that the conformation parameters can provide, and tuning the parameters for a specific face is difficult.

10.2. Scattered data interpolation

Radial basis functions are mathematically proven to closely approximate smooth hyper-surfaces [187] such as human facial shapes. Some approaches morph a generic mesh into specific shapes with scattered data interpolation techniques based on radial basis functions. The advantages of this approach are as follows. First, the morph does not require equal numbers of nodes in the target meshes since missing points are interpolated [122]. Second, mathematical support ensures that a morphed mesh approaches the target mesh, if appropriate correspondences are selected [107, 187].

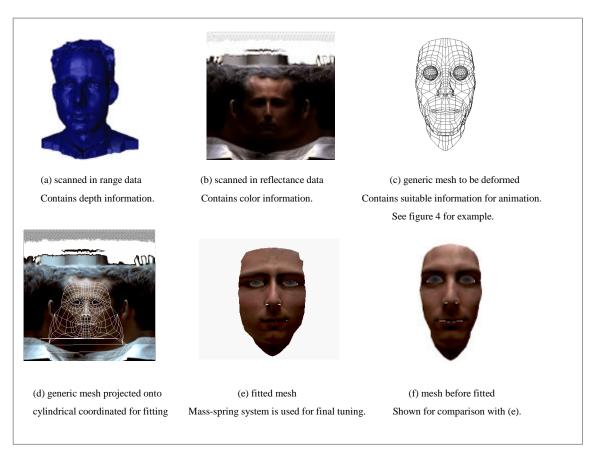


Fig. 9 - Example construction of a person specific model for animation from a generic model and laser scanned mesh.

Ulgen [122] uses 3D-volume morphing [38] to obtain a smooth transition from a generic facial model to a target model. First, biologically meaningful landmark points are selected (manually) around the eyes, nose, lips, and perimeters of both face models. Second, the landmark points define the coefficients of the Hardy multi-quadric radial basis function used to morph the volume. Finally, points in the generic mesh are interpolated using the coefficients computed from the landmark points. An example uses a generic face with 1251 polygons and a target face of 1157 polygons. Manually, 150 vertices are selected as landmark points, more than 50 around the nose. The success of the morphing depends strongly on the selection of the landmark points. Animation of the final fitted model is based on Platt's [104] muscle model facial animation system.

Pighin et al. also employed a scattered data interpolation technique for a three stage fitting process [103]. In the first stage, camera parameters (position, orientation, and focal length) are estimated while the 3D coordinates of feature points on the face are recovered from manual feature correspondences. In the second stage, the radial basis function coefficients are determined for the morph. In the third stage, additional correspondences facilitate fine-tuning. A sample generic mesh of under 400 polygons is morphed with 13 initially selected correspondence points, and 99 additional selected points for final tweaking.

10.3. Automatic correspondence points detection

Fitting intrinsically requires accurate corresponding features between two models. Incorrect correspondences result in poor facial fitting. Manual correspondence selection is tedious at best, and error prone as the number of features points increases. Several efforts at using the known properties of faces attempt to automate correspondence detection, and thereby automate the fitting process.

Yin et al. [136] acquires two views of a person's face and identifies several fiducial points defined on a generic facial model. In the profile and front views, the head is segmented from the background with a threshold operation, and the profile of the head is extracted with an edge detector. The vertical positions of the fiducial points are determined by analyzing the profile curve with local maximum curvature tracking (LMCT), described in [137]. The vertical positions of fiducial points limit the correspondence search area and the computation cost. The interior fiducial points are located by the relative positions of strong edges in each search area. Finally, the fiducial points extracted from the im ages are matched with predefined fiducial points in the generic mesh.

In Yin's work [136], the generic model is modified separately for the 2D front and profile views. A 3D individual model is obtained by combining the two modified 2D models. Fiducial points in the generic model are moved to the positions of the corresponding points in the modified images, defining displacement vectors. Positions of the non-fiducial points are derived by interpolations of the displacement vectors. To complete the model, two texture maps are blended based on the approximate orientation of localized facial regions. Animation of the complete individual model uses a Layered Force Spreading Method (LFSM). In LFSM, vertices are layered from a center point to the periphery . The center points are the fiducial points and the minimum number of polygon edges traversed to reach the center point defines the layer of periphery points. Spring forces propagate non-linearly from the center to the periphery layers based on the pre-assigned layer weights to generate facial expression animation.

Lee et al. [60, 61] also show the automatic construction of individual models of the heads using the laser-scanned range⁵ and reflectance⁶ data. To make facial features more evident for automatic detection, a modified Laplacian operator is first applied to the range map producing a Laplacian field map. Mesh adaptation procedures use on the Laplacian field map to automatically identify feature points. The generic model with all the facial features labeled prior to the adaptation step is now conformed to the 3D mesh geometry and texture. The heuristic mesh adaptation procedures are summarized below.

- 1) Locate nose tip (highest range data point).
- 2) Locate chin tip (the greatest value of the positive Laplacian of range below the nose).
- 3) Locate mouth contour (the greatest positive Laplacian between the nose and the chin).
- 4) Locate chin contour (similar to above).
- 5) Locate ears.
- 6) Locate eyes.
- 7) Activate spring forces.
- 8) Adapt hair mesh.
- 9) Adapt body mesh.
- 10) Store texture coordinates.

In the fitting process, contractile muscles are automatically inserted at anatomically plausible positions within a dynamic skin model and rooted to an estimated skull structure with a hinged jaw.

10.4. Anthropometry

In individual model acquisition, laser scanning and stereo images are widely used for their ability to generate detailed geometry and fine textures. However, as mentioned earlier, these methods also suffer from some drawbacks. Scanned data or stereo images often miss regions that must be interpolated using neighborhood information. Outlier and perimeter artifacts must be touched up by hand. A large set of scanned data or stereo photographs are needed to describe a broad sampling of face geometry. Existing

⁵ data with depth information

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⁶ data with color information

methods of automatically finding corresponding feature points are not robust, they still require manual adjustment if the facial feature are not salient in the measured data.

A generation of individual models using anthropometry ⁷ is proposed to solve these problems for cases where facial variations are desirable, but absolute appearance is not important. Kuo et al. [58] proposed a method to synthesize a lateral face from only one 2D gray-level image of a frontal face with no depth information. As a first step, a database is constructed, which contains facial parameters measured based on the anthropomorphic definitions. This database serves as a priori knowledge. Secondly, the lateral facial parameters are estimated from frontal facial parameters by using minimum mean square error (MMSE) estimation rules operated on the already constructed database. Specifically, the depth of one lateral facial parameter is determined by the linear combination of several frontal facial parameters. The 3D generic facial model is then adapted according to both the plane coordinates extracted from the 2D frontal image and their estimated depth. Finally, the lateral face is texture -mapped and synthesized.

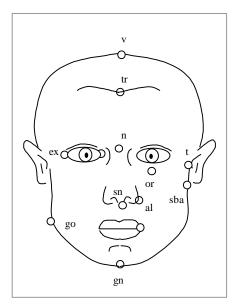


Fig. 10 - Some of the anthropometric landmarks on the face. The selected landmarks are widely used as measurements for describing the human face. (adapted from [39])

Whereas [58] used anthropometry with one frontal image, Decarlo et al. [26] went one step further and constructs various facial models purely based on anthropometry with no assistance from images. This system constructs a new face model in two steps. The first step generates a random set of measurements that characterize the face. The form and values of these measurements are computed according to face anthropometry (see figure 10). The second step constructs the best surface that satisfies the geometric constraints using a variational constrained optimization technique [41, 117, 130]. In this technique, one imposes a variety of constraints on the surface and then tries to create a smooth and fair surface while minimizing the deviation from a specified rest shape, subject to the constrains. In the case of [26], anthropometric measurements are the constraints, and the remainder of the face is determined by minimizing the deviation from the given surface objective function. Variational modeling enables the system to capture the shape similarities among various faces, while allowing anthropometric differences. Although the constructed models are not yet realistic, this technique may have potential for rapidly generating plausible facial variations.

⁷ the science dedicated to the measurements of the human face

10.5. Other Methods

Essa et al. [34] tackled the fitting problem using Modular Eigenspace methods⁸ [73, 98]. This method enables the automatic extraction of the positions of feature points such as the eyes, nose, and lips in the image. These features define the warping of a specific face image to match the generic face model. After warping, deformable nodes are extracted from the image for further refinement.

DiPaola's Facial Animation System (FAS) [28] is an extension of Parke's approach. New facial models are generated by digitizing live subjects or sculptures, or by manipulating existing models with free form deformations, stochastic noise deformations, or vertex editing. Akimoto, et al. [3] uses front and profile images of a subject to automatically create 3D facial models. Some other fitting techniques are described in [57, 116, 135].

11. Animation using Tracking

The difficulties in achieving life-like facial animations led to the performance driven approach where tracked human actors control the animation. Real time video processing allows interactive animations where the actors observe the animations they create with their motions and expressions. Accurate tracking of feature points or edges is important to maintain a life-like quality. Often the tracked 2D or 3D feature motions are filtered or transformed to generate the motion data needed for a specific animation system . Figure 11 shows an animation driven by a real time feature tracking system.



Fig. 11 - Real time tracking is performed without markups on the face using Eyematic Inc.'s face tracking system. Real time animation of the synthesized avatar is achieved based on the 11 tracked features.

11.1. Snakes – markups on the face

Snakes, or deformable curves, are widely used to track intentionally marked-up facial features. The recognition of facial features with snakes is primarily based on color sample identification of the highlighted features and edge detection. Many systems use snakes coupled with underlying muscles mechanisms to drive facial animation [51, 68, 118, 119, 120, 128]. Muscle contraction parameters are estimated by tracked facial feature displacements in video sequences.

⁸ Modular Eigenspace Methods are primarily used for the recognition and detection of rigid, roughly convex objects, i.e. faces. It is modular in that it allows the incorporation of important facial features such as eyes, nose, and mouth. It is an eigenspace method in that the matching is performed using the image eigenvectors. For a detailed description of this approach, refer to [73, 98].

Tracking errors often accumulate over long image sequences. Consequently, a snake is prone to losing the contour it is supposed to follow. In [83], data extracted from a previous frame is used only for the features that are relatively easy to track. A reliability test is performed whenever necessary and the snakes are reinitialized if error accumulations are detected. For real time performance (10 frames/sec on a SGI Indy), only a few features are tracked

11.2. Optical flow – no markups on the face

Colored markers painted on the face or lips [18, 56, 65, 76, 80, 92, 113, 131] are extensively used to ease the process of creating facial expressions or speech recognition from the video sequences. However, marking on the face is intrusive and repulsive to some people. Also relying on the marks restricts the scope of the geometric information to be retrieved only to the marked points. Optical flow [46] and spatio-temporal normalized correlation measurements ¹⁰ [25] can be utilized for the feature tracking, which obviate the cumbersome markings on the face [34, 37].

Essa et al. [34] coupled the pixel by pixel measurements of the surface motion to a physically based face model and a muscle control model. In doing so, optical flow technique [46] provided the detailed spatio-temporal records of both the displacement of each point on the facial surface and the muscle control required to generate the facial animation from the video sequences. It is an improvement over the other systems by removing the need for intrusive marking on the face, allowing more detailed geometric measurement to be recovered. Robust head tracking is achieved using ellipsoid model¹¹ in this system.

They identified the problems of using the most state-of-the-art facial description tool, FACS [32], as follows.

The action units (Aus) are purely local spatial patterns. Real facial motion is rarely completely localized.

There is no time component of the description, or only a heuristic one.

Performance driven approach using the video-based technique showed the superior results than the system using FACS in the comparisons shown in [34]. In terms of the spatial aspect, both models showed the very similar deformations in the primary activation regions. In the distant are as of the face, however, the video-based model showed far more deformations than FACS model. In the case of the temporal aspect, video-based technique showed co-articulation effects, which are rarely observed in the system using FACS. The limitations specific to this system was that it was unable to represent wrinkles and its animation was confined to limited set of facial motions.

Similar idea is found in Essa and his colleague's earlier work [37]. It also utilized the optical flow with a physically based optimal observation. The primary visual measurements of the system are set of peak normalized correlation scores using a set of previously trained 2D templates. The normalized correlation matching [25] process allows the user to move freely side-to-side and up-and-down, and minimizes the effects of illumination changes. The matching is also insensitive to small changes in viewing distance ($-15\% \sim +15\%$) and small head rotations (-15 degrees $\sim +15$ degrees). For speed up, matching process is made more efficient by limiting the search area to the neighborhood of where the eye and mouth, etc are last observed. When there is no exact match between the image and the existing template expression, interpolation based on a weighted combination of expressions is performed using the Radial Basis Function (RBF) method¹² [107] with the linear basis functions. The continuous time Kalman filter (CTKF) is

⁹ an approximated vector representation of the displacements of group of pixels from one frame to the next frame

¹⁰ Mean and variance (normalized correlation) of each pixel in the view (spatio) are computed for each frame in an image sequece (temoral).

¹¹ A head is approximated as an ellipsoid.

 $^{^{12}}$ a subset of neural networks by which a hyper-surface, i.e. human face, can be smoothly interpolated given a set of feature points.

incorporated to reduce the noise. 3D finite element mesh is adapted for facial model, onto which muscles are attached based on the work of Pieper [100] and Waters [128]. As an offline process, the muscle parameters associated with each facial expression is first determined using Finite Element Methods [7] (see section 12.3. for the brief description of FEM).

For motion estimation and facial expression analysis, Eisert and Girod [31] used the whole face image instead of discrete feature point correspondences when computing the gradients of the intensity. Since estimating 3D motion vectors from 2D images is a difficult task, the errors of the estimates tend to accumulate as frame sequences are processed. To minimize and eventually eliminate the error accumulation, a feedback loop is utilized hierarchically in the motion estimation process. First performed is an approximation for the motion parameters between two consecutive frames of the low resolutions. Due to the motion compensation, the differences between the newly created approximated frame and the current frame decreases. Then the procedure is repeated at higher resolutions, each time producing more accurate parameter sets. This iterative repetition is the result of the feedback loop and the moving to higher resolution constitutes the hierarchy. With this scheme, a large displacement vectors (30 pixels) between two successive video frames could be obtained. In addition, edge forces used in snakes was obviated to avoid an error accumulation.

11.3. Some other works

Kato et al. [52] exploited isodensity maps for the description and the synthesis of facial expressions. The isodensity map is constructed from the gray level histogram of the image based on the brightness of the region. The lightest gray level area is called level one isodensity line and the darkest called level eight isodensity line, which altogether represent 3D structure of the face. This method is proposed as an alternative of difficult feature tracking techniques.

Saji et al. [110] introduced the notion of Lighting Switch Photometry to extract 3D shapes from the moving face. The idea is to take the time sequence images illuminated in turn by separate light sources from the same view point. The normal vector at each point on the surface is computed by measuring the intensity of radiance. The 3D shape of the face at a particular instant is then determined from these normal vectors. Even if the human face moves, the detailed facial shapes such as the wrinkles on a face are extracted by Lighting Switch Photometry.

Azarbayejani et al. [5] used extended Kalman filter formulation to recover motion parameters of an object. Motion parameters are limited to only head position and orientation, though. Saulnier et al. [111] reported a template-based method for tracking and animation. Li et al. [63] used the Candid model for 3D-motion estimation for model based image coding. Masse et al. [72] used optical flow and principal direction analysis for automatic lip reading.

12. Mouth Animation

Among all the organs on the face, a mouth is the most complicated one in terms of the anatomical structure and its deformation behavior. Its inherent complexity made many researchers consider the modeling and animation of the mouth independent realm separate from those of complete face. Facial animation researchers started to concentrate only on the mouth and the lip animation to achieve better realism especially on the mouth region. Basic ideas and methods to tackle the problem associated with mouth region are not much different, if not the same, from those of complete facial animation with a little bit of variation. In this section, researche's specifically involved in the modeling and the animation of the mouth are introduced. They may be categorized as the approaches based on the muscle modeling using mass spring system, Finite Element Method, and parameterization, etc.

12.1. Muscle modeling using mass spring system

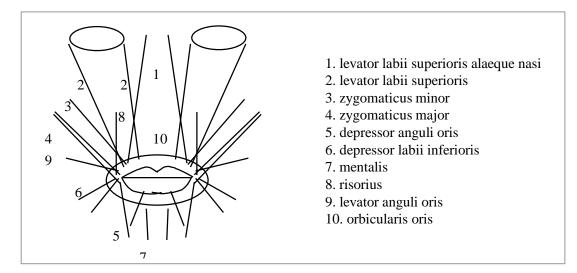
In mouth modeling and speech animation, mass-spring systems are often utilized to correctly model the phonetic structure of the speech animation [55]. Kelso et al. [55] qualitatively analyzed the real person's face in reiterant speech production and modeled it using a simple spring mass system. Browman et al. [17] showed the control of vocal-tract simulation with two mass spring systems. One spring controlled the lip aperture and the other the protrusion.

Exploiting the characteristics of simplicity and generality of mass-spring systems, Waters et al. [126] presented the physics based two dimensional muscle modeling and animation. Since mouth animation is generated through the muscle behavior rather than surface or skin behavior, orthogonality is obtained between the image quality and the number of model elements. Consequently, realism is achieved with fewer elements than a comparable number of surface elements, resulting in significant savings. Texture mapping is also utilized to improve the realism on top of fewer geometric elements.

Attempt to automatically synchronize computer generated faces with the synthetic speech was made by Waters et al. [127] when input text speech in the form of ASCII file is fed into the system. Two di fferent approaches are used and analyzed as proposed mouth animation methods. First, each viseme ¹³ mouth node is defined with positions in the topology of the mouth. Intermediate position of each node between consecutive viseme is interpolated using a cosine function rather than a linear function to give the effect of acceleration and deceleration at the start and end of the each viseme animation. However, during fluent speech, mouth shape rarely converges to discrete viseme targets due to the short interval between positions and the physical properties of the mouth. To emulate fluent speech, the calculation of the co-articulated visemes is needed. Hence, second method exploits Newtonian physics, Hookean elastic force, and velocity dependent damping coefficient to construct the dynamic equation of the nodal displacements. The dynamic system adapts itself as the rate of speech increases, thus reducing the lip displacement as it tries to accommodate the new position. This behavior is characteristics of real lip motion. The real time (15 frames / sec) animation was achieved using two dimensional wire frame of 200 polygons representing only the frontal view on DEC Alpha AXP 3000/500 workstation (150MHz).

12.2. Muscle represented as layered spring mesh

Another muscle modeling approach was proposed to produce realistic mouth animation by Sera et al. [113]. Main contribution of this work is that the mouth shape control mechanism is added to existing method [60] in order to produce speech animation. Facial skin is represented as three-layered spring mesh with appropriately chosen elasticity coefficients following the idea of [60]. Muscle contraction value for each phoneme is then determined by comparison of the position of the makers on the photos and the model (see figure 11 for muscle placements around the mouth). For the animation of the speech, an intermediate mouth shape is defined by a linear interpolation of the basic mouth shape's muscle parameters. Linear interpolation is performed based on the spring muscle force instead of the each vertex position in order to achieve more realism. However, high computation cost keeps this system from working in real time.



 $^{^{\}rm 13}$ a group of phonemes with similar mouth shapes when pronounced.

Fig. 11 - Muscle placements around the mouth. Although 5, 6, and 7 are attached to the mouth radially in reality, they are modeled linearly here.

12.3. Modeling using Finite Element Method

The finite element method (FEM) is a numerical method used to approximate the physics of an arbitrary complex body [7]. It implicitly defines interpolation functions so that the description of the physical property of the material, typically a stress-strain relationship, between node is possible. To achieve the stress-strain relationship, an arbitrary complex body is decomposed into piecewise elements with simple physics and the overall stress-strain relationship is found by integrating the piecewise components over the entire body.

Basu et al. [6] built the Finite Element Method (FEM) 3D model of the lips. Constructed model is trained to cover only the subspace of lip motions in such a way that given the training data set, the solution minimizes the strain felt throughout the linear elastic FEM structure. The goal was to extend the idea of [15, 66] to 3D. 2D models in [15, 66] suffered from the fact that the changes in lip shapes resulting from rigid rotations had to be modeled as complicated changes of the lip shape. By modeling the true three-dimensional nature of the lips, variations that looked complex and nonlinear from a 2D perspective became simple and linear [6]. Using FEM alleviated the difficult control problem occurring at muscle-based approaches [36, 126] because the muscle contraction parameters are automatically approximated in training stage as well as the accuracy problem resulting from using only key-frames for mouth animation [126]. The use of normalized or chromatic color information (r' = r / (r + g + b)) made the system robust against wide variations in lighting conditions in tracking of the subject with markups.

12.4. Parameterization

Parametric techniques for facial animation, or more specifically, mouth animation usually require a significant number of input parameters for a synthetic mouth model to be faithfully controlled. However, manipulating a large number of parameters is an unwieldy task. Mouth animation using only two parameters was proposed by Moubaraki et al. [75] for easy manipulation of parameters while maintaining visual realism. Main idea is to compute the opening angle at the corner of the mouth as well as the protrusion coefficients using a radial basis function given two parameters — width and height of the mouth. Lip shape is then obtained using a piecewise spline interpolation. For each of the scanned facial expression, opening angle at the lip corner and the z -component for protrusion are computed and are associated with measured height and widths of the mouth. This set of association is fed into the radial basis neural network for training. Detected feature points from the video sequences are input to the trained network to perform lip shape computation and protrusion, and then animate the model. Teeth are also modeled using two texture-mapped portions of a cylinder.

12.5. Tongue modeling

In most facial animation system, tongue movement is often ignored, or oversimplified. In most c ases, it is modeled by a simple parallelepiped [21, 62, 71, 86]. Although only a small portion of the tongue is visible during normal speech, considering the tongue shape will greatly enhance the visual quality of synthesized mouth animation system.

Stone [115] proposed a 3-D model of the tongue by defining five segments in the coronal plane and five segments in the sagittal plane ¹⁴. This tongue model was capable of being bent, twisted, asymmetrically shaped, and groove shaped. This relatively accurate model of the tongue is carefully simplified by Pelachaud et al. [94] to be used for normal speech animation.

Pelachaud et al. [94] proposed to model the tongue based on the soft object technique [134]. This technique assumes so-called skeleton, comprising of few geometric primitives (9 triangles) that serves as a charge

¹⁴ In anatomy, coronal plane divides the body into front and back halves while sagittal plane cuts through the center of the body dividing the body into right and left halves.

distribution causing a spatial potential field. Modifying the skeleton will modify the equi-potential surface, the soft object, and hence shape of the tongue. The palate is modeled as a semi-sphere and the upper teeth are simulated by a finite strip—shaped plane. Collision detection mechanism is also proposed using implicit function and the shape of the tongue changes based on the volume preservation. Equi-potential surfaces are expensive to render directly. Hence, they are converted into the triangular mesh in adaptive way when they are rendered. Here adaptive way means that the triangle shapes adapt to the local curvature of the equi-potential surface in such a way that flat area is represented by large triangles while curved area by small triangles. In addition, isotropically curved surface area is represented by equilateral triangles and anisotropically curved surface area by acute triangles [82]. Lip animation as well as tongue animation was performed based on FACS [32] taking co-articulation ¹⁵ into account.

12.6. Some other works

Modeling of the lips based on the algebraic functions was proposed [45]. The lip model was developed by adjusting a set of continuous functions best fitting the contours of 22 reference lip shapes. To predict all the algebraic equations of the contours of the model of the various lip shapes, five parameters are used, which are measured from the real speaker in the video sequences. The model is able to compute the cont act forces during the interaction between the lips and the world by virtue of volumetric model of the lips created by a implicit surface technique. High resolution of the realistic lip model was successfully achieved by this method.

As an image based approach, Duchnowski et al. [30] fed the raw pixel intensities into a neural net to classify lip shapes, thus lip reading. Adjoudani et al. [2] associated a small set of observed mouth shape parameters with the polygonal lip mesh. Petajan [99] exploited several image features in order to parameterize the lip shape. Some other simulations of the process of synthesized speaking and modeling of lip shapes can also be found in [1, 11, 16, 22, 23, 64, 74, 93, 96, 109].

Conclusion

In this paper, we have introduced the issues associated with facial modeling and animation, presented the variety of approaches to tackle the problems, and classified them into categories based on their approaches. Two big realms in creating facial modeling and animation are geometry man ipulations and image manipulations. They are often coupled together to achieve the realistic synthesized facial animations.

Generation of facial modeling and animation can be summarized as follows. First, an individual specific model is obtained using a laser scanner or stereo images and fitted into the prearranged prototype mesh by scattered data interpolation technique or by some others as discussed in section 8. Second, the constructed individual facial model is deformed to produce facial expression based on (simulated) muscles mechanism, Finite Element Method, or 2D & 3D morphing technique etc. Wrinkles and vascular effects are also considered for added realism. Third, the complete facial animation is performed by Facial Action Coding System or by tracking human actor in the video footage.

Although not mentioned in this paper, there have been approaches in facial animation and expression synthesis in the context of neural networks [53], using genetic algorithms [97] and many more. The goal of the research related to the synthesis of the face, achieving realism in real time in automated way, has not been reached yet. However, the success in each realm is recently reported.

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¹⁵ Rapid sequences of speech require that the posture for one phoneme anticipate the posture for the next phonemes. Conversely, the posture for the current phoneme is modified by the previous phonemes. This overlap between phonetic segments is referred to as co-articulation [54].

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