BIOM300 Final Project

Future of Human in a Zombie Outbreak

Group 12

Final Draft

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Introduction

This report provides a brief overview of the modeling of a zombie attack, which has captured the popular imagination in recent years through its prominent depiction in novels, movies, and video games. While zombies do not exist in reality, it is worth exploring their hypothetical outbreak scenarios. In this report, we aim to develop a model that simulates the spread of zombie infection, then generate potential solutions to help people gain insights into the dynamics of infectious diseases and the effectiveness of various response strategies. The report also provides a brief outline to accomplish these objectives, demonstrating the simulations with different states and concluding with possible results.

Background

According to popular belief, an unknown source of infection caused the first zombie when a fraudulent corpse emerged from its coffin and rapidly infected the humans involved in its burial and some bodies in the cemetery. However, the infected people did not begin to turn into zombies until they killed the first zombie.

Description of Biological System

In this model, humans have a constant birth rate and death rate. When a zombie outbreak happens, zombies will become insane and aggressive, and bite will bite anyone at will. Humans who are attacked by zombies will turn into new zombies in a short time and attack more humans afterward. Moreover, dead humans could resurrect and become zombies. However, humans can kill zombies to avoid being infected and survive. At last, we assume that:

- Only humans will be infected by zombies' biting.
- Zombies only attack humans who have not been infected before.
- Killed zombies cannot resurrect forever and they will be permanently removed.
- There is no treatment. Infected people will either become zombies or die naturally.

The model was derived based on assumptions about the virus's behaviour and population interactions. The model equations were developed via mathematical tools such as differential equations to describe the changes in each population over time.

Derivation of the Model

There are five state variables:

H—healthy humans

I—infected humans

Z—zombies

R—removed humans

R^*—removed zombies

Six important parameters:

α, the rate at which zombies are killed;

β, the probability of humans being bitten by zombies;

γ, the rate at which dead humans become zombies;

δ, the natural death rate(non-zombie related) of humans;

π, the birth rate of humans;

ρ, the period that an infected person needs to become a real zombie.

$$\frac{dH}{dt} = \pi - \beta HZ - \delta H$$

This equation describes the rate of change of healthy humans, which is determined by π, β, and δ. As more humans become infected (HZ), the rate of change in healthy humans decreases.

$$\frac{dI}{dt} = \beta HZ - (\rho + \delta)I$$

This equation describes the rate of change of infected humans, determined by β, ρ, and δ. As more humans become infected (HZ), the rate of change of infected humans increases.

$$\frac{dZ}{dt} = \rho I + \gamma R - \alpha HZ$$

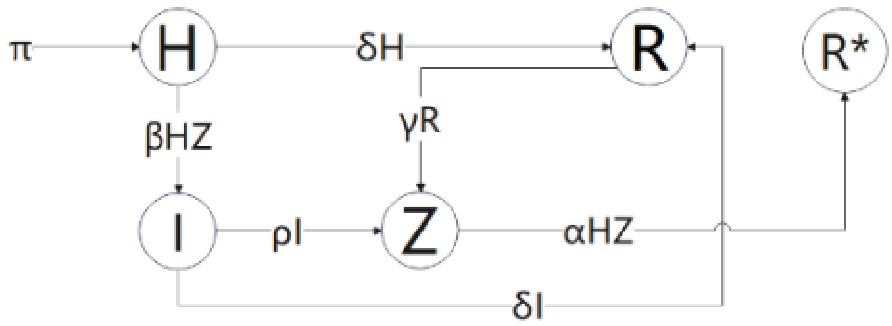
This equation describes the rate of change of zombies, ρ, γ, and α. As more humans become infected (βHZ), the rate of change of zombies increases.

$$\frac{dR}{dt} = \delta(H + I) - rR$$

This equation describes the rate of change of removed individuals, which is determined by α, γ, δ, and the number of healthy and infected humans ($H + I$).

$$\frac{dR^*}{dt} = \alpha HZ$$

This equation describes the rate of change of removed zombies, only determined by α and interaction between humans and zombies (HZ). Below is the flow diagram of our ecosystem.



Simulations of the Model

We stimulate the model by choosing the following initial conditions and parameter values: Initially, there are 500 healthy humans, 1 infected human, 1 zombie, 100 removed humans, and 0 killed zombies. The birth and death rate of humans, which have a dispensable influence on the population on a huge scale of a zombie outbreak, could be ignored. That's why we set both of them as 0 in our simulation.
The rate of humans eliminating zombies (α) is 0.001
The probability of a human being bitten by a zombie (β) is 0.005
The rate of dead zombies being replaced by new zombies from dead humans (γ) is 0.002
The natural death rate of humans (δ) is 0
The birth rate of humans (π) is 0
The rate of infected humans turning into zombies (ρ) is 1.0

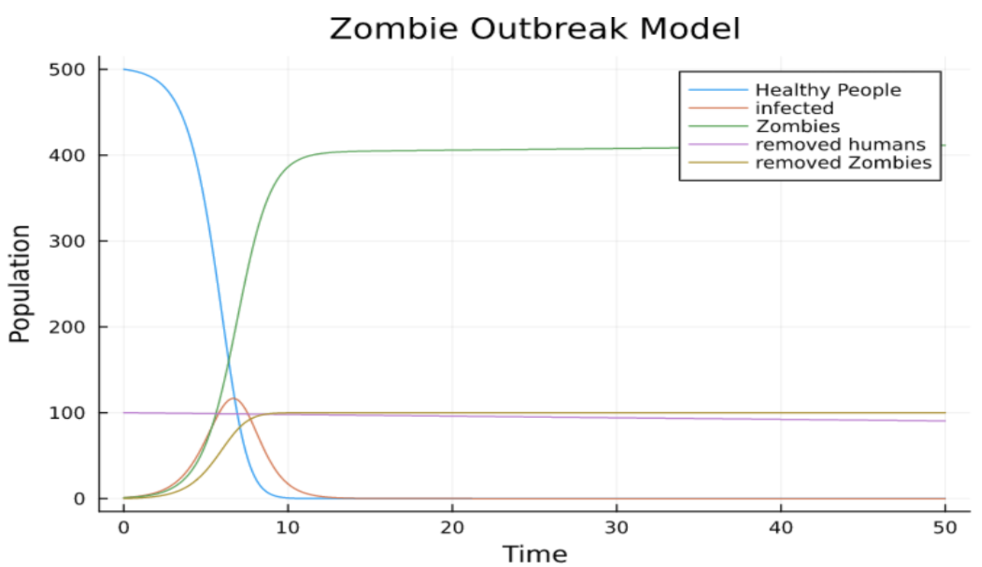


Figure 1: The first model is a classic case in which zombies defeat humans in a short period. However, the number of zombies is still increasing because only a few dead humans resurrect to zombies at this time.

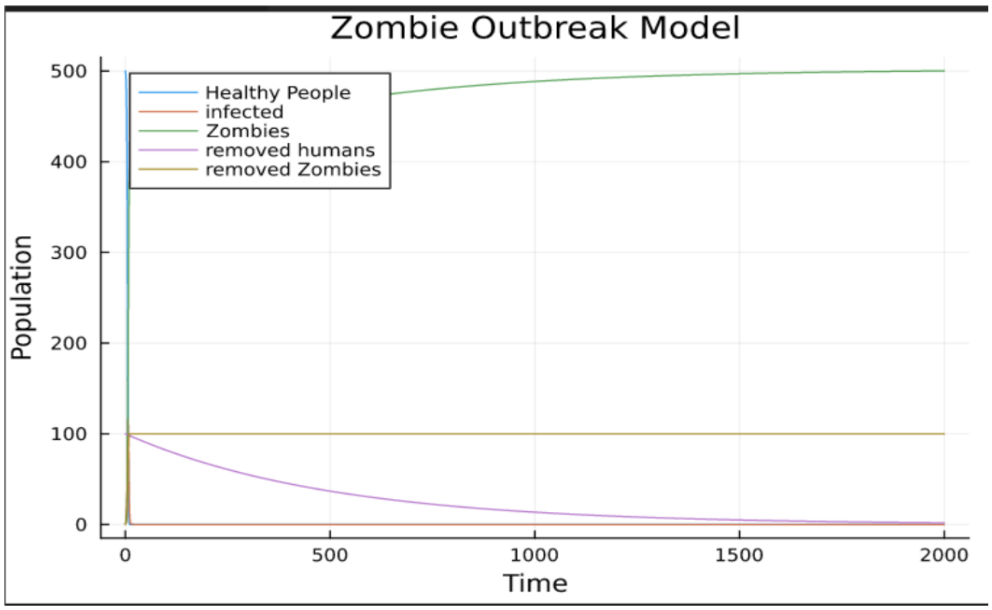


Figure 2: We extend the length of the time axis to 2000 days to wait all removed humans resurrect. It's clear that almost all dead humans resurrect and the number of zombies is stable at this moment.

Don't be too pessimistic about the future of humanity. If survivors acquire plentiful firearms and protective gear, their ability to hunt and neutralize zombies will immensely improve. Since other nonzero parameters are hard to control by humans, by determining the killing rate required to overcome the zombie threat, we can better prepare for and increase our chances of survival.

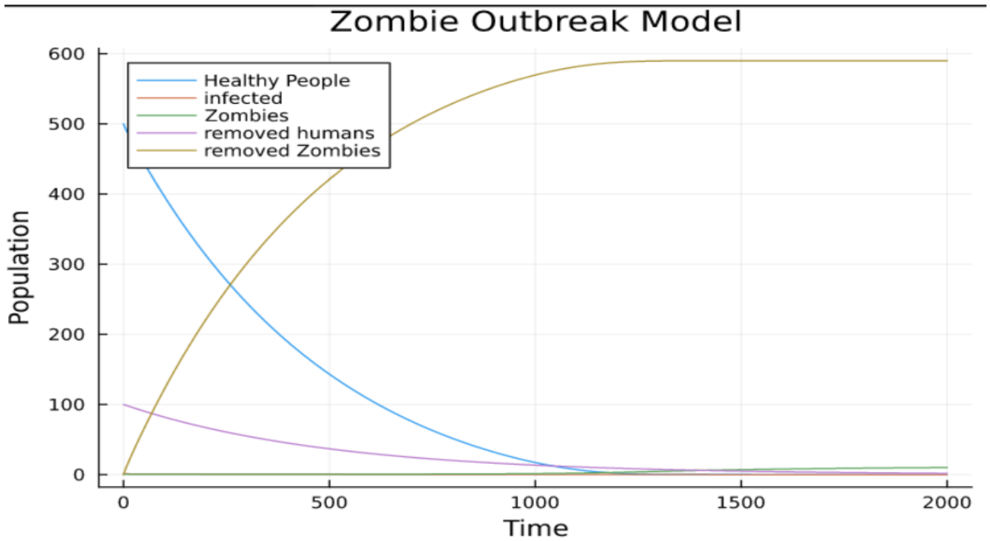


Figure 3: This plot clearly shows that although zombies win. They almost become extinct due to the more fierce insistence of humans. We keep all parameters the same as before but α, which increases to 0.0059.

We find that the sum of all groups is always constant to the sum of initial values. It could express in formulas: $N(0) = H(0) + Z(0) + I(0) + R(0) + R^*(0) = H(t) + Z(t) + I(t) + R(t) + R^*(t)$ and $H' + Z' + I' + R' + R^{*'} = N' = 0$, which N is the total population and bounded all the time, equal to 602. If we consider the sum of all the differential equations with birth and death rate both not 0, the sum will equal to π, which also proves the system is bounded.

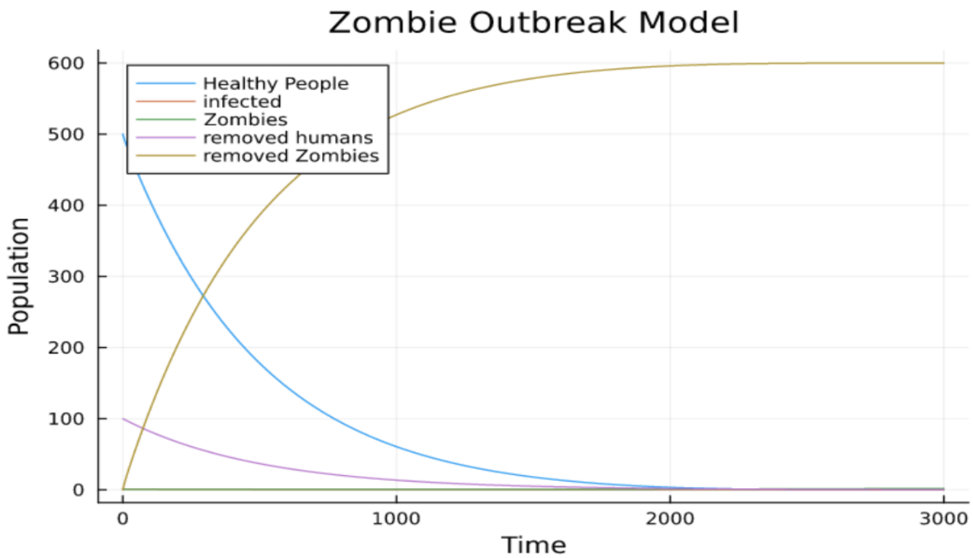


Figure 4: As you see above, both humans and zombies almost perish together at last when α is 0.006, which means if survivors could improve their efficiency more than five times, they will save humans' future.

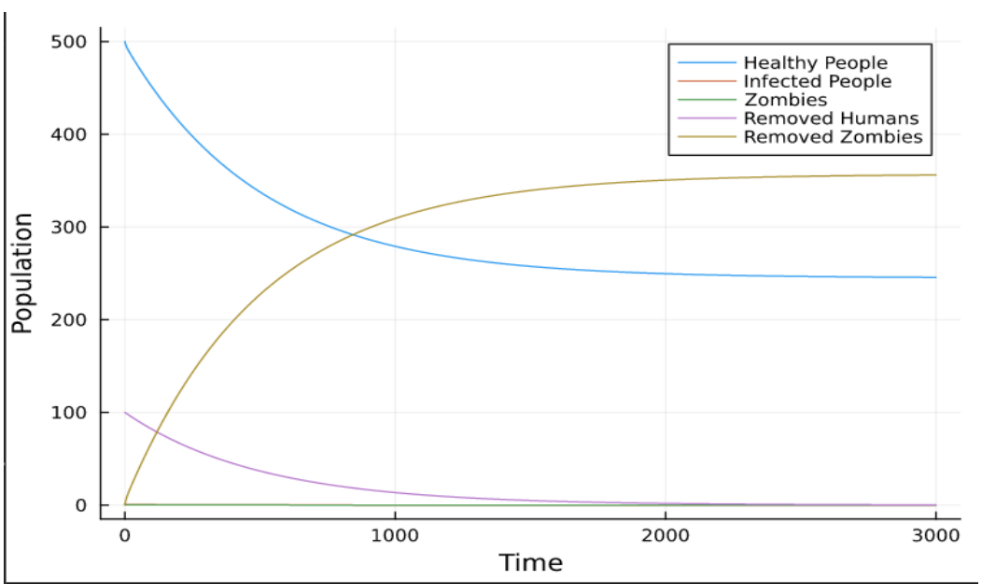


Figure 5: Humans win when α is 0.007. Excellent armed forces kill all zombies at birth.

Qualitative Description

From the simulations, we can observe the following behaviours of the model:

In Figures 1 and 2, from the beginning, the number of infected individuals increases rapidly due to the presence of a single infected individual and the high probability of being bitten by a zombie. As the number of infected individuals increases, the number of zombies also increases, decreasing the number of healthy individuals. After about seven days, lines of zombies and healthy people intersect, with infected people nearly reaching the maximum simultaneously. From the mathematical aspect, the rate of zombies and human change becomes slower after the intersection, which indicates that the intersection or point nearby is the second derivative zero point. Also, the first derivative of infected people achieves 0 a few seconds later, which makes sense since the rate of zombie increase reaches the maximum. On the ninth day, humans are extinct with a stable line equal to 0. Simultaneously, removed zombies become stable at 100 since zombie killers are gone. On about the tenth day, only a small number of struggling infected people existed. On the twelfth day, zombies finally master the world and are still enlarging from removed humans until the 2000th day. After that, the number of zombies will keep constant.

In Figure 3, the same scenario happens as in Figures 1 and 2. However, since the efficiency of killing zombies improves rapidly, the intersection comes much later than before, which means survivors persist much longer, although they lose at last. On the 2300th day, humans lose this battle, and the removed zombies stop increasing. As time passes, the number of zombies stabilizes and nearly approaches 0.

In Figure 4, at the end of the simulation, the number of removed zombies stabilizes, and the number of zombies and humans approaches zero.

In Figure 5, although many humans are infected, they have developed an advanced armed force that could kill zombies at birth, which is why the line of zombies keeps zero. On the 2000th day, humans and removed zombies become stable since there are no potential zombie threats anymore.

This simulation indicates the results of an outbreak depending on different levels of humans' insisting strength and hunting efficiency, which shows the importance of armed force in this kind of doom. This simulation could also solve the question asked in the proposal.

Analysis of the Model

First of all, we find the equilibrium for the differentiation equations. Let us call this the HIZR model. The model equations are:

$$H' = \pi - \beta HZ - \delta H$$

$$I' = \beta HZ - (\rho + \delta)I$$
$$Z' = \rho I + \gamma R - \alpha HZ$$
$$R' = \delta(H + I) - rR$$
$$R^{*\prime} = \alpha HZ$$

We set the equations above to zero and ignore the birth rate and death rate.

$$0 = -\beta HZ$$

$$0 = \beta HZ - \rho I$$
$$0 = \rho I + \gamma R - \alpha HZ$$
$$0 = -rR$$
$$0 = \alpha HZ$$

We could get either $H = 0$ or $Z = 0$ from the first equation. Hence, I in the second equation is 0 since ρ is always positive. Next, either $I = 0$ or $R = 0$ or $\rho I = -\gamma R$. However, both sides of the equation should have the same sign in our setup, so the latter situation is impossible. From the last two equations, it is clear that all state variables are 0 except H or Z, which are two equilibrium points in this model.

$$J = \begin{bmatrix} -\beta Z & 0 & -\beta H & 0 & 0 \\ \beta Z & -\rho & \beta H & 0 & 0 \\ -\alpha Z & \rho & -\alpha H & \gamma & 0 \\ 0 & 0 & 0 & -\gamma & 0 \\ \alpha Z & 0 & \alpha H & 0 & 0 \end{bmatrix}$$

In this case, we calculate the Jacobian Matrix. Then we discuss the stability of $(N, 0, 0, 0, R^*)$. The characteristic equation at this equilibrium point is $-\lambda(\beta N\lambda\rho(-\lambda-\gamma)-\lambda(-\lambda-\rho)(-\lambda-\gamma)(-\lambda-N\alpha))$. Since $\beta N\rho$ is larger than 0, an eigenvalue with positive real part exists. By Hadamard-Perron Theorem, this point is unstable, which means this result is not likely to happen.

Now we will determine the second equilibrium point. For the equilibrium point, which represents zombie wins, we find that the determinant for this case is $\lambda^2(-\lambda-\gamma)(-\beta\bar{Z}-\lambda)$, and $\lambda = 0,\ -\beta,\ -\gamma,\ -\beta\bar{Z}$.

This result cannot tell us the stability since the eigenvalue could be 0, so the point is not hyperbolic. However, if we ignore the first differential equation and set $H = N - I - R - R * -Z$, we could get four new equations and Jacobian Matrix:

$$I' = \beta Z\left(N - I - R - R^* - Z\right) - \left(\rho + \delta\right) I$$
$$Z' = \rho I + \gamma R - \alpha Z\left(N - I - R - R^* - Z\right)$$
$$R' = \delta\left(N - R - R^* - Z\right) - \gamma R$$
$$R^{*\prime} = \alpha\left(N - I - R - R^* - Z\right) Z$$

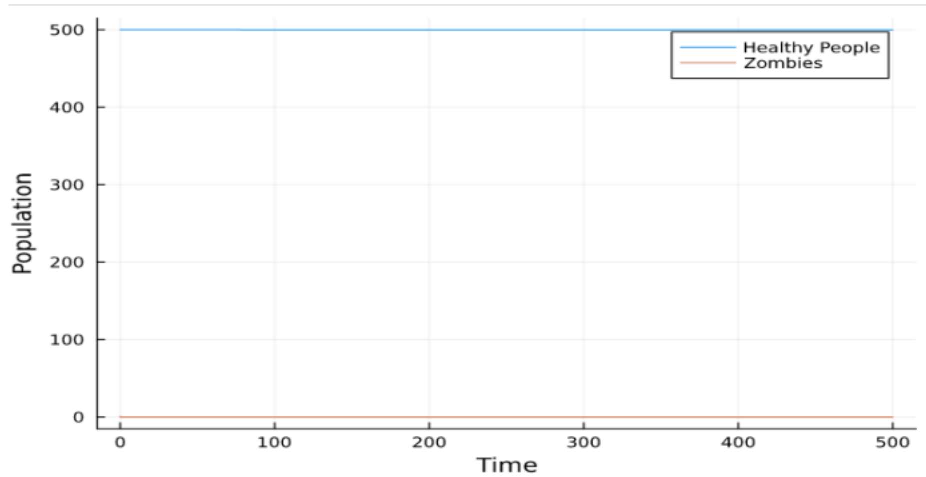


Figure 6: Humans win when α is 0.007. Excellent armed forces kill all zombies at birth.

$$J = \begin{bmatrix} -\beta Z - \rho & \beta N - 2\beta Z & -\beta Z & -\beta Z \\ \alpha Z + \rho & 2\alpha Z - \alpha N & \alpha Z + \gamma & \alpha Z \\ 0 & 0 & -\gamma & -\gamma \\ -\alpha Z & \alpha N - a\alpha Z & -\alpha Z & -\alpha Z \end{bmatrix}$$

The determinant above gives us four eigenvalues $\lambda1 = -2\alpha Z + \alpha N + \beta N - 2\beta Z - \gamma - \rho$, $\lambda2 = -\alpha Z - \beta Z - \rho$, $\lambda3 = -\alpha Z - \beta Z - \rho$, $\lambda4 = -\alpha Z - \beta Z - \gamma$. Since $2Z$ is larger than N because zombie wins when the other variables are 0, all eigenvalues are negative. Hence, the equilibrium $(0, 0, Z, 0, R^*)$ of zombie-win is stable.

For the third equilibrium, in which all state variables are 0 except R^* (R^* could be any non-positive value), we use the same method as the first equilibrium point. The characteristic equation at this equilibrium point is
$-\lambda^3(-\lambda - \rho)(-\lambda - \gamma)$, and the eigenvalues are $\lambda = 0,\ -\rho,\ -\gamma$.

We see that the eigenvalues contain 0, which means we cannot determine the stability from the Jacobian Matrix of this point. In reality, from the description of our model, the situation where humans and zombies perish together never happens because the last zombie will always convert the last human to a zombie, or the last human kills the last zombie and becomes the only one who survives the outbreak.

In conclusion, we talked about three equilibrium points here: $(N, 0, 0, 0, R^*)$, $(0, 0, Z, 0, R^*)$ and $(0, 0, 0, 0, R^*)$

Discussion

Unless humans employ highly aggressive tactics against the undead, a zombie outbreak infecting humans can be disastrous. By analyzing and simulating the model, we can gain insights into the factors that influence the spread of the disease.

Simulations of the model revealed that the disease could exhibit different patterns of spread depending on the initial conditions and the values of the parameters. In some cases, zombies may spread rapidly and infect a large portion of the population before eventually declining due to the depletion of susceptible individuals. In other cases, the disease may exhibit a slower and more sustained spread, with a significant portion of the population remaining infected for an extended period.

Our zombie outbreak model shows that as time passes, the populations of zombies and humans are bounded while removing the infected from the system. The equilibrium point of the model reveals that the people of humans, zombies, and susceptible individuals remain constant, indicating a steady state where the rate of new infections equals the rate of removal of infected individuals from the system. However, external factors such as the availability of weapons, medical supplies, and the behavior of individuals involved can significantly affect the infection rate and the overall outcome of the outbreak. Therefore, it is important to consider external factors that could influence the spread of a zombie outbreak, like mutation and strong weapons, while the total population is proven to be bounded. Besides, our model contained too many variables, so using differential tools like phase diagram is difficult to perform due to a large amount of computing. This is also why we set humans' birth and death rates to zero – to reduce computing.

In summary, our analysis and simulation of the zombie outbreak model showed that unless taking aggressive tactics(e.g., heavy weapons), a zombie outbreak can have disastrous consequences for humans. The model revealed that zombies tend to win in most situations, and the equilibrium point indicates a steady state where the rate of new infections equals the rate of removal of infected individuals. However, external factors such as the availability of weapons, medical supplies, and individual behavior can significantly impact the infection rate and overall outcome. Therefore, it is vital to consider these factors and take appropriate action to mitigate the spread of a zombie outbreak.

Response Letter

In peer review, we received many reasonable and useful suggestions from another group. We modified our contents based on many valuable suggestions like rationalizing and enriching the description of the resurrection of dead humans into zombies, formatting the state variables and parameters into a list to facilitate the reading, and adding thoughts of improvements and shortcomings of the model in the discussion.

Moreover, for mathematical errors and unclarities in our analysis, we changed our pictures into typed equations which facilitate reading. We also replaced wrong equilibrium notations with correct ones and listed them together.

Finally, we enlarged the figures' size and rectified the caption to make our observations easy to read. With the help of grammar correction tools, we improve our correctness of grammar and spelling. However, there are still some minor grammar errors since lack of time, a heavy workload, and English is not our first language.

We referenced almost all suggestions from this group in the revision process, and in reality, they improved the quality of our draft tremendously from aspects of mathematics, logic, and typography. For example, adding a background of the resurrection process eliminates the confusion of zombie conversion and specifies the relationship among variables. Also, modifying the minor errors in analysis helps students judge the model without confusion from a mathematical aspect. At last, the supplementary information on shortcomings and improvements implements our considerations for constructing a more realistic model.

In the end, we abandoned a few suggestions in peer review feedback. For instance, we have explained the reason for setting both natural birth and death rate as 0. They ignore these pieces of words when reading our draft. Also, Kate advised us to elaborate on whether humans should be optimistic or pessimistic about the future, which is a good idea. However, as she said, we need more time and professional epidemiological background to improve our model.