Lecture 8 Logistic Regression II

ECE 625: Data Analysis and Knowledge Discovery

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Outline

Logistic Regression with indicator variable

- We can predict if an individual default by checking if she is a student or not. Thus we can use a qualitative variable `Student` coded as (Student $= 1$, Non-student $= 0$).

```
> glm.fit=glm(default~student,data=defaultData,family=binomial)
> summary(glm.fit)
Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         -3.50413    0.07071  -49.55  < 2e-16 ***
factor(student)Yes   0.40489    0.11502    3.52 0.000431 ***
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1
```

- β_1 is positive. This indicates students tend to have higher default probabilities than non-students.

```
> predict(glm.fit, list(student = c('Yes','No')),type="response")
         1          2
0.04313859 0.02919501
```

Multiple logistic Regression

- Logistic Regression with several covariates

$$\text{P(Y=1|X)} = p(X) = \frac{e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}{1+e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}.$$

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \boxed{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p.}$$

a linear model for the logit

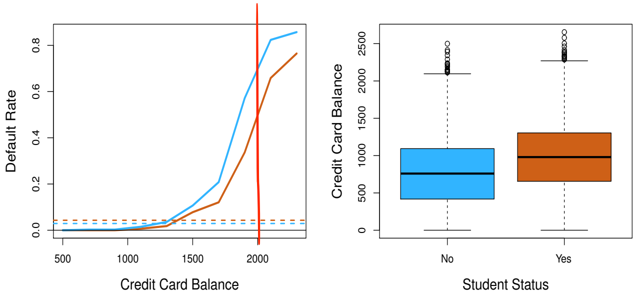
Generalized linear model

Use a linear model to predict the transform of the response

```
> glm.fit=glm(default~balance+income+student,data=defaultData,fam
> summary(glm.fit)
Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.087e+01  4.923e-01 -22.080  < 2e-16 ***
balance      5.737e-03  2.319e-04  24.738  < 2e-16 ***
income       3.033e-06  8.203e-06   0.370  0.71152
studentYes  -6.468e-01  2.363e-01  -2.738  0.00619 **
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1
```

- Why is coefficient for `student` negative, while it was positive before?

Confounding



- Students tend to have higher balances than non-students, so their marginal default rate is higher than for non-students.
- But for each level of balance, students default less than non-students.
- *Confounding*: the results obtained using one predictor may be quite different from those obtained using multiple predictors, especially when there is correlation among the predictors.

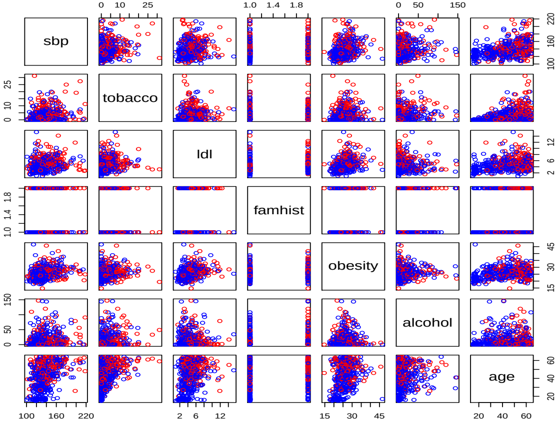
South African Heart Disease

- 160 cases of MI (myocardial infarction) and 302 controls (all male in age range $15-64$), from Western Cape, South Africa in early 80s.
- Overall prevalence very high in this region: 5.1%.
- Measurements on seven predictors (risk factors), shown in scatterplot matrix.
- Goal is to identify relative strengths and directions of risk factors.
- This was part of an intervention study aimed at educating the public on healthier diets.

Pair Plots

pairs(heartData1[,-8],col=cols[heartData1[,8]+1])
1 or 2 -> red or blue

8th column
is response



Scatterplot matrix of the South African Heart Disease data. The response is color coded. The cases (MI) are red, the controls blue. `famhist` is a binary variable, with 1 indicating family history of MI.

South African Heart Disease

Generalized linear model   if binomial, it means LR

```
> glm.fit=glm(chd~.,data=heartData1,family=binomial)
>
> summary(glm.fit)

Call:
glm(formula = chd ~ ., family = binomial, data = heartData1)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.7517  -0.8378  -0.4552   0.9292   2.4434

Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)   -4.1295997  0.9641558  -4.283 1.84e-05 ***
sbp            0.0057607  0.0056326   1.023  0.30643
tobacco        0.0795256  0.0262150   3.034  0.00242 **
ldl            0.1847793  0.0574115   3.219  0.00129 **
famhistPresent 0.9391855  0.2248691   4.177 2.96e-05 ***
obesity       -0.0345434  0.0291053  -1.187  0.23529
alcohol        0.0006065  0.0044550   0.136  0.89171
age            0.0425412  0.0101749   4.181 2.90e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 596.11  on 461  degrees of freedom
Residual deviance: 483.17  on 454  degrees of freedom
AIC: 499.17
```

sbp: not useful
obesity: not useful
alcohol: not useful

the generalizability of the model

Logistic regression with more than two classes

P(Y=1|X=x) = p(x) = e(beta x)/(1+e(beta x)) = 1/(1+ e(-beta x))

- So far we have discussed logistic regression with two classes. It can easily be generalized to more than two classes.
- One version (used in the `R` package `glmnet` or `nnet`) has the symmetric form.

beta is now a matrix

sum_k Pr (Y=k|X) = 1

$$\Pr(Y=k|X) = \frac{e^{\beta_{0k}+\beta_{1k}X_1+\cdots+\beta_{pk}X_p}}{\sum_{k=1}^{K}\left(e^{\beta_{0k}+\beta_{1k}X_1+\cdots+\beta_{pk}X_p}\right)}.$$

softmax

- There is a linear function for each class.
- Only $K-1$ linear functions are needed. Usually set $e^{\beta_{0K}+\beta_{1K}X_1+\cdots+\beta_{pK}X_p} = 1$.
- Multi-class logistic regression is also referred to as multinomial regression.

this becomes the simplest neural network

losing the interpretability

Simulated Example

```
> library(nnet)
> x=matrix(rnorm(100*5),100,5)
>
> y=rnorm(100)
> #multinomial
> g4=sample(1:4,100,replace=TRUE)
> fit3=multinom(g4~x)
# weights:  28 (18 variable)
initial  value 138.629436
iter  10 value 130.910132
iter  20 value 130.869074
final  value 130.868827
converged
> summary(fit3)
Call:
multinom(formula = g4 ~ x)

Coefficients:
  (Intercept)        x1          x2         x3        x4        x5
2 -0.09206107 0.7771141 -0.07521353 0.48808850 0.3695944 0.4197601
3  0.19450922 0.1198007  0.21709470 0.27615848 0.2629457 0.1542603
4 -0.14379965 0.2477509 -0.29897262 0.01837793 0.2444425 0.2160098

Std. Errors:
  (Intercept)        x1        x2        x3        x4        x5
2   0.3219613 0.3626664 0.3128671 0.3311019 0.3513568 0.3064434
3   0.2923117 0.3150980 0.2862657 0.3169007 0.3333487 0.2906013
4   0.3133926 0.3645049 0.3230845 0.3515986 0.3620371 0.3227726

Residual Deviance: 261.7377
AIC: 297.7377
>
```

Summary and Remarks

- Multiple logistic regression
- Multi-class logistic regression
- Read textbook Chapter 4 and `R` code
- Do `R` lab