Image Processing

Lecture 7

Introducing Image Processing

Image Enhancement

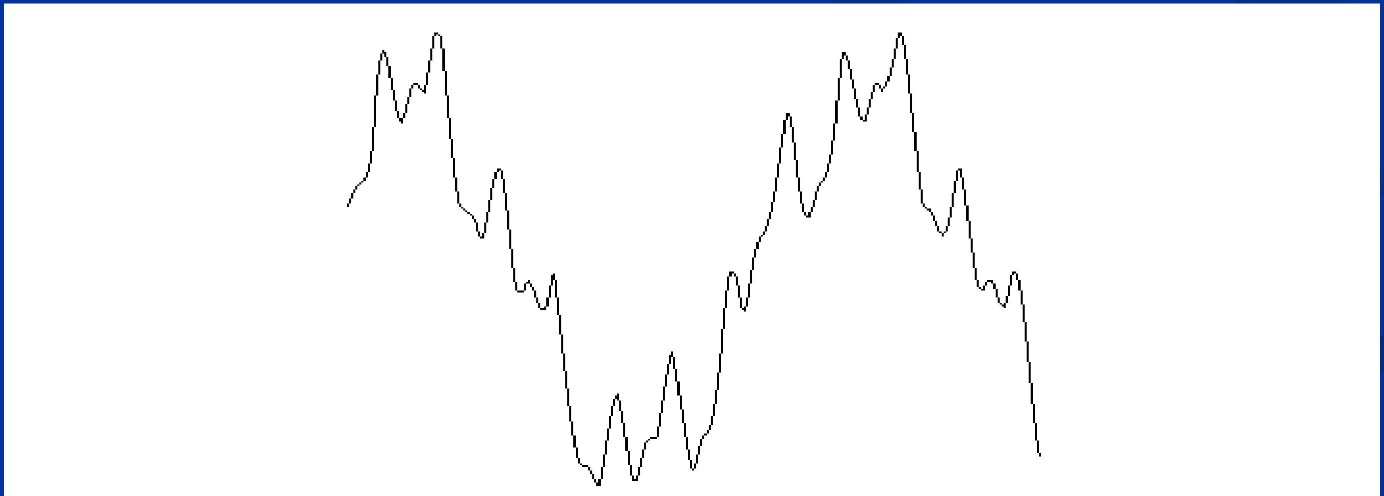
in the frequency Domain

Introduction

- So far we processed the image directly.
- This means that the transformation was a function of the image itself depending on (coordinates and gray levels).
- We called this the SPATIAL domain.
- So what's the FREQUENCY domain ?

Fourier Transform

- Any function that periodically repeats itself can be expressed as the sum of sines/cosines of different frequencies, each multiplied by a different coefficient.



Fourier Transform

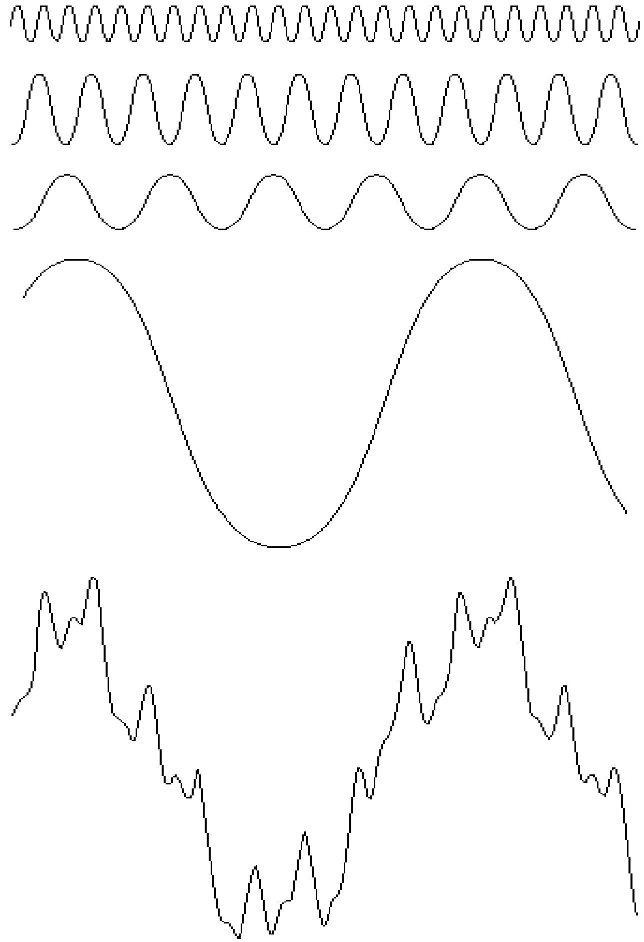


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

Fourier Transform

1-D

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

2-D

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx\, dy$$

- With : $e^{ix} = cos(x) + i\ sin(x)$

Inverse Fourier Transform

- Inverse 1D :

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

- Inverse 2D :

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du\, dv$$

Fourier Transform

1-D

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

2-D

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx\, dy$$

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} du\, dv$$

1D
Discrete Fourier Transform (DFT)

- Suppose $\{f(0), f(1), \ldots, f(M-1)\}$ is a sequence/vector/1-D image of length M. Its M-point DFT is defined as

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j\frac{2\pi}{M}ux}, u = 0,1,2,\ldots\ldots, M-1$$

- Inverse DFT

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j\frac{2\pi}{M}ux}, x = 0,1,2,\ldots\ldots, M-1$$

- Recall: $e^{j\theta} = \cos\theta + j\sin\theta$

Example

- Example: Let $f(x) = \{1, -1, 2, 3\}$. (Note that M=4.)

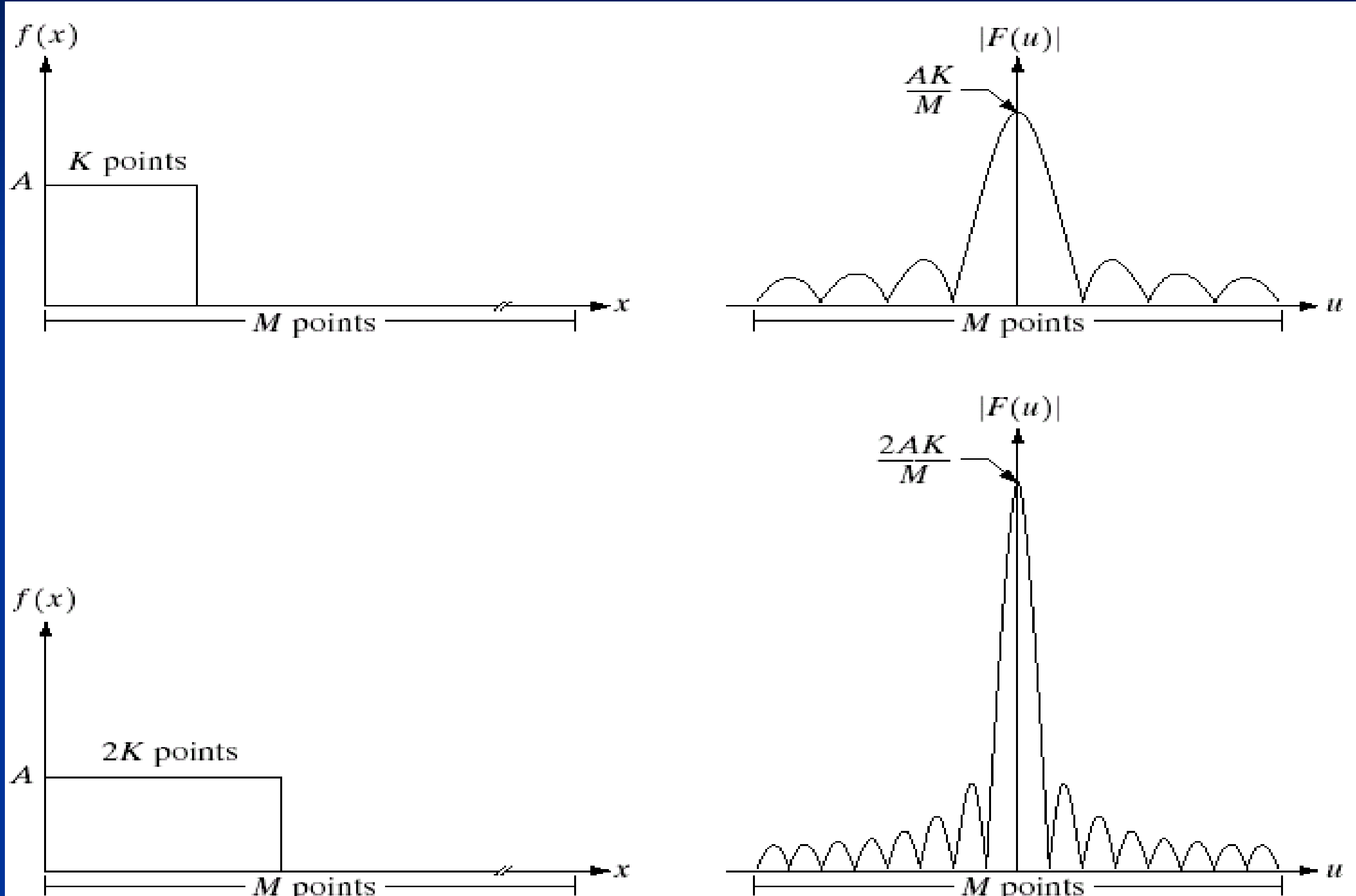
$$F(0) = \sum_{x=0}^{3} f(x) e^{-j\frac{2\pi}{4}x*0} = 5$$

$$F(1) = \sum_{x=0}^{3} f(x) e^{-j\frac{2\pi}{4}x*1} = -1 + 4j$$

$$F(2) = \sum_{x=0}^{3} f(x) e^{-j\frac{2\pi}{4}x*2} = 1$$

$$F(3) = \sum_{x=0}^{3} f(x) e^{-j\frac{2\pi}{4}x*3} = -1 - 4j$$

Fourier Spectrum



a b
c d

FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

Magnitude, Phase and Power Spectrum

$$F(u) = R(u) + jI(u)$$

Magnitude: $$\left|F(u)\right| = \sqrt{R^2(u) + I^2(u)}$$

Phase: $$\phi(u) = \tan^{-1}\left(\frac{I(u)}{R(u)}\right)$$

Power Spectrum: $$P(u) = \left|F(u)\right|^2$$

2D DFT

DFT

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$u = 0,1,2,\ldots,M-1, \quad v = 0,1,2,\ldots,N-1$$

IDFT

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$x = 0,1,2,\ldots,M-1, \quad y = 0,1,2,\ldots,N-1$$

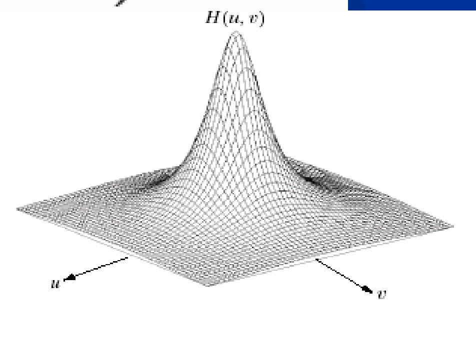
Magnitude, Phase and Power Spectrum

$$F(u,v) = R(u,v) + jI(u,v)$$

Magnitude: $|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$

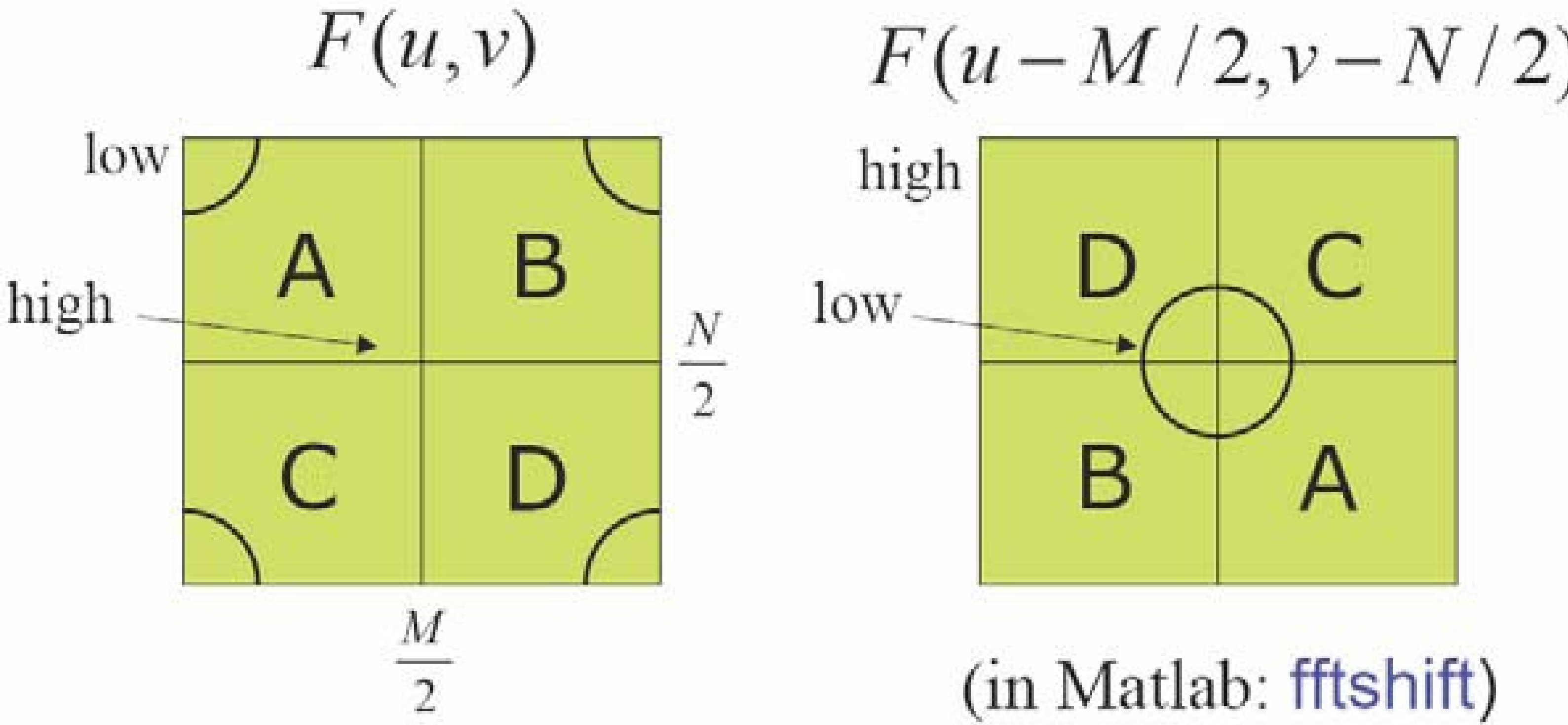
Phase: $\phi(u,v) = \tan^{-1}\left(\dfrac{I(u,v)}{R(u,v)}\right)$

Power Spectrum: $P(u,v) = |F(u,v)|^2$

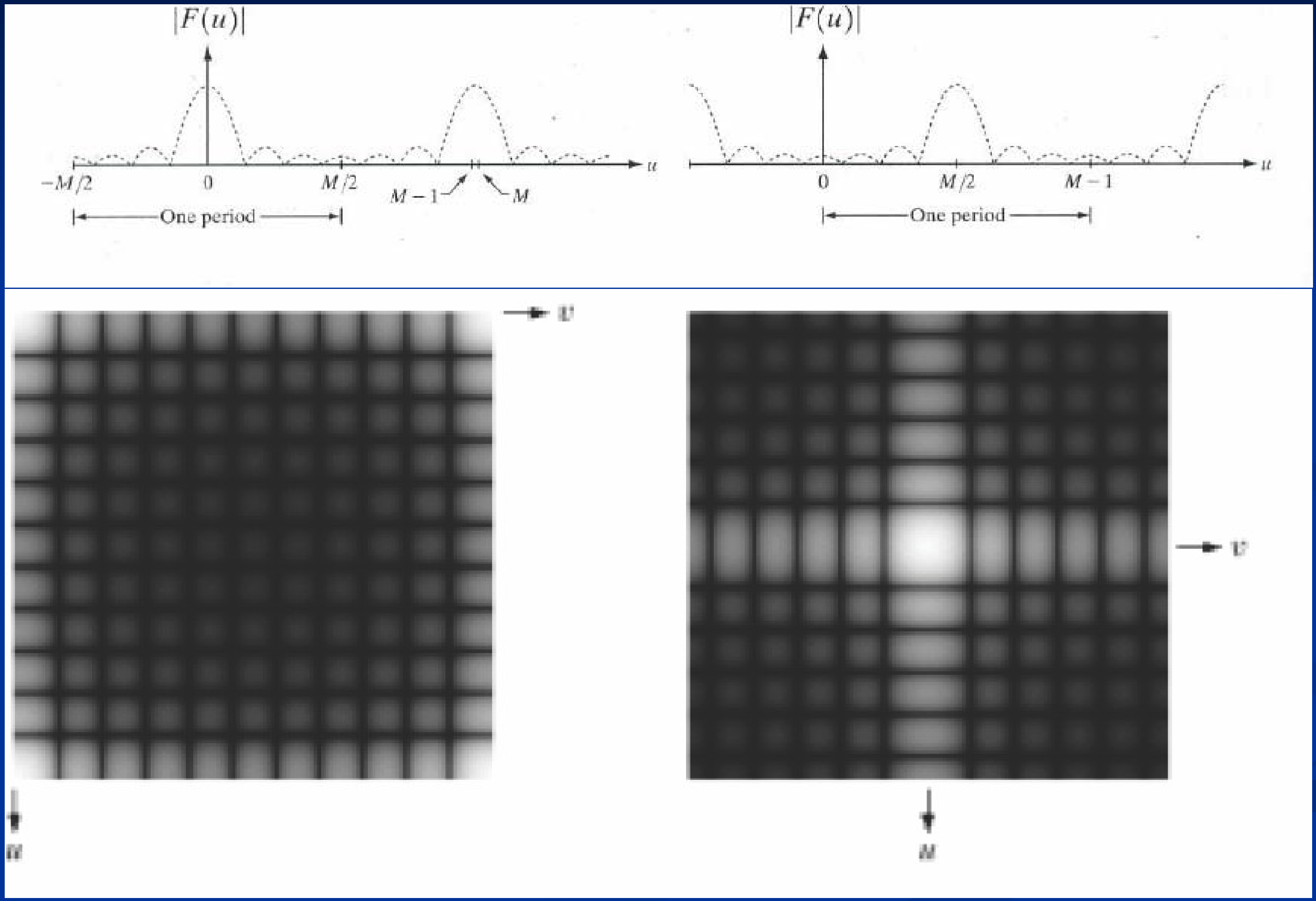


Displaying the 2-D DFT

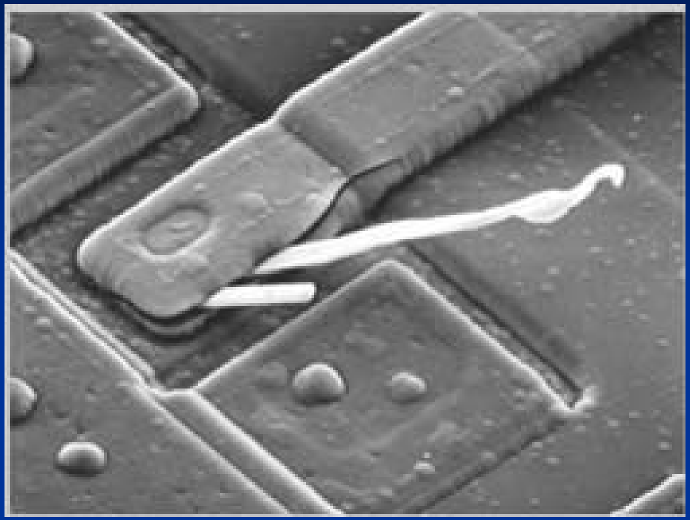
$$f(x,y)(-1)^{x+y} \longleftrightarrow F(u-M/2, v-N/2)$$



Displaying the 2-D DFT



2D DFT : Example



Basic steps for filtering in the frequency domain

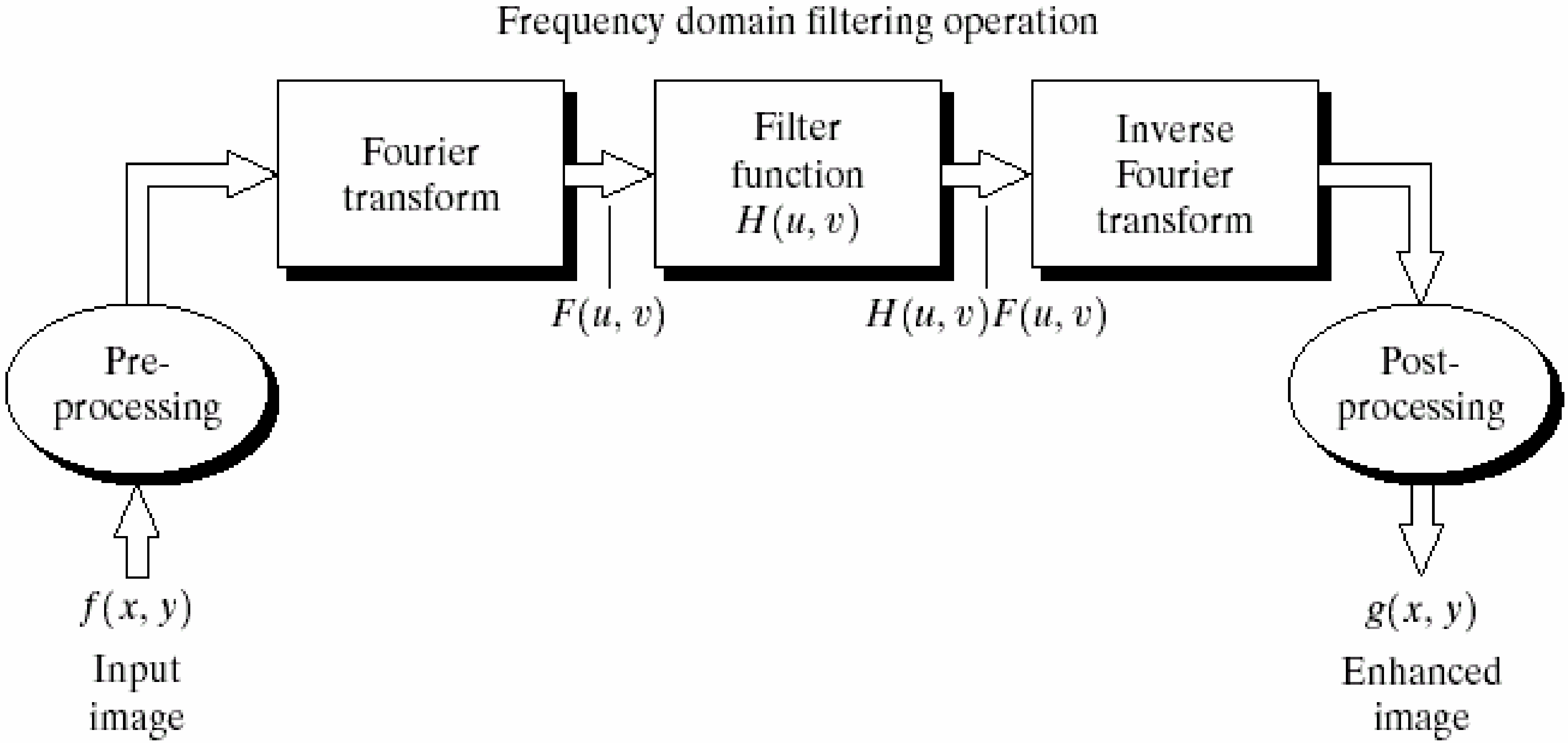


FIGURE 4.5 Basic steps for filtering in the frequency domain.

$$G(u,v) = H(u,v)F(u,v)$$

Basic steps for filtering in the frequency domain

1. multiply the input image by $(-1)^{x+y}$ to center the transform to $u = M/2$ and $v = N/2$
2. compute $F(u,v)$, the 2-D DFT of the image from (1)
3. multiply $F(u,v)$ by a filter function $H(u,v)$
4. compute the inverse DFT of the result in (3)
5. obtain the real part of the result in (4)
6. multiply the result in (5) by $(-1)^{x+y}$ to cancel the multiplication of the input image.

Lowpass Filter (LPF)

- Edges and sharp transitions in gray values in an image contribute significantly to high-frequency content of its Fourier transform.
- Regions of relatively uniform gray values in an image contribute to low-frequency content of its Fourier transform.
- Hence, an image can be smoothed in the Frequency domain by attenuating the high-frequency content of its Fourier transform. This would be a lowpass filter!

Ideal lowpass filter

G(u,v) = H(u,v) F(u,v)

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_o \\ 0 & \text{if } D(u,v) > D_o \end{cases}$$

- Where D(u,v) is the distance from point (u,v) to the origin of the frequency rectangle

$$D(u,v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$$

Ideal lowpass filter

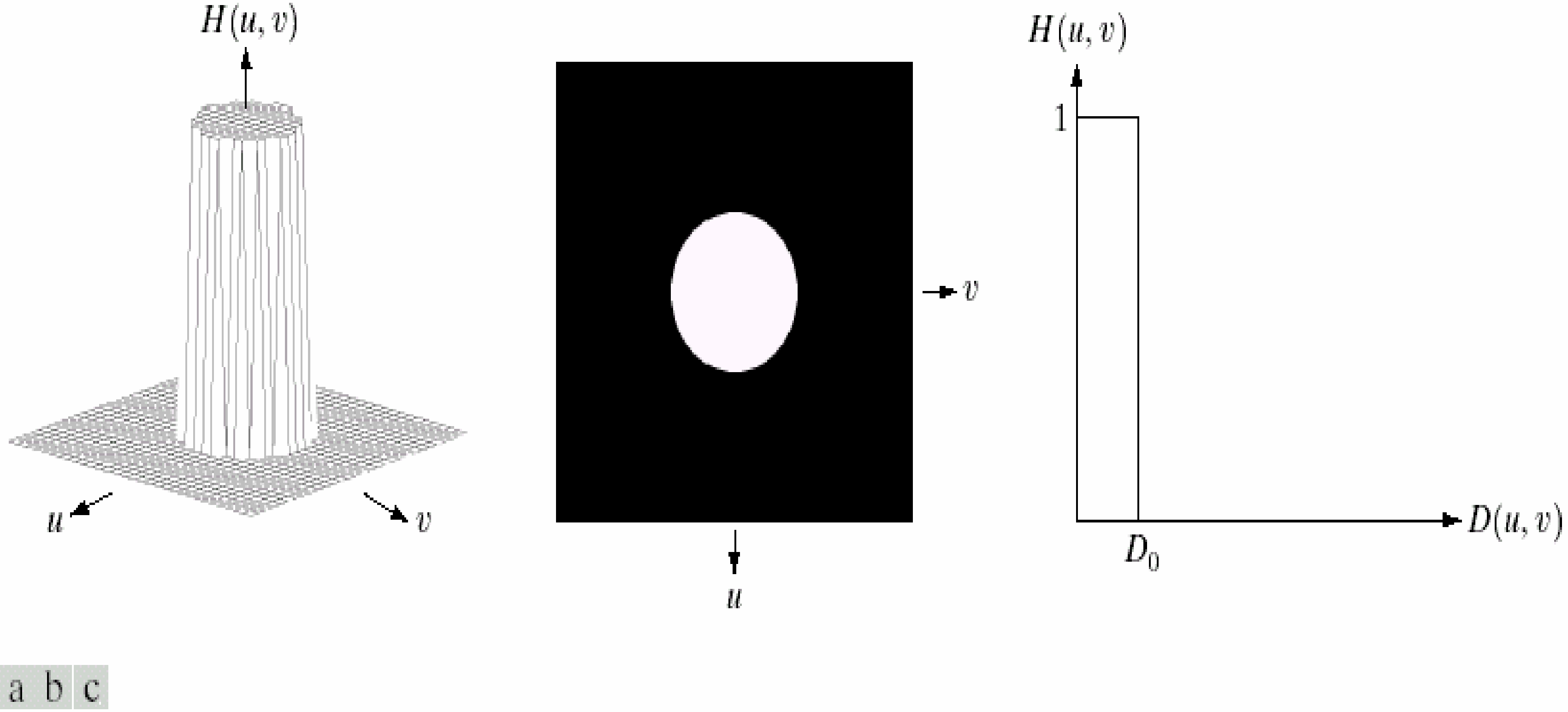
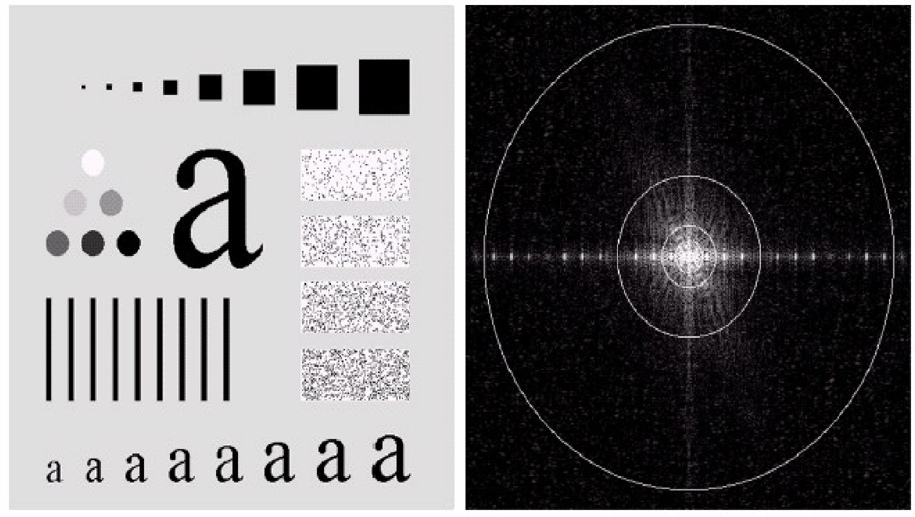


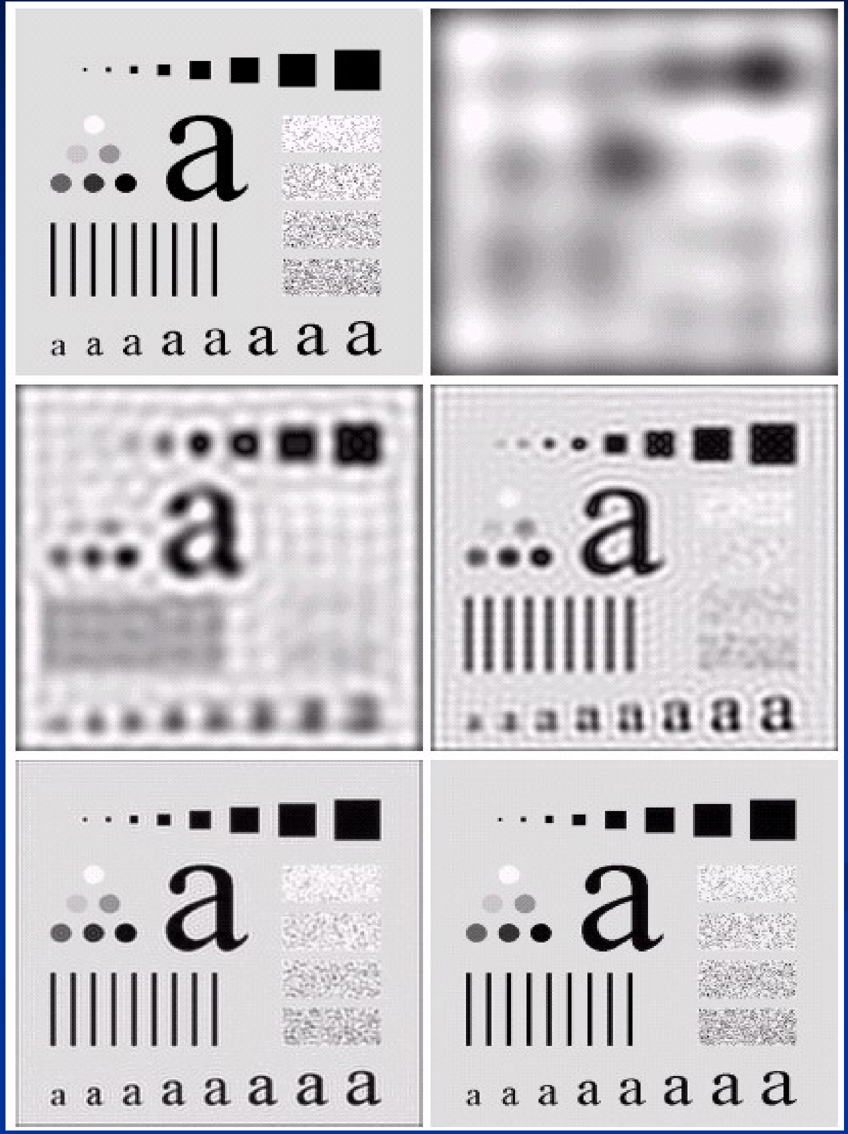
FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Ideal lowpass filter



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.



a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

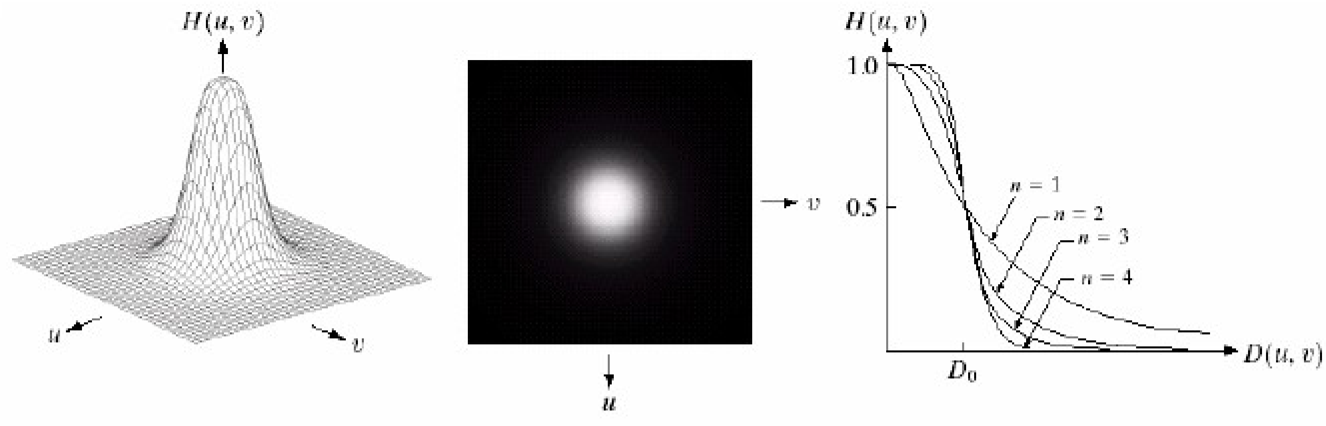
Other LowPass Filter

- Butterworth Lowpass Filter)BLPF)

$$H(u,v) = \frac{1}{1+\left[D(u,v)/D_o\right]^{2n}}$$

- Frequency response does not have a sharp transition
- more appropriate for image smoothing
- not introduce ringing.

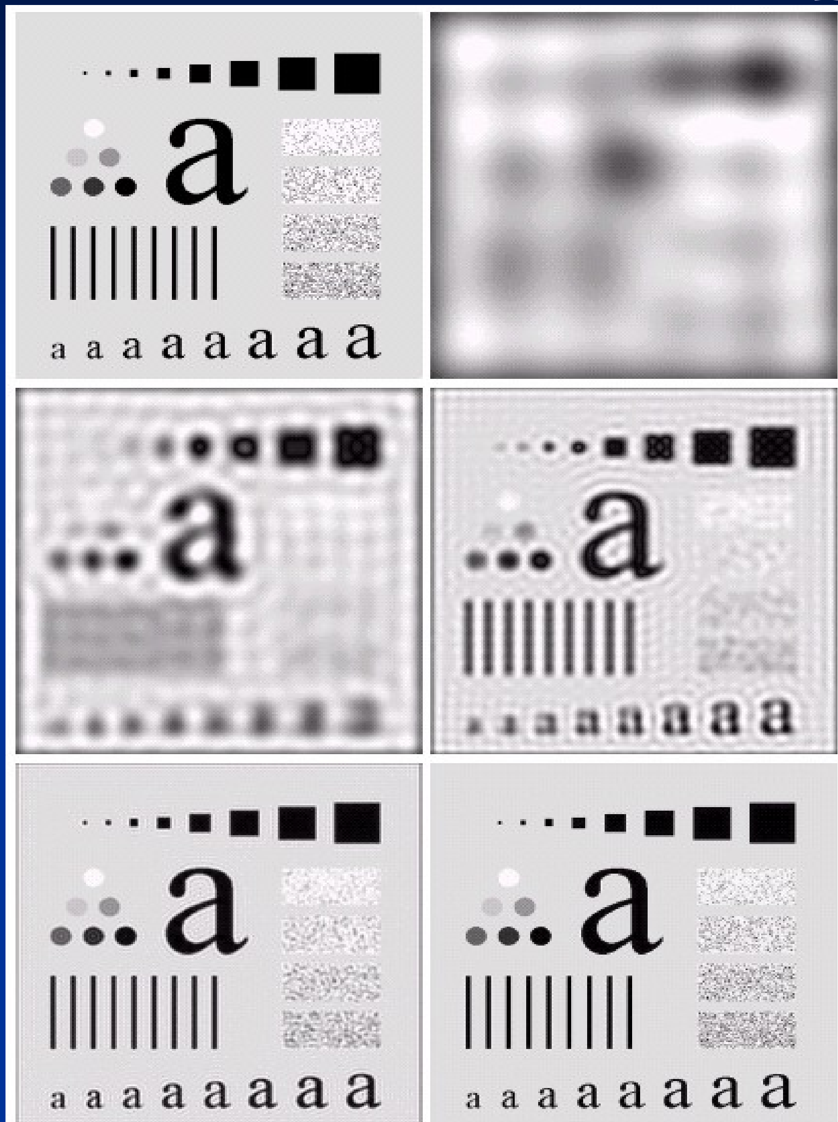
- *n* : filter order
- *D0* : cutoff frequency

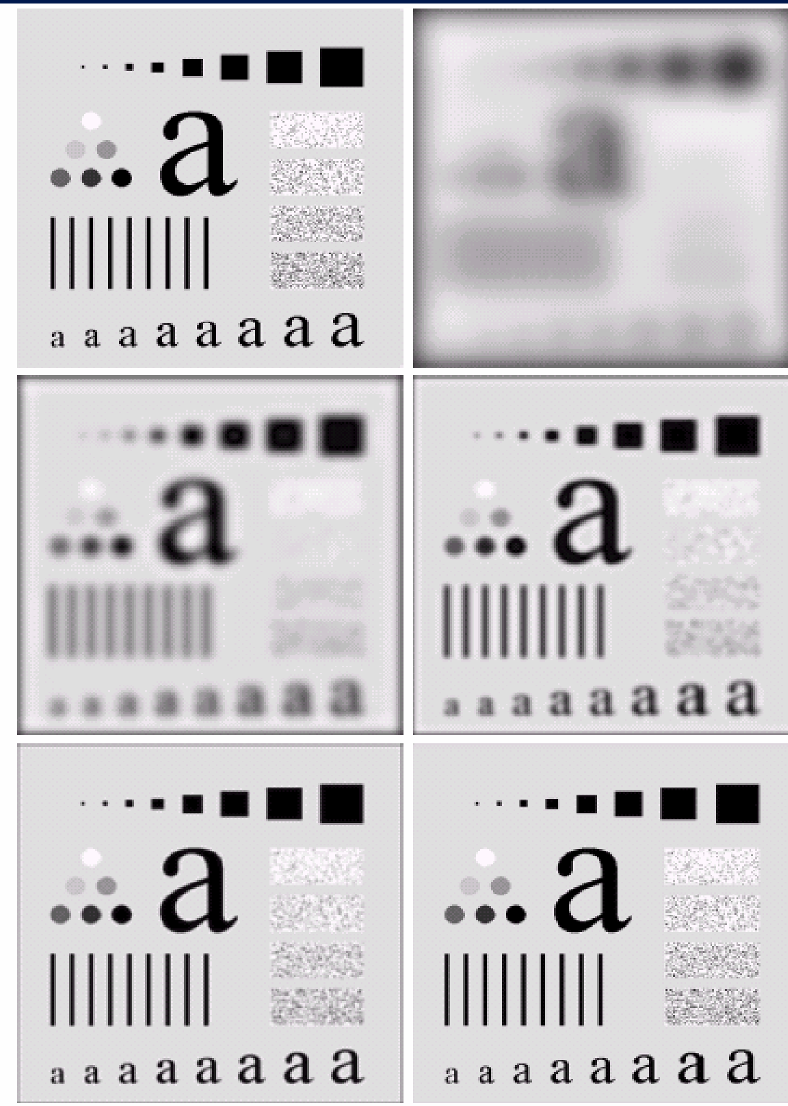


a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Lowpass Filter)BLPF)



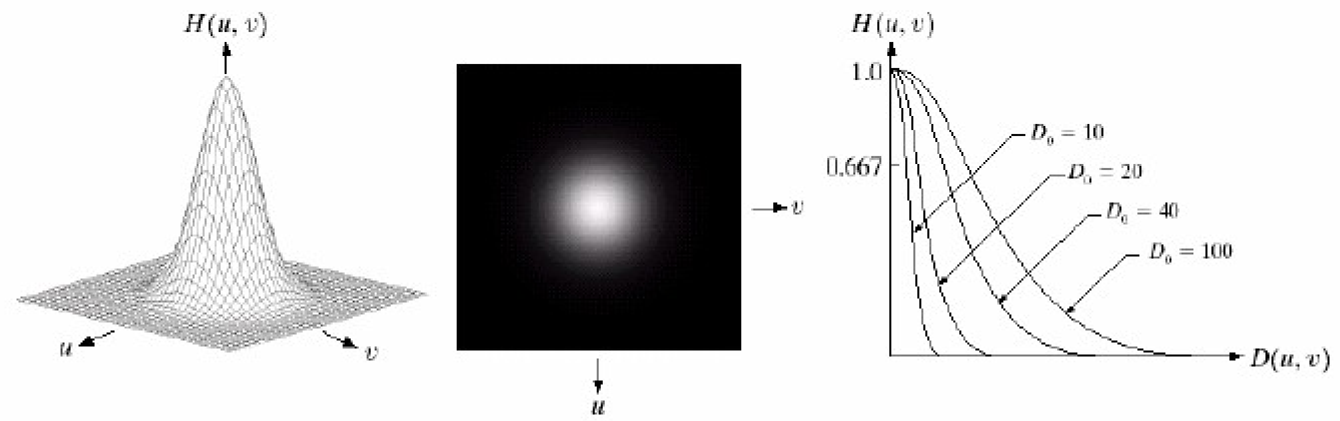


a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Gaussian Lowpass Filter: GLPF

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

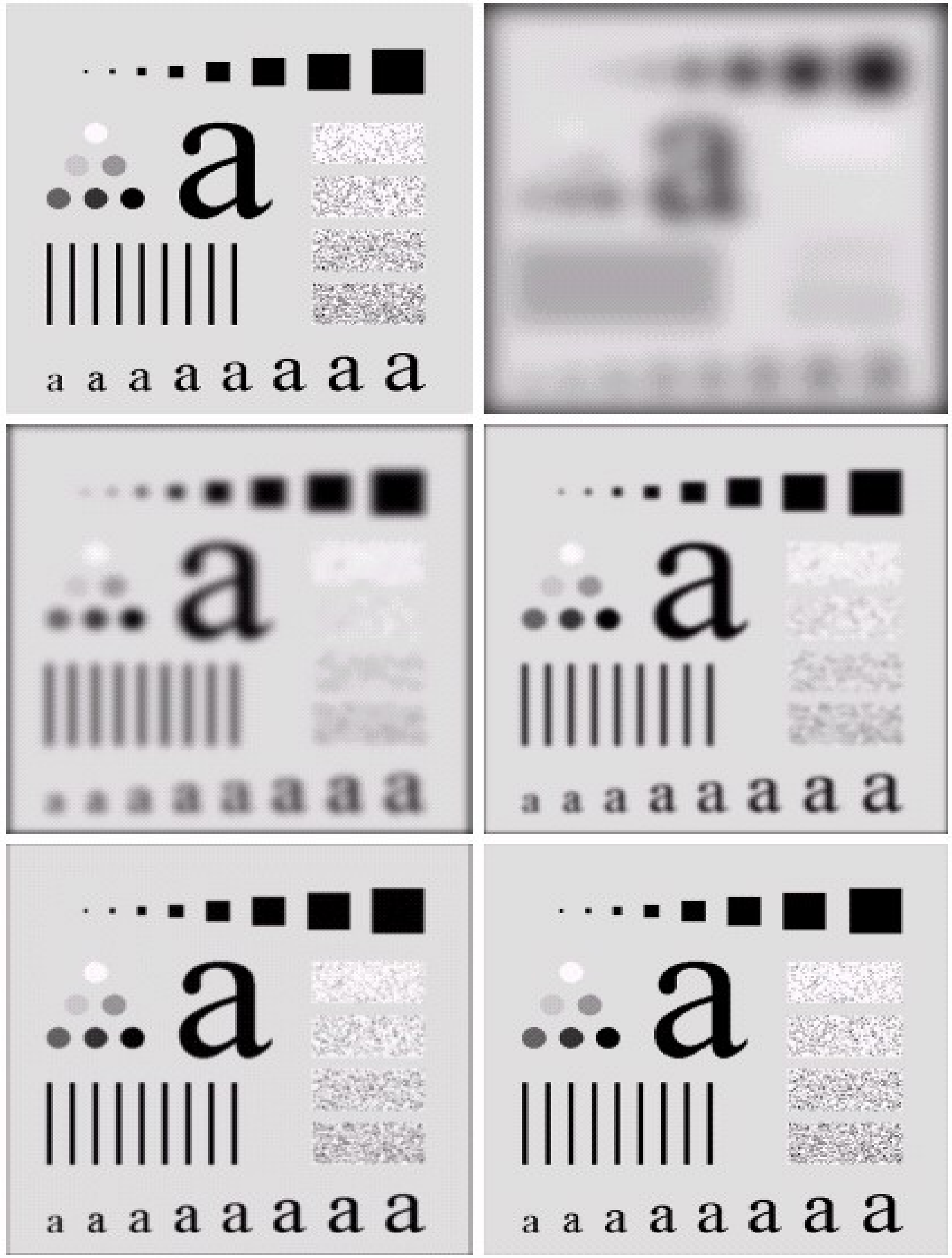


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a b
c d
e f

HighPass Filter

Sharpening Frequency Domain Filter:

Ideal highpass filter

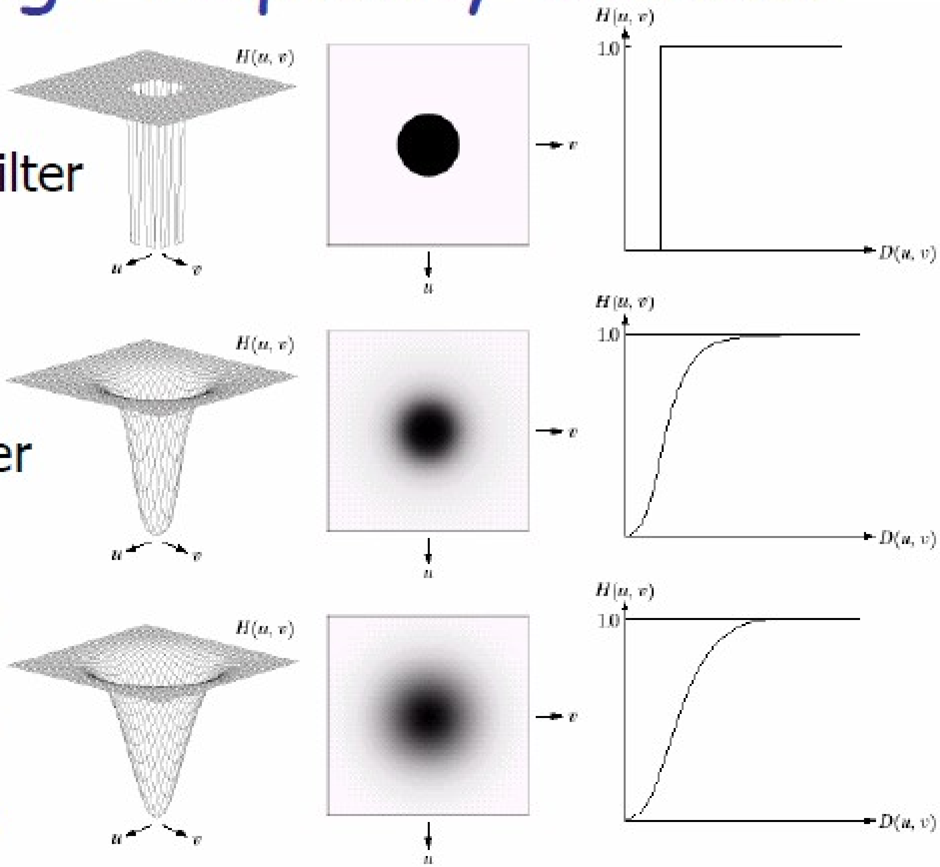
$$H(u,v)=\begin{cases}0 & \text{if } D(u,v)\le D_0\\ 1 & \text{if } D(u,v)>D_0\end{cases}$$

Butterworth highpass filter

$$H(u,v)=\frac{1}{1+\left[D_0/D(u,v)\right]^{2n}}$$

Gaussian highpass filter

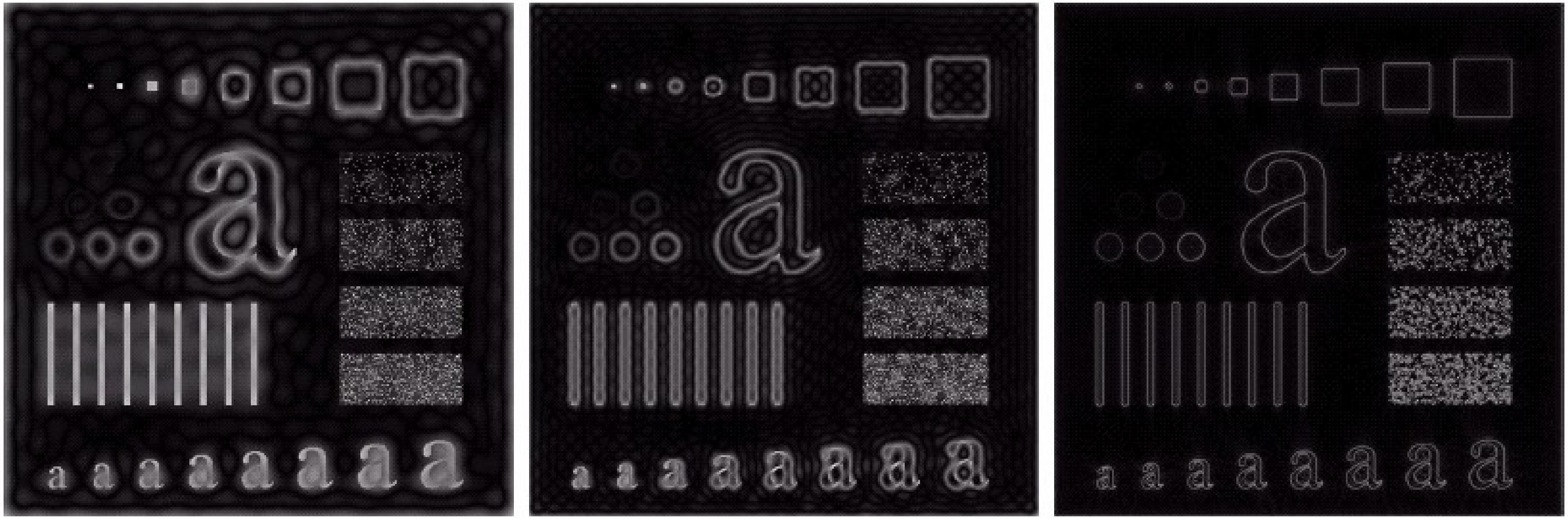
$$H(u,v)=1-e^{-D^2(u,v)/2D_0^2}$$



a	b	c
d	e	f
g	h	i

FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

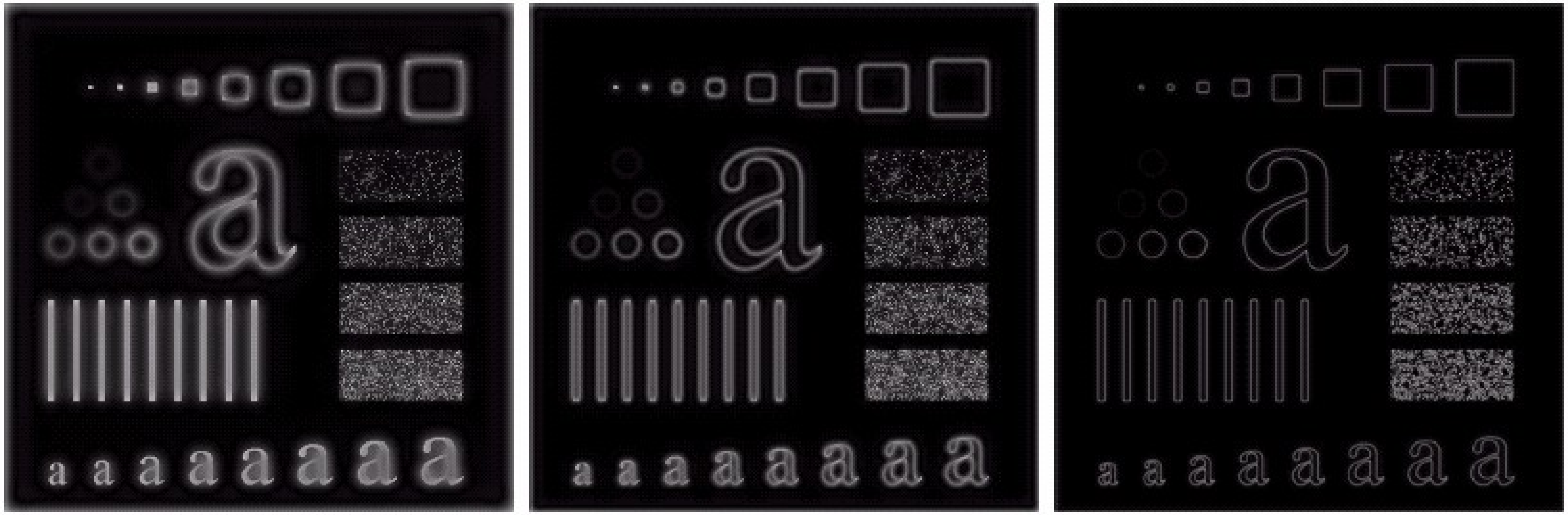
Example : IHPF



a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

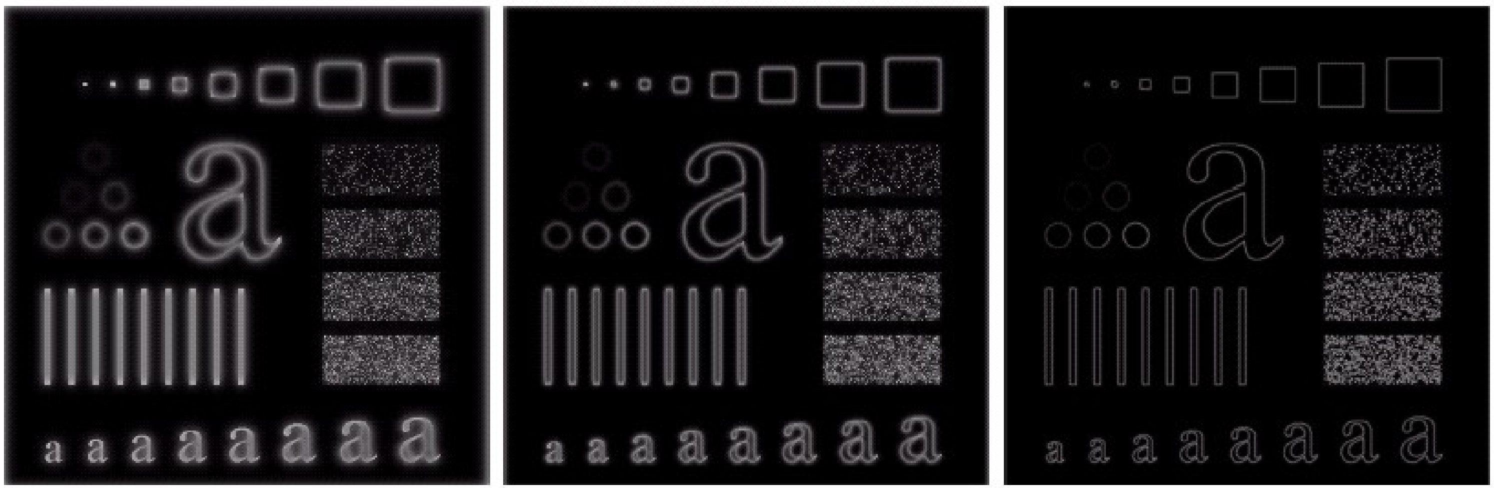
Example : BHPF



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

Example : GHPF



a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.