Real-Time Imaging and Control

Practice

MSCV/ESIREM

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Basics 1 - Boolean Logic

Problem 1

Remark 1. *Reminder on hexadecimal and two's complement:*

- *A 4-bit word can be converted to hexadecimal easily.*
- *Two's complement on n bits: if $m < 0$ and $|m|$ can be expressed on $n-1$ bits, m can be encoded as the unsigned integer $2^n - m$. This is useful to get the negative of a known positive number m*
- *When dealing with negative binary numbers encoded in two's complement, we can interpret it as a sign-bit concatenated to a $(n-1)$ bit word representing $2^{n-1} - |m^\star|$ where $m^\star$ is the number $|m|$ with the MSB removed. Ex :* $1000 = -2^{4-1} + 0b000 = -8 + 0; 1010 = -2^{4-1} + 0b010 = -6$

1. Give decimal values of those unsigned integers: 1001 1011 , 0011 1100 et 0101 0101.
2. Same question but with signed integers in two's complement.
3. Give binaries values of the decimal values -100, 83 et -29 (in two's complement).

1. $0x9b = 155$, $0x3c = 60$,$0x55 = 85$
2. -101, the other two do not change
3. $0b1001.1100$, $0b0101.0011$, $0b1110.0011$

Problem 2

Remark 2. *Some results We can define the dual of an expression e as the expression built with all the bits negated and with sums and products inverted.*
Example: $x\bar{y} + \bar{x}z \to (\bar{x} + y)(x + \bar{z})$
Notably, if two expressions e and f are equal, their dual are equal as well.

Simplify each expression with algebraic manipulation. when applicable, $f(a, b, c) = a + b + c$.

1. $a + 0$
2. $\bar{a} \cdot 0$
3. $a + \bar{a}$
4. $a + a$
5. $a + ab$
6. $a + \bar{a}b$
7. $a \cdot (\bar{a} + b)$
8. $ab + \bar{a}b$
9. $(\bar{a} + \bar{b}) \cdot (\bar{a} + b)$
10. $a \cdot (a + b + c + \cdots)$
11. $f(a, b, ab)$
12. $f(a, b, \bar{a}\bar{b})$
13. $f(a, b, \overline{ab})$
14. $a + a\bar{a}$
15. $ab + a\bar{b}$
16. $\bar{a} + \bar{a}b$
17. $(d + \bar{a} + b + \bar{c})b$
18. $(a + \bar{b})(a + b)$
19. $d + (d + da)$
20. $a(a + ab)$
21. $\overline{(\bar{a} + \bar{a})}$
22. $\overline{(a + \bar{a})}$
23. $d + d\bar{a}bc$
24. $\bar{d}\overline{(dabc)}$
25. $ac + \bar{a}b + bc$
26. $(a + c)(\bar{a} + b)(c + b)$
27. $\bar{a} + \bar{b} + ab\bar{c}$

1. $a + 0 = a$
2. $\bar{a} \cdot 0 = 0$
3. $a + \bar{a} = 1$
4. $a + a = a$
5. $a + ab = a$
6. $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$ (use the dual)
7. $a \cdot (\bar{a} + b) = ab$
8. $ab + \bar{a}b = b$
9. $(\bar{a} + \bar{b}) \cdot (\bar{a} + b) = \bar{a}$
10. $a \cdot (a + b + c + \cdots) = a$
11. $f(a, b, ab) = a + b$
12. $f(a, b, \bar{a}\bar{b}) = 1$
13. $f(a, b, \overline{ab}) = 1$
14. $a + a\bar{a} = a$
15. $ab + a\bar{b} = a$
16. $\bar{a} + \bar{a}b = \bar{a}$
17. $(d + \bar{a} + b + \bar{c})b = b$
18. $(a + \bar{b})(a + b) = a$
19. $d + (d + da) = d$

20. $a(a+ab) = a$

21. $\overline{(\bar{a}+\bar{a})} = a$

22. $\overline{(a+\bar{a})} = 0$

23. $d + d\bar{a}bc = d$

24. $\bar{d}\overline{(dabc)} = \bar{d}$

25. $ac + \bar{a}b + bc = ac + \bar{a}b$

26. $(a+c)(\bar{a}+b)(c+b) = (a+c)(\bar{a}+b)$ with the dual of the consensus theorem

27. $\bar{a}+\bar{b}+ab\bar{c} = \bar{a}+\bar{b}+\bar{c}$ (use the dual).

For number 27, the direct method would be $\bar{a}+\bar{b}+ab\bar{c} = \overline{ab}(1+\bar{c})+ab\bar{c} = \overline{ab}+(ab+\overline{ab})\bar{c} = \bar{a}+\bar{b}+\bar{c}$

Problem 3

Use De Morgan laws to simplify the following :

1. $\overline{\overline{(\bar{a}+c)}\cdot\overline{(b+c)}}$
2. $\overline{ab\bar{c}}$
3. $\overline{b+\bar{c}}\cdot\overline{c+\bar{a}}\cdot\overline{\bar{a}+\bar{b}}$

1. $\overline{\overline{(\bar{a}+c)}\cdot\overline{(b+c)}} = \overline{\overline{(\bar{a}+c)}} + \overline{\overline{(b+c)}} = \bar{a}+b+c$
2. $\bar{a}+\bar{b}+c$
3. 0

Problem 4

Remark 3. *In a Karnaugh map, there are two possible groupings :*

- *One with the 1, called "Minimum Sum of Products" (MSP)*
- *One with the 0, called "Minimum Product of Sums"(MPS)*

Note : in the slides, the notation was MSB in the rows and LSB in the columns. The other notation (with MSB and LSB transposed) exists and will be used in the following Karnaugh maps.

c \ ab	00	01	11	10
0	0	X	X	1
1	1	1	1	X

1. In the Karnaugh map above, find the MSP. A careful choice for the Do Not Care ("X") values is advised.
2. Same question with the MPS.
3. Are the equations equal ?

Both are equal to $a + c$

Problem 5

Find the logic equations described by the Karnaugh maps. Empty cells mean 0.
It is highly suggested to use colored pencils to circle the groupings.

1.

cd \ ab	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11		1	1	
10		1	1	

2.

cd \ ab	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

3.

cd \ ab	00	01	11	10
00			1	
01	1		1	1
11	1	1	1	1
10			1	

4.

cd \ ab	00	01	11	10
00		1		1
01	1		1	1
11		1		1
10	1	1	1	1

1. $b + \bar{c}$
2. $bd + \bar{b}\bar{d}$
3. $cd + \bar{b}d + ab$
4. $a\bar{b} + c\bar{d} + \bar{a}bc + a\bar{c}d + \bar{a}b\bar{d} + \bar{b}\bar{c}d$

Problem 6

Simplify the following expressions with a Karnaugh map. It is suggested to start with a truth table first.

1. $(a+\bar{b}+\bar{c})\cdot(\bar{c}+(a+b+d)\cdot(\bar{a}+\bar{b}+\bar{d}))$
2. $(\bar{c}+ab)(\bar{c}+(a+\bar{d})(b+\bar{d}))(\bar{c}+(a+\bar{b})(b+\bar{d}))$
3. $\bar{w}y+w\bar{x}y+\bar{w}x\bar{z}$

1. $\bar{c}+\bar{b}d+a\bar{d}$
2. $ab+(\bar{b}\bar{c}\bar{d})$
3. $\bar{w}x\bar{z}+\bar{x}y+\bar{w}y$

Problem 7

Four people have access to a safe: Albert, Bernard, Carolyn and David.
Since they do not have the same role, some rules have been set :

- Albert can open the safe if Bernard or Carolyn are present
- The others can open the safe if two other persons are present.

What is the boolean equation for the safe? The binary variables used as inputs are "x is present".
$a(b+c)+bad+cbd+dbc$

Problem 8

Remark 4. *The NAND operator is functionally complete i.e $\vee$, $\wedge$ and $\neg$ can be expressed with the NAND operator only.*

1. Write the truth table of the NAND operator. As a reminder, a NAND $b=\overline{a\cdot b}$.
2. Write the operator $\vee$, $\wedge$ and $\neg$ in terms of NAND.
3. Propose a logic gate architecture implementing the OR, AND and NOT gates in terms of NAND gates.

If we write NAND as $\uparrow$, we have :

- $\neg a = a\uparrow a$
- $a\wedge b=\neg(a\uparrow b)=(a\uparrow b)\uparrow(a\uparrow b)$

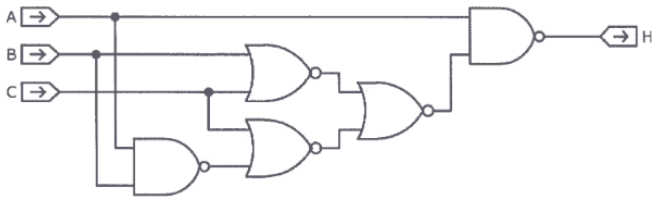

Figure 1: An arrangement of logic gates

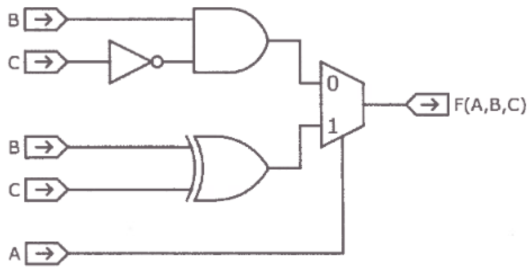

Figure 2: Unknown gibberish.

- $a \vee b = \bar{a} \uparrow \bar{b}$

Problem 9

1. Derive the truth table of the circuit shown on figure 1.
2. Derive a simplified boolean expression from the truth table. By any means necessary.

$\bar{a} + \bar{c}$

Problem 10

A drug dealer wrote what is shown on figure 2 before getting gunned down in a shooting. The police needs the help of MSCV and ESIREM students to understand what was written.

1. Derive the truth table of the circuit shown on figure 2
2. Derive a simplified boolean expression from the truth table. By any means necessary.

$a\bar{b}c + b\bar{c}$