Machine Learning & Deep Learning

Tutorial

MSCV/ESIREM

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VAE, GANs and Similarity learning

Any question or exercise marked with a "*" is typically more technical or goes further into developing the tools and notions seen during class.

Problem 1

Basic concepts.

1. Describe quickly how a (deterministic) auto-encoder works. Same question with a variational auto-encoder.
2. Describe briefly how Generative Adversarial Networks (GAN) work.
3. Describe briefly what similarity learning is.

Problem 2

VAE Check the last tutorial for an exercise on this.

Problem 3

Condition of optimality for a GAN We will prove here some results found in [Goodfellow et al., 2014].

1. Recall the GAN loss function.
2. The generator loss can "saturate" during the early phases of the training process. What could be the reason, and what improvement can be made to the optimization process to avoid this problem?

Global Optimality of $p_g = p_{data}$. The generator G implicitly defines a probability distribution p_g as the distribution of the samples $G(z)$ is obtained when $z \sim p_z$. Therefore, we would like the GAN training algorithm to converge to a good estimator of p_{data} if given enough capacity and training time.

Recall that the GAN training criterion is:

$$\min_G \max_D V(D,G) = E_{x \sim p_{data}}(x)[\log D(x)] + E_{z \sim p_z}(z)[\log(1 - D(G(z)))].$$

1. Show that the function $y \mapsto a\log(y) + b\log(1-y)$ achieves its maximum in $[0,1]$ at $\frac{a}{a+b}$.
2. We first consider the optimal discriminator D for any given generator G. Write the training criterion V as a set of integrals.
3. Deduce an expression for the optimal discriminator D_G^* given an arbitrary generator G.
4. Note that the training objective for D can be interpreted as maximizing the log-likelihood for estimating the conditional probability $P(Y = y|x)$, where Y indicates whether x comes from p_{data} (with $y = 1$) or from p_g (with $y = 0$).

   Rewrite the training criterion for the discriminiator $C(G) = \max_D V(G,D)$ function of p_{data} and p_g
5. We will show that the global minimum of the virtual training criterion $C(G)$ is achieved if and only if $p_g = p_{data}$. $C(G)$ achieves the value $-\log 4$ at that point.
   (a) Show that when $p_g = p_{data}$, we have $D_G^*(x) = \frac{1}{2}$. Then, deduce that we have $C(G) = -\log 4$
   (b) Recall the expression of the Kullback-Leibler divergence. Calculate its value for p_g and p_{data}.
   (c) Show that $C(G) = -\log(4) + 2JSD(p_{data} \parallel p_g)$.
   (d) How to interpret this result?

Problem 4

Evolution of GANs (*)

LSGAN Least Square GAN [Mao et al., 2017] (LSGAN) optimizes the following :

$$\min_D V_{LSGAN}(D) = \frac{1}{2}E_{x \sim p_{data}}(x)[(D(x) - b)^2] + \frac{1}{2}E_{z \sim p_z}(z)[(D(G(z)) - a)^2]$$
$$\min_G V_{LSGAN}(G) = \frac{1}{2}E_{z \sim p_z}(z)[(D(G(z)) - c)^2]$$

We are assumed to use the $a - b$ coding scheme for the discriminator, where a and b are the fake and real data labels, respectively. c denotes the value G wants D to believe for the fake data.

1. Show that for a fixed G, the optimal discriminator D is

$$D^*(x) = \frac{bp_{data}(x) + ap_g(x)}{p_{data}(x) + p_g(x)}$$

2. Start by remarking that adding a term $E_{x\sim p_{data}}(x)[(D(x)-c)^2]$ to $V_{LSGAN}(G)$ causes no change in the optimal values. Show that:

$$2C(G) = \int_{\mathcal{X}} \frac{((b-c)(p_{data}(x)+p_g(x)) - (b-a)p_g(x))^2}{p_{data}(x)+p_g(x)} dx.$$

3. Show that optimizing LSGANs yields minimizing the Pearson χ^2 divergence between $p_{data}+p_g$ and $2p_g$, if a, b, and c satisfy the conditions of $b-c=1$ and $b-a=2$. We define the Pearson χ^2 divergence as:

$$\chi^2_{Pearson}(P \parallel Q) = \int \frac{(p(x)-q(x))^2}{q(x)} dx$$

The Kullback-Leibler divergence is widely used in variational inference due to the convenient evidence lower bound (ELBO, see VAE), and the Jensen-Shannon divergence has very similar properties. However, optimizing $KL(p_g \parallel p_d)$ has the problem of mode-seeking behavior or under-dispersed approximations. This problem also appears in GANs learning, known as the mode collapse problem. In other words, the GAN learns to reproduce only one or a few modes of the training data.

The advantage of using the Pearson χ^2 divergence instead of the Kullback-Leibler/Jensen-Shannon divergence is that it makes LSGANs less mode-seeking and alleviates the mode collapse problem.

Reading exercise - Wasserstein GAN This exercise is a *reading* exercise. The questions are only here to guide you somewhat to extract the information and make your conclusions.

Wasserstein GAN uses the Wasserstein distance (a distance between probability distributions) instead of the KL or Pearson divergence.

1. Find the definition of the Wasserstein distance and the Kantorovich-Rubenstein duality theorem. Note the special hypothesis for the optimization in the duality theorem.
2. Find the training algorithm for the Wasserstein GAN From the original article. What is striking with the new loss compared to the original GAN or LSGAN?
3. The original version uses weight clipping to enforce a special hypothesis. What is this special hypothesis, and is clipping a good thing?
4. An improvement was found in [Gulrajani et al., 2017] but there is a problem. Which one?
5. Propose an improvement and compare it to what was introduced in [Adler and Lunz, 2018].

Problem 5

Reading exercise - GAN in audio: DrumGAN vs. StyleWaveGAN This exercise is a *reading* exercise. The questions are only here to guide you somewhat to extract the information and make your conclusions. Also, these articles are not about vision or robotics to push you outside of your comfort zone. This might also imply you will need to read some of the bibliography.

1. Prerequisite: Read [Karras et al., 2018] and [Karras et al., 2020]

2. Make a table identifying the common points and the differences between DrumGAN [Nistal et al., 2020] and StyleWaveGAN [Lavault et al., 2022].

3. One of the common points is the use of perceptual controls in the synthesis. Describe in more detail what are the similarities and differences between the approaches.

4. A staple in model evaluations, especially in audio, is subjective evaluation. Only one article explores the just-noticeable difference in perceptual controls with GANs. Find it.

References

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