[1]

$$f(z) = z^4 + az^3 + bz^2 + cz + d$$

where a, b, c and d are real constants.

Given that $-1 + 2i$ and $3 - i$ are two roots of the equation $f(z) = 0$

(a) show all the roots of $f(z) = 0$ on a single Argand diagram,

(4)

(b) find the values of a, b, c and d.

(5)

[2] In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin. The point A represents the complex number $6 + 2\mathrm{i}$.

(a) Find the complex numbers represented by the points B and C, giving your answers in the form $x + \mathrm{i}y$, where x and y are real and exact.

(6)

The points D, E and F are the midpoints of the sides of triangle ABC.

(b) Find the exact area of triangle DEF.

(3)

[3] The loci C_1 and C_2 are given by $|z+2| = 2$ and $\arg(z+2) = \frac{5}{6}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[4]**

(ii) Find the complex number represented by the intersection of C_1 and C_2. **[2]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z+2| \leqslant 2 \text{ and } \tfrac{5}{6}\pi \leqslant \arg(z+2) \leqslant \pi.$$ **[2]**

[4] Given that

$$z_1 = 3\left(\cos\left(\frac{\pi}{3}\right) + \mathrm{i}\sin\left(\frac{\pi}{3}\right)\right)$$
$$z_2 = \sqrt{2}\left(\cos\left(\frac{\pi}{12}\right) - \mathrm{i}\sin\left(\frac{\pi}{12}\right)\right)$$

(a) write down the exact value of

(i) $|z_1 z_2|$

(ii) $\arg(z_1 z_2)$

(2)

Given that $w = z_1 z_2$ and that $\arg(w^n) = 0$, where $n \in \mathbb{Z}^+$

(b) determine

(i) the smallest positive value of n

(ii) the corresponding value of $|w^n|$

(3)

[5] On a single Argand diagram, shade the region, R, that satisfies both

$$|z - 2\mathrm{i}| \leqslant 2 \qquad \text{and} \qquad \frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{3}\pi$$

(2)

[6] $\mathrm{f}(z) = z^3 + az + 52$ where a is a real constant

Given that $2 - 3\mathrm{i}$ is a root of the equation $\mathrm{f}(z) = 0$

(a) write down the other complex root.

(1)

(b) Hence

(i) solve completely $\mathrm{f}(z) = 0$

(ii) determine the value of a

(4)

(c) Show all the roots of the equation $\mathrm{f}(z) = 0$ on a single Argand diagram.

(1)

[7] Given that $z = a + b\text{i}$ is a complex number where a and b are real constants,

(a) show that zz^* is a real number.

(2)

Given that

- $zz^* = 18$
- $\dfrac{z}{z^*} = \dfrac{7}{9} + \dfrac{4\sqrt{2}}{9}\text{i}$

(b) determine the possible complex numbers z

(5)

[8] A student was asked to answer the following:

For the complex numbers $z_1 = 3 - 3\mathrm{i}$ and $z_2 = \sqrt{3} + \mathrm{i}$, find the value of $\arg\left(\frac{z_1}{z_2}\right)$

The student's attempt is shown below.

Line 1 → $\arg(z_1) = \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$

Line 2 → $\arg(z_2) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

Line 3 → $\arg\left(\frac{z_1}{z_2}\right) = \frac{\arg(z_1)}{\arg(z_2)}$

Line 4 → $= \left(\frac{\pi}{4}\right) \Big/ \left(\frac{\pi}{6}\right) = \frac{3}{2}$

The student made errors in line 1 and line 3

Correct the error that the student made in

(a) (i) line 1

(ii) line 3

(2)

(b) Write down the correct value of $\arg\left(\frac{z_1}{z_2}\right)$

(1)

[8]

$$f(z) = z^4 + az^3 + 6z^2 + bz + 65$$

where a and b are real constants.

Given that $z = 3 + 2\mathbf{i}$ is a root of the equation $f(z) = 0$, show the roots of $f(z) = 0$ on a single Argand diagram.

(9)