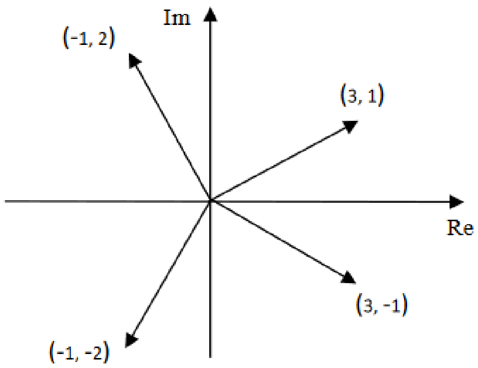
[1]

Question	Scheme		Marks	AOs
1(a)	$z=-1-2\text{i}$ or $z=3+\text{i}$		M1	1.2
	$z=-1-2\text{i}$ and $z=3+\text{i}$		A1	1.1b
			B1 B1	1.1b 1.1b
			(4)	
(b) **Way 1**	$(z-(-1+2\text{i}))(z-(-1-2\text{i}))=\ldots$ or $(z-(3+\text{i}))(z-(3-\text{i}))=\ldots$	$\text{f}(z)=(z-(-1+2\text{i}))(z-(-1-2\text{i}))$ $(z-(3+\text{i}))(z-(3-\text{i}))=\ldots$	M1	3.1a
	z^2+2z+5 or $z^2-6z+10$	e.g. $\text{f}(z)=(z^2+2z+5)(\ldots)$	A1	1.1b
	z^2+2z+5 and $z^2-6z+10$	$\text{f}(z)=(z^3+z^2(-1-\text{i})+z(-1+2\text{i})-15-5\text{i})(\ldots)$	A1	1.1b
	$\text{f}(z)=(z^2+2z+5)(z^2-6z+10)$	Expands the brackets to forms a quartic	M1	3.1a
	$\text{f}(z)=z^4-4z^3+3z^2-10z+50$ or States $a=-4, b=3, c=-10, d=50$		A1	1.1b
			(5)	

Question	Scheme	Marks	AOs
Way 2	$\text{sum roots} = \alpha + \beta + \gamma + \delta = (-1+2\mathrm{i}) + (-1-2\mathrm{i}) + (3+\mathrm{i}) + (3-\mathrm{i}) = \ldots$ $\text{pair sum} = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$ $= (-1+2\mathrm{i})(-1-2\mathrm{i}) + (-1+2\mathrm{i})(3-\mathrm{i}) + (-1+2\mathrm{i})(3+\mathrm{i}) + (-1-2\mathrm{i})(3-\mathrm{i})$ $+ (-1-2\mathrm{i})(3+\mathrm{i}) + (3+\mathrm{i})(3-\mathrm{i}) = \ldots$ $\text{triple sum} = \alpha\beta\gamma + \alpha\beta\delta + \beta\gamma\delta + \alpha\gamma\delta$ $= (-1+2\mathrm{i})(-1-2\mathrm{i})(3-\mathrm{i}) + (-1+2\mathrm{i})(-1-2\mathrm{i})(3+\mathrm{i}) + (-1+2\mathrm{i})(3+\mathrm{i})(3-\mathrm{i})$ $+ (-1-2\mathrm{i})(3+\mathrm{i})(3-\mathrm{i}) = \ldots$ $\text{Product} = \alpha\beta\gamma\delta = (-1+2\mathrm{i})(-1-2\mathrm{i})(3-\mathrm{i})(3+\mathrm{i}) = \ldots$	M1	3.1a
	sum = 4, pair sum = 3, triple sum = 10 and product = 50	A1 A1	1.1b 1.1b
	$a = -(\text{their sum roots}) = -4$ $b = +(\text{their pair sum}) = 3$ $c = -(\text{triple sum}) = -10$ $d = +(\text{product}) = 50$	M1 A1	3.1a 1.1b
		(5)	
Way 3	$\mathrm{f}\ z\ = \ -1+2\mathrm{i}\ ^4 + a\ -1+2\mathrm{i}\ ^3 + b\ -1+2\mathrm{i}\ ^2 + c\ -1+2\mathrm{i}\ + d = 0$ $\mathrm{f}\ z\ = \ 3+\mathrm{i}\ ^4 + a\ 3+\mathrm{i}\ ^3 + b\ 3+\mathrm{i}\ ^2 + c\ 3+\mathrm{i}\ + d = 0$ Leading to $-7 + 11a - 3b - c + d = 0 \qquad 24 - 2a - 4b + 2c = 0$ $28 + 18a + 8b + 3c + d = 0 \qquad 96 + 26a + 6b + c = 0$	M1 A1 A1	3.1a 1.1b 1.1b
	Solves their simultaneous equation to find a value for one of the constants	M1	3.1a
	$a = -4, b = 3, c = -10, d = 50$	A1	1.1b
		(5)	

[2]

Question	Scheme	Marks	AOs
6(a)	Examples: $\begin{pmatrix}\cos 120 & -\sin 120\\ \sin 120 & \cos 120\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix}=\ldots$ **or** $(6+2\mathrm{i})\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}\right)$ **or** $\sqrt{40}\left(\cos\arctan\left(\frac{2}{6}\right)+\mathrm{i}\sin\arctan\left(\frac{2}{6}\right)\right)\left(\cos\left(\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(\frac{2\pi}{3}\right)\right)$ **or** $\sqrt{40}\left(\cos\left(\arctan\left(\frac{2}{6}\right)+\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(\arctan\left(\frac{2}{6}\right)+\frac{2\pi}{3}\right)\right)$ **or** $\sqrt{40}\mathrm{e}^{\mathrm{i}\arctan\left(\frac{2}{6}\right)}\mathrm{e}^{\mathrm{i}\left(\frac{2\pi}{3}\right)}$	M1	3.1a
	$\left(-3-\sqrt{3}\right)$ **or** $\left(3\sqrt{3}-1\right)\mathrm{i}$	A1	1.1b
	$\left(-3-\sqrt{3}\right)+\left(3\sqrt{3}-1\right)\mathrm{i}$	A1	1.1b
	Examples: $\begin{pmatrix}\cos 240 & -\sin 240\\ \sin 240 & \cos 240\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix}=\ldots$ **or** $(6+2\mathrm{i})\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{i}\right)$ **or** $\sqrt{40}\left(\cos\arctan\left(\frac{2}{6}\right)+\mathrm{i}\sin\arctan\left(\frac{2}{6}\right)\right)\left(\cos\left(\frac{4\pi}{3}\right)+\mathrm{i}\sin\left(\frac{4\pi}{3}\right)\right)$ **or** $\sqrt{40}\left(\cos\left(\arctan\left(\frac{2}{6}\right)+\frac{4\pi}{3}\right)+\mathrm{i}\sin\left(\arctan\left(\frac{2}{6}\right)+\frac{4\pi}{3}\right)\right)$ **or** $\sqrt{40}\mathrm{e}^{\mathrm{i}\arctan\left(\frac{2}{6}\right)}\mathrm{e}^{\mathrm{i}\left(\frac{4\pi}{3}\right)}$	M1	3.1a
	$\left(-3+\sqrt{3}\right)$ **or** $\left(-3\sqrt{3}-1\right)\mathrm{i}$	A1	1.1b
	$\left(-3+\sqrt{3}\right)+\left(-3\sqrt{3}-1\right)\mathrm{i}$	A1	1.1b
		(6)	
(b) Way 1	Area $ABC = 3\times\frac{1}{2}\sqrt{6^2+2^2}\sqrt{6^2+2^2}\sin 120°$ or Area $AOB = \frac{1}{2}\sqrt{6^2+2^2}\sqrt{6^2+2^2}\sin 120°$	M1	2.1
	Area $DEF = \frac{1}{4}ABC$ or $\frac{3}{4}AOB$	dM1	3.1a
	$=\frac{3}{8}\times 40\times\frac{\sqrt{3}}{2}=\frac{15\sqrt{3}}{2}$	A1	1.1b
		(3)	

(b) Way 2	$D\left(\frac{3-\sqrt{3}}{2}, \frac{3\sqrt{3}+1}{2}\right)$ $OD = \sqrt{\left(\frac{3-\sqrt{3}}{2}\right)^2 + \left(\frac{3\sqrt{3}+1}{2}\right)^2} = \sqrt{10}$ Area $DOF = \frac{1}{2}\sqrt{10}\sqrt{10}\sin 120°$	M1	2.1
	Area $DEF = 3DOF$	dM1	3.1a
	$= 3 \times \frac{1}{2} \times \sqrt{10}\sqrt{10} \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 3	$AB = \sqrt{\left(9+\sqrt{3}\right)^2 + \left(3-3\sqrt{3}\right)^2} = \sqrt{120}$ Area $ABC = \frac{1}{2}\sqrt{120}\sqrt{120}\sin 60°\left(=30\sqrt{3}\right)$	M1	2.1
	Area $DEF = \frac{1}{4}ABC$	dM1	3.1a
	$= \frac{1}{4} \times 30\sqrt{3} = \frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 4	$D\left(\frac{3-\sqrt{3}}{2}, \frac{3\sqrt{3}+1}{2}\right)$, $E(-3,-1)$, $F\left(\frac{3+\sqrt{3}}{2}, \frac{-3\sqrt{3}+1}{2}\right)$ $DE = \sqrt{\left(\frac{3-\sqrt{3}}{2}+3\right)^2 + \left(\frac{3\sqrt{3}+1}{2}+1\right)^2}\left(=\sqrt{30}\right)$ Area $DEF = \frac{1}{2}\sqrt{30}\sqrt{30}\sin 60°$	M1 dM1	2.1 3.1a
	$= \frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 5	Area $ABC = \frac{1}{2}\begin{vmatrix} 6 & -3-\sqrt{3} & \sqrt{3}-3 & 6 \\ 2 & 3\sqrt{3}-1 & -3\sqrt{3}-1 & 2 \end{vmatrix} = 30\sqrt{3}$	M1	2.1
	Area $DEF = \frac{1}{4}ABC$	dM1	3.1a
	$= \frac{1}{4} \times 30\sqrt{3} = \frac{15\sqrt{3}}{2}$	A1	1.1b

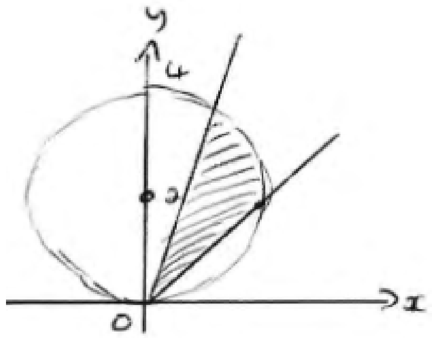
[3]

Question		Answer	Marks	Guidance
5	(i)		B1 B1 B1 B1 **[4]**	Circle centre (–2, 0) or circle centre (2, 0) Touching y-axis at origin Half line with negative slope upwards Completely correct diagram
5	(ii)	$-2-\sqrt{3}+i$	B1ft B1ft **[2]**	Correct real part and correct imaginary part of a complex number, ft for their half line from centre of their circle, allow decimals (-3.73 or better) or trig expressions
5	(iii)		B1ft B1 **[2]**	Shade inside their circle Completely correct diagram and shading **S.C. allow last B1 for radius or complete line**

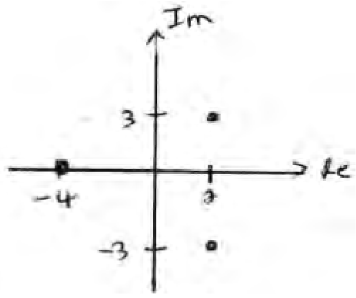
[4]

Question	Scheme	Marks	AOs
1(a) (i)	$\lvert z_1 z_2\rvert = 3\sqrt{2}$	B1	1.1b
(ii)	$\arg(z_1 z_2) = \frac{\pi}{3} + \left(-\frac{\pi}{12}\right) = \frac{\pi}{4}$ o.e.	B1	1.1b
		(2)	
(b) (i)	$n = 8$	B1ft	2.2a
(ii)	$\lvert w^n\rvert = (\text{'their}\lvert z_1 z_2\rvert\text{'})^{\text{their } n}$	M1	1.1b
	$\lvert w^n\rvert = 104\,976$	A1	1.1b
		(3)	
			(5 marks)

[5]

Circle centre (0, 2) and radius 2 or  with the point on the origin	B1	1.1b
Fully correct 	B1	1.1b
	(2)	

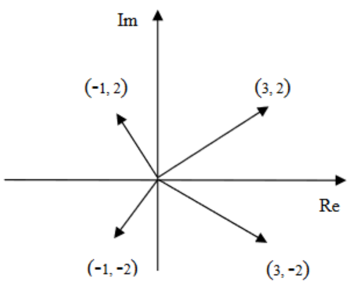
[6]

Question	Scheme	Marks	AOs
1(a)	$2+3i$	B1	1.1b
		(1)	
(b) (i)	$z*=2+3i$ so $z+z*=4$, $zz*=13$ $z+z*+\alpha=0 \Rightarrow \alpha=\ldots$ or $\alpha zz*=-52 \Rightarrow \alpha=-\frac{52}{"13"}=\ldots$ or $z^2-(\text{sum roots})z+(\text{product roots})=0$or $\big(z-(2+3i)\big)\big(z-(2-3i)\big)=\ldots$ $\Rightarrow (z^2-4z+13)\underline{(z+4)} \Rightarrow z=\ldots$	M1	3.1a
	$z=2\pm 3\text{i},\ -4$	A1	1.1b
(ii)	$(z^2-4z+13)(z+4)$ expands the brackets to find value for a Or a = pair sum = $-4(2+3i+2-3i)+13=\ldots$ Or $f(-4)/f(2\pm 3i)=0 \Rightarrow \ldots \Rightarrow a=\ldots$	M1	1.1b
	$a=-3$	A1	2.2a
		(4)	
(c)		B1ft	1.1b
		(1)	
			(6 marks)

[7]

Question	Scheme	Marks	AOs
7(a)	$z *= a - bi$then $zz *= (a+bi)(a-bi) = \ldots$	M1	1.1b
	$zz *= a^2 + b^2$ therefore, a real number	A1	2.4
		(2)	
(b)	$\frac{z}{z*} = \frac{a+bi}{a-bi} = \frac{(a+bi)(a+bi)}{(a-bi)(a+bi)} = \frac{(a^2-b^2)+2abi}{a^2+b^2} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$ or $\frac{z}{z^*} = \frac{z^2}{zz^*} = \frac{z^2}{18} \Rightarrow$ $z^2 = 14 + 8\sqrt{2}i$ or $a + bi = \left(\frac{7}{9} + \frac{4\sqrt{2}i}{9}\right)(a - bi) = ..+..i$	M1	1.1b
	Forms two equations from $a^2 + b^2 = 18$ or $\frac{a^2-b^2}{18} = \frac{7}{9}$ or $\frac{a^2-b^2}{a^2+b^2} = \frac{7}{9}$ or $\frac{2ab}{18} = \frac{4\sqrt{2}}{9}$ or $\frac{2ab}{a^2+b^2} = \frac{4\sqrt{2}}{9}$ or $a = \frac{7}{9}a + \frac{4\sqrt{2}}{9}b$ oe	M1 A1	3.1a 1.1b
	Solves the equations simultaneously e.g. $a^2 + b^2 = 18$ and $a^2 - b^2 = 14$ leading to a value for a or b	dM1	1.1b
	$z = \pm(4 + \sqrt{2}i)$	A1	2.2a
		(5)	
			(7 marks)

[8]

Question	Scheme	Marks	AOs
3	$z = 3 - 2\mathrm{i}$ is also a root	B1	1.2
	$(z-(3+2\mathrm{i}))(z-(3-2\mathrm{i})) = \ldots$ **or** Sum of roots = 6, Product of roots = 13 $\Rightarrow \ldots$	M1	3.1a
	$= z^2 - 6z + 13$	A1	1.1b
	$(z^4 + az^3 + 6z^2 + bz + 65) = (z^2 - 6z + 13)(z^2 + cz + 5) \Rightarrow c = \ldots$	M1	3.1a
	$z^2 + 2z + 5 = 0$	A1	1.1b
	$z^2 + 2z + 5 = 0 \Rightarrow z = \ldots$	M1	1.1a
	$z = -1 \pm 2\mathrm{i}$	A1	1.1b
		B1 $3 \pm 2\mathrm{i}$ Plotted correctly	1.1b
		B1ft $-1 \pm 2\mathrm{i}$ Plotted correctly	1.1b
			(9 marks)