[1] $f(z) = z^4 + az^3 + bz^2 + cz + d$

where a, b, c and d are real constants.

Given that -1 + 2i and 3 - i are two roots of the equation f(z) = 0

(a) show all the roots of f(z) = 0 on a single Argand diagram,

(4)

(b) find the values of a, b, c and d.

(5)

[2]	In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle
L—J	with its centre at the origin. The point A represents the complex number $6 + 2i$.

(a) Find the complex numbers represented by the points B and C, giving your answers in the form x + iy, where x and y are real and exact.

(6)

The points D, E and F are the midpoints of the sides of triangle ABC.

(b) Find the exact area of triangle DEF.

(3)

- [3] The loci C_1 and C_2 are given by |z+2|=2 and $\arg(z+2)=\frac{5}{6}\pi$ respectively.
 - (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [4]
 - (ii) Find the complex number represented by the intersection of C_1 and C_2 . [2]
 - (iii) Indicate, by shading, the region of the Argand diagram for which

$$|z+2| \le 2$$
 and $\frac{5}{6}\pi \le \arg(z+2) \le \pi$. [2]

[4] Given that

$$z_1 = 3\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$$
$$z_2 = \sqrt{2}\left(\cos\left(\frac{\pi}{12}\right) - i\sin\left(\frac{\pi}{12}\right)\right)$$

- (a) write down the exact value of
 - (i) $|z_1 z_2|$
 - (ii) $arg(z_1z_2)$

(2)

Given that $w = z_1 z_2$ and that $\arg(w^n) = 0$, where $n \in \mathbb{Z}^+$

- (b) determine
 - (i) the smallest positive value of n
 - (ii) the corresponding value of $|w^n|$

(3)

[5] On a single Argand diagram, shade the region, R, that satisfies both

$$|z-2i| \leqslant 2$$
 and $\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{3}\pi$ (2)

[6] $f(z) = z^3 + az + 52 \qquad \text{where } a \text{ is a real constant}$

Given that 2-3i is a root of the equation f(z) = 0

(a) write down the other complex root.

(1)

- (b) Hence
 - (i) solve completely f(z) = 0
 - (ii) determine the value of a

(4)

(c) Show all the roots of the equation f(z) = 0 on a single Argand diagram.

(1)

[7] Given that z = a + bi is a complex number where a and b are real constants,

(a) show that zz^* is a real number.

(2)

Given that

•
$$zz^* = 18$$

$$\bullet \quad \frac{z}{z^*} = \frac{7}{9} + \frac{4\sqrt{2}}{9}i$$

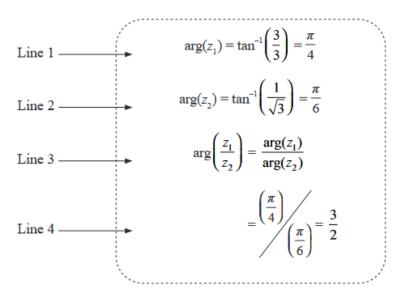
(b) determine the possible complex numbers z

(5)

[8] A student was asked to answer the following:

For the complex numbers $z_1 = 3 - 3i$ and $z_2 = \sqrt{3} + i$, find the value of $\arg\left(\frac{z_1}{z_2}\right)$

The student's attempt is shown below.



The student made errors in line 1 and line 3

Correct the error that the student made in

- (a) (i) line 1
 - (ii) line 3

(2)

(b) Write down the correct value of $\arg\left(\frac{z_1}{z_2}\right)$

(1)

[8]
$$f(z) = z^4 + az^3 + 6z^2 + bz + 65$$

where a and b are real constants.

Given that z = 3 + 2i is a root of the equation f(z) = 0, show the roots of f(z) = 0 on a single Argand diagram.

(9)