ECO2411. Assignment 3

Andriy Levitskyy (ID:1003874701), Yukuo Liu(ID:998402232)

The answer to Exercise 1 is after the answer to Exercise 2

Exercise 2

2.2

The ACF of VIX suggest absence of any autocorrelation dynamics for the series. ACF for S&P looks similar to an AR(1) process with small autocorrelation coefficient. Cross-ACF suggest possibility of some Granger-Causality between variables, as the cross-autocorrelations are significant at lags 1,2 and 3.

2.4

The most persistant combination is given by the eigenvector on the largest in magnitude eigenvalue of AR(1) coefficient matrix: [1 S&P,0.00063 VIX] The least risky portfolio can be found by renormalizing the eigenvector on the smalles eigenvalue of the covariance matrix [-1 S&P,0.00013 VIX] in such a way that the sum of entries are equal to 1 (for example by dividing by the sum of the entries). The portfolio shares I got are: [1.0001300169, -0.0001300169], implying that one needs to short VIX to decrease variance. As the eigenvalue is very close to zero, this may imply that this portfolio is almost riskless.

2.6

As squared return on S&P is a good indicator of the volatility of stock market prices, I may decide to increase my exposure to the stock market, as the VAR forecast predicts decrease in risk. Given the results of the spectral decomposition I would also buy some VIX options, so I can be compensated if the stock market volatility increases.

Exercise 1

1.1:

$\mathrm{Y}_t = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}$

The mean vector of Y_t is:

$\mathrm{E}(\mathrm{Y}_t) = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} E(a_{11})E(\epsilon_{1t}) \\ E(a_{21}\epsilon_{1t}) + E(a_{22}\epsilon_{2t}) \end{bmatrix}$

$\mathrm{E}(\mathrm{Y}_t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

The variance-covariance matrix of Y_t is:

$\mathrm{V}(\mathrm{Y}_t) = \begin{bmatrix} E[\mathrm{y}_{1t} - E(y_{1t})]^2 & E[(\mathrm{y}_{1t} - E(y_{1t}))(y_{2t} - E(y_{2t}))] \\ E[(\mathrm{y}_{2t} - E(y_{2t}))(y_{1t} - E(y_{1t}))] & E[\mathrm{y}_{2t} - E(y_{2t})]^2 \end{bmatrix}$

$\mathrm{V}(\mathrm{Y}_t) = \begin{bmatrix} E[\mathrm{y}_{1t} - E(y_{1t})]^2 & E[(\mathrm{y}_{1t} - E(y_{1t}))(y_{2t} - E(y_{2t}))] \\ E[(\mathrm{y}_{2t} - E(y_{2t}))(y_{1t} - E(y_{1t}))] & E[\mathrm{y}_{2t} - E(y_{2t})]^2 \end{bmatrix}$

$\mathrm{V}(\mathrm{Y}_t) = \begin{bmatrix} E[y_{1t}^2] - E[y_{1t}]^2 & E[(\mathrm{y}_{1t})(y_{2t})] + E[\mathrm{y}_{1t}]E[y_{2t}] \\ E[(\mathrm{y}_{1t})(y_{2t})] + E[\mathrm{y}_{1t}]E[y_{2t}] & E[y_{2t}^2] - E[y_{2t}]^2 \end{bmatrix}$

$\mathrm{V}(\mathrm{Y}_t) = \begin{bmatrix} E[a_{11}^2\epsilon_{1t}^2] - E[a_{11}\epsilon_{1t}]^2 & E[(a_{11}\epsilon_{1t})(a_{21}\epsilon_{1t} + a_{22}\epsilon_{2t})] + E[a_{11}\epsilon_{1t}]E[a_{21}\epsilon_{1t} + a_{22}\epsilon_{2t}] \\ E[(a_{11}\epsilon_{1t})(a_{21}\epsilon_{1t} + a_{22}\epsilon_{2t})] + E[a_{11}\epsilon_{1t}]E[a_{21}\epsilon_{1t} + a_{22}\epsilon_{2t}] & E[(a_{21}\epsilon_{1t} + a_{22}\epsilon_{2t})^2] + E[a_{21}\epsilon_{1t} + a_{22}\epsilon_{2t}]^2 \end{bmatrix}$

$\mathrm{V}(\mathrm{Y}_t) = \begin{bmatrix} E[a_{11}^2] & E[(a_{11})(a_{21})] \\ E[(a_{11})(a_{21})] & E[a_{21}^2] + E[a_{22}^2] \end{bmatrix}$

$\mathrm{V}(\mathrm{Y}_t) = \begin{bmatrix} a_{11}^2 & (a_{11})(a_{21}) \\ (a_{11})(a_{21}) & a_{21}^2 + a_{22}^2 \end{bmatrix}$

1.2:

$\mathrm{Y}_t = A^*\eta_t$

$\epsilon_t = Q^{-1}\eta_t$

$\mathrm{Y}_t = A\epsilon_t = AQ^{-1}\eta_t$

where $\mathrm{A}^* = AQ^{-1}$

1.3:

If $\mathrm{Q}= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then,

$$\eta_t = \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \epsilon_{2t} \\ \epsilon_{1t} \end{bmatrix}$$

and

$$\mathrm{Y}_t = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{2t} \\ \epsilon_{1t} \end{bmatrix}$$

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} a_{11}\epsilon_{1t} \\ a_{22}\epsilon_{2t} + a_{21}\epsilon_{1t} \end{bmatrix}$$

In this case, the structural shock (η_{1t}) is linearly related to the next period's reduced-form innovation ϵ_{2t} and η_{2t} is linearly related to ϵ_{1t}.

1.4

a) No, because this is a reduced-form VAR model. In order to establish a causal link, this model needs to be transformed into the structural form where the error terms (η_t) can be interpreted as structual shocks that are uncorrelated between independent variables (Killian, 2011).

b) False, since $\sum$ is a non-diagonal matrix, ϵ_{1t} is a shock to both y_{1t} and y_{2t}. ϵ_{2t} can be interpreted as a reduced-form innovation (Killian, 2011).

c) Yes, if we know $V(Y_t) = \sum$, then we can find A and Q by Cholesky decomposition of the variance-covariance matrix.

d) No, they are not the same since the structural form can distinguish each of these separately (Killian, 2011).

References

Kilian (2013). Structural Vector Autoregressions. Handbook of Research Methods and Applications in Empirical Macroeconomics, Chapter 22, 515-554, Edward Elgar.

Appendix

Making the dataset

```
library(lubridate)
library(zoo)
library(ggplot2)
library(vars)
setwd("/Users/andriylevitskyy/Desktop/MA/Fin Metr/assignment3")
vix<-read.csv("vixcurrent.csv")
vix<-vix[,c(1,2)]
vix$Date<-mdy(vix$Date)
colnames(vix)<-c("date","vix")
sp500<-read.csv("SP500.csv")
sp500$DATE<-ymd(sp500$DATE)
colnames(sp500)<-c("date","sp")
final_dataset<-merge(vix,sp500,by="date")
final_dataset$sp<-as.numeric(as.character(final_dataset$sp))
final_dataset<-read.zoo(final_dataset)
sp2<-diff(log(final_dataset$sp))^2
vix2<-diff(log(final_dataset$vix))
final_dataset<-cbind(final_dataset,sp2)
final_dataset<-cbind(final_dataset,vix2)
final_dataset<- data.frame(index(final_dataset), as.data.frame(final_dataset))
colnames(final_dataset)[1]<-"date"
```

Plotting the series

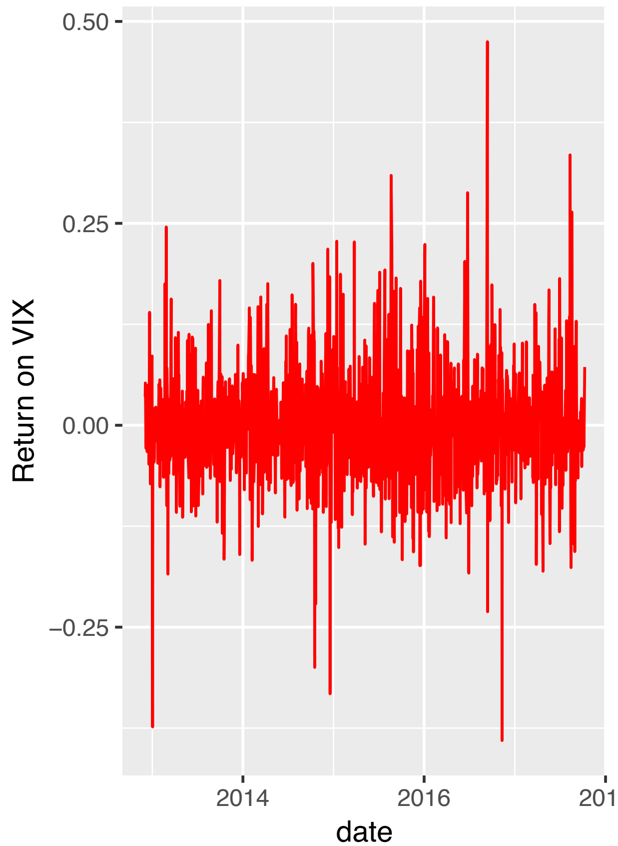
```
p1<-ggplot() +
  geom_line(data = final_dataset, aes(x = date, y = vix2), color = "red")+
  xlab('date') +
  ylab('Return on VIX')
p2<-ggplot() +
  geom_line(data = final_dataset, aes(x = date, y = sp2), color = "blue") +
  xlab('data') +
  ylab('Squared return on S&P')
require(gridExtra)
```

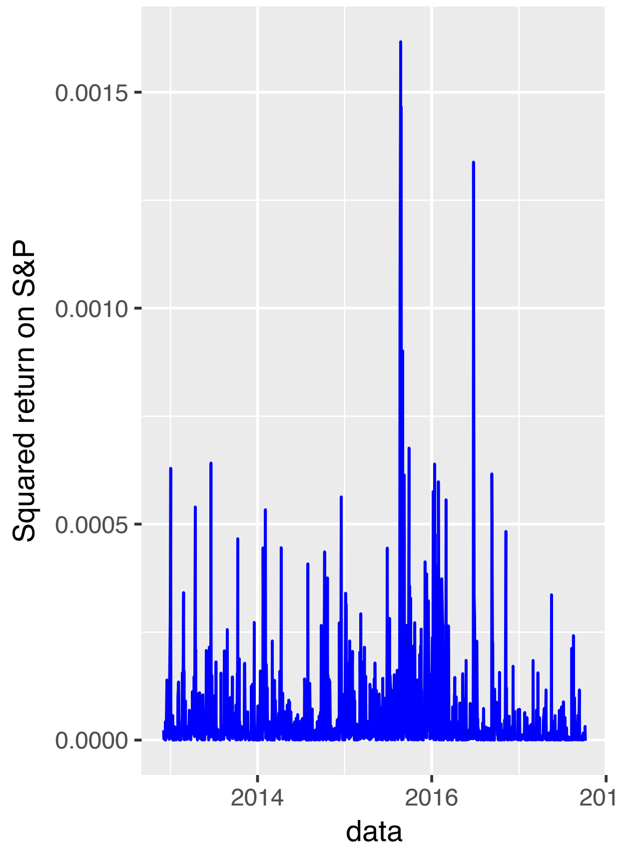
```
## Loading required package: gridExtra
```

```
grid.arrange(p1,p2, ncol=2)
```

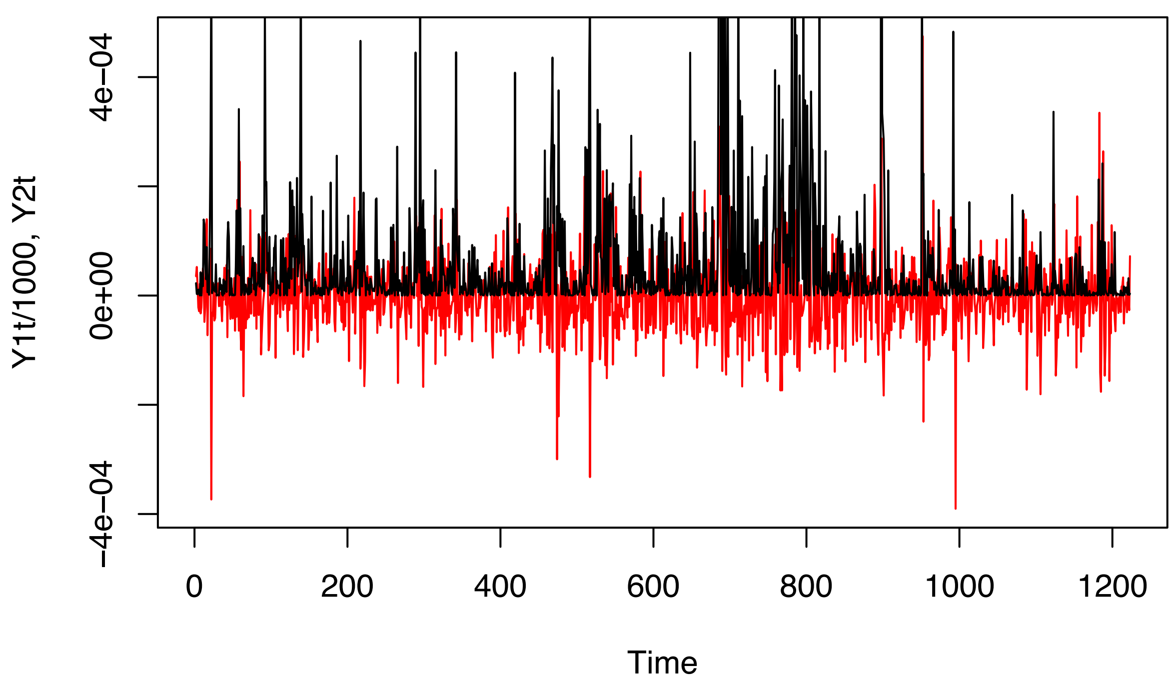
```
## Warning: Removed 1 rows containing missing values (geom_path).

## Warning: Removed 1 rows containing missing values (geom_path).
```



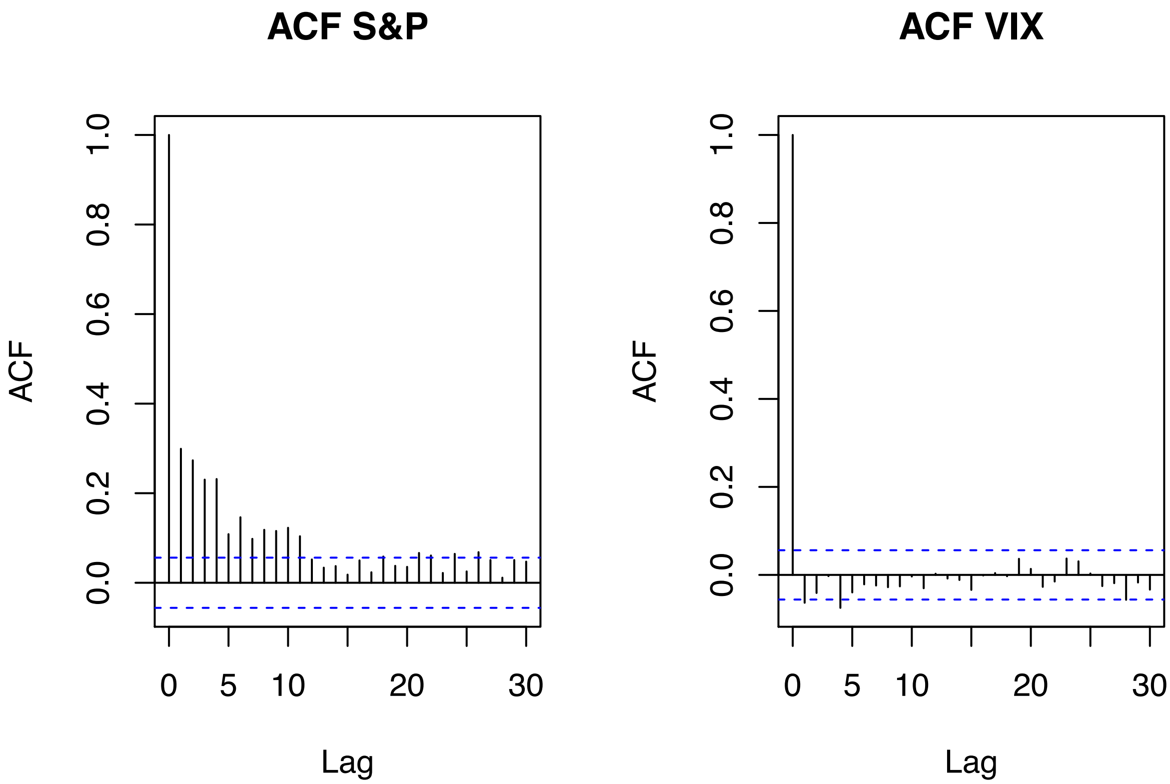



```
plot(ts(final_dataset$vix2/1000),col="red",ylab="Y1t/1000, Y2t")
lines(ts(final_dataset$sp2),col="black")
```

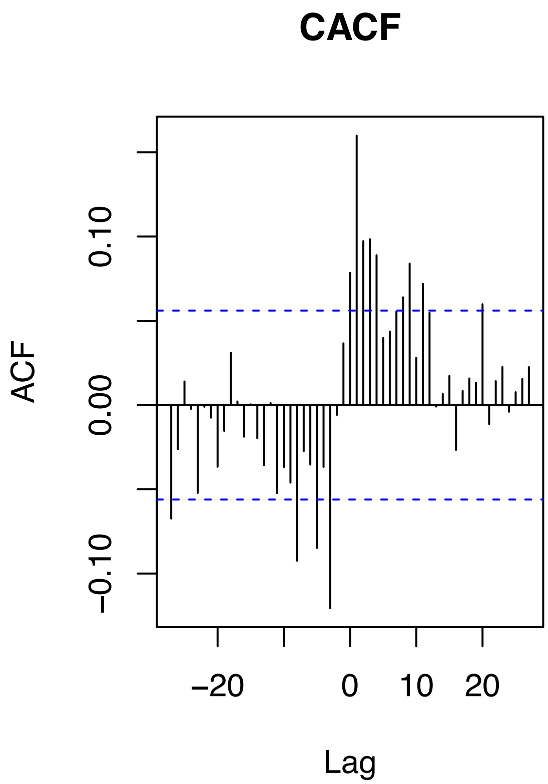


Autocorrelation functions

```
par(mfrow=c(1,2))
acf(final_dataset$sp2,na.action=na.pass,main="ACF S&P")
acf(final_dataset$vix2,na.action=na.pass,main="ACF VIX")
```




```
ccf(final_dataset$sp2,final_dataset$vix2,na.action=na.pass,main="CACF")
```



VAR modelling

```
final_dataset<-na.omit(final_dataset)
var_mod<-VAR(final_dataset[,c(4,5)])
summary(var_mod)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: sp2, vix2
## Deterministic variables: const
## Sample size: 1221
## Log Likelihood: 10780.584
## Roots of the characteristic polynomial:
## 0.3039 0.08222
## Call:
## VAR(y = final_dataset[, c(4, 5)])
##
##
## Estimation results for equation sp2:
## ====================================
## sp2 = sp2.l1 + vix2.l1 + const
##
##          Estimate Std. Error t value Pr(>|t|)
## sp2.l1  2.884e-01  2.714e-02  10.624  < 2e-16 ***
## vix2.l1 2.347e-04  4.642e-05   5.055 4.95e-07 ***
## const   4.168e-05  3.736e-06  11.157  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
##
## Residual standard error: 0.0001181 on 1218 degrees of freedom
## Multiple R-Squared: 0.1082,  Adjusted R-squared: 0.1068
## F-statistic: 73.91 on 2 and 1218 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation vix2:
## =====================================
## vix2 = sp2.l1 + vix2.l1 + const
##
##          Estimate Std. Error t value Pr(>|t|)
## sp2.l1  24.482651  16.763030   1.461   0.1444
## vix2.l1 -0.066711   0.028669  -2.327   0.0201 *
## const   -0.001840   0.002307  -0.798   0.4252
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.07296 on 1218 degrees of freedom
## Multiple R-Squared: 0.005759,    Adjusted R-squared: 0.004127
## F-statistic: 3.528 on 2 and 1218 DF,  p-value: 0.02967
##
##
##
## Covariance matrix of residuals:
##            sp2      vix2
## sp2  1.396e-08 7.027e-07
## vix2 7.027e-07 5.323e-03
##
## Correlation matrix of residuals:
##          sp2    vix2
## sp2  1.00000 0.08152
## vix2 0.08152 1.00000
```

Spectral Decompositions

```r
covar_mat<-matrix(data=c(1.396e-08,7.027e-07,7.027e-07,5.323e-03),nrow=2,ncol=2)
coef_mat<-matrix(data=c(2.884e-01,2.347e-04,24.482651,-0.066711 ),nrow=2,ncol=2)
eigen(covar_mat)
```

```
## $values
## [1] 5.323000e-03 1.386723e-08
##
## $vectors
##               [,1]          [,2]
## [1,] 0.0001320124 -0.9999999913
## [2,] 0.9999999913  0.0001320124
```

```r
eigen(coef_mat)
```

```
## $values
## [1]  0.30390416 -0.08221516
##
## $vectors
```

```
##                [,1]        [,2]
## [1,] 0.9999997995 -0.99988544
## [2,] 0.0006332713  0.01513613
```

Predictions

```
prediction<-predict(var_mod,n.ahead=50)
par(mfrow=c(1,2))
plot(prediction$fcst$sp2[,1],type="l",ylab="S&P prediction")
plot(prediction$fcst$vix2[,1],type="l",ylab="VIX prediction")
```