Methods 2 – Portfolio Assignment 2

- *Type:* Individual assignment
- *Due:* 10 April 2022, 23:59

1. Square root function

Along the lines of section 6.4.2 (p. 247ff) in Gill's book, write an *R* function that calculates the square root of a given positive number. Your solution should contain:

a) A quick introduction into what the function does and why it works

Using Newton's method, this function returns the square root of an integer.

In situations where the absolute value of $(r^2 - x) >= delta,$, the operation = $r = (r + x/r)/2$ is repeated to find the ideal output of the square root of the given x value.

b) A discussion of the choices you made (e.g., starting point of the algorithm)

My first step is to name the function "square.root"

This includes the x values, which represent the value from which I would like to obtain the square root, and r, which represents my starting point and determines my output.

In order to avoid complicating the functions, I define the starting point for each instance.

When the function reaches the accurate square root of the given x value, a "while" statement is used to ensure it stops running without proceeding any further.

This new value of r is returned as a result of my square root function.

c) A range of examples

```
square.root<-function(x,r)
{delta<-0.1e-6
while(abs(r^2-x)>=delta)
{r=(r+x/r)/2
 }
output<-list(r)
names(output)<-("x.sqrt")
return(output)
}

square.root(6,2)

## $x.sqrt
## [1] 2.44949

square.root(99,2)

## $x.sqrt
## [1] 9.949874

square.root(54,2)

## $x.sqrt
## [1] 7.348469

square.root(32,2)

## $x.sqrt
## [1] 5.656854
```

2. Power series derivatives

The power series definitions ($:=$ means "is defined as") of the exponential, sine, and cosine functions are

$$\begin{aligned}
\exp(x) \quad &:= \sum_{n=0}^{\infty} \frac{x^n}{n!}, \\
\sin(x) \quad &:= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \\
\cos(x) \quad &:= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}.
\end{aligned}$$

Using these definitions, show that

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\exp(x) &= \exp(x),\\ \frac{\mathrm{d}}{\mathrm{d}x}\sin(x) &= \cos(x),\\ \frac{\mathrm{d}}{\mathrm{d}x}\cos(x) &= -\sin(x).\end{aligned}$$

Where $exp(x)$ denotes the exponential function e^x, I will show that $\frac{d}{dx}exp(x) = exp(x)$.

As shown in the following example, the function can be expressed as a power series:

$$exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots \frac{x^n}{n!}$$

As a result, I can calculate the derivative of the expansion term by term.

$$\frac{d}{dx}exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots \frac{x^{n-1}}{(n-1)!}$$

Every term in the root power series is equal to its left-hand side neighbor. The first term, however, is equal to 0. Therefore:

$$\frac{d}{dx}exp(x) = exp(x)$$

This is how sine can be expanded:

$$sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \pm \ldots$$

I once again take the derivatives of the expansion term by term, thus resulting in the series:

$$\frac{d}{dx}sin(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \pm \ldots$$

Since $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \pm \ldots$

It is hereby shown, that

$$\frac{d}{dx}sin(x) = cos(x)$$

The cosine function can be expanding like this:

$$cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \pm \ldots$$

The derivatives can now be taken term by term, thus resulting in the following series:

$$\frac{d}{dx}cos(x) = -x + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} \pm \ldots$$

Since $sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \pm \ldots$

it is hereby shown, that

$$\frac{d}{dx}cos(x) = -sin(x)$$