# Study Material - Youtube

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# Video Overview

This tutorial, "Forward Pass and Backpropagation," from the *Mathematical Foundations of Generative AI* course, provides a foundational, step-by-step walkthrough of the core training mechanism of a simple neural network. The instructor uses a Multi-Layer Perceptron (MLP) to demonstrate how a network processes input data to make a prediction (the **forward pass**) and how it uses the resulting error to adjust its internal parameters (the **backward pass** or **backpropagation**). The lecture is framed around a simple regression task, making the concepts tangible and easy to follow.

### Learning Objectives

Upon completing this tutorial, a student will be able to: - **Understand the fundamental workflow of supervised learning** in the context of neural networks. - **Differentiate conceptually and mathematically between the forward pass and the backward pass.** - **Calculate the output of a simple MLP** for a given input by performing a forward pass. - **Grasp the role of key components** like input features, weights, biases, and activation functions. - **Understand the purpose of a loss function**, specifically the Mean Squared Error (MSE), in quantifying prediction error. - **Follow the application of the chain rule** to calculate the gradient of the loss with respect to network weights. - **Comprehend the gradient descent update rule** for optimizing network parameters.

### Prerequisites

To fully benefit from this tutorial, students should have a basic understanding of: - **Neural Network Basics:** Familiarity with the concept of a Multi-Layer Perceptron (MLP) or a Fully Connected (FC) network, including layers and nodes. - **Regression:** Knowledge of regression as a supervised learning task where the goal is to predict a continuous value. - **Basic Calculus:** A foundational understanding of derivatives and the chain rule is essential for the backpropagation section. - **Basic Linear Algebra:** Concepts like vectors and dot products are implicitly used.

**Key Concepts Covered in This Video**

- Supervised Learning Framework
- Forward Pass
- Backward Pass (Backpropagation)
- Multi-Layer Perceptron (MLP)
- Activation Functions (Sigmoid, ReLU)
- Loss Function (Mean Squared Error)
- Gradient Descent
- Chain Rule of Differentiation

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# Fundamentals of Supervised Learning and Neural Networks

The instructor begins by framing the discussion within the paradigm of supervised learning, which is the foundation for many neural network applications.

## The Goal: Learning a Mapping Function

**Intuitive Foundation (01:36):** In supervised learning, we have a dataset where we know the "right answers." For each input, we have a corresponding correct output. The goal of the neural network is to learn the underlying relationship, or **mapping function**, that transforms the inputs into the correct outputs.

Imagine you are teaching a child to identify fruits. You show them an apple (input) and say "apple" (output). You show them a banana (input) and say "banana" (output). After many examples, the child learns the mapping from the visual characteristics of a fruit to its name. A neural network learns in a similar way, but with numerical data.

**Mathematical Formulation (02:19):** This mapping is represented by a function, which we can call $f(\cdot)$. The network's objective is to approximate this true underlying function. - **Input:** $x_i$ - **Output:** $y_i$ - **Learned Mapping:** The network learns a function $\hat{f}$ (parameterized by its weights $\theta$) that approximates the true mapping, such that $\hat{y}_i = \hat{f}(x_i) \approx y_i$.

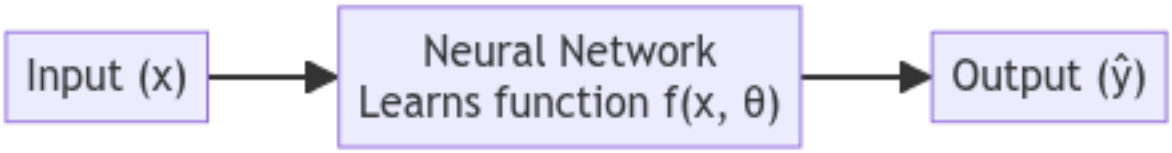


*Figure 1: The basic goal of a neural network is to learn a function that maps inputs to outputs.*

## Representing Data: Inputs and Outputs

**Training Data (01:50):** The data used to teach the network is called the **training data**. It consists of pairs of input features and their corresponding target labels. - **Input Feature ($x_i$):** A vector of numerical values that describe a single data point. For example, if we are predicting house prices, the features might be `[square_footage, number_of_bedrooms]`. - **Target Label ($y_i$):** The "correct answer" for the input $x_i$. In a regression task, this is a continuous number (e.g., the actual price of the house).

The entire training dataset, $D$, is a collection of these pairs (02:38):

$$D = \{(x_i, y_i)\}_{i=1}^{m}$$

where $m$ is the total number of training examples.

## The Training Process: Forward and Backward Pass

The training of a neural network is an iterative process involving two main steps: the forward pass and the backward pass.

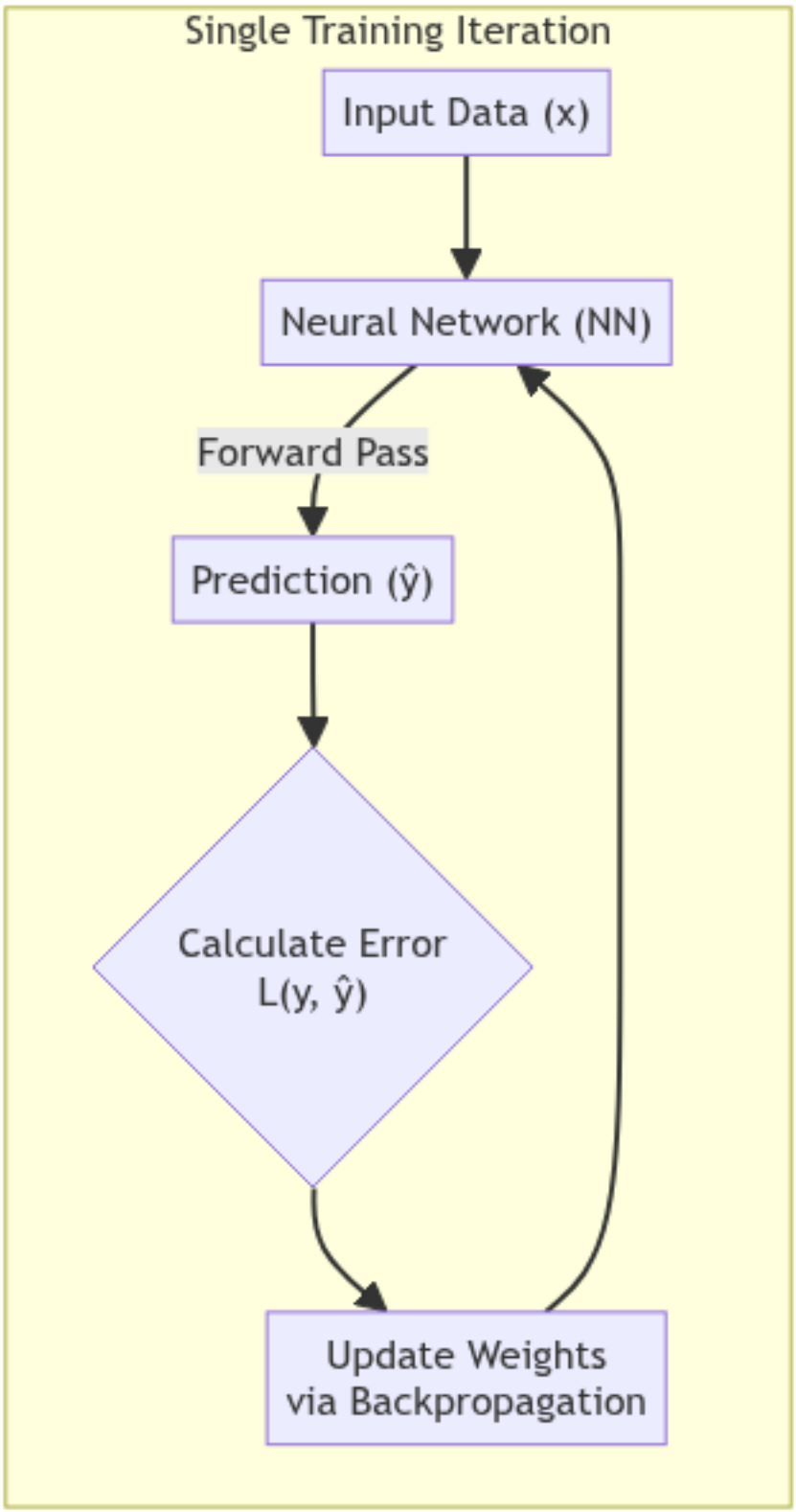


*Figure 2: The iterative cycle of forward pass, error calculation, and backward pass for training a neural network.*

**1. Forward Pass: From Input to Prediction**

**Intuitive Explanation (04:12):** The forward pass is the process of information flowing "forward" through the network. An input sample $x_i$ is fed into the first layer, and its values are passed from layer to layer, with computations happening at each node, until a final prediction $\hat{y}_i$ is produced at the output layer.

**Mathematical Formulation (04:15):** The prediction $\hat{y}_i$ is a function of the input $x_i$ and the network's parameters $\theta$ (all the weights and biases).

$$\hat{y}_i = f(x_i; \theta)$$

This process involves a series of linear combinations (multiplying inputs by weights and adding biases) followed by non-linear activation functions at each layer.

**2. Backward Pass (Backpropagation): Learning from Error**

**Intuitive Explanation (04:23):** After the forward pass, we compare the network's prediction $\hat{y}_i$ with the true target label $y_i$ to calculate the error. Backpropagation is the clever algorithm that takes this error and propagates it "backward" through the network. It determines how much each weight and bias in the network contributed to the final error.

**Goal (03:45):** The ultimate goal is to **minimize the error**. To do this, we need to adjust the network's parameters. Backpropagation tells us the *direction* and *magnitude* of the adjustment needed for each parameter.

**Mathematical Formulation (30:33):** Backpropagation uses the **chain rule** from calculus to compute the gradient of the loss function $L$ with respect to each parameter in $\theta$. This gradient, $\frac{\partial L}{\partial w}$ for a weight $w$, tells us how a small change in the weight will affect the loss. We then use an optimization algorithm like **Gradient Descent** to update the weights in the direction that minimizes the loss.

The update rule for a single weight $w_{ij}^k$ is:

$$w_{ij}^k(\text{new}) \leftarrow w_{ij}^k(\text{old}) - \alpha \frac{\partial L}{\partial w_{ij}^k}$$

- $\alpha$ is the **learning rate**, a small positive number that controls the step size of the update.

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# A Simple MLP for Regression: A Walkthrough

The instructor demonstrates these concepts with a simple, fully-connected neural network.

## Network Architecture

The example network has two input nodes, one hidden layer with two nodes, and one output node.

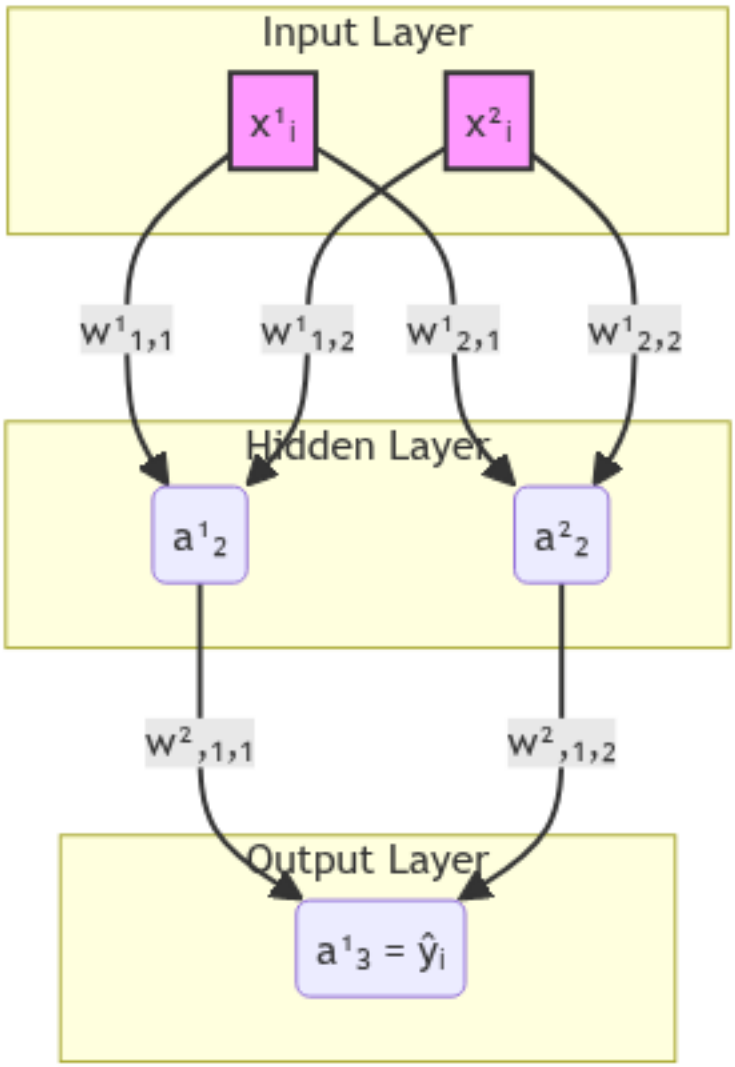


*Figure 3: A simple 2-2-1 MLP architecture as described in the tutorial. The notation $w^k_{i,j}$ represents the weight in layer $k$ from source node $j$ to destination node $i$. The instructor uses a slightly different notation in the video, which is clarified in the mathematical sections.*

**The Forward Pass: Step-by-Step Calculation**

Let's trace the computation from input to output.

**1. Pre-computation (Input Layer):** The activations of the input layer nodes are simply the input features themselves. For clarity, we can denote the input layer as layer 1. - Activation of first input node: $a^1_1 = x^1_i$ - Activation of second input node: $a^1_2 = x^2_i$

**2. Hidden Layer (Layer 2) Computation:** Each node in the hidden layer computes a weighted sum of its inputs and then applies a non-linear activation function.

- **Pre-activation for hidden node 1 ($z^2_1$):**

$$z^2_1 = (w^1_{1,1} \cdot a^1_1 + w^1_{1,2} \cdot a^1_2) + b^1_1$$

- **Activation for hidden node 1 ($a^2_1$):**

$$a^2_1 = g(z^2_1)$$

- **Pre-activation for hidden node 2 ($z_2^2$):**

$$z_2^2 = (w_{2,1}^1 \cdot a_1^1 + w_{2,2}^1 \cdot a_2^1) + b_2^1$$

- **Activation for hidden node 2 ($a_2^2$):**

$$a_2^2 = g(z_2^2)$$

**3. Output Layer (Layer 3) Computation:** The output node takes the activations from the hidden layer as its inputs.

- **Pre-activation for output node 1 ($z_1^3$):**

$$z_1^3 = (w_{1,1}^2 \cdot a_1^2 + w_{1,2}^2 \cdot a_2^2) + b_1^2$$

- **Final Prediction ($\hat{y}_i$):**

$$\hat{y}_i = a_1^3 = g(z_1^3)$$

**Note on Activation Functions:** For regression tasks, the final layer's activation function is often the **identity function** ($g(z) = z$), as the output is not constrained. However, for this tutorial, the instructor uses a sigmoid function throughout for demonstration purposes.

### Activation Functions Explained

Activation functions introduce non-linearity into the network, allowing it to learn complex patterns. Without them, a neural network would just be a series of linear operations, equivalent to a single linear model.

- **Sigmoid Function (18:02):** The sigmoid function squashes its input into a range between 0 and 1.

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Its derivative is conveniently expressed in terms of the function itself:

$$\frac{d\sigma(z)}{dz} = \sigma(z)(1 - \sigma(z))$$

- **ReLU (Rectified Linear Unit) (18:24):** ReLU is a very popular activation function. It is computationally efficient and helps mitigate the vanishing gradient problem.

$$g(z) = \begin{cases} z & \text{if } z > 0 \\ 0 & \text{if } z \leq 0 \end{cases}$$

### Numerical Example of Forward Pass

The instructor provides a concrete example at (38:14) to make the process clear. - **Input:** $x_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ - **Parameters:** All weights and biases are initialized to 1. - **Activation Function:** Sigmoid ($\sigma$)

1. **Hidden Layer Activations:**
   - $a_1^2 = \sigma(w_{1,1}^1 x_i^1 + w_{1,2}^1 x_i^2 + b_1^1) = \sigma(1 \cdot 1 + 1 \cdot 0 + 1) = \sigma(2) \approx 0.88$
   - $a_2^2 = \sigma(w_{2,1}^1 x_i^1 + w_{2,2}^1 x_i^2 + b_2^1) = \sigma(1 \cdot 1 + 1 \cdot 0 + 1) = \sigma(2) \approx 0.88$
2. **Output Layer Activation (Prediction):**
   - $\hat{y}_i = a_1^3 = \sigma(w_{1,1}^2 a_1^2 + w_{1,2}^2 a_2^2 + b_1^2) = \sigma(1 \cdot 0.88 + 1 \cdot 0.88 + 1) = \sigma(2.76) \approx 0.94$

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## Backpropagation: Learning from Error

Once we have the prediction $\hat{y}_i$, we can compute the error and use backpropagation to update the weights.

### The Loss Function: Mean Squared Error (MSE)

For a regression task, a common choice for the loss function is the **Mean Squared Error (MSE)**. For a single training example, the loss $L$ is (25:31):

$$L = \frac{1}{2}(y_i - \hat{y}_i)^2$$

The goal is to find the parameters $\theta$ that minimize this loss.

### The Chain Rule: The Engine of Backpropagation

To minimize the loss using gradient descent, we need to find the partial derivative of the loss with respect to every weight and bias in the network. The **chain rule** allows us to do this by breaking down the complex dependency into a chain of simpler derivatives.

**Intuitive Explanation (32:05):** Imagine we want to know how changing a weight in the first layer, say $w^1_{1,1}$, affects the final loss $L$. This weight doesn't affect the loss directly. Instead, it affects the pre-activation $z^2_1$, which affects the activation $a^2_1$, which in turn affects $z^3_1$, then the final prediction $a^3_1$ (or $\hat{y}_i$), and finally the loss $L$. The chain rule lets us multiply the rates of change at each step in this path to find the overall effect.

**Mathematical Derivation (35:53):** Let's compute the gradient for the weight $w^1_{1,1}$. The path of influence is: $w^1_{1,1} \to z^2_1 \to a^2_1 \to z^3_1 \to a^3_1 \to L$

Using the chain rule, we get:

$$\frac{\partial L}{\partial w^1_{1,1}} = \frac{\partial L}{\partial a^3_1} \cdot \frac{\partial a^3_1}{\partial z^3_1} \cdot \frac{\partial z^3_1}{\partial a^2_1} \cdot \frac{\partial a^2_1}{\partial z^2_1} \cdot \frac{\partial z^2_1}{\partial w^1_{1,1}}$$

Let's compute each term: 1. $\frac{\partial L}{\partial a^3_1} = \frac{\partial L}{\partial \hat{y}_i} = \frac{\partial}{\partial \hat{y}_i}\left[\frac{1}{2}(y_i - \hat{y}_i)^2\right] = -(y_i - \hat{y}_i) = (\hat{y}_i - y_i)$ 2. $\frac{\partial a^3_1}{\partial z^3_1} = \frac{\partial g(z^3_1)}{\partial z^3_1} = g'(z^3_1)$. For sigmoid, this is $a^3_1(1 - a^3_1)$. 3. $\frac{\partial z^3_1}{\partial a^2_1} = \frac{\partial}{\partial a^2_1}(w^2_{1,1}a^2_1 + w^2_{1,2}a^2_2 + b^2_1) = w^2_{1,1}$ 4. $\frac{\partial a^2_1}{\partial z^2_1} = \frac{\partial g(z^2_1)}{\partial z^2_1} = g'(z^2_1)$. For sigmoid, this is $a^2_1(1 - a^2_1)$. 5. $\frac{\partial z^2_1}{\partial w^1_{1,1}} = \frac{\partial}{\partial w^1_{1,1}}(w^1_{1,1}a^1_1 + w^1_{1,2}a^1_2 + b^1_1) = a^1_1 = x^1_i$

By multiplying these five terms, we get the gradient for a single weight. This process is repeated for all parameters.

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## Overall Training Procedure

The instructor summarizes the entire training loop in pseudocode format (48:27).

```
Initialize all weights and biases ( ) randomly or to small values.

for e in range(max_epochs):
  Shuffle the training data D.
  Divide D into k batches: d , d , ..., d .

  for each batch d  in D:
    // 1. Forward Pass
    For each example (x, y) in the batch:
      Compute prediction ŷ = f(x;  )

    // 2. Loss Computation
    Compute the average loss L for the batch.

    // 3. Backward Pass (Backpropagation)
```

```
Compute gradients of the loss with respect to all parameters: L/

// 4. Update Weights
Update all parameters using gradient descent:
 _new =  _old -   * ( L/ )
```

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# Key Mathematical Concepts Summary

| Concept | Formula | Description |
|---|---|---|
| **Forward Pass (Activation)** | $a_i^k = g\left(\sum_j w_{i,j}^k a_j^{k-1} + b_i^k\right)$ | Activation of the i-th node in layer k. |
| **Mean Squared Error (MSE) Loss** | $L = \frac{1}{2}(y - \hat{y})^2$ | Measures the squared difference between the true value and the prediction. |
| **Sigmoid Activation** | $\sigma(z) = \frac{1}{1+e^{-z}}$ | Squashes values to a (0, 1) range. |
| **Sigmoid Derivative** | $\sigma'(z) = \sigma(z)(1 - \sigma(z))$ | Derivative of the sigmoid function. |
| **ReLU Activation** | $g(z) = \max(0, z)$ | Outputs the input if positive, otherwise zero. |
| **Gradient Descent Update** | $w \leftarrow w - \alpha \frac{\partial L}{\partial w}$ | Rule for updating a weight to minimize loss. |

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# Visual Elements from the Video

- **Supervised Learning Mapping (02:02):** A simple diagram showing an "input" circle mapping to an "output" circle via a function $f(\cdot)$. This visually represents the core goal of supervised learning.
- **Neural Network Architecture (11:22):** The instructor draws a 2-input, 2-hidden-node, 1-output-node network. This diagram is crucial for understanding the flow of calculations and the notation for weights and activations.
- **Error Analogy (05:44):** A drawing of a soccer goalpost is used to provide an analogy for error. The desired trajectory is the "correct answer," and the actual trajectory is the "prediction." The distance between where the ball lands and the goal is the "error," which the training process aims to reduce.
- **Backpropagation Path Highlighting (34:15):** The instructor highlights the specific path of dependencies from a single weight in the first layer all the way to the final output, illustrating the chain of computations that the chain rule must traverse.

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# Self-Assessment for This Video

1. **Conceptual Question:** What is the fundamental difference between the forward pass and the backward pass in a neural network? What is the primary purpose of each?
2. **Calculation Problem:** Using the same 2-2-1 network architecture from the video and assuming all weights and biases are `0.5`, calculate the final prediction $\hat{y}_i$ for the input $x_i = [0.5, 1.0]^T$. Assume the activation function is Sigmoid.
3. **Intuition Question:** Why is the chain rule essential for backpropagation? Why can't we just calculate the derivative of the loss with respect to a weight directly?
4. **Parameter Question:** In the gradient descent update rule $w \leftarrow w - \alpha \frac{\partial L}{\partial w}$, what does the term $\frac{\partial L}{\partial w}$ represent intuitively? What is the role of the learning rate $\alpha$?

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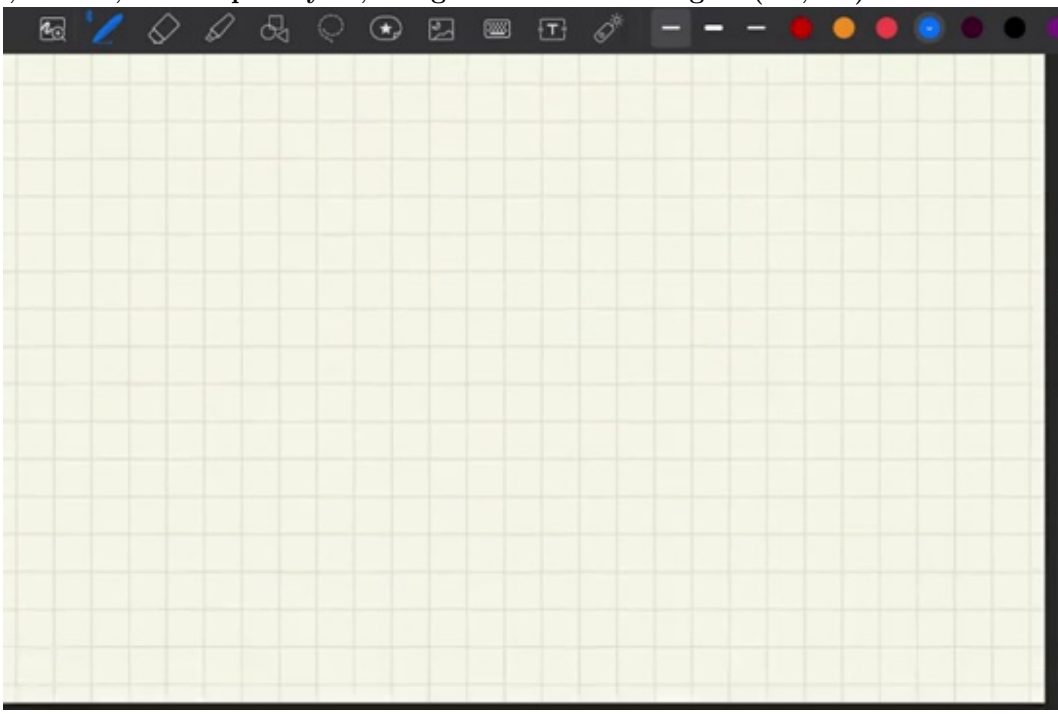
## Key Takeaways from This Video

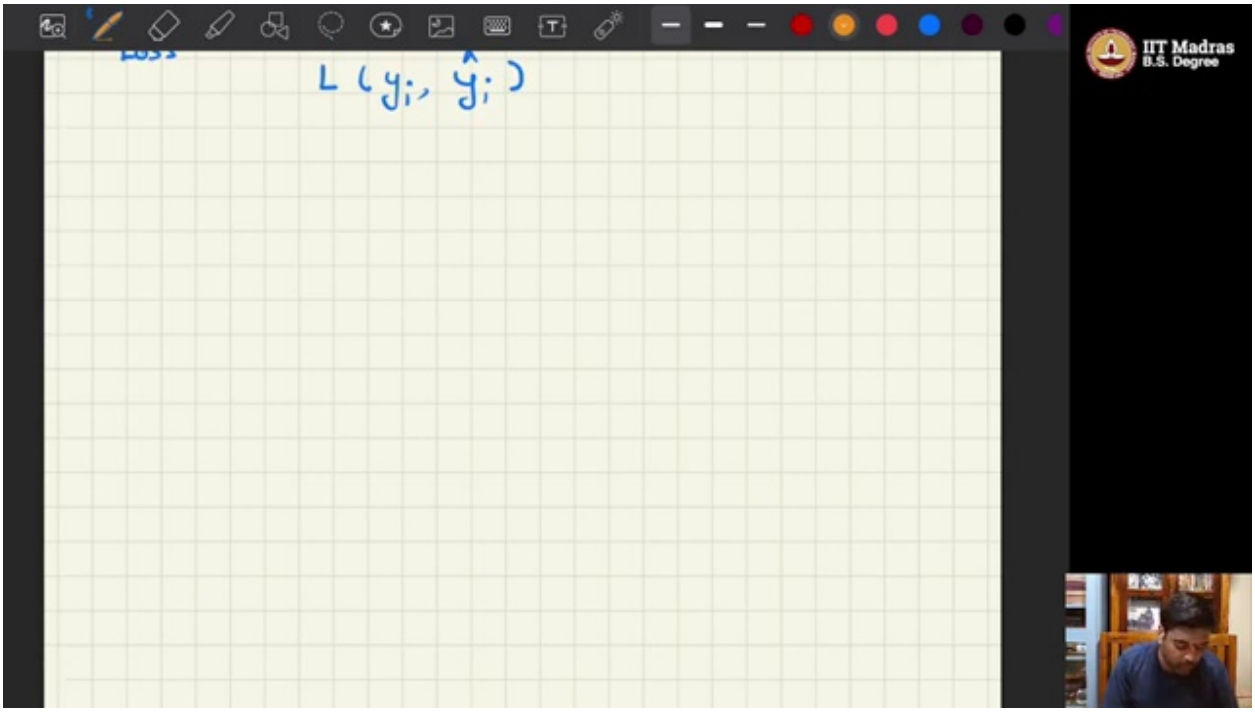
- **Two-Step Process:** Neural network training is a two-step dance: a **forward pass** to make a prediction and a **backward pass** to learn from the error.
- **Forward Pass is Prediction:** It's a cascade of simple calculations (weighted sums and activation functions) flowing from the input layer to the output layer.
- **Backpropagation is Learning:** It uses the chain rule to calculate how much each parameter (weight/bias) is responsible for the final error.
- **Gradient Descent is Correction:** The computed gradients are used to update the parameters in a way that is likely to reduce the error in the next iteration.
- **Everything is Differentiable:** The entire process, from the loss function back through every activation function and linear combination, must be differentiable for backpropagation to work.

### Visual References

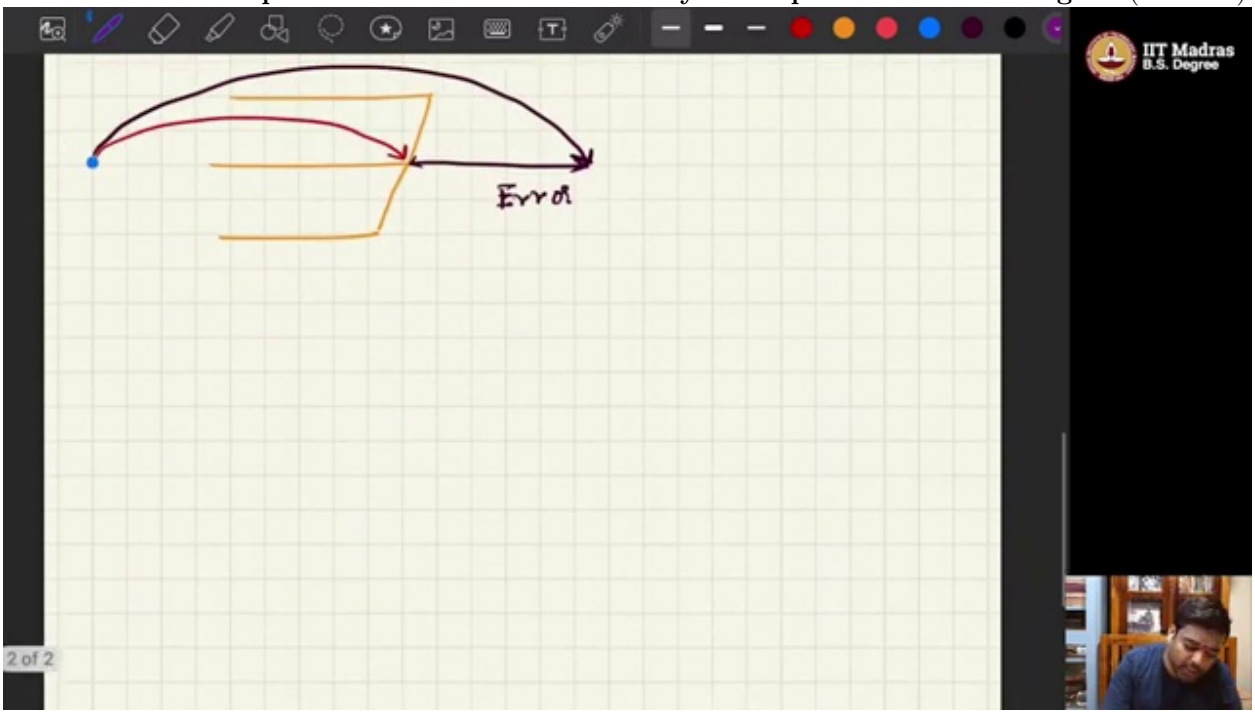
**A diagram of the simple Multi-Layer Perceptron (MLP) architecture used throughout the example. It clearly labels the input, hidden, and output layers, along with all initial weights (w1, w2) and biases (b1, b2).** (at 03:15):



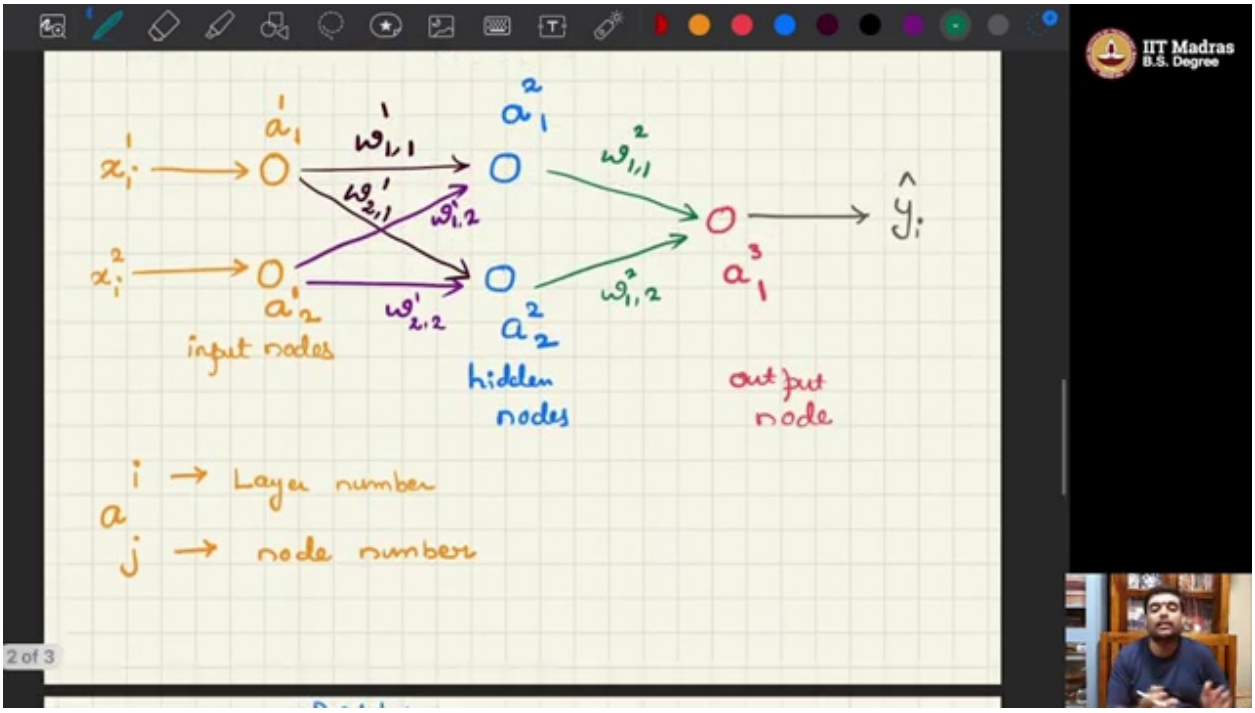
**The MLP diagram fully annotated with the results of the forward pass calculation. This visual shows the step-by-step flow of the input value through the network to produce the final prediction (y_hat), including all intermediate calculations.** (at 05:45):




**A visual breakdown of the chain rule applied to the network for backpropagation. This screenshot illustrates how the gradient of the loss is calculated and propagated backward from the output to determine its sensitivity to a specific network weight.** (at 08:30):




**A final summary slide that consolidates all the key mathematical formulas covered in the lecture. It provides a side-by-side view of the forward pass equations, the loss function, the chain rule derivatives, and the weight update rule.** (at 15:40):


IIT Madras
B.S. Degree
input nodes
hidden nodes
output node
i → Layer number
j → node number
2 of 3