Study Material - Youtube

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Video Overview

This video provides a comprehensive tutorial on the mathematical foundations and practical implementation of Wasserstein Generative Adversarial Networks (WGANs). The lecture begins by outlining the training instabilities inherent in standard GANs, such as the saddle point problem and gradient saturation. It then introduces the Wasserstein distance, also known as the Earth Mover's Distance, as a more stable alternative for the loss function. The instructor derives the mathematical formulation for the Wasserstein-1 distance, explaining the core concepts of "transport plans" and the Lipschitz constraint. The second half of the video is a hands-on coding tutorial in PyTorch, demonstrating how to build and train a WGAN from scratch. This includes setting up the Generator and Critic architectures, defining the WGAN-specific loss functions, and implementing the training loop with a focus on the weight clipping technique used to enforce the Lipschitz constraint.

Learning Objectives

Upon completing this lecture, a student will be able to: - Understand the key limitations of standard GANs that motivate the development of WGANs. - Grasp the intuitive and mathematical definition of the Wasserstein-1 distance (Earth Mover's Distance). - Understand the role of the "Critic" in WGANs and how it differs from a standard GAN's "Discriminator." - Explain the importance of the 1-Lipschitz constraint and how weight clipping is used to approximate it. - Implement a complete WGAN in PyTorch, including the Generator, Critic, optimizers, and the unique training loop. - Analyze and interpret the WGAN loss functions for both the Critic and the Generator.

Prerequisites

To fully benefit from this tutorial, students should have: - A foundational understanding of the standard Generative Adversarial Network (GAN) architecture and training process. - Familiarity with neural network

concepts, including convolutional layers, transposed convolutional layers, and activation functions. - Practical programming experience in Python and the PyTorch deep learning framework. - Basic knowledge of probability theory, including distributions and the concept of expectation.

Key Concepts Covered

- **Wasserstein GAN (WGAN):** An alternative to the original GAN that minimizes an approximation of the Wasserstein distance, leading to more stable training.
- **Wasserstein Distance (Earth Mover's Distance):** A metric that measures the distance between two probability distributions.
- **Transport Plan:** A joint distribution that describes the optimal way to "move" mass from one probability distribution to another to transform it.
- **Critic:** The WGAN equivalent of the discriminator, which outputs a real-valued score rather than a probability.
- **1-Lipschitz Constraint:** A mathematical constraint on the critic function, essential for the WGAN formulation.
- **Weight Clipping:** A practical technique to enforce the Lipschitz constraint by clamping the critic's weights to a small range.

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1. The Problem with Standard GANs: Training Instability

The instructor begins by highlighting the core challenges that arise when training standard Generative Adversarial Networks (00:29). These issues are the primary motivation for developing more advanced architectures like WGAN.

1.1. The Saddle Point Problem

In a standard GAN, the training process is a minimax game where the generator and discriminator have competing objectives. This can lead to a difficult optimization landscape. The instructor refers to this as the "saddle point problem" (00:38), where the model can get stuck in an equilibrium that is not optimal, leading to unstable training and poor convergence.

1.2. Gradient Saturation

A common issue in standard GANs is that if one network (e.g., the discriminator) becomes much stronger than the other, the gradients provided to the weaker network (the generator) can become very small or "saturate" (00:47). This is particularly problematic when the discriminator uses a sigmoid activation function. If the discriminator becomes too confident, its output will be close to 0 or 1, where the sigmoid function is flat, resulting in vanishing gradients. This effectively stops the generator from learning.

> **Key Insight:** WGANs were designed to address these training instabilities by replacing the Jensen-Shannon divergence (the implicit loss in standard GANs) with the Wasserstein distance, which provides smoother and more reliable gradients.

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2. Wasserstein Distance: A New Foundation for GAN Loss

To overcome the issues of standard GANs, WGAN introduces the **Wasserstein distance** as the loss function. It is also famously known as the **Earth Mover's Distance (EMD)** (01:08).

2.1. Intuitive Understanding: The Earth Mover's Distance

Conceptually, the Wasserstein distance measures the minimum "cost" or "work" required to transform one probability distribution into another.

Imagine you have two different piles of dirt, representing two probability distributions. The EMD is the minimum effort needed to move the dirt from the first pile's shape to the second pile's shape. The "cost" is calculated as the amount of dirt moved multiplied by the distance it is moved. The goal is to find the most efficient way to move the dirt, which corresponds to the minimum total cost.

This can be visualized as finding the optimal "transport plan."

```
graph TD
    A["Distribution P<br/>(Initial pile of dirt)"] -- "Transport Plan ( )<br/>How to move the dirt" -->
    B -- "Cost = Σ (mass × distance)" --> C["Wasserstein Distance<br/>(Minimum cost to transform P to Q
```

This diagram illustrates the core idea of the Wasserstein distance as the minimum cost to transform one distribution into another, guided by an optimal transport plan.

22.2. Mathematical Formulation of Wasserstein-1 Distance

The instructor introduces the formal definition of the Wasserstein-1 distance between two probability distributions, p_x (the real data distribution) and $\hat{p}_x$ (the generated data distribution) at 01:47.

The formula is given as:

$$W(p_x, \hat{p}_x) = \inf_{\lambda \in \Pi(p_x, \hat{p}_x)} \mathbb{E}_{(x,\hat{x})\sim\lambda}[\|x - \hat{x}\|]$$

Let's break down this equation: - p_x **and** $\hat{p}_x$: The two probability distributions we want to measure the distance between. In the context of GANs, p_x is the distribution of real data, and $\hat{p}_x$ is the distribution of data generated by the generator. - $\Pi(p_x, \hat{p}_x)$: This represents the set of all possible joint probability distributions $\lambda(x, \hat{x})$ whose marginal distributions are p_x and $\hat{p}_x$. - $\lambda(x, \hat{x})$: A single "transport plan." It tells us how much probability mass should be moved from a point x in the first distribution to a point $\hat{x}$ in the second. - $\mathbb{E}_{(x,\hat{x})\sim\lambda}[\|x - \hat{x}\|]$: This is the expected cost of moving mass according to a specific transport plan λ. It's the average distance mass has to travel to transform p_x into $\hat{p}_x$ under that plan. - inf **(Infimum)**: This means we are looking for the "greatest lower bound" over all possible transport plans in Π. In simple terms, we want to find the transport plan λ that results in the **minimum possible expected cost**. This minimum cost is the Wasserstein distance.

Marginalization Property At 02:49, the instructor explains the marginalization property that any valid transport plan λ must satisfy: - $\int \lambda(x, \hat{x}) d\hat{x} = p_x(x)$ - $\int \lambda(x, \hat{x}) dx = p_{\hat{x}}(\hat{x})$

This ensures that if we sum up all the mass moved *from* a point x, we get the original mass at that point, and if we sum up all the mass moved *to* a point $\hat{x}$, we get the target mass at that point.

2.3. The Kantorovich-Rubinstein Duality and the WGAN Loss

The infimum in the original Wasserstein formula is intractable. However, the Kantorovich-Rubinstein duality provides an alternative, more practical formulation:

$$W(p_x, p_\theta) = \sup_{\|f\|_L \leq 1} \left(\mathbb{E}_{x\sim p_x}[f(x)] - \mathbb{E}_{\hat{x}\sim p_\theta}[f(\hat{x})] \right)$$

Here: - f: Is a function that must be **1-Lipschitz**. In WGAN, this function is approximated by the critic network, T_w. - sup **(Supremum)**: We search for the 1-Lipschitz function f that maximizes this expression. - p_θ: The distribution of the generator's output, $G_\theta(z)$.

This dual formulation leads directly to the WGAN objective function (13:52):

$$L = \mathbb{E}_{x\sim p_x}[T_w(x)] - \mathbb{E}_{z\sim p_z}[T_w(G_\theta(z))]$$

- The **critic** tries to **maximize** this loss. - The **generator** tries to **minimize** this loss (which is equivalent to minimizing $-\mathbb{E}_{z\sim p_z}[T_w(G_\theta(z))]$).

2.4. Enforcing the Lipschitz Constraint: Weight Clipping

For the duality to hold, the critic network T_w must be 1-Lipschitz. The original WGAN paper proposed a simple, though imperfect, method to enforce this: **weight clipping** (21:21).

After each gradient update, the weights of the critic network are "clamped" or "clipped" to lie within a small, fixed range, such as `[-0.01, 0.01]`. - **Purpose:** This forces the function learned by the critic to have bounded gradients, which is a necessary condition for being Lipschitz continuous. - **Implementation:** In PyTorch, this is done by iterating through the critic's parameters and using the `.clamp_()` method.

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3. Practical Implementation of WGAN in PyTorch

The second half of the lecture (from 15:49) is a walkthrough of a WGAN implementation in a Google Colab notebook.

3.1. Setup and Hyperparameters

The code begins by importing necessary libraries and defining key hyperparameters.

```
# 1. Import necessary libraries
import torch
import torch.nn as nn
import torch.optim as optim
import torchvision
from torchvision.utils import save_image
from torch.utils.data import DataLoader
import os

# 2. Setup hyperparameters
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
z_dim = 100          # Dimensionality of the latent space
batch_size = 128     # Batch size for training
epochs = 20          # Number of training epochs
lr = 5e-5            # Learning rate
n_critic = 5         # Number of critic updates per generator update
clip_value = 0.01    # The range for weight clipping [-c, c]
img_size = 28        # MNIST image size
channels = 1         # MNIST is grayscale
```

3.2. Network Architectures

Generator The Generator takes a latent vector `z` and upsamples it to produce an image. It uses `ConvTranspose2d` layers. The final `Tanh` activation ensures the output pixel values are in the range `[-1, 1]`, matching the normalized input data.

```
# As shown at 17:51
class Generator(nn.Module):
    def __init__(self):
        super().__init__()
        self.init_size = img_size // 4 # Initial size: 7x7 for MNIST
        self.fc = nn.Linear(z_dim, 128 * self.init_size * self.init_size)
```

```
        self.conv_blocks = nn.Sequential(
            nn.BatchNorm2d(128),
            nn.ConvTranspose2d(128, 64, 4, stride=2, padding=1), # Upsamples to 14x14
            nn.BatchNorm2d(64),
            nn.ReLU(True),
            nn.ConvTranspose2d(64, channels, 4, stride=2, padding=1), # Upsamples to 28x28
            nn.Tanh()
        )

    def forward(self, z):
        x = self.fc(z)
        x = x.view(x.size(0), 128, self.init_size, self.init_size)
        return self.conv_blocks(x)
```

Critic The Critic takes an image and outputs a single, unbounded real number (a raw score). It uses standard `Conv2d` layers and `LeakyReLU` activations. **Crucially, it does not have a final sigmoid layer.**

```
# As shown at 19:06
class Critic(nn.Module):
    def __init__(self):
    ().__init__()
        self.model = nn.Sequential(
            nn.Conv2d(channels, 64, 4, stride=2, padding=1), # Downsamples to 14x14
            nn.LeakyReLU(0.2),
            nn.Conv2d(64, 128, 4, stride=2, padding=1), # Downsamples to 7x7
            nn.BatchNorm2d(128),
            nn.LeakyReLU(0.2),
            nn.Flatten(),
            nn.Linear(128 * 7 * 7, 1) # Output is a single raw score
        )

    def forward(self, x):
        return self.model(x)
```

3.3. The WGAN Training Loop

The training loop (20:03) alternates between training the critic and the generator. A key difference from standard GANs is that the critic is trained for `n_critic` steps for every one step of the generator.

```
flowchart TD
    A["Start Epoch"] --> B{For each batch in DataLoader}
    B --> C["--- Train Critic ---"]
    C --> D{For n_critic iterations}
    D --> E["1. Get real images batch"]
    E --> F["2. Generate fake images batch<br/>(Detach from graph)"]
    F --> G["3. Calculate Critic Loss:<br/>L = E[critic(fake)] - E[critic(real)]"]
    G --> H["4. Backpropagate loss"]
    H --> I["5. Update Critic weights (RMSprop)"]
    I --> J["6. Clip Critic weights"]
    J --> D
    D -- After n_critic steps --> K["--- Train Generator ---"]
    K --> L["7. Generate new fake images"]
    L --> M["8. Calculate Generator Loss:<br/>L = -E[critic(fake)]"]
    M --> N["9. Backpropagate loss"]
```

```
    N --> O["10. Update Generator weights (RMSprop)"]
    O --> B
    B -- End of Epoch --> P["Save sample images"]
    P --> A
```

This flowchart details the WGAN training process, highlighting the separate, iterative updates for the Critic and Generator, and the crucial weight clipping step.

Critic Loss Calculation The critic's goal is to make the score for real images as high as possible and the score for fake images as low as possible. The loss is calculated to be minimized, so the signs are flipped.

```
# As shown at 20:15
loss_C = -torch.mean(critic(real_imgs)) + torch.mean(critic(fake_imgs))
```

Generator Loss Calculation The generator's goal is to produce images that the critic scores highly. Therefore, it tries to maximize the critic's output for fake images, which is equivalent to minimizing the negative of that output.

```
# As shown at 22:01
loss_G = -torch.mean(critic(gen_imgs))
```

Self-Assessment for This Video

1. **Conceptual Questions:**
   - Why is the discriminator in a WGAN called a "critic"? What is the fundamental difference in its output compared to a standard GAN's discriminator?
   - Explain the "Earth Mover's Distance" using an analogy. What does a "transport plan" represent in this analogy?
   - What is the 1-Lipschitz constraint, and why is it crucial for the WGAN loss function to be a valid approximation of the Wasserstein distance?
   - Describe the technique of "weight clipping." What is its purpose, and what are its potential drawbacks?
2. **Mathematical Problems:**
   - Given the two discrete distributions from the lecture, $p_x = [0.4, 0.3, 0.2, 0.1]$ and $p_{\hat{x}} = [0.1, 0.2, 0.3, 0.4]$ over the points $\{1, 2, 3, 4\}$, propose an optimal transport plan (matrix) that you believe minimizes the transport cost. Justify your choice.
   - Write down the complete loss function for the WGAN critic and generator. Explain why one is a maximization problem and the other is a minimization problem.
3. **Coding Exercises:**
   - In the critic's training loop, why is it important to call `.detach()` on the `fake_imgs` tensor? What would happen if you omitted this step?
   - Modify the provided `Critic` network code to remove the `BatchNorm2d` layers. Why might this be a good idea in the context of WGANs? (Hint: Consider how batch statistics might interact with the critic's scoring function).
   - The original WGAN paper used RMSprop. Modify the code to use the Adam optimizer instead. What hyperparameter adjustments might you need to consider?

Key Takeaways from This Video

- **WGANs offer more stable training** than standard GANs by using the Wasserstein distance, which provides smoother gradients and avoids the mode collapse and gradient saturation problems.

- The **Wasserstein distance** measures the minimum cost to transform one distribution into another, providing a more meaningful metric for the distance between the real and generated distributions.
- The WGAN **critic** does not use a sigmoid function; it outputs a raw score, and its function is constrained to be 1-Lipschitz.
- **Weight clipping** is a simple method to enforce the Lipschitz constraint, which is a cornerstone of the WGAN algorithm.
- The WGAN training loop is distinct: the critic is trained more frequently than the generator (e.g., 5 critic updates for every 1 generator update) to ensure it provides a reliable estimate of the Wasserstein distance.