Study Material - Youtube

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Video Overview

This video lecture, titled “Bi-directional GANs,” is part of the “Mathematical Foundations of Generative AI” series. The instructor, Prof. Prathosh A P, addresses a key limitation of standard Generative Adversarial Networks (GANs): the difficulty of inverting the generator. The lecture introduces the Bi-directional GAN (BiGAN) architecture as a solution to this problem, enabling a mapping from the data space back to the latent space. This capability is crucial for tasks like data manipulation and editing. The video details the components of a BiGAN, its modified training objective, and the theoretical underpinnings that allow it to learn this inverse mapping.

Learning Objectives

Upon completing this lecture, students will be able to: - Understand the concept of GAN inversion and its practical applications, such as data editing. - Identify the limitations of a standard (naive) GAN in performing inversion. - Describe the architecture of a Bi-directional GAN (BiGAN), including its three core components: the Generator, the Encoder, and the Discriminator. - Explain the role of the Encoder in mapping data from the data space (X) to the latent space (Z). - Understand how the Discriminator in a BiGAN is modified to operate on joint distributions of data and latent vectors. - Formulate and interpret the minimax objective function for training a BiGAN. - Explain how the BiGAN framework ensures that the learned encoder is the inverse of the generator.

Prerequisites

To fully grasp the concepts in this video, students should have a solid understanding of: - **Standard GANs**: Familiarity with the generator and discriminator, the adversarial training process, and the standard GAN objective function. - **Probability and Distributions**: Concepts of probability distributions, joint distributions, and expectation. - **Neural Networks**: Basic knowledge of neural network architecture and training using backpropagation. - **Calculus and Optimization**: Understanding of gradients, optimization problems (minimax), and how parameters are updated during training.

Key Concepts

- **GAN Inversion**: The process of finding the latent vector z that produces a given data sample x through the generator, i.e., finding z such that $g(z) \approx x$.
- **Bi-directional GAN (BiGAN)**: An extension of the GAN framework that learns a mapping from the data space to the latent space (an encoder) simultaneously with the generator.
- **Encoder Network (E_ϕ)**: A neural network introduced in BiGANs that learns the inverse mapping from the data space X to the latent space Z.
- **Joint Distribution Discrimination**: The BiGAN discriminator is trained to distinguish between the joint distribution of real data and its encoded latent representation, and the joint distribution of generated data and its original latent vector.

Bi-directional GANs (BiGANs) for Invertibility

The Problem of GAN Inversion

(00:11) The lecture begins by posing a fundamental question: How can we modify a standard GAN to make the inversion process possible? In a standard GAN, we have a generator g_θ that maps a latent vector z to a data sample x. However, there is no direct way to go in the reverse direction—from a data sample x back to the latent vector z that generated it.

This inversion is crucial for many applications, such as **data manipulation and editing**. For example, if we want to edit an image x_i, the process would be: 1. **Invert**: Find the latent vector z_i such that $x_i = g_\theta(z_i)$. 2. **Edit in Latent Space**: Apply an editing function f_{edit} to the latent vector to get an edited latent vector, $z_{edit} = f_{edit}(z_i)$. 3. **Generate Edited Data**: Pass the edited latent vector through the generator to get the edited image, $x_{edit} = g_\theta(z_{edit})$.

Without a reliable inversion mechanism, step 1 is computationally expensive and often inaccurate. BiGANs are designed to solve this problem by explicitly learning an inverse mapping.

Architecture of a Bi-directional GAN (BiGAN)

(00:41) A BiGAN modifies the standard GAN architecture by introducing a third component: an **Encoder**. This creates a system with three neural networks working together.

1. Standard GAN Components (Recap)

(00:48) In a naive GAN, we have two networks: - **Generator (g_θ)**: Maps from the latent space Z to the data space X. - **Mathematical Form**: $g_\theta : Z \to X$. It takes a vector z sampled from a prior distribution (e.g., Gaussian $\mathcal{N}(0, I)$) and outputs a data sample $\hat{x} = g_\theta(z)$. - **Discriminator (D_ω)**: Classifies whether an input sample is real (from the true data distribution P_x) or fake (from the generator). - **Mathematical Form**: $D_\omega : X \to [0, 1]$. It takes a data sample x and outputs the probability that it is real.

2. The Encoder: Learning the Inverse Mapping

(01:55) The key innovation of BiGAN is the introduction of an **Encoder network**, denoted as E_ϕ. - **Purpose**: The encoder's job is to learn the inverse of the generator's mapping. It takes a data sample from the data space X and maps it to a corresponding vector in the latent space Z. - **Mathematical Form**: $E_\phi : X \to Z$. It takes a real data sample $x \sim P_x$ and produces a latent representation $\hat{z} = E_\phi(x)$.

This creates a "bi-directional" structure where the generator maps $Z \to X$ and the encoder maps $X \to Z$.

3. The Modified Discriminator

(05:30) In a BiGAN, the discriminator's role is expanded. Instead of just looking at data samples x, it now evaluates **pairs** or **tuples** of data, consisting of a data sample and a latent vector.

The discriminator, D_ω, is designed to distinguish between two types of tuples: 1. **Tuple from the Encoder Path (Real Data, Encoded Latent)**: This pair consists of a real data sample x and its corresponding latent representation $\hat{z}$ produced by the encoder. The tuple is $(x, E_\phi(x))$. 2. **Tuple from the Generator Path (Generated Data, Original Latent)**: This pair consists of a generated data sample $\hat{x}$ and the original latent vector z that produced it. The tuple is $(g_\theta(z), z)$.

The discriminator's goal is to determine if a given (x, z) pair comes from the joint distribution of the encoder or the generator.

The overall architecture can be visualized as follows:

```
graph TD
    subgraph "Generator Path (Fake)"
        Z["Latent Vector<br/>z ~ p(z)"] --> G["Generator<br/>g<sub> </sub>(z)"];
        G --> x_hat["Generated Data<br/>x̂"];
        Z --> D_input_fake["(z, x̂)"];
    end

    subgraph "Encoder Path (Real)"
        X["Real Data<br/>x ~ p(x)"] --> E["Encoder<br/>E<sub> </sub>(x)"];
        E --> z_hat["Encoded Latent<br/>ẑ"];
        X --> D_input_real["(ẑ, x)"];
    end

    subgraph "Discriminator"
        D_input_real --> D{"Discriminator<br/>D<sub> </sub>(z, x)"};
        D_input_fake --> D;
        D --> Output["Real or Fake?"];
    end

    style Z fill:#cde4ff,stroke:#333,stroke-width:2px
    style X fill:#d5fdd5,stroke:#333,stroke-width:2px
    style G fill:#f9f,stroke:#333,stroke-width:2px
    style E fill:#f9f,stroke:#333,stroke-width:2px
    style D fill:#fcf,stroke:#333,stroke-width:2px
```

Figure 1: Architecture of a Bi-directional GAN (BiGAN). The discriminator learns to distinguish between tuples from the generator path (latent vector and generated data) and the encoder path (encoded latent vector and real data).

Training Objective of a BiGAN

(11:45) The training of a BiGAN is formulated as a minimax game involving all three networks: the generator g_θ, the encoder E_ϕ, and the discriminator D_ω.

Intuitive Goal

The core idea is to force the joint distribution of the encoder, $P_E(x, z) = P_x(x) \cdot P_\phi(z|x)$, to match the joint distribution of the generator, $P_G(x, z) = P_z(z) \cdot P_\theta(x|z)$. If these two joint distributions are identical, their marginals must also be identical. This implies that the distribution of encoded vectors $P_\phi(z)$ will match the prior distribution $P_z(z)$, and the distribution of generated data $P_\theta(x)$ will match the real data distribution $P_x(x)$.

Crucially, if the joint distributions match, it implies that E_ϕ is the inverse of g_θ.

Mathematical Formulation

The objective function for BiGAN is a modification of the standard GAN loss. The optimization problem is:

$$\min_{\theta,\phi} \max_{w} L_{BiGAN}(\theta, \omega, \phi)$$

The loss function L_{BiGAN} is defined as:

$$L_{BiGAN}(\theta, \omega, \phi) = \mathbb{E}_{x \sim P_x} \left[\mathbb{E}_{\hat{z} \sim P_\phi(\cdot|x)} [\log D_\omega(x, \hat{z})] \right] + \mathbb{E}_{z \sim P_z} \left[\mathbb{E}_{\hat{x} \sim P_\theta(\cdot|z)} [\log(1 - D_\omega(\hat{x}, z))] \right]$$

Let's break this down:

- **First Term**: $\mathbb{E}_{x \sim P_x} \left[\mathbb{E}_{\hat{z} \sim P_\phi(\cdot|x)} [\log D_\omega(x, \hat{z})] \right]$
  - This term corresponds to the **encoder path**.
  - We sample a real data point x from the true data distribution P_x.
  - We pass it through the encoder to get $\hat{z} = E_\phi(x)$.
  - The discriminator D_ω evaluates the pair $(x, \hat{z})$.
  - The discriminator is trained to maximize this term (output close to 1), while the encoder is trained to minimize it (by producing latent vectors that fool the discriminator).
- **Second Term**: $\mathbb{E}_{z \sim P_z} \left[\mathbb{E}_{\hat{x} \sim P_\theta(\cdot|z)} [\log(1 - D_\omega(\hat{x}, z))] \right]$
  - This term corresponds to the **generator path**.
  - We sample a latent vector z from the prior distribution P_z.
  - We pass it through the generator to get a fake sample $\hat{x} = g_\theta(z)$.
  - The discriminator D_ω evaluates the pair $(\hat{x}, z)$.
  - The discriminator is trained to maximize this term (by making $D_\omega(\hat{x}, z)$ close to 0), while the generator is trained to minimize it (by producing realistic data that fools the discriminator).

  Key Insight: The discriminator's task is to distinguish between the joint distribution of real data and its encoded representation, $P_E(x, z)$, and the joint distribution of generated data and its latent source, $P_G(x, z)$. By training the generator and encoder to fool the discriminator, we force these two joint distributions to become indistinguishable.

Training Process Summary

1. **Discriminator Update**: Maximize L_{BiGAN} with respect to its parameters ω. This involves showing it batches of "real" tuples $(x, E_\phi(x))$ and "fake" tuples $(g_\theta(z), z)$ and updating ω to improve its classification accuracy.
2. **Generator and Encoder Update**: Minimize L_{BiGAN} with respect to their parameters θ and ϕ. This involves updating θ and ϕ to produce tuples that the discriminator classifies as "real".

How BiGAN Achieves Inversion

(16:52) Once the BiGAN is trained and has reached its optimum, the following holds: - The generator g_θ^* can be used for data generation, just like in a standard GAN. - The encoder E_ϕ^* can be used for **inversion**. For any given data point x, $E_\phi^*(x)$ provides the corresponding latent vector z.

The theoretical justification is that at the optimum of the minimax game, the joint distribution of the encoder path matches the joint distribution of the generator path:

$$P(x, z) = P_x(x) P_\phi(z|x) = P_z(z) P_\theta(x|z)$$

This alignment ensures that E_ϕ^* effectively learns the inverse of g_θ^*.

Key Mathematical Concepts

- **Naive GAN Architecture** (00:48):
  - Generator: $g_\theta(z) : Z \to X$
  - Discriminator: $D_\omega(x) : X \to [0, 1]$
- **BiGAN Architecture** (01:22):
  - Generator: $g_\theta(z) : Z \to X$
  - Encoder: $E_\phi(x) : X \to Z$
  - Discriminator: $D_\omega(x, z) : X \times Z \to [0, 1]$
- **BiGAN Objective Function** (11:45):

$$L_{BiGAN}(\theta, \omega, \phi) = \mathbb{E}_{(x,\hat{z})\sim P_E}[\log D_\omega(x, \hat{z})] + \mathbb{E}_{(\hat{x},z)\sim P_G}[\log(1 - D_\omega(\hat{x}, z))]$$

  where the optimization is $\min_{\theta,\phi} \max_\omega L_{BiGAN}$.
- **Joint Distributions at Optimality** (18:28):
  - The training objective forces the joint distribution of the encoder path, $P_E(x, z) = P_x(x)P_\phi(z|x)$, to match the joint distribution of the generator path, $P_G(x, z) = P_z(z)P_\theta(x|z)$.

Visual Elements from the Video

- **BiGAN Architecture Diagram** (02:48, 15:36): The instructor draws a diagram illustrating the two paths of the BiGAN.
  - The **generator path** starts with a latent vector $z \sim \mathcal{N}(0, I)$, goes through the generator g_θ to produce $\hat{x}$, and forms the tuple $(z, \hat{x})$.
  - The **encoder path** starts with a real data sample $x \sim P_x$, goes through the encoder E_ϕ to produce $\hat{z}$, and forms the tuple $(x, \hat{z})$.
  - The **discriminator** D_ω takes these tuples as input and tries to distinguish between them.

Practical Examples and Applications

- **Data Manipulation/Editing** (00:11): The primary application discussed is editing an image. By inverting an image to its latent code, one can perform manipulations (e.g., adding a smile, changing hair color) in the semantically meaningful latent space and then generate the modified image. BiGAN provides the necessary inversion tool (E_ϕ) to make this process efficient.

Self-Assessment for This Video

1. **Question**: What is the primary limitation of a standard GAN that BiGANs aim to solve?
   - **Answer**: Standard GANs do not provide a direct or efficient way to perform inversion, i.e., to find the latent vector z that corresponds to a given data sample x.
2. **Question**: What are the three main components of a BiGAN, and what is the function of each?
   - **Answer**:
     1. **Generator (g_θ)**: Maps a latent vector z to a data sample x ($Z \to X$).
     2. **Encoder (E_ϕ)**: Maps a data sample x to a latent vector z ($X \to Z$). It learns the inverse of the generator.
     3. **Discriminator (D_ω)**: Distinguishes between joint tuples from the generator path $(g_\theta(z), z)$ and the encoder path $(x, E_\phi(x))$.
3. **Question**: How does the discriminator's input in a BiGAN differ from that in a standard GAN?

- **Answer**: In a standard GAN, the discriminator takes a single data sample (x or $\hat{x}$) as input. In a BiGAN, the discriminator takes a tuple containing both a data sample and a latent vector, i.e., (x, z), as input.

4. **Question**: Explain the minimax game in BiGAN training. What is being minimized and what is being maximized?
   - **Answer**: The training is a minimax game where the discriminator's parameters (ω) are updated to maximize the objective function (better distinguish between real/encoded and fake/latent tuples), while the generator's (θ) and encoder's (ϕ) parameters are updated to minimize the objective function (fool the discriminator).
5. **Application Exercise**: Once a BiGAN is trained, describe the steps you would take to find the latent representation of a new, unseen image and then generate a new image from that latent code.
   - **Answer**:
     1. Take the new image x_{new}.
     2. Feed it into the trained encoder network E^*_ϕ to get its latent representation: $z_{new} = E^*_\phi(x_{new})$.
     3. Feed this latent vector z_{new} into the trained generator network g^*_θ to get the reconstructed image: $x_{reconstructed} = g^*_\theta(z_{new})$.

Key Takeaways from This Video

- **Invertibility is Key**: The ability to map from data space back to latent space is a powerful feature for generative models, enabling applications like data editing and feature extraction.
- **BiGAN Solves Inversion**: BiGANs extend the GAN framework by adding an encoder network that learns the inverse mapping of the generator.
- **Joint Distribution Matching**: The core principle of BiGAN is to train a discriminator on the joint space of data and latent vectors, forcing the joint distribution of real data and its encoded version to match that of generated data and its latent source.
- **Three-Player Game**: The training involves a minimax game between three networks (Generator, Encoder, Discriminator), leading to a system where the generator and encoder are approximate inverses of each other.