High Performance Computing
Homework 2: due 3/21/22
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Question 2

I am running the code on my laptop which has an Intel Core i9-10980HK Processor.

Below are the outputs with and withot openmp with block sizes 16, 8, and 32. We see a small speed up with openmp for block size 16, but with a block size of 8 there is a slowdown because the work in organizing the threads is larger than the work needed to be done. For a block size of 32 we see a great speedup and this gets greater as the block size increase. The code submitted has the default block size of 16 with omp.

Output with blocksize equal to 16 Blocked version with no openmp:

Dimension	Time	Gflop/s	GB/s	Error
16	0.479507	4.170954	35.453112	0.000000e+00
64	0.494874	4.041750	32.839221	0.000000e+00
112	0.490088	4.082157	32.948840	0.000000e+00
160	0.489992	4.096064	32.973318	0.000000e+00
208	0.511574	3.940302	31.673970	0.000000e+00
256	0.512991	3.924565	31.519159	0.000000e+00
304	0.492610	4.106292	32.958398	0.000000e+00
352	0.490100	4.093556	32.841486	0.000000e+00
400	0.498517	4.108187	32.947656	0.000000e+00
448	0.553213	3.900794	31.276007	0.000000e+00
496	0.580283	3.785105	30.341889	0.000000e+00
544	0.550685	4.092808	32.802649	0.000000e+00
592	0.521065	3.981740	31.907726	0.000000e+00
640	0.530063	3.956420	31.700817	0.000000e+00
688	0.638600	4.079681	32.684884	0.000000e+00
736	0.578048	4.138290	33.151303	0.000000e+00
784	0.705514	4.098204	32.827449	0.000000e+00
832	0.570982	4.034666	32.316126	0.000000e+00
880	0.667935	4.081065	32.685623	0.000000e+00
928	0.778205	4.107806	32.897858	0.000000e+00
976	0.923151	4.028438	32.260525	0.000000e+00
1024	0.650377	3.301908	26.441064	0.000000e+00
1072	0.600172	4.105241	32.872561	0.000000e+00
1120	0.704796	3.986768	31.922619	0.000000e+00
1168	0.781531	4.077671	32.649299	0.000000e+00
1216	0.916448	3.923946	31.417381	0.000000e+00
1264	1.005110	4.018442	32.172970	0.000000e+00
1312	1.147429	3.936457	31.515663	0.000000e+00

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1360   1.261393   3.988379  31.930492 0.000000e+00
1408   1.414849   3.945736  31.588305 0.000000e+00
1456   1.638780   3.766981  30.156545 0.000000e+00
1504   1.757668   3.871120  30.989551 0.000000e+00
1552   2.021703   3.698179  29.604491 0.000000e+00
1600   2.072999   3.951763  31.633866 0.000000e+00
1648   2.334594   3.834337  30.693311 0.000000e+00
1696   2.532625   3.852447  30.837750 0.000000e+00
1744   2.782259   3.813044  30.521844 0.000000e+00
1792   3.113534   3.696497  29.588481 0.000000e+00
1840   3.518625   3.540874  28.342388 0.000000e+00
1888   3.608982   3.729505  29.851845 0.000000e+00
1936   3.839208   3.780110  30.256499 0.000000e+00
1984   4.093367   3.815701  30.540994 0.000000e+00
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with openmp

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Dimension       Time    Gflop/s       GB/s        Error
   16   0.501572   3.987466  33.893464 0.000000e+00
   64   0.506178   3.951494  32.105885 0.000000e+00
  112   0.440331   4.543439  36.672043 0.000000e+00
  160   0.603885   3.323545  26.754533 0.000000e+00
  208   0.620597   3.248093  26.109669 0.000000e+00
  256   0.602076   3.343875  26.855497 0.000000e+00
  304   0.807424   2.505254  20.107962 0.000000e+00
  352   0.893625   2.245074  18.011620 0.000000e+00
  400   0.620588   3.300096  26.466770 0.000000e+00
  448   0.631123   3.419254  27.415089 0.000000e+00
  496   0.854702   2.569820  20.600007 0.000000e+00
  544   0.783621   2.876198  23.051881 0.000000e+00
  592   0.726867   2.854371  22.873537 0.000000e+00
  640   0.599071   3.500676  28.049165 0.000000e+00
  688   0.695951   3.743490  29.991446 0.000000e+00
  736   0.549600   4.352490  34.867232 0.000000e+00
  784   0.670643   4.311295  34.534356 0.000000e+00
  832   0.520653   4.424681  35.439990 0.000000e+00
  880   0.623278   4.373473  35.027546 0.000000e+00
  928   0.713427   4.480789  35.884940 0.000000e+00
  976   0.821420   4.527353  36.255931 0.000000e+00
 1024   0.522266   4.111859  32.926994 0.000000e+00
 1072   0.577306   4.267842  34.174587 0.000000e+00
 1120   0.693564   4.051331  32.439585 0.000000e+00
 1168   0.746000   4.271885  34.204342 0.000000e+00
 1216   0.858916   4.186780  33.521784 0.000000e+00
 1264   0.980534   4.119157  32.979329 0.000000e+00
 1312   1.087841   4.152083  33.241979 0.000000e+00
 1360   1.278357   3.935450  31.506752 0.000000e+00
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1408	1.295831	4.308138	34.489581	0.000000e+00
1456	1.468567	4.203590	33.651819	0.000000e+00
1504	1.783572	3.814898	30.539476	0.000000e+00
1552	1.847806	4.046212	32.390556	0.000000e+00
1600	2.002932	4.090004	32.740481	0.000000e+00
1648	2.139268	4.184430	33.495755	0.000000e+00
1696	2.328424	4.190304	33.542196	0.000000e+00
1744	2.589858	4.096317	32.789325	0.000000e+00
1792	2.889852	3.982616	31.878704	0.000000e+00
1840	3.040034	4.098311	32.804310	0.000000e+00
1888	3.247172	4.145058	33.178029	0.000000e+00
1936	3.486015	4.163099	33.321996	0.000000e+00
1984	4.024973	3.880539	31.059956	0.000000e+00

With block size 8 without openmp

Dimension	Time	Gflop/s	GB/s	Error
8	0.484535	4.127675	37.149077	0.000000e+00
56	0.496167	4.031437	32.827419	0.000000e+00
104	0.504369	3.965364	32.027939	0.000000e+00
152	0.501347	3.992709	32.151813	0.000000e+00
200	0.516641	3.902128	31.373108	0.000000e+00
248	0.507198	3.969641	31.885177	0.000000e+00
296	0.513863	3.936608	31.599262	0.000000e+00
344	0.522884	3.892606	31.231370	0.000000e+00
392	0.518867	3.947128	31.657579	0.000000e+00
440	0.520077	3.930990	31.519394	0.000000e+00
488	0.542882	3.853246	30.889138	0.000000e+00
536	0.543685	3.965289	31.781493	0.000000e+00
584	0.601263	3.975170	31.855812	0.000000e+00
632	0.512058	3.943867	31.600857	0.000000e+00
680	0.648020	3.881755	31.099707	0.000000e+00
728	0.573154	4.039003	32.356406	0.000000e+00
776	0.697356	4.020516	32.205573	0.000000e+00
824	0.570233	3.924543	31.434443	0.000000e+00
872	0.694426	3.819295	30.589401	0.000000e+00
920	0.788504	3.950204	31.635981	0.000000e+00
968	0.916909	3.956944	31.688252	0.000000e+00
1016	0.533025	3.935173	31.512366	0.000000e+00
1064	0.613787	3.924980	31.429348	0.000000e+00
1112	0.693219	3.967107	31.765395	0.000000e+00
1160	0.787971	3.961809	31.721797	0.000000e+00
1208	0.899387	3.919982	31.385814	0.000000e+00
1256	1.029975	3.847443	30.804050	0.000000e+00
1304	1.163591	3.811207	30.513034	0.000000e+00
1352	1.288107	3.837143	30.719850	0.000000e+00
1400	1.481601	3.704102	29.653980	0.000000e+00

1448	1.670314	3.635278	29.102307	0.000000e+00
1496	1.899542	3.525136	28.219942	0.000000e+00
1544	2.681334	2.745497	21.978198	0.000000e+00
1592	2.552253	3.161808	25.310355	0.000000e+00
1640	3.044705	2.897452	23.193754	0.000000e+00
1688	3.382639	2.843752	22.763494	0.000000e+00
1736	3.377233	3.098261	24.800370	0.000000e+00
1784	3.493815	3.250235	26.016451	0.000000e+00
1832	4.057936	3.030409	24.256505	0.000000e+00
1880	4.252464	3.125093	25.014042	0.000000e+00
1928	4.464600	3.210470	25.697077	0.000000e+00
1976	4.703562	3.280681	26.258729	0.000000e+00

with openmp

Dimension	Time	Gflop/s	GB/s	Error
8	3.484161	0.574027	5.166240	0.000000e+00
56	3.506087	0.570512	4.645601	0.000000e+00
104	3.711086	0.538928	4.352880	0.000000e+00
152	3.685955	0.543070	4.373141	0.000000e+00
200	4.547864	0.443285	3.564012	0.000000e+00
248	4.020530	0.500779	4.022382	0.000000e+00
296	3.869262	0.522807	4.196588	0.000000e+00
344	3.483038	0.584369	4.688541	0.000000e+00
392	3.488771	0.587036	4.708268	0.000000e+00
440	3.497598	0.584520	4.686788	0.000000e+00
488	3.607585	0.579850	4.648303	0.000000e+00
536	4.229333	0.509742	4.085545	0.000000e+00
584	4.539232	0.526547	4.219592	0.000000e+00
632	3.822211	0.528356	4.233536	0.000000e+00
680	4.732841	0.531490	4.258170	0.000000e+00
728	4.508574	0.513460	4.113319	0.000000e+00
776	5.388279	0.520339	4.168076	0.000000e+00
824	4.118010	0.543443	4.352822	0.000000e+00
872	5.149818	0.515012	4.124823	0.000000e+00
920	5.935011	0.524810	4.203042	0.000000e+00
968	6.866150	0.528412	4.231664	0.000000e+00
1016	4.000894	0.524269	4.198279	0.000000e+00
1064	4.713246	0.511134	4.092915	0.000000e+00
1112	5.241937	0.524629	4.200809	0.000000e+00
1160	5.987260	0.521406	4.174842	0.000000e+00
1208	6.866609	0.513439	4.110909	0.000000e+00
1256	7.597373	0.521597	4.176102	0.000000e+00
1304	8.520469	0.520474	4.166987	0.000000e+00
1352	8.816910	0.560588	4.488020	0.000000e+00
1400	10.421287	0.526614	4.215925	0.000000e+00
1448	13.014182	0.466572	3.735155	0.000000e+00

1496	18.797491	0.356225	2.851708	0.000000e+00
1544	14.341918	0.513292	4.108997	0.000000e+00
1592	15.504743	0.520469	4.166365	0.000000e+00
1640	17.142579	0.514618	4.119458	0.000000e+00
1688	18.578526	0.517769	4.144606	0.000000e+00
1736	19.775150	0.529126	4.235449	0.000000e+00
1784	21.352124	0.531831	4.257031	0.000000e+00
1832	22.954740	0.535715	4.288062	0.000000e+00
1880	26.692823	0.497862	3.985015	0.000000e+00
1928	27.671522	0.517986	4.146037	0.000000e+00
1976	29.011354	0.531891	4.257283	0.000000e+00

With block size 32 without openmp

Dimension	Time	Gflop/s	GB/s	Error
32	0.566817	3.528523	29.110317	0.000000e+00
64	0.584123	3.424209	27.821696	0.000000e+00
96	0.573867	3.487346	28.189381	0.000000e+00
128	0.577684	3.463282	27.922707	0.000000e+00
160	0.578060	3.472030	27.949843	0.000000e+00
192	0.572769	3.509481	28.222077	0.000000e+00
224	0.565662	3.536774	28.420504	0.000000e+00
256	0.681717	2.953228	23.718111	0.000000e+00
288	0.581095	3.453107	27.720772	0.000000e+00
320	0.578903	3.509425	28.163136	0.000000e+00
352	0.584902	3.430066	27.518485	0.000000e+00
384	0.645978	3.155577	25.310357	0.000000e+00
416	0.580423	3.472911	27.850073	0.000000e+00
448	0.623212	3.462659	27.763103	0.000000e+00
480	0.641413	3.448389	27.644588	0.000000e+00
512	0.864462	2.484187	19.912308	0.000000e+00
544	0.656525	3.432998	27.514472	0.000000e+00
576	0.666932	3.438485	27.555637	0.000000e+00
608	0.652257	3.445814	27.611853	0.000000e+00
640	0.632070	3.317914	26.584783	0.000000e+00
672	0.697927	3.478466	27.869141	0.000000e+00
704	0.608567	3.440022	27.559265	0.000000e+00
736	0.695833	3.437791	27.539696	0.000000e+00
768	0.873561	3.111298	24.922797	0.000000e+00
800	0.594008	3.447764	27.616593	0.000000e+00
832	0.670008	3.438349	27.539851	0.000000e+00
864	0.741795	3.477901	27.855414	0.000000e+00
896	0.831848	3.458916	27.702213	0.000000e+00
928	0.926018	3.452107	27.646619	0.000000e+00
960	1.025743	3.450127	27.629765	0.000000e+00
992	1.136468	3.435879	27.514744	0.000000e+00
1024	0.886720	2.421828	19.393543	0.000000e+00

1056	0.679976	3.463606	27.735083	0.000000e+00
1088	0.759668	3.390728	27.150753	0.000000e+00
1120	0.824052	3.409805	27.302793	0.000000e+00
1152	0.904640	3.379961	27.063157	0.000000e+00
1184	0.957405	3.467283	27.761688	0.000000e+00
1216	1.053733	3.412717	27.324187	0.000000e+00
1248	1.132240	3.433484	27.489884	0.000000e+00
1280	1.365738	3.071091	24.587920	0.000000e+00
1312	1.327810	3.401698	27.234324	0.000000e+00
1344	1.421640	3.415372	27.343309	0.000000e+00
1376	1.545213	3.372073	26.996191	0.000000e+00
1408	1.674955	3.332995	26.682901	0.000000e+00
1440	1.769216	3.375488	27.022655	0.000000e+00
1472	1.871449	3.408596	27.287294	0.000000e+00
1504	1.984645	3.428394	27.445387	0.000000e+00
1536	2.892141	2.506018	20.061197	0.000000e+00
1568	2.256945	3.416231	27.347274	0.000000e+00
1600	2.397611	3.416734	27.350958	0.000000e+00
1632	2.571498	3.380681	27.062021	0.000000e+00
1664	2.726298	3.379999	27.056242	0.000000e+00
1696	2.848756	3.424935	27.415636	0.000000e+00
1728	3.006785	3.432092	27.472622	0.000000e+00
1760	3.180529	3.428220	27.441340	0.000000e+00
1792	3.760903	3.060215	24.495380	0.000000e+00
1824	3.545763	3.422905	27.398250	0.000000e+00
1856	3.790173	3.373687	27.004041	0.000000e+00
1888	3.992866	3.370942	26.981817	0.000000e+00
1920	4.233768	3.343541	26.762257	0.000000e+00
1952	4.378457	3.397413	27.193231	0.000000e+00
1984	4.595429	3.398826	27.204314	0.000000e+00

with openmp

Dimension	Time	Gflop/s	GB/s	Error
32	0.268476	7.449567	61.458926	0.000000e+00
64	0.278754	7.175357	58.299775	0.000000e+00
96	0.126598	15.808129	127.782380	0.000000e+00
128	0.115237	17.361494	139.977045	0.000000e+00
160	0.124822	16.079230	129.437799	0.000000e+00
192	0.113579	17.697945	142.320973	0.000000e+00
224	0.114262	17.509005	140.697364	0.000000e+00
256	0.149921	13.428854	107.850486	0.000000e+00
288	0.114448	17.532674	140.748412	0.000000e+00
320	0.115313	17.618350	141.387260	0.000000e+00
352	0.117136	17.127528	137.409488	0.000000e+00
384	0.140169	14.542630	116.644013	0.000000e+00
416	0.117271	17.188829	137.841184	0.000000e+00

448	0.127302	16.951562	135.915200	0.000000e+00
480	0.140369	15.757280	126.320865	0.000000e+00
512	0.146393	14.669356	117.584054	0.000000e+00
544	0.130223	17.307594	138.715277	0.000000e+00
576	0.132843	17.262700	138.341357	0.000000e+00
608	0.160604	13.994412	112.139432	0.000000e+00
640	0.121449	17.267801	138.358257	0.000000e+00
672	0.136114	17.835899	142.899526	0.000000e+00
704	0.133786	15.648002	125.361831	0.000000e+00
736	0.134313	17.810072	142.674163	0.000000e+00
768	0.160675	16.915590	135.500922	0.000000e+00
800	0.119945	17.074549	136.767141	0.000000e+00
832	0.130059	17.712868	141.873262	0.000000e+00
864	0.144349	17.872625	143.146487	0.000000e+00
896	0.172750	16.655866	133.395642	0.000000e+00
928	0.180921	17.669140	141.505443	0.000000e+00
960	0.208461	16.976520	135.953630	0.000000e+00
992	0.257593	15.158641	121.391378	0.000000e+00
1024	0.153352	14.003651	112.138609	0.000000e+00
1056	0.143675	16.392325	131.262781	0.000000e+00
1088	0.146593	17.571342	140.699937	0.000000e+00
1120	0.155927	18.020318	144.291264	0.000000e+00
1152	0.181458	16.850480	134.920855	0.000000e+00
1184	0.187922	17.664787	141.437651	0.000000e+00
1216	0.209620	17.155312	137.355361	0.000000e+00
1248	0.250858	15.496928	124.074766	0.000000e+00
1280	0.259212	16.180979	129.548965	0.000000e+00
1312	0.260806	17.318639	138.654712	0.000000e+00
1344	0.322564	15.052618	120.510543	0.000000e+00
1376	0.323157	16.123981	129.085589	0.000000e+00
1408	0.326806	17.082378	136.756086	0.000000e+00
1440	0.360197	16.579727	132.729927	0.000000e+00
1472	0.384322	16.598092	132.874947	0.000000e+00
1504	0.414227	16.426112	131.496271	0.000000e+00
1536	0.514514	14.086611	112.766258	0.000000e+00
1568	0.514397	14.988888	119.987576	0.000000e+00
1600	0.503711	16.263297	130.187693	0.000000e+00
1632	0.622180	13.972514	111.848604	0.000000e+00
1664	0.588561	15.656637	125.328368	0.000000e+00
1696	0.676169	14.429525	115.504262	0.000000e+00
1728	0.630365	16.370760	131.041871	0.000000e+00
1760	0.757187	14.400088	115.266156	0.000000e+00
1792	0.737065	15.614873	124.988694	0.000000e+00
1824	0.851664	14.250707	114.068159	0.000000e+00
1856	0.882725	14.485662	115.947731	0.000000e+00
1888	0.923829	14.569486	116.617624	0.000000e+00

```
1920   0.895409  15.809294 126.540222 0.000000e+00
1952   1.068626  13.920138 111.418153 0.000000e+00
1984   1.050173  14.872851 119.042781 0.000000e+00
```

Question 4

Jacobi

Below are the results for N = 100 and N=1000 and for number of threads equal to 1, 4, and 16. We see that more threads results in faster execution and the speed up is greater for larger N. (with the exception that while 4 threads is faster than 1 for N = 100 it is slower with 16 threads, this is because there is more overhead with more threads).

N = 100, Threads = 1

```
initial residual = 100.000000 so tolerance = 0.010000
after 18622 iterations the residual is 0.009997
Total time was 0.712711s with a total of 1 threads
```

N = 100, Threads = 4

```
initial residual = 100.000000 so tolerance = 0.010000
after 18622 iterations the residual is 0.009997
Total time was 0.340120s with a total of 4 threads
```

N = 100, Threads = 16

```
intial residual = 100.000000 so tolerance = 0.010000
after 18622 iterations the residual is 0.009997
Total time was 0.343957s with a total of 16 threads
```

N = 1000, Threads = 1

```
initial residual = 1000.000000 so tolerance = 0.100000
after 5000 iterations the residual is 888.157673
Total time was 33.089935s with a total of 1 threads
```

N = 1000, Threads = 4

```
initial residual = 1000.000000 so tolerance = 0.100000
after 5000 iterations the residual is 888.157673
Total time was 18.381664s with a total of 4 threads
```

N = 1000, Threads = 16

```
initial residual = 1000.000000 so tolerance = 0.100000
after 5000 iterations the residual is 888.157673
Total time was 12.717927s with a total of 16 threads
```

Gauss-Seidel

Below are the resuts for the same N and thread values as the Jacobi method. We see similar results as before with increase N and threads but additionally we note that this method is much faster than the Jacobi method. Additionally, because only one copy of u is needed this method uses less memory. However, oddly for N=100 while 4 threads is worse than 1 thread 16 threads is better. Perhaps the speed up from 4 threads is small while the overhead is a lot but with 16 threads the overhead is not much more while the speedup is much greater.

N = 100, Threads = 1

```
initial residual = 100.000000 so tolerance = 0.010000
after 9769 iterations the residual is 0.010000
Total time was 0.319787s with a total of 1 threads
```

N = 100, Threads = 4

```
initial residual = 100.000000 so tolerance = 0.010000
after 9769 iterations the residual is 0.010000
Total time was 4.957361s with a total of 4 threads
```

N = 100, Threads = 16

```
initial residual = 100.000000 so tolerance = 0.010000
after 9769 iterations the residual is 0.010000
Total time was 0.290278s with a total of 16 threads
```

N = 1000, Threads = 1

```
initial residual = 1000.000000 so tolerance = 0.100000
after 5000 iterations the residual is 1190.659893
Total time was 13.150385s with a total of 1 threads
```

N = 1000, Threads = 4

```
initial residual = 1000.000000 so tolerance = 0.100000
after 5000 iterations the residual is 1190.659893
Total time was 8.486498s with a total of 4 threads
```

N = 1000, Threads = 16

```
initial residual = 1000.000000 so tolerance = 0.100000
after 5000 iterations the residual is 1190.659893
Total time was 5.444008s with a total of 16 threads
```