Applied Probability I Group Project Report

Part 1

To calculate the proportion of time during which the temperature was above 0 using a Monte Carlo Simulation, we used a stochastic process. The number of steps for the simulation (or the number of discrete time intervals) was calculated using the formula below:

$$steps = 1/\Delta t$$

Through this process, for various step sizes, we generated a standard random variable, Z, using numpy. Through the formula below, we incremented the temperature for every step.

$$T(t) = T(t-1) + \sigma\sqrt{\Delta}Z\,.$$

For each step, if the temperature was larger than the current max temperature, this temperature became the new max temperature. At the end of the simulation, the final max temp is the true max temperature overall.

Additionally, if the temperature was above zero, the above zero counter was incremented by 1, and the probability of the temperature being above zero (Probability, P) was calculated using the formula below:

$$Probability\ of\ temperature\ above\ zero\ (P) = steps\ with\ T(t) > 0\ /\ number\ of\ steps$$

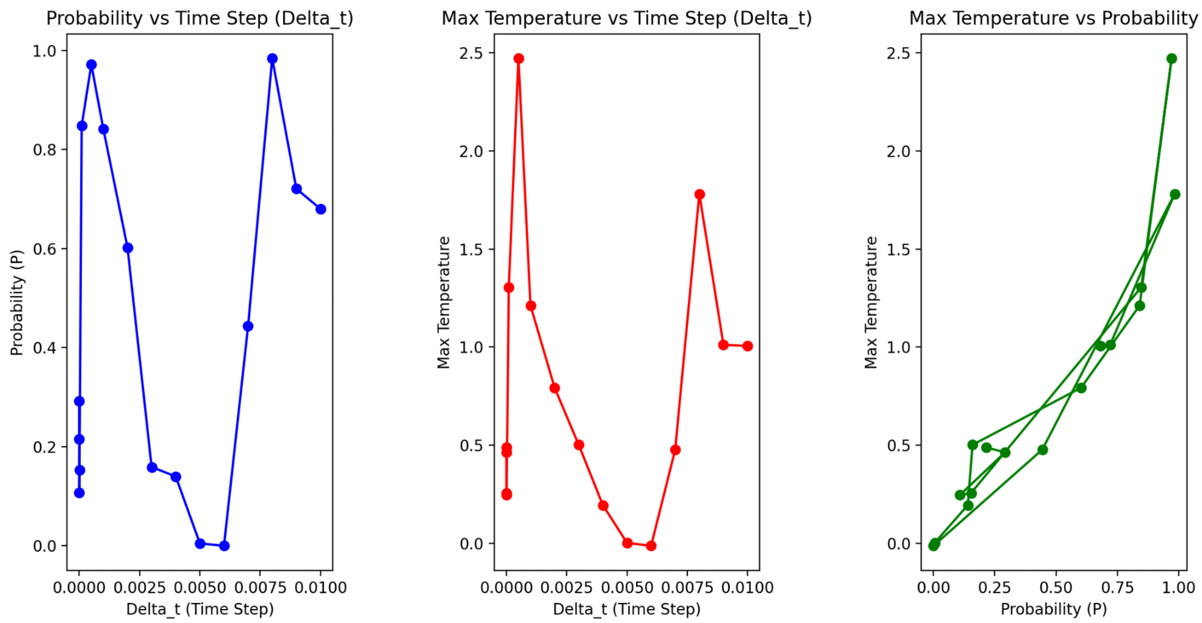
This process was repeated for multiple trials to obtain an estimate of the proportion of time the temperature was above zero.

For a trial run of the process, please see the results below (additional trials were excluded for brevity).

Note that for this trial run, the max temperature and probability are strongly correlated, with a value of 0.9237 for the correlation coefficient, *R*. This suggests that higher maximum temperatures are associated with a greater proportion of time the temperature remains above zero.

For the larger value of Δt, the results began to converge more towards a higher correlation coefficient *R.* This is because there is more sample point to the temperature evolution, making the simulation more precise, and making the irregularities be evened out. For a smaller value of

Δt, where $\lim_{\Delta t \to -\infty}$, we could get a correlation coefficient near R=1, however this would be very computationally expensive as the more precise the simulation is, the more computing power it would require.



Delta_t	Probability	Max Temperature
0.0100	0.6800	1.0063
0.0090	0.7207	1.0126
0.0080	0.9840	1.7816
0.0070	0.4437	0.4777
0.0060	0.0000	-0.0123
0.0050	0.0050	0.0028
0.0040	0.1400	0.1955
0.0030	0.1592	0.5025
0.0020	0.6020	0.7934
0.0010	0.8410	1.2124
0.0005	0.9715	2.4726
0.0001	0.8477	1.3056
0.00001	0.1534	0.2568

0.000001	0.1074	0.2466
0.0000001	0.2919	0.4642
0.00000001	0.2152	0.4895

Sources

https://medium.com/analytics-vidhya/monte-carlo-simulations-for-predicting-stock-prices-python-a64f53585662
https://matplotlib.org/stable/index.html
https://numpy.org/